Polynomials 2

Objective Section (1 mark each)

Multiple Choice Questions

Q. 1. The zeroes of the polynomial $x^2 - 3x - m(m+3)$ are [CBSE OD, Set 1, 2020]

(a) $m, m+3$ (b) $-m, m+3$

(c) $m, -(m+3)$ (d) $-m, -(m+3)$

Ans. $x^2 - 3x - m(m+3)$

$= x^2 - (m+3-m)x - m(m+3)$

$= x^2 - (m+3)x + mx - m(m+3)$

$= x\{x-(m+3)\} + m\{x-(m+3)\}$

$= (x+m)\{x-(m+3)\}$

$\therefore$ Its zeroes are $-m, (m+3)$.

$\therefore$ Option (b) is correct. **Ans.**

Q. 2. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is [CBSE Delhi, Set 1, 2020]

(a) 10 (b) – 10

(c) – 7 (d) – 2

Ans. Let $p(x) = x^2 + 3x + k$

$\because$ $x = 2$ is a zero of $p(x)$

$\therefore$ $p(2) = 0$

$\Rightarrow$ $(2)^2 + 3(2) + k = 0$

$\Rightarrow$ $4 + 6 + k = 0$

$\Rightarrow$ $10 + k = 0$

$\Rightarrow$ $k = -10$

$\therefore$ Option (b) is correct. **Ans.**

Q. 3. The quadratic polynomial, the sum of whose zeroes is – 5 and their product is 6, is [CBSE Delhi, Set 1, 2020]

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Ans. Let the polynomial be

$p(x) = k[x^2 - (\text{Sum of zeroes})\, x + \text{Product of zeroes}]$

Sum of zeroes $= -5$

Product of zeroes $= 6$

$\therefore$ Required polynomial is

$p(x) = k[x^2 - (-5)x + (6)]$

$= k[x^2 + 5x + 6]$

Let $k = 1$

$\therefore$ $p(x) = x^2 + 5x + 6$

$\therefore$ Option (a) is correct. **Ans.**

Short Answer Type Questions-I (3 marks each)

Q. 1. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x + 3,\ 3x^2 + 7x + 2,\ 4x^3 + 3x^2 + 2,\ x^3 + \sqrt{3x} + 7,$

$7x + \sqrt{7},\ 5x^3 - 7x + 2,\ 2x^2 + 3 - \frac{5}{x},\ 5x - \frac{1}{2},\ ax^3 + bx^2 + cx + d,\ x + \frac{1}{x}$

Answer the following questions :

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

[CBSE OD, Set 1, 2020]

Ans. **(i)** 3

$\left(x^2 + \sqrt{3x} + 7, 2x^2 + 3 - \frac{5}{x} \text{ and } x + \frac{1}{x}\right)$

(ii) There is only one quadratic polynomial which is $3x^2 + 7x + 2$.

Short Answer Type Questions-II (3 marks each)

Q. 1. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

[CBSE Delhi, Set 1, 2020]

Ans. Given, $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Let α and β be the zeroes of $f(x)$

$\therefore$ $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now let the zeroes of required polynomial be α' and β'.

So, $\alpha' = \frac{1}{\alpha}$, $\beta' = \frac{1}{\beta}$

Now, sum of zeroes (S) = $\alpha' + \beta'$

$$= \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{-b/a}{c/a}$$

$$= -\frac{b}{c}$$

Product of zeroes (P) = $\alpha' \times \beta'$

$$= \frac{1}{\alpha} \times \frac{1}{\beta}$$

$$= \frac{1}{\alpha\beta}$$

$$= \frac{1}{c/a}$$

$$= \frac{a}{c}$$

$\therefore$ Required polynomial

$$g(x) = k[x^2 - Sx + P]$$

$$= k\left[x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}\right]$$

$$= k\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right]$$

$$= k\left[\frac{cx^2 + bx + a}{c}\right]$$

$\because$ k/c = constant

$\therefore$ $g(x) = cx^2 + bx + a$ is the required polynomial. **Ans.**

Q. 2. Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.
[CBSE Delhi, Set 1, 2020]

Given, $f(x) = 3x^2 - x^3 - 3x + 5$

$= -x^3 + 3x^2 - 3x + 5$

and, $g(x) = x - 1 - x^2$

$= -x^2 + x - 1$

Dividing $f(x)$ by $g(x)$

$$-x^2 + x - 1 \overline{)\ -x^3 + 3x^2 - 3x + 5\ (\ x - 2}$$

$$\underline{-x^3 + x^2 - x}$$

(+) (−) (+)

$$2x^2 - 2x + 5$$

$$\underline{2x^2 - 2x + 2}$$

(−) (+) (−)

$$3$$

$\therefore$ $q(x) = (x - 2)$, $r(x) = 3$

Now,

Dividend = Divisor × Quotient + Remainder.

