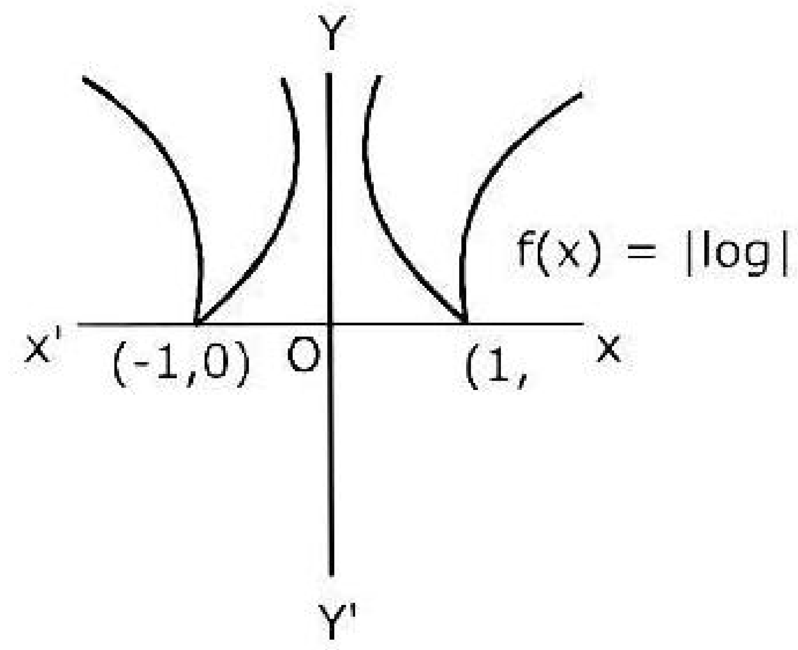
DIFFERENTIABLITY
DPP - 1

1. If $f(x) = |\log|x||$, then



(A) f(x) is continuous and differentiable for all x in its domain
(B) f(x) is continuous for all x in its domain but not differentialble at $x = \pm 1$.
(C) f(x) is neither continuous nor differentialble at $x = \pm1$.
(D) None of these

2. The left hand derivative of $f(x) = [x]\ \text{sins}(\pi x)$ at $x = k$ (k is integer), is
(A) $(-1)^k(k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$ (C) $(-1)^k k\pi$ (D) $(-1)^{k-1} k\pi$

3. Let $f(x) = |x - 1| + |x + 1|$, then the function is
(A) continuous (B) differentiable except $x = \pm1$
(C) Both (a) and (b) (D) None of these

4. Let f be twice differentiable funciton such that $f'(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to
(A) 22 (B) 11 (C) 0 (D) none of thses

5. The function $f(x) = \begin{cases} 2x - 3[x], & x \ge 1 \\ \sin\left(\frac{\pi x}{2}\right), & x < 1 \end{cases}$

(A) is continuous at $x = 2$
(B) is differentiable at $x = 1$
(C) is continuous but not differentiable at $x = 1$
(D) None of these

6. The set of points where the funciton $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable
(A) $(-\infty, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$ (C) $(-1, \infty)$ (D) None of these

7. The funciton $f(x) = e^{-|x|}$ is
(A) Continuous everywhere but not differentiable at $x = 0$
(B) Continuous at $x = 0$
(C) Not continuous at $x = 0$
(D) None of these

8. If $f(x) = x + |x| + \cos([\pi^2]x)$ and $g(x) = \sin x$, where [.] denotes the greatest integer function, then
(A) $f(x) + g(x)$ is continuous function, then
(B) $f(x) + g(x)$ is differentiable everywhere
(C) $f(x) \times g(x)$ is differentiable everywhere
(D) $f(x) \times g(x)$ is continuous but not differentiable at $x = 0$

