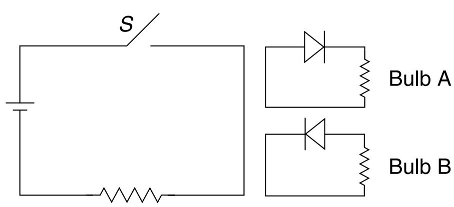
Chapter 11

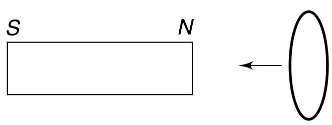
Electromagnetic Induction

Level 1

Q. 1: In the Figure shown, when the switch S is closed, one of the two bulbs glows momentarily. Which one?



Q. 2: A short circuited coil is moved towards a fixed magnet at a constant velocity. When the coil is at a distance x from the magnet it experiences a magnetic force F. Now the number of turns in the coil is doubled and the experiment is repeated with the coil moving with same constant velocity towards the magnet. What will be the magnetic force on the coil when it is at a distance x from the magnet?



Q. 3: A horizontal conducting loop of radius R is fixed in air. A uniformly charged rod having charge Q on it is held vertically above the conducting loop at a height h above it. The rod is released and it begins to fall along the axis of the loop. Calculate the emf induced in the conducting loop.

Q. 4: (a) When a small magnet is moved towards a solenoid, an emf is induced in it. However, if a magnet is moved inside the hole of a toroid, no emf is induced. Explain.

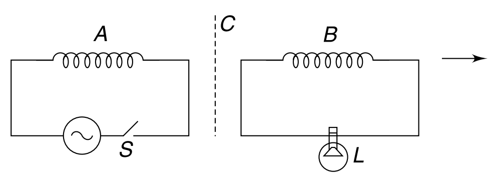
(b) A wire is kept horizontal along North-South direction. It is allowed to fall freely. How much emf will be induced across the ends of the wire due to presence of Earth's magnetic field.

Q. 5: Coil A is connected to an ac source through a switch S. Coil B, kept close to A, is connected to a low voltage bulb (L). Explain the following observations.

(i) When S is closed, the bulb lights up.

(ii) With S closed if B is moved away from A, the bulb gets dimmer.

(iii) With S closed, if a copper plate C is inserted between the coils, the bulb gets dimmer.



Q. 6: A coil is rotated with constant angular speed in a uniform magnetic field about an axis that is perpendicular to the field. Tell, with reason, if the following statement is true–

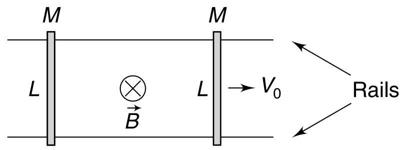
"The emf induced in the coil is maximum at the instant the magnetic flux through the coil is zero".

Q. 7: ABC is a triangular frame made of a conducting wire and is right angled at B. Its side BC is vertical and AB is horizontal. The frame is placed in a uniform magnetic field B in vertically upward direction. The frame is rotated about its side BC with constant angular speed ω. Resistance of the entire frame is R. Neglect gravity. Length $AB = L$; $BC = 2L$.

(a) Find the torque needed to keep the frame rotating with constant angular speed.

(b) Find the potential difference between points A and C.

Q. 8: Two parallel conducting rails are separated by a distance L. Two identical conducting wires are placed on the rails perpendicular to them. Each wire has mass M. One of the wires is given a velocity v_0 parallel to the rails away from the other wire. There is a uniform magnetic field directed into the plane of the figure everywhere. Answer following questions for the system comprising of two wires. [Ignore friction and self inductance]

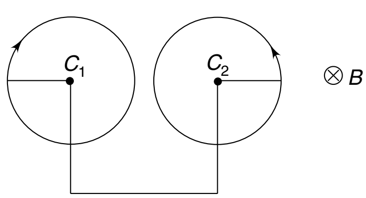


(i) Will the momentum of the system decrease with time?

(ii) What is kinetic energy of the system in steady state?

Q. 9: A stretchable conducting ring is in the shape of a circle. It is kept in a uniform magnetic field (B) that is perpendicular to the plane of the ring. The ring is pulled out uniformly from all sides so as to increase its radius at a constant rate $\frac{dr}{dt} = V$ while maintaining its circular shape. Calculate the rate of work done by the external agent against the magnetic force when the radius of the ring is r_0. Resistance of the ring remains constant at R.

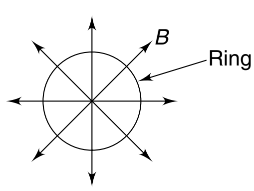
Q. 10: Two rods, each of length $L = 0.6$ m, are rotating in same plane about their ends C_1 and C_2. The distance between C_1 and C_2 is 1.201 m. The rods rotate in opposite directions with same angular speed ω and they are found to be in position shown in the Figure at a given instant of time. The ends C_1 and C_2 are connected using a conducting wire. There exists a uniform magnetic field perpendicular to the figure having magnitude $B = 5.0$ T. At what angular speed ω do we expect to see sparks in air? The dielectric breakdown of air happens if electric field in it exceeds 3×10^6 Vm^{-1}.



Q. 11: A vertical cylindrical region has a horizontal radial magnetic field inside it. A wire ring is made of a wire of cross section S, density d and resistivity ρ. Radius of the ring is r. The ring is kept horizontal with its centre on the axis of the cylindrical region and released. The field strength at all points on the circumference of the ring is B. At a certain instant velocity of the ring is v downward. Find

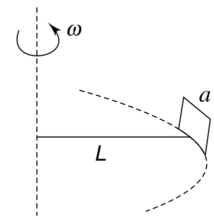
(a) Current in the ring and

(b) Acceleration of the ring at the instant.

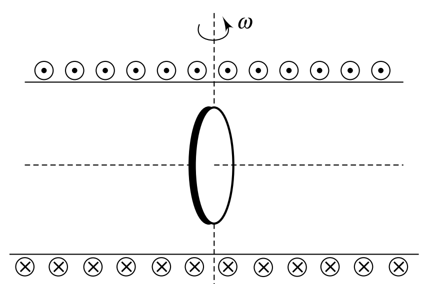


Top view

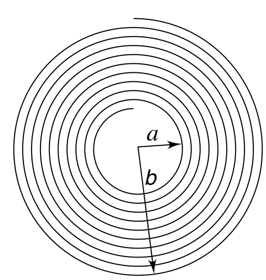
Q. 12: A conducting square loop of side length a is held vertical with the help of a non conducting rod of length L. The rod is made to rotate in horizontal circle about a vertical axis through its end. The loop rotates with the rod while its plane always remains perpendicular to the rod. The resistance of the loop is R and angular speed of the rod is ω. There is a uniform horizontal magnetic field B in the entire space. Find the average rate of heat dissipation in the loop during one rotation.



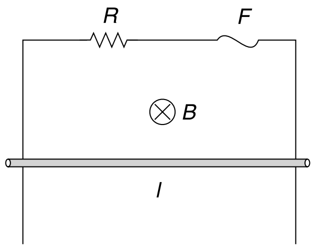
Q. 13: A long solenoid has n turns per unit length and carries a current $i = i_0 \sin \omega t$. A coil of N turns and area A is mounted inside the solenoid and is free to rotate about its diameter that is perpendicular to the axis of the solenoid. The coil rotates with angular speed ω and at time $t = 0$ the axis of the coil coincides with the axis of the solenoid as shown in figure. Write emf induced in the coil as function of time.



Q. 14: A flat coil, in the shape of a spiral, has a large number of turns N. The turns are wound tightly and the inner and outer radii of the coil are a and b respectively. A uniform external magnetic field (B) is applied perpendicular to the plane of the coil. Find the emf induced in the coil when the field is made to change at a rate $\frac{dB}{dt}$.



Q. 15: A pair of long conducting rails are held vertical at a separation $l = 50$ cm. The top ends are connected by a resistance of $R = 0.5\,\Omega$ and a fuse (F) of negligible resistance. A conducting rod is free to slide along the rails under gravity. The whole system is in a uniform horizontal magnetic field $B = 1.5T$ as shown. Resistance of the rod and rails are negligible and the rod remains horizontal as it moves. The rod is released from rest. Find the minimum mass of the rod that will ensure that the fuse blows out. It is known that rating of the fuse is $4A$.

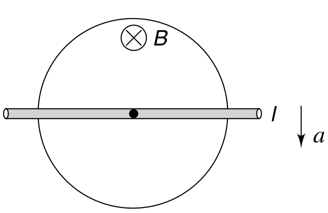


Q. 16: A thin uniform conducting rod of mass M and length L oscillates in a vertical plane about a fixed horizontal axis passing through its top end. The rod is oscillating with angular amplitude θ_0. A uniform horizontal magnetic field B, perpendicular to the plane of the oscillation is switched on.

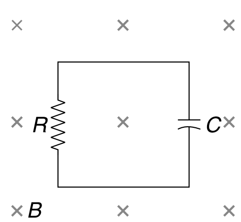
(a) Calculate the maximum emf induced between the ends of the rod.

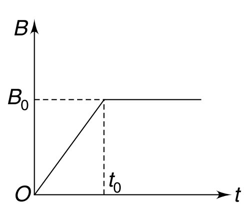
(b) If the rod has a finite thickness (any real life rod will definitely have a thickness), what difference in oscillation is expected in absence of magnetic field and in presence of magnetic field. Describe qualitatively.

Q. 17: A wire ring of radius R is fixed in a horizontal plane. The wire of the ring has a resistance of $\lambda \Omega m^{-1}$. There is a uniform vertical magnetic field B in entire space. A perfectly conducting rod (l) is kept along the diameter of the ring. The rod is made to move with a constant acceleration a in a direction perpendicular to its own length. Find the current through the rod at the instant it has travelled through a distance $x = \frac{R}{2}$.

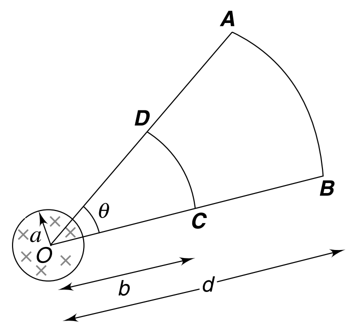


Q. 18: A conducting loop having resistance R contains a capacitor of capacitance C. A uniform magnetic field B is applied perpendicular to the plane of the loop. The magnetic field is made to change with time as depicted in the graph [$t_0 = 2RC$]. Plot the variation of charge on the capacitor as a function of time (t).





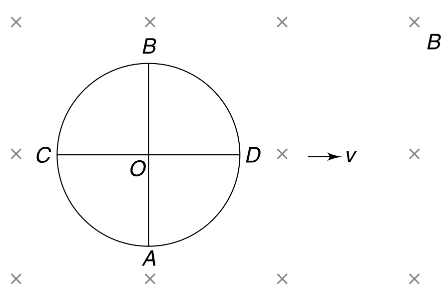
Q. 19: There is a uniform magnetic field (B) perpendicular to the plane of the figure in a circular region of radius a centred at O. $ABCD$ is a conducing loop in the plane of the figure with its arms BC and DA along two radial lines from O having an angle θ between them. AB is circular arc of radius $OB = d$ centred at O. CD is also a circular arc of radius $OC = b$ centred at O. The magnetic field is changed at a rate $\frac{dB}{dt}$.



(a) Find emf induced in the loop $ABCD$.

(b) Find emf induced in arc AB.

Q. 20: A uniform conducting wire is used to make a ring and its two diameters AB and CD. The ring is placed in a uniform magnetic field B perpendicular to its plane. The resistance of the wire per unit length is λ Ω/m.



(a) Find the current in BO and BC when the ring is moved with constant velocity v in its plane.

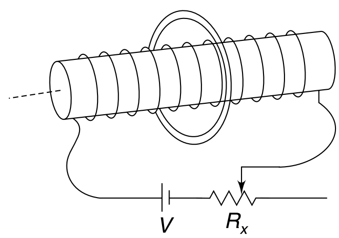
(b) Find current in BO and BC when the ring is kept stationary but the magnetic field is increased at a constant rate of $\frac{dB}{dt} = \alpha$ T/s. Assume that magnetic field is confined inside the circular ring only. Take radius of the ring to be a.

Q. 21: A bar magnet is kept along the axis of a conducting loop. When the magnet is moved along the axis, a current is seen in the loop. Which force is responsible for driving electrons in the loop if the observer is in

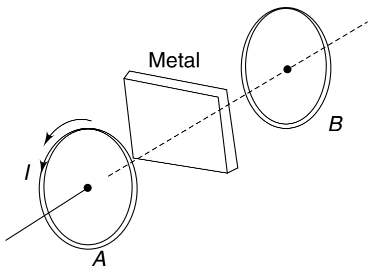
(a) Reference frame attached to the loop.

(b) Reference frame attached to the magnet.

Q. 22: A long narrow solenoid of radius a has n turns per unit length and resistance of the wire wrapped on it is R. The solenoid is connected to a battery of emf V through a variable resistance R_x. There is a conducting ring of radius $2a$ held fixed around the solenoid with its axis coinciding with that of the solenoid. The relaxation time of free electrons inside the material of the conducting ring at given temperature is τ and specific charge of an electron is α. The variable resistance R_x is changed linearly with time from zero to R in an interval Δt. Calculate the drift speed of the free electrons in the ring during this interval. [Neglect inductance]



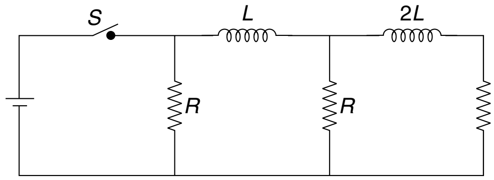
Q. 23: Two coils A and B are mounted co-axially some distance apart. Coil A is given a current that changes sinusoidally with time. A current gets induced in coil B. How does the magnitude of current in coil B change if a metal plate is placed between the two coils.



Q. 24: A thin conducting ring of radius R has a thin layer of insulation on its inner face. A superconducting ring of radius R is pressed into it. The insulation layer separates the two rings. Find the self inductance of the conducting ring.

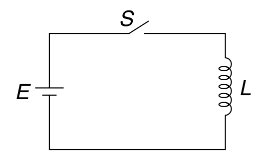
Q. 25: A certain volume of copper is drawn into a wire of radius a and is wrapped in shape of a helix having radius r (>> a). The windings are as close as possible without overlapping. Self inductance of the inductor so obtained is L_1. Another wire of radius $2a$ is drawn using same volume of copper and wound in the fashion as described above. This time the inductance is L_2. Find $\frac{L_1}{L_2}$.

Q. 26: In the circuit shown in figure, the current through each resistor is I. Find the currents through the resistors immediately after the switch 'S' is opened. How much heat will get dissipated in the circuit after the switch is opened?

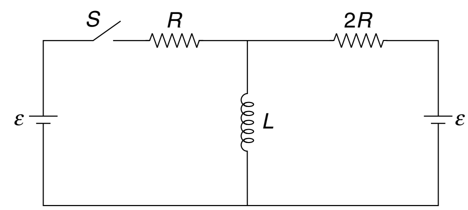


Q. 27: "The current through an inductor cannot change instantaneously. However, the potential difference across an inductor can change abruptly". Do you agree with this statement. Give reason.

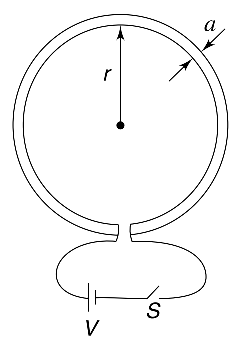
Q. 28: Consider an ideal inductor (having no resistance) of inductance L which is connected to an ideal cell (no resistance) of emf E by closing the switch S at time $t = 0$. Plot the variation of current through the inductor versus time.



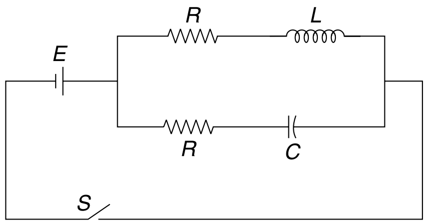
Q. 29: The circuit shown in the Figure is in steady state. Find the rate of change of current through L immediately after the switch S is closed.



Q. 30: A circular loop of radius r is made of a wire of circular cross section of diameter 'a'. When current I flows through the loop the magnetic flux linked with the loop due to self induced magnetic field is given by $\phi = \mu_0 r\left[\ln\left(\frac{16r}{a}\right) - \frac{7}{4}\right] I$. The resistivity of the material of the wire is ρ and $r >> a$. Switch S is closed at time $t = 0$ so as to connect the loop to a cell of emf V. Find the current in the loop at time t.

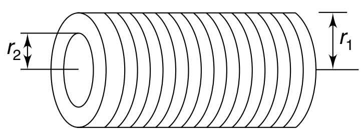


Q. 31: In the circuit shown, find the value of resistance R in terms of inductance L and capacitance C such that the current through the cell remains constant forever after the switch is closed.



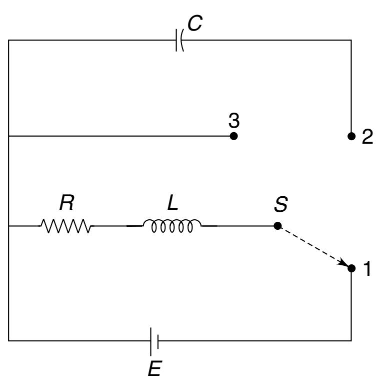
Q. 32: A proton beam has a circular cross sectional area A and it carries a current I distributed uniformly over its cross section. Calculate the magnetic energy per unit length of the beam within the beam.

Q. 33: Two long co-axial solenoids have radii, and number of turns per unit length equal to r_1, r_2 and n_1, n_2 respectively where suffix 1 refers to the outer solenoid and 2 refers to the inner solenoid. Length of both is l. The current in the outer solenoid is made to grow as $I_1 = kt$ where t is time. Resistance of the wire used in inner solenoid is R. Write the current induced in the inner solenoid assuming that it is shorted.



Q. 34: A capacitor of capacitance C is having a charge Q_0. It is connected to a pure inductor of inductance L. The inductor is a solenoid having N turns. Find the magnitude of magnetic flux through each of the N turns in the coil at the instant charge on the capacitor becomes $\frac{Q_0}{2}$.

Q. 35: In the circuit arrangement shown in Figure, the three way switch S is kept in position 1 for a long time.



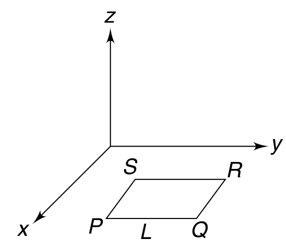
(a) Find the potential difference across the inductor immediately after the switch is thrown from position 1 to position 2.

(b) After being left in position 2 for a long time, the switch is moved to position 3. Find the potential difference across R immediately after the switch is moved to position 3.

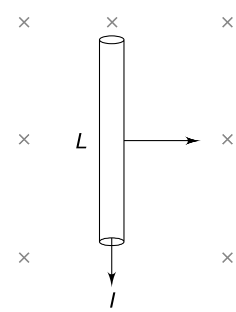
Assume no time lag in moving the switch from one position to another.

LEVEL 2

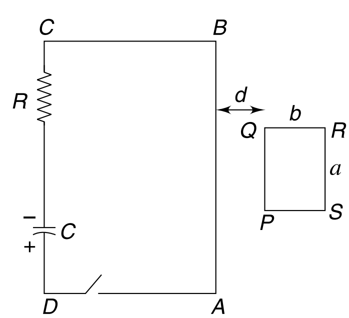
Q 36: A rectangular conducting loop $PQRS$ is kept in xy plane with its adjacent sides parallel to x and y axes. A magnetic field $\vec{B}$ is switched on in the region which varies in position and time as $\vec{B} = [B_0 \sin(ky - \omega t)]\,\hat{k}$. It was found that the emf induced in the loop is always zero. Express the length (L) of the loop in terms of constant k and ω.



Q. 37: A conducting rod of length L is carrying a constant current I. There exists a magnetic field B perpendicular to the rod. Due to the magnetic force the rod moved through a distance x in a direction perpendicular to the field (B) as well as its own length. The rod acquires a kinetic energy. A student says that a magnetic force ILB acted on the rod and it performed a work $W = ILBx$ on the rod. But we know that magnetic force is always perpendicular to the velocity of the charge and it cannot perform work on a moving charge. Which agency has actually spent energy to impart kinetic energy to the rod?



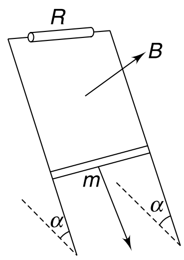
Q. 38: In a rectangular circuit $ABCD$ a capacitor having capacitance $C = 20\ \mu$F is charged to a potential difference of 100 V. Resistance of circuit is $R = 10\ \Omega$. Another rectangular conducting loop $PQRS$ is kept side by side to the first circuit with sides AB and PQ parallel and close to each other. The length and width of rectangle $PQRS$ is $a = 10$ cm and $b = 5$ cm respectively and AB as well as BC is large compared to a. PQ is located near the centre of side AB with $d = 5$ cm. The loop $PQRS$ has 25 turns and the wire used has resistance of $1\ \Omega\,\text{m}^{-1}$. The switch S is closed at time $t = 0$. Neglect self inductance of loops.



(a) Find the current in *ABCD* at $t = 200\ \mu s$

(b) Find the current in *PQRS* at $t = 200\ \mu s$.

