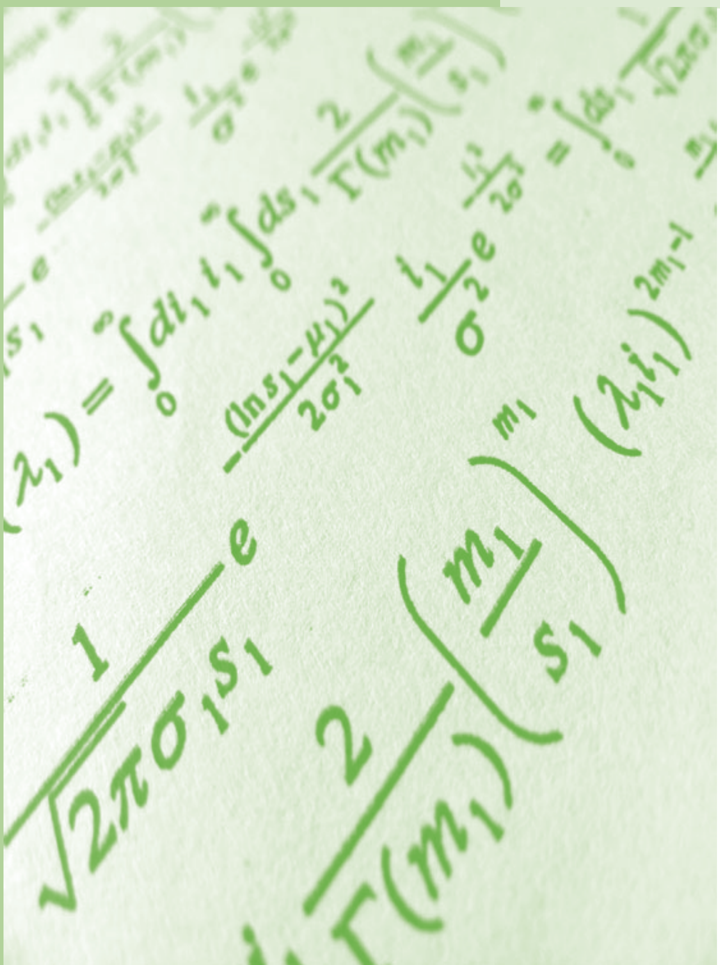
Chapter

18

Linear Programming



REMEMBER

Before beginning this chapter, you should be able to:

- Have basic knowledge of linear equations and inequations
- Represent equations and inequations by graphs

KEY IDEAS

After completing this chapter, you would be able to:

- Study about convex sets and objective functions
- Understand open and closed convex polygon
- State and prove the fundamental theorem
- Study graphical method of solving linear programming problem

INTRODUCTION

In business and industry, certain problems arise, the solutions of which depend on the way in which a change in one variable may affect the other. Hence, we need to study the interdependence between variables, such as cost of labour, cost of transportation, cost of material, availability of labour and profit. In order to study the interdependence, we need to represent these variables algebraically. The conditions, which these variables have to satisfy, are represented as a set of linear inequations. We try to find the best or the optimum condition by solving these inequations. This process is called 'linear programming'. In a situation where only two variables exist, the graphical method can be used to arrive at the optimal solution. To start with, let us define some important terms related to linear equations and inequations.

Convex Set

A subset X of a plane is said to be convex, if the line segment joining any two points P and Q in X, is contained in X.

Example: The triangular region ABC (shown below) is convex, as for any two points P and Q in the region, the line segment joining P and Q, i.e., $\overline{PQ}$ is contained in the triangular region ABC.

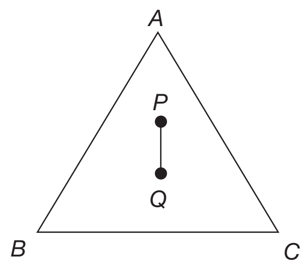


Figure 18.1

Example: The polygonal region $ABCDE$ (shown below) is not convex, as there exist two points P and Q, such that these points belong to the region $ABCDE$, but the line segment $\overline{PQ}$ is not wholly contained in the region.

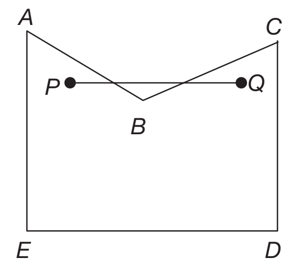


Figure 18.2

Objective Function

Every linear programming problem involving two variables, consists of a function of the form $f = ax + by$, which is to be either maximized or minimized subject to certain constraints. Such a function is called the objective function or the profit function.

Closed-Convex Polygon

Closed convex polygon is the set of all points within, or on a polygon with a finite number of vertices.

Example: If l_1 and l_2 are two lines, which meet the coordinate axes at A, E and D, C respectively, and B is the point of intersection of the lines, then $OABC$ is a closed convex polygon.

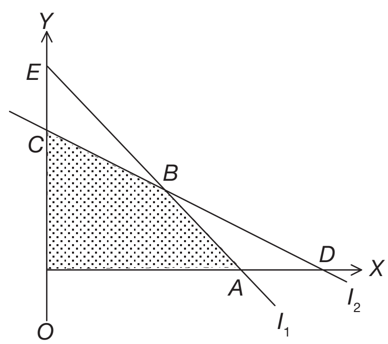


Figure 18.3

Open-Convex Polygon

Consider an infinite region bounded by two non-intersecting rays and a number of line segments, such that the endpoint of each segment is also the endpoint of another segment or one of the rays. If the angle between any two intersecting segments, or between a segment and the intersecting ray, measured through the region, is less than 180°, the region is an open-convex polygon.

For example, in the figure above, the region bounded by the rays $\overrightarrow{AP}$, $\overrightarrow{DQ}$ and the segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ is an open convex polygon, because each of the angle $\angle A$, $\angle B$, $\angle C$ and $\angle D$ is less than 180°.

The Fundamental Theorem

When the values of the expression, $f = ax + by$, are considered over the set of points forming a non-empty closed convex polygon, the maximum or minimum value of f occurs on at least one of the vertices of the polygon. To solve any linear programming problem, we use the fundamental theorem.

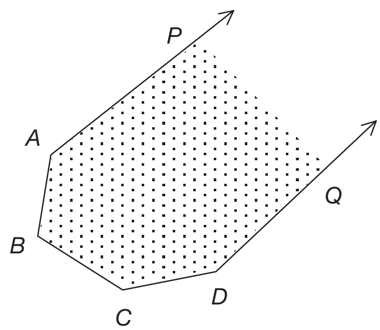


Figure 18.4

Feasible Region

The common region determined by all the constraints of a linear programming problem is called the feasible region.

Feasible Solution

Every point in the feasible region is called a feasible solution.

Optimum Solution

A feasible solution which maximizes or minimizes the objective function is called an optimum solution.