Multiple correct

9. If $f(x) = \begin{cases} (\sin^{-1} x)^2 \cos(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

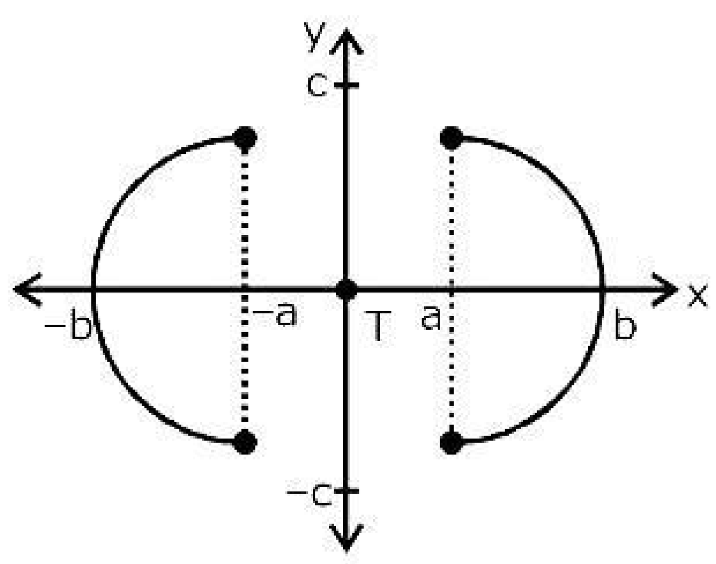
(A) f(x) is continuous everywhere in $x \in (-1,1)$
(B) f(x) is discontinuous in $x \in [-1, 1]$
(C) f(x) is differentiable everywhee in $x \in (-1, 1)$
(D) f(x) is non - differentiable anywhere in $x \in [-1, 1]$

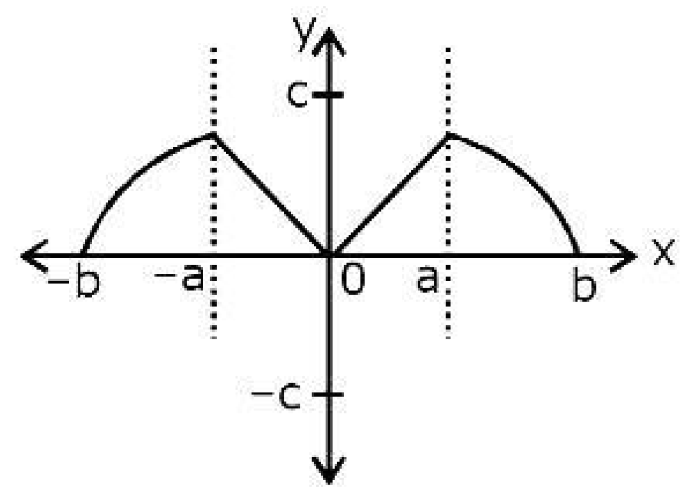
10. Lef f(x) = $f(x) = \begin{cases} \dfrac{5e^{1/x} + 2}{3 - e^{1/x}}, & x \neq 0 \\ 0, & x \neq 0 \end{cases}$

Column I	Column II
(A) y = f(x) is	(p) Continuous at x = 0
(B) y = xf(x) is	(q) discontinuous at x = 0
(C) y = x^2 f(x) is	(r) differentiable at x = 0
(D) y =x^{-1}f(x) is	(s) non - differentiable x = 0

DIFFERENTIABILITY
DPP - 2

1. If graphs of $|y| = f(x)$ and $y = |f(x)|$ are given as below $(a, b > 0)$.





$|y| = f(x)$ $y = |f(x)|$

Then identify the correct statement.
(A) f(x) is discontinuous at 2 points in [−b, b] and non-differentiable at 2 points in (−b, b).
(B) f(x) is discontinuous at 2 points in [−b, b] and non-differentiable at 3 points in (−b, b).
(C) f(x) is discontinuous at 3 points in [−b, b] and non-differentiable at 3 points in (−b, b).
(D) f(x) is discontinuous at 3 points in [−b, b] and non-differentiable at 4 points in (−b, b).

