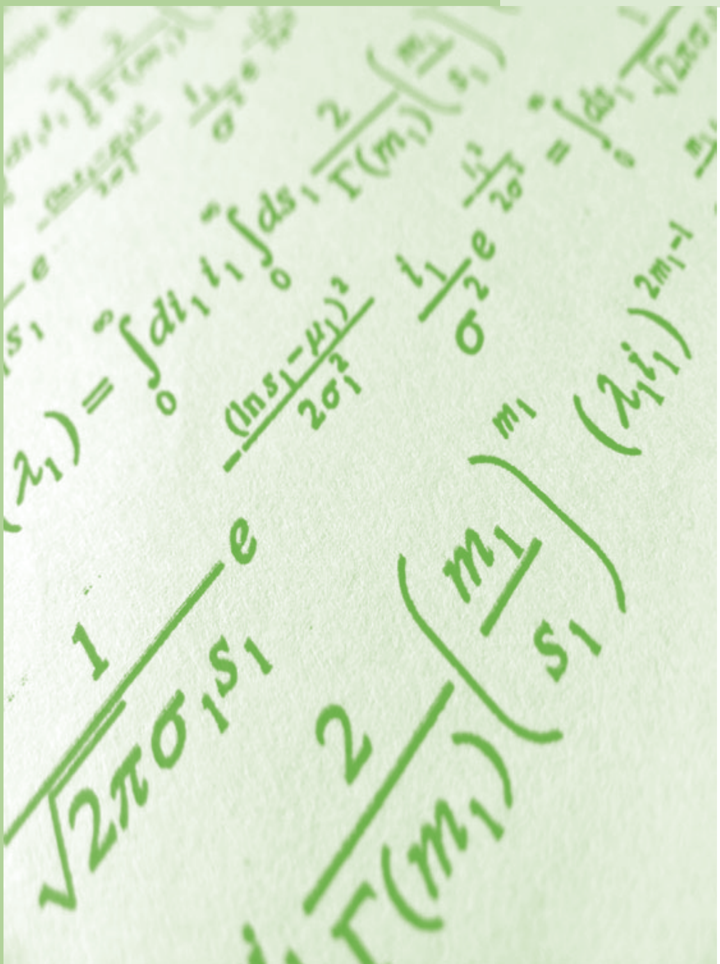
Chapter 14

Mensuration



REMEMBER

Before beginning this chapter, you should be able to:

- Explain polygons and its properties
- Use circles, circumference of a circle, sector of a circle–in mathematical problems

KEY IDEAS

After completing this chapter, you would be able to:

- Calculate perimeters and areas of plane figures
- Study types of prisms, and pyramids and calculate lateral, total surface areas and volumes of prisms and pyramids
- Compute curved, total surface area and volume of cylinder and cone
- Obtain surface area and volume of sphere, hemisphere and hollow sphere

INTRODUCTION

Mensuration is a branch of mathematics that deals with the computation of geometric magnitudes, such as the length of a line, the area of a surface and the volume of a solid. In this chapter we will deal with the areas and volumes of three dimensional figures like prisms, pyramids, cones, spheres, hemispheres, etc. However, some problems on plane figures like circles, sectors, segments, etc., as an exercise of revision.

Circle and Semi-Circle

1. Area of circle = πr^2 sq. units.
2. Area of the semi circle = $\dfrac{\pi r^2}{2}$ sq. units.
3. Circumference of the circle = $2\pi r$ units = πd units
4. Circumference of the semicircle = $(\pi + 2)r$ units
   $= \dfrac{36r}{7}$ units

   (Where r is radius and d is diameter)

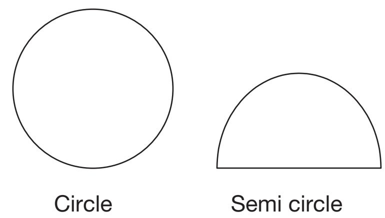


Figure 14.1

Circular Ring

Area of the ring = $\pi(R^2 - r^2) = \pi(R + r)(R - r)$
(Where R and r are outer radius and inner radius of a ring and $(R - r)$ is the width of the ring)

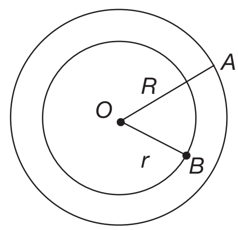


Figure 14.2

Sectors and Segements

1. Length of the arc ACB
   $l = \left(\dfrac{\theta}{360°}\right) 2\pi r$ units.
2. Area of the sector $AOBC$
   $A = \left(\dfrac{\theta}{360°}\right)\pi r^2$ sq. units.
3. Perimeter of the sector = $(l + 2r)$ units.
4. Area of the segment ACB = (A − Area of the ΔAOB) sq. units.
5. Perimeter of the segment ACB = (length of arc $ACB + AB$) units.

   (Where r is the radius of the circle and θ is sector angle)

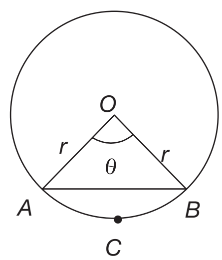
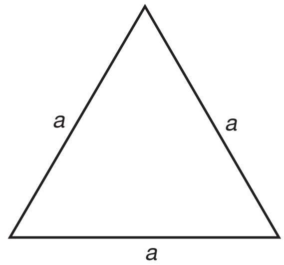


Figure 14.3

Rotations Made by a Wheel

1. Distance covered by a wheel in one revolution = Circumference of the wheel.
2. Number of rotations made by a wheel in unit time $= \dfrac{\text{Distance covered by it in unit time}}{\text{Circumference of the wheel}}$.
3. Angle made by minute hand in one minute $= \dfrac{360°}{60} = 6°$.
4. Angle made by hour hand in one minute $= \dfrac{30°}{60} = \left(\dfrac{1}{2}\right)^{\circ}$.

Equilateral Triangle



1. Circumference of an equilateral triangle = $3a$ units.
2. Area of the equilateral triangle $= \dfrac{\sqrt{3}a^2}{4}$ sq. units.
3. Height of the equilateral triangle $= \dfrac{\sqrt{3}a}{2}$ units.
4. Radius of in-circle of equilateral triangle $= \dfrac{1}{3}\left(\dfrac{\sqrt{3}a}{2}\right) = \dfrac{a}{2\sqrt{3}}$ units.
5. Circum-radius of equilateral triangle $= \dfrac{2}{3}\left(\dfrac{\sqrt{3}a}{2}\right) = \dfrac{a}{\sqrt{3}}$ units.

(Where a is side of the triangle)

EXAMPLE 14.1

The hour hand of a clock is 6 cm long. Find the area swept by it between 11:20 am and 11:55 am. (in cm^2)

(a) 2.75 **(b)** 5.5 **(c)** 11 **(d)** None of these

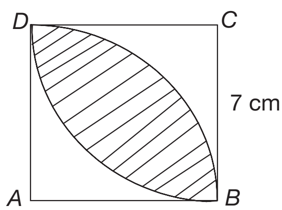
SOLUTION

Angle made by the hours hand of the clock in 35 minutes is 17.5° (angle of sector).

EXAMPLE 14.2

In the figure given below, $ABCD$ is a square of side 7 cm. BD is an arc of a circle of radius AB. What is the area of the shaded region?

(a) 14 cm^2 **(b)** 21 cm^2 **(c)** 28 cm^2 **(d)** 35 cm^2



SOLUTION

Area of shaded region = 2(Area of sector $\overline{BAD}$ $-$ Area of ΔABD).

PRISMS

Prism is a solid in which two congruent and parallel polygons form the top and the bottom faces. The lateral faces are parallelograms.

The line joining the centres of the two parallel polygons is called the axis of the prism and the length of the axis is referred to as the height of prism.

If two parallel and congruent polygons are regular and if the axis is perpendicular to the base, then the prism is called a right prism. The lateral surfaces of a right prism are rectangles.

Consider two congruent and parallel triangular planes ABC and PQR. If we join the corresponding vertices of both the planes, i.e., A to P, B to Q and C to R, then the resultant solid formed is a triangular prism. A right prism, the base of which is a rectangle is called a cuboid and the one, the base of which is a pentagon is called a pentagonal prism. If all the faces of the solid are congruent, it is a cube. In case of a cube or a cuboid, any face may be the base of the prism. A prism whose base and top faces are squares but the lateral faces are rectangular is called a square prism.

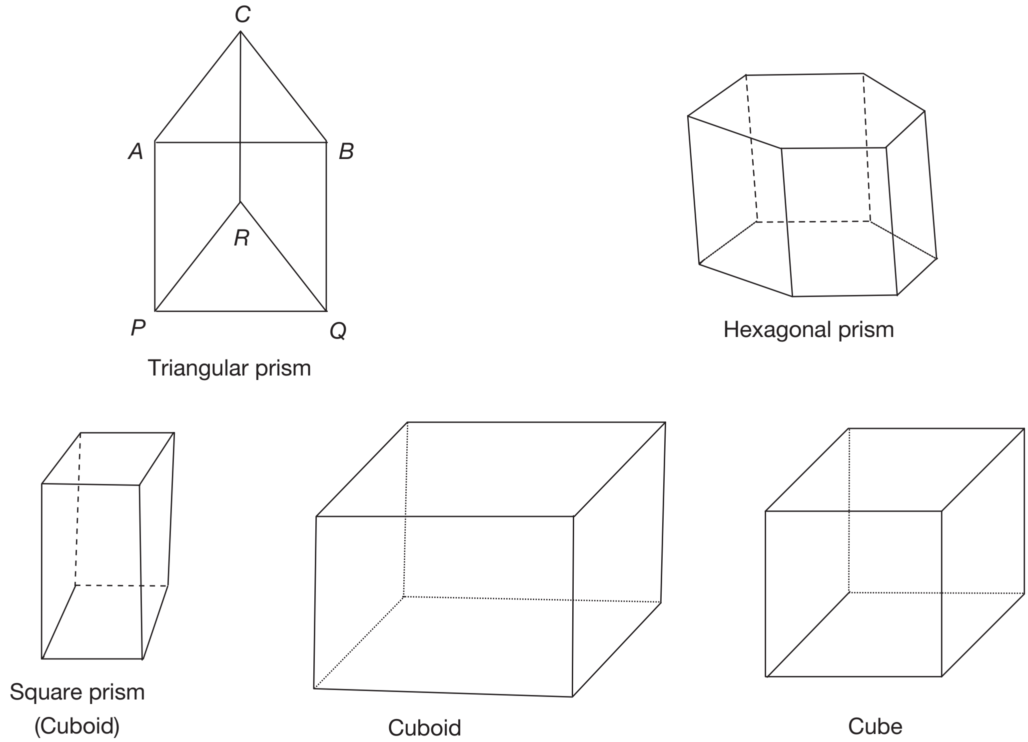


Figure 14.4

Notes The following points hold good for all prisms.