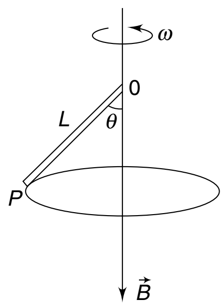
Q. 39: A copper connector of mass m, starting from rest, slides down two conducting bars set at angle α to the horizontal, due to gravity (see Figure). At the top the bars are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistance of the bars the connector and the sliding contacts, as well as the self inductance of the loop, are assumed to be negligible. The coefficient of friction between the connector and the bars is equal to $\frac{1}{2}\tan\alpha$



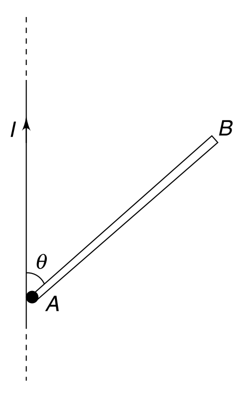
(a) Find the steady-state velocity of the connector.

(b) How will your answer differ if the magnetic field was in opposite direction.

Q. 40: A conducting rod (OP) of length L rotates in form of a conical pendulum with an angular velocity ω about a fixed vertical axis passing through its end O. There is a uniform magnetic field B in vertically downward direction. The rod makes an angle θ with the direction of the magnetic field. Calculate the emf induced across the ends of the rod. Which end is positive?



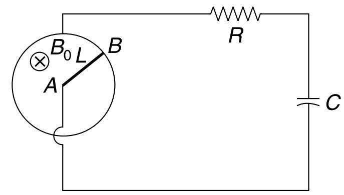
Q. 41: A conducting rod AB of mass M and length L is hinged at its end A. It can rotate freely in the vertical plane (in the plane of the Figure). A long straight wire is vertical and carrying a current I. The wire passes very close to A. The rod is released from its vertical position of unstable equilibrium. Calculate the emf between the ends of the rod when it has rotated through an angle θ (see Figure).



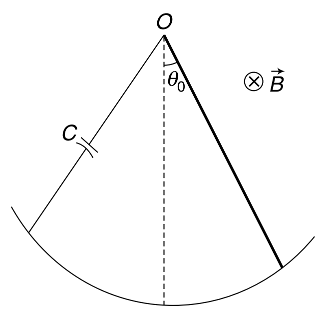
Q. 42: A perfectly conducting ring of radius L is kept fixed on a horizontal surface. A vertical uniform magnetic field B_0 exists in the region. A conducting rod (AB) of length L is hinged at the centre of the ring at A and its other end (B) touches the ring. The ring and the end A of the rod are connected to an external circuit having resistance R and capacitance C. The rod is made to rotate at a constant angular speed ω_0. Neglect friction and self inductance of the circuit.

(i) Find work done by the external agent in rotating the rod by the time the capacitor acquires a charge q_0.

(ii) Find heat generated in resistance R by the time the capacitor acquires a charge q_0.



Q. 43: A conducting rod of mass M and length L can oscillate like a pendulum in a vertical plane about point O. The lower end of the rod glides smoothly on a circular conducting arc of radius L. The circular arc is connected to point O of the rod through a capacitor of capacitance C. The entire device is kept in a uniform horizontal magnetic field B directed into the plane of the Figure. Disregard resistance of any component. The rod is deflected through a small angle θ_0 from vertical position and released at time $t = 0$.

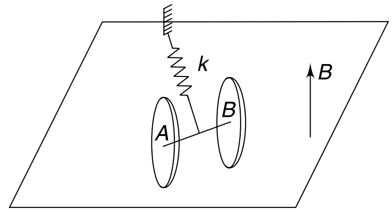


(a) Write the deflection angle (θ) of the rod as a function of time t.

(b) If the capacitor is replaced with a resistor what kind of motion do you expect?

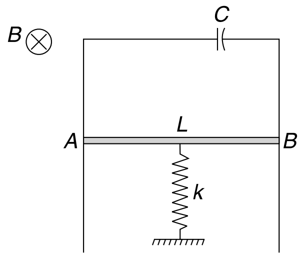
Give qualitative description only.

Q. 44: Two identical thin circular, metal plates are at a small separation d. They are connected by a thin conducting rod (AB) of length d. Each plate has area A. An ideal spring of stiffness k is connected to a rigid support and midpoint of rod AB as shown in the figure. Spring is made of insulating material. The system is on a smooth horizontal surface. The entire region has a uniform vertical upward magnetic field B.



The discs are pulled away from the support and released. Find time period of oscillations. Mass of the two disc plus rod system is M. Neglect any eddy current.

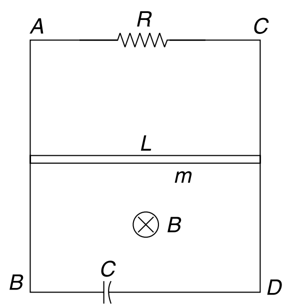
Q. 45: Two metal bars are fixed vertically and are connected on top by a capacitor of capacitance C. A sliding conductor AB can slide freely on the two bars. Length of conductor AB is L and its mass is m. It is connected to a vertical spring of force constant k. The conductor AB is released at time $t = 0$, from a position where the spring is relaxed. Taking initial position of the conductor as origin and downward direction as positive x axis, write the x co-ordinate of the conductor as a function of time. The entire space has a uniform horizontal magnetic field B. Neglect resistance and inductance of the circuit and assume that the bar AB always remains horizontal.



Q. 46: Two long fixed parallel vertical conducting rails AB and CD are separated by a distance L. They are connected by resistance R and a capacitance C at two ends as shown in the figure. There is a uniform magnetic field B directed horizontally into the plane of the figure. A horizontal metallic bar of length L and mass m can slide without friction along the rails. The bar is released from rest at $t = 0$. Neglect resistance of bar and rails and also neglect the self inductance of the loop.

(a) Find the maximum speed acquired by the bar after it is released.

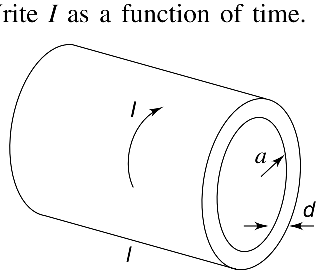
(b) Find the speed of the bar as a function of time t.



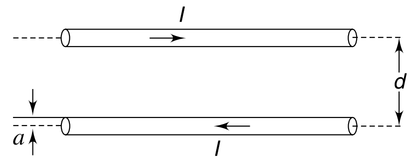
Q. 47: A hollow cylinder made of material of resistivity ρ has length ℓ, radius a and wall thickness d ($\ell >> a >> d$). A current I flows through the cylinder in tangential direction and is uniformly distributed along its length.

(a) Find the emf developed along the circumference of the cylinder if the current changes at a rate $\frac{dI}{dt} = \beta$

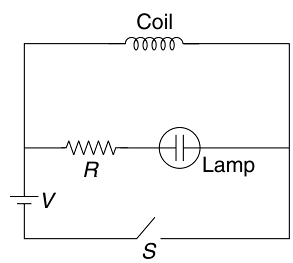
(b) Assume that no external source is present and the current at time $t = 0$ is I_0. The current decays with time. Write I as a function of time.



Q. 48: A long straight wire of cross sectional radius a carries a current I. The return current is carried by an identical wire which is parallel to the first wire. The centre to centre distance between the two wires is d. Find the inductance (L) of a length x of this arrangement. Neglect magnetic flux inside the wires.

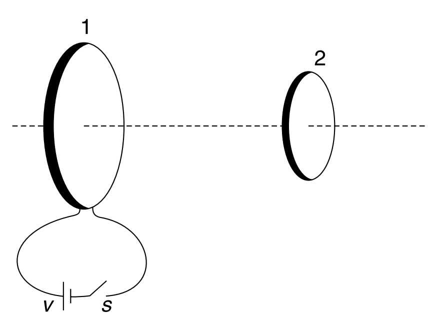


Q. 49: In the circuit shown in the figure the neon lamp lights up if potential difference across it becomes 60 V and goes out if the potential difference falls below 30 V. The inductor coil has a very small resistance and emf of the cell is $V = 4$ volt. The lamp does not light when the switch is closed. The neon light flashes once when the switch is opened. Explain why?

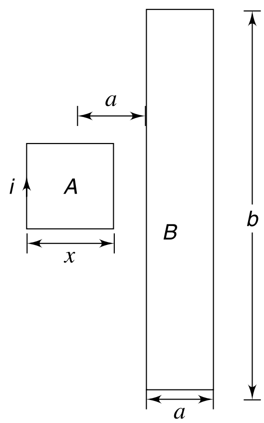


Q. 50: Determine the mutual inductance of a toroid and an infinite straight wire located along the central axis of the toroid. The toroid has a rectangular cross section with inner and outer radii a and b respectively. The width of the rectangular cross section parallel to the straight wire is h. Total turns in the toroid is N.

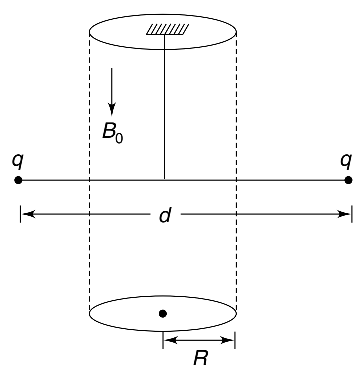
Q. 51: Two coils –1 and 2 – are mounted co axially as shown in the figure. The resistance of the two coils are R_1 and R_2 and their self inductances are L_1 and L_2 respectively. Switch S is closed at time $t = 0$ to connect the coil 1 to an ideal cell of emf V. It is observed that by the time current reaches its steady value in coil 1, the quantity of charge that flows in coil 2 is Q_0. Calculate the mutual inductance (M) between the two coils.



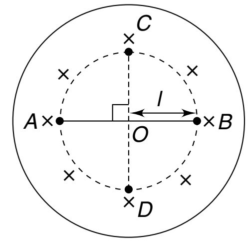
Q. 52: A rectangular conducting loop B has its side lengths equal to b and a ($<< b$). In the plane of the loop there is another very small loop A in shape of a square of side length x. Loop A is placed symmetrically with respect to B with its centre at a distance a from one of its longer side (see Figure). There is a current (i) in loop A which is made to increase at a constant rate of $\frac{di}{dt} = \alpha\, As^{-1}$. Calculate the emf induced in the bigger loop.



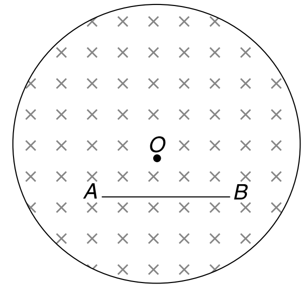
Q. 53: Two identical small balls, made of insulting material are attached to the ends of a light insulating rod of length d. The rod is suspended from the ceiling by a thin torsion free fibre as shown in figure. Each ball is given a charge q. There is a uniform magnetic field B_0, pointing vertically down, in a cylindrical region of radius R. The fibre is along the axis of the cylindrical region. The system is initially at rest. Now the magnetic field is suddenly switched off. Calculate the angular velocity acquired by the system. Each ball has mass m.



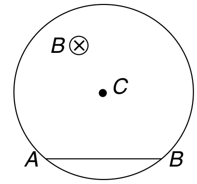
Q. 54: A massless non conducting rod AB of length $2l$ is placed in uniform time varying magnetic field confined in a cylindrical region of radius ($R > l$) as shown in the figure. The center of the rod coincides with the centre of the cylindrical region. The rod can freely rotate in the plane of the Figure about an axis coinciding with the axis of the cylinder. Two particles, each of mass m and charge q are attached to the ends A and B of the rod. The time varying magnetic field in this cylindrical region is given by $B = B_0\left[1 - \frac{t}{2}\right]$ where B_0 is a constant. The field is switched on at time $t = 0$. Consider: $B_0 = 100T$, $l = 4$ cm, $\frac{q}{m} = \frac{4\pi}{100}$ C/kg. Calculate the time in which the rod will reach position CD shown in the figure for the first time. Will end A be at C or D at this instant?



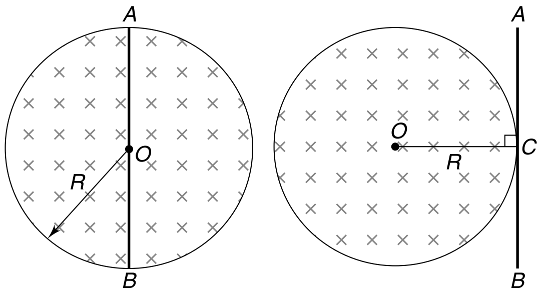
Q. 55: A cylindrical region of radius R is filled with a uniform magnetic field B as shown in the figure. A metal wire (AB) of length L is placed inside the field such that its ends are symmetrically located with respect to the centre (O) of the circular cross section of the region. If the magnetic field is changed at a rate $\frac{dB}{dt}$ the emf induced in the metal wire is ε. Find change in value of ε if the wire is displaced by a small distance ΔL parallel to its own length. Assume that the wire remains inside the field region.



Q. 56: A cylindrical volume of radius R has a uniform axial magnetic field B, which is increasing at a rate of $\frac{dB}{dt} = \alpha\, \text{Ts}^{-1}$. A chord ($AB$) of the circular cross section of the cylindrical region has length L. Calculate the line integral of induced electric field $\left(\int_A^B \vec{E}\,\vec{dl}\right)$ as one moves along the chord from A to B. Try to find the answer without actually performing the integration. Is the value of integral same if one moves along the arc from A to B?



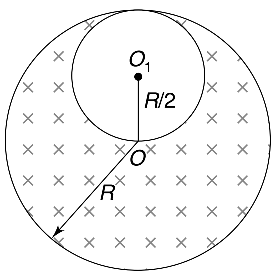
Q. 57: A uniform magnetic field B exists in a circular region of radius R. The field is perpendicular to the plane and is increasing at a constant rate of $\frac{dB}{dt} = \alpha$. There is a straight conducting rod AB of length $2R$. Find the emf induced in the rod when it is placed as shown in figure (a) & (b). Point C is midpoint of the rod in figure (b)



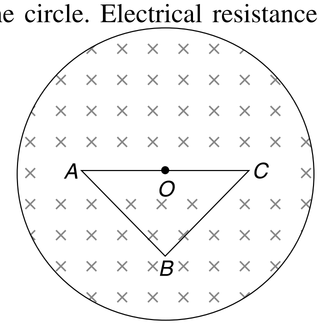
(a) (b)

Q. 58: A thin beam of n identical positively charged particle are constrained to move in a circular orbit of radius R in a particle accelerator. Each particle has charge q and mass m and the current in the circular orbit is I_0. The magnetic flux through the circular path is made to increases at a constant rate of β Wbs^{-1}. Calculate the current after the particles complete one turn.

Q. 59: There is a long cylinder of radius R having a cylindrical cavity of radius $R/2$ as shown in the figure. Apart from the cavity, the entire space inside the cylinder has a uniform magnetic field parallel to the axis of the cylinder. The magnetic field starts changing at a uniform rate of $\frac{dB}{dt} = k$ T/s. Find the induced electric field at a point inside the cavity.



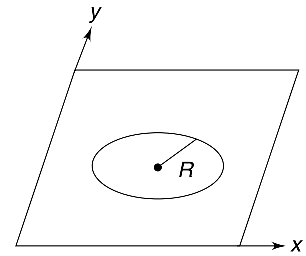
Q. 60: A uniform magnetic field B exists in a region of circular cross section of radius R. The field is directed perpendicularly into the plane of the figure. There is a triangular circuit ABC made of a uniform wire placed in the circular region. The triangle is a right angled isosceles triangle with equal sides $AB = BC = \ell$. The hypotenuse AC has its midpoint at the centre of the circle. Electrical resistance of the triangular circuit per unit length is $\frac{r_0}{\ell}$. If the magnetic field is changed at a constant rate of $\frac{dB}{dt} = \alpha$, find the potential difference between points C and A and that between B and A.



Q. 61: A conducting ring of mass $m = \pi$ kg and radius $R = \frac{1}{2}$ m is kept on a flat horizontal surface (xy plane). A uniform magnetic field is switched on in the region which changes with time (t) as $\vec{B} = (2\hat{j} + t^2\hat{k})$ T. Resistance of the ring is $r = \pi\ \Omega$ and $g = 10$ ms^{-2}.

(a) Calculate the induced electric field at the circumference of the ring at the instant it begins to topple.

(b) Calculate the heat generated in the ring till the instant it starts to topple.



Q. 62: A tightly wound solenoid of length ℓ and cross sectional area A_1 is partly inserted co-axially into another tightly wound solenoid of length ℓ and cross section A_2 ($> A_1$). The centres of the two solenoids are separated by x ($< \ell$). Assume that $A_2 << x^2$, $A_2 << l^2$. Calculate the total magnetic field energy inside the solenoid when both of them carry same current (I) in the same sense. Number of turns in each solenoid is N.

Q. 63: In the last problem calculate the emf induced in the outer coil if the inner coil is pulled out at a speed v. How much emf will be induced in the inner coil?

Q. 64: In the last problem, calculate the force needed to pull out the inner coil at constant speed. The outer coil remains fixed.

Q. 65: Two ends of an inductor of inductance L is connected to two parallel conducting rails. A conducting wire of length ℓ (that is equal to separation between the rails) can slide on the rails without friction. The wire has mass m. It is projected with a velocity v_0 parallel to the rails (see Figure). Neglect self inductance and resistance of the loop.

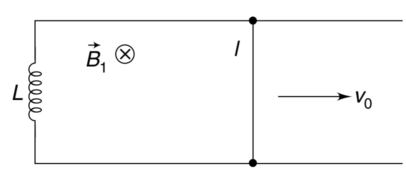
(a) Find velocity of the wire as a function of time.

(b) Write current through the wire at time t.

(c) Find speed of the wire as a function of its displacement.

(d) Is the current in the conductor zero when it stops? If no, find this current.

(e) Will the conductor move after it stops?

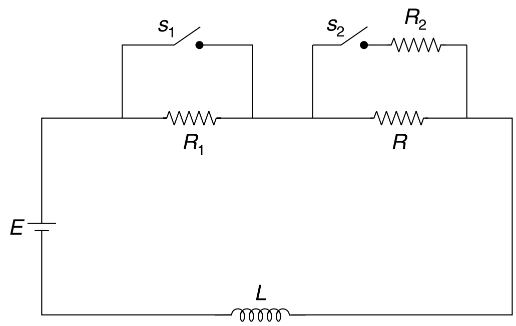


Q. 66: In the circuit shown in Figure, switch S_1 is kept closed and S_2 open for a long time. Resistance $R_1 = 1000$R and $R_2 = 10^{-3}$R.

(a) Switch S_1 is opened at time $t = 0$. Write the current and potential drop across R_1 immediately after S_1 is opened. What will be value of current after a long time?

(b) Switch S_2 is closed (S_1 is already closed since long) at time $t = 0$. Write current and potential drop across

R_2 immediately after this operation. Write current through the inductor as a function of time.



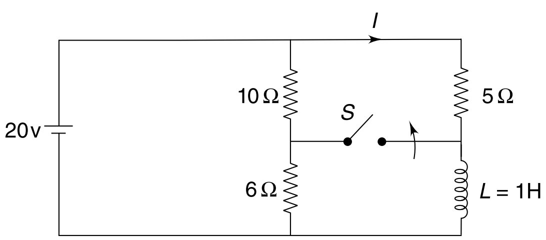
Q 67: A ring is made of a nearly superconducting material. Inductance of the ring is $L = 0.5$ H. A current allowed to decay in the ring was observed to remain constant for a month. The instrument used to measure current could detect any change if it is greater than 1%. Estimate the resistance of the ring considering it as a LR circuit with decaying current.

Q. 68: In the circuit shown, the switch '*S*' has been closed for a long time and then opens at $t = 0$.

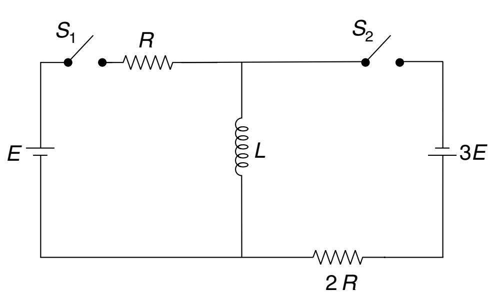
(a) Find the current through the inductor just before the switch is opened.

(b) Find the current I a long time after the switch is opened.

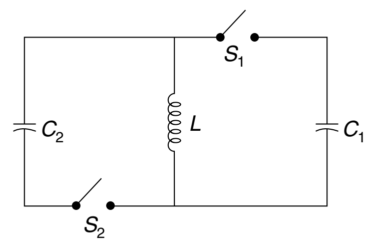
(c) Find current I as a function of time after the switch is opened. Also write the current through the cell as a function of time.



Q. 69: In the circuit shown, switch S_2 is open and S_1 is closed since long. Take $E = 20$ V, $L = 0.5$ H and $R = 10\ \Omega$. Find the rate of change of energy stored in the magnetic field inside the inductor, immediately after S_2 is closed.



Q. 70: The circuit shown in the figure is used to transfer energy from one capacitor to another. Initially capacitor of capacitance $C_1 = C_0$ is charged to a potential difference of V_0. Switch S_1 is closed at time $t = 0$. After some time S_1 is opened and S_2 is closed simultaneously. At time $t = T$, S_2 was opened and it was found that the potential difference across capacitor of capacitance $C_2 = \frac{C_0}{9}$ was $3V_0$. Find the smallest possible value of time T. The coil has inductance L. Assume no resistance.

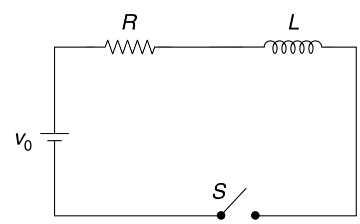


Q. 71: A pure inductor coil having inductance L is connected to a resistance R and a cell of emf V_0 as shown in the figure. Switch '*S*' is closed at $t = 0$.

