CHAPTER 01

Composition, Resolution and Equilibrium of Forces

1.1 Force

Force is the action of one body on another. It may be defined as an action which changes or tends to change the state of rest or of uniform motion of body. For representing the force acting on the body, the magnitude of the force, its point of action and direction of its action should be known. There are different types of forces such as gravitational, frictional, magnetic, inertia or those caused by mass and acceleration.

According to Newton's second law of motion, we can write force as

$$F = ma = \text{mass} \times \frac{\text{length}}{\text{time}^2}$$

One Newton force is defined as that which gives an acceleration of 1 m/s^2 to a body of mass of 1 kg in the direction of force.

Thus, $1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/s}^2 = 1 \text{ kg-m/s}^2$

The action of one body and another, which changes or tends to change the state of rest or of uniform motion of body is called as force.

The three requisites for representing the force acting on the body are:

- Magnitude of force
- It points of action, and
- Direction of its action

there are various types of forces such as gravitational

1.2 Effects of a Force

A force may produce the following effects in a body, on which it acts:

1. It may change the motion of a body i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or retard it.
2. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
3. It may give rise to the internal stresses in the body, on which it acts.

1.3 Characteristics of a Force

To know the effect of force on a body, the following elements of force should be known.

1. Magnitude (i.e. 2 N, 5 kN, 10 kN etc.)
2. Direction or line of action.

3. Sense or nature (push or pull).
4. Point of application.

1.4 Force Systems

A force system is collection of forces acting on a body in one or more planes. According to the relative position of the lines of action of the forces, the forces may be classified as follows:

1. **Collinear:** The forces whose lines of action lie on the same line are known as collinear forces.
2. **Concurrent:** The forces, which meet at one point, are known as concurrent forces. Concurrent forces may or may not be collinear.
3. **Coplanar:** The forces whose line of action lie on the same plane are known as coplanar forces.
4. **Coplanar concurrent:** The forces, which meet at one point and their line of action lie on the same plane, are known as coplanar concurrent forces.
5. **Non-coplanar concurrent:** The forces, which meet at one point but their lines of action do not lie on the same plane, are known as coplanar non-concurrent forces.
6. **Coplanar non-concurrent:** The forces, which do not meet at one point but their line of action lie on the same plane, are known as coplanar non-concurrent forces.
7. **Non-coplanar non-concurrent:** The forces, which do not meet at one point and their line of action do not lie on the same plane, are known as non-coplanar non-concurrent forces.

1.5 Resultant Force

A single force which produces same effect on the body as the system of forces is called as resultant force.

1.6 Parallelogram Law of Forces

This law is used for finding the resultant of two forces acting at a point.

If two forces F_1 and F_2 are acting at a point and are represented in magnitude and direction by two sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram both in magnitude and direction.

Consider a parallelogram *OACB* as shown in figure 1.1 where sides *OA* and *OB* represent the forces F_1 F_2 acting at a point *O*. According to the parallelogram law of forces, the resultant *R* is represented by a diagonal *OC*.

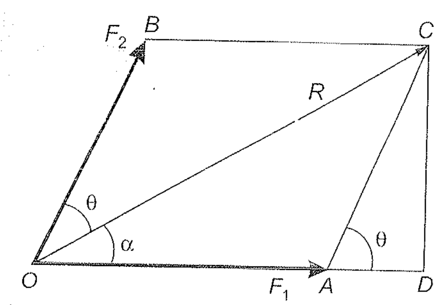


Fig. 1.1

Let θ be the angle between the forces F_1 and F_2 and α be the angle made by *R* with force F_1.

From the figure 1.1 we can write

$$BC = OA = F_1$$
$$AC = OB = F_2$$
$$\angle BOA = \theta = \angle CAD$$

and ΔODC and ΔADC are right angle triangles.

From triangle *ADC*, we can write

$$AD = AC\cos\theta = F_2\cos\theta$$
$$CD = AC\sin\theta = F_2\sin\theta$$

From triangle *ODC*, we can write

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$$

$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta$$

$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2(\cos^2\theta + \sin^2\theta)$$

$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2$$

$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2} \quad \text{... (i)}$$

From triangle *ODC*,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \quad \text{... (ii)}$$

Thus $$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

and $$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

1.7 Triangle Law of Forces

This law states that:

If two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle taken in order then their third side will represent the resultant of two forces in the direction and magnitude taken in opposite order.

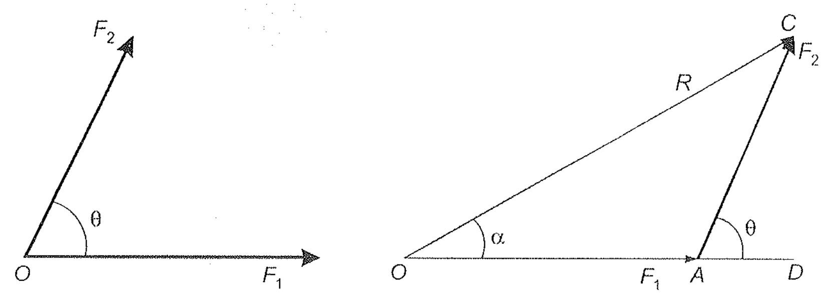


Fig. 1.2

If three forces are acting on a body and they are represented by three sides of the triangle in magnitude and direction, then the body will be in equilibrium condition.

1.8 Polygon Law of Forces

When two more forces are acting on the body, the triangle law can be extended to polygon law.

If a number coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then their resultant can be represented by closing side of the polygon in magnitude and direction in the opposite order.