1. The number of lateral faces = the number of sides of the base.
2. The number of edges of a prism = number of sides of the base × 3.
3. The sum of the lengths of the edges = 2(perimeter of base) + number of sides × height.

Lateral Surface Area (LSA) of a Prism

LSA = Perimeter of base × height = ph

Total Surface Area (TSA) of a Prism

TSA = LSA + 2(area of base)

Volume of a Prism

Volume = Area of base × height = Ah

Note The volume of water flowing in a canal = The cross section area of the canal × The speed of water.

EXAMPLE 14.3

The base of a right prism is a right angled triangle. The measure of the base of the right angled triangle is 3 m and its height 4 m. If the height of the prism is 7 m, then find

(a) the number of edges of the prism.

(b) the volume of the prism.

(c) the total surface area of the prism.

SOLUTION

(a) The number of the edges = The number of sides of the base × 3 = 3 × 3 = 9.

(b) The volume of the prism = Area of the base × Height of the prism $= \frac{1}{2}(3 \times 4) \times 7 = 42$ m^3.

(c) TSA = LSA + 2(area of base)

= ph + 2(area of base)

where, p = perimeter of the base = sum of lengths of the sides of the given triangle.

As, hypotenuse of the triangle $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ m

∴ Perimeter of the base = 3 + 4 + 5 = 12 m

$\Rightarrow$ LSA = ph = 12 × 7 = 84 m^2.

TSA = LSA + 2(area of base) = $84 + 2\left(\frac{1}{2} \times 3 \times 4\right) = 84 + 12 = 96$ m^2.

CUBES AND CUBOIDS

Cuboid

In a right prism, if the base is a rectangle, then it is called a cuboid. A match box, a brick, a room, etc., are in the shape of a cuboid.

The three dimensions of the cuboid, its length (l), breadth (b) and height (h) are generally denoted by $l \times b \times h$.

1. The lateral surface area of a cuboid = $ph = 2(l + b)h$ sq. units, where p is the perimeter of the base.
2. The total surface area of a cuboid = LSA + 2(base area) = $2(l + b)h + 2lb = 2(lb + bh + lh)$ sq. units.
3. The volume of a cuboid = $Ah = (lb)h = lbh$ cubic units, where A is the area of the base.
4. Diagonal of cuboid $= \sqrt{l^2 + b^2 + h^2}$ units

Note If a box made of wood of thickness t has inner dimensions of l, b and h, then

the outer length = $l + 2t$,

the outer breadth = $b + 2t$ and

the outer height = $h + 2t$.

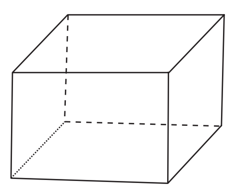


Figure 14.5 Cuboid

Cube

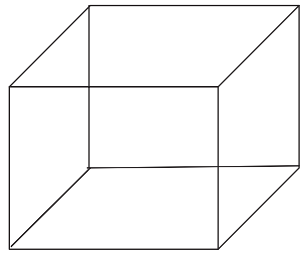


Figure 14.6 cube

In a cuboid, if all the dimensions, i.e., its length, breadth and height are equal, then the solid is called a cube.

All the edges of a cube are equal in length and each edge is called the side of the cube.

Thus, the size of a cube is completely determined by its side.

If the side of cube is 'a' units, then

1. The lateral surface area of a cube = $4a^2$ sq. units.
2. The total surface area of a cube = LSA + 2(area of base) = $4a^2 + 2a^2 = 6a^2$ sq. units.
3. The volume of a cube = a^3 cubic units.
4. The diagonal of a cube $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3}\, a$ units.

Note If the inner edge of a cube made of wood of thickness 't' is 'a' units, then the outer edge of the cube is given by $(a + 2t)$ units.

EXAMPLE 14.4

The dimensions of a room are 12 m × 7 m × 5 m. Find

(a) the diagonal of the room.

(b) the cost of flooring at the rate of ₹2 per m^2.

(c) the cost of whitewashing the room excluding the floor at the rate of ₹3 per m^2.

SOLUTION

(a) The diagonal of the room $= \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 7^2 + 5^2} = \sqrt{144 + 49 + 25} = \sqrt{218}$ m.

(b) To find the cost of the flooring, we should know the area of the base.

Base area = $lb = 12 \times 7 = 84$ m^2

$\therefore$ The cost of flooring = 84×2 = ₹168.

(c) The total area that is to be whitewashed

= LSA + Area of roof = $2(l + b)h + lb$

$= 2(12 + 7)5 + 12 \times 7 = 2(19)\ (5) + 84$

$= 190 + 84$

$= 274$ m^2

$\therefore$ The cost of whitewashing = 274×3 = ₹822.

EXAMPLE 14.5

A box is in the form of a cube. Its edge is 5 m long. Find

(a) the total length of the edges.

(b) the cost of painting the outside of the box, on all the surfaces, at the rate of ₹5 per m^2.

(c) the volume of liquid which the box can hold.

SOLUTION

(a) Length of edges = Number of edges of base $\times$ 3 $\times$ Length of each edge = $4 \times 3 \times 5 = 60$ m.

(b) To find the cost of painting the box, we need to find the total surface area.

TSA = $6a^2 = 6 \times 5^2 = 6 \times 25 = 150$ m^2

$\therefore$ Cost of painting = 150×5 = ₹750.

(c) Volume = $a^3 = 5^3 = 125$ m^3.

EXAMPLE 14.6

The sum of the length, breadth and the height of a cuboid is $5\sqrt{3}$ cm and length of its diagonal is $3\sqrt{5}$ cm. Find the total surface area of the cuboid.

(a) 30 cm^2 **(b)** 20 cm^2 **(c)** 15 cm^2 **(d)** 18 cm^2

HINTS

(i) Use suitable algebraic identity to find the LSA of the cuboid.

(ii) $l + b + h = 5\sqrt{3}$ and $l^2 + b^2 + h^2 = 3\sqrt{5}$

(iii) Square the first equation and evaluate $2(lb + bh + hl)$.

RIGHT CIRCULAR CYLINDER

A cylinder has two congruent and parallel circular planes which are connected by a curved surface. Each of the circular planes is called the base of the cylinder. A road roller, water pipe, power cables, round pillars are some of the objects which are in the shape of a cylinder.

In the Fig. 14.7, a right circular cylinder is shown. Let A be the centre of the top face and A^1 be the centre of the base. The line joining the centres (i.e., AA^1) is called the axis of cylinder. The length AA^1 is called the height of the cylinder. If the axis is perpendicular to the base, then it is a right circular cylinder. The radius r of the base of the cylinder and the height h, completely describe the cylinder.

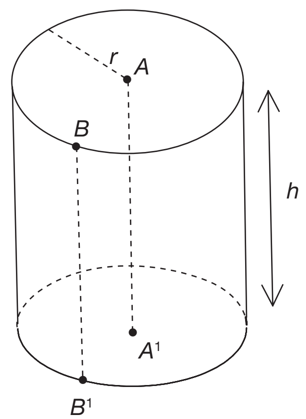


Figure 14.7

Lateral (curved) surface area = Perimeter of base $\times$ Height $= 2\pi rh$ sq. units.

The total surface area = LSA + 2(Base area) = $2\pi rh + 2(\pi r^2)$ $= 2\pi r(h + r)$ sq. units.

Volume = Area of base $\times$ Height = $\pi r^2 h$ cubic units.

Hollow Cylinder

The part of a cylinder from which a smaller cylinder of the same axis is cut out is a hollow cylinder. Let R and r be the external and internal radii of the hollow cylinder and h be the height.

Volume of the material used = $\pi R^2 h - \pi r^2 h = \pi h(R + r)(R - r)$ cubic units.

Curved surface area = $2\pi Rh + 2\pi rh = 2\pi h(R + r)$ sq. units.

Total surface area = Curved surface area + Area of the two ends.

$= 2\pi h(R + r) + 2\pi(R^2 - r^2) = 2\pi(R + r)(R - r + h)$ sq. units.

Note If a plastic pipe of length l is such that its outer radius is R and the inner radius is r, then the volume of the plastic content of the pipe = $l\pi(R^2 - r^2)$ cubic units.

EXAMPLE 14.7

A closed cylindrical container, the radius of which is 7 cm and height 10 cm is to be made out of a metal sheet. Find

(a) the area of metal sheet required.

(b) the volume of the cylinder made.

(c) the cost of painting the lateral surface of the cylinder at the rate of ₹4 per cm^2.

SOLUTION

(a) The area of the metal sheet required = The total surface area of the cylinder = $2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7(7 + 10) = 44(17) = 748 \text{ cm}^2.$$

(b) Volume = $\pi r^2 h = \dfrac{22}{7} \times 7 \times 7 \times 10 = 22 \times 70 = 1540 \text{ cm}^3$.

(c) To find the cost of painting the lateral surface, we need to find the curved (lateral) surface area.

$$\therefore \text{LSA} = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 10 = 440 \text{ cm}^2.$$

Cost of painting = 440 × 4 = ₹1760.