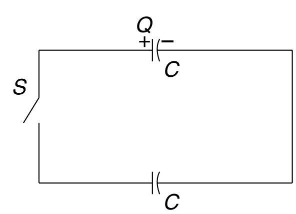
(a) Plot the variation of voltage across the resistance and the inductance as a function of time

(b) Find the time (t_1) when the two curves, obtained in (a) intersect.

(c) A student decides to start counting time from the instant the current becomes half its maximum value. Show the graphical plot of current vs time as obtained by this student.



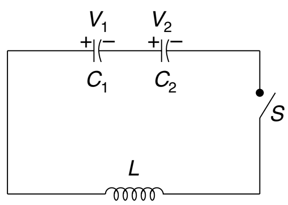
Q. 72: The circuit shown in figure has two identical capacitors one of which carries a charge Q and the other is having no charge. Switch S is closed. Find the maximum value of current in the circuit if self inductance of the loop is L and each capacitor has capacitance C. Neglect resistance of the loop.



Q. 73: The capacitors shown in the circuit have capacitance $C_1 = C$ and $C_2 = 3C$ and they have been charged to potentials $V_1 = 2V_0$ and $V_2 = 3V_0$ respectively. Switch S is closed to connect them to the inductor L.

(a) Find the maximum current through the inductor

(b) Find potential difference across C_1 and C_2 when the current in the inductor is maximum.

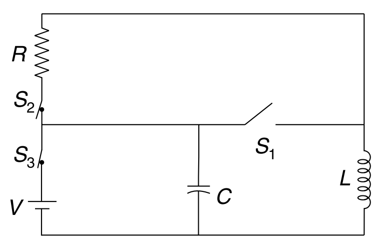


Q. 74. In the circuit shown in figure S_1 is open and, S_2 and S_3 are closed. The circuit is in steady state. At time $t = 0$, switch S_1 is closed and S_2 and S_3 are opened simultaneously.

$V = 100$ volt, $R = 10\ \Omega$, $C = 100\ \mu$F, $L = 0.03$ H

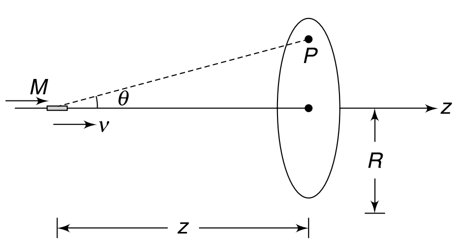
(a) Find the maximum charge that will appear on the capacitor at any time.

(b) Find the time at which the charge on the capacitor will become zero for the first time.



LEVEL 3

Q. 75: A short bar magnet having magnetic dipole moment M is moving along the axis of a fixed conducting (non magnetic) ring of radius R. The axis of the ring is along z direction.

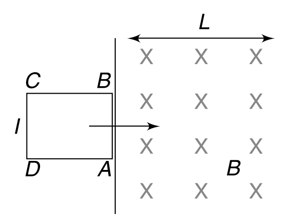


(a) Write the z component of magnetic field due to the magnetic dipole at a point P in the plane of the ring, at the instant the magnet is at a distance z from the centre of the ring. Position of point P can be defined in terms of angle θ as shown.

(b) Write the magnetic flux through the ring due to the magnetic field produced by the magnet as a function of z.

(c) Write the magnitude of emf induced in the ring at the instant shown if speed of the magnet at the moment is v.

Q. 76: A region of width L contains a uniform magnetic field B directed into the plane of the figure. A square conducting loop of side length ℓ ($< L$) is kept with its side AB at the boundary of the field region (see Figure). The loop is pushed into the field region with a speed such that it just manages to exit the field region. Calculate the time needed for the entire loop to enter the field region after it is pushed. Mass and resistance of the loop is M and R respectively.

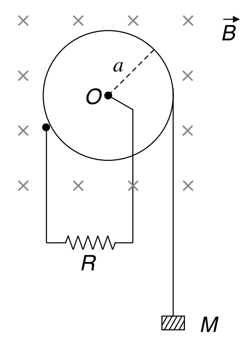


Q. 77: A metallic pulley is in the shape of a disc of radius a. It can rotate freely about a horizontal axis passing through its centre. The moment of inertia of the pulley about this axis is I. A light string is tightly wrapped around the pulley with its one end connected to a block of mass M. The centre of the pulley and its circumference are connected to a resistance R as shown. The contact of resistance at the circumference does not cause any friction. A uniform magnetic field B is switched on which is parallel to the axis of rotation of the pulley (see Figure). The mass M is allowed to fall. Assume that resistivity of the material of the pulley is negligible.

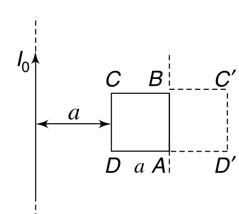
(a) Find the acceleration of the block of mass M at the instant its velocity becomes v_0.

(b) Assuming that the block can fall through a large distance, calculate the terminal speed (v_T) that it will acquire.

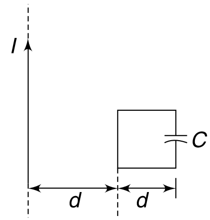
(c) Find the rate of change of kinetic energy of the pulley at the instant speed of the falling block is $\frac{v_T}{2}$.



Q.78: Figure shows a square conducting frame and a long wire-both lying in the same plane. The side length of the square loop is 'a' and it is at a distance 'a' from the long wire which is having a steady current I_0. The inductance and resistance of the square loop are L and R respectively. The loop is turned by 180° about its side AB so as to bring it to final position $ABC'D'$ at rest. Calculate the net charge that flow past a side of the loop.



Q.79: A square loop of side length d has a capacitor of capacitance C and the resistance of the loop is R. In the plane of the loop there is a long straight current carrying wire having current I. The distance of the straight wire from the loop is d as shown in figure. The current in the straight wire is made to grow with time as $I = \alpha t$ where $\alpha = 2\ \text{As}^{-1}$. Neglect self inductance of the loop.

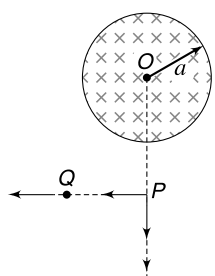


(a) Find the charge on the capacitor as a function of time (t).

(b) Find the heat generated in the loop as a function of time.

(c) Who supplies energy for heat dissipation?

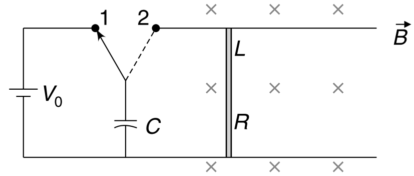
Q.80: There exists a uniform magnetic field perpendicular to the plane of the figure in a cylindrical region of radius a. The magnetic field is increasing at a constant rate of αTs^{-1}. A particle having charge q is at a point P outside the field region. The particle is slowly moved to infinity. Calculate work done by the external agent on the particle if-



(a) the particle is moved in radial direction OP.

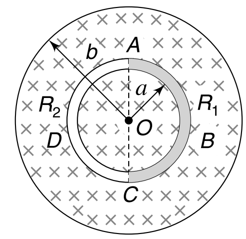
(b) the particle is moved in a direction perpendicular to OP along PQ.

Q.81: The Figure shows an electromagnetic gun. A bar of mass m, resistance R and length L is free to slide on two smooth rails separated by a distance L. A uniform magnetic field B is present perpendicular to the plane of the figure. A capacitor of capacitance C is charged using a battery of emf V_0 by placing switch (S) at 1. To fire the gun (i.e., to impart a kinetic energy to the rod) the switch is shifted to position 2 after the capacitor is fully charged. The rails end abruptly at the point where the speed of the rod becomes maximum. The efficiency of the gun can be defined as the kinetic energy imparted to the bar divided by the energy spent by the battery while charging the capacitor. Calculate the efficiency of the gun. Neglect self inductance of the circuit.



Q.82: A thin solenoid is made of a large number of turns of very thin wire tightly wound in several layers. The radii of innermost and outermost layers are a and b respectively and the length of the solenoid is L ($L >> a, b$). The total number of turns is N. Calculate the self inductance of the solenoid. Neglect edge effects.

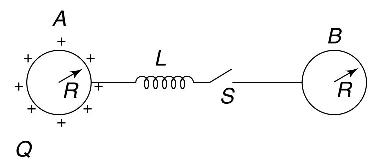
Q.83: A uniform magnetic field B exists perpendicular to the plane of the Figure in a circular region of radius b with its centre at O. A circular conductor of radius a ($< b$) and centre at O is made by joining two semicircular wires ABC and ADC. The two segments have same cross section but different resistances R_1 and R_2 respectively. The magnetic field in increased with time and there is an induced current in the conductor.



(a) Find the ratio of electric fields inside the conductors ABC and ADC.

(b) Explain why the electric field in two conductors is different despite the fact that the magnetic field is symmetrical.

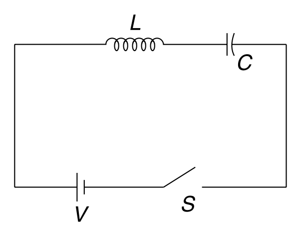
Q.84: Two conducting sphere of radius R are placed at a large distance from each other. They are connected by a coil of inductance L, as shown in the figure. Neglect the resistance of the coil. The sphere A is given a charge Q and the switch 'S' is closed at time $t = 0$. Find charge on sphere B as a function of time. At what time charge on B is $\dfrac{Q}{2}$?



Q.85: In the circuit shown in the figure, switch S is closed at time $t = 0$.

(a) Write current in the circuit and charge on capacitor as a function of time. Draw the graphical plot for the same.

(b) Find maximum charge on the capacitor. What is potential difference across the inductor when charge on the capacitor is maximum?

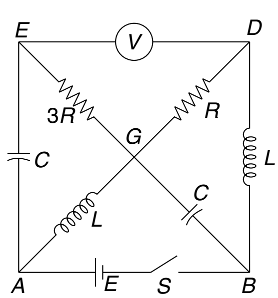


Q.86: In the circuit shown, switch S is kept closed and the circuit is in steady state.

(a) Find reading of the ideal voltmeter

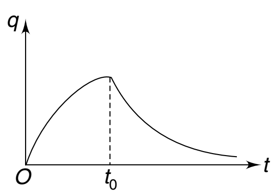
(b) Now the switch is opened. Find the reading of the voltmeter immediately after the switch is opened.

(c) Fid the heat dissipated in resistance R after the switch is opened.



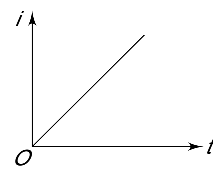
ANSWERS

1. Bulb A
2. 2F
3. Zero
4. (b) Zero
6. True
7. (a) Zero (b) $\frac{1}{2} B\omega L^2$
8. (i) No (ii) $\frac{1}{4} mV_0^2$
9. $\frac{4\pi^2 B^2 r^2 v^2}{R}$
10. $\omega = 1667 \text{ rads}^{-1}$
11. (a) $\frac{BvS}{\rho}$ (b) $g - \frac{B^2 v}{\rho d}$
12. $\frac{a^4 B^2 \omega^2}{2R}$
13. $\mu_0 i_0 NnA \cos(2\omega t)$
14. $\frac{\pi N}{3}(a^2 + b^2 + ab)\frac{dB}{dt}$
15. 0.3 kg
16. (a) $\frac{B}{2}\sqrt{3gL^3(1 - \cos\theta_0)}$

    (b) In presence of field the oscillations will die
17. $\frac{18B}{5\pi}\frac{\sqrt{3aR}}{\lambda}$
18. 

19. (a) Zero (b) $\frac{a^2\theta}{2}\frac{dB}{dt}$
20. (a) Zero in both BO and BC

    (b) Zero in BO and $\frac{a\alpha}{2\lambda}$ in BC
21. (a) Induced electric field applies the force

    (b) Magnetic force.
22. $\frac{\alpha\mu_0 anV\tau}{8R\Delta t}$
23. Current decreases.
24. Zero
25. 8 : 1
26. $2I, I, I, 3LI^2$
27. Statement is true.
28. 

29. $\frac{2\varepsilon}{3L}$
30. $i = \frac{Va^2}{8\rho r}[1 - e^{-t/\tau}]$

    Where $\tau = \frac{\mu_0 a^2}{8\rho}\left[\ln\left(\frac{16r}{a}\right) - \frac{7}{4}\right]$
31. $R = \sqrt{\frac{L}{C}}$
32. $\frac{\mu_0 I^2}{16\pi}$
33. $i = \frac{k\pi\mu_0 n_1 n_2 r_2^2 l}{R}(1 - e^{-t/\tau})$ where $\tau = \frac{\pi\mu_0 n_2^2 r_2^2 l}{R}$
34. $\frac{1}{2}\sqrt{\frac{3L}{C}}\frac{Q_0}{N}$
35. (a) E (b) zero
36. $L = n\frac{2\pi}{k}$; $n = 1, 2, 3, \ldots$
37. The source that is driving current through the rod.
38. (a) 3.7A (b) 35 μA
39. (a) $v = \frac{mgR\sin\alpha}{2B^2\ell^2}$ (b) No difference
40. $\frac{1}{2} B\omega L^2 \sin^2\theta$; O is positive
41. $\frac{\mu_0 I}{2\pi\sin\theta}\sqrt{3gL(1 - \cos\theta)}$
42. (i) $\frac{1}{2} q_0 B_0 \omega_0 L^2$ (ii) $\frac{1}{2} q_0\left[B_0\omega_0 L^2 - \frac{q_0}{C}\right]$
43. (a) $\theta = \theta_0 \cos\omega t$ where $\omega = \left[\frac{6Mg}{4ML + 3B^2L^3C}\right]^{1/2}$

    (b) Oscillations die after some time.
44. $T = 2\pi\sqrt{\frac{M + \epsilon_0 AB^2 \cdot d}{k}}$
45. $x = \frac{mg}{k}(1 - \cos\omega t)$ where $\omega = \sqrt{\frac{k}{m + B^2L^2C}}$

46. (a) $\dfrac{mgR}{B^2L^2}$

(b) $v = \dfrac{mgR}{B^2L^2}\left[1 - e^{-\frac{B^2L^2t}{R(m + B^2L^2C)}}\right]$

47. (a) $\dfrac{\mu_0 \pi a^2 \beta}{\ell}$ (b) $I = I_0 e^{-\frac{2\rho t}{\mu_0 a d}}$

48. $L = \dfrac{\mu_0 x}{\pi}\,\ell n\left(\dfrac{d - a}{a}\right)$

50. $M = \dfrac{\mu_0 N h\,\ell n\,(b/a)}{2\pi}$

51. $\dfrac{R_1 R_2 Q_0}{V}$

52. $\dfrac{\mu_0 a \alpha}{4\pi}$

53. $\omega = \dfrac{2qR^2B_0}{md^2}$

54. 1 s, end A

55. No change

56. $\dfrac{L\alpha}{4}\sqrt{4R^2 - L^2}$

57. (a) zero (b) $\dfrac{\pi R^2}{4}\alpha$

58. $I = \sqrt{I_0^2 + \dfrac{n^2 q^3 \beta}{2\pi^2 R^2 m}}$

59. $\dfrac{kR}{4}$ perpendicular to OO_1

60. $V_{CA} = \dfrac{\ell^2\alpha}{2(\sqrt{2} + 1)}$; $V_{BA} = \dfrac{\ell^2\alpha}{4(\sqrt{2} + 1)}$

61. (a) 10 V/m (b) $\dfrac{2\pi}{3}$ kJ

62. $\dfrac{\mu_0 I^2 N^2}{2\ell^2}\,[3A_1 - 2A_1 x + A_2 \ell]$

63. $\dfrac{\mu_0 N^2 A_1 v I}{\ell^2}$; Same as in the outer coil

64. $F = \dfrac{\mu_0 N^2 I^2 A_1}{\ell^2}$

65. (a) $v = v_0 \cos \omega t$

(b) $i = v_0\sqrt{\dfrac{m}{L}}\,\sin \omega t$ where $\omega = \dfrac{B\ell}{\sqrt{mL}}$

(c) $v = \sqrt{v_0^2 - \dfrac{B^2\ell^2}{mL}x^2}$

(d) No, $v_0\sqrt{\dfrac{m}{L}}$

(e) Yes

66. (a) Current $= \dfrac{E}{R}$, $V_{R1} = 1000\,E$, $I_\infty = \dfrac{E}{1001R} \simeq 10^{-3}\dfrac{E}{R}$

(b) At $t = 0^+ : i_0 = \dfrac{E}{R}$, $V_{R_2} = 10^{-3}E$,

$i = \dfrac{1000E}{R}\left[1 - e^{-\frac{Rt}{1000L}}\right]$

67. $1.9 \times 10^{-9}\ \Omega$

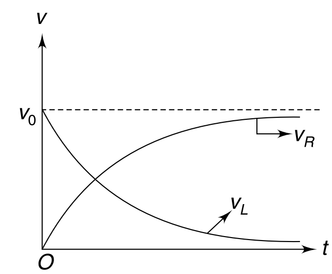
68. (a) $6A$ (b) $4A$

(c) $I = 4 + 2e^{-5t}$, current through cell $= \dfrac{21}{4} + 2e^{-5t}$

69. -40 J/s

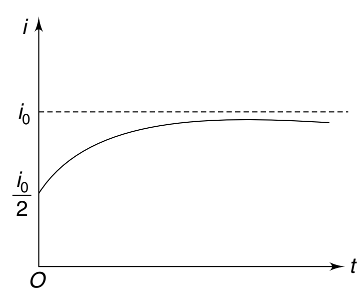
70. $T_{min} = \dfrac{2\pi}{3}\sqrt{LC_0}$

71. (a)



(b) $t_1 = \dfrac{L}{R}\,\ell n 2$

(c)



72. $i_{max} = \dfrac{Q}{\sqrt{2LC}}$

73. (a) $\dfrac{5V_0}{2}\sqrt{\dfrac{3C}{L}}$ (b) $\dfrac{7V_0}{4}, \dfrac{7V_0}{4}$

74. (a) 0.02 C (b) 0.9 ms

75 (a) $B_z = \frac{\mu_0}{4\pi r^3}[2\cos^2\theta - \sin^2\theta]$

(b) $\phi = \frac{\mu_0 MR^2}{2(R^2 + z^2)^{3/2}}$

(c) $E_{in} = \frac{3\mu_0 MR^2 vz}{2(R^2 + z^2)^{5/2}}$

76. $\frac{MR\ell n2}{B^2\ell^2}$

77. (a) $\frac{Mga^2 - \frac{B^2a^4v_0}{4R^2}}{I + Ma^2}$ (b) $\frac{4MgR^2}{B^2a^2}$

(c) $\frac{IM^2g^2R^2}{B^2a^2(I + Ma^2)}$

78. $\frac{\mu_0 I_0 a}{2\pi R}\ell n3$

79. (a) $q = \varepsilon C(1 - e^{-t/RC})$

(b) $H = \frac{\varepsilon^2 C}{2}\left(1 - e^{-\frac{2t}{RC}}\right)$ where $\varepsilon = \frac{\mu_0 \alpha \cdot d\,\ell n2}{2\pi}$

80. (i) zero (ii) $\frac{\pi}{4}qa^2\alpha$

81. $\frac{mB^2L^2C}{2(m + B^2L^2C)^2}$

82. $\frac{\mu_0 \pi N^2(3a^2 + 2ab + b^2)}{6L}$

83. (a) $\frac{E_1}{E_2} = \frac{R_1}{R_2}$

84. $q = \frac{Q}{2}\left[1 - \cos\left(\sqrt{\frac{2K}{LR}}\,t\right)\right]$; $t = \frac{\pi}{2}\sqrt{\frac{LR}{2K}}$ where $K = \frac{1}{4\pi\epsilon_0}$

85. $q = 2CV\sin^2\left(\frac{\omega t}{2}\right)$, $i = i_0 \sin\omega t$ Where $\omega = \frac{1}{\sqrt{LC}}$

86. (a) E

(b) $2E$ with polarity reversed

(c) $\frac{1}{2}E^2\left(\frac{L}{R^2} + C\right)$

SOLUTIONS

2. If number of turns is doubled the total flux doubles and hence emf induced in the coil doubles. But this doubles the resistance also so the current wouldn't change. The force doubles since number of turns doubled.

3. The lines of magnetic field produced by the falling rod lie in horizontal plane. The flux through the area bound by the loop is zero at any instant.

4. (a) Moving magnet inside a toroid does not cause a change in flux through any turn.

(b) Neither horizontal nor vertical component of earth's magnetic field is intercepted by the wire.

5. (i) Coil creates a magnetic field that is time changing (since A is connected to an ac source). This causes the flux through B to change. Emf is induced in B.

(ii) With B moving away, the flux linked with B decreases. Thus emf induce in it also decreases.

(iii) The magnetic field strength at B will get reduced substantially due to eddy current in the Cu plate. The bulb will get dimmer.

6. When angle between the normal to the coil and $\vec{B}$ is θ.

$$\phi = BA\cos\theta$$

and $$-\frac{d\phi}{dt} = BA\sin\theta\cdot\frac{d\theta}{dt}$$

∴ $$\varepsilon_{in} = BA\omega\sin\theta \qquad \left[\frac{d\theta}{dt} = \omega\right]$$

∴ ε_{in} is maximum when $\theta = 90°$

Flux $\phi = BA\cos 90° = 0$ at this instant.

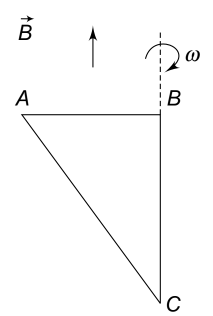
7. (a) There is no flux change in loop ABC.