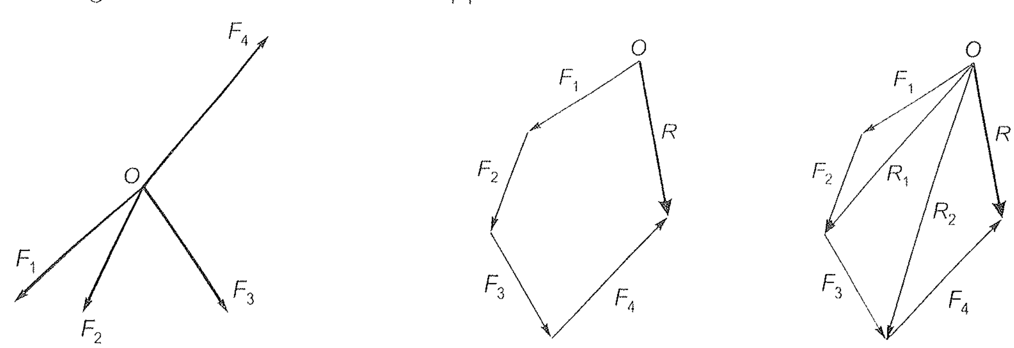


Fig. 1.3

Consider the forces F_1, F_2 and F_3 acting at a point O as shown in figure 1.3. As per the polygon law of forces the resultant force R is as shown in figure 1.3. According to parallelogram law, then the resultant of F_1 and F_2 is represented by R_1 and resultant of R_1 and F_3 is represented by R_2. The resultant R is the resultant of F_4 and R_2. This procedure can be extended to any number of forces acting at a point in a plane.

1.9 Composition of Forces

Conversion of system of forces into an equivalent single force system is known as the composition of forces. The effect of single equivalent force will be same as the effect produced by number of forces action on a body.

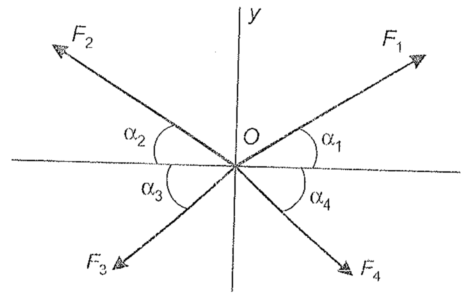


Fig. 1.4

Let the forces F_1, F_2, F_3, F_4 are acting on a body in a plane making angle α_1, α_2, α_3 and α_4 with x-axis as shown in figure 1.4. Let R be the resultant force of all the forces acting at the point making an angle θ with horizontal as shown in figure. Resolving the forces along x-axis and y-axis, we get

$$\Sigma F_x = F_1 \cos \alpha_1 - F_2 \cos \alpha_2 - F_3 \cos \alpha_3 + F_4 \cos \alpha_4$$

$$\Sigma F_y = F_1 \sin \alpha_1 + F_2 \sin \alpha_2 - F_3 \sin \alpha_3 - F_4 \sin \alpha_4$$

Component of R along x-axis = $R\cos\theta$

Component of R along y-axis = $R\sin\theta$

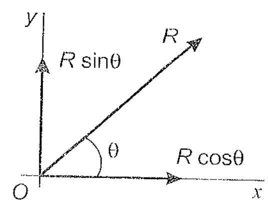


Fig. 1.5

$$R \cos \theta = \Sigma F_x$$

and $$R \sin \theta = \Sigma F_y$$

$$R^2 (\sin^2 \theta + \cos^2 \theta) = (\Sigma F_x)^2 + (\Sigma F_y)^2$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and $$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

A body which is under co-planar system of concurrent forces is in equilibrium if $R = 0$ or

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

1.10 Resolution of Forces

Replacing force F by two forces along x and y axis acting on the same body is called resolution of forces. Resolution is the reverse process of composition.

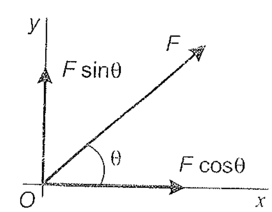


Fig. 1.6

Case I: A force F acting at a point 'O' making angle θ with horizontal as shown in figure 1.6. Then its components along x and y axis are given by

$F_x = F\cos\theta$ and $F_y = F\sin\theta$

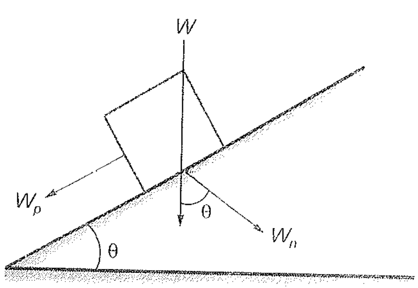


Fig. 1.7

Case II: The resolution of force W when the body is on an inclined plane. The components of the body force W are given by

$W_n = W\cos\theta$ and $W_p = W\sin\theta$

where W_n is normal component to inclined plane and W_p is parallel component to inclined plane.

1.11 Equilibrium of Forces

If a body is moving at a constant velocity or the body is at rest then the body is said to be in equilibrium in a state. If a number of forces are acting on the body and its resultant comes out to be zero, then the body is said to be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

1.12 Principles of Equilibrium

Three important principles of equilibrium are:

1. **Two force principle.** If a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
2. **Three force principal.** If body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force or in other words forces must be coplanar and concurrent.
3. **Four force principle.** If a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

1.13 Lami's Theorem

If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two.

Mathematically,
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q and R are three forces and α, β and γ are the angles as shown in figure 1.8.