2. Number of points of non - differentiability of the function $g(x) = [x^2] \{\cos^2 4x\} + \{x^2\} [\cos^2 4x] + x^2 \sin^2 4x + [x^2] [\cos^2 4x] + \{x^2\} \{\cos^2 4x\}$ in $(-50, 50)$ where $[x]$ and $\{x\}$ denotes the greatest integer function and fractional part function of x respectively, is equal to
(A) 98 (B) 99 (C) 100 (D) 0

3. Let f be differentiable at x = 0 and f′ (0) = 1. Then $\lim_{h \to 0} \frac{f(h) - f(-2h)}{h} =$
(A) 3 (B) 2 (C) 1 (D) -1

4. Let $f(x) = \begin{cases} x^2 \left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in R$, then f is
(A) differentiable both at x = 0 and at x = 2
(B) differentiable at x = 0 but not differentiable at x = 2
(C) not differentiable at x = 0 but differentiable at x = 2
(D) differentiable neither at x = 0 nor at x = 2

Multiple Type

5. Let the function f be difined by $f(x) = \begin{cases} p + qx + x^2, & x < 2 \\ 2px + 3qx^2, & x \geq 2 \end{cases}$. Then,
(A) f(x) is continuous in R if 3p + 10q = 4
(B) f(x) is differentiable in R if $p = q = \frac{4}{13}$
(C) f(x) is continuous in R if p = -2, q = 1
(D) f(x) is differentiable in R if 2p + 22q = 4

6. Select the correct statements.

(A) The function f defined by

$f(x)=\begin{cases} 2x^2+3 & \text{for } x \le 1 \\ 3x+2 & \text{for } x > 1 \end{cases}$ is neither differentiable nor continuous at x=1.

(B) The function $f(x)=x^2|x|$ is twice differentiable at x=0.

(C) If f is continuous at x = 5 and f(5) = 2 then $\lim_{x\to 2} f(4x^2-11)$ exists.

(D) If $\lim_{x\to a}(f(x)+g(x)) = 2$ and $\lim_{x\to a}(f(x)-g(x)) = 1$ then $\lim_{x\to a} f(x)\cdot g(x)$ need not exist.

7. Two functions f & g have first & second derivatives at x = 0 & satisfy the relations,

$f(0) = \frac{2}{g(0)}$, $f'(0) = 2g'(0) = 4g(0)$, $g''(0) = 5f''(0) = 6f(0) = 3$ then

(A) if $h(x) = \frac{f(x)}{g(x)}$ then $h'(0) = \frac{15}{4}$

(B) if $k(x) = f(x) \cdot g(x) \sin x$ then $k'(0) = 2$

(C) $\lim_{x\to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$

(D) none

8.

Column-I	**Column-II**
(A) $f(x) = \begin{cases} \ln(1+x^3)\cdot\sin\frac{1}{x}, & \text{if } x>0 \\ 0, & \text{if } x\le 0 \end{cases}$	(P) continuous everywhere but not differentiable at x = 0
(B) $g(x) = \begin{cases} \ln^2(1+x)\cdot\sin\frac{1}{x}, & \text{if } x>0 \\ 0, & \text{if } x\le 0 \end{cases}$	(Q) differentiable at x = 0 but derivative is discontinuous at x = 0
(C) $u(x) = \begin{cases} \ln\left(1+\frac{\sin x}{2}\right), & \text{if } x>0 \\ 0, & \text{if } x\le 0 \end{cases}$	(R) differentiable and has continuous derivative
(D) $v(x) = \lim_{t\to 0} \frac{2x}{\pi}\tan^{-1}\left(\frac{2}{t^2}\right)$	(S) continuous and differentiable at x = 0

9.

Column I	**Column II**
(A) $f(x) = \lvert x^3\rvert$ is	(p) Continuous in (-1, 1)
(B) $f(x) = \sqrt{\lvert x\rvert}$ is	(q) differentiable in (-1, 1)
(C) $f(x) = \lvert\sin^{-1}x\rvert$ is	(r) differentiable in (0, 1)
(D) $f(x) = \cos^{-1}\lvert x\rvert$	(s) non - differentiable at least one point in (-1, 1)

Integer Type

10. A function f is defined as, $f(x) = \begin{cases} \frac{1}{|x|} & \text{if } |x| \ge \frac{1}{2} \\ a+bx^2 & \text{if } |x| < \frac{1}{2} \end{cases}$. If f(x) is derivable at $x = \frac{1}{2}$, then find (a – b).