# Chapter 22 Ray Optics & Optical Instruments

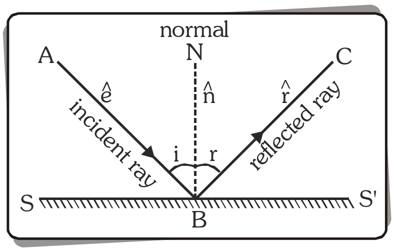
## REFLECTION

### LAWS OF REFLECTION

The incident ray (AB), the reflected ray (BC) and normal (NB) to the surface (SS') of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also plane of reflection).

The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal

$$\angle i = \angle r$$




In vector form $\hat{\mathbf{r}} = \hat{\mathbf{e}} - 2(\hat{\mathbf{e}}.\hat{\mathbf{n}})\hat{\mathbf{n}}$

### OBJECT :

- **Real :** Point from which rays actually diverge.
- **Virtual:** Point towards which rays appear to converge

### IMAGE :

- Image is decided by reflected or refracted rays only. The point image for a mirror is that point towards which the rays reflected from the mirror, actually converge (real image).

**OR**

- From which the reflected rays appear to diverge (virtual image) .

### CHARACTERISTICS OF REFLECTION BY A PLANE MIRROR :

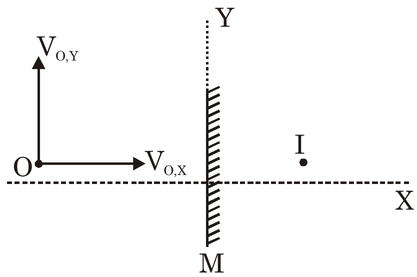
- The size of the image is the same as that of the object.
- For a real object the image is virtual and for a virtual object the image is real.
- For a fixed incident light ray, if the mirror be rotated through an angle θ the reflected ray turns through an angle 2θ in the same sense.

**Number of images (n) in inclined mirror**

Find $\frac{360}{\theta} = m$

- ❒ If m even, then n = m – 1, for all positions of object.
- ❒ If m odd , then n = m, If object not on bisector and n = m – 1, If object at bisector

### VELOCITY OF IMAGE OF MOVING OBJECT (PLANE MIRROR)




(i) Velocity component along X-axis

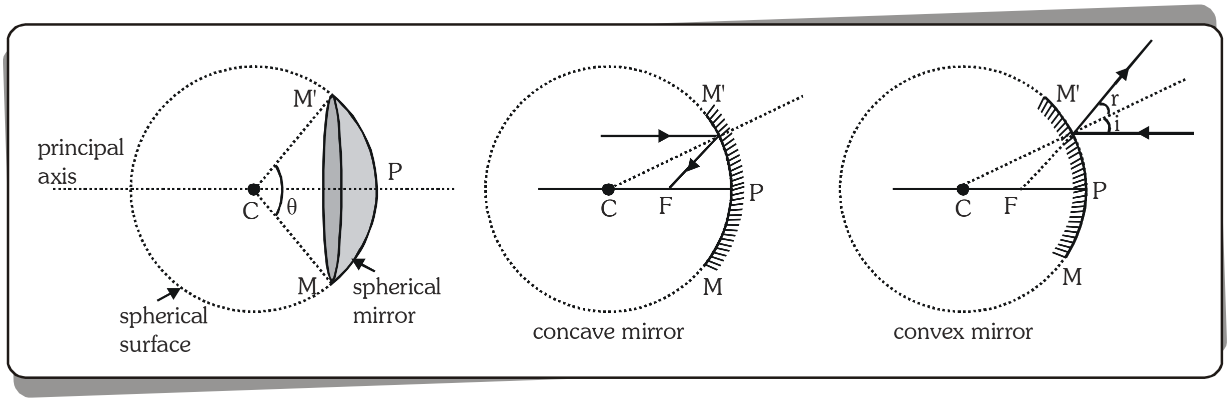
$$\vec{V}_{I,M} = -\vec{V}_{O,M}$$

$$\Rightarrow \quad \vec{V}_{I,X} = 2\vec{V}_{M,X} - \vec{V}_{O,X}$$

(ii) Along Y-axis

$$\vec{V}_{I,Y} = \vec{V}_{O,Y}$$

## SPHERICAL MIRRORS




### PARAXIAL RAYS :

Rays which forms very small angle with princiapl axis are called paraxial rays. All formulae are valid for paraxial ray only.