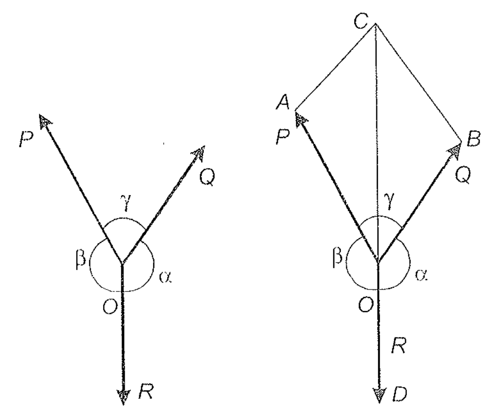


Fig. 1.8

Proof of Lami's Theorem

Consider three coplanar forces P, Q and R acting at a point O as shown in figure 1.8. Now complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure 1.9. The resultant of two forces P and Q is diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure,

$$BC = P \text{ and } AC = Q$$

$$\angle AOC = (180° - \beta)$$

and $$\angle ACO = \angle BOC = (180° - \alpha)$$

$$\angle CAO = 180° - (\angle AOC + \angle ACO) = 180° - [(180° - \beta) + (180° - \alpha)]$$

$$= 180° - 180° + \beta - 180° + \alpha$$

$$\angle CAO = \alpha + \beta - 180° \quad \text{... (i)}$$

But $$\alpha + \beta + \gamma = 360°$$

or $$\alpha + \beta + \gamma - 180° = 360° - 180° = 180°$$

$$(\alpha + \beta - 180°) + \gamma = 180° \quad \text{... (ii)}$$

From equation (i) and (ii) we get,

$$\angle CAO = 180° - \gamma$$

We know that in triangle *AOC*

$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

$$\frac{OA}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin(180^\circ - \beta)} = \frac{OC}{\sin(180^\circ - \gamma)}$$

or $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$ Hence Proved

1.14 Free Body Diagram

A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces. This diagram of the isolated element of a portions of the body along with the net effects of the system on it is called free body diagram.

The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed. If appreciable body force are present, such as gravitational or magnetic attraction, then these force must also be shown on the free-body diagram of the isolated system.

The free-body diagram is the most important single step in the solution of problems in mechanics.

Example 1.1 **Two forces F_1 and F_2 acting at a point have resultant R. If F_2 be doubled, R is doubled. Again if the direction of F_2 is reversed, then R is doubled. Show that**

$F_1 : F_2 : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$

Solution:

According to parallelogram of forces

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

or $$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \quad \text{... (i)}$$

If F_2 is doubled the resultant is also doubled, therefore

$$(2R)^2 = F_1^2 + 2(F_2)^2 + 2F_1(2F_2)\cos\theta$$

or $$4R^2 = F_1^2 + 4F_2^2 + 4F_1F_2\cos\theta \quad \text{... (ii)}$$

If the direction of F_2 is reversed, the resultant is again doubled.

Therefore

$$(2R)^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta$$

or $$4R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta \quad \text{... (iii)}$$

Adding equation 1 and 3 we get,

$$5R^2 = 2F_1^2 + 2F_2^2 \quad \text{... (iv)}$$

Multiplying equation (iii) by 2 and adding to equation (ii) we get,

$$12R^2 = 3F_1^2 - 6F_2^2$$

or $$4R^2 = F_1^2 + 2F_2^2 \quad \text{... (v)}$$

Subtracting equation 5 from equation 4 we get,

$$R^2 = F_1^2$$

or $$R = F_1$$

Now substituting the value of F_1 in equation (iv) we get,

$$5R^2 = +2R^2 + 2F_2^2$$

or $$F_2^2 = \frac{3}{2}R^2$$

or $$F_2 = \sqrt{\frac{3}{2}}R$$

Thus $$F_1 : F_2 : R = R : \sqrt{\frac{3}{2}}R : R$$

or $$F_1 : F_2 : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$$ Hence Proved.

Example 1.2 The resultant of two forces F_1 and F_2 is $(2k+1)\sqrt{F_1^2 + F_2^2}$ when the angle between forces is θ and it becomes $(2k-1)\sqrt{F_1^2 + F_2^2}$ when angle between forces is $(90° - \theta)$.

Prove that $$\tan\theta = \frac{k-1}{k+1}$$

Solution:

According to parallelogram law of forces

$$R = \sqrt{F_1^2 + F^2 + 2F_1F_2\cos\theta}$$

or $$R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta$$

In first case,

Thus, $$(2k+1)\sqrt{F_1^2 + F_2^2} = \sqrt{F_1^2 + F^2 + 2F_1F_2\cos\theta}$$

or $$(2k+1)^2 (F_1^2 + F_2^2) = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

or $$(4k^2 + 4k + 1)(F_1^2 + F_2^2) = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

or $$(4k^2 + 4k + 1 - 1)(F_1^2 + F_1^2) = 2F_1F_2\cos\theta$$

or $$(4k^2 + 4k)(F_1^2 + F_2^2) = 2F_1F_2\cos\theta \quad \ldots \text{(i)}$$

In second case q is replaced by $90° - \theta$,

$$R_2 = (2k-1)\sqrt{F_1^2 + F_2^2}$$

Thus $$(2k-1)^2 (F_1^2 + F_2^2) = F_1^2 + F_2^2 + 2F_1 F_2 \cos(90° - \theta)$$

or $$(4k^2 - 4k + 1)(F_1^2 + F_2^2) = F_1^2 + F_2^2 + 2F_1F_2\cos(90° - \theta)$$

or $$(4k^2 - 4k + 1 - 1)(F_1^2 + F_2^2) = 2F_1 F_2 \sin\theta$$

or $$(4k^2 - 4k)(F_1^2 + F_2^2) = 2F_1F_2\sin\theta \quad \ldots \text{(ii)}$$

Dividing equation (ii) by (i) we get,

$$\frac{4k^2 - 4k}{4k^2 + 4k} = \frac{\sin\theta}{\cos\theta}$$

or $$\tan\theta = \frac{4k(k-1)}{4k(k+1)} = \frac{k-1}{k+1}$$ Hence Proved.