$f(x) = g(x) \times q(x) + r(x)$

$= (-x^2 + x - 1)(x - 2) + 3$

$= -x^2(x - 2) + x(x - 2) - 1(x - 2) + 3$

$= -x^3 + 2x^2 + x^2 - 2x - x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

$= f(x)$ **Hence Verified.**

Q. 3. If 4 is zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes. **[CBSE Delhi, Set 1, 2020]**

Ans. Let $p(x) = x^3 - 3x^2 - 10x + 24$

$\because$ $x = 4$ is a zero of $p(x)$.

$\Rightarrow$ $(x - 4)$ is a factor of $p(x)$

Dividing $p(x)$ by $(x - 4)$,

$$x - 4 \overline{)\ x^3 - 3x^2 - 10x + 24\ (\ x^2 + x - 6}$$

$$x^3 - 4x^2$$

(−) (+)

$$\underline{\qquad\qquad}$$

$$x^2 - 10x$$

$$x^2 - 4x$$

(−) (+)

$$\underline{\qquad\qquad}$$

$$-6x + 24$$

$$-6x + 24$$

(+) (−)

$$\underline{\qquad\qquad}$$

$$\times$$

Now, to find other zeroes of $p(x)$,

Put $x^2 + x - 6 = 0$

$\Rightarrow$ $x^2 + 3x - 2x - 6 = 0$

$\Rightarrow$ $x(x + 3) - 2(x + 3) = 0$

$\Rightarrow$ $(x + 3)(x - 2) = 0$

$\Rightarrow$ $x = -3, 2$

$\therefore$ -3 and 2 are the other two zeroes of the given polynomial. **Ans.**

Q. 4. For what value of k, is the polynomial $f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$ completely divisible by $3x^2 - 5$?
[CBSE OD, Set 1, 2019]

Ans. Given,

$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$

It is completely divisible by $3x^2 - 5$

Let $g(x) = 3x^2 - 5$

$\therefore\ 3x^2 - 5\overline{)3x^4 - 9x^3 + x^2 + 15x + k}\ (x^2 - 3x + 2$

$$\begin{array}{r} 3x^4 + 0 - 5x^2 \\ \hline -9x^3 + 6x^2 + 15x \\ -9x^3 + 15x \\ \hline 6x^2 + k \\ 6x^2 - 10 \\ \hline k + 10 \end{array}$$

$\because f(x)$ is completely divisible by $g(x)$ then

$$k + 10 = 0$$
$$k = -10$$

Q. 5. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients. [CBSE OD, Set 1, 2019]

Ans. The given polynomial is

$$P(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$$

$\because \quad P(y) = 0$

$$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$
$$21y^2 - 11y - 2 = 0$$
$$21y^2 - 14y + 3y - 2 = 0$$
$$7y\,(3y - 2) + 1\,(3y - 2) = 0$$
$$(3y - 2)\,(7y + 1) = 0$$
$$y = \frac{2}{3}, -\frac{1}{7}$$

So zeroes of $P(y)$ are $\frac{2}{3}, -\frac{1}{7}$

Verification: On comparing $7y^2 - \frac{11}{3}y - \frac{2}{3}$ with $ax^2 + bx + c$, we get

$$a = 7,\ b = \frac{11}{3},\ c = \frac{2}{3}$$

$$\text{Sum of zeroes} = \frac{-b}{a}$$

$$\frac{2}{3} + \left(-\frac{1}{7}\right) = \frac{-\left(\frac{-11}{3}\right)}{7}$$

$$\frac{14 - 3}{21} = \frac{11}{3 \times 7}$$

$$\frac{11}{21} = \frac{11}{21}$$

$$\text{Product of zeroes} = \frac{c}{a}$$

$$\frac{2}{3} \times \left(-\frac{1}{7}\right) = \frac{\left(-\frac{2}{3}\right)}{7}$$

$$\frac{-2}{21} = \frac{-2}{3 \times 7}$$

$$\frac{-2}{21} = \frac{-2}{21}$$

Hence Verified.