EXAMPLE 14.8

A cylindrical tank with radius 60 cm is being filled by a circular pipe with internal diameter of 4 cm at the rate of 11 m/s. Find the height of the water column in 18 minutes.

(a) 66 m **(b)** 12.2 m **(c)** 13.2 m **(d)** 6.1 m

HINT

Volume of water in the tank = Area of the cross sections of the pipe × rate × time.

PYRAMID

A pyramid is a solid obtained by joining the vertices of a polygon to a point in the space by straight lines. The base of the solid obtained is the polygon and lateral faces are triangles. The fixed point in space where all the triangles (i.e., lateral faces) meet is called its vertex.

In the Fig. 14.8, the base *ABCD* is a quadrilateral. All the vertices of the base are joined to a fixed point *O* in space, by straight lines. The resultant solid obtained is called a pyramid.

The straight line joining the vertex and the centre of the base is called the axis of the pyramid. If the axis is not perpendicular to the base, it is an oblique pyramid.

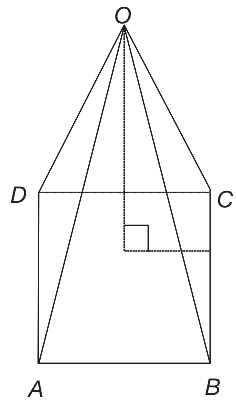


Figure 14.8

Right Pyramid

If the base of a pyramid is a regular polygon and if the line joining the vertex to the centre of the base is perpendicular to the base, then the pyramid is called a right pyramid.

The length of the line segment joining the vertex to the centre of the base of a right pyramid is called the height of the pyramid and is represented by 'h'.

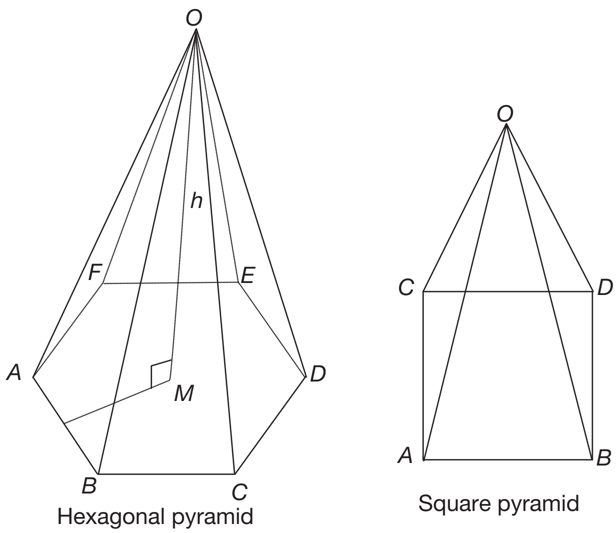


Figure 14.9

The perpendicular distance between the vertex and the mid-point of any of the sides of the base (i.e., regular polygon) of a right pyramid is called its slant height and is represented by 'l'.

For a right pyramid with perimeter of base = p, height = h and slant height = l,

1. Lateral surface area = $\frac{1}{2}$ (Perimeter of base) × (Slant height) = $\frac{1}{2}pl$.
2. Total surface area = Lateral surface area + Area of base.
3. Volume of a pyramid = $\frac{1}{3}$ × Area of base × Height.

EXAMPLE 14.9

An hexagonal pyramid is 20 m high. Side of the base is 5 m. Find the volume and the slant height of the pyramid.

SOLUTION

Given $h = 20$ m,

Side of base = $a = 5$ m

$\therefore$ Area of base $= \dfrac{\sqrt{3}}{4} \times a^2 \times 6 = \dfrac{6\sqrt{3}}{4} \times 5^2 = \dfrac{3\sqrt{3}}{2} \times 25 \text{ m}^2.$

Volume $= \dfrac{1}{3} Ah$, where A = area of the base and h = height

$= \dfrac{1}{3} \times \dfrac{3\sqrt{3}}{2}(25) \times 20 = \sqrt{3} \times 250 = 250\sqrt{3} \text{ m}^3.$

To find slant height, refer to the figure shown. In the figure,

O is the vertex of the pyramid and G is the centre of the hexagonal base. H is the mid-point of AB.

OG is the axis of the pyramid.

OH is the slant height of the pyramid.

ΔOGH is a right angled triangle.

$\therefore\ OH^2 = GH^2 + OG^2$

GH = altitude of $\Delta AGB = \dfrac{\sqrt{3}}{2} a = \dfrac{\sqrt{3}}{2} \times 5 = \dfrac{5\sqrt{3}}{2}$ m

$$\therefore\ OH^2 = \left(\frac{5\sqrt{3}}{2}\right)^2 + (20)^2 = \frac{25 \times 3}{4} + 400$$

$$= \frac{75 + 1600}{4} = \frac{1675}{4}$$

$$\Rightarrow \quad OH = \frac{\sqrt{1675}}{2} \text{ m}$$

$\therefore$ Slant height $= \dfrac{\sqrt{1675}}{2}$ m.

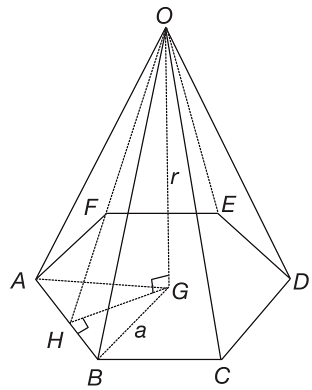


Figure 14.10

CONE

A cone is a solid pointed figure with a circular base. A cone is a kind of pyramid whose base is a circle.

A cone has one vertex, one plane surface (i.e., the base) and a curved surface. (i.e., the lateral surface).

The line joining the vertex to the centre of base (i.e., AO) is called the axis of the cone. The length of the line segment AO is called the height or perpendicular height of the cone. An ice cream cone and a conical tent are some of the examples of conical objects.

Right Circular Cone

In a cone, if the line joining the vertex and the centre of the base of the cone is perpendicular to the base, then it is a right circular cone. In other words, if the axis of the cone is perpendicular to the base of the cone,then it is a right circular cone. We generally deal with problems on right circular cones.

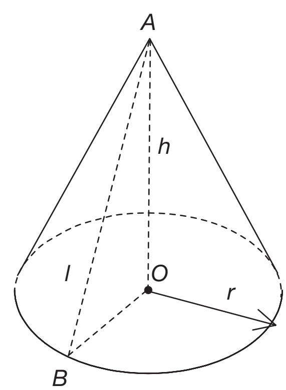


Figure 14.11 Cone

A cone is generally defined as a solid obtained by the revolution of a right angled triangle about one of its two perpendicular sides.

If we consider any point B on the periphery of the base of the cone, then the line joining B and the vertex A is called the slant height of the cone and is denoted by 'l'.

From the figure it is clear that ΔAOB is right angled.

$\therefore l = \sqrt{r^2 + h^2}$.

Hollow Cone

In earlier classes we have studied about sector. We may recall that sector is an area bounded by an arc of a circle and its two radii. (as shown in Fig. 14.12)

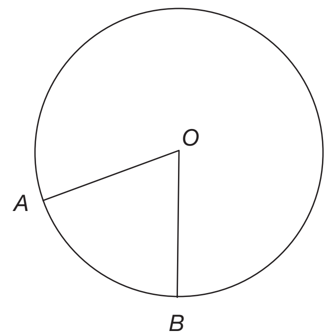


Figure 14.12

Now consider the sector AOB. If we roll the sector up and bring (join) together the radii OA and OB such that they coincide, then the figure formed is called a hollow cone. The radius of the circle becomes the slant height of the cone and the length of the arc of the sector becomes the perimeter of the base of the cone.

For a cone of radius r, height h and slant height l,

1. Curved surface area of a cone = πrl sq. units.
2. Total surface area of a cone = curved surface area + area of base = $\pi rl + \pi r^2 = \pi r(r + l)$ sq. units.
3. Volume of a cone = $\frac{1}{3}\pi r^2 h$ cubic units.

EXAMPLE 14.10

Find the volume of the greatest right circular cone, which can be cut from a cube of a side 4 cm. (in cm^3).

(a) $\frac{12\pi}{5}$ **(b)** $\frac{20\pi}{3}$ **(c)** $\frac{18\pi}{5}$ **(d)** $\frac{16\pi}{3}$

SOLUTION

Let the diameter of cone be the edge of the square

$\therefore l = 4$ cm

$h = 4$ cm

$r = 2$ cm

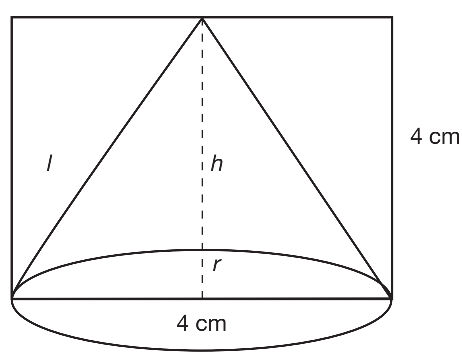
Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi (2)^2 \cdot 4$

$\Rightarrow V = \frac{1}{3}\pi \cdot 16$

$\therefore V = \frac{16}{3}\pi$ cm^3

$\therefore$ Correct option is (d).



Cone Frustum (or a Conical Bucket)

If a right circular cone is cut by a plane perpendicular to its axis (i.e., a plane parallel to the base), then the solid portion containing the base of the cone is called the frustum of the cone.

From the Fig. 14.13, we observe that a frustum is in the shape of a bucket.