Example 1.3 Two forces F_1 and F_2 acting at angle θ have a resultant R. If each force is increased by R. Prove that the new resultant made with R an angle ϕ such that

$$\tan\alpha = \frac{(F_1 - F_2)\sin\theta}{R + (F_1 + F_2)(1 + \cos\theta)}$$

Solution:

The figure will be as shown in figure (a)

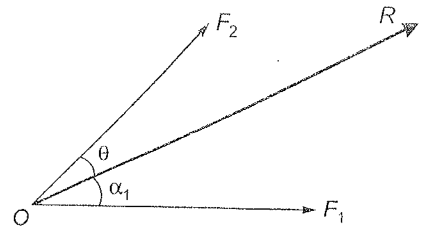


Fig. (a)

In first case,
$$\tan\alpha_1 = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

In second case as shown in figure (b),
$$\tan\alpha_2 = \frac{(F_2+R)\sin\theta}{(F_1+R)+(F_2+R)\cos\theta}$$

where,
$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha$$

angle between R and R',
$$\phi = \alpha_2 - \alpha_1$$

or
$$\tan\phi = \tan(\alpha_2 - \alpha_1) = \frac{\tan\alpha_2 - \tan\alpha_1}{1+\tan\alpha_2\tan\alpha_1}$$

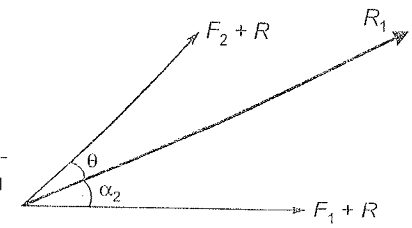


Fig. (b)

Substituting the value of tan α_1 and tan α_2

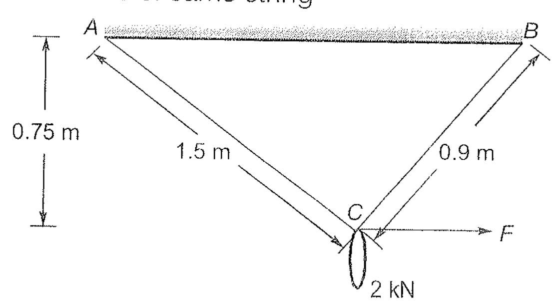
$$\tan\alpha = \frac{\dfrac{F(F_2+R)\sin\theta}{(F_1+R)+(F_2+R)\cos\theta} - \dfrac{F_2\sin\theta}{F_1+F_2\cos\theta}}{1+\dfrac{(F_2+R)\sin\theta}{(F_1+R)+(F_2+R)\cos\theta}\cdot\dfrac{F_2\sin\theta}{F_1+F_2\cos\theta}}$$

Solving
$$\tan\phi = \frac{\sin\theta(F_1-F_2)}{R+(F_1+F_2)(1+\cos\theta)}$$
Hence Proved.

Example 1.4 A string of 2.4 m in length is tied to two points A and B in the horizontal plane. A ring of 2 kN weight which can slide freely along ACB is at C, in equilibrium condition when a horizontal force F is applied at point C. If the point C is 75 cm below of AB find out the magnitude of F. The length of AC is 1.5 m.

Solution:

As per statement of problem the configuration is shown in figure (a). All forces and angle has been shown in figure (b). Tension in AC and CB will be equal because of same string



$$\cos\theta_1 = \frac{DC}{AC} = \frac{75}{150} = \frac{1}{2}$$

and
$$\sin\theta_1 = \frac{\sqrt{3}}{2}$$

$$\cos\theta_2 = \frac{DC}{BC} = \frac{75}{90} = \frac{5}{6}$$

and
$$\sin\theta_2 = \frac{\sqrt{11}}{6}$$

Resolving all forces vertical we get,
$$T\cos\theta_1 + T\cos\theta_2 = 2$$

or
$$T\left(\frac{1}{2}+\frac{5}{6}\right) = 2$$

or $T = 1.5$ kN

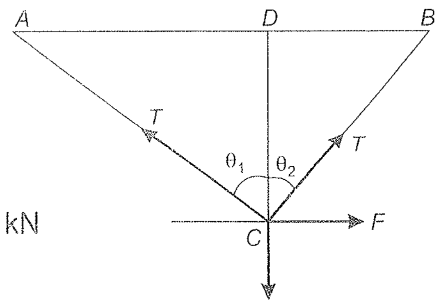
Resolving all forces horizontal,

$$T \sin \theta_1 = F + T \sin \theta_2$$

or

$$F = T(\sin\theta_1 - \sin\theta_2)$$

$$= 1.5\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{11}}{6}\right) \qquad = 0.47 \text{ kN}$$



Example 1.5 Four smooth pegs *ABCD* are fixed in a vertical plane so that they form the four highest corners of a regular hexagon with the side BC horizontal. A string is thrown over the pegs which supports a weight *W*. The length of the loop is such that the angles formed by it at the lower pegs are right angles. Determine the tension in the string and the pressure on the pegs.

Solution:

As per statement of problem the configuration is shown in figure (a)

We know that interior angle of hexagon are 120°.