∴ Induced emf = 0

Hence there is no current and no torque.

(b) $\varepsilon_{AB} = \varepsilon_{AC} = \frac{1}{2}B\omega L^2$

[End B is positive]



8. (i) The current induced in the loop will cause a magnetic force to act on both the wires. This force is equal and opposite on the two wires. Hence, momentum of the system will remain conserved.

(ii) Due to magnetic force the wire on the left speeds up and that on the right slows down.

When speed of both becomes same, emf induced in both of them will be equal. This will make the net emf in the loop equal to zero. There will be no current after this.

$\therefore$ Final speed of both wires is $V = \dfrac{V_0}{2}$

$$\therefore \quad KE = \frac{1}{2}M\left(\frac{V_0}{2}\right)^2 \times 2 = \frac{1}{4}MV_0^2$$

9. Flux: $\phi = B \cdot \pi r^2$

Induced emf: $\varepsilon = \left|\dfrac{d\phi}{dt}\right| = 2\pi Br\dfrac{dr}{dt} = 2\pi Brv.$

Induced current: $I = \dfrac{\varepsilon}{R} = \dfrac{2\pi Brv}{R}$

Rate of work done = Rate of heat dissipated in Joule heating = I^2R

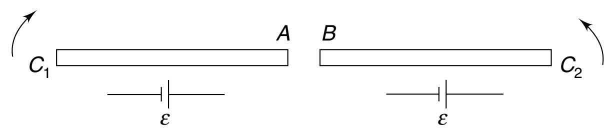
$$= \frac{4\pi^2B^2r^2v^2}{R}$$

10. The emf induced in both rods is $\varepsilon = \dfrac{1}{2}B\omega L^2$ with polarity as shown in the figure.

Since C_1 and C_2 are at same potential, the difference in potential between the tips A and B will be

$$V_{AB} = 2\varepsilon = B\omega L^2$$

In the position shown in the Figure, the gap between the tips A and B is smallest (= 0.001 m). A spark will jump across the gap if



$$\frac{V_{AB}}{0.001} \geq 3 \times 10^6$$

$$\Rightarrow \quad B\omega L^2 \geq 3 \times 10^3$$

$$\Rightarrow \quad \omega \geq \frac{3 \times 10^3}{5 \times (0.6)^2} = 1667 \text{ rads}^{-1}$$

11. (a) Induced emf in a small element of length dl is $d\varepsilon = Bvdl$

$$d\varepsilon = Bvdl$$

Total emf in the ring is $\varepsilon = Bv\int dl = BvL$

Where L is circumference of the ring.

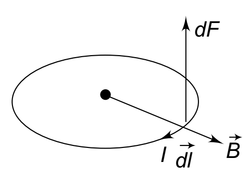
Resistance of the ring $R = \dfrac{\rho L}{S}$

$\therefore$ Induced current $I = \dfrac{\varepsilon}{R} = \dfrac{BvS}{\rho}$

The current will be clockwise (seen from top).

(b) Magnetic force on each small element of the ring is vertically upward.

$$\therefore \quad F_B = IB\int dl = IBL$$



$$\Rightarrow \quad F_B = \frac{B^2vSL}{\rho}$$

Mass of the ring $m = dLS$

$$\therefore \quad ma = mg - F_B$$

$$\Rightarrow \qquad dLS\,a = dLS\,g - \frac{B^2 vSL}{\rho}$$

$$\Rightarrow \qquad a = g - \frac{B^2 v}{\rho d}$$

12. Let the area vector of the loop be parallel to $\vec{B}$ at time $t = 0$.

Angle between the two vectors at time t will be $\theta = \omega t$.

Flux through the loop at time t will be

$$\phi = a^2 B\cos(\omega t)$$

$\therefore$ Induced emf in the loop is

$$\varepsilon = -\frac{d\phi}{dt} = a^2 B\omega \sin(\omega t)$$

Current in the loop $\quad I = \dfrac{\varepsilon}{R} = \dfrac{a^2 B\omega}{R}\sin(\omega t)$

Rate of heat dissipation

$$P = I^2 R = \frac{a^4 B^2 \omega^2}{R}\sin^2(\omega t)$$

Average rate of heat dissipation in one rotation

$$P_{av} = \frac{a^4 B^2 \omega^2}{R} < \sin^2(\omega t) >$$

$$= \frac{1}{2}\frac{a^4 B^2 \omega^2}{R}$$

Note: Average of $\sin^2\theta$ in one cycle (i.e., $0 \le \theta \le 2\pi$) is $\frac{1}{2}$.

13. Field inside the solenoid at time t is

$$B = \mu_0 ni = \mu_0 ni \sin\omega t$$

Let at time $t = 0^+$, the area vector of the coil be in the direction of $\vec{B}$.

At time t, the area vector has rotated by $\theta = \omega t$.

$\therefore$ Flux linked with one turn of coil is

$$\phi = BA\cos(\omega t) = \mu_0 n i_0 A \sin\omega t \cdot \cos\omega t$$

$$= \frac{1}{2}\mu_0 i_0 nA\sin(2\omega t)$$

$\therefore$ Emf induced in the coil is

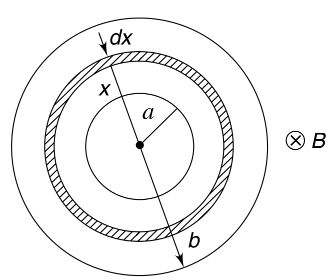
$$\varepsilon = N\frac{d\phi}{dt} = \mu_0 i_0 nNA\cos(2\omega t)$$

14. Consider a circular strip of radius x and width dx.

Number of turns in the strip $\quad dN = \dfrac{N\,dx}{(b-a)}$

Flux linked with a circular loop of radius x is $\phi = B\cdot\pi x^2$

$$\Rightarrow \qquad \frac{d\phi}{dt} = \pi x^2 \frac{dB}{dt}$$



Emf induced in coil of width dx will be

$$d\varepsilon = (dN)\left(\frac{d\phi}{dt}\right) = \frac{N\,dx}{b-a}\cdot\pi x^2\frac{dB}{dt}$$

$\therefore$ Emf induced in the complete coil is

$$\varepsilon = \int d\varepsilon = \frac{\pi N}{(b-a)} \frac{dB}{dt} \int_a^b x^2 dx$$

$$= \frac{\pi N}{(b-a)} \frac{dB}{dt} \cdot \frac{(b^3 - a^3)}{3}$$

$$= \frac{\pi N}{3} (a^2 + b^2 + ab) \frac{dB}{dt}$$

15. When current in the circuit is $I = 4A$, magnetic force on the rod is

$$F = IlB = 4 \times 0.5 \times 1.5 = 3 \text{ N}$$

The weight of the rod must exceed F to blow out the fuse.

$$\therefore \quad mg \geq 3$$

$$\therefore \quad m \geq \frac{3}{10} = 0.3 \text{ kg}$$

16. (a) Angular speed of the rod will be maximum when it is vertical.

Apply energy conservation between the extreme position of the rod and its vertical position.

$$\frac{1}{2} I\omega^2 = Mg \frac{L}{2} (1 - \cos\theta_0)$$

$$\Rightarrow \quad \frac{1}{3} ML^2\omega^2 = MgL(1 - \cos\theta_0)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{L}(1 - \cos\theta_0)}$$

$\therefore$ From the concept of motional emf

$$\varepsilon = \frac{1}{2} B\omega L^2$$

$$\varepsilon = \frac{B}{2} \sqrt{3gL^3(1 - \cos\theta_0)}$$

(b) If the rod has finite thickness, eddy currents will be induced in it. Energy will get dissipated as heat. The amplitude of oscillation will decrease and the rod will stop oscillating after some time.

17. Speed of the rod when it has travelled a distance $x = \frac{R}{2}$ is

$$v = \sqrt{2ax} = \sqrt{aR}$$

Length of rod inside the ring $\quad L = AC = 2\sqrt{R^2 - \frac{R^2}{4}} = \sqrt{3}R$

Emf induced in the rod at this instant is

$$E = BvL = B \cdot \sqrt{aR}\sqrt{3} \cdot R = \sqrt{3a}\, B \cdot R^{3/2}$$

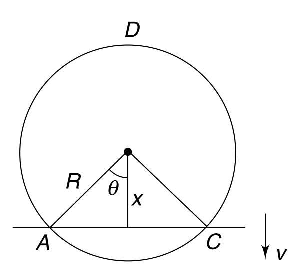
$$\sin\theta = \frac{x}{R} = \frac{1}{2} \quad \Rightarrow \quad \theta = 30°$$

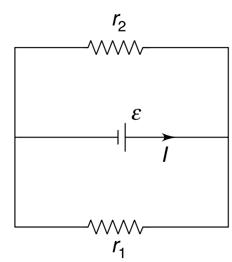
$\therefore$ Length of arc $\quad ABC = \frac{2\pi R}{6} = \frac{\pi R}{3}$

Length of arc $\quad ADC = \frac{5}{6} \cdot 2\pi R = \frac{5\pi R}{3}$

Resistance of arc ABC; $\quad r_1 = \frac{\pi R}{3} \lambda$

Resistance of arc ADC; $\quad r_2 = \frac{5\pi R}{2} \lambda$





Equivalent resistance $\quad r = \frac{r_1 r_2}{r_1 + r_2} = \frac{5\pi}{18} \lambda R$

∴ Current through the rod $$I = \frac{E}{r} = \frac{18B}{5\pi}\frac{\sqrt{3aR}}{\lambda}$$

18. Flux through the loop $$\phi = BA$$

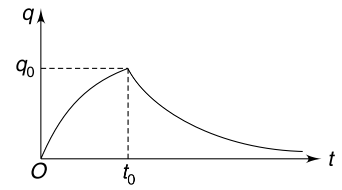
Emf induced $$|\varepsilon| = \frac{d\phi}{dt} = A\frac{dB}{dt}$$

For $0 < t < t_0$

$$\varepsilon = A\frac{B_0}{t_0} = \text{a constant}$$

For $t > t_0$

$$\varepsilon = 0$$



The charge acquired by the capacitor in time t_0 is

$$q_0 = C\varepsilon(1 - e^{-2}) \qquad [\because \; t_0 = 2\tau]$$

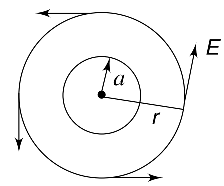
After this there is no emf in the loop and the capacitor begins to discharge. Therefore, the graph will be as shown in the figure.

19. (a) There is no flux linked to the loop. Hence no emf is induced.

(b) Electric field at a distance r from centre O is given by

$$E2\pi r = \pi a^2 \frac{dB}{dt}$$

$$\Rightarrow \qquad E = \frac{a^2}{2r}\frac{dB}{dt}$$



Emf induced in arc AB

$$\varepsilon_{AB} = \left(\frac{a^2}{2d}\frac{dB}{dt}\right)(d\theta) = \frac{a^2\theta}{2}\frac{dB}{dt}$$

This does not depend on d. This implies that same emf is induced in CD. Hence there is no emf in loop $ABCD$.

Note: Electric field lines are normal to arms BC and AD. There is no emf in these arms.

20. (a) No emf is induced in the loop. There will be no current anywhere. [But there is an emf between B and A. There is an emf between B and D.]

(b) Emf is induced in the ring. However, there is no emf induced in BA and CD [because induced electric field has circular field lines which are normal to BC and CD].

$$\varepsilon_{\text{in}} = \pi a^2 \cdot \frac{dB}{dt} = \pi a^2 \cdot \alpha$$

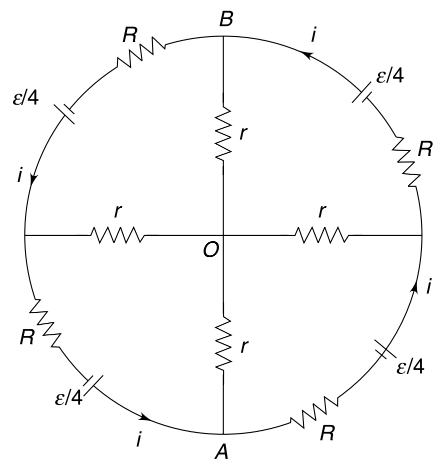
The effective circuit is as shown in figure.

In the figure- $R = \lambda\frac{\pi a}{2}$ and $r = \lambda a$

$$iR = \frac{\varepsilon}{4}$$

$$\Rightarrow \qquad i\frac{\lambda\pi a}{2} = \frac{\pi a^2\alpha}{4}$$

$$i = \frac{a\alpha}{2\lambda}$$



22. Rate of change of current $$\frac{\Delta I}{\Delta t} = \frac{\frac{V}{R} - \frac{V}{2R}}{\Delta t} = \frac{V}{2R\Delta t}$$

Flux through ring

$$\phi = \pi a^2(\mu_0 nI)$$

$$\frac{d\phi}{dt} = \mu_0\pi a^2 n\frac{dI}{dt} = \mu_0\pi a^2 n\frac{V}{2R\Delta t}$$

∴ Induced electric field in the ring can be calculated as

$$E_{in} \cdot 2\pi(2a) = \frac{\pi\mu_0 a^2 nV}{2R\Delta t}$$

$$\therefore \quad E_{in} = \frac{\mu_0 anV}{8R\Delta t}$$

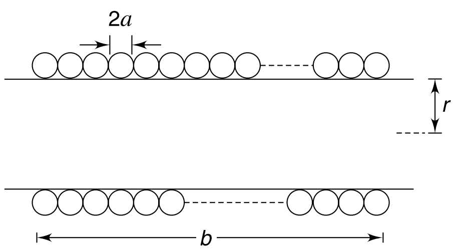
$$\therefore \quad v_d = \frac{eE_{in}}{m}\tau = \frac{\alpha\mu_0 anVt}{8R\Delta t}$$

23. Eddy currents are induced in the metal plate due to time changing magnetic field created by the current in A. As per Lenz's law the field produced by the eddy current will oppose the cause of induction. The coil B now faces the resultant field due to coil A and that due to the eddy current. Therefore, the rate of change of flux through B gets reduced. Hence current in B reduces.

24. When a current (I) is given to the conducting ring, a current is induced in the superconducting ring such that it cancels the flux due to current I. Thus flux linked with the ring is always zero. Hence self inductance is always zero.

25. Length of wire $l = \dfrac{\text{volume}\,(V)}{\pi a^2}$

Winding is as shown in the Figure.



Number of turns $\quad N = \dfrac{l}{2\pi r}$

Length of the helix $\quad b = 2a \cdot N = \dfrac{al}{\pi r}$

Number of turns per meter length $\quad n = \dfrac{1}{2a}$

$\therefore$ Self inductance $\quad L = \pi\mu_0 n^2 r^2 b$

$$= \pi\mu_0 \cdot \left(\frac{1}{2a}\right)^2 \cdot r^2 \cdot \frac{al}{\pi r}$$

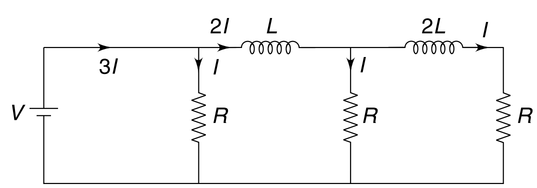
$$= \frac{1}{4}\mu_0 r \frac{l}{a} = \frac{1}{4}\mu_0 r \frac{V}{a(\pi a^2)}$$

$$= \frac{\mu_0 r}{4\pi}\frac{V}{a^3}$$

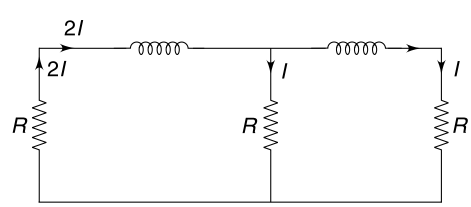
$$\therefore \quad L \propto \frac{1}{a^3}$$

$$\therefore \quad \frac{L_1}{L_2} = \left(\frac{2a}{a}\right)^3 = \frac{8}{1}$$

26. In steady state the current in different branches are as shown.



Immediately after opening the switch, the current through inductors will not change. Current will be as shown below.



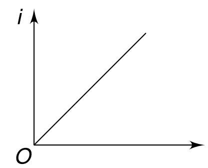
Note that current through a resistor can change abruptly.

Heat loss = Energy stored in magnetic field

$$= \frac{1}{2}L(2I)^2 + \frac{1}{2}2L(I)^2 = 3LI^2$$

28. At time t after closing the switch

$$V - L\frac{di}{dt} = 0$$



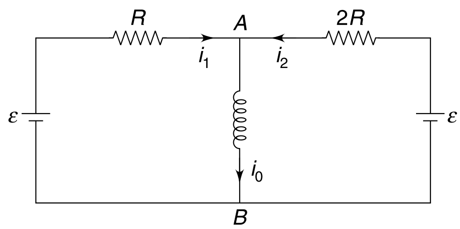
$$\Rightarrow \quad \int_0^i di = \frac{V}{L}\int_0^i dt$$

$$\Rightarrow \quad i = \frac{V}{L}t$$

The current will approach to infinity as $t \to \infty$. In practice, this never happens as there indeed is some resistance in the circuit.

29. In steady state the current through the inductor is $i_0 = \frac{\varepsilon}{2R}$.

The current through the inductor will not change immediately after the switch is closed.



Let $\quad V_{AB} = V$

In left loop $\quad Ri_1 + V = \varepsilon \quad$ ...(i)

In right loop $\quad 2Ri_2 + V = \varepsilon \quad$ ...(ii)

From (i) and (ii)

$$i_1 + i_2 = \frac{\varepsilon - V}{R} + \frac{\varepsilon - V}{2R}$$

Since $\quad i_1 + i_2 = i_0 = \frac{\varepsilon}{2R} \quad \left[\text{Because } i_0 = \frac{\varepsilon}{2R}\right]$

$$\therefore \quad \frac{\varepsilon - V}{R} + \frac{\varepsilon - V}{2R} = \frac{\varepsilon}{2R}$$

$$\Rightarrow \quad V = \frac{2\varepsilon}{3}$$

$$\Rightarrow \quad L\frac{di}{dt} = \frac{2\varepsilon}{3} \Rightarrow \frac{di}{dt} = \frac{2\varepsilon}{3L}$$

At this instant the potential difference across R is $\varepsilon - \frac{2\varepsilon}{3} = \frac{\varepsilon}{3}$. This potential difference will eventually become ε. It means current is increasing.

$\frac{di}{dt}$ is positive.

30. Self inductance of the loop is

$$L = \mu_0 r\left[\ln\left(\frac{16r}{a}\right) - \frac{7}{4}\right]$$

Resistance of the loop is $\quad R = \frac{\rho \cdot 2\pi r}{\pi\left(\frac{a}{2}\right)^2} = \frac{8\rho r}{a^2}$

The circuit is a simple $L - R$ circuit, hence

$$i = \frac{V}{R}[1 - e^{-t/\tau}] = \frac{Va^2}{8\rho r}[1 - e^{-t/\tau}]$$

Where $$\tau = \frac{L}{R} = \frac{\mu_0 a^2}{8\cdot\rho}\left[\ln\left(\frac{16r}{a}\right) - \frac{7}{4}\right]$$

31. Cell is connected in parallel to $R - L$ path and $R - C$ path.

Current through inductor and capacitor at time t after the switch is closed is

$$i_L = \frac{E}{R}\left[1 - e^{-\frac{Rt}{L}}\right]$$

$$i_C = \frac{E}{R}e^{-\frac{t}{RC}}$$

Current through the cell is

$$i = i_L + i_C = \frac{E}{R}\left[1 - e^{-\frac{Rt}{L}} + e^{-\frac{t}{RC}}\right]$$

This will be independent of time if $e^{-\frac{Rt}{L}} = e^{-\frac{t}{RC}}$

$\Rightarrow$ $$\frac{R}{L} = \frac{1}{RC} \Rightarrow R = \sqrt{\frac{L}{C}}$$

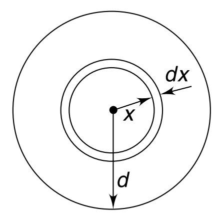
32. Let a = radius of circular area A

Current density $$j = \frac{I}{A} = \frac{I}{\pi a^2}$$

Magnetic field at a distance x from the axis $(0 < x < a)$ is given by Ampere's law as $B\cdot 2\pi x = \mu_0 j\cdot \pi x^2$

$\Rightarrow$ $$B = \left(\frac{\mu_0 j}{2}\right)x = \frac{\mu_0 I}{2\pi a^2}x$$

Energy density at radial distance x is $u_B = \frac{1}{2\mu_0}B^2 = \frac{\mu_0 I^2}{8\pi^2 a^4}x^2$



Magnetic energy in annulus region of thickness dx and length L is

$$dU = u_B\cdot 2\pi x\cdot dx\cdot L$$

$$= \frac{\mu_0 I^2 x^2}{8\pi^2 a^4}\cdot 2\pi x\cdot dx\cdot L = \frac{\mu_0 I^2 L}{4\pi a^4}x^3\cdot dx$$

$\therefore$ Energy stored in cylindrical region of radius a and length L is

$$U = \int dU = \frac{\mu_0 I^2 L}{4\pi a^4}\int_0^a x^3\cdot dx = \frac{\mu_0 I^2 L}{16\pi}$$

Energy per unit length is $= \frac{\mu_0 I^2}{16\pi}$

33. Mutual inductance $$M = \pi\mu_0 n_1 n_2 r_2^2 l$$

Emf induced in inner coil is

$$|\varepsilon_2| = M\frac{dI_1}{dt} = k\pi\mu_0 n_1 n_2 r_2^2 l$$

Thus a constant emf is induced in the inner coil which has a resistance R and self inductance

$$L_2 = \pi\mu_0 n_2^2 r_2^2 l$$

Therefore, current in it at time t will be $i = i_0(1 - e^{-t/\tau})$

Where $$i_0 = \frac{\varepsilon_2}{R} = \frac{k\pi\mu_0 n_1 n_2 r_2^2 l}{R}$$

And $$\tau = \frac{L_2}{R} = \frac{\pi\mu_0 n_2^2 r_2^2 l}{R}$$

34. $$Q = Q_0 \cos(\omega t) \quad \text{where} \quad \omega = \frac{1}{\sqrt{LC}}$$

When $$Q = \frac{Q_0}{2};\ \cos(\omega t) = \frac{1}{2} \qquad \text{...(i)}$$

Flux linked to the coil is $$\phi = LI = L\frac{dQ}{dt}$$

$\therefore$ $$|\phi| = LQ_0\omega|\sin(\omega t)|$$

When $$\cos(\omega t) = \frac{1}{2};\ \sin(\omega t) = \frac{\sqrt{3}}{2}$$

$\therefore$ $$|\phi| = \frac{\sqrt{3}}{2}LQ_0\omega$$

$$\text{Flux through each turn} = \frac{|\phi|}{N} = \frac{\sqrt{3}LQ_0\omega}{2N} = \frac{1}{2}\sqrt{\frac{3L}{C}}\frac{Q_0}{N}$$

35. (a) Long time after switch is placed in position 1, current through the circuit is $I = \dfrac{E}{R}$

Potential difference across R is $V_R = E$

When switch is moved to position 2, current (through L) cannot change instantly. Drop across R remains E. Capacitor is uncharged, therefore, emf induced in the inductor must be E.