### SIGN CONVENTION :

- We follow cartesian co-ordinate system convention according to which the pole of the mirror is the origin.
- The direction of the incident rays is considered as positive x-axis. Vertically up is positive y-axis.
- All distance are measured from pole.

**Note :** According to above convention radius of curvature and focus of concave mirror is negative and of convex mirror is positive.

### MIRROR FORMULA :

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$

f = x-coordinate of focus

u = x-coordinate of object

v = x-coordinate of image

**Note :** Valid only for paraxial rays.

### TRANSVERSE MAGNIFICATION :

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$h_2$ = y co-ordinate of image

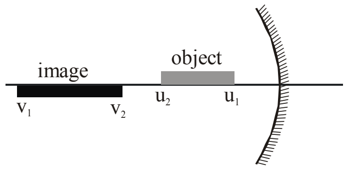
$h_1$ = y co-ordinate of the object

(both perpendicular to the principal axis of mirror)

| Magnification | Image |
|---|---|
| $\lvert m \rvert > 1$ | enlarged |
| $\lvert m \rvert < 1$ | diminished |
| $m < 0$ | inverted |
| $m > 0$ | erect |

### LONGITUDINAL MAGNIFICATION

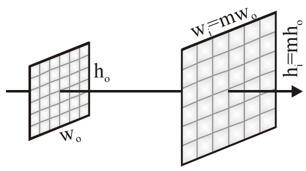
$$m_L = \frac{\text{length of image}}{\text{length of object}} = \frac{|v_2 - v_1|}{|u_2 - u_1|}$$




For small objects only : $m_L = -\frac{dv}{du} = m^2$

### SUPERFICIAL MAGNIFICATION

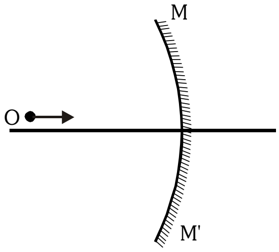
Linear magnification $m = \frac{h_i}{h_o} = \frac{w_i}{w_o}$




$$m_s = \frac{\text{area of image}}{\text{area of object}} = \frac{(ma)\times(mb)}{(a \times b)} = m^2$$

### VELOCITY OF IMAGE OF MOVING OBJECT (SPHERICAL MIRROR)

Velocity component along axis (Longitudinal velocity)




When an object is coming from infinite towards the focus of concave mirror

$$\because \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \therefore -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0$$

$$\Rightarrow \vec{v}_{IM}=-\frac{v^2}{u^2}\vec{v}_{ox}=-m^2\,\vec{v}_{OM}$$

- $v_{IM}=\frac{dv}{dt}$ = velocity of image with respect to mirror
- $v_{OM}=\frac{du}{dt}$ = velocity of object with respect to mirror.

**(b)** **<u>Velocity component perpendicular to axis</u> (Transverse velocity)**

$$m=\frac{h_I}{h_0}=-\frac{v}{u}=\frac{f}{f-u} \Rightarrow h_I=\left(\frac{f}{f-u}\right)h_0$$

$$\frac{dh_I}{dt} = \left(\frac{f}{f-u}\right)\frac{dh_0}{dt}+\frac{f\,h_0}{(f-u)^2}\frac{du}{dt} \quad ;$$

$$\vec{v}_{iy}=\left[m\vec{v}_{oy}+\frac{m^2h_0}{f}\vec{v}_{ox}\right]\hat{j}$$

$$\left[\begin{array}{l}\frac{dh_I}{dt}=\text{velocity of image}\perp^{r}\text{ to principal-axis}\\ \frac{dh_o}{dt}=\text{velocity of object }\perp^{r}\text{ to principal-axis}\end{array}\right]$$

**Note :** Here principal axis has been taken to be along x–axis.

## NEWTON'S FORMULA :

Applicable to a pair of real object and real image position only . They are called conjugate positions or foci, $X_1$, $X_2$ are the distance along the principal axis of the real object and real image respectively from the principal focus

$$\boxed{X_1X_2=f^2}$$

## OPTICAL POWER :

Optical power of a mirror (in Diopters) $=-\frac{1}{f}$

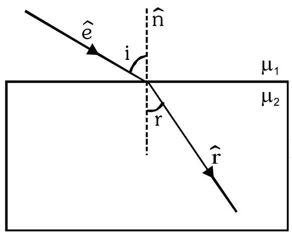
where f = focal length (in meters) with sign .

## REFRACTION - PLANE SURFACE

## LAWS OF REFRACTION (at any refracting surface)

**Laws of Refraction**

(i) Incident ray, refracted ray and normal always lie in the same plane.