Let,

Radius of upper base be R,

Radius of lower base = r,

Height of frustum = h,

Slant height of frustum = l

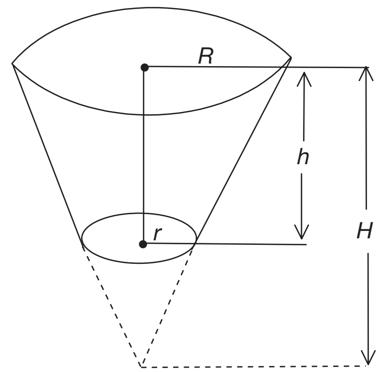


Figure 14.13

1. Curved surface area of a frustum = $\pi l(R + r)$ sq. units.
2. Total surface area of a frustum = curved surface area + area of upper base + area of lower base = $\pi l(R + r) + \pi r^2 + \pi R^2$ sq. units.
3. Volume of a frustum = $\frac{1}{3}\pi h(R^2 + Rr + r^2)$ cubic units.
4. Slant height (l) of a frustum $= \sqrt{(R-r)^2 + h^2}$ units.

EXAMPLE 14.11

A joker's cap is in the form of a cone of radius 7 cm and height 24 cm. Find the area of the cardboard required to make the cap.

SOLUTION

Area of the cardboard required = curved surface area of the cap (or cone) = πrl

Now, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm

$\Rightarrow$ Curved surface area = $\frac{22}{7} \times 7 \times 25 = 550$ cm^2

$\therefore$ Area of the cardboard required = 550 cm^2.

EXAMPLE 14.12

The diameter of an ice-cream cone is 7 cm and its height is 12 cm. Find the volume of ice-cream that the cone can contain.

SOLUTION

Volume of ice-cream $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 12 = 22 \times 7 = 154$ cm^3.

EXAMPLE 14.13

The diameters of top and bottom portions of a milk can are 56 cm and 14 cm respectively. The height of the can is 72 cm. Find the

(a) area of metal sheet required to make the can (with out lid).

(b) volume of milk which the container can hold.

SOLUTION

The milk can is in the shape of a frustum with $R = 28$ cm, $r = 7$ cm and $h = 72$ cm.

(a) Area of metal sheet required = curved surface area + area of bottom base

$= \pi l(R + r) + \pi r^2$.

Now, $l = \sqrt{(R - r^2) + h^2} = \sqrt{(28-7)^2 + 72^2} = \sqrt{21^2 + 72^2} = \sqrt{9(7^2 + 24^2)}$

$= 3\sqrt{49 + 576} = 3 \times \sqrt{625} = 3 \times 25 = 75$ cm

$\therefore$ Area of metal sheet $= \dfrac{22}{7} \times 75(28+7) + \dfrac{22}{7} \times 7^2 = 22 \times 75 \times 5 + 22 \times 7$

$= 22(375 + 7)$

$= 22(382) = 8404 \text{ cm}^2$.

(b) Amount of milk which the container can hold $= \dfrac{1}{3}\pi h(R^2 + Rr + r^2)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 72(28^2 + 7 \times 28 + 7^2)$

$= \dfrac{22}{7} \times 24(7 \times 4 \times 28 + 7 \times 28 + 7 \times 7)$

$= \dfrac{22}{7} \times 24 \times 7(112 + 28 + 7)$

$= 22 \times 24 \times (147) = 77616 \text{ cm}^3$.

EXAMPLE 14.14

From a circular canvas of diameter 56 m, a sector of 270° was cut out and a conical tent was formed by joining the straight ends of this piece. Find the radius and the height of the tent.

SOLUTION

As shown in the figure, when the free ends of the torn canvas are joined to form a cone, the radius of sector becomes slant height.

$\therefore l = \dfrac{56}{2} = 28$ m.

The length of the arc of the sector becomes the circumference of the base of the cone.

Let the radius of the base of the cone $= r$.

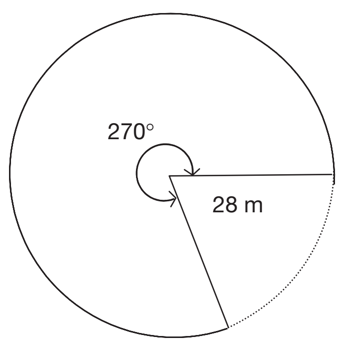


Figure 14.14

$$\Rightarrow \quad 2\pi r = 2 \times \frac{22}{7} \times \frac{56}{2} \times \left(\frac{270}{360}\right)$$

$$\Rightarrow \quad 2\pi r = 2 \times \frac{22}{7} \times 28 \times \frac{3}{4} \quad \Rightarrow \quad r = 21 \text{ m}$$

$$\therefore \text{height } h = \sqrt{l^2 - r^2} = \sqrt{28^2 - 21^2} = \sqrt{7^2(4^2 - 3^2)} = 7\sqrt{16 - 9} = 7\sqrt{7} \text{ m}$$

$\therefore h = 7\sqrt{7}$ m and $r = 21$ m.

SPHERE

Sphere is a set of points in the space which are equidistant from a fixed point. The fixed point is called the centre of the sphere, and the distance is called the radius of the sphere. A lemon, a foot ball, the moon, globe, the Earth, small lead balls used in cycle bearings are some objects which are spherical in shape.

A line joining any two points on the surface of sphere and passing through the centre of the sphere is called its diameter.

The size of sphere can be completely determined by knowing its radius or diameter.

Solid Sphere

A solid sphere is the region in space bounded by a sphere. The centre of a sphere is also a part of solid sphere whereas the centre is not a part of hollow sphere. Marbles, lead shots, etc., are the examples of solid spheres while a tennis ball is a hollow sphere.

Hollow Sphere

From a solid sphere a smaller sphere having the same centre of the solid sphere, is cut off, then we obtain a hollow sphere. This can also be called a spherical shell.

Hemisphere

If a sphere is cut into two halves by a plane passing through the centre of sphere, then each of the halves is called a hemisphere.

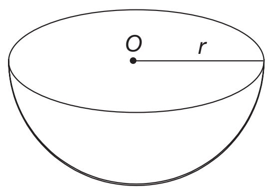


Figure 14.15 Hemisphere

Hemispherical Shell

A hemispherical shell is shown in the figure given below.

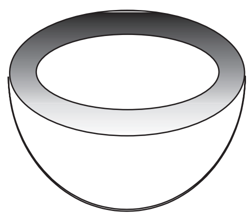


Figure 14.16 Hemispherical shell

Formulae to Memorize

Sphere

1. Surface area of a sphere = $4\pi r^2$ sq. units.
2. Volume of a sphere = $\frac{4}{3}\pi r^3$ sq. units.

Spherical Shell/Hollow Sphere

1. Thickness = $R - r$, where R = outer radius, r = inner radius.
2. Volume $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$ cubic units.
3. Total surface area of a hemi-spherical shell $= \frac{1}{2}$(surface area of outer hemisphere + surface area of inner hemisphere + area of ring).

Hemisphere

1. Curved surface area of a hemisphere $= 2\pi r^2$ sq. units.
2. Total surface area of a hemisphere $= 3\pi r^2$ sq. units.
3. Volume of a hemisphere $= \frac{2}{3}\pi r^3$ cubic units.

EXAMPLE 14.15

The cost of painting a solid sphere at the rate of 50 paise per square metre is ₹1232. Find the volume of steel required to make the sphere.

SOLUTION

Cost of painting = Surface area × Rate of painting.

$$\therefore \text{Surface area} = \frac{\text{Cost of painting}}{\text{Rate of painting}} = \frac{1232}{0.5} = 2464 \text{ m}^2.$$

$$\Rightarrow \quad 4\pi r^2 = 2464 \quad \Rightarrow \quad r^2 = \frac{2464}{4\pi} = \frac{616}{\left(\frac{22}{7}\right)} = \frac{616 \times 7}{22} = 28 \times 7$$

$$\Rightarrow \quad r = 7 \times 2 = 14 \text{ m}$$

$$\therefore \text{Volume of steel required} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 = \frac{34496}{3} \text{m}^3.$$

EXAMPLE 14.16

A hollow hemispherical bowl of thickness 1 cm has an inner radius of 6 cm. Find the volume of metal required to make the bowl.

SOLUTION

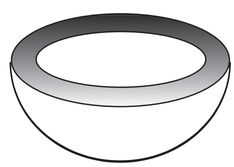


Figure 14.17

Inner radius, $r = 6$ cm

thickness, $t = 1$ cm

$\therefore$ Outer radius, $R = r + t = 6 + 1 = 7$ cm

$\therefore$ Volume of steel required $= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 7^3 - \frac{2}{3} \times \frac{22}{7} \times 6^3 = \frac{44}{21}(7^3 - 6^3)$$

$$= \frac{44}{21}(343 - 216) = \frac{44}{21} \times 127 = \frac{5588}{21} \text{ cm}^2.$$

EXAMPLE 14.17

A thin hollow hemispherical sailing vessel is made of metal covered by a conical canvas tent. The radius of the hemisphere is 14 m and total height of vessel (including the height of tent) is 28 m. Find area of metal sheet and the canvas required.

SOLUTION

The vessel (with the conical tent) is shown in figure.

Total height, $H = 28$ m

Radius of hemisphere $= r = 14$ m

$\therefore$ height of conical tent $= h = H - r$
$= 28 - 14 = 14$ m.

We can observe that radius of base of cone = radius of the hemisphere = 14 m.

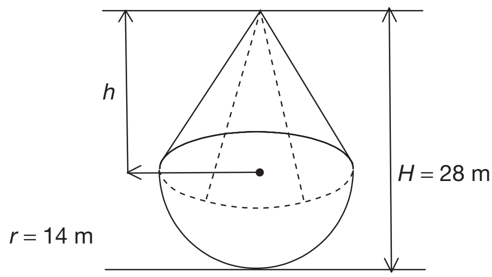


Figure 14.18

$\therefore$ Area of canvas required $= \pi rl = \frac{22}{7} \times 14 \times \sqrt{14^2 + 14} = 44 \times 14\sqrt{2} = 616\sqrt{2}$ m^2

Area of metal sheet required = surface area of hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times 14 \times 14 = 1232$ m^2.

EXAMPLE 14.18

A wafer cone is completely filled with ice cream forms a hemispherical scoop, just covering the cone. The radius of the top of the cone, as well as the height of the cone are 7 cm each. Find the volume of the ice cream in it (in cm^3). (Take $\pi = 22/7$ and ignore the thickness of the cone).