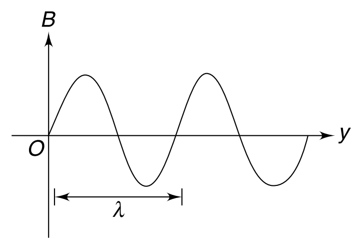
(b) When left in position 2 the LCR circuit will oscillate with decreasing energy. After a long time, the circuit will lose all its energy.

$\therefore$ There is no current after the switch is moved to position 3.

36. At any time (say at $t = 0$) the magnetic field varies sinusoidally with y (see Figure).

The pattern progresses to right with time.

The wavelength of this sinusoidal B is



$$\lambda = \frac{2\pi}{k}$$

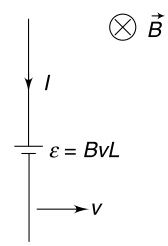
If the length (L) of the loop is such that there are exactly integral number of wave inside it, the total flux will remain constant (= 0) with time

$\Rightarrow$ $$L = n\lambda$$

$\Rightarrow$ $$L = n\frac{2\pi}{k};\ n = 1, 2, 3, \ldots$$

37. From the theory of motional emf we know that an emf will be induced in the rod with its upper end positive.

The source driving the current is doing work against this emf. Rate at which work is being performed is



$$\varepsilon I = BvLI$$

$\therefore$ Work done in time 't' is

$$W = BLI\int v\,dt = BLIx$$

38. (a) $PQRS$ is a discharging RC circuit

$\therefore$ $$q = q_0 e^{-t/RC}$$

And $$i = \frac{dq}{dt} = \frac{q_0}{RC}e^{-t/RC} = \frac{CV}{RC}e^{-t/RC}$$

$$i = \frac{V}{R}e^{-t/RC} = \frac{100}{10}e^{-t/RC} = 10e^{-t/RC}$$

Time constant $\tau = RC = 10 \times 20 \times 10^{-6} = 200\ \mu s$

$\therefore$ $$i = 10e^{-\frac{t}{200}} \text{ where } t \text{ is in } \mu s$$

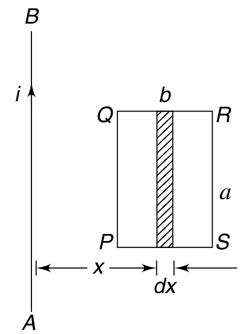
At $t = 200\ \mu s$

$$i = 10e^{-1} = 0.37 \times 10 = 3.7A$$

(b) Current in wire AB will have meaningful field inside the loop $PQRS$. All other arms of large loop are far away from $PQRS$.

Field at a distance x from AB is

$$B \simeq \frac{\mu_0 i}{2\pi x}$$



Flux through an element of width dx is

$$d\phi = Ba\,dx = \frac{\mu_0 ia}{2\pi}\frac{dx}{x}$$

$\therefore$ Flux through $PQRS$ is

$$\phi = \frac{\mu_0 ia}{2\pi}\int_d^{d+b}\frac{dx}{x} = \frac{\mu_0 ia}{2\pi}\ell n\left(\frac{d+b}{d}\right)$$

$$\Rightarrow \quad \frac{d\phi}{dt} = \frac{\mu_0 a}{2\pi}\ell n\left(\frac{d+b}{d}\right)\frac{di}{dt}$$

$$\Rightarrow \quad \frac{d\phi}{dt} = \frac{\mu_0 a}{2\pi}\ell n\left(\frac{d+b}{d}\right)\left(-\frac{10}{200 \times 10^{-6}}e^{-\frac{t}{200\mu s}}\right)$$

At $t = 200\ \mu s$

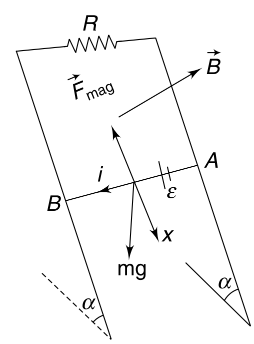
$$\frac{d\phi}{dt} = -2 \times 10^{-7} \times 0.1\ \ell n\left(\frac{5+5}{5}\right)\frac{3.7}{200 \times 10^{-6}}$$

$$\therefore \quad \varepsilon_{in} = 3.7\ell n2 \times 10^{-4} = 0.26\text{ mV}$$

Resistance of loop $PQRS = 1 \times 25 \times 0.3 = 7.5\ \Omega$

$$\therefore \text{ Current} \quad I = \frac{\varepsilon_{in}}{7.5} = \frac{0.26\text{ mV}}{7.5} = 35\,\mu\text{A}$$

39. From lenz's law the current through the connector is directed from A to B. Here $\varepsilon = vB\ell$ between A and B. Where v is the velocity of the connector at any moment.



For the connector, $F_x = ma_x$

Or, $mg\sin\alpha - i\ell B - \mu mg\cos\alpha = ma_x$

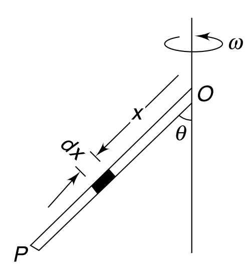
For steady state, acceleration of the rod must be equal to zero

Hence, $$\frac{1}{2}mg\sin\alpha = i\ell B \quad \text{...(i)}$$

But $$i = \frac{\varepsilon_{in}}{R} = \frac{vB\ell}{R} \quad \text{...(ii)}$$

From (i) and (ii) $$v = \frac{mgR\sin\alpha}{2B^2\ell^2}$$

40. Consider an elemental length dx. Velocity of the element is perpendicular to the plane of the figure and its magnitude is



$$v = \omega(x\sin\theta)$$

Emf induced in the element is

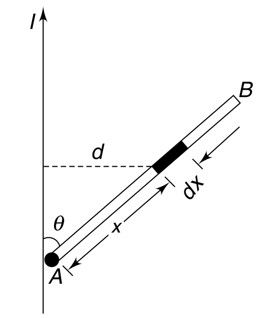
$$d\varepsilon = Bv(dx)\sin\theta \quad [\theta \text{ is angle between } dx \text{ and } B]$$

$$= B\omega\sin^2\theta\, x\,dx$$

$\therefore \qquad \varepsilon = B\omega\sin^2\theta \int_0^L x dx = \frac{1}{2} B\omega\sin^2\theta (L^2)$

The free electrons experience force towards P. Hence O is positive.

41. We will apply energy conservation to find the angular speed (ω) of the rod.



$$\frac{1}{2} I_A\omega^2 = \text{loss in gravitational } PE$$

$$\frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2 = Mg\,\frac{L}{2}(1-\cos\theta)$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{3g(1-\cos\theta)}{L}} \qquad \text{...(i)}$$

Now consider an element of length dx on the rod.

Speed of the element is $\qquad v = \omega x$

Magnetic field at the location of the element is

$$B = \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{2\pi x \sin\theta}$$

$\therefore$ *Emf* induced in the element is

$$d\varepsilon = Bv\,dx = \frac{\mu_0 I\omega}{2\pi\sin\theta}\,dx$$

$\therefore$ *Emf* in the rod is

$$\varepsilon = \int d\varepsilon = \frac{\mu_0 I\omega}{2\pi\sin\theta}\int_0^L dx = \frac{\mu_0 I\omega L}{2\pi\sin\theta}$$

$$= \frac{\mu_0 I}{2\pi\sin\theta}\sqrt{3gL(1-\cos\theta)}$$

42. (i) *Emf* induced in the rod $\qquad \varepsilon = \frac{1}{2} B_0\omega_0 L^2$

This is constant since ω_0 is constant. If charge q_0 is forced through this emf, work done (by imaginary cell created in the rod is)

$$W = q_0\varepsilon = \frac{1}{2} q_0 B_0\omega_0 L^2$$

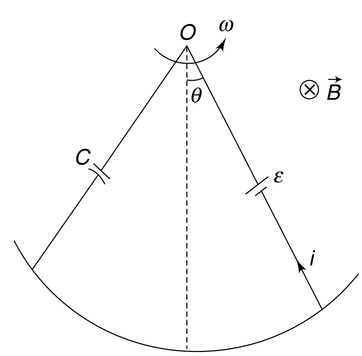
This is the work done by the external agent in keeping the rod rotating with a constant speed. Mechanical work by the agent gets converted into the electrical energy.

(ii) W = energy stored in capacitor + Heat dissipated in R

$$\therefore \qquad \frac{1}{2} q_0 B_0\omega_0 L^2 = \frac{q_0^2}{2C} + H$$

$$\therefore \qquad H = \frac{1}{2} q_0\left[B_0\omega_0 L^2 - \frac{q_0}{C}\right]$$

43. (a) For any position θ of the rod, let its angular speed be $\omega\left(=\frac{d\theta}{dt}\right)$



emf induced $\qquad \varepsilon = \frac{1}{2} B\omega L^2$

Charge on the capacitor is

$$q = C\varepsilon = \frac{1}{2} BC\omega L^2$$

Current in the circuit

$$i = \frac{dq}{dt} = \frac{1}{2} BCL^2 \frac{d\omega}{dt} = \frac{1}{2} BCL^2 \cdot \alpha \qquad [\alpha = \text{angular acceleration}]$$

Using $\tau = I\alpha$ for rod we get

$$\frac{1}{3} ML^2 \cdot \alpha = -Mg \frac{L}{2} \sin\theta - (BLi) \frac{L}{2}$$

$$\frac{1}{3} ML^2 \cdot \alpha = -Mg \frac{L}{2} \sin\theta - \frac{1}{4} B^2L^4C \cdot a$$

$$\therefore \qquad \left[\frac{1}{3} ML^2 + \frac{1}{4} B^2L^4C\right] \alpha \simeq -Mg \frac{L}{2} \theta$$

$$\alpha = -\left(\frac{6Mg}{4ML + 3B^2L^3C}\right) \theta$$

Thus motion of the rod is simple harmonic. Initially, it is at positive extreme.

$$\therefore \qquad \theta = \theta_0 \cos\omega t \text{ where } \omega = \left(\frac{6Mg}{4ML + 3B^2L^3C}\right)^{1/2}$$

(b) In presence of a resistor the mechanical energy of the system will get dissipated as heat and the oscillations will die.

44. Capacitance of disc system $\qquad C = \dfrac{\in_0 A}{d}$

When speed of the system is v, the emf induced between two plates is

$$\varepsilon = Bvd$$

$\therefore$ Charge on the capacitor $\qquad q = C\varepsilon = \in_0 ABv$

Energy stored in the capacitor is $\qquad U = \dfrac{1}{2} C\varepsilon^2$

$$\Rightarrow \qquad U = \frac{\in_0 A}{2} B^2v^2 \cdot d$$

Force associated with this energy is

$$F = -\frac{dU}{dx} = -\frac{\in_0 A}{2} B^2 d\, 2v \frac{dv}{dx}$$

$$= -\in_0 AB^2 \cdot d \cdot v \frac{dv}{dt} \cdot \frac{dt}{dx}$$

But $\qquad \dfrac{dt}{dx} = \dfrac{1}{v}$

$$\therefore \qquad F = -\in_0 AB^2 d \frac{dv}{dt}$$

Alternatively, this force can be worked out as magnetic force on the rod (= Bid) when the current through it is $i = \dfrac{dq}{dt}$

When the system is displaced by x from its equilibrium and has a speed v; we have

$$M \frac{dv}{dt} = -kx - \in_0 AB^2 \cdot d \frac{dv}{dt}$$

$$\Rightarrow \qquad (M + \in_0 AB^2 \cdot d) \frac{dv}{dt} = -kx$$

$$\Rightarrow \qquad \frac{dv}{dt} = -\left(\frac{k}{M + \in_0 AB^2 \cdot d}\right) \cdot x \quad [SHM]$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{M + \epsilon_0 AB^2 \cdot d}{k}}$$

45. At any time t let the velocity of the conductor be v

Induced emf, $\varepsilon = BLv$

If charge on the capacitor at time t is q then

$$q = C\varepsilon = BLCv$$

Current through the conductor is $i = \frac{dq}{dt} = BLC\frac{dv}{dt}$

Force equation for the conductor is

$$m\frac{dv}{dt} = mg - kx - BiL \qquad \text{[where } x = \text{displacement]}$$

$$m\frac{dv}{dt} = mg - kx - B^2L^2C\frac{dv}{dt}$$

$$\Rightarrow \qquad \frac{dv}{dt}[m + B^2L^2C] = mg - kx$$

In equilibrium $\frac{dv}{dx} = 0 \Rightarrow kx_0 = mg$ ...(i)

If x' is displacement of the bar measured from the equilibrium position then $x' = x - x_0$

$$[m + B^2L^2C]\frac{dv}{dt} = mg - k(x_0 + x')$$

$$\Rightarrow \qquad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m + B^2L^2C}\right)x' \qquad [\because mg = kx_0]$$

Solution to this equation is of the form

$$x' = A\sin(\omega t + \delta) \text{ where } \omega = \sqrt{\frac{k}{m + B^2L^2C}}$$

$$\therefore \qquad x = x_0 + x' = \frac{mg}{k} + A\sin(\omega t + \delta)$$

$$\therefore \qquad v = \frac{dx}{dt} = A\omega\cos(\omega t + \delta)$$

At $t = 0,\ v = 0$ $\therefore\ \delta = \frac{\pi}{2}$

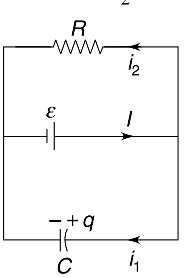
$$\therefore \qquad x = \frac{mg}{k} + A\cos\omega t$$

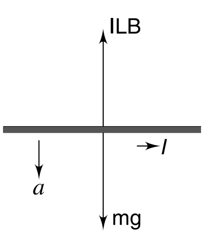
At $t = 0;\ x = 0 \Rightarrow A = -\frac{mg}{k}$

$$\therefore \qquad x = \frac{mg}{k}(1 - \cos\omega t)$$

46. When speed of the bar is v emf induced in it is $\varepsilon = BLv$

Let charge on the capacitor and current through it be q and i_1. Current in R be i_2. Current through the bar is $I = i_1 + i_2$





$$i_2 = \frac{BLv}{R}$$

and $q = BLvC \Rightarrow \frac{dq}{dt} = BLC\frac{dv}{dt}$

$$\Rightarrow \qquad i_1 = BLC\frac{dv}{dt}$$

Force equation for the bar is

$$m\frac{dv}{dt} = mg - ILB$$

$$\Rightarrow \quad m\frac{dv}{dt} = mg - \left(\frac{BLv}{R} + BLC\frac{dv}{dt}\right)LB \qquad [\because \; I = i_2 + i_1]$$

$$\Rightarrow \quad (m + B^2L^2C)\frac{dv}{dt} = mg - \frac{B^2L^2}{R}v$$

$$\Rightarrow \quad \int_0^v \frac{dv}{mg - \frac{B^2L^2}{R}v} = \left(\frac{1}{m + B^2L^2C}\right)\int_0^t dt$$

$$\Rightarrow \quad \left[\ell n\left(mg - \frac{B^2L^2}{R}v\right)\right]_0^v = -\frac{B^2L^2t}{R(m + B^2L^2C)}$$

$$\Rightarrow \quad \ell n\left[\frac{mg - \frac{B^2L^2v}{R}}{mg}\right] = -\frac{B^2L^2t}{R(m + B^2L^2C)}$$

$$\therefore \quad 1 - \frac{B^2L^2v}{mgR} = e^{-\frac{B^2L^2t}{R(m + B^2L^2C)}}$$

$$\Rightarrow \quad v = \frac{mgR}{B^2L^2}\left[1 - e^{-\frac{B^2L^2t}{R(m + B^2L^2C)}}\right]$$

when $t \to \infty$; $v \to \dfrac{mgR}{B^2L^2}$

47. (a) Drawing analogy from the case of an ideal solenoid we can write the magnetic field inside the cylinder as:

$$B = \mu_0\frac{I}{\ell} \qquad \left[\because \; \begin{array}{l} B = \mu_0 ni \\ ni = I/\ell \end{array}\right]$$

Flux linked with the cylinder is

$$\phi = BA = \pi a^2\frac{\mu_0 I}{\ell}$$

Self inductance is $\quad L = \mu_0\dfrac{\pi a^2}{\ell}$

Induced emf $\quad \varepsilon = \left|\dfrac{d\phi}{dt}\right| = \dfrac{\mu_0\pi a^2}{\ell}\dfrac{dI}{dt} = \dfrac{\mu_0\pi a^2\beta}{\ell}$

(b) Resistance of cylinder $R = \rho\dfrac{2\pi a}{\ell d}$

$$\therefore \quad \varepsilon = IR$$

$$-\frac{\mu_0\pi a^2}{\ell}\frac{dI}{dt} = I\frac{\rho 2\pi a}{\ell d}$$

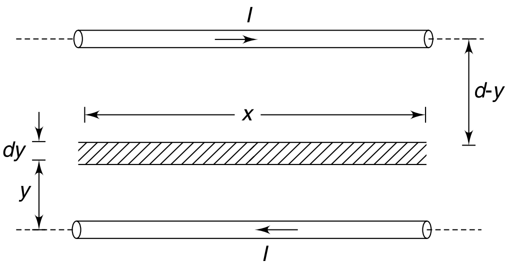
$$\Rightarrow \quad \frac{dI}{dt} = -\frac{2\rho}{\mu_0 ad}I$$

$$\therefore \quad \int_{I_0}^{I}\frac{dI}{I} = -\frac{2\rho}{\mu_0 ad}\int_0^t dt$$

$$\Rightarrow \quad \ell n\left(\frac{I}{I_0}\right) = \frac{-2\rho t}{\mu_0 ad}$$

$$\Rightarrow \quad I = I_0 e^{-\frac{2\rho t}{\mu_0 ad}}$$

48. Consider a strip of width dy as shown in figure.



Magnetic field at the strip due to current in two wires is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right) \otimes$$

$\therefore$ Flux through the strip of area xdy is

$$d\phi = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right) x\,dy$$

Flux linked with the area between two wires in a length x will be

$$\phi = \frac{\mu_0 Ix}{2\pi}\left[\int_a^{d-a}\frac{dy}{y} + \int_a^{d-a}\frac{dy}{d-y}\right]$$

$$= \frac{\mu_0 Ix}{2\pi}\left[\ell n\left(\frac{d-a}{a}\right) - \ell n\left(\frac{d-d+a}{d-a}\right)\right]$$

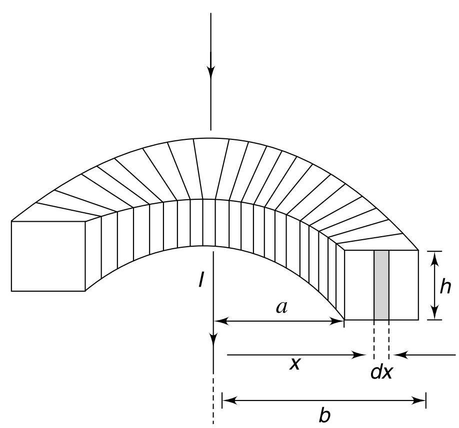
$$= \frac{\mu_0 Ix}{2\pi}[2\ell n(d-a) - 2\ell na]$$

$$= \frac{\mu_0 Ix}{\pi}\,\ell n\left(\frac{d-a}{a}\right)$$

$$\therefore \quad L = \frac{\phi}{I} = \frac{\mu_0 x}{\pi}\,\ell n\left(\frac{d-a}{a}\right)$$

49. When switch is closed the induced emf in the inductor is 4 volt and begins to fall thereafter. In steady state there is some large current (i_0) flowing through it. The bulb will not light as the potential difference across it never reaches 60 V. When the switch is opened the cell gets disconnected and the current in the inductor must go to zero. The decay path of the current has a huge resistance in form of the neon lamp. The time constant is small which means that $\frac{di}{dt}$ is large. A big emf is induced in the inductor which causes the lamp to flash.

50. We have drawn half of the toroid and the straight wire. This diagram will give you a fair idea of the situation. Consider a current I in the straight wire. Consider a strip of width dx on the cross section of the toroid at radial distance x.