In vector form $\boxed{(\hat{e}\times\hat{n}).\hat{r}=0}$

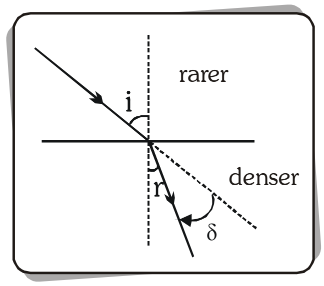
(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

**Snell's law** : $\boxed{\frac{\text{Sin } i}{\text{Sin } r}={}_1\mu_2=\frac{\mu_2}{\mu_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$

In vector form $\boxed{\mu_1|\hat{\mathbf{e}}\times\hat{\mathbf{n}}|=\mu_2|\hat{\mathbf{r}}\times\hat{\mathbf{n}}|}$

**Note :** Frequency of light does not change during refraction .

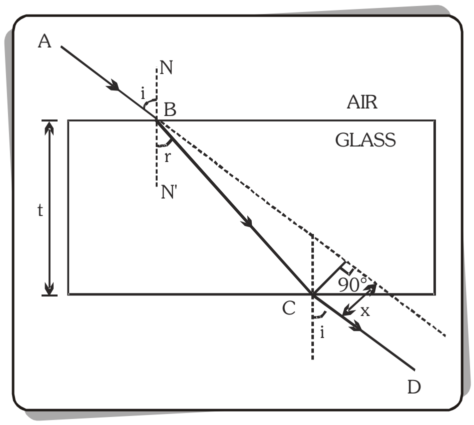
## DEVIATION OF A RAY DUE TO REFRACTION




$\boxed{\text{angle of deviation, } \delta=i-r}$

## REFRACTION THROUGH A PARALLEL SLAB

Emergent ray is parallel to the incident ray, if medium is same on both sides.

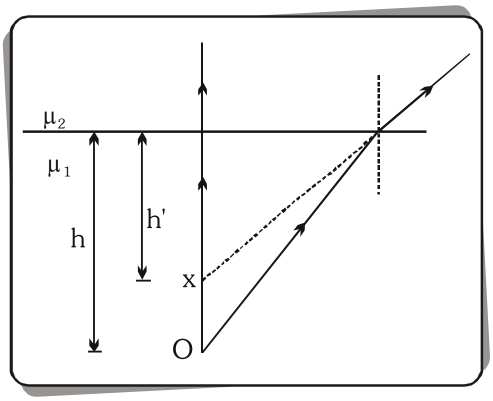



Lateral shift $x=\frac{t\sin(i-r)}{\cos r}$ ; t = thickness of slab

**Note :** Emergent ray will not be parallel to the incident ray if the medium on both the sides are different.

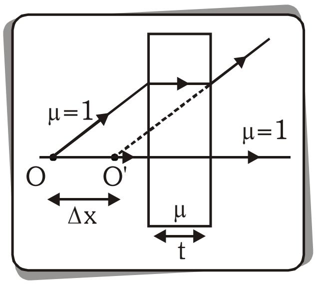
## APPARENT DEPTH OF SUBMERGED OBJECT

$(h' < h)\ \mu_1 > \mu_2$




For near normal incidence $h' = \frac{\mu_2}{\mu_1} h$

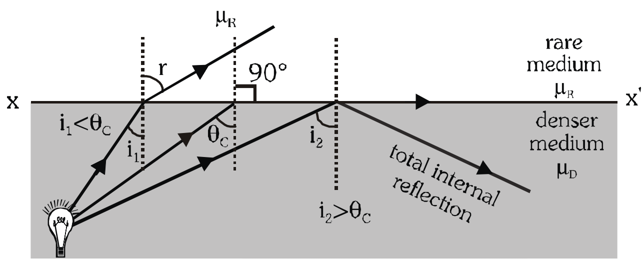
**Note :** h and h' are always measured from surface.




$\Delta x$ = Apparent normal shift = $t\left(1 - \frac{1}{\mu}\right)$

Note : Shift is always in direction of incidence ray.