(a) 1176 **(b)** 1980 **(c)** 1078 **(d)** 1274

SOLUTION

Required volume = Volume of the ice cream forming the hemisphere + Volume of the ice cream within the cone.

Radius of the hemisphere shape = Radius of the cone = 7 cm.

$\therefore$ Required volume $= \frac{2}{3}\pi(7)^3 + \frac{1}{3}\pi(7)^3$

$$= \pi(7)^3 = \frac{22}{7}(7)^3 = (22)(49)$$

$$= 1078 \text{ cm}^3.$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If the length of the side of an equilateral triangle is 12 cm, then what is its in-radius?

2. The radius of a circle is 8 cm and O is its centre. If $\angle AOB = 60°$ and AB is a chord, then what is the length of the chord AB?

3. The circum radius of an equilateral triangle is x cm. What is the perimeter of the triangle in terms of x?

4. If the difference between the outer radius and the inner radius of a ring is 14 cm, then what is the difference between its outer circumference and inner circumference?

5. The area of a ring is 22 cm^2. What is the difference of the square of the outer radius and the square of the inner radius?

6. A cone is formed by joining together the two straight edges of a sector, so that they coincide with each other. The length of the arc of the sector becomes the _______ of the circular base and radius of sector becomes the _______ of the cone.

7. The volume of a cube with diagonal d is _______.

8. If the total surface area of a cube is $\frac{50}{3}$ m^3, then find its side.

9. Find the maximum number of soaps of size 2 cm × 3 cm × 5 cm that can be kept in a cuboidal box of dimensions 6 cm × 3 cm × 15 cm.

10. Total number of faces in a prism which has 12 edges is _______.

11. W, P, H and A are whole surface area, perimeter of base, height and area of the base of a prism respectively. The relation between W, P, H and A is _______.

12. If s is the perimeter of the base of a prism, n is the number of sides of the base, S is the total length of the edges and h is the height, then $S =$ _______.

13. If the number of lateral surfaces of a right prism is equal to n, then the number of edges of the base of the prism is _______.

14. If f, e, and v represent the number of rectangular faces, number of edges and number of vertices respectively of a cuboid, then the values of f, e, and v respectively are _______.

15. Find the number of vertices of a pyramid, whose base is a pentagon.

16. A and B are the volumes of a pyramid and a right prism respectively. If the pyramid and the prism have the same base area and the same height, then what is the relation between A and B?

17. If the ratio of the base radii of two cones having the same curved surface areas is 6 : 7, then the ratio of their slant heights is _______.

18. The heights of two cones are equal and the radii of their bases are R and r. The ratio of their volumes is _______.

19. If the heights of two cylinders are equal and their radii are in the ratio of 7 : 5, then the ratio of their volumes is _______.

20. Volumes of two cylinders of radii R, r and heights H, h respectively are equal. Then $R^2H =$ _______.

21. The volumes of two cylinders of radii R, r and heights H, h respectively are equal. If $R : r = 2 : 3$, then $H : h =$ _______.

22. A sector of a circle of radius 6 cm and central angle 30° is folded into a cone such that the radius of the sector becomes the slant height of the cone. What is the radius of the base of the cone thus formed?

23. If R and r are the external and the internal radii of a hemispherical bowl, then what is the area of the ring, which forms the edge of the bowl (in sq. units)?

24. What is the volume of a hollow cylinder with R, r and h as outer radius, inner radius and height respectively?

25. The side of a cube is equal to the radius of the sphere. Find the ratio of their volumes.

26. A sphere and the base of a cylinder have equal radii. The diameter of the sphere is equal to the height of the cylinder. The ratio of the curved surface area of the cylinder and surface area of the sphere is _______.

27. A road roller of length $3l$ m and radius $\frac{l}{3}$ m can cover a field in 100 revolutions, moving once over. The area of the field in terms of l is ______ m^3.

28. What is the volume of sand to be spread uniformly over a ground of dimensions. $10x$ m × $8x$ m up to a height of $0.1x$ m?

29. The outer radius and the inner radius of a hollow cylinder are $(2 + x)$ cm and $(2 - x)$ cm. What is its thickness?

30. The slant height, outer radius and inner radius of a cone frustum are $2a$ cm, $(a + b)$ cm and $(a - b)$ cm. What is its curved surface area?

Short Answer Type Questions

31. A circle is inscribed in an equilateral triangle. If the in-radius is 21 cm, what is the area of the triangle?

32. Three cubes each of side 3.2 cm are joined end to end. Find the total surface area of the resulting cuboid.

33. A square is drawn with the length of side equal to the diagonal of a cube. If the area of the square is 72075 cm^2, then find the side of the cube.

34. What is the area of a ground that can be levelled by a cylindrical roller of radius 3.5 m and 4 m long by making 10 rounds?

35. A square of side 28 cm is folded into a cylinder by joining its two sides. Find the base area of the cylinder thus formed.

36. Find the number of cubes of side 2 m to be dropped in a cylindrical vessel of radius 14 m in order to increase the water level by 5 m.

37. Find the capacity of a closed cuboidal cistern whose length is 3 m, breadth is 2 m and height is 6 m. Also find the area of iron sheet required to make the cistern.

38. An open metallic conical tank is 6 m deep and its circular top has diameter of 16 m. Find the cost of tin plating its inner surface at the rate of ₹0.8 per 100 cm^2. (Take $\pi = 3.14$)

39. The total surface area of a hemisphere is 3768 cm^2. Find the radius of the hemisphere. (Take $\pi = 3.14$)

40. The base radius of a conical tent is 120 cm and its slant height is 750 cm. Find the area of the canvas required to make 10 such tents (in m^2). (Take $\pi = 3.14$)

41. From a cylindrical wooden log of length 30 cm and base radius $7\sqrt{2}$ cm, biggest cuboid of square base is made. Find the volume of wood wasted.

42. A right circular cone is such that the angle at its vertex is 90° and its base radius is 49 cm, then find the curved surface area of the cone.

43. The base of a right pyramid is an equilateral triangle, each side of which is $6\sqrt{3}$ cm long and its height is 4 cm. Find the total surface area of the pyramid in cm^2.

44. If the thickness of a hemispherical bowl is 12 cm and its outer diameter is 10.24 m, then find the inner surface area of the hemisphere. (Take $\pi = 3.14$)

45. A spherical piece of metal of diameter 6 cm is drawn into a wire of 4 mm in diameter. Find the length of the wire.

Essay Type Questions

46. The cost of the canvas required to make a conical tent of base radius 8 m at the rate of ₹40 per m^2 is ₹10048. Find the height of the tent. (Take $\pi = 3.14$)

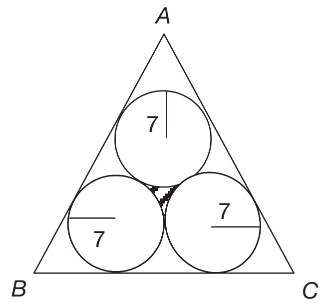
47. A hollow sphere which has internal and external diameter as 16 cm and 14 cm respectively is melted into a cone with a height of 16 cm. Find the diameter of the base of the cone.

48. A drum in the shape of a frustum of a cone with radii 24 ft and 15 ft and height 5 ft is full of water. The drum is emptied into a rectangular tank of

base 99 ft × 43 ft. Find the rise in the height of the water level in the tank.

49. A cylindrical tank of radius 7 m, has water to some level. If 110 cubes of side 7 dm are completely immersed in it, then find the rise in the water level in the tank. (in metres)

50. Find the area of the shaded portion in the figure given below, where *ABC* is an equilateral triangle and the radius of each circle is 7 cm.



CONCEPT APPLICATION

Level 1

1. The area of a sector whose perimeter is four times its radius (r units) is

(a) $\sqrt{r}$ sq. units. (b) r^4 sq. units.

(c) r^2 sq. units. (d) $\frac{r^2}{2}$ sq. units.

2. A chord of a circle of radius 28 cm makes an angle of 90° at the centre. Find the area of the major segment.

(a) 1456 cm^2 (b) 1848 cm^2

(c) 392 cm^2 (d) 2240 cm^2

3. The area of a circle inscribed in an equilateral triangle is 48π square units. What is the perimeter of the triangle?

(a) $17\sqrt{3}$ units (b) 36 units

(c) 72 units (d) $48\sqrt{3}$ units

4. Two circles touch each other externally. The distance between the centres of the circles is 14 cm and the sum of their areas is 308 cm^2. Find the difference between radii of the circles. (in cm)

(a) 1 (b) 2

(c) 0 (d) 0.5

5. If the outer and the inner radii of a circular track are 7 m and 3.5 m respectively, then the area of the track is

(a) 100 m^2 (b) 178 m^2

(c) 115.5 m^2 (d) 135.5 m^2

6. The base of a right pyramid is an equilateral triangle of perimeter 8 dm and the height of the pyramid is $30\sqrt{3}$ cm. Find the volume of the pyramid.

(a) 16000 cm^3 (b) 1600 cm^3

(c) $\frac{16000}{3}$ cm^3 (d) $\frac{5}{4}$ cm^3

7. The volume of a cuboid is $20\sqrt{42}$ m^3. Its length is $5\sqrt{2}$ m, breadth and height are in the ratio $\sqrt{3} : \sqrt{7}$. Find its height.

(a) $\sqrt{7}$ m (b) $3\sqrt{7}$ m

(c) $4\sqrt{7}$ m (d) $2\sqrt{7}$ m

8. A metal cube of edge $\frac{3\sqrt{2}}{\sqrt{5}}$ m is melted and formed into three smaller cubes. If the edges of the two smaller cubes are $\frac{3}{\sqrt{10}}$ m and $\frac{\sqrt{5}}{\sqrt{2}}$ m, find the edge of the third smaller cube.