Thus $BAF = 120°$ and line *AD* bisect the angle *BAF*

Therefore,

$$BAD = 60° = CDA$$

$$DAO = BAO - BAD$$

$$= 90° - 60° = 30° \; ADO$$

$$AOD = 180° - 30° - 30° = 120°$$

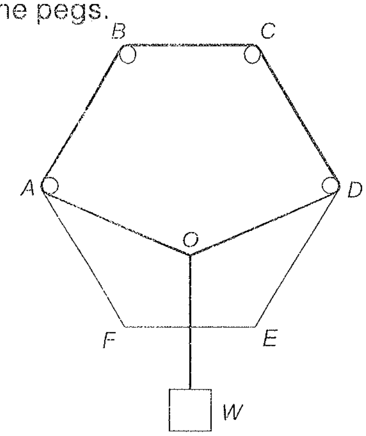


Fig. (a)

Now at point *O* all forces are shown in figure (b)

Due to symmetry,

$$\theta_1 = \theta_2 = \theta$$

$$120° + \theta + \theta = 360°$$

or

$$\theta = 120°$$

Applying Lami's theorem,

$$\frac{W}{\sin 120°} = \frac{T}{\sin 120°} = \frac{T}{\sin 120°}$$

or

$$T = W$$

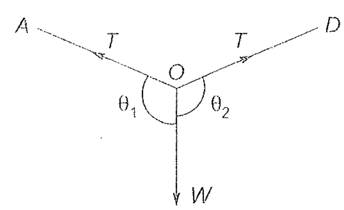


Fig. (b)

Consider the pressure or reaction force R_A at peg *A*. It is resultant of two tension force in string

Thus,

$$R_A = \sqrt{T^2 + T^2 + 2TT\cos 90°} = \sqrt{2}\, T = \sqrt{2} W$$

Due to symmetry,

$$R_A = R_D$$

Similarly R_B and R_C,

$$R_B = \sqrt{T^2 + T^2 + 2TT\cos 120°} = T = W$$

Example 1.6 A smooth sphere of weight W is supported in contact with a smooth vertical wall by string fastened to a point on its surface, the end being attached to a point on the wall. If the length of the string is equal to the radius of sphere, find tension in the string and reaction on the wall

Solution:

As per statement of problem the configuration is shown in figure (a).

Here

$$\cos\theta = \frac{r}{2r} = \frac{1}{2} \text{ or } \theta = 60°$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

The sphere is in equilibrium under the action of following forces:

(1) Weight W, (2) Tension T in string, (3) Reaction R of the wall

All forces are shown in figure (b) and force diagram is shown in figure (c)

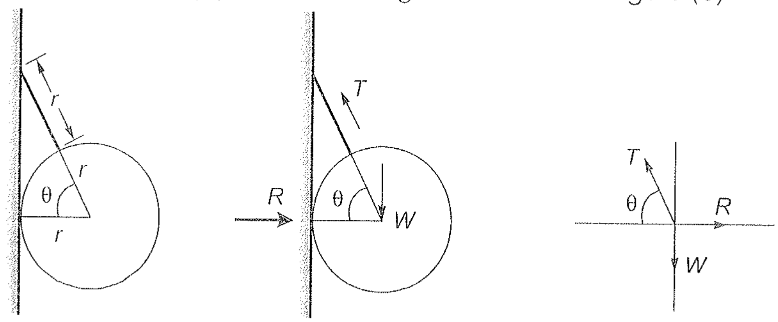


Fig. (a) Fig. (b) Fig. (c)

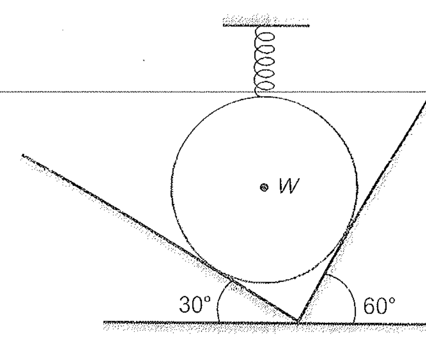
Applying Lami's theorem we get,

$$\frac{T}{\sin 90^\circ} = \frac{R}{\sin(90^\circ + \theta)} = \frac{W}{\sin(180^\circ - \theta)}$$

$$T = \frac{R}{\cos\theta} = \frac{W}{\sin\theta} \quad \text{or } T = \frac{W}{\sin\theta} = \frac{W}{\sin\sqrt{3}/2} = \frac{2W}{\sqrt{3}}$$

$$R = \frac{W\cos\theta}{\sin\theta} = \frac{W}{\tan\theta} = \frac{W}{\sqrt{3}}$$

Example 1.7 The figure shows a sphere resting in a smooth V shaped groove and subjected to a spring force. The spring is compressed to a length of 125 mm from its free length of 150 mm. If the stiffness of spring $k = 10$ N/mm and weight of sphere is 100 N. Determine the contact reactions at the point A and B.



Solution:

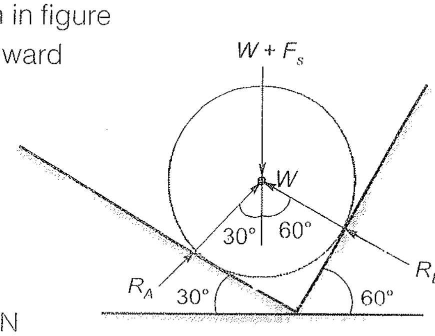
Deflection of the spring,

$$x = 150 - 125 = 25 \text{ mm}$$

$$F_s = kx = 25 \times 10 = 25 \text{ N}$$

This force will act on sphere vertically downward. Beside this spring force reaction R_A at point A, reaction R_B at B and weight of sphere W will be there as shown in figure

$$F_s + W = 250 + 100 = 350 \text{ N vertically downward}$$



Applying Lami's theorem, we get,

$$\frac{W + F_s}{\sin(60^\circ + 30^\circ)} = \frac{R_B}{\sin(180^\circ - 30^\circ)} = \frac{R_A}{\sin(180^\circ - 60^\circ)}$$