## CRITICAL ANGLE & TOTAL INTERNAL REFLECTION (TIR)




### CONDITIONS

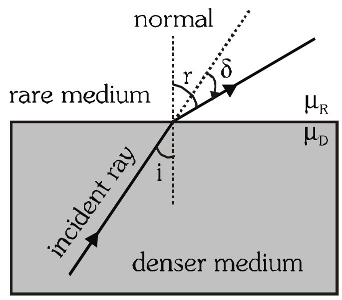
- Angle of incident > critical angle $[i > \theta_c]$
- Light should travel from denser to rare medium
  $\Rightarrow$ Glass to air, water to air, Glass to water
  Snell's Law at boundary xx', $\mu_D \sin\theta_C = \mu_R \sin 90°$

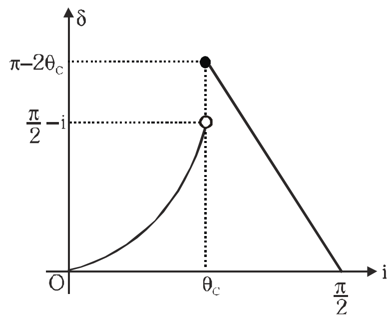
$$\Rightarrow \sin\theta_C = \frac{\mu_R}{\mu_D}$$

**Graph between angle of deviation (δ) and angle of incidence (i) as rays goes from denser to rare medium**

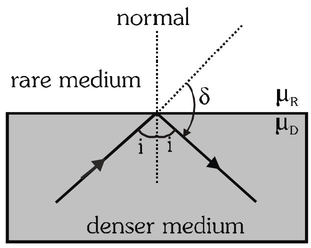
- If $i < \theta_c$ $\mu_D \sin i = \mu_R \sin r$; $r = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right)$ so

$$\delta = r - i = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right) - i$$



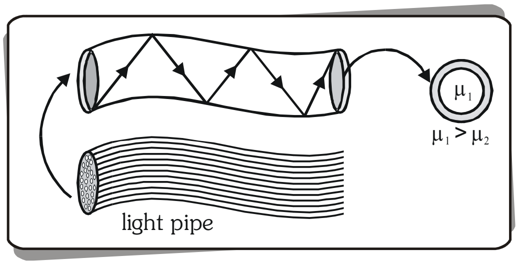

- If $i > \theta_c$; $\delta = \pi - 2i$



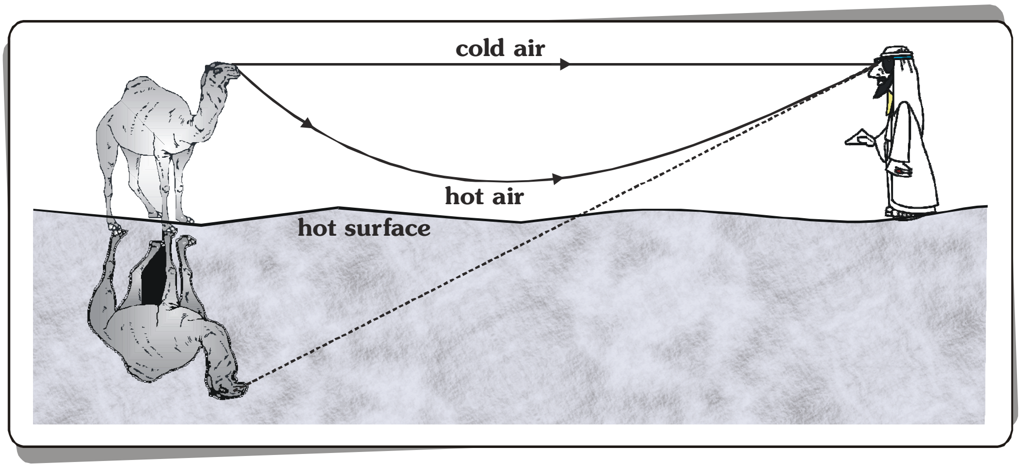

## SOME ILLUSTRATIONS OF TOTAL INTERNAL REFLECTION

- **Sparkling of diamond :** The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so C= 24°. Now the cutting of diamond are such that i > C. So TIR will take place again and again inside it. The light which beams out from a few places in some specific directions makes it sparkle.