(a) $\frac{3}{\sqrt{7}}$ m (b) $\frac{6}{\sqrt{15}}$ m

(c) $\frac{5}{\sqrt{11}}$ m (d) $\frac{4}{\sqrt{10}}$ m

9. Find the volume of the space covered by rotating a rectangular sheet of dimensions 16.1 cm × 7.5 cm along its length.

(a) 2846.25 cm^3 (b) 2664 cm^3

(c) 2864.25 cm^3 (d) 2684 cm^3

10. The base of a right prism is an equilateral triangle of edge 12 m. If the volume of the prism is $288\sqrt{3}$ m^3, then its height is

(a) 6 m (b) 8 m

(c) 10 m (d) 12 m

11. A roller levelled an area of 165000 m^2 in 125 revolutions, whose length is 28 m. Find the radius of the roller.

(a) 7.5 m (b) 8.5 m

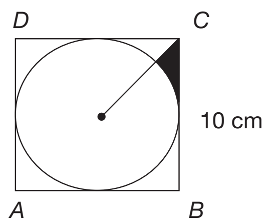
(c) 6.5 m (d) 7 m

12. A large sphere of radius 3.5 cm is carved from a cubical solid. Find the difference between their surface areas.

(a) 122 cm^2 (b) 80.5 cm^2

(c) 144.5 cm^2 (d) 140 cm^2

13. In the figure given below, $ABCD$ is a square of side 10 cm and a circle is inscribed in it. Find the area of the shaded part as shown in the figure.



(a) $\left(\dfrac{100-36\pi}{41}\right)$ cm^2

(b) $\left(\dfrac{100-25\pi}{8}\right)$ cm^2

(c) $\left(\dfrac{100+25\pi}{8}\right)$ cm^2

(d) None of these

14. The outer curved surface area of a cylindrical metal pipe is 1100 m^2 and the length of the pipe is 25m. The outer radius of the pipe is

(a) 8 m (b) 9 m

(c) 7 m (d) 6 m

15. The volume of a hemisphere is 2.25π cm^3. What is the total surface area of the hemisphere?

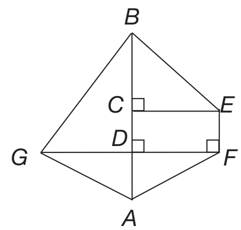
(a) 2.25π cm^2 (b) 5π cm^2

(c) 6.75π cm^2 (d) 4.5π cm^2

16. Find the area of the figure given below, in which $AB = 100$ m, $CE = 30$ m, C is mid-point of $\overline{AB}$ and D is mid-point of $\overline{AC}$ and $\overline{GF}$.

(a) 5250 m^2 (b) 3750 m^2

(c) 3375 m^2 (d) 3175 m^2



17. The area of the base of a right equilateral triangular prism is $16\sqrt{3}$ cm^2. If the height of the prism is 12 cm, then the lateral surface area and the total surface area of the prism respectively are

(a) 288 cm^2, $(288 + 32\sqrt{3})$ cm^2

(b) 388 cm^2, $(388 + 32\sqrt{3})$ cm^2

(c) 288 cm^2, $(288 + 24\sqrt{3})$ cm^2

(d) 388 cm^2, $(388 + 24\sqrt{3})$ cm^2

18. A metallic cone of diameter 32 cm and height 9 cm is melted and made into identical spheres, each of radius 2 cm. How many such spheres can be made?

(a) 72 (b) 64

(c) 52 (d) 48

19. A cylindrical vessel open at the top has a base radius of 28 cm. If the total cost of painting the outer part of the vessel is ₹357 at the rate of ₹0.2 per 100 cm^2, then find the height of the vessel. (approximately)

(a) 10 m (b) 9 m

(c) 8 m (d) 4 m

20. The radii of the ends of a bucket 16 cm high are 20 cm and 8 cm. Find the curved surface area of the bucket.

(a) 1760 cm^2 (b) 2240 cm^2

(c) 880 cm^2 (d) 3120 cm^2

21. A cylindrical vessel of radius 8 cm contains water. A solid sphere of radius 6 cm is lowered into the water until it is completely immersed. What is the rise in the water level in the vessel?

(a) 3 cm (b) 3.5 cm

(c) 4 cm (d) 4.5 cm

22. What is the difference in the areas of the regular hexagon circumscribing a circle of radius 10 cm and the regular hexagon inscribed in the circle?

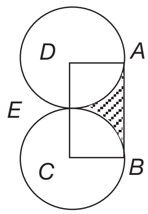
(a) 50 cm^2 (b) $50\sqrt{3}$ cm^2

(c) $100\sqrt{3}$ cm^2 (d) $100\sqrt{3}$ cm^2

23. In the shown figure, two circles of radii of 7 cm each, are shown. *ABCD* is rectangle and *AD* and *BC* are the radii. Find the area of the shaded region (in cm^2).

(a) 20 (b) 21

(c) 19 (d) 18



24. There is a closed rectangular shed of dimensions 10 m × 4 m inside a field. A cow is tied at one corner of outside of the shed with a 6 m long rope. What is the area that the cow can graze in the field?

(a) 66 m^2 (b) 88 m^2

(c) 0.8π m^2 (d) 27π m^2

25. The base of a right prism is a square of perimeter 20 cm and its height is 30 cm. What is the volume of the prism?

(a) 700 cm^3 (b) 750 cm^3

(c) 800 cm^3 (d) 850 cm^3

26. A conical cup when filled with ice cream forms a hemispherical shape on its open end. Find the volume of ice cream (approximately), if radius of the base of the cone is 3.5 cm, the vertical height of cone is 7 cm and width of the cone is negligible.

(a) 120 cm^3 (b) 150 cm^3

(c) 180 cm^3 (d) 210 cm^3

27. A hemispherical bowl of internal diameter 24 cm contains water. This water is to be filled in cylindrical bottles, each of radius 6 cm and height 8 cm. How many such bottles are required to empty the bowl?

(a) 3 (b) 4

(c) 5 (d) 6

28. A dome of a building is in the form of a hemisphere. The total cost of white washing it from inside, was ₹1330.56. The rate at which it was white washed is ₹3 per square metre. Find the volume of the dome approximately.

(a) 1150.53 m^3 (b) 1050 m^3

(c) 1241.9 m^3 (d) 1500 m^3

Level 2

29. A circular garden of radius 15 m is surrounded by a circular path of width 7 m. If the path is to be covered with tiles at a rate of ₹10 per m^2, then find the total cost of the work. (in ₹)

(a) 8410 (b) 7140

(c) 8140 (d) 7410

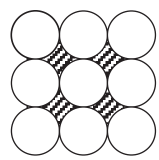
30. Find the area of the shaded region, given that the radius of each circle is equal to 5 cm.

(a) $(400 - 100\pi)$ cm^2

(b) $(360 - 100\pi)$ cm^2

(c) 231 cm^2

(d) $(400 - 50\pi)$ cm^2



31. The volume of a right prism, whose base is an equilateral triangle, is $1500\sqrt{3}$ cm^3 and the height of the prism is 125 cm. Find the side of the base of the prism.

(a) $8\sqrt{3}$ cm (b) $4\sqrt{3}$ cm

(c) $16\sqrt{3}$ cm (d) $24\sqrt{3}$ cm

32. A right circular cylinder of volume 1386 cm^3 is cut from a right circular cylinder of radius 4 cm and height 49 cm, such that a hollow cylinder of uniform thickness, with a height of 49 cm and an outer raidus of 4 cm is left behind. Find the thickness of the hollow cylinder left behind.

(a) 0.5 cm (b) 2 cm

(c) 1.5 cm (d) 1 cm

33. The volume of a hemisphere is 18π cm^3. What is the total surface area of the hemisphere?

(a) 18π cm^2

(b) 27π cm^2

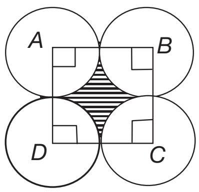
(c) 21π cm^2

(c) 24π cm^2

34. The diagram shown above has four circles of 7 cm radius with centres at A, B, C and D. If the quadrilateral $ABCD$ represents a square, then find the area of the shaded region.

(a) 42 cm^2 (b) 21 cm^2

(c) 63 cm^2 (d) 84 cm^2



35. Find the total surface area of a hollow metallic hemisphere whose internal radius is 14 cm and the thickness of the metal is 7 cm.

(a) 4774 cm^2

(b) 4477 cm^2

(c) 4747 cm^2

(d) 7744 cm^2

36. A metal cube of edge $\frac{3}{10}$ m is melted and formed into three smaller cubes. If the edges of the two smaller cubes are $\frac{1}{5}$ m and $\frac{1}{4}$ m, find the edge of the third smaller cube.

(a) $\frac{7}{20}$ m (b) $\frac{1}{20}$ m

(c) $\frac{3}{20}$ m (d) None of these

37. Two hemispherical vessels can hold 10.8 litres and 50 litres of liquid respectively. The ratio of their inner curved surface areas is

(a) 16 : 25 (b) 25 : 9

(c) 9 : 25 (d) 4 : 3

38. A cylindrical drum 1.5 m in diameter and 3 m in height is full of water. The water is emptied into another cylindrical tank in which water rises by 2 m. Find the diameter of the second cylinder up to 2 decimal places.

(a) 1.74 m (b) 1.94 m

(c) 1.64 m (d) 1.84 m

39. Curved surface area of a conical cup is $154\sqrt{2}$ cm^2 and base radius is 7 cm. Find the angle at the vertex of the conical cup.

(a) 90° (b) 60°

(c) 45° (d) 30°

40. An equilateral triangle has a circle inscribed in it and is circumscribed by a circle. There is another equilateral triangle inscribed in the inner circle. Find the ratio of the areas of the outer circle and the inner equilateral triangle.