Q. 6. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half to their product. [CBSE Delhi, Set 1, 2019]

Ans. The given quadratic polynomial is

$$x^2 - (k + 6)x + 2(2k - 1)$$

Comparing with $ax^2 + bx + c$, we get

$a = 1$, $b = -(k + 6)$ and $c = 2(2k + 1)$

Let the zeroes of the polynomial be α and β

we know that

$$\alpha + \beta = -\frac{b}{a}$$
$$= \frac{k + 6}{1}$$

or $\quad \alpha + \beta = k + 6 \quad$ ...(i)

Also, $\quad \alpha\beta = \frac{c}{a}$

$$= \frac{2\,(2k - 1)}{1}$$

or $\quad \alpha\beta = 2\,(2k - 1) \quad$ ...(ii)

Given,

$$\text{Sum of zeroes} = \frac{1}{2} \text{ of their product}$$

$\therefore \quad \alpha + \beta = \frac{1}{2}\alpha\beta$

or $\quad k + 6 = \frac{1}{2} \times 2\,(2k - 1)$

[using equations (i) & (ii)]

or $\quad k + 6 = 2k - 1$

$$k = 7$$

Q. 7. Find all zeroes of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1. [CBSE Delhi, Set 3, 2019]

Ans. Given,

$$P(x) = 3x^3 + 10x^2 - 9x - 4$$

$x = 1$ is a zero of $P(x)$

$\therefore$ $(x - 1)$ is a factor of $P(x)$

To find other zeroes, we divide $P(x)$ by $(x - 1)$

$$
\begin{array}{r|l}
x - 1 \overline{)3x^3 + 10x^2 - 9x - 4} & 3x^2 + 13x + 4 \\
3x^3 - 3x^2 & \\
\hline
13x^2 - 9x & \\
13x^2 - 13x & \\
\hline
4x - 4 & \\
4x - 4 & \\
\hline
0 &
\end{array}
$$

$$P(x) = (x - 1)(3x^2 + 13x + 4)$$

$$= (x - 1)(3x^2 + 12x + x + 4)$$

$$= (x - 1)\{3x(x + 4) + 1(x + 4)\}$$

$$= (x - 1)(x + 4)(3x + 1)$$

other zeroes are $x + 4 = 0$

$$x = -4,$$

and $3x + 1 = 0$

$$x = -\frac{1}{3}$$

$\therefore$ All zeroes are $x = 1$, $x = -4$ and $x = -\frac{1}{3}$

Q. 8. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$. [CBSE 2018]

Topper's Answers

Ans) Given, polynomial $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$.

Two zeroes → $2+\sqrt{3}$ and $2-\sqrt{3}$.

⇒ Product of two zeroes is also a zero.

⇒ $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$. $[(a+b)(a-b) = a^2 - b^2]$.

As 1 is a zero, ⇒ $x-1$ is a factor.

Dividing,

$$
\begin{array}{r|l}
x - 1 \overline{)2x^4 - 9x^3 + 5x^2 + 3x - 1} & 2x^3 - 7x^2 - 2x + 1 \\
2x^4 - 2x^3 & \\
\hline
-7x^3 + 5x^2 & \\
-7x^3 + 7x^2 & \\
\hline
-2x^2 + 3x & \\
-2x^2 + 2x & \\
\hline
x - 1 & \\
x - 1 & \\
\hline
0 &
\end{array}
$$

⇒ By division algorithm,

$p(x) = (x-1)(2x^3 - 7x^2 - 2x + 1)$. → $g(x)$.

Now, in a cubic polynomial, we know:

Sum of roots $= \frac{-\text{coeff. of } x^2}{\text{coeff. of } x^3}$.

The roots of $g(x)$ are $2+\sqrt{3}$, $2-\sqrt{3}$ and α.

→ $\alpha + 2 + \sqrt{3} + 2 - \sqrt{3} = -\frac{(-7)}{2}$

$\alpha + 4 = \frac{7}{2}$

$\alpha = -\frac{1}{2}$. which is hence a zero of $p(x)$

⇒ All zeroes are $-\frac{1}{2}$, 1, $2+\sqrt{3}$ and $2-\sqrt{3}$.