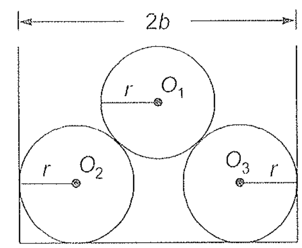
$$W + F_s = \frac{R_B}{\sin 30^\circ} = \frac{R_A}{\sin 60^\circ}$$

or

$$R_B = (W + F_s)\sin 30^\circ = 350 \times \frac{1}{2} = 175 \text{ N}$$

$$R_A = (W + F_s)\sin 60^\circ = 350 \times \frac{\sqrt{3}}{2} = 303.1 \text{ N}$$

Example 1.8 Three cylinders weighing W each and r in radius are placed in a channel of $2b$ width as shown in figure. Determine the pressure exerted by the cylinder.



(1) O_1 on cylinder O_2, (2) O_2 on the horizontal wall, (3) O_3 on the vertical wall

Determine all the values if $2b = 36$ cm, $r = 8$ cm and $W = 400$ N.

Solution:

From the geometry of figure (a) we get,

$$\sin\alpha = \frac{O_2N}{O_1O_2} = \frac{b-r}{2r}$$

$$\cos\alpha = \frac{\sqrt{(r+b)(3r+b)}}{2r}$$

Due to symmetry of figure equal reaction forces may be observe easily. All forces acting on system is shown in figure (b). Here

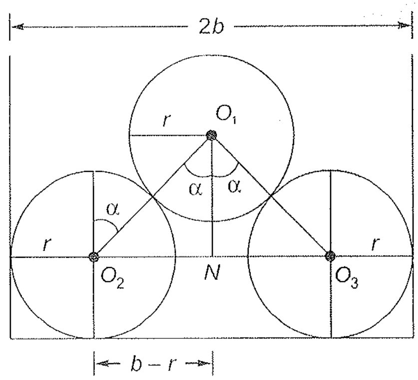


Fig. (a)

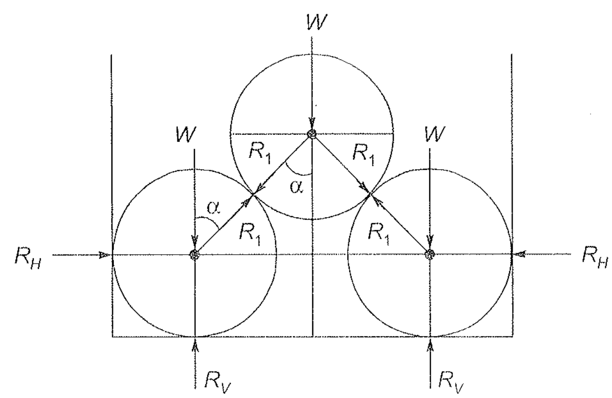


Fig. (b)

(1) R_1 is pressure exerted by the cylinder O_1 on cylinder O_2.

(2) R_V is pressure exerted by the cylinder O_2 on the horizontal wall

(3) R_H is pressure exerted by the cylinder O_3 on the vertical wall

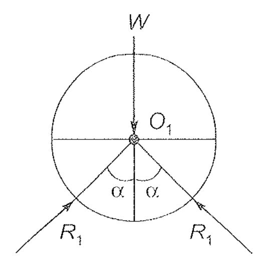


Fig. (c)

Now consider FBD of O_1 as shown in figure (c),

Apply Lami's theorem,

$$\frac{W}{\sin 2\alpha} = \frac{R_1}{\sin(180^\circ - \alpha)} = \frac{R_1}{\sin(180^\circ - \alpha)}$$

or
$$\frac{W}{2\sin\alpha\cos\alpha} = \frac{R_1}{\sin\alpha}$$

Thus
$$R_1 = \frac{W}{2\cos\alpha}$$

Now consider the FBD of O_2 as shown in figure,

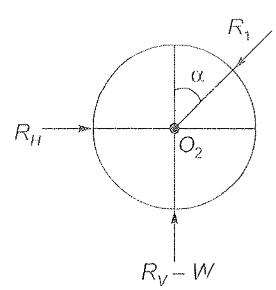


Fig. (d)

Applying Lami's theorem,
$$\frac{R_V - W}{\sin(90^\circ + \alpha)} = \frac{R_H}{\sin(180^\circ - \alpha)} = \frac{R_1}{\sin 90^\circ}$$

or
$$\frac{R_V - W}{\cos\alpha} = \frac{R_H}{\sin\alpha} = R_1 = \frac{W}{2\cos\alpha}$$

Thus, $$R_H = R_1 \sin\alpha = \frac{W\sin\alpha}{2\cos\alpha} = 0.5\, W\tan\alpha$$

$$R_v - W = R_1 \cos\alpha$$

or $$R_v = \frac{W\cos\alpha}{2\cos\alpha} + W = 1.5\, W$$

Now for $2b = 36$ cm, $r = 80$ mm, $W = 200$ N we get,

$$\sin\alpha = \frac{18-8}{16} = \frac{5}{8} = 0.625$$

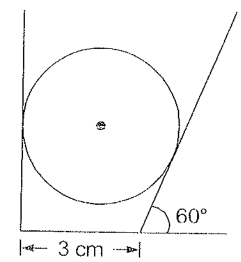
$$\cos\alpha = 0.78, \tan\alpha = 0.80$$

Thus $$R_1 = \frac{200}{2\times 0.78} = 128 \text{ N}$$

$$R_H = \frac{200\times 0.80}{2} = 80 \text{ N}$$

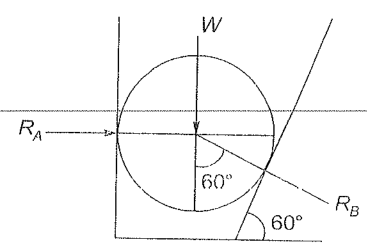
$$R_v = 1.5 \times 200 = 300 \text{ N}$$

Example 1.9 A smooth sphere of 2 kN weight and 2 cm radius is resting against the walls as shown in figure. Determine the reaction at the supporting points.