(a) $\frac{16\pi}{3\sqrt{3}}$ (b) $\frac{8\pi}{2\sqrt{3}}$

(c) $\frac{24\pi}{3\sqrt{3}}$ (d) None of these

41. A triangle has sides of 48 cm, 14 cm and 50 cm. Find its circum-radius (in cm).

(a) 25 (b) 12.5

(c) 20 (d) 17.5

42. The base of a pyramid is an n-sided regular polygon of area 360 cm^2. The total surface area of the pyramid is 900 cm^2. Each lateral face of the pyramid has an area of 30 cm^2. Find n.

(a) 20 (b) 18

(c) 16 (d) 24

43. In a right prism, the base is an equilateral triangle. Its volume is $80\sqrt{3}$ cm^3 and its lateral surface area is 240 cm^2. Find its height (in cm).

(a) 10 (b) 5

(c) 15 (d) 20

44. A goat is tied to one corner of a field of dimensions 16 m × 10 m with a rope 7 m long. Find the area of the field that the goat can graze.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) Required area = 38.5 m^2.

(B) Area of the field that the goat can graze

= Area of the sector of radius 7 m and a sector angle of 90°.

(C) $\frac{90°}{360} \times \frac{22}{7} \times (7)^2$

(a) ABC (b) BCA

(c) BAC (d) CBA

45. A right prism has a triangular base. If its perimeter is 24 cm and lateral surface area is 192 cm^2, find its height.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $192 = 24 \times h$

(B) Given LSA = 192 cm^2, perimeter of the base = 24 cm

(C) $h = 8$ cm

(D) Lateral surface area of a prism = perimeter of the base × height

(a) BADC (b) BCAD

(c) DABC (d) BDAC

Level 3

46. An ink pen, with a cylindrical barrel of diameter 2 cm and height 10.5 cm, and completely filled with ink, can be used to write 4950 words. How many words can be written using 400 ml of ink?
(Take 1 litre = 1000 cm^3)

(a) 40000 (b) 60000

(c) 45000 (d) 80000

47. Each of height and side of the base of a regular hexagonal pyramid is equal to x cm. Find its lateral surface area in terms of x (in cm^2).

(a) $\frac{9\sqrt{7}}{2}x^2$ (b) $\frac{7\sqrt{7}}{2}x^2$

(c) $\frac{5\sqrt{7}}{2}x^2$ (d) $\frac{3\sqrt{7}}{2}x^2$

48. The diameters of the top and the bottom portions of a bucket are 42 cm and 28 cm. If the height of the bucket is 24 cm, then find the cost of painting its outer surface at the rate of 5 paise/cm^2.

(a) ₹158.25

(b) ₹172.45

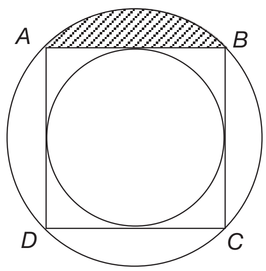
(c) ₹168.30

(d) ₹164.20

49. In the following figure, a circle is inscribed in square $ABCD$ and the square is circumscribed by a circle. If the radius of the smaller circle is r cm, then find the area of the shaded region (in cm^2).

(a) $\left(\frac{\pi - 2}{4}\right)r^2$ (b) $\left(\frac{3\pi - 4}{2}\right)r^2$

(c) $\left(\frac{\pi + 2}{4}\right)r^2$ (d) $\left(\frac{\pi - 2}{2}\right)r^2$

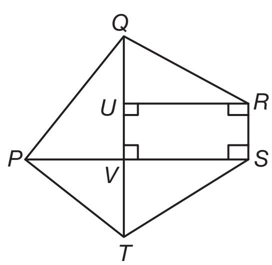


50. $ABCD$ is a square of side 4 cm. If E is a point in the interior of the square such that ΔCED is equilateral, then find the area of ΔACE (in cm^2).

(a) $2(\sqrt{3} - 1)$ (b) $4(\sqrt{3} - 1)$

(c) $6(\sqrt{3} - 1)$ (d) $8(\sqrt{3} - 1)$

51.



In the given figure (not to scale), $QT = 90$ m and $UR = 50$ m. $QU : UT = UV : VT = 1 : 2$. $PV : VS = 4 : 5$. Find the area of the figure. (in m^2)

(a) 4550 (b) 4200

(c) 4250 (d) 4100

52. H_1 is a regular hexagon circumscribing a circle. H_2 is a regular hexagon inscribed in the circle. Find the ratio of areas of H_1 and H_2.

(a) 4 : 3 (b) 2 : 1

(c) 3 : 1 (d) 3 : 2

53. A dish, in the shape of a frustum of a cone, has a height of 6 cm. Its top and its bottom have radii of 24 cm and 16 cm respectively. Find its curved surface area (in cm^2).

(a) 240π (b) 400π

(c) 180π (d) 160π

54. Two circles touch each other externally. The sum of their areas is 490π cm^2. Their centres are separated by 28 cm. Find the difference of their radii (in cm).

(a) 14 (b) 7

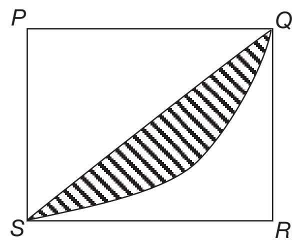
(c) 10.5 (d) 3.5

55. A closed rectangular shed has dimensions 21 m × 14 m. It is inside a field. A cow is tied outside the shed at one of its corners with a 21 m rope. Find the area over which the cow can graze (in m^2).

(a) 342π (b) 294π

(c) 343π (d) 441π

56. In the given figure, $PQRS$ is a square of diagonal $7\sqrt{2}$ cm. With P as the centre, the arc QS is drawn. Find the area of the shaded region (in cm^2).



(a) $\frac{49}{4}(\pi - 2)$ (b) $\frac{49}{4}(\pi - 1)$

(c) $\frac{49}{4}(\pi - 3)$ (d) $\frac{49}{2}(\pi - 2)$

57. Three solid cubes have a face diagonal of $4\sqrt{2}$ cm each. Three other solid cubes have a face diagonal of $8\sqrt{2}$ cm each. All the cubes are melted together to form a cube. Find the side of the cube formed (in cm).

(a) $\sqrt[3]{324}$ (b) $\sqrt[3]{576}$

(c) 12 (d) 24

58. The outer radius and inner radius of a 30 cm long cylindrical gold pipe are 14 cm and 7 cm respectively. It is filled with bronze. The densities of gold and bronze are 20 gm/cm^3 and 30 gm/cm^3 respectively. Find the weight of the cylinder formed. (in gm).

(a) 66150π (b) 99225π

(c) 132300π (d) 198450π

59. A rectangular sump has an inner length and breadth of 24 m and 20 m respectively. Water flows through an inlet pipe at 180 m per minute. The cross-sectional area of the pipe is 0.5 m^2. The tank takes half an hour to get filled. Find the depth of the sump (in m).

(a) 4.625 (b) 6.125

(c) 5.625 (d) 5.125

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $2\sqrt{3}$ cm
2. 8 cm
3. $3\sqrt{3}x$ cm
4. 88 cm
5. 7 cm.
6. 0.5 cm
7. $\dfrac{d^3}{3\sqrt{3}}$ cu units.
8. $\dfrac{5}{3}$ m
9. 9
10. 6
11. $W = P \times H + 2A$
12. $nh + 2s$
13. n
14. 6, 12 and 8
15. 6
16. $A = \dfrac{B}{3}$
17. 7 : 6
18. $R^2 : r^2$
19. 49 : 25
20. r^2h
21. 9 : 4
22. circumference, slant height
23. $\pi(R^2 - r^2)$
24. $\pi(R^2 - r^2)h$
25. 21 : 88
26. 1 : 1.
27. $200\pi l^2$ cm^2
28. $8x^3$ m^3
29. $2x$ cm
30. $4\pi a^2$ cm^2

Short Answer Type Questions

31. $1323\sqrt{3}$ sq. units
32. 143.36 cm^2
33. 155 cm
34. 880 m^2
35. $\dfrac{686}{11}$ cm^2
36. 385
37. 72 m^2
38. ₹20096
39. 157 m^2
40. 282.6 m^2
41. 3360 cm^3
42. $7546\sqrt{2}$ cm^2
43. $72\sqrt{3}$ cm^2
44. 20 cm
45. 900 cm

Essay Type Questions

46. 251.2 m^2, 6 m
47. 13 cm
48. $1\dfrac{3}{7}$ ft
49. 0.245
50. 7.87 cm^2

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (b)
11. (a)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (a)
21. (d)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (c)		

Level 2

29. (c)	**30.** (a)	**31.** (b)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (c)	**38.** (d)
39. (a)	**40.** (a)	**41.** (a)	**42.** (b)	**43.** (d)	**44.** (b)	**45.** (d)			

Level 3

46. (b)	**47.** (d)	**48.** (c)	**49.** (d)	**50.** (b)	**50.** (a)	**51.** (a)	**53.** (b)	**54.** (a)	**55.** (c)
56. (a)	**57.** (c)	**58.** (c)	**59.** (c)						

CONCEPT APPLICATION

Level 1

1. Perimeter of a sector = $l + 2r = 4r$.
2. Find the area of triangle and the area of sector formed by the chord.
3. Radius of the circle $= \frac{1}{3}$ of the median.
4. Use $(a + b)^2 = a^2 + b^2 + 2ab$ and $a - b = \sqrt{(a+b)^2 - 4ab}$.
5. Area of the track = Area of outer circle − Area of inner circle
6. Volume of the pyramid $= \frac{1}{3}$ area of the base × height.
7. Breadth $= \sqrt{3}x$ m and Height $= \sqrt{7}x$ m.
8. Volume of large cube is equal to the sum of volumes of three small cubes.
9. Find the volume of cylinder. Whose radius is breadth of rectangle and height is equal to length of the rectangle.
10. Volume of the prism = Area of the base × height.
11. Area levelled by the roller in one revolution = CSA of the cylinder.
12. Diameter of the sphere = length of the edge of a cube.
13. Draw $\overline{OP}$ from the centre to the midpoint of $\overline{BC}$.
14. CSA of a cylinder $= 2\pi Rh$.
15. Find 'r' by using volume formula.
16. Find individual areas of different parts of the figure.
17. Find the edge of the prism.
18. Volume of the cone = Volume of all the spheres formed.
19. TSA of outer part $= 357 \times \frac{100}{0.2}$ cm^2.
20. CSA of bucket $= \pi l(R + r)$.
21. Equate the two volumes.
22. Find the area of two hexagon.
23. Required area is the difference of areas of rectangle and sum of areas of two sectors.
24. Draw the diagram and proceed.
25. Volume of the prism = Area of the base × height.
26. Required volume = Volume of cone + Volume of sphere.
27. Volume of bowl = Volume of a bottle × Number of bottles.
28. Volume of a hemisphere $= \frac{3}{2}\pi r^3$ cubic units.