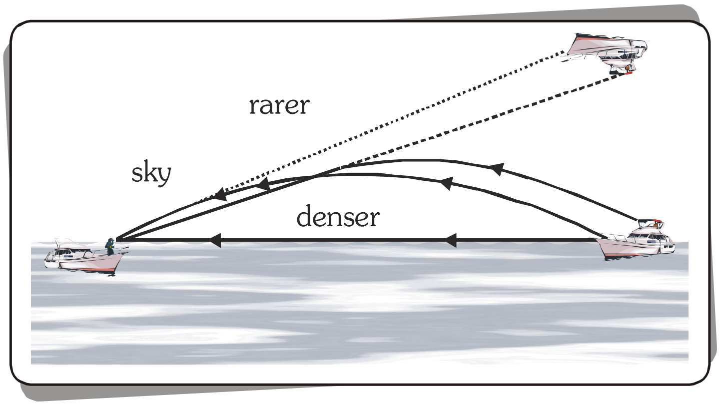
- **Optical Fibre :** In it light through multiple total internal reflections is propagated along the axis of a glass fibre of radius of few microns in which index of refraction of core is greater than that of surroundings (cladding)



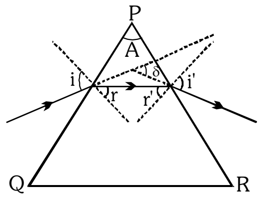

- **Mirage and looming** : Mirage is caused by total internal reflection in deserts where due to heating of the earth, refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with $i > \theta_C$ so that TIR will take place and we see the image of an object along with the object as shown in figure.

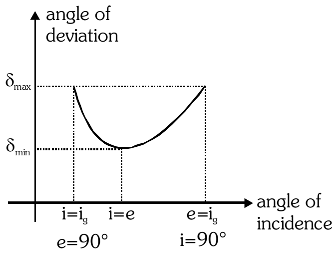



Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here $\mu$ decreases with height and so the image of an object is formed in air if ($i>\theta_C$) as shown in figure.



## REFRACTION THROUGH PRISM



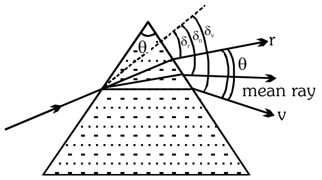

- $\delta = (i + i') - (r + r')$
- $r + r' = A$
- There is one and only one angle of incidence for which the angle of deviation is minimum.

  When $\delta = \delta_m$ then $i = i'$ & $r = r'$ , the ray passes symetrically about the prism, & then

  $$n = \frac{\sin\left[\frac{A+\delta m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$ , where n = w.r.t. surroundings R.I. of glass.
- For a thin prism ( $A \le 10°$) ; $\delta=(n-1)A$
- **Dispersion Of Light :** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called ***Dispersion of Light.***
- **Angle of Dispersion :** Angle between the rays of the extreme colours in the refracted (dispersed) light is called Angle of Dispersion.

  $$\theta = \delta_v - \delta_r$$
- Dispersive power ($\omega$) of the medium of the material of prism.

$$\omega = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$




For small angled prism ( $A \le 10°$);

$$\omega = \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n-1} \quad ; n = \frac{n_v + n_R}{2}$$

$n_v$, $n_R$ & n are R. I. of material for violet, red & yellow colours respectively .

## COMBINATION OF TWO PRISMS

- **Achromatic Combination :**

  It is used for deviation without dispersion . Condition for this $(n_v - n_r) A + (n'_v - n'_r) A' = 0$

  $\omega\delta + \omega'\delta' = 0$ where $\omega$, $\omega'$ are dispersive powers for the two prisms & $\delta$ , $\delta'$ are the mean deviation.

  Net mean deviation

  $$= \left[\frac{n_v + n_R}{2} - 1\right] A + \left[\frac{n'_v + n'_R}{2} - 1\right] A'$$
- **Direct Vision Combination :**

  It is used for producing disperion without deviation condition for this

  $$\left[\frac{n_v + n_R}{2} - 1\right] A = -\left[\frac{n'_v + n'_R}{2} - 1\right] A'$$

  Net angle of dispersion = $(n_v - n_r) A + (n_v' - n_r') A'$.

- $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

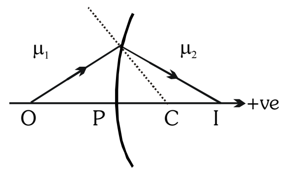
  v, u & R are to be kept with sign as

  v = PI

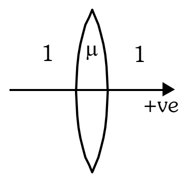
  u = –PO

  R = PC

  **(Note : Radius is with sign)**




- $$m = \frac{\mu_1 v}{\mu_2 u}$$
- **Lens Formula :**

- 


  $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
- $$m = \frac{v}{u}$$
- **Power of Lenses**

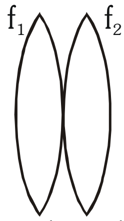
  Reciprocal of focal length in meter is known as power of lens.