Ans. Here, $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

And two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

Quadratic polynomial with zeroes is given by,

$\{x-(2+\sqrt{3})\}.\ \{x-(2-\sqrt{3})\}$

$\Rightarrow \quad (x-2-\sqrt{3})\ (x-2+\sqrt{3})$

$\Rightarrow \quad (x-2)^2 - (\sqrt{3})^2$

$\Rightarrow \quad x^2 - 4x + 4 - 3$

Let, $\quad x^2 - 4x + 1 = g(x)$

Now, $g(x)$ will be a factor of $p(x)$ so $g(x)$ will be divisible by $p(x)$

$$
\begin{array}{r}
\quad\quad 2x^2 - x - 1 \\
x^2 - 4x + 1 \overline{)\ 2x^4 - 9x^3 + 5x^2 + 3x - 1\ (} \\
\quad 2x^4 - 8x^3 + 2x^2 \\
\quad - \quad + \quad - \\
\hline
\quad\quad - x^3 + 3x^2 + 3x \\
\quad\quad - x^3 + 4x^2 - x \\
\quad\quad + \quad - \quad + \\
\hline
\quad\quad\quad - x^2 + 4x - 1 \\
\quad\quad\quad - x^2 + 4x - 1 \\
\quad\quad\quad + \quad - \quad + \\
\hline
\quad\quad\quad\quad \times
\end{array}
$$

For other zeroes,

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

or $\quad 2x\,(x-1) + 1\,(x-1) = 0$

$$(x-1)\,(2x+1) = 0$$

$$x - 1 = 0 \ \text{ and } \ 2x + 1 = 0$$

$$x = 1,\ x = \frac{-1}{2}$$

Zeroes of $p(x)$ are

$$1,\ \frac{-1}{2},\ 2+\sqrt{3} \ \text{ and } \ 2-\sqrt{3}\,.$$

Q. 9. If three zeroes of a polynomial $x^4 - x^3 - 3x^2 + 3x$ are 0, $\sqrt{3}$ and $-\sqrt{3}$, then find the fourth zero. [CBSE Term 1, 2016]

Ans. Let, $\quad P(x) = x^4 - x^3 - 3x^2 + 3x$

Given, $0, \sqrt{3}, -\sqrt{3}$ are three zeroes, so

$$x = 0,$$

$$x = \sqrt{3} \text{ and } x = -\sqrt{3}$$

$$\Rightarrow \quad (x-\sqrt{3}) = 0 \text{ and } x + \sqrt{3} = 0$$

Here, $x(x+\sqrt{3})(x-\sqrt{3})$ will also be the factor of $P(x)$.

Or, $x(x^2 - 3)$ will be the factor of $P(x)$.

then $\quad x^3 - 3x \ \overline{)\ x^4 - x^3 - 3x^2 + 3x\ (} x - 1$

$$
\begin{array}{r}
\quad x^4 \quad - 3x^2 \\
- \quad\quad + \\
\hline
\quad\quad - x^3 + 3x \\
\quad\quad - x^3 + 3x \\
\quad\quad + \quad - \\
\hline
\quad\quad\quad 0
\end{array}
$$

Quotient $= (x - 1)$

So fourth zero $\Rightarrow x - 1 = 0$

$$\Rightarrow \quad x = 1$$

Q. 10. Using division algorithm, find the quotient and remainder on dividing $f(x)$ by $g(x)$ where $f(x) = 6x^3 + 13x^2 + x - 2$ and $g(x) = 2x + 1$. [CBSE Term 1, 2016]

Ans. Given, $f(x) = 6x^3 + 13x^2 + x - 2$

and $g(x) = 2x + 1$,

$f(x) \div g(x) \Rightarrow$

$$
\begin{array}{r}
\quad\quad 3x^2 + 5x - 2 \\
2x + 1 \overline{)\ 6x^3 + 13x^2 + x - 2\ (} \\
\quad 6x^3 + 3x^2 \\
\quad - \quad - \\
\hline
\quad\quad 10x^2 + x - 2 \\
\quad\quad 10x^2 + 5x \\
\quad\quad - \quad - \\
\hline
\quad\quad\quad - 4x - 2 \\
\quad\quad\quad - 4x - 2 \\
\quad\quad\quad + \quad + \\
\hline
\quad\quad\quad\quad 0
\end{array}
$$