Magnetic field at this location $\quad B = \frac{\mu_0 I}{2\pi x}$

Flux through the strip $= B(hdx) = \frac{\mu_0 Ihdx}{2\pi x}$

Flux through the entire cross section is

$$\phi = \frac{\mu_0 hI}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 h}{2\pi}\,\ell n\left(\frac{b}{a}\right)\cdot I$$

Total flux linked with all turns of the coil

$$\phi_0 = N\phi = \frac{\mu_0 Nh\,\ell n(b/a)}{2\pi}\cdot I$$

Comparing with $\quad \phi = MI$

$$M = \frac{\mu_0 Nh\,\ell n(b/a)}{2\pi}$$

51. Current in coil 1 will attain its steady value $I_{01} = \frac{V}{R_1}$ after infinite time.

Let current in the two coils be i_1 and i_2 at time t. Using Kirchhoff's loop rule to the second coil we can write-

$$M\frac{di_1}{dt} + L_2\frac{di_2}{dt} - R_2 i_2 = 0$$

$\Rightarrow$ $$Mdi_1 + L_2 di_2 = R_2 i_2 dt$$

$\Rightarrow$ $$M\int_0^{I_{01}} di + L_2\int_0^{I_{02}} di_2 = R_2\int_{t=0}^{\infty} i_2 dt$$

$\Rightarrow$ $$MI_{01} + L_2 I_{02} = R_2 Q_0$$

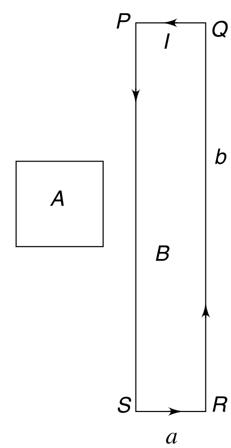
But $$I_{02} = 0 \quad \text{and} \quad I_{01} = \frac{V}{R_1}$$

$\therefore$ $$M\frac{V}{R_1} = R_2 Q_0$$

$\Rightarrow$ $$M = \frac{R_1 R_2 Q_0}{V}$$

52. To calculate the mutual inductance of the pair of loops, let us assume that there is a current I in the bigger loop. Field due to QP and SR will be very small at the location of loop A due to large distance. Loop A is very close to arm PS and RQ. We can consider PS and RQ to be of infinite length for writing magnetic field at location of A.

$$B_{PS} = \frac{\mu_0 I}{2\pi a} \otimes \quad \text{and} \quad B_{RQ} = \frac{\mu_0 I}{2\pi(2a)} \odot$$



$\therefore$ $$B = \frac{\mu_0 I}{4\pi a} \otimes$$

Since loop A is small, we can consider the field to be uniform throughout its area.

$\therefore$ $$\phi_A = x^2 \cdot B = \frac{\mu_0 I}{4\pi a} x^2$$

Comparing with $\phi = MI$

Mutual inductance of coils $M = \dfrac{\mu_0 x^2}{4\pi a}$

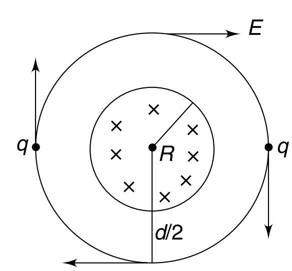
M does not change if loop A is made primary.

$\therefore$ emf induced in B when current is made to change in A is

$$\varepsilon_B = M\frac{di_A}{dt} = \frac{\mu_0 x^2 \alpha}{4\pi a}$$

53. When the magnetic field is switched off, an electric field is induced. The electric field (E) on a circle of radius $\frac{d}{2}$ is given by $E \cdot 2\pi\left(\frac{d}{2}\right) = \pi R^2 \frac{\Delta B}{\Delta t}$

$\therefore$ $$E = \frac{R^2}{d}\frac{\Delta B}{\Delta t}$$



Electric force on each ball; $F_e = Eq$

Impulse on each ball = $Eq\Delta t = q \cdot \dfrac{R^2}{d}(\Delta B)$

Angular impulse on each ball = $q \cdot \dfrac{R^2}{d}(\Delta B) \cdot \dfrac{d}{2}$

Total angular impulse on both balls = $qR^2 \Delta B$

Since $\Delta B = B_0$

$\therefore$ Angular Impulse $= qR^2 B_0$

Change in angular momentum = qR^2B_0

$$2 \cdot m\left(\frac{d}{2}\right)^2 \omega = qR^2B_0$$

$$\Rightarrow \qquad \omega = \frac{2qR^2B_0}{md^2}$$

54. The magnetic field is decreasing with time. Using Faraday's law(or Lenz's law) one can see that the emf induced in a closed path will be in clockwise sense. It means that the induced electric field is clockwise. Therefore, end A will reach point C.

Magnitude of induced electric field is-

$$E = \frac{l}{2}\left|\frac{dB}{dt}\right| = \frac{l}{2}\frac{B_0}{2} = \frac{B_0 l}{4} = \frac{B_0 \times 0.04}{4} = 1 \text{ V/m.}$$

$$\tau = 2Eq \times l$$

$$\therefore \qquad \alpha = \frac{2Eql}{I} = \frac{2Eql}{2 \times ml^2} = \frac{Eq}{ml}$$

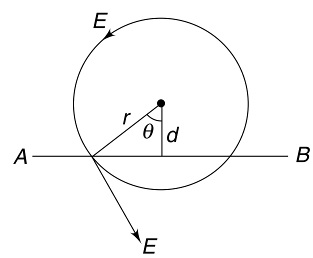
$$\therefore \qquad \theta = \frac{1}{2}\alpha t^2$$

$$t^2 = \frac{2\theta}{\alpha} = \frac{2 \times \frac{\pi}{2}}{\alpha} = \frac{\pi ml}{Eq}$$

$$= \frac{\pi l}{E} \times \frac{1}{q/m} = \frac{\pi \times 0.04}{1} \times \frac{1}{\frac{4\pi}{100}}$$

$$\therefore \qquad t^2 = 1 \qquad \therefore \quad t = 1 \text{ s.}$$

55. The electric field induced at a distance r from the centre is given by



$$\oint \vec{E}\,\vec{dl} = -\frac{d\phi}{dt}$$

$$\Rightarrow \qquad E2\pi r = \pi r^2 \frac{dB}{dt} \quad \Rightarrow \quad E = \frac{r}{2}\frac{dB}{dt}$$

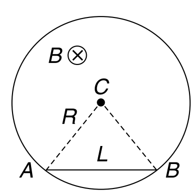
Component of E along AB is

$$E_{AB} = E\cos\theta = \frac{1}{2}(r\cos\theta)\frac{dB}{dt} = \frac{d}{2}\left(\frac{dB}{dt}\right) = a \text{ constant.}$$

The emf induced in wire AB is given by $E_{AB}\,L$ and it will not change even if the wire is displaced by a small amount parallel to itself.

56. Consider a triangular closed loop ABC

Area of the loop



$$A = \frac{1}{2} \times L \times \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

Flux through loop ABC is

$$\phi = \frac{L}{4}\sqrt{4R^2 - L^2} \cdot B$$

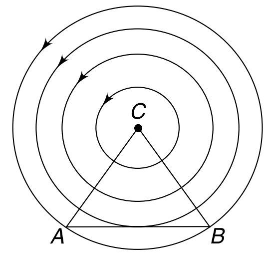
Emf induced in the loop is $\qquad \varepsilon = \frac{d\phi}{dt} = \frac{L}{4}\sqrt{4R^2 - L^2} \cdot \alpha$

Because of symmetry the induced field lines will be circular. It means field lines of E are perpendicular to CA and CB. The emf in the loop ABC can also be written as

$$\int_C^A \vec{E}\cdot\vec{dl} + \int_A^B \vec{E}\cdot\vec{dl} + \int_B^C \vec{E}\cdot\vec{dl} = \frac{L\alpha}{4}\sqrt{4R^2 - L^2}$$

$$\Rightarrow \quad 0 + \int_A^B \vec{E}\cdot\vec{dl} + 0 = \frac{L\alpha}{4}\sqrt{4R^2 - L^2}$$

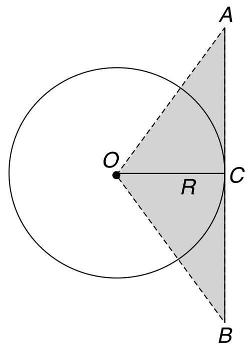
$$\therefore \quad \int_A^B \vec{E}\cdot\vec{dl} = \frac{L\alpha}{4}\sqrt{4R^2 - L^2}$$



Note: Alternative way of finding the answer can be found in the result obtained in last problem.

57. (a) Electric field lines are perpendicular to the rod at all points in Figure (a)

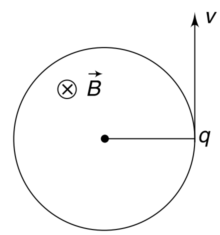
(b) Emf induced in AB = rate of change of flux through the shaded region.



$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\pi R^2}{4}\cdot\alpha$$

58. $I_0 = nqf_0$. Where f_0 = frequency of rotation

$$I_0 = nq\frac{\omega_0}{2\pi} \qquad \text{...(i)}$$



Due to change in magnetic flux an electric field is induced.

$$2\pi RE = \frac{d\phi}{dt} \quad \Rightarrow \quad E = \frac{\beta}{2\pi R}$$

It will be wise to assume that the magnetic field is assisting in providing the necessary centripetal force. The situation can be as shown in the figure.

If B is made to increase, it will create an electric field in anticlockwise sense. This field will speed up the particle.

Tangential acceleration $a_t = \dfrac{qE}{m} = \dfrac{qB}{2\pi Rm}$

Angular acceleration $\quad a = \dfrac{a_t}{R} = \dfrac{qB}{2\pi R^2 m}$

Angular speed after one rotation is given by

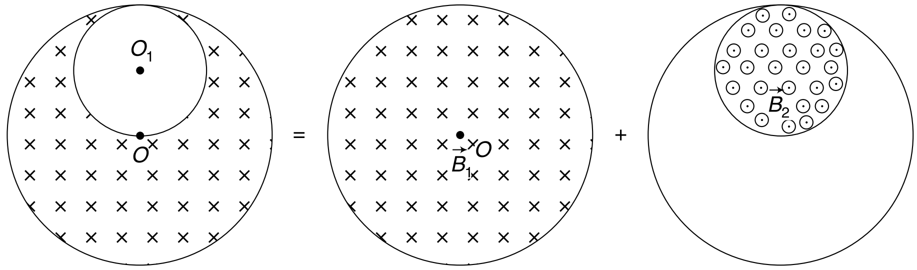
$$\omega^2 = \omega_0^2 + 2\alpha(2\pi)$$

Using (i)

$$\left(\frac{2\pi I}{nq}\right)^2 = \left(\frac{2\pi I_0}{nq}\right)^2 + \frac{2q\beta}{R^2 m}$$

$$\therefore \quad I = \sqrt{I_0^2 + \frac{n^2 q^3 \beta}{2\pi^2 R^2 m}}$$

59. We can consider the entire system of magnetic field to be superposition of fields as shown below.



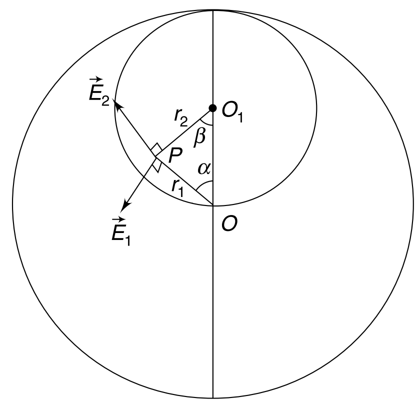
$$\frac{dB_1}{dt} = \frac{dB_2}{dt} = \alpha$$

Say, both fields are increasing.

Induced electric field inside a cylindrical region having time changing magnetic field is given by

$$E = \frac{r}{2}\left(\frac{dB}{dt}\right) \text{ where } r \text{ is distance from the axis of the cylinder.}$$

Induced Field at point P (inside cavity)



(i) Due to changing $\vec{B}_1$ is $\vec{E}_1$ (perpendicular to OP)

$$E_1 = \frac{r_1}{2}\frac{dB_1}{dt} = \frac{r_1}{2}k$$

(ii) Due to changing $\vec{B}_2$ is $\vec{E}_2$ (perpendicular to O_1P)

$$E_2 = \frac{r_2}{2}\frac{dB_2}{dt} = \frac{r_2}{2}k$$

Vector sum of E_1 and E_2 is the resultant field at P

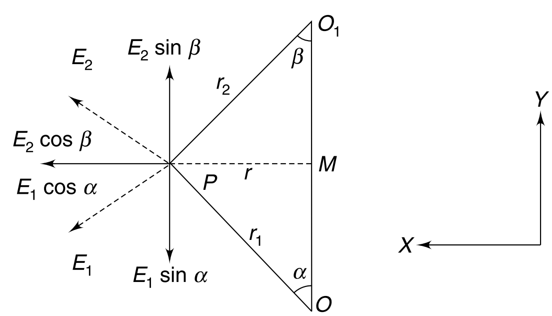
$$E_y = E_2 \sin\beta - E_1 \sin\alpha$$

$$= \frac{r_2}{2}k\frac{r}{r_2} - \frac{r_1}{2}k\frac{r}{r_1} = 0$$

$$E_x = E_1\cos\alpha + E_2\cos\beta$$

$$= \frac{r_1}{2}k\frac{OM}{r_1} + \frac{r_2}{2}k\frac{O_1M}{r_2}$$

$$= \frac{k}{2}(OM + O_1M) = \frac{kR}{4}$$



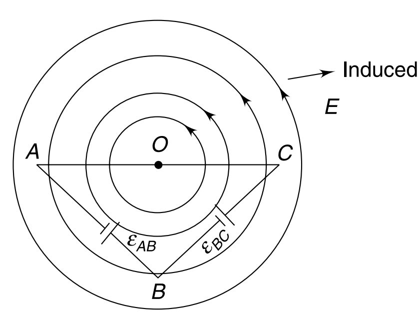
Hence, resultant field inside the cavity is uniform perpendicular to line OO_1

60. Faraday's law gives-

Induced $$\text{emf} = \left|-\frac{d\phi}{dt}\right|$$

$$\varepsilon = \frac{d}{dt}(B \times \text{Area})$$

$$= (\text{Area})\left(\frac{dB}{dt}\right) = \frac{1}{2}\ell^2 \cdot \alpha$$



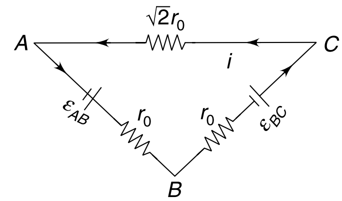
The time changing magnetic field induces an electric field having circular lines. Because there is no radial component of induced field E, there is no emf induced in the segment AC.

Segment AB and BC are symmetrical

$\therefore$ emf induced in AB and BC

$$\varepsilon_{AB} = \varepsilon_{BC} = \frac{\varepsilon}{2} = \frac{\ell^2\alpha}{4}$$

From point of view of current electricity, the circuit is as shown in figure.



Current in the triangle

$$i = \frac{\varepsilon}{2r_0 + \sqrt{2}r_0} = \frac{\ell^2\alpha}{2\sqrt{2}r_0(\sqrt{2}+1)}$$

$$V_{CA} = i\sqrt{2}r_0 = \frac{\ell^2\alpha}{2(\sqrt{2}+1)}$$

$$V_{BA} = \varepsilon_{AB} - ir_0$$

$$= \frac{\ell^2\alpha}{4} - \frac{\ell^2\alpha}{2\sqrt{2}(\sqrt{2}+1)} = \frac{\ell^2\alpha}{4(\sqrt{2}+1)}$$

61. Flux through the ring is $\phi = \vec{B}\cdot\vec{A} = (2\hat{j} + t^2\hat{k})\cdot(\pi R^2\hat{k}) = \pi R^2 t^2$

$\therefore$ Induced emf $|\varepsilon| = \dfrac{d\phi}{dt} = 2\pi R^2 t$

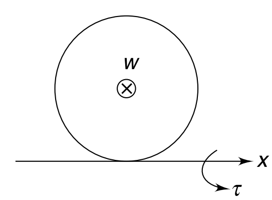
If induced electric field is E_{in} then

$$E_{in} 2\pi R = \varepsilon$$

$$E_{in} 2\pi R = 2\pi R^2 t$$

$$\therefore \quad E_{in} = Rt \qquad \text{...(i)}$$

Current in the ring is $i = \dfrac{\varepsilon}{r} = \dfrac{2\pi R^2 t}{r}$ [Clockwise]



Magnetic dipole moment $\vec{\mu} = (\pi R^2 i)(-\hat{k}) = \dfrac{2\pi^2 R^4 t}{r}(-\hat{k})$

Torque on the ring $\vec{\tau} = \vec{\mu}\times\vec{B} = \dfrac{4\pi^2 R^4 t}{r}(\hat{i})$

The ring will begin to topple when magnetic torque exceeds the torque due to weight about an axis tangential to the ring and parallel to x axis. In the Figure the weight (W) of the ring is acting into the plane of the Figure.

$$\frac{4\pi^2 R^4 t}{r} = mgR$$

$$\Rightarrow \quad t_0 = \frac{mg\cdot r}{4\pi^2 R^3} = \frac{\pi\times 10\times\pi}{4\pi^2\times\left(\frac{1}{2}\right)^3} = 20 \text{ sec.}$$

Induced field at t_0 from equation (i) is

$$E_{in} = \frac{mgr}{4\pi^2 R^2} = \frac{\pi\times 10\times\pi}{4\pi^2\left(\frac{1}{2}\right)^2} = 10 \text{ V/m}$$

Heat generated till t_0 is

$$H = \int_0^{t_0} i^2 r\,dt = \frac{4\pi^2 R^4}{r}\int_0^{t_0} t^2\,dt = \frac{4}{3}\frac{\pi^2 R^4}{r}t_0^3$$

$$= \frac{4}{3}\frac{\pi^2}{\pi}\times\left(\frac{1}{2}\right)^4\times(20)^3 = \frac{2\pi}{3} \text{ kJ}$$

62. Field due to one coil is $B = \mu_0\left(\dfrac{N}{\ell}\right) I$

Field in overlapping region of length $(\ell - x)$ is $= 2B$

$\therefore$ Energy = energy in overlapping region + energy in non overlapping region of outer solenoid + energy in non overlapping region of inner solenoid

$$E = \frac{(2B)^2}{2\mu_0}\cdot A_1(\ell - x) + \frac{B^2}{2\mu_0}[A_2x + (A_2 - A_1)(\ell - x)] + \frac{B^2}{2\mu_0}A_1x$$

$$\therefore \quad E = \frac{B^2}{2\mu_0}[3A_1\ell - 2A_1x + A_2\ell] = \frac{\mu_0N^2I^2}{2\ell^2}[3A_1\ell - 2A_1x + A_2\ell]$$

63. The outer coil is intersected by the inner coil's flux = BA_1

In time dt this flux is intercepted by fewer turns of the outer coil. Number of turns of outer coil which lose the flux due to inner coil in interval dt is = $\frac{N}{\ell}v\,dt$

$\therefore$ emf induced in outer coil

$$\varepsilon_{in} = BA_1\frac{N}{\ell}v = \frac{\mu_0NIA_1Nv}{\ell^2} = \frac{\mu_0N^2A_1vI}{\ell^2}$$

The other coil will also have same emf.

Put your own arguments for this.

64. Mechanical power supplied by external agent = Rate of change of magnetic field energy in the solenoids + Rate of work done by the power sources driving current in the two coils.

$$F\cdot v = \frac{dE}{dt} + \varepsilon_{in}\cdot I + \varepsilon_{in}\cdot I$$

$$= \frac{dE}{dx}\cdot\frac{dx}{dt} + 2\varepsilon_{in}\cdot I \qquad \left(\text{from Q. 62 } E = \frac{\mu_0N^2I^2}{2\ell^2}[3A_1\ell - 2A_1x + A_2\ell]\right)$$

$$= -\frac{\mu_0N^2I^2A_1}{\ell^2}\cdot v + 2\frac{\mu_0N^2I^2A_1v}{\ell^2} = \frac{\mu_0N^2I^2A_1v}{\ell^2} \quad (\text{from Q. 63 we get } \varepsilon_{in})$$

$$\therefore \quad F = \frac{\mu_0N^2I^2A_1}{\ell^2}$$

65. (a) Let the velocity at time t be v. Induced emf $\varepsilon = B\ell v$

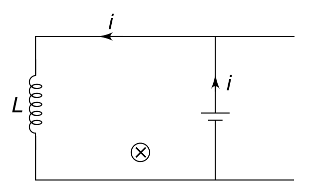
$$\therefore \quad L\frac{di}{dt} = B\ell v \qquad \text{...(i)}$$

Magnetic force on the wire is = $i\ell B$

$$\therefore \quad m\frac{dv}{dt} = -i\ell B \qquad \text{...(ii)}$$

Differentiating equation (ii) with respect to time t

$$B\ell\frac{di}{dt} = -m\frac{d^2v}{dt^2}$$



Putting in (i)

$$L\frac{m}{B\ell}\frac{d^2v}{dt^2} = -B\ell v$$

$$\therefore \quad \frac{d^2v}{dt^2} = -\frac{B^2\ell^2}{mL}v$$

Solution to this equation is

$$v = v_0\sin(\omega t + \delta) \text{ where } \omega = \frac{B\ell}{\sqrt{mL}}$$

At $t = 0$; $v = v_0$

This means $\quad \delta = \frac{\pi}{2}$ and $v = v_0\cos(\omega t)$

(b) From (ii)
$$i = -\frac{m}{\ell B}\frac{dv}{dt}$$
$$= \frac{mv_0\omega}{\ell B}\sin\omega t = \frac{mv_0}{\ell B}\frac{B\ell}{\sqrt{mL}}\sin\omega t$$
$$= v_0\sqrt{\frac{m}{L}}\sin\omega t$$

(c)
$$L\frac{di}{dt} = B\ell v \quad \Rightarrow \quad L\frac{di}{dt} = B\ell\frac{dx}{dt}$$

$$\Rightarrow \quad L\,di = B\ell\,dx \quad \Rightarrow \quad L\int_0^i di = B\ell\int_0^x dx$$

$$\Rightarrow \quad Li = B\ell x \text{ where } x = \text{displacement}$$

Magnetic force on the conductor will be
$$F = ilB = \frac{B^2\ell^2}{L}x$$

$$\therefore \quad mv\frac{dv}{dx} = -\frac{B^2\ell^2}{L}\cdot x$$

$$\Rightarrow \quad \int_{v_0}^{v} v\,dv = -\frac{B^2\ell^2}{mL}\int_0^x x\,dx$$

$$\Rightarrow \quad v^2 = v_0^2 - \frac{B^2\ell^2}{mL}x^2 \quad \Rightarrow \quad v = \sqrt{v_0^2 - \frac{B^2\ell^2}{mL}x^2}$$

(d) $v = 0$ when
$$x = \frac{v_0}{B\ell}\sqrt{mL}$$

At this instant
$$\frac{di}{dt} = 0$$

But
$$i = \frac{B\ell}{L}x = v_0\sqrt{\frac{m}{L}}$$

(e) Since $i \neq 0$, the conductor will experience magnetic force and will move towards left.