Graphical Method of Solving a Linear Programming Problem

When the solution set of the constraints is a closed convex polygon, the following method can be adopted:

1. Draw the graphs of the given system of constraints (system of inequations).
2. Identify the common region. If it is a closed convex polygon, find its vertices.
3. Find the value of the objective function at each of these vertices.
4. The vertex at which the objective function has the maximum or minimum value, gives the required solution.

The following examples explain the method in detail.

EXAMPLE 18.1

Maximize the function $f = 4x + 5y$ subject to the constraints $3x + 2y \leq 18$, $x + y \leq 7$ and $x \geq 0$, $y \geq 0$.

SOLUTION

Given, $f = 4x + 5y$

$3x + 2y \leq 18 \rightarrow$ (A)

$x + y \leq 7 \rightarrow$ (B)

and $x \geq 0$, $y \geq 0$.

As $x \geq 0$ and $y \geq 0$, the feasible region lies in first quadrant. The graph of $3x + 2y \leq 18$ is shown with horizontal lines and the graph of $x + y \leq 7$ is shown with vertical lines.

$\therefore$ The feasible region is the part of the first quadrant in which there are both horizontal and vertical lines.

The feasible region is a closed-convex polygon $OAED$ in the above graph. The vertices of the closed polygon $OAED$ are $O(0, 0)$, $A(6, 0)$, $E(4, 3)$ and $D(0, 7)$.

Now, $f = 4x + 5y$

At $O(0, 0)$, $f = 0$

At $A(6, 0)$, $f = 4(6) + 5(0) = 24$

At $E(4, 3)$, $f = 4(4) + 5(3) = 31$

At $D(0, 7)$, $f = 4(0) + 5(7) = 35$

$\therefore$ f is maximum at $D(0, 7)$ and the maximum value is 35.

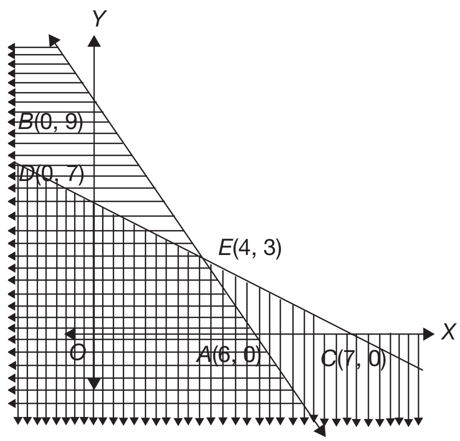


Figure 18.5

EXAMPLE 18.2

A manufacturer makes two models A and B of a product. Each model is processed by two machines. To complete one unit of model A, machines I and II must work 1 hour and 3 hours. To complete one unit of model B, machine I and II must work 2 hours and 1 hour. Machine I may not operate for more than 8 hours per day, and machine II for not more than 9 hours per day. If profits on model A and B per unit are ₹300 and ₹350, then how many units of each model should be produced, per day, to maximize the profit?

SOLUTION

Let the manufacturer produce x units of model A and y units of model $B \Rightarrow x \geq 0$, $y \geq 0$.

$\therefore$ Profit function $f = 300x + 350y$.

To make x and y units of models A and B, machine I should be used only 8 hours per day.

$\therefore$ $x + 2y \leq 8$ and machine II should be used for at the most 9 hours per day.

$3x + y \leq 9$ and $x \geq 0$, $y \geq 0$

Hence, we maximize $f = 300x + 350y$, subject to the constraints.

$$x + 2y \leq 8$$
$$3x + y \leq 9$$
$$x \geq 0,\ y \geq 0$$

As $x \geq 0$ and $y \geq 0$, the feasible region lies in the first quadrant.

The graph of $x + 2y \leq 8$ is shown with horizontal lines and the graph of $3x + y \leq 9$ is shown with vertical lines.

∴ The feasible region is the part of the first quadrant in which there are both horizontal and vertical lines.

The shaded region is the closed polygon having vertices $O(0, 0)$, $A(3, 0)$, $B(2, 3)$ and $C(0, 4)$.

Profit function $f = 300x + 350y$

At vertex $O(0, 0)$, $f = 300(0) + 350(0) = 0$

At vertex $A(3, 0)$, $f = 300(3) + 350(0) = 900$

At vertex $B(2, 3)$, $f = 300(2) + 350(3) = 600 + 1050 = 1650$

At vertex $C(0, 4)$, $f = 300(0) + 350(4) = 1400$

∴ f is maximum at the vertex $B(2, 3)$.

Hence, in order to get the maximum profit, the manufacturer has to produce 2 units of model A and 3 units of model B per day.

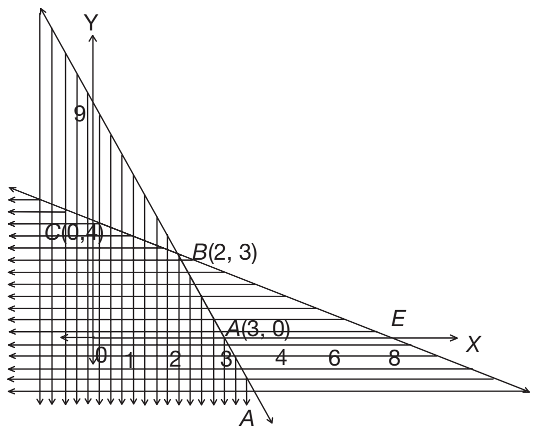


Figure 18.6

EXAMPLE 18.3

A transport company has two main depots P and Q, from where buses are sent to three sub-depots A, B and C in different parts of a region. The number of buses available at P and Q are 12 and 18. The requirements of A, B and C are 9, 13 and 8 buses. The distance between the two main depots and the three sub-depots are given in the following table.

To / From	A	B	C
P	15	40	50
Q	20	30	70

How should the buses be sent from P and Q to A, B and C, so that the total distance covered by the buses is the minimum?

SOLUTION

Let x buses be sent from P to A, and y buses be sent from P to B.

Since, P has only 12 buses, so $12 - (x + y)$ buses are sent from P to C.

Since, 9 buses are required for A, and x buses are sent from P, so $(9 - x)$ buses should be sent from Q to A. Similarly, the number of buses to be sent from Q to B is $13 - y$, and the number of buses to be sent from Q to C is $18 - (9 - x + 13 - y)$ or $x + y - 4$.

The above can be represented in the following table:

The Number of Buses to be Sent

to→ / From↓	A (9)	B (13)	C (8)
P (12)	x	y	$12 - (x + y)$
Q (18)	$9 - x$	$13 - y$	$x + y - 4$

All the variables are non-negative.

The given conditions are:

$x \geq 0$ and $x \leq 9$

$y \geq 0$ and $y \leq 13$

The region which satisfies the above set of inequalities is the rectangle $OGBC$.

The graph of $x + y \geq 4$ is shown with horizontal lines and the graph of $x + y \leq 12$ is shown with vertical lines.

∴ The feasible region is the area with both horizontal and vertical lines within the rectangular region $OGBC$, i.e., the polygonal region $FGHDE$.

The distance covered by the buses

$$d = 15x + 40y + 50(12 - (x + y)) + 20(9 - x) + 30(13 - y) + 70(x + y - 4)$$

$$= 15x + 40y + 600 - 50x - 50y + 180 - 20x + 390 - 30y + 70x + 70y - 280$$

$$= 15x + 30y + 890$$

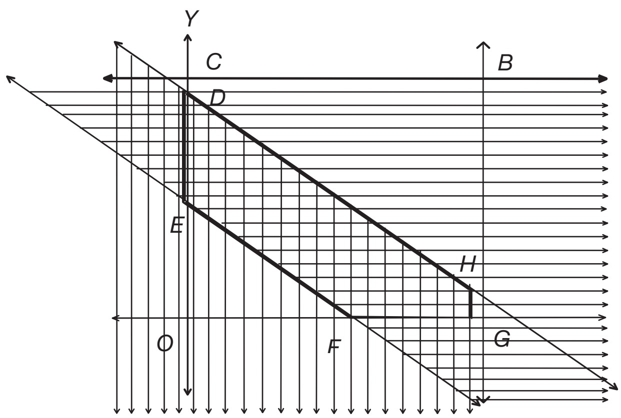


Figure 18.7

∴ We have to minimize the distance d.

In the graph, $DEFGH$ is a closed-convex polygon with vertices $F(4, 0)$, $G(9, 0)$, $H(9, 3)$, $D(0, 12)$ and $E(0, 4)$ objective function $d = 15x + 30y + 890$.

At vertex $F(4, 0)$, $d = 15(4) + 30(0) + 890 = 60 + 890 = 950$

At vertex $G(9, 0)$, $d = 15(9) + 30(0) + 890 = 1025$

At vertex $H(9, 3)$, $d = 15(9) + 30(3) + 890 = 1115$

At vertex $D(0, 12)$, $d = 15(0) + 30(12) + 890 = 1250$

At vertex $E(0, 4)$, $d = 15(0) + 30(4) + 890 = 1010$

$\therefore$ d is minimum at $F(4, 0)$

$\therefore$ $x = 4$; $y = 0$

$\therefore$ 4 buses have to be sent from P to A, and 8 buses have to be sent from P to C. Also, 5 buses have to be sent from Q to A, and 13 buses have to be sent from Q to B to minimize the distance travelled.

The Number of Buses to be Sent

From ↓ / to →	A	B	C
P	4	0	8
Q	5	13	0

General Graphical Method for Solving Linear Programming Problems

If the polygon is not a closed-convex polygon, the above method is not applicable. So, we apply the following method:

1. In this method, first we draw graphs of all systems of inequations representing the constraints.
2. Let the objective function be $ax + by$. Now, for different values of the function, we draw the corresponding lines $ax + by = c$. These lines are called the isoprofit lines.
3. Take different values of c, so that the line $ax + by = c$ moves away from the origin till we reach a position where the line has at least one point in common with the feasible region. At this point, the objective function has the optimum value.

This is explained with the following example.

EXAMPLE 18.4

Minimize $3x + 2y$ subject to the constraints $x + y \geq 5$ and $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$.

SOLUTION

As $x \geq 0$ and $y \geq 0$, the feasible region lies in the I quadrant.

The graph of $x + y \geq 5$ is shown with horizontal lines, and the graph of $x + 2y \geq 6$ is shown with vertical lines.

$\therefore$ The feasible region is that part of the I quadrant in which there are both horizontal and vertical lines.

At $B(4, 1)$, $f(x, y) = 3(4) + 2(1) = 14$

At $A(0, 5)$, $f(x, y) = 3(0) + 2(5) = 10$

At $C(6, 0)$, $f(x, y) = 3(6) + 2(0) = 18$.

Clearly, $f(x, y)$ is the minimum at A.

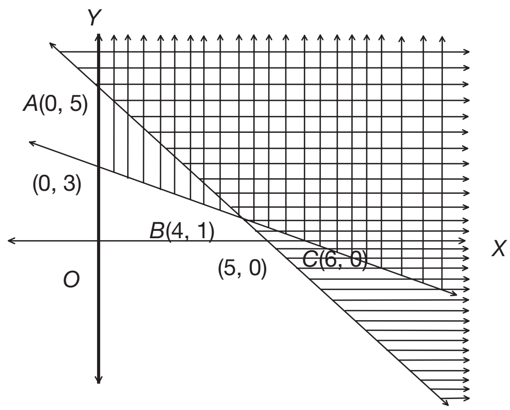


Figure 18.8

EXAMPLE 18.5

In an examination, the marks obtained by Rohan in two subjects are x and y. The total marks in the two subjects is less than or equal to 150. The maximum of marks of each subject is 100. Find the sum of the minimum and the maximum values of $3x + 4y$. (No negative marks in the exam.)

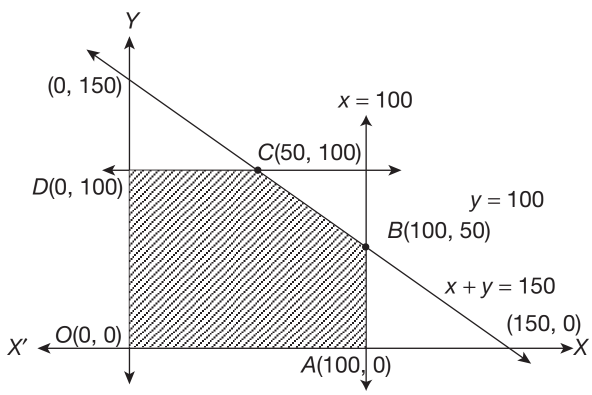
(a) 400 **(b)** 550 **(c)** 500 **(d)** 300

SOLUTION

$x + y \leq 150$

$x \geq 0$, $y \geq 0$ and $x \leq 100$

$y \leq 100$



The shaded region is a closed convex polygon whose vertices are:

$O(0, 0)$, $A(100, 0)$, $B(100, 50)$, $C(50, 100)$ and $D(0, 100)$.

$f = 3x + 4y$

The value off at $O(0, 0)$ is $= 3 \times 0 + 4 \times 0 = 0$.

The value off at $A(100, 0)$ is $= 300$.

The value off at $B(100, 50)$ is $300 + 200 = 500$.

The value off at $C(50, 100)$ is

$= 3 \times 50 + 4 \times 100 = 550$

The value of f at $D(0,100)$ is $= 0 + 400 = 400$.

The sum of minimum value and maximum value $= 0 + 550 = 550$.

EXAMPLE 18.6

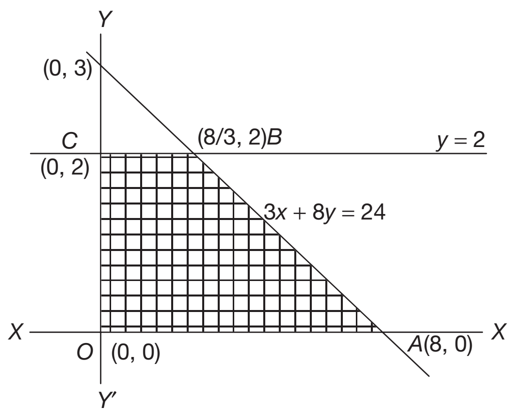
Find the maximum value of $4x + 7y$ with the conditions $3x + 8y \leq 24$, $y \leq 2$, $x \geq 0$ and $y \geq 0$.

(a) 14 **(b)** $\frac{74}{3}$ **(c)** 21 **(d)** 32

SOLUTION

Given condition $3x + 8y \leq 24$

$y \leq 2$; $x \geq 0$, $y \geq 0$.



The vertices of the feasible polygon region are (0, 0), (8, 0), (8/3, 2) and (0, 2).

The maximum value of the objective function attains at (8, 0).

$f = 4x + 7y$

The maximum value $= 4 \times 8 + 7 \times 0 = 32$.

EXAMPLE 18.7

Which of the following is a point in the feasible region determined by the linear inequations $3x + 2y \geq 6$ and $8x + 7y \leq 56$?

(a) (3, 1) **(b)** (–3, 1) **(c)** (1, –3) **(d)** (–3, –1)

HINT

(3, 1) satisfies both the given inequations.

EXAMPLE 18.8

The solution of the system of inequalities $x \geq 0$, $y \geq 0$, $5x + 2y \geq 10$, $6x + 5y \leq 30$ is a polygonal region with the vertices ______.

(a) (0, 0), (2, 0), (0, 5), (0, 6)

(b) (0, 0), (5, 0), (6, 0), (0, 2)

(c) (2, 0), (5, 0), (0, 6), (0, 5)

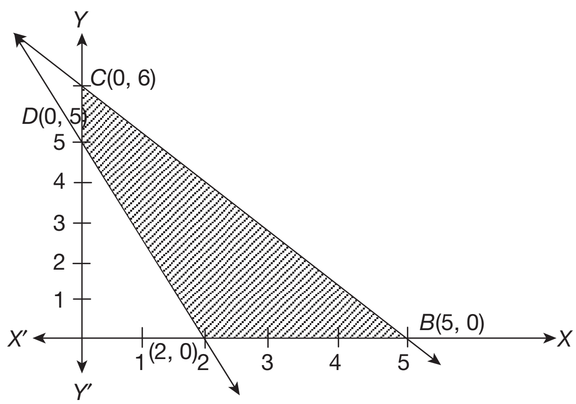
(d) (0, 0), (0, 5), (6, 0), (2, 0)

SOLUTION

Given constraints are:

$5x + 2y \geq 10$; $6x + 5y \leq 30$ and $x \geq 0$, $y \geq 0$

The given lines form a closed-convex polygon with the vertices (2, 0), (5, 0), (0, 6) and (0, 5)



EXAMPLE 18.9

The length and breadth of a rectangle (in cm) are x and y respectively $x \leq 30$, $y \leq 20$, $x \geq 0$ and $y \geq 0$. If a rectangle has a maximum perimeter, then its area is ______.

(a) 400 cm^2 **(b)** 600 cm^2 **(c)** 900 cm^2 **(d)** None of these

SOLUTION

The perimeter function is $p = 2(x + y)$

p is maximum, when $x = 30$ and $y = 20$

$\Rightarrow \quad p = 2(50) = 100$

$\therefore$ The area of the rectangle = $30 \times 20 = 600$ cm^2.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Does the point (1, 3) lie in the region specified by $x - y + 2 > 0$?

2. The region specified by the inequality $4x + 6y \leq 12$ contains the origin. (True/False)

3. Does the point (0, 0) lie in the region specified by $x + y > 6$?

4. In a rectangular coordinate system, the region specified by the inequality $y \geq 1$ lies below the X-axis. (True/False)

5. The feasible solution that maximizes an objective function is called ______.

6. If the line segment joining any two points A and B, belonging to a subset Y of a plane, is contained in the subset Y, then Y is called ______.

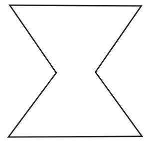
7. Does the line $y - x + 3 = 0$ pass through the point (3, 0)?

8. If $x > 0$ and $y < 0$, then the point (x, y) lies in the ______ quadrant of a rectangular co-ordinate system.

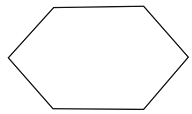
9. State whether the point (–3, 4) lies on the line, $3x + 2y + 1 = 0$ or not?

10. State which of the following figures are convex?

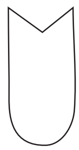
(i)



(ii)



(iii)



11. Define the feasible region.

12. Define the feasible solution.

13. The distances of a point P from the positive X axis and positive Y axis are 3 units and 5 units, respectively. Find the coordinates of P.

14. State which of the following points belong to the region specified by the corresponding inequations. (1, 2), $3x + 4y < 4$.

15. State which of the following points belong to the region specified by the corresponding inequations. (–4, 8), $5x + 6y + 30 > 0$.

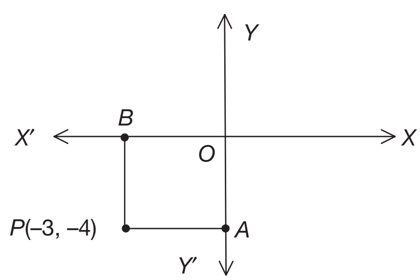
Short Answer Type Questions

16. If (0, 0), (0, 4), (2, 4) and (3, 2) are the vertices of a polygonal region subject to certain constraints, then the maximum value of the objective function $f = 3x + 2y$ is ______.

17. If (3, 2), (2, 3), (4, 2) and (2, 4) are the vertices of a polygonal region subject to certain constraints, then the minimum value of the objective function $f = 9x + 5y$ is ______.

18. A profit of ₹300 is made on class I ticket, and ₹800 is made on class II ticket. If x and y are the number of tickets of class I and class II sold, then the profit function is ______.

19. In the following figure, find AP and BP.



20. Draw the graphs of the following inequations.

$x - 4y + 8 \geq 0$

21. Draw the graphs of the following inequations.

$4x - 5y - 20 \le 0$

22. Draw the polygonal region represented by the given systems of inequations.

$x \ge 1,\ y \ge 1,\ x \le 4,\ y \le 4$

23. Minimize $x + y$, subject to the constraints:

$2x + y \ge 6$

$x + 2y \ge 8$

$x \ge 0$ and $y \ge 0$

24. Define the following:

Convex set

25. Define the following:

Feasible solution

Essay Type Questions

26. A dietician wishes to mix two types of items in such a way that the mixture contains at least 9 units of vitamin A and at least 15 units of vitamin C. Item (1) contains 1 unit/kg of vitamin A and 3 units/kg of vitamin C while item (2) contains 3 units/kg of vitamin A and 5 units/kg of vitamin C. Item (1) costs ₹6.00/kg and item (2) costs ₹9.00/kg. Formulate the above information as a linear programming problem.

27. A manufacturer produces pens and pencils. It takes 1 hour of work on machine A and 2 hours on machine B to produce a package of pens while it takes 2 hours on machine A and 1 hour on machine B to produce a package of pencils. He earns a profit of ₹4.00 per package on pens and ₹3.00 per package on pencils. How many packages of each should he produce each day so as to maximize his profit, if he operates his machines for at most 12 hours a day? Formulate the above information mathematically and then solve.

28. Santosh wants to invest a maximum of ₹150,000 in saving certificates and national saving bonds, which are in denominations of ₹4000 and ₹5000, respectively. The rate of interest on saving certificate is 10% per annum, and the rate of interest on national saving bond is 12% per annum. Formulate the above information as a linear programming problem.

29. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 8 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 10 minutes each for cutting and 15 minutes each for assembling. There are 4 hours available for cutting and 5 hours available for assembling. The profit is 60 paise on each item of type A and 75 paise on each item of type B. Formulate the above information as a linear programming problem.

30. Find the ratio of the maximum and minimum values of the objective function $f = 3x + 5y$ subject to the constraints: $x \ge 0,\ y \ge 0,\ 2x + 3y \ge 6$ and $9x + 10y \le 90$.

CONCEPT APPLICATION

Level 1

1. If an isoprofit line coincides with the edge of the polygon, then the problem has

(a) no solution (b) one solution

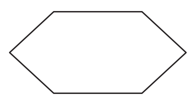
(c) infinite solutions (d) None of these

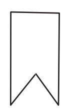
2. Which of the following is a convex set?

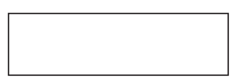
(a) A triangle (b) A square

(c) A circle (d) All of these

3. Which of the following is not a convex set?

(a) 

(b) 

(c) 

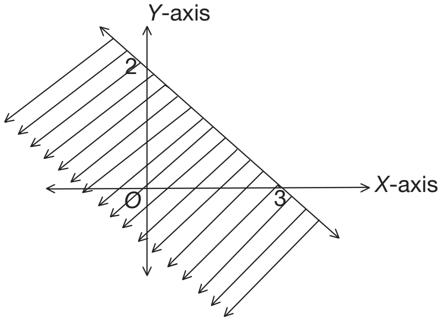
(d) None of these

4. Which of the following points belongs to the region indicated by the inequation $2x + 3y < -6$?

(a) (0, 2) (b) (–3, 8)

(c) (3, –2) (d) (–2, –2)

5. The inequation represented by the following graph is



(a) $2x + 3y + 6 \leq 0$

(b) $2x + 3y - 6 \geq 0$

(c) $2x + 3y \leq 6$

(d) $2x + 3y + 6 \geq 0$

6. The minimum value of $2x + 3y$ subjected to the conditions $x + 4y \geq 8$, $4x + y \geq 12$, $x \geq 0$ and $y \geq 0$ is

(a) $\frac{28}{3}$ (b) 16

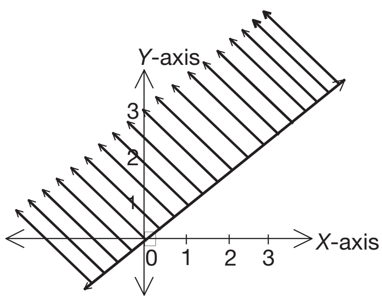
(c) $\frac{25}{3}$ (d) 10

7. Find the maximum value of $x + y$ subject to the conditions $4x + 3y \leq 12$, $2x + 5y \leq 10$, $x \geq 0$, $y \geq 0$.

(a) 3 (b) $\frac{20}{7}$

(c) 4 (d) $\frac{23}{7}$

8. The inequation represented by the graph given below is:



(a) $x \geq y$ (b) $x \leq y$

(c) $x + y \geq 0$ (d) $x + y \leq 0$

9. The solution of the system of inequalities $x \geq 0$, $x - 5 \leq 0$ and $x \geq y$ is a polygonal region with the vertices as

(a) (0, 0), (5, 0), (5, 5)

(b) (0, 0), (0, 5), (5, 5)

(c) (5, 5), (0, 5), (5, 0)

(d) (0, 0), (0, 5), (5, 0)

10. If the isoprofit line moves away from the origin, then the value of the objective function ______.

(a) increases

(b) decreases

(c) does not change

(d) becomes zero

11. The solutions of the inequations $x \geq 0$, $y \geq 0$, $y = 2$ and $x = 2$ form the polygonal region with the vertices (0, 0) (0, 2) (2, 0) and (2, 2) and the polygon so formed by joining the vertices is a _____.

(a) parallelogram

(b) rectangle

(c) square

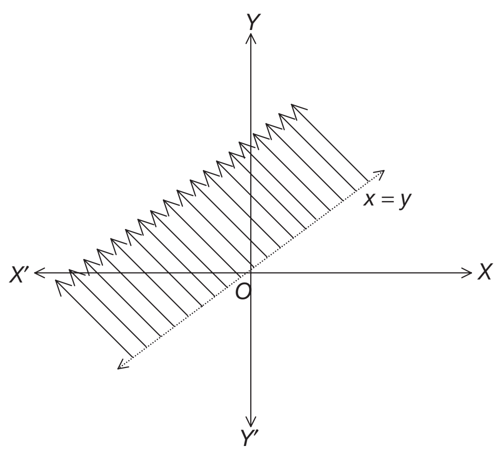
(d) rhombus

12. Maximize $5x + 7y$, subject to the constraints $2x + 3y \leq 12$, $x + y \leq 5$, $x \geq 0$ and $y \geq 0$.

(a) 29 (b) 30

(c) 28 (d) 31

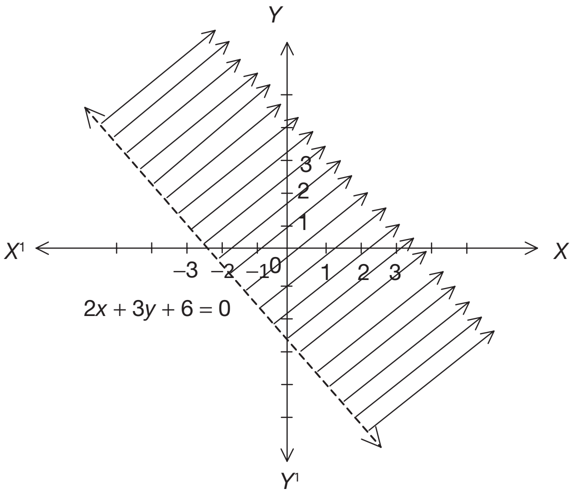
13. The inequation that best describes the graph given below is _______.



(a) $x > y$ (b) $x < y$

(c) $x \leq y$ (d) $x \geq y$

14. The inequation that best describes the following graph is ________.



(a) $2x + 3y + 6 \leq 0$

(b) $2x + 3y + 6 \geq 0$

(c) $2x + 3y + 6 > 0$

(d) $2x + 3y + 6 < 0$

15. The vertices of a closed-convex polygon determined by the inequations $7x + 9y \leq 63$ and $5x + 7y \geq 35$, $x \geq 0$, $y \geq 0$ are:

(a) (7, 0) (5, 0) (9, 0) (0, 9)

(b) (7, 0) (9, 0) (0, 7) (0, 5)

(c) (9, 0) (6, 0) (5, 0) (0, 8)

(d) (0, 9) (0, 5) (7, 0) (3, 0)

Level 2

16. The vertices of a closed convex polygon representing the feasible region of the objective function are (6, 2), (4, 6), (5, 4) and (3, 6). Find the maximum value of the function $f = 7x + 11y$.

(a) 64 (b) 79

(c) 94 (d) 87

17. If the vertices of a closed-convex polygon are A(8, 0), O(0, 0), B(20, 10), C(24, 5) and D(16, 20), then find the maximum value of the objective function $f = \frac{1}{4}x + \frac{1}{5}y$.

(a) $7\frac{1}{2}$ (b) 8

(c) 6 (d) 7

18. Find the profit function p, if it yields the values 11 and 7 at (3, 7) and (1, 3), respectively.

(a) $p = -8x + 5y$ (b) $p = 8x - 5y$

(c) $p = 8x + 5y$ (d) $p = -(8x + 5y)$

19. A shopkeeper can sell upto 20 units of both books and stationery. If he makes a profit of ₹2 on each book and ₹3 on each unit of stationery, then the profit function is ______, if x and y denote the number of units of books and stationery sold.

(a) $p = 2x - 3y$ (b) $p = 2x + 3y$

(c) $p = 3x - 2y$ (d) $p = 3x + 2y$

20. The vertices of a closed-convex polygon representing the feasible region of the objective function f are (4, 0), (2, 4), (3, 2) and (1, 4). Find the maximum value of the objective function $f = 7x + 8y$.

(a) 39 (b) 46

(c) 49 (d) 38

21. The cost of each table and each chair cannot exceed ₹7. If the cost of 3 tables and 4 chairs cannot exceed ₹30, form the inequations for the above data.

(a) $x > 0$, $y > 0$, $x \leq 7$, $y \geq 7$, $3x + 4y \leq 30$

(b) $x < 0$, $y < 0$, $x \leq 7$, $y \leq 7$, $3x + 4y \leq 30$

(c) $0 < x < 7$, $0 < y \leq 7$, $3x + 4y \leq 30$

(d) $x > 0$, $\leq y > 0$, $x \geq 7$, $y \geq 7$ and $3x + 4y \leq 30$

22. The vertices of the closed-convex polygon determined by the inequations $3x + 2y \geq 6$, $4x + 3y \leq 12$, $x \geq 0$ and $y \geq 0$ are

(a) (1, 0), (2, 0), (0, 2), (0, 1)

(b) (2, 0), (3, 0), (0, 4) and (0, 3)

(c) (1, 0), (2, 0), (0, 2) and (2, 2)

(d) (1, 0), (0, 2), (2, 2) and (1, 1)

23. Which of the following is a point in the feasible region determined by the linear inequations $2x + 3y \leq 6$ and $3x - 2y \leq 16$?

(a) (4, –3) (b) (–2, 4)

(c) (3, –2) (d) (3, –4)

24. The maximum value of the function $f = 5x + 3y$ subjected to the constraints $x \geq 3$ and $y \geq 3$ is ______.

(a) 15 (b) 9

(c) 24 (d) Does not exist

25. A telecom company manufactures mobile phones and landline phones. They require 9 hours to make a mobile phone and 1 hour to make a landline phone. The company can work not more than 1000 hours per day. The packing department can pack at most 600 telephones per day. If x and y are the sets of mobile phones and landline phones, respectively, then the inequalities are:

(a) $x + y \geq 600$, $9x + y \leq 1000$, $x \geq 0$, $y \geq 0$

(b) $x + y \leq 600$, $9x + y \geq 1000$, $x \geq 0$, $y \geq 0$

(c) $x + y \leq 600$, $9x + y \leq 1000$, $x \leq 0$, $y \leq 0$

(d) $9x + y \leq 1000$, $x + y \leq 600$, $x \geq 0$, $y \geq 0$

26. If the isoprofit line moves towards the origin, then the value of the objective function ______.

(a) increases

(b) does not change

(c) becomes zero

(d) decreases

27. The minimum cost of each tablet is ₹10 and each capsule is ₹10. If the cost of 8 tablets and 5 capsules is not less than ₹150, frame the inequations for the given data.

(a) $x \geq 10$, $y \geq 10$, $8x + 5y \geq 150$

(b) $x \geq 10$, $y \geq 10$, $8x + 5y \leq 150$

(c) $x \leq 10$, $y \leq 10$, $8x + 5y \geq 150$

(d) $x \leq 10$, $y \leq 10$, $8x + 5y \leq 150$

28. Find the profit function p in two variables x and y, if it yields the values 23 and 7 at (3, 2) and (2, 3) respectively.

(a) $p = 11x + 5y$

(b) $p = 5x + 11y$

(c) $p = 11x - 5y$

(d) $p = 5x - 11y$

29. The maximum value of the function, $f = 3x + 5y$, subject to the constraints $x \geq 5$ and $y \geq 5$, is ______.

(a) 40 (b) 24

(c) 8 (d) Does not exist.

30. The vertices of a closed-convex polygon representing the feasible region of the objective function $f = 5x + 3y$, are (0, 0), (3, 0), (3, 1), (1, 3) and (0, 2). Find the maximum value of the objective function.

(a) 6 (b) 18

(c) 14 (d) 15

Level 3

31. The cost of each table or each chair cannot exceed ₹9. If the cost of 4 tables and 5 chairs cannot exceed ₹120, then the inequations which best represents the above information are:

(a) $x < 9$, $y < 9$, $5x + 4x \geq 120$

(b) $x > 9$, $y > 9$, $4x + 5y \geq 120$

(c) $0 < x \leq 9$, $0 < y \leq 9$, $4x + 5y \leq 120$

(d) $0 < x \leq 9$, $0 < y \leq 9$, $5x + 4y \geq 120$

32. The vertices of a closed-convex polygon determined by the inequations $5x + 4y \leq 20$, $3x + 7y \leq 21$, $x \geq 0$ and $y \geq 0$ are

(a) (0, 0)(7, 0)(0, 3)$\left(\frac{148}{69}, \frac{45}{23}\right)$

(b) (4, 0)(0, 3)(0, 5)$\left(\frac{148}{69}, \frac{45}{23}\right)$

(c) (0, 0), (4, 0)(0, 3)$\left(\frac{56}{23}, \frac{45}{23}\right)$

(d) (0, 0)(7, 0)(4, 0)(0, 3)

33. The profit function p which yields the values 61 and 57 at (4, 7) and (5, 6), respectively, is ________.

(a) $2x + 5y$ (b) $7x + 3y$

(c) $5x + 2y$ (d) $3x + 7y$

34. The vertices of a closed-convex polygon representing the feasible region of the objective function f are (5, 1) (3, 5) (4, 3) and (2, 5). Find the maximum value of the function $f = 8x + 9y$.