Quotient $= 3x^2 + 5x - 2$ and Remainder $= 0$

Q. 11. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $-\frac{3}{5}$ respectively. Hence find the zeroes. [CBSE Term 1, 2015]

Ans. Quadratic polynomial

$= x^2 - (\text{Sum of zeroes})\,x + \text{Product of zeroes}$

$$= x^2 - (0)\,x + \left(\frac{-3}{5}\right) = x^2 - \frac{3}{5}$$

$$= (x)^2 - \left(\sqrt{\frac{3}{5}}\right)^2$$

$$= \left(x - \sqrt{\frac{3}{5}}\right)\left(x + \sqrt{\frac{3}{5}}\right)$$

Zeroes are, $x-\sqrt{\frac{3}{5}}=0$ or $x+\sqrt{\frac{3}{5}}=0$

$\Rightarrow \quad x=\sqrt{\frac{3}{5}}$ or $x=-\sqrt{\frac{3}{5}}$

$\Rightarrow \quad x=\sqrt{\frac{3}{5}\times\frac{5}{5}}$ or $x=-\sqrt{\frac{3}{5}\times\frac{5}{5}}$

$\therefore \quad x=\frac{\sqrt{15}}{5}$ or $x=\frac{-\sqrt{15}}{5}$

Long Answer Type Questions (4 marks each)

Q. 1. Obtain all other zeroes or the polynomial $x^4+6x^3+x^2-24x-20$, if two of its zeroes are +2 and –5. [CBSE Term 1, 2016]

Ans. Given, 2 and –5 are the zeroes of polynomial

$p(x)=x^4+6x^3+x^2-24x-20$

So, $(x-2)$ and $(x+5)$ are factors of $p(x)$

$\Rightarrow (x-2)(x+5)$ is also a factor of $p(x)$

So, $(x-2)(x+5)=x^2+3x-10$

$$x^2+3x-10\overline{)x^4+6x^3+x^2-24x-20}\,(x^2+3x+2$$

$$\begin{array}{r} x^4+3x^3-10x^2 \\ \hline 3x^3+11x^2-24x-20 \\ 3x^3+9x^2-30x \\ \hline 2x^2+6x-20 \\ 2x^2+6x-20 \\ \hline 0 \end{array}$$

So, by remainder theorem,

Dividend = Divisor × Quotient + Remainder

$x^4+6x^3+x^2-24x-20$

$=(x^2+3x-10)\times(x^2+3x+2)+0$

$=(x^2+3x-10)(x^2+2x+x+2)$

$=(x^2+3x-10)[x(x+2)+1(x+2)]$

$=(x^2+3x-10)(x+2)(x+1)$

So other zeros are – 2 and – 1.

Q. 2. If the polynomial

$(x^4+2x^3+8x^2+12x+18)$ is divided by another polynomial (x^2+5), the remainder comes out to be $(px+q)$, find the values of p and q.

[CBSE Term 1, Set 1, 2015]

Ans.

$$x^2+5\overline{)x^4+2x^3+8x^2+12x+18}\,\begin{array}{l}x^2+2x+3\end{array}$$

$$\begin{array}{r} +x^4 \quad +5x^2 \\ \hline 2x^3+3x^2+12x+18 \\ +2x^3 \quad +10x \\ \hline 3x^2+2x+18 \\ +3x^2 \quad +15 \\ \hline 2x+3 \end{array}$$

Remainder $=2x+3$

i.e., $px+q=2x+3$

$\therefore \quad p=2,\ q=3$

Q. 3. What must be subtracted from $p(x)=8x^4+14x^3-2x^2+8x-12$ so that $4x^2+3x-2$ is factor of $p(x)$? This question was given to group of students for working together.

Do you think teacher should promote group work?

[CBSE Term 1, Set 1, 2015]

Ans. For this,

$$4x^2+3x-2\overline{)8x^4+14x^3-2x^2+8x-12}\,\begin{array}{l}2x^2+2x-1\end{array}$$

$$\begin{array}{r} +8x^4+6x^3-4x^2 \\ \hline 8x^3+2x^2+8x-12 \\ +8x^3+6x^2-4x \\ \hline -4x^2+12x-12 \\ -4x^2-3x+2 \\ \hline 15x-14 \end{array}$$

Polynomial to be subtracted is $(15x-14)$.