Solution:

Sphere is in contact with wall at two point. At these point reaction force is exerted by wall. In figure (a) all forces has been shown.



Applying Lami's theorem we get,

$$\frac{W}{\sin(90° + 60°)} = \frac{R_A}{\sin(180° - 60°)} = \frac{R_B}{\sin 190°}$$

or $$\frac{2}{\cos 60°} = \frac{R_A}{\sin 60°} = R_B$$

Thus $$R_A = \frac{2\sin 60°}{\cos 60°} = 2\tan 60° = 2\sqrt{3} \text{ kN}$$

$$R_B = \frac{2}{\cos 60°} = \frac{2}{1/2} = 4 \text{ kN}$$

Example 1.10 A uniform rod AB remains in equilibrium in a vertical plane, resting on smooth inclined place AC and BC, which are at right angles. If the plane BC is at α with the horizontal find the inclination θ of the rod with the plane AC.

Solution:

As per statement of problem the configuration is shown in figure (a). All angles and forces is shown in figure (b).

The rod is in equilibrium under action of the following forces.

1. Weight W of the rod acting vertical downward through the middle point G of the rod.
2. Reaction R_A at point of contact with plane AC normal to the plane AC.
3. Reaction R_B at point of contact with plane BC normal to the plane BC.

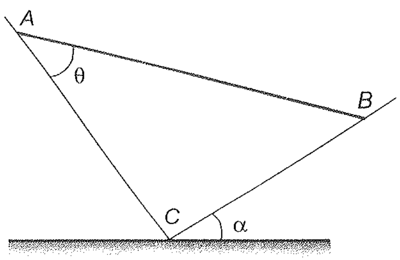


Fig. (a)

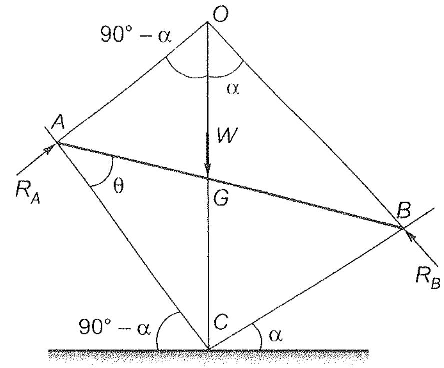


Fig. (b)

If rod is in equilibrium then, three force must be concurrent. Let these forces meet at O.

since $\quad AO \perp AC \quad$ and $\quad BO \perp BC$

Thus $\quad AO \parallel BC \quad$ and $\quad BO \parallel AC$

$$AOB = 90°$$

Now the figure $AOBC$ is a rectangle whose diagonal OGC is vertical.

Also $\quad GA = GO$

Thus $\quad GAC = GCA$

$$\theta = \alpha \qquad \text{Hence Proved.}$$

Example 1.11 A rod whose center of gravity divide it into two portion of length a and b rests inside a smooth sphere in a position inclined to the horizontal. Show that if θ be the inclination to the horizontal and 2 α the angle that it subtends at the center of sphere

$$\tan\theta = \frac{b-a}{b+a}\tan\alpha$$

Solution:

As per statement of problem the configuration is shown in figure (a). The rod AB is equilibrium under these forces as shown in figure (b).

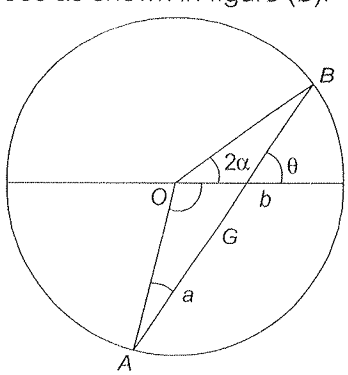


Fig. (a)

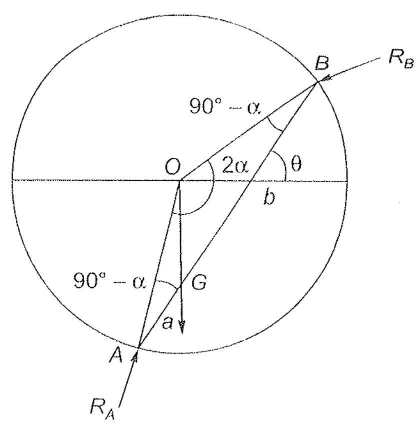


Fig. (b)

1. Weight W of the rod acting vertically downward.
2. Reaction R_A at point of contact A and normal to sphere where passes through O.