66. (a) Initial current in the circuit is $i_0 = \frac{E}{R}$

After S_1 is opened, the resistance in the circuit suddenly rises to $R_1 + R = R_1'$ (say) $= 1001R$
But the current will not change suddenly.

$\therefore$ Drop across R_1' is $= i_0R_1' = \frac{E}{R}1001R = 1001E$

$\therefore$ emf induced in the inductor $= 1000E$

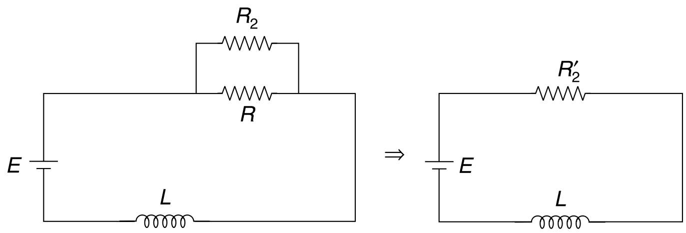
Potential drop across $R_1 = 1000E$

[$1000E$ plus a drop of E across R makes it $1001E$]

At $t \to \infty$:
$$I = \frac{E}{1001R}$$

(b) Initial current
$$i_0 = \frac{E}{R}$$

With S_1 closed and S_2 closed, the circuit changes to as shown in figure.



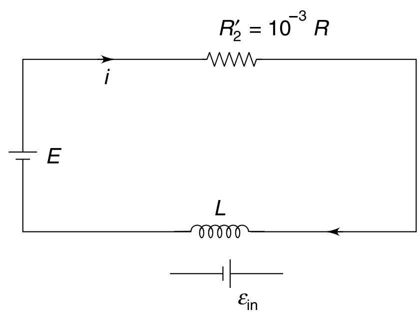
$$R_2' = \frac{RR_2}{R + R_2} = \frac{R_2}{1 + \dfrac{R_2}{R}} \simeq R_2 = 10^{-3} R$$

At time $t = 0^+$, the current is $\quad i_0 = \dfrac{E}{R}$

$\therefore$ Potential drop across R_2'

$$V_{R_2'} = \frac{E}{R}(10^{-3} R) = 10^{-3} E$$

At time '*t*'



i will increase to acquire final value of $I_\infty = \dfrac{E}{10^{-3} R} = 1000 \dfrac{E}{R}$

$$E - R_2' i - L\frac{di}{dt} = 0$$

$$\Rightarrow \qquad L\frac{di}{dt} = E - R_2' i$$

$$\Rightarrow \qquad \int_{\frac{E}{R}}^{i} \frac{di}{E - R_2' i} = \frac{1}{L}\int_0^t dt$$

$$\Rightarrow \qquad \ell n(E - R_2' i) - \ell n\left(E - \frac{E R_2'}{R}\right) = -\frac{R_2'}{L} t$$

$$\Rightarrow \ell n(E - 10^{-3} Ri) - \ell n(E - 10^{-3} E) = 10^{-3} \frac{Rt}{L}$$

$$\Rightarrow \qquad \ell n(E - 10^{-3} Ri) - \ell n(E) = -\frac{Rt}{1000 L}$$

$$\Rightarrow \qquad 1 - \frac{10^{-3} Ri}{E} = e^{-\frac{Rt}{1000 L}}$$

$$\Rightarrow \qquad i = \frac{E}{10^{-3} R}\left[1 - e^{-\frac{Rt}{1000 L}}\right]$$

Hence $\qquad i = \dfrac{1000 E}{R}\left[1 - e^{-\frac{Rt}{1000 L}}\right]$

67. Decaying current in a LR circuit is given by

$$I = I_0 e^{-t/\tau} \simeq I_0\left[1 - \frac{\tau}{t}\right] \quad \text{...(i)}$$

[$\because$ $e^{-x} \simeq 1 - x$ for small x]

Notice that $\tau = \frac{L}{R}$ is expected to be a very large number owing to a small resistance of the superconducting ring.

From (i) $\frac{\Delta I}{\Delta t} = -\frac{I_0}{\tau}$

Negative sign indicates that ΔI is negative. Let's talk only about magnitude of change in current.

$$\frac{\Delta I}{I_0} = \frac{\Delta t}{\tau}$$

As per the question, no change in current was observed for a month.

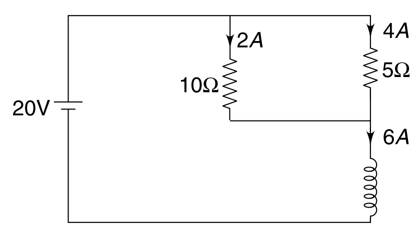
It means maximum value of $\frac{\Delta I}{I_0}$ is 0.01 when $\Delta t = 1$ month

$$\therefore \quad 0.01 = \frac{30 \times 24 \times 60 \times 60}{\frac{L}{R}}$$

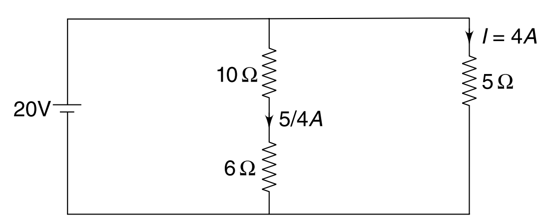
$$\therefore \quad R = \frac{0.01 \times 0.5}{30 \times 24 \times 60 \times 60} = 1.9 \times 10^{-9}\ \Omega$$

68. (a) When the switch 'S' is in closed position for a long time the circuit is in steady condition and the inductor acts as a short circuit.

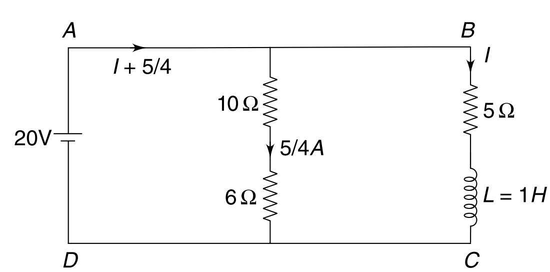
There is no potential difference across the inductor. It means no potential difference across 6Ω resistance. There is no current through 6Ω resistance. Effective circuit at $t = 0^-$ is as shown in Figure. Required answer is $6A$.



(b) At $t \to \infty$ once again the circuit will be in steady condition. $I = 4A$.



(c) At time 't' the circuit parameters are as shown in figure.



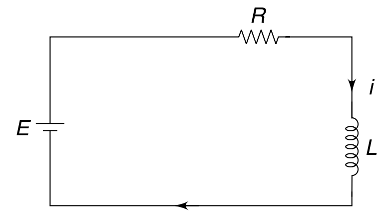
Applying Kirchhoff's Voltage law (KVL) in loop *ABCDA*:

$$5I + L\frac{dI}{dt} = 20$$

$$\int_6^I \frac{dI}{5I - 20} = -\int_0^t dt \qquad \text{[At } t = 0 \text{ current through inductor is } 6A]$$

Solving $\qquad I = 4 + 2e^{-5t}$

Current through cell $= \frac{5}{4} + I = \frac{21}{4} + 2e^{-5t}$



69. Just before closing S_2, the effective circuit is as shown.

$$i = \frac{E}{R} = \frac{20}{10} = 2A$$

Immediately, after closing S_2, the current through inductor will remain $i = 2A$. The effective circuit has been shown in figure.

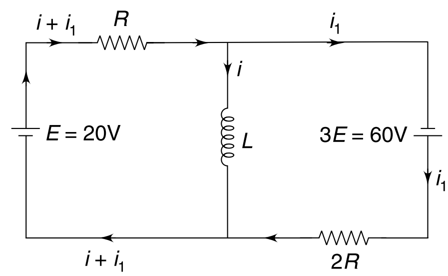
Applying KVL to the bigger loop

$$(i + i_1)\,R + i_1\,(2R) = 4E$$

$$3iR = 4E - iR$$

$$\Rightarrow \quad 30i_1 = 80 - 20 \qquad [\because \; iR = 2 \times 10 = 20]$$

$$\Rightarrow \quad i_1 = 2A$$



Applying KVL to loop containing L and cell of emf $3E$

$$3E + L\frac{di}{dt} = 2Ri_1$$

$$60 + 0.5\frac{di}{dt} = 40$$

$$\Rightarrow \quad \frac{di}{dt} = -40 \text{ A/s}$$

Negative sign says that current through the inductor is decreasing

Energy stored in magnetic field

$$U_B = \frac{1}{2}Li^2$$

$$\frac{dU_B}{dt} = Li\frac{di}{dt} = 0.5 \times 2 \times (-40) = -40 \text{ J/s}$$

Negative sign says that energy stored in the inductor is decreasing.

70. Initial energy in C_1 is $\qquad U_1 = \frac{1}{2}C_0V_0^2$

Final energy in C_2 is $\qquad U_2 = \frac{1}{2}\frac{C_0}{9}(3V_0)^2 = \frac{1}{2}C_0V_0^2$

Therefore, complete energy of C_1 gets transferred to C_2

The time period of LC circuit consisting of L and C_1 is

$$T_1 = 2\pi\sqrt{LC_0}$$

The entire energy of the capacitor can get transferred to the inductor in smallest interval of $t_1 = \frac{T_1}{4} = \frac{\pi}{2}\sqrt{LC_0}$

At time t_1 the current through L is maximum and switch S_1 is opened and S_2 is closed.

Now the time period of LC circuit having L and C_2 is

$$T_2 = 2\pi\sqrt{\frac{LC_0}{9}} = \frac{2\pi}{3}\sqrt{LC_0}$$

The inductor will transfer its entire energy to C_2 in smallest time given by

$$t_2 = \frac{T_2}{4} = \frac{\pi}{6}\sqrt{LC_0}$$

$$\therefore \quad T_{\min} = t_1 + t_2 = \frac{2\pi}{3}\sqrt{LC_0}$$

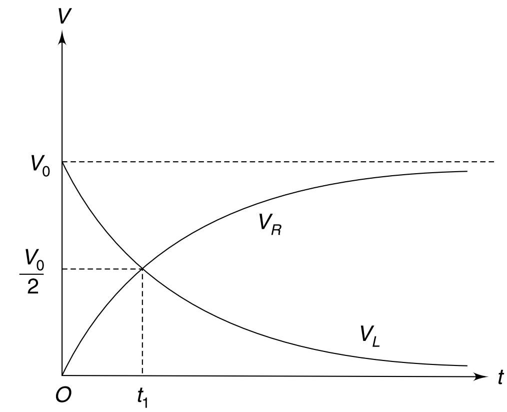
71. (a) Current $\quad i = i_0[1 - e^{-t/\tau}]$

Where $\quad t_0 = \frac{V_0}{R}$ and $\tau = \frac{L}{R}$

$$V_R = iR = i_0R\,[1 - e^{-t/\tau}] = V_0[1 - e^{-t/\tau}]$$

$$V_L = V_0 - V_R = V_0e^{-t/\tau}$$

Graph is as shown.



(b) The two curves intersect when

$$V_R = V_L \Rightarrow V_0[1 - e^{-t/\tau}] = V_0e^{-t/\tau}$$

$$\Rightarrow \quad e^{-t/\tau} = \frac{1}{2} \Rightarrow \frac{t}{\tau} = \ell n 2$$

$$\Rightarrow \quad t_1 = \tau\ell n2 = \frac{L}{R}\ell n2$$

(c)

$$L\frac{di}{dt} + Ri = V_0$$

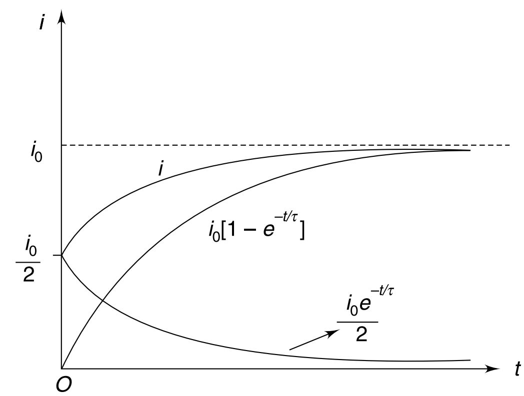
$$\int \frac{di}{V_0 - R_i} = \frac{1}{L}\int dt$$

$$[\ell n[V_0 - Ri]]_{\frac{i_0}{2}}^{i} = -\frac{R}{L}[t]_0^t \qquad \left[\because \text{at } \begin{array}{l} t = 0 \\ i = \frac{i_0}{2} \end{array}\right]$$

$$\Rightarrow \quad \ell n\left(\frac{V_0 - Ri}{V_0 - R\frac{i_0}{2}}\right) = -\frac{t}{\tau} \qquad \left[\tau = \frac{L}{R}\right]$$

$$\Rightarrow \quad V_0 - Ri = \left(V_0 - R\frac{i_0}{2}\right)e^{-t/\tau}$$

$$\Rightarrow \quad i = i_0[1 - e^{-t/\tau}] + \frac{i_0}{2}e^{-t/\tau} \qquad \left[\text{where } i_0 = \frac{V_0}{R}\right]$$



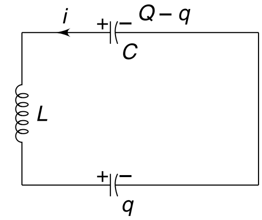
72. Let charge on two capacitor at some time be $(Q - q)$ and q as shown in Figure.

Energy is conserved in the circuit and magnetic energy will be maximum (i.e. the current will be maximum) when energy stored in the two capacitor is minimum.

Energy stored in capacitors is

$$E = \frac{(Q - q)^2}{2C} + \frac{q^2}{2C} = \frac{1}{2C}[2q^2 - 2Qq + Q^2]$$

E is minimum when $\quad \frac{dE}{dq} = 0$



$$\Rightarrow \qquad 4q - 2Q = 0 \Rightarrow q = \frac{Q}{2}$$

$$E_{min} = \frac{Q^2}{4C}$$

Magnetic energy is $\frac{1}{2} Li^2_{max} = \frac{Q^2}{2C} - \frac{Q^2}{4C}$

$$\Rightarrow \qquad i_{max} = \frac{Q}{\sqrt{2LC}}$$

73. When current through L is maximum $\frac{dI}{dt} = 0$

$\Rightarrow$ emf induced in L is zero at that instant. At this time a total charge $Q = 9CV_0 - 2CV_0 = 7CV_0$ is distributed on the two capacitors so that potential difference across them is same (V)

$$CV + 3CV = 7CV_0$$

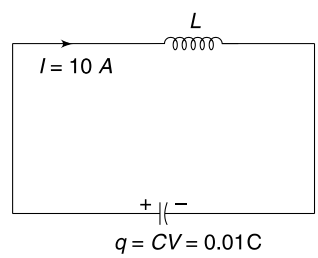
$$\Rightarrow \qquad V = \frac{7V_0}{4}$$

Energy conservation

$$\frac{1}{2} LI^2_{max} = \left[\frac{1}{2}C(2V_0)^2 + \frac{1}{2}3C(3V_0)^2\right] - \left[\frac{1}{2}CV^2 + \frac{1}{2}(3C)V^2\right]$$

Solving $I_{max} = \frac{5V_0}{2}\sqrt{\frac{3C}{L}}$

74. (a) At time $t = 0^+$, situation is as shown below



$$q = CV = 100 \times 10^{-6} \times 100 = 0.01 \text{ C}$$

Energy stored in the capacitor

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 100^2 = 0.5 \text{J}$$

Energy stored in the Inductor

$$U_2 = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.03 \times 10^2 = 1.5 \text{J}$$

When charge on capacitor become maximum energy stored in is $= U_1 + U_2 = 2 \text{J}$

$$\frac{q_0^2}{2C} = 2$$

$$\Rightarrow \qquad q_0^2 = 4 \times 100 \times 10^{-6}$$

$$\Rightarrow \qquad q_0 = 2 \times 10^{-2} = 0.02 C$$

(b) Angular frequency of oscillation $\omega = \frac{1}{\sqrt{LC}} = \frac{10^3}{\sqrt{3}}$ rad/s

Charge on capacitor changes with time according to $q = q_0 \sin(\omega t + \delta)$

At $t = 0$, $q = 0.01$

$$\therefore \qquad 0.01 = 0.02 \sin \delta$$

$$\Rightarrow \qquad \delta = \pi/6 \text{ or } 5\pi/6$$

We take $\delta = \frac{5\pi}{6}$ as charge on the capacitor is initially decreasing (look at the direction of current initially)

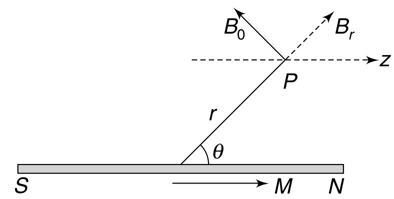
$$\therefore \qquad q = 0.02 \sin\left(\omega t + \frac{5\pi}{6}\right)$$

q is zero when $\omega t + \frac{5\pi}{6} = \pi$

$$\Rightarrow \qquad \omega t = \pi/6$$

$$\frac{10^3}{\sqrt{3}} t = \frac{\pi}{6} \Rightarrow t = \frac{3.14 \times 1.73}{6 \times 10^3} = 0.9 \text{ ms}$$

75. (a) We know the two components of field due to a dipole as



$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \quad \text{and} \quad B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

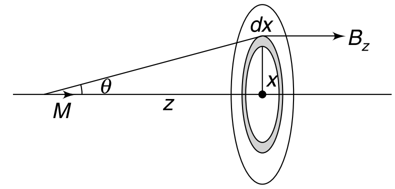
$$\therefore \quad B_z = B_r\cos\theta - B_\theta\sin\theta$$

$$= \frac{\mu_0}{4\pi r^3}[2\cos^2\theta - \sin^2\theta] \quad \text{...(i)}$$

(b) If we consider the point P to be at a distance x from the centre of the ring, then

$$\sin\theta = \frac{x}{\sqrt{z^2 + x^2}}; \ \cos\theta = \frac{z}{\sqrt{z^2 + x^2}}$$

∴ equation (i) becomes



$$B_z = \frac{\mu_0 M}{4\pi(z^2 + x^2)^{3/2}}\left[\frac{3z^2}{x^2 + z^2} - 1\right]$$

Flux through a region lying between radius $x + dx$ and x is

$$d\phi = 2\pi x dx B_z$$

∴ Flux through ring is

$$\phi = \int_{x=0}^{x=R} 2\pi x\, dx \cdot B_z$$

$$= \frac{\mu_0 M}{2}\left[3z^2\int_0^R \frac{x\,dx}{(z^2 + x^2)^{5/2}} - \int_0^R \frac{x\,dx}{(z^2 + x^2)^{3/2}}\right]$$

$$= \frac{\mu_0 M}{2}\frac{R^2}{(R^2 + z^2)^{3/2}}$$

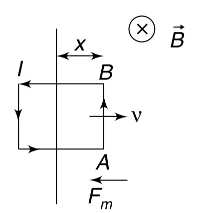
[Integration can be done by method of substitution. Put $z^2 + x^2 = t$]

(c)
$$E_{in} = \left|\frac{d\phi}{dt}\right| = \left|\frac{d\phi}{dz}\frac{dz}{dt}\right| = v\left|\frac{d\phi}{dz}\right|$$

$$= \frac{3\mu_0 MR^2 vz}{2[R^2 + z^2]^{5/2}}$$

76. Let the speed of the loop be v when it penetrates x inside the field region. Motional emf in arm AB is $\varepsilon = B\ell v$

$$\therefore \quad \text{Current} \quad I = \frac{B\ell v}{R}$$



Magnetic force (opposing the motion) on arm AB is $F_m = I\ell B = \dfrac{B^2\ell^2 v}{R}$

$$\therefore \quad M\frac{dv}{dt} = -\frac{B^2\ell^2 v}{R}$$

$$\therefore \quad \frac{dv}{dt} = -\frac{B^2\ell^2 v}{MR} \quad \text{...(i)}$$

If we write acceleration as $\dfrac{dv}{dt} = v\dfrac{dv}{dx}$ then

$$v\frac{dv}{dx} = -\frac{B^2\ell^2}{MR}v$$

$$\Rightarrow \quad dv = -\frac{B^2\ell^2}{MR}dx$$

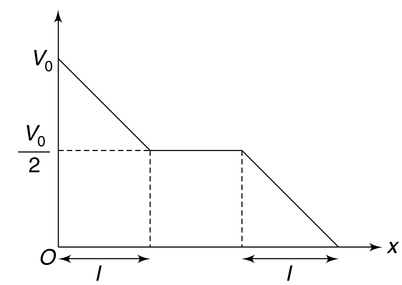
$$\Rightarrow \quad \int_{v_0}^{v} dv = -\left(\frac{B^2\ell^2}{MR}\right)\int_0^x dx \qquad [v_0 = \text{initial speed}]$$

$$v = v_0 - \left(\frac{B^2\ell^2}{MR}\right)x \qquad \text{...(ii)}$$

Speed decreases linearly with x.