  - **SI unit** : dioptre (D)
  - **Power of lens :** $P = \frac{1}{f(m)} = \frac{100}{f(cm)}$ dioptre

## Combination of Lenses

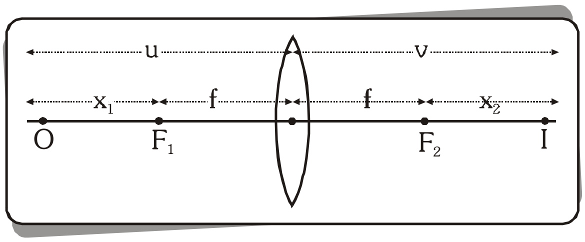
**Two thin lens are placed in contact to each other**

Power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$




Use sign convention when solving numericals

## Newton's Formula



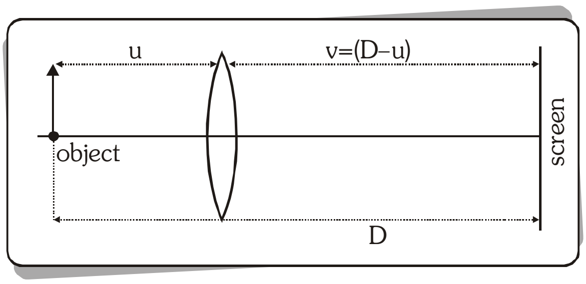

$$\boxed{f = \sqrt{x_1 x_2}}$$

$x_1$ = distance of object from focus

$x_2$ = distance of image from focus

## Displacement Method

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart.



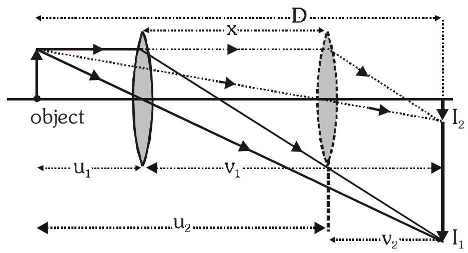

**(i) For D<4f:** u will be imaginary hence physically no position of lens is possible

**(ii) For D = 4f :** $u = \frac{D}{2} = 2f$ so only one position of lens is possible and since $v = D - u = 4f - 2f = u$

**(iii) For D > 4f :**

$$u_1 = \frac{D - \sqrt{D(D-4f)}}{2} \text{ and } u_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

So there are two positions of lens for which real image will be formed on the screen.(for two distances $u_1$ and $u_2$ of the object from lens)




If the distance between two positions of lens is x then

$x = u_2 - u_1$

$$= \frac{D + \sqrt{D(D-4f)}}{2} - \frac{D - \sqrt{D(D-4f)}}{2} = \sqrt{D(D-4f)}$$

$$\Rightarrow x^2 = D^2 - 4Df \Rightarrow f = \frac{D^2 - x^2}{4D}$$

Distance of image corresponds to two positions of the lens :

$$v_1 = D - u_1 = D - \frac{1}{2}[D - \sqrt{D(D-4f)}]$$

$$= \frac{1}{2}[D + \sqrt{D(D-4f)}] = u_2 \Rightarrow v_1 = u_2$$

$$v_2 = D - u_2 = D - \frac{1}{2}[D + \sqrt{D(D-4f)}]$$

$$= \frac{1}{2}[D - \sqrt{D(D-4f)}] = u_1 \Rightarrow v_2 = u_1$$

Distances of object and image are interchangeable. for the two positions of the lens. Now

$x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + u_1$ $[\because v_1 = u_2]$

so $u_1 = v_2 = \frac{D - x}{2}$ and $u_2 = v_1 = \frac{D + x}{2}$;

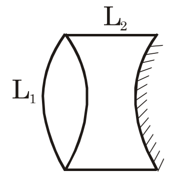
$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \frac{D+x}{D-x} \text{ and } m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \frac{D-x}{D+x}$$

Now $m_1 \times m_2 = \frac{D+x}{D-x} \times \frac{D-x}{D+x} \Rightarrow \frac{I_1 I_2}{O^2} = 1 \Rightarrow O = \sqrt{I_1 I_2}$

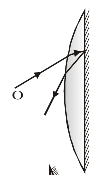
## Silvering of one surface of lens

$P_{eff} = 2P_{L1} + 2P_{L2} + P_M$

$$\frac{1}{f_{eff}} = \frac{2}{f_{L1}} + \frac{2}{f_{L2}} - \frac{1}{f_M}$$




When plane surface is silvered $f = \frac{R}{2(\mu - 1)}$




When convex surface is silvered $f = \frac{R}{2\mu}$