3. Reaction R_B at point of contact B and normal to sphere which passes through O.
For rod to be in equilibrium these forces must be concurrent. So line of acting of center of gravity will pass through O.
Now from geometry of figure (b)

$$BOG = 180° - (90° - \alpha) - (90° - \theta) = \alpha + \theta$$
$$AOG = 2\alpha - (\alpha + \theta) = \alpha - \theta$$

From the ΔAOG

$$\frac{AG}{\sin AOG} = \frac{AO}{\sin AGO}$$

or
$$\frac{a}{\sin(\alpha - \theta)} = \frac{r}{\sin(90 + \theta)}$$

or
$$\frac{a}{\sin(\alpha - \theta)} = \frac{r}{\cos\theta} \quad \text{... (i)}$$

From the ΔBOG

$$\frac{BG}{\sin BOG} = \frac{OB}{\sin OGB}$$

or
$$\frac{b}{\sin(\alpha + \theta)} = \frac{r}{\cos\theta} \quad \text{... (ii)}$$

From equation (i) and (ii) we get

$$\frac{a}{\sin(\alpha - \theta)} = \frac{b}{\sin(\alpha + \theta)}$$

or
$$b(\sin\alpha\cos\theta - \cos\alpha\sin\theta) = a(\sin\alpha\cos\theta - \cos\alpha\sin\theta)$$

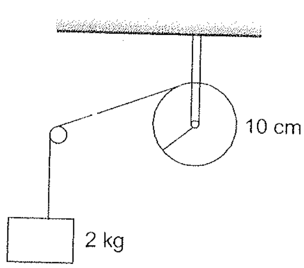
or
$$(b - a)\sin\alpha\cos\theta = (a + b)\cos\alpha\sin\theta$$

or
$$\frac{(b - a)\sin\alpha}{(b + a)\cos\alpha} = \frac{\sin\theta}{\cos\theta}$$

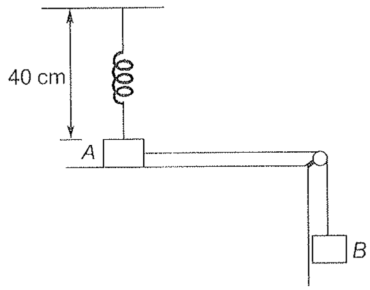
or
$$\tan\theta = \frac{b - a}{b + a}\tan\alpha$$ Hence proved

Objective Brain Teasers

Q.1 A string is wrapped on a wheel of moment of inertia 0.2 kg/m^2 and radius 10 cm and goes through a light pulley to support a block of mass 2.0 kg as shown in figure. The acceleration of block is _______ m/s^2.

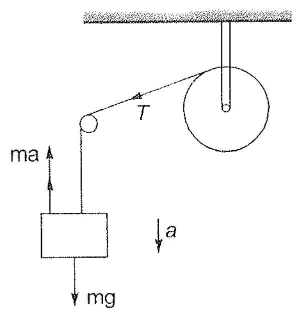


Q.2 Figure shows two blocks A and B, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which block A slides is smooth. The block A is attached to a spring of spring constant 40 N/m. Initially spring is vertical and unstretched. The extension in the spring at the instant when block A breaks off is __________ cm. (Take g = 10 m/s^2)



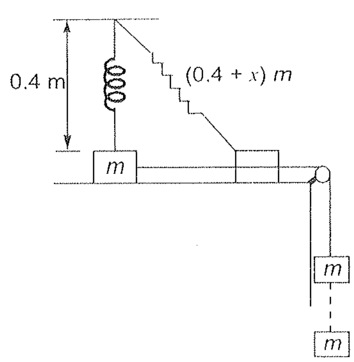
Hints & Explanation

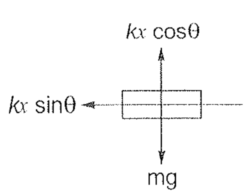
1. (0.89)(0.87 to 0.91)



$$I = 0.2 \text{ kgm}^2$$
$$r = 0.1$$
$$m = 2 \text{ kg}$$
$$\therefore \quad mg - T = ma$$
Now $$T = \frac{Ia}{r^2}$$
$$\Rightarrow \quad mg = \left(m + \frac{I}{r^2}\right)a$$
$$\Rightarrow \quad 2 \times 9.81 = \left(2 + \frac{0.2}{0.01}\right)a$$
$$\Rightarrow \quad a = 0.89 \text{ m/s}^2$$

2. (10)





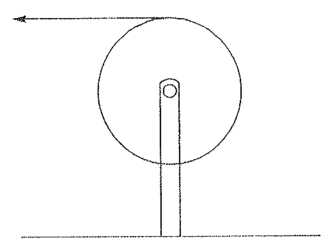
Given: $m = 0.32$ kg, $k = 40$ N/m, $h = 0.4$ m, $g = 10 \text{ m/s}^2$

From *FBD*, $kx\cos\theta = mg$

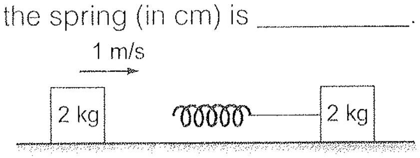
$$\Rightarrow \quad kx\cos\theta = \frac{mg}{kx}$$
$$\Rightarrow \quad \frac{0.4}{0.4+x} = \frac{mg}{kx}$$
$$\Rightarrow \quad \frac{0.4}{0.4+x} = \frac{0.32\times 10}{40x}$$
On solving $x = 0.1 \text{ m} = 10 \text{ cm}$

Student's Assignments

Q.1 A string is wrapped around the rim of a wheel of moment of Inertia 0.2 kg m^2 and radius 20 cm. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled with a force of 20 N. The angular velocity of wheel after 5 second is ____________ rad/s.



Q.2 A block of mass 2 kg is moving on a horizontal frictionless surface with velocity of 1 m/s, towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end is 100 N/m. The maximum compression in the spring (in cm) is ___________.



Q.3 A particle of mass m is kept on a fixed, smooth sphere of radius 10 cm at a position, where the radius through the particle makes an angle 30° with the vertical. The particle is released from this position. The distance travelled by the particle before it leaves the contact with sphere is ____________ cm.