When the loop in completely inside the field region; there is no current, no force and hence no change in speed. When arm AB begins to exit the field region, emf is induced again and the loop beings to retard once again.

If we wish that the loop just manages to come out of the field region then its $v - x$ graph will look as shown.



v vs x graph will have the same slope as given by equation (ii), even when the loop is exiting. Hence, the speed when it is completely inside the field region is $\frac{v_0}{2}$

From (i) $$\frac{dv}{dt} = -\frac{B^2\ell^2}{MR}v$$

$$\therefore \quad \frac{dv}{v} = -\frac{B^2\ell^2}{MR}dt$$

$$\int_{v_0}^{v_0/2}\frac{dv}{v} = -\frac{B^2\ell^2}{MR}\int_0^t dt$$

$$\ell n\left(\frac{1}{2}\right) = -\frac{B^2\ell^2}{MR}t$$

$$\therefore \quad t = \frac{MR\ell n2}{B^2\ell^2}$$

77. (a) velocity of block = v_0

Angular velocity of the pulley $\omega = \frac{v_0}{a}$

Emf developed across the circumference and centre $\varepsilon = \frac{1}{2}B\omega a^2$

Current in the resistance, $i = \frac{B\omega a^2}{2R}$

Power dissipated in R is

$$P_R = i^2R = \frac{B^2\omega^2a^4}{4R^2}$$

Mechanical power delivered to pulley = rate of change of its kinetic energy

$$P_{\text{mech}} = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\omega\frac{d\omega}{dt} \qquad \text{...(i)}$$

$P_R + P_{\text{mech}}$ = power delivered by string tension to the pulley

$$= T\cdot a\cdot\omega$$

$$= \left[Mg - M\frac{dv}{dt}\right]a\omega \qquad \left[\because \; T = Mg - M\frac{dv}{dt}\right]$$

$$\therefore \quad \frac{B^2\omega^2a^4}{4R^2} + I\omega\frac{d\omega}{dt} = Mga\omega - Ma\omega\frac{d(\omega a)}{dt} \qquad [\because \; v = \omega a]$$

$$\Rightarrow \quad [I + M\cdot a^2]\frac{d\omega}{dt} = Mga - \frac{B^2a^4\omega}{4R^2}$$

Acceleration of mass M is $\left(a\frac{d\omega}{dt}\right)$

$$\therefore \qquad a_M = \frac{Mga^2 - \dfrac{B^2a^5\omega}{4R^2}}{I + Ma^2}$$

When $$\omega = \frac{v_0}{a}$$

$$a_M = \frac{Mga^2 - \dfrac{B^2a^4v_0}{4R^2}}{I + Ma^2} \qquad \ldots\text{(i)}$$

(b) When the block acquires terminal speed $a_M = 0$

$$\Rightarrow \qquad Mga^2 = \frac{B^2a^4v_T}{4R^2}$$

$$\Rightarrow \qquad v_T = \frac{4MgR^2}{B^2a^2}$$

(c) When $\omega \cdot a = \dfrac{v_T}{2}$

$$\omega = \frac{2MgR^2}{B^2a^3}$$

$$\frac{d\omega}{dt} = \frac{a_M}{a} = \frac{Mga - \dfrac{B^2a^3}{4R^2}\cdot\dfrac{2MgR^2}{B^2a^2}}{I + Ma^2}$$

$$= \frac{Mga}{2(I + Ma^2)}$$

From (i) $P_{\text{mech}} = I\omega \dfrac{d\omega}{dt}$

$$= \frac{2IMgR^2}{B^2a^3}\cdot\frac{Mga}{2(I + Ma^2)}$$

$$= \frac{I\cdot M^2g^2R^2}{B^2a^2(I + Ma^2)}$$

78. Let ϕ = flux through the loop due to current I_0 and I = instantaneous current in the loop. The emf induced in the loop will be

$$\varepsilon = -\frac{d\phi}{dt} - L\frac{dI}{dt}$$

$$\Rightarrow \qquad IR = -\frac{d\phi}{dt} - L\frac{dI}{dt} \Rightarrow I\,dt = -\frac{1}{R}\,d\phi - \frac{L}{R}\,dI$$

$$\Rightarrow \qquad \int I\,dt = -\frac{1}{R}\int d\phi - \frac{L}{R}\int dI$$

$$\Rightarrow \qquad \Delta q = -\frac{\Delta\phi}{R} - \frac{L\Delta I}{R}$$

$$|\Delta q| = \frac{\Delta\phi + L\Delta I}{R}$$

The loop is at rest initially as well as finally; hence

$$\Delta I = I_f - I_i = 0 - 0 = 0$$

$$\therefore \qquad |\Delta q| = \left|\frac{\Delta\phi}{R}\right| = \frac{|\phi_f - \phi_i|}{R} \qquad \text{...(i)}$$

Flux in initial as well as final position can be obtained by integrating the flux over a strip of width dx.

$$d\phi = \frac{\mu_0 I_0}{2\pi x}\, a\, dx$$

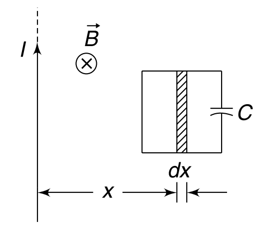
$$\phi_i = \frac{\mu_0 I_0 a}{2\pi}\int_a^{2a}\frac{dx}{x} = \frac{\mu_0 I_0 a}{2\pi}\,\ell n 2$$

$$\phi_f = -\frac{\mu_0 I_0 a}{2\pi}\int_{2a}^{3a}\frac{dx}{x} = -\frac{\mu_0 I_0 a}{2\pi}\,\ell n\left(\frac{3}{2}\right) = \frac{\mu_0 I_0 a}{2\pi}\,\ell n\left(\frac{2}{3}\right)$$

$$|\phi_f - \phi_i| = \frac{\mu_0 I_0 a}{2\pi}\,\ell n 3$$

$$\therefore \qquad |\Delta q| = \frac{\mu_0 I_0 a\,\ell n 3}{2\pi \cdot R}$$

79. (a) Magnetic field due to straight wire at a distance x from it is $B = \dfrac{\mu_0 I}{2\pi x}$



Flux linked with a strip of area $d\cdot(dx)$ [see figure] will be

$$d\phi = \frac{\mu_0 I\cdot d}{2\pi}\,\frac{dx}{x}$$

Flux through the loop

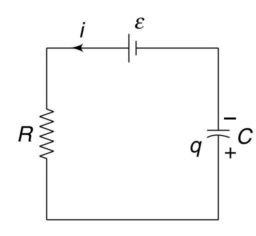
$$\phi = \int d\phi = \frac{\mu_0 I\cdot d}{2\pi}\int_a^{2a}\frac{dx}{x} = \frac{\mu_0 I\cdot d}{2\pi}\,\ell n 2$$

$$\therefore \qquad \phi = \frac{\mu_0 \alpha d\,\ell n 2}{2\pi}\cdot t \qquad [\because\ I = \alpha t]$$

Flux is time dependent. An emf is induced in the loop. Magnitude of emf is

$$\varepsilon = \frac{d\phi}{dt} = \frac{\mu_0 \alpha\cdot d\cdot \ell n 2}{2\pi}$$

From Lenz's law the sense of emf is so as to drive an anticlockwise current in the loop. Let charge on the capacitor and current in the loop be q and i at any time t



$$iR + \frac{q}{C} = \varepsilon$$

$$\therefore \qquad R\frac{dq}{dt} + \frac{q}{C} = \varepsilon$$

$$\therefore \qquad \int_0^q \frac{dq}{\varepsilon - \dfrac{q}{C}} = \frac{1}{R}\int_0^t dt$$

$$\Rightarrow \qquad \left[\ell n\left(\varepsilon - \frac{q}{C}\right)\right]_0^q = -\frac{1}{RC}\,t$$

$$\Rightarrow \qquad \ell n\left(\varepsilon - \frac{q}{C}\right) - \ell n\varepsilon = -\frac{t}{RC} \quad \Rightarrow \quad \ell n\left(1 - \frac{q}{C\varepsilon}\right) = -\frac{t}{RC}$$

$$\therefore \qquad q = C\varepsilon[1 - e^{-t/RC}] = \frac{\mu_0 \alpha\cdot d\cdot \ell n 2 C}{2\pi}(1 - e^{-t/RC})$$

(b) $$i = \frac{dq}{dt} = \frac{\mu_0 \alpha \cdot d \cdot \ell n2}{2\pi R} e^{-t/RC}$$

∴ Heat generated

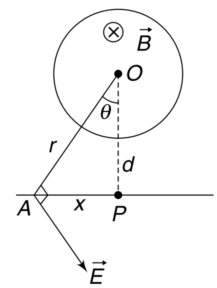
$$H = \int_0^t i^2 R\, dt = \left(\frac{\mu_0 \alpha \cdot d \cdot \ell n2}{2\pi R}\right)^2 \cdot R \int_0^t e^{-\frac{2t}{RC}} dt$$

$$= \left(\frac{\mu_0 \alpha \cdot d \cdot \ell n2}{\pi}\right)^2 \cdot \frac{C}{8}\left[1 - e^{-\frac{2t}{RC}}\right] \Rightarrow H = \frac{\varepsilon^2 C}{2}\left(1 - e^{-\frac{2t}{RC}}\right)$$

(d) The time changing current in the loop opposes the current in straight wire. The source driving the current in the straight wire has to do more work maintaining or increasing the current. This work done provides energy to the capacitor and produces heat as well.

80. (i) The induced electric field will be perpendicular to OP at all points in this path. Hence the particle experiences no force and work done in moving it to infinity is zero.

(ii) Induced electric field at a point A shown in the figure can be calculated as



$$E \cdot 2\pi r = \pi a^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{a^2 \alpha}{2r}$$

Electric force on the charge along AP is

$$F_e = qE\cos\theta = \frac{qa^2 \alpha \cdot d}{2r^2}$$

The external agent must apply equal and opposite force to keep the charge moving without gaining any kinetic energy. Work done by the external agent in small displacement dx will be

$$dW = \frac{qa^2 \alpha d}{2r^2} dx$$

But $$x = d\tan\theta \Rightarrow dx = d\sec^2\theta\, d\theta \text{ and } r = d\sec\theta$$

∴ $$dW = \frac{qa^2 \alpha d}{2(d\sec\theta)^2} \cdot d\sec^2\theta\, d\theta = \frac{qa^2\alpha}{2} d\theta$$

∴ $$W = \frac{qa^2\alpha}{2}\int_0^{\pi/2} d\theta = \frac{\pi q a^2 \alpha}{4}$$

81. Charge on capacitor $Q_0 = CV_0$

After the switch is put to position 2, there is a current through the bar and thereby a magnetic force acts on it, pushing it away from the capacitor. The bar accelerates, its speed increases. Now, there is an induced emf in the bar which is increasing with increasing speed. At the instant induced emf becomes equal to the potential difference across the capacitor, the current will become zero. At this instant speed of the bar is maximum. As per the question, the rails come to an end and the bar is thrown out.

Let q_0 = charge on the capacitor when the bar acquires maximum speed (u_{max}).

$$BLu_{max} = \frac{q_0}{C} \quad ...(i)$$

Force equation for moving bar is

$$m\frac{du}{dt} = BiL$$

$$\Rightarrow \quad m\frac{du}{dt} = BL\left(-\frac{dq}{dt}\right) \Rightarrow mdu = -BLdq$$

$$\Rightarrow \quad m\int_0^{u_{max}} du = -BL\int_{Q_0}^{q_0} dq \Rightarrow mu_{max} = BL(Q_0 - q_0)$$

$$\Rightarrow \quad mu_{max} = BLQ_0 - B^2L^2Cu_{max}$$

$$\Rightarrow \quad u_{max} = \frac{BLQ_0}{m + B^2L^2C} = \frac{BLCV_0}{m + B^2L^2C}$$

$\therefore$ Kinetic energy

$$K = \frac{1}{2}mu_{max}^2 = \frac{m}{2}\left(\frac{BLCV_0}{m + B^2L^2C}\right)^2$$

Energy spent by the battery in charging the capacitor is $Q_0V_0 = CV_0^2$

$\therefore$ Efficiency is

$$\eta = \frac{K}{CV_0^2} = \frac{mB^2L^2C}{2(m + B^2L^2C)^2}$$

82. Number of turns in a layer of radius r and thickness dr is $dN = \frac{N\,dr}{(b-a)}$

Field produced due to such a layer is

$$dB = \frac{\mu_0 dNI}{L} = \frac{\mu_0 NI}{(b-a)L}dr$$

For $r < a$, the resultant field due to all layers is

$$B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{(b-a)L}(b-a) = \frac{\mu_0 NI}{L}$$

For $a < r < b$

$$B = \int_r^b dB = \frac{\mu_0 NI}{(b-a)L}(b-r)$$

For $r > b$ $\quad B = 0$

$\therefore$ Flux linked with a single turn of radius r is

$$\phi_0 = \frac{\mu_0 NI}{L}\pi a^2 + \frac{\mu_0 NI}{(b-a)L}\int_a^r (b-r)\,2\pi r\,dr$$

$$= \frac{\mu_0 \pi NI(3br^2 - 2r^3 - a^3)}{3(b-a)L}$$

$\therefore$ Total flux linked with the coil is

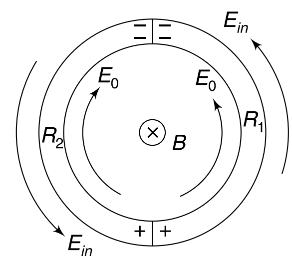
$$\phi = \int_{r=a}^{b} \phi_0\,dN = \frac{\mu_0 \pi N^2 I}{3(b-a)^2 L}\int_a^b (3br^2 - 2r^3 - a^3)\,dr$$

$$= \frac{\mu_0 \pi N^2(3a^2 + 2ab + b^2)}{6L} I$$

$$\Rightarrow \quad L_{self} = \frac{\mu_0 \pi N^2(3a^2 + 2ab + b^2)}{6L}$$

83. (a) Current (and hence current density) in the entire loop must be same.

From microscopic form of Ohm's law we can write



$$\sigma_1 E_1 = \sigma_2 E_2$$

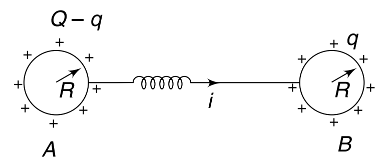
$$\Rightarrow \quad R_2 E_1 = R_1 E_2 \quad \Rightarrow \quad \frac{E_1}{E_2} = \frac{R_1}{R_2}$$

(b) The induced electric field must be uniform everywhere in the circular conductor. It is given by

$$2\pi a E_{in} = \pi a^2 \frac{dB}{dt} \quad \Rightarrow \quad E_{in} = \frac{a}{2}\left(\frac{dB}{dt}\right)$$

There is accumulation of charge at the junctions which produce additional electric field in the conductor. If $R_1 > R_2$ then $E_1 > E_2$. In this case the charge at the upper junction is negative and charge at the lower junction will be positive. In figure E_0 is electric field due to charge.

84. Let q = charge on B at time 't'



$$V_A - V_B = L\frac{di}{dt}$$

$$\Rightarrow \quad K\frac{Q-q}{R} - K\frac{q}{R} = L\frac{d^2q}{dt^2}$$

[Spheres are at large distance. Hence charge on one does not affect the potential of other]

$$\Rightarrow \quad \frac{d^2q}{dt^2} = -\frac{2K}{LR}\left[q - \frac{Q}{2}\right] \qquad \text{...(i)}$$

Let $q - \frac{Q}{2} = x$ which means $\frac{d^2q}{dt^2} = \frac{d^2x}{dt^2}$

In terms of x, the differential equation (i) becomes

$$\frac{d^2x}{dt^2} = -\frac{2K}{LR}x$$

Solution to this equation is

$$x = x_0 \sin(\omega t + \delta) \qquad \left[\omega = \sqrt{\frac{2K}{LR}}\right]$$

$$\Rightarrow \quad q - \frac{Q}{2} = x_0 \sin(\omega t + \delta)$$

$$\Rightarrow \quad q = \frac{Q}{2} + x_0 \sin(\omega t + \delta)$$

Current $$i = \frac{dq}{dt} = x_0 \omega \cos(\omega t + \delta)$$

Just after closing the switch (at $t = 0^+$) the current is zero

$$\therefore \quad \delta = \frac{\pi}{2}$$

$$\therefore \quad q = \frac{Q}{2} + x_0 \cos \omega t$$

Also, at $t = 0$, $q = 0$

$$\therefore \quad x_0 = -Q/2$$

$$\therefore \quad q = \frac{Q}{2}[1 - \cos \omega t]$$

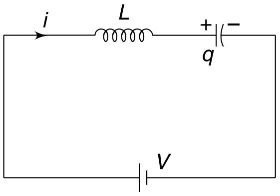
$$q = \frac{Q}{2} \text{ means } \cos \omega t = 0$$

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad \sqrt{\frac{2K}{LR}}\, t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{\pi}{2}\sqrt{\frac{LR}{2K}}$$

85. Let i be current and q charge on the capacitor at time 't'



$$L\frac{di}{dt} + \frac{q}{C} = V$$

Differentiating wrt time we get $L\dfrac{d^2i}{dt^2} = -\dfrac{i}{C}$

Solution to this equation is $i = i_0 \sin(\omega t + \delta)$ $\left[\omega = \dfrac{1}{\sqrt{LC}}\right]$

At $t = 0$, $i = 0$ $\therefore$ $\delta = 0$

$$\therefore \quad i = i_0 \sin \omega t \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{dq}{dt} = i_0 \sin \omega t$$

$$\Rightarrow \quad \int dq = i_0 \int \sin \omega t\, dt$$

$$\Rightarrow \quad q = -\frac{i_0}{\omega}\cos \omega t + c$$

At $t = 0$, $q = 0$ $\therefore$ $c = \dfrac{i_0}{\omega}$

$$\therefore \quad q = \frac{i_0}{\omega}[1 - \cos \omega t] \qquad \text{...(ii)}$$

Maximum charge on capacitor $q_0 = \dfrac{2i_0}{\omega}$

Charge is maximum when entire work done by the battery is stored as energy in the electric field of the capacitor.

$$Vq_0 = \frac{q_0^2}{2C}$$

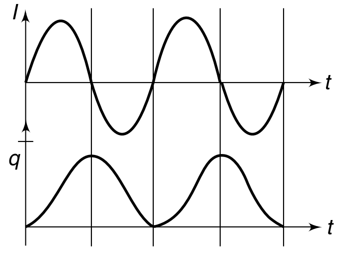
$$\therefore \quad q_0 = 2CV$$

When charge on capacitor is maximum, potential difference across it is 2 V. Induced emf in the inductor at this instant is V

$$\therefore \quad q = CV[1 - \cos \omega t] = 2CV\sin^2\left(\frac{\omega t}{2}\right)$$

$$i = i_0 \sin \omega t$$

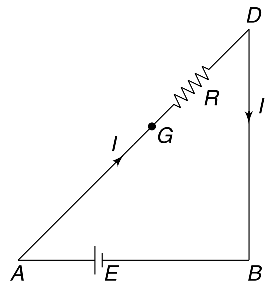
Graphs are as shown.



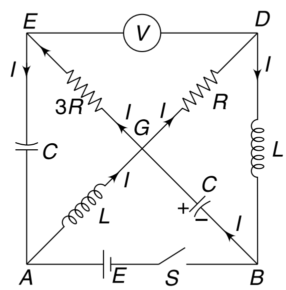
86. (a) There is no current through capacitors, voltmeters and inductors are zero resistance, when the circuit is in steady state. The effective circuit is as shown.

$$V_E = V_G \quad \text{and} \quad V_G - V_D = E$$

$\therefore$ Reading of voltmeter $= E$



(b) The current through inductors and the voltages across the capacitors cannot change immediately. Current, before the switch is opened, is $I = \frac{E}{R}$ through both inductors.



After opening the switch, the current in R and the inductor between B and D must be same (since voltmeter does not conduct). It implies that current through R is still I and $V_G - V_D = RI = E$

The current I in GD will loop through $GDBG$. The current I in the inductor between A and G must loop through $AGEA$.

$$\therefore \quad V_{GE} = 3R \cdot I = 3E$$

$$\therefore \quad V_D - V_E = 2E$$

(c) When switch was opened the capacitor between A and E was uncharged. The circuit is effectively two disjoint loops – BGD and AGE

Energy stored in L and C gets dissipated in R.

$$\therefore \quad U_R = \frac{1}{2}LI^2 + \frac{1}{2}CE^2$$

$$= \frac{1}{2}L\left(\frac{E}{R}\right)^2 + \frac{1}{2}CE^2 = \frac{1}{2}E^2\left(\frac{L}{R^2} + C\right)$$