Level 2

29. (i) Area of circular path = Area of ring.
 (ii) Area of path $= \pi(R^2 - r^2)$.
 (iii) Total cost = Area × cost/m^2.
30. (i) Find the area of the square formed by joining the centers of all outer circles.
 (ii) The required area = Area of the square − 16 $\left(\frac{1}{4}\text{area of circle}\right)$.
31. (i) Volume of prism $= \frac{\sqrt{3}}{4}a^2 \times$ height.
 (ii) Use the above formula and get the value of a.
32. Find the radius of the cylinder which is cut. (i.e., $\pi r^2 h = 1386$).
33. Radius of the hemisphere = Radius of the sphere.
34. Area of shaded region = Area of square ABCD − 4 $\left(\frac{1}{4}\text{ area of circle}\right)$.
35. TSA $= 3\pi R^2 + \pi r^2$.
36. Volume of big cube = Sum of the volumes of smaller cubes.
37. Ratio of their CSA's is $r_1^2 : r_2^2$.
38. Volume of water in the cylindrical drum = Volume of the second cylinder up to the water risen.
39. Use, $\tan\left(\frac{\theta}{2}\right) = \frac{r}{h}$ and find θ.

40. (i) Circum radius $= \dfrac{a}{\sqrt{3}}$ and inradius $= \dfrac{a}{2\sqrt{3}}$, where a is the side of the outer equilateral triangle.

(ii) For an equilateral triangle of side a, if an in-circle and circum circle are drawn whose radii are r and R then $r = \dfrac{a}{2\sqrt{3}}$ and $R = \dfrac{a}{\sqrt{3}}$.

41. $48^2 = 2304$

$14^2 = 196$

$50^2 = 2500$

$48^2 + 14^2 = 50^2$

$\therefore$ The triangle is right angled and its hypotenuse is 50 cm. Its circum radius $= \dfrac{50}{2}$ cm $= 25$ cm.

42. Total area of the lateral faces $= 900 - 360 = 540$ cm^2.

Number of lateral faces it has $\dfrac{540}{30} = 18$.

43. Let the side of the base be a cm. Let the height of the prism be h cm.

$$\left(\frac{\sqrt{3}}{4}a^2\right)(h) = 80\sqrt{3} \qquad (1)$$

$$(3a)(h) = 240 \qquad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\sqrt{3}}{12}a = \frac{\sqrt{3}}{3}$$

$a = 4$

from Eq. (2), $\Rightarrow$ $h = 20$

44. BCA

45. BDAC

Level 3

46. (i) 1 ml = 1 cm^3

(ii) Find the number of words per 1 ml of ink.

47. CSA of a pyramid $= \dfrac{1}{2}$ perimeter of the base $\times$ slant height.

48. CSA of a bucket $= \pi l(R + r)$.

49. Diagonals of the square meet at the centre of the circles.

50. (i) Draw the figure according to the data and draw $\overline{EF} \parallel \overline{CD}$.

(ii) Area of ΔACE = Area of the triangle ABC – {Area of ΔECD + Area of $ABED$}.

51. $QT = 90$ m

$QU : UT = 1 : 2$

$\therefore UT = 60$ m

$UV : VT = 1 : 2$ and $QU = 30$ m

$\therefore UV = 20$ m and $VT = 40$ m

$URSV$ is a rectangle.

$\therefore VS = UR = 50$ m.

$PV : VS = 4 : 5$

$\therefore PV = 40$ m.

Area of the figure = Area of ΔPQT + Area of $QRST$

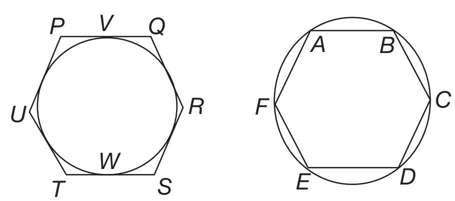
$$= \frac{1}{2}(PV)(QT) + \frac{1}{2}(QT + RS)(UR)$$

$$= \frac{1}{2}(40)(90) + \frac{1}{2}(90 + 20)(50)$$

($\because RS = UV = 20$ m)

$= 1800 + 2750 = 4550$ m^2.

52.



Let H_1 be $PQRSTU$ and H_2 be $ABCDEF$.

Let R be the radius of the circle.

$$\therefore R = \frac{\sqrt{3}(PU)}{2} \Rightarrow PU = \frac{2R}{\sqrt{3}}$$

$AB = R$ ($\because$ A hexagon inscribed in a circle must have its side equal to the radius of the circle)

Area of $H_1 = \dfrac{3\sqrt{3}}{2}\left(\dfrac{2}{\sqrt{3}}R\right)^2$

And area of $H_2 = \dfrac{3\sqrt{3}}{2}(R)^2$

$(\because$ Area of hexagon $= \dfrac{3\sqrt{3}}{2}$ (Its side)^2)

Required ratio $= \dfrac{\left(\dfrac{2}{\sqrt{3}}R\right)^2}{R^2} = \dfrac{4}{3}.$

53. The bucket is in shape of a frustum. The slant height of a frustum

$= \sqrt{(\text{Top radius} - \text{Bottom Radius})^2 + (\text{Height})^2}.$

$\therefore$ Slant height of the bucket

$1 = \sqrt{(24-16)^2 + 6^2} = 10$ m.

$\therefore$ Its curved surface area $= \pi l(R + r) = \pi$

$(10)(24 + 16) = 400\pi\text{ m}^2$.

54. Let the radii of the circles be denoted by r_1 cm and r_2 cm where $r_1 \geq r_2$. As the circles touch each other externally, distance between their centres = sum of their radii.

$\therefore r_1 + r_2 = 28$ (1)

Also $\pi(r_1)^2 + \pi(r_2)^2 = 490\pi$

$\therefore (r_1)^2 + (r_2)^2 = 490$ (2)

Squaring both sides of Eq. (1), we get

$(r_1)^2 + (r_2)^2 + 2(r_1)(r_2) = 784$

from Eq. (2), $490 + 2(r_1)(r_2) = 784$

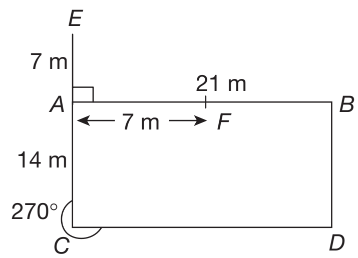
$r_1 \cdot r_2 = 147 - (3)$

Solving Eqs. (1) and (3)

$r_1 = 21$ and $r_2 = 7$. $(\therefore\ r_1 \geq r_2)$

$\therefore$ Required difference = 14 cm.

55.



Suppose the cow was tied at C. Total area it can graze

= Area of the sector ECD with central angle 270° and radius 21 m + Area of the sector EAF with central angle 90°

and radius 7 m $= \dfrac{270°}{360°}\pi(21)^2 + \dfrac{90°}{360°}\pi(7)^2$

$= \dfrac{3}{4}\pi(441) + \dfrac{1}{4}\pi(49) = 343\pi\ \text{m}^2.$

56. Area of the shaded region

= Area of the sector PQS − Area of ΔPQS

$QS = \sqrt{2}$ (side) $= 7\sqrt{2}$ cm.

Side = 7 cm

$\therefore$ Area of sector PQS

$= \dfrac{90°}{360°}\pi\,(7)^2 = \dfrac{49\pi}{4}\text{cm}^2$

Area of $\Delta PQS = \dfrac{1}{2}(PQ)(PS)$

$= \dfrac{1}{2}(7)^2 = \dfrac{49}{2}\text{cm}^2$

$\therefore$ Required area $= \left(\dfrac{49\pi}{4} - \dfrac{49}{2}\right)\text{cm}^2$

$= \dfrac{49}{4}(\pi - 2)\ \text{cm}^2.$

57. Side of each of the first three cubes $= \dfrac{4\sqrt{2}}{\sqrt{2}} = 4$ cm.

Side of each of the other three cubes $= \dfrac{8\sqrt{2}}{\sqrt{2}}$ = 8 cm.

Let the side of the cube formed be a cm.

Total volume of the six cubes = $3(4^3 + 8^3) = 3(64 + 512) = 1728$ cm^3.

$\therefore a^3 = 1728$

$a = 12$.

58. Volume of the pipe = $\pi(14^2 - 7^2)(30)$ cm^3.

$\therefore$ Volume of the gold = $\pi(14^2 - 7^2)(30)$ cm^3

Volume of the bronze in the pipe

= $\pi(7)^2$ (30) cm^3.

Weight of the pipe (in gms) = Weight of gold in it (in gm) + Weight of bronze in it (in gm)

= $[\pi(14^2 - 7^2)(30)]$ [20] + $[\pi(7)^2 (30)]$ [30]

= $30\pi[(196 - 49)(20) + (49)(30)]$

= $30\pi[2940 + 1470] = 132300\,\pi$ gm.

59. Let the depth of the sump be h m.

Volume of water flowing through the pipe = (180)(0.5) m^3/minute = 90 m^3/minute.

Total volume of water which must flow through to fill the sump = (90)(30) = 2700 cm^3.

$\therefore$ (24)(20)(h) = 2700

$h = 5.625$.

HINTS AND EXPLANATION