(a) 61 (b) 69

(c) 59 (d) 49

35. The cost of each table or each chair cannot exceed ₹13. If the cost of 5 tables and 7 chairs cannot exceed ₹250, then the inequations which best represents the above information are

(a) $x > 13,\ y > 13,\ 5x + 7y > 250$

(b) $x > 0,\ y > 0,\ 5x + 7y < 250$

(c) $x < 13,\ y < 13,\ 5x + 7y \leq 25$

(d) $0 < x \leq 13,\ 0 < y \leq 13,\ 5x + 7y \leq 250$

36. The minimum value of $f = x + 4y$ subject to the constraints $x + y \geq 8,\ 2x + y \geq 10,\ x \geq 0,\ y \geq 0$ is ______.

(a) 4 (b) 26

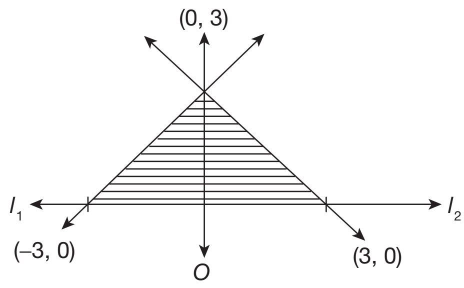
(c) 5 (d) 8

37. A tailor stitches trousers and shirts and each piece is completed by two machines I and II. To complete each trousers, machines I and II must work 3/2 hours and 2 hours respectively, and to complete each shirt, machines I and II must work 2 hours and 1 hour respectively. Machine I may not operate for more than 12 hours per day and machine II not more than 11 hours per day. If the profit on each trouser and each shirt is ₹150 and ₹100 respectively, then the maximum profit is ______.

(a) ₹900 (b) ₹500

(c) ₹375 (d) ₹600

38. Which of the following inequations represent the shaded region in the given figure



(a) $y \geq 0,\ x + y \leq 3,\ x - y \geq -3$

(b) $x \geq 0,\ x \pm y \leq 3$

(c) $y \geq 0,\ x \pm y \geq -3,\ -3 \leq x$

(d) $x \geq 0,\ x \pm y \leq -3$

39. A telecom company offers calls for day and night hours. Calls can be availed 8 hours during the day and 4 hours at night and at most 10 hours a day. The profit on the day calls is ₹60 per hour, and on night calls ₹50 per hour. How many hours during the day and at night a customer must use to fetch a maximum profit to the company?

(a) 6 hours during day and 4 hours at night

(b) 5 hours during day and 5 hours at night

(c) 8 hours during day and 2 hours at night

(d) 6 hours during day and 2 hours at night

40. On a rainy day, a shopkeeper sells two colours (black and red) of umbrellas. He sells not more than 20 umbrellas of each colour. At least twice as many black ones are sold as the red ones. If the profit on each of the black umbrellas is ₹30 and that of the red ones is ₹40, then how many of each kind must be sold to get a maximum profit?

(a) 20, 10 (b) 30, 15

(c) 40, 20 (d) 10, 5

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. No
2. True
3. No
4. False
5. optimum solution
6. a convex set
7. Yes
8. fourth (Q_4)
9. The point lies on the line
10. (ii) Convex set
13. $P = (5, 3)$
15. −4, 8

Short Answer Type Questions

16. 14
17. 33
18. $f = 300x + 800y$
19. $AP = 3$ units, $BP = 4$ units
23. $\frac{14}{3}$

Essay Type Questions

26. $x + 3y \geq 9$,
    $3x + 5y \geq 15$ and
    $x \geq 0,\ y \geq 0$
27. The manufacturer can manufacture 4 packages of each pens and pencils daily to obtain the maximum profit of ₹28
28. $4x + 5y \leq 150$ and $x \geq 0,\ y \geq 0$.
29. $8x + 10y \leq 240$
    $10x + 15y \leq 300$
    $x \geq 0,\ y \geq 0$
30. 5 : 1

CONCEPT APPLICATION

Level 1

1. (c) **2.** (d) **3.** (b) **4.** (d) **5.** (c) **6.** (a) **7.** (d) **8.** (b) **9.** (a) **10.** (a)
11. (c) **12.** (a) **13.** (b) **14.** (c) **15.** (b)

Level 2

16. (c) **17.** (b) **18.** (a) **19.** (b) **20.** (b) **21.** (c) **22.** (b) **23.** (c) **24.** (d) **25.** (d)
26. (d) **27.** (a) **28.** (c) **29.** (d) **30.** (b)

Level 3

31. (c) **32.** (c) **33.** (d) **34.** (b) **35.** (d) **36.** (d) **37.** (a) **38.** (a) **39.** (c) **40.** (a)

CONCEPT APPLICATION

Level 1

2. Recall the definition of convex set.

3. Recall the definition of convex set.

4. Check the point which satisfies the given inequation.

5. The corresponding equation of the line is $\frac{x}{3}+\frac{y}{1}=1$.

6. Find the open-convex polygon formed by the given inequations and proceed.

7. Find the closed-convex polygon formed by the given inequations and proceed.

8. The corresponding equation of the line is $x = y$.

9. Check the points which belongs the given inequations.

10. Increases (standard result).

11. Check which quadrilateral is formed by the points (0, 0), (0, 2), (2, 0) and (2, 2).

12. Represent the regions of given inequations and identify the vertices of the convex polygons.

17. (i) Substitute the given points in f.

(ii) The maximum/minimum value of f occurs at one of the vertices of the closed-convex polygon.

(iii) Substitute the given points in f.

(iv) Identify the maximum value of f.

18. (i) Profit function $p = ax + by$.

(ii) Take the profit function as $p = ax + by$.

(iii) Substitute the given points and obtain the equations in a and b.

(iv) Solve the above equations to get a and b.

19. (i) $p = ax + by$ – find a and b.

(ii) Profit on books is ₹$2x$ and profit on stationary is ₹$3y$.

(iii) Profit function = Profit on books + Profit on stationary.

20. (i) Substitute the given points.

(ii) The maximum/minimum value of f occurs at one of the vertices of the closed-convex polygon.

(iii) Substitute the given points in f.

(iv) Identify the maximum value of f.

21. (i) Let the cost of each table and chair be x and y.

(ii) Consider the cost of each table and each chair as ₹x and ₹y respectively.

(iii) Frame the inequations according to the given conditions.

22. (i) Check the points which belong to the given inequation.

(ii) Represent the given inequations on the graph.

(iii) Detect the closed-convex polygon and its vertices.

23. (i) Check the point which satisfies the given inequation.

(ii) Substitute the values in the options in the given inequations.

(iii) The point which satisfies the given inequations is the required point.

24. (i) Find the convex polygon.

(ii) Represent the given inequations on the graph.

(iii) Find the vertices of the closed convex polygon.

(iv) Substitute the vertices of the polygon in f and check for the maximum value of f.

25. (i) Time taken to manufacture x mobiles and y landlines is $9x$ and y hours respectively.

(ii) $x \geq 0$ and $y \geq 0$. As the number of mobiles cannot be negative.

(iii) Use the above information and frame the inequations.

26. If the isoprofit line moves towards the origin, then the value of the objective function decreases.

27. Let the cost of each tablet be x and the cost of each capsule be y. Minimum cost of each tablet and each capsule is ₹10.

$\therefore$ $x \geq 10$ and $y \geq 10$.

The cost of 8 tablets and 5 capsules should be greater than or equal to 150, i.e., $8x + 5y \geq 150$.

28. Let the profit function be $p = ax + by$

Given at (3, 2), p attains the value 23.

$3a + 2b = 23$ (1)

At (2, 3), p attains the value 7.

$2a + 3b = 7$ (2)

Solving Eqs. (1) and (2), we get $a = 11$, $b = -5$

$\therefore$ Profit function (p) = $11x - 5y$.

29. The given constraints $x \geq 5$ and $y \geq 5$ form an open-convex polygon.

$\therefore$ The maximum value does not exist.

30. Objective function $f = 5x + 3y$

The vertices of the convex polygon are (0, 0), (3, 0), (3, 1), (1, 3) and (0, 2).

The maximum value attains at (3, 1).

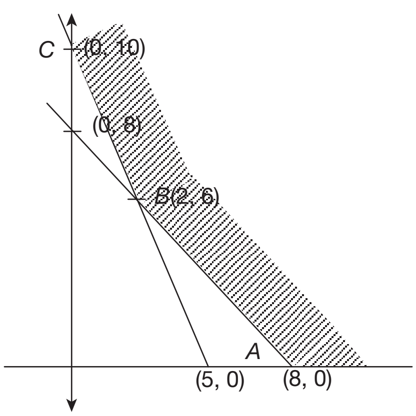
$\therefore$ The maximum value = $5 \times 3 + 3 \times 1$

= 15 + 3 = 18.

36. $x + y \geq 8$; $2x + y \geq 10$, $x \geq 0$; $y \geq 0$

Solving $x + y = 8$; $2x + y = 10$, we get $(x, y) = (2, 6)$

ABC is an open-convex polygon.



The vertices are $A(8, 0)$, $B(2, 6)$, $C(0, 10)$

$F = x + 4y$

$A(8, 0), f = 8 + 4(0) = 8$

$C(0, 10), f = 0 + 4(10) = 40$

$B(2, 6), f = 2 + 4(6) = 26$

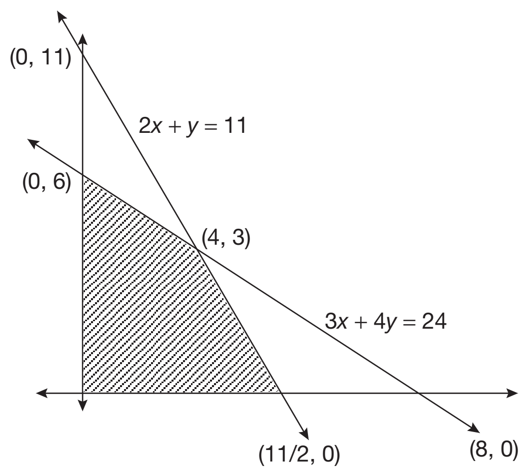
$\therefore$ The minimum value of f is 8.

It attains at $A(8, 0)$.

37. Let the number of trousers = x

Let the number of shirts = y

	Machine I (≤ 12 hours)	Machine II (≤ 11 hours)
Trousers (x)	$\frac{3}{2}$ hours	2 hours
Shirts (y)	2 hours	1 hour



According to the given condition:

$\Rightarrow \frac{3x}{2} + 2y \leq 12$ and $2x + y \leq 11$

$\Rightarrow 3x + 4y \leq 24$ and $2x + y \leq 11$

$x \geq 0$, $y \geq 0$

The profit function, $p = 15x + 10y$

The shaded region is a closed-convex polygon with vertices $\left(\frac{11}{2}, 0\right)$, (4, 3), (0, 6) (0, 0).

$\therefore$ $ABCD$ is the feasible reason.

$\therefore$ The maximum profit is at $C(4, 3)$, i.e.,

$P = 150 \times 4 + 100 \times 3 = 900$.

38. In the shaded region, y is non-negative.

$\therefore$ $y \geq 0$

Intercepts of l_1 are 3, 3.

And the region contains origin $\Rightarrow x + y \leq 3$.

Intercepts of l_2 are −3, 3, and the region contains the origin.

$x - y \geq -3$.

39. Let the number of hours used during the day = x

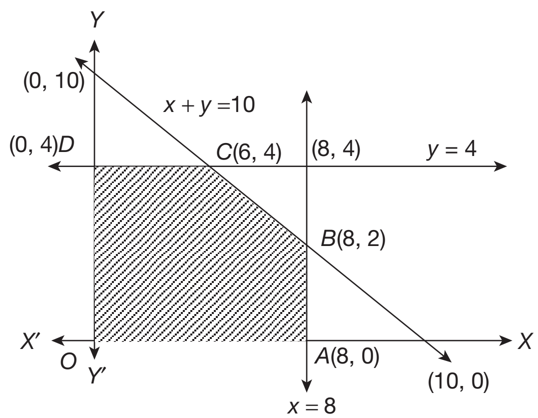
Let the number of hours used at night = y

Given, $x + y \leq 10$, $x \leq 8$ and $y \leq 4$.

Profit on day calls = ₹60 per hour

Profit on night calls = ₹50 per hour

Profit function $p = 60x + 50y$



The maximum profit is at $B(8, 2)$

$\therefore\ P = 60x + 50y$

$= 60 \times 8 + 50 \times 2 = 480 + 100$

$= 580$

∴ 8 hours during day and 2 hours at night.

40. Let the number of black umbrellas sold be x and that of red umbrellas be y

$x \geq 0$ and $x \leq 20$

$y \geq 0$ and $y \leq 20$

$x \geq 2y$.

Profit function $p = 30x + 40y$

The maximum profit attains at $c(20, 10)$.

The maximum profit = $30 \times 20 + 40 \times 10$

$= 600 + 400 =$ ₹1000.

Number of black umbrellas = 20

Number of red umbrellas = 10.