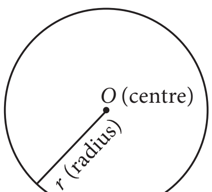
CHAPTER 3

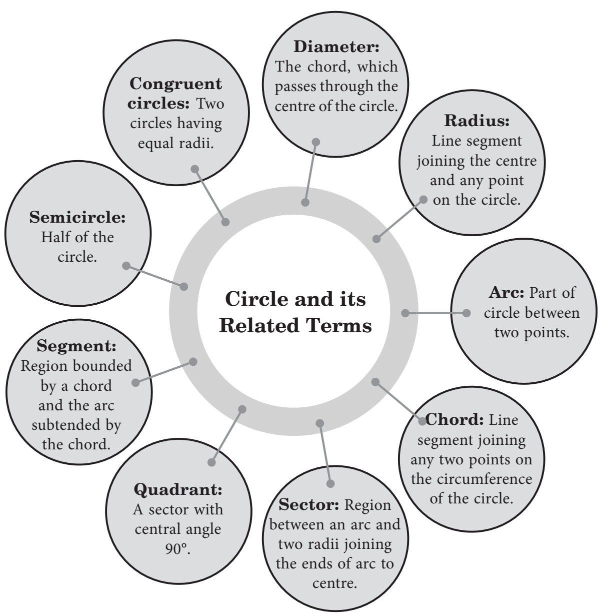
Circles

Recap Notes

- **Circle :** A circle is a collection of all those points in a plane which are at a constant distance from a fixed point in the plane. The fixed point (say O) is called the centre of the circle and the constant distance r is called the radius of the circle. A circle with centre 'O' and radius 'r' is usually denoted by $C(O, r)$.

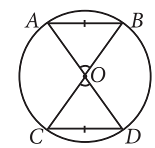


- Some basic terms of circle are as given below:



EQUAL CHORDS OF A CIRCLE SUBTEND EQUAL ANGLES AT THE CENTRE

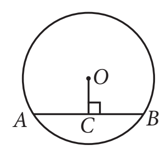
- In the given figure, it is given that O is the centre of the circle and AB and CD are its two equal chords. Then, $\angle AOB = \angle COD$



Converse of the above result is also true *i.e.,* if the angles subtended by two chords of a circle at the centre are equal, then the chords are equal.

THE PERPENDICULAR FROM THE CENTRE OF A CIRCLE TO A CHORD BISECTS THE CHORD

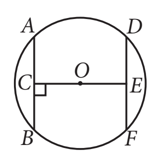
- In the given figure, O is the centre of the circle and a perpendicular from it is drawn to the chord AB. Then, $AC = CB$



Converse of the above result is also true *i.e.,* A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

EQUAL CHORDS OF A CIRCLE ARE EQUIDISTANT FROM THE CENTRE

- In the given figure, if AB and DF are two equal chords.

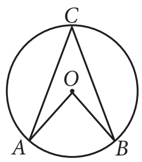


Then, $OC = OE$.

Converse of above result is also true *i.e.*, Chords equidistant from the centre of a circle are equal.

THE ANGLE SUBTENDED BY AN ARC AT THE CENTRE IS DOUBLE THE ANGLE SUBTENDED BY IT AT ANY POINT ON THE REMAINING PART OF CIRCLE

- In the given figure, arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at a point C on the remaining part of the circle.
  Then, $\angle AOB = 2\angle ACB$

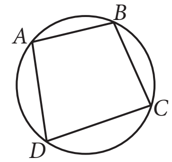


CYCLIC QUADRILATERAL

- A quadrilateral $ABCD$, whose all the vertices lie on a circle is called a cyclic quadrilateral.

THE SUM OF THE EITHER PAIR OF OPPOSITE ANGLES OF A CYCLIC QUADRILATERAL IS 180°

- In the given figure, $ABCD$ is a cyclic quadrilateral.
  Therefore, $\angle ABC + \angle ADC = 180°$ and $\angle DAB + \angle BCD = 180°$



Converse of above result is also true. *i.e.*, If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

IMPORTANT POINTS

- Angles in the same segment of a circle are equal.
- Angle in a semicircle is a right angle.
- Exterior angle of a cyclic quadrilateral is equal to its interior opposite angle.
- The chord that makes larger angle at the centre is longer than the chord that makes smaller angle.

Practice Time

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. The collection of all points in a plane which are at a fixed distance from a given point of the plane is called

(a) Centre (b) Circumference
(c) Circle (d) Radius

2. Which of the following is the angle type in the major sector of a circle?

(a) acute angle (b) obtuse angle
(c) reflex angle (d) none of these

3. The two parts of the circle formed by a chord whose mid-point coincides with the centre are called

(a) Quadrants (b) Semicircles
(c) Segments (d) Sectors

4. End points of chord lie

(a) Outside the circle (b) Inside the circle
(c) On the circle (d) None of these

5. The points lying in which of the following don't belong to the circular region of a circle

(a) Exterior of circle (b) On the circle
(c) Interior of circle (d) Both (a) and (c)

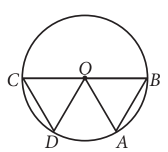
6. The degree measure of circle is

(a) 180° (b) 360° (c) 90° (d) 270°

7. The minute hand of a clock is at 12 and the hour hand is at 4. The angle between the hands of the clock is

(a) 120° (b) 20° (c) 30° (d) 60°

8. In the given figure, O is the centre of the circle. COD is an equilateral triangle. $CD = AB$, then $\angle AOB$ =



(a) 30° (b) 45° (c) 60° (d) 90°

9. AB, BC, CD and DA are four chords of a circle with centre O. If $\angle AOB = 50°$, $\angle BOC = 150°$ and $\angle COD = 85°$, then the longest among the four chords is

(a) BC (b) AB (c) DA (d) CD

10. Given a circle with centre O and smaller chord AB of length 5 cm and the longest chord CD of the circle of length 12 cm. The radius of the circle is

(a) 15 cm (b) 6 cm (c) 5 cm (d) 3.5cm

11. AD is a diameter of a circle and AB is a chord. If AD = 36 cm and AB = 26 cm, the distance of AB from the centre of the circle is

(a) 17.35 cm (b) 15.45 cm
(c) 12.45 cm (d) 8.35 cm

12. The length of the perpendicular from the centre of a circle of radius 13 cm on a chord of length 24 cm is

(a) 6 cm (b) 5 cm (c) 4 cm (d) 3 cm

13. Number of circles that can be drawn passing through two points are

(a) one
(b) two
(c) infinite
(d) no circle can be drawn

14. Equal chords of a circle are equidistant from

(a) the centre
(b) an extremity of a diameter
(c) any point on the circumference
(d) any point on the diameter

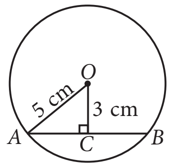
15. The centre of the circle passing through the vertices of a right angled triangle lies

(a) inside the triangle (b) outside the triangle
(c) on the hypotenuse (d) none of these

16. $ABCD$ is a cyclic quadrilateral. If $\angle A = 4x$, $\angle B = 3y$, $\angle C = 2x$ and $\angle D = 15y$, then x and y are equal to

(a) $x = 25°, y = 20°$ (b) $x = 30°, y = 10°$
(c) $x = 20°, y = 18°$ (d) $x = 10°, y = 25°$

17. In the following figure, O is the centre of the circle. OA = 5 cm and perpendicular OC on chord AB is 3 cm.Then the length of the chord AB is

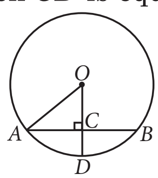


(a) 8 cm (b) 10 cm (c) 12 cm (d) 16 cm

18. The angle in a semi circle measures

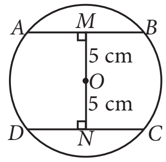
(a) 45° (b) 60° (c) 90° (d) 36°

19. In the given figure, if OA = 13 cm, AB = 24 cm and $OD \perp AB$, then CD is equal to



(a) 3 cm (b) 8 cm (c) 4 cm (d) 5 cm

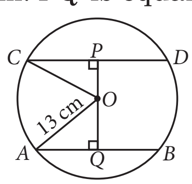
20. In the adjacent figure, what is the relation between AB and CD?



(a) $AB > CD$ (b) $AB < CD$

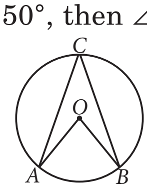
(c) $AB = CD$ (d) $AB = 2CD$

21. In the given figure, O is the centre of the circle. Its radius is 13 cm, chord AB = 10 cm and chord CD = 24 cm. PQ is equal to



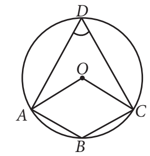
(a) 18 cm (b) 25 cm (c) 11.5 cm (d) 17 cm

22. In the given figure, O is the centre of the circle. If $\angle ACB$ = 50°, then $\angle AOB$ is equal to



(a) 60° (b) 100° (c) 90° (d) 105°

23. In the given figure, $\angle AOC$ = 130°, then $\angle ABC$ equals



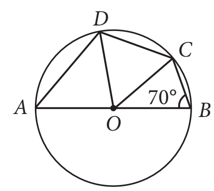
(a) 65° (b) 130° (c) 115° (d) 125°

24. $ABCD$ is a cyclic quadrilateral. If $\angle A = (2x + 4)°$, $\angle B = (y + 10)°$, $\angle C = (4x - 4)°$ and $\angle D = (5y + 20)°$, then $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively are

(a) 70°, 122°, 110°, 58° (b) 60°, 105°, 120°, 75°

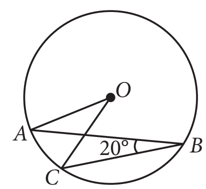
(c) 84°, 50°, 96°, 130° (d) 64°, 35°, 116°, 145°

25. In the given figure, AB is diameter of the circle, $OC = DC$ and $\angle OBC$ = 70° Then $\angle ODA$ is equal to



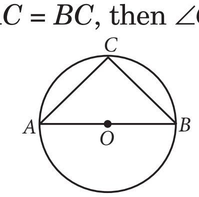
(a) 30° (b) 60° (c) 50° (d) 80°

26. In the given figure, if $\angle ABC$ = 20°, then $\angle AOC$ is equal to



(a) 20° (b) 40° (c) 60° (d) 10°

27. In the given figure, if AOB is a diameter of the circle and $AC = BC$, then $\angle CAB$ is equal to



(a) 30° (b) 60° (c) 90° (d) 45°

28. If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then the radius of the circle is

(a) 15 cm (b) 17 cm (c) 16 cm (d) 34 cm

29. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is

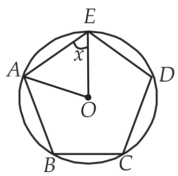
(a) 17 cm (b) 15 cm (c) 4 cm (d) 8 cm

30. The line drawn through the centre of a circle to bisect a chord is _______ to the chord.

(a) Perpendicular (b) Parallel

(c) Both (a) and (b) (d) Can't say

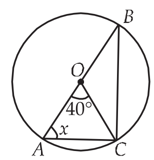
31. In the given pentagon $ABCDE$, $AB = BC = CD = DE = AE$.



The value of x is

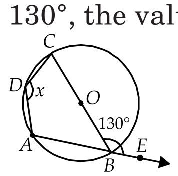
(a) 36° (b) 54° (c) 72° (d) 108°

32. In the given figure, AB is diameter, $\angle AOC = 40°$. The value of x is



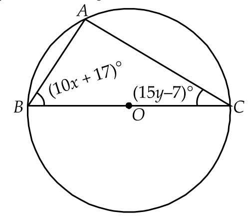
(a) 50° (b) 60° (c) 70° (d) 80°

33. $ABCD$ is a cyclic quadrilateral with centre O in the given figure. Chord AB is produced to E where $\angle CBE = 130°$, the value of x is equal to



(a) 130° (b) 260° (c) 140° (d) 280°

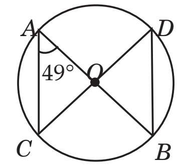
34. In the given figure, BC passes through the centre of a circle where points A, B and C from a triangle and $\angle B$ is 44° more than $\angle C$. The values of x and y respectively are



(a) $x = 4; y = 3$ (b) $x = 3; y = 5$

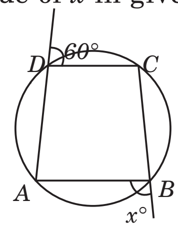
(c) $x = 7; y = 2$ (d) $x = 5; y = 2$

35. In the given figure, O is the centre of a circle. If $\angle OAC = 49°$, then $\angle ODB =$



(a) 40° (b) 49° (c) 60° (d) 75°

36. Find the value of x in given figure.



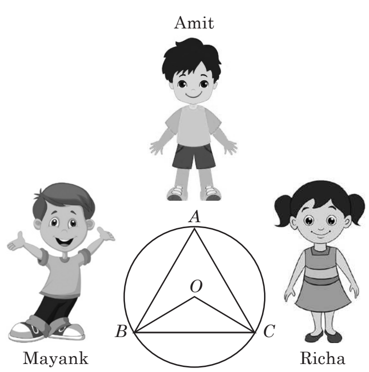
(a) 120° (b) 100°

(c) 140° (d) None of these

Case Based MCQs

Case I : Read the following passage and answer the questions from 37 to 41.

Three friends Amit, Mayank and Richa were playing with ball by standing on a circle at A, B and C point respectively. Richa throws a ball to Amit, Amit to Mayank and Mayank to Richa. They all are equidistant from each other as shown in the figure.



Considering O as the centre of the circle, answer the following questions.

37. Which type of ΔABC is given in figure?

(a) Right angled triangle

(b) Equilateral triangle

(c) Isosceles triangle

(d) Scalene triangle

38. Measure of $\angle ABC$ is

(a) 45° (b) 60°

(c) 30° (d) 90°

39. If $AB = 6$ cm, then $BC + CA$ is equal to

(a) 12 cm (b) 14 cm

(c) 15 cm (d) 18 cm

40. Measure of $\angle BOC$ is

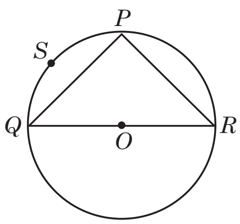
(a) 90° (b) 100° (c) 120° (d) 150°

41. Value of $\angle OBC + \angle OCB$ is

(a) 30° (b) 45° (c) 90° (d) 60°

Case II : Read the following passage and answer the questions from 42 to 46.

Ankit visited in a mall with his father. He sees that three shops are situated at P, Q, R as shown in the figure from where they have to purchase things according to their need.
Distance between shop P and Q is 8 m, that of between shop Q and R is 10 m and between shop P and R is 6 m.



Considering O as the centre of the circle, answer the following questions.

42. Find the radius of the circle.
(a) 5 m (b) 7 m (c) 14 m (d) 8 m

43. Measure of $\angle QPR$ is
(a) 60° (b) 90° (c) 120° (d) 180°

44. Area of ΔPQR is
(a) 18 m^2 (b) 20 m^2 (c) 22 m^2 (d) 24 m^2

45. Length of the longest chord of the circle is
(a) 6 m (b) 8 m (c) 10 m (d) 24 m

46. In figure, $PSQP$ is known as
(a) Major segment (b) Minor segment
(c) Major sector (d) Minor sector

Case III : Read the following passage and answer the questions from 47 to 51.

Rohan is class 9th student. He did not attend school as his health is not fine since few days. At home, he open his Maths textbook and observe chapter circle. He was so much curious to know about the properties of circle. His father Mr Raman, who is also a good Mathematician helps Rohan to understand all properties one by one. At last, he ask some questions to Rohan for quick revision. Help Rohan to answer the following questions.

47. A circle can circumscribe a
(a) Rectangle (b) Trapezium
(c) Triangle (d) All of these

48. The perpendicular distance from the centre of the circle decreases when
(a) the length of the chord increases
(b) the length of the chord decreases
(c) Radius of circle is decreased
(d) Radius of circle is increased

49. The number of radii that can be drawn in a circle is
(a) 10 (b) 100 (c) 1000 (d) infinite

50. The distance from the centre of the circle to the longest chord is equal to
(a) $\frac{22}{7}$ units (b) 0 unit
(c) 1 unit (d) 3 units

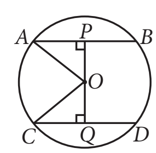
51. Angle formed by the equal arcs on the circumference of the circle
(a) are supplementary (b) are equal
(c) are complimentary (d) None of these

Assertion & Reasoning Based MCQs

Directions (Q.52 to 60) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

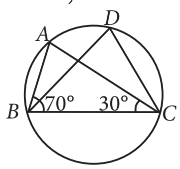
(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

52. Assertion : In the given figure, O is the centre of the circle of radius 5 cm. If $OP \perp AB$, $OQ \perp CD$, $AB \parallel CD$, AB = 6 cm and CD = 8 cm, then PQ = 6 cm.



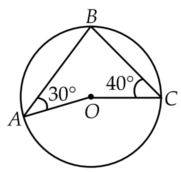
Reason : Perpendicular bisectors of two chords of a circle intersect at its centre.

53. Assertion : In the given figure, $\angle ABC = 70°$ and $\angle ACB = 30°$. Then, $\angle BDC = 80°$.



Reason : Angles in the same segment of a circle are equal.

54. Assertion : In the given figure, $\angle BAO = 30°$ and $\angle BCO = 40°$. Then the measure of $\angle AOC = 60°$.

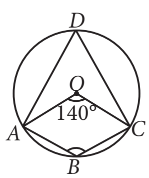


Reason : Angle subtended by an arc of a circle at the centre of the circle is twice the angle subtended by that arc on the remaining part of the circle.

55. Assertion : AB and CD are two parallel chords of a circle whose diameter is AC. Then $AB \neq CD$.

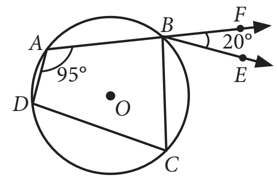
Reason : Perpendicular from the centre of a circle bisects the chord.

56. Assertion : In the given figure, O is the centre of circle. If $\angle AOC = 140°$, then $\angle ABC = 110°$.



Reason : In cyclic quadrilateral, opposite angles are supplementary.

57. Assertion : In the given figure, $ABCD$ is a cyclic quadrilateral in which AB is extended to F and $BE \mid\mid DC$. If $\angle FBE = 20°$ and $\angle DAB = 95°$, then $\angle ADC = 105°$.



Reason : A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

58. Assertion : The length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm is 17.32 cm.

Reason : The perpendicular from the centre of a circle to a chord bisects the chord.

59. Assertion : In a cyclic quadrilateral $ABCD$, $\angle A - \angle C = 60$, then the smaller of two is 60

Reason : Opposite angles of cyclic quadrilateral are supplementary.

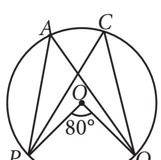
60. Assertion : In an isosceles triangle ABC with $AB = AC$, a circle is passing through B and C intersects the sides AB and AC at D and E respectively. Then $DE \parallel BC$.

Reason : Exterior angle of a cyclic quadrilateral is equal to interior opposite angle of that quadrilateral.

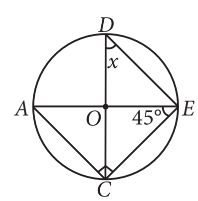
SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

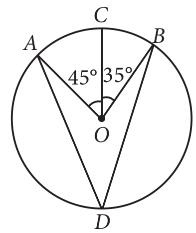
1. In the given figure, if $\angle POQ = 80°$, then find $\angle PAQ$ and $\angle PCQ$.



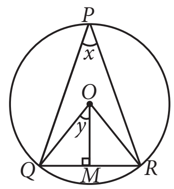
2. If O is the centre of the circle, then find the value of x in the given figure.



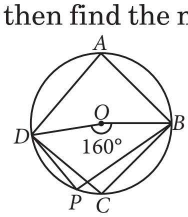
3. In the given figure, O is the centre of the circle, $\angle AOC = 45°$ and $\angle COB = 35°$. Find the measure of $\angle ADB$.



4. In the given figure, O is the centre of circle. If $\angle QPR = x$, OM bisects $\angle QOR$ and $\angle QOM = y$, then find the relation between x and y.



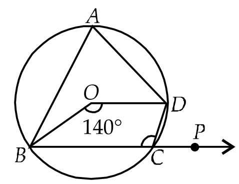
5. In the given figure, $ABCD$ is a cyclic quadrilateral and O is the centre of the circle. If $\angle BOD = 160°$, then find the measure of $\angle BPD$.



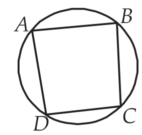
6. If $ABCD$ is a cyclic quadrilateral in which $AD \parallel BC$, then prove that $\angle B = \angle C$.

7. Two chords AB and CD of a circle are parallel and a line l is the perpendicular bisector of AB. Show that l bisects CD.

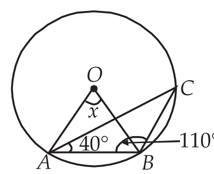
8. In the given figure, O is the centre of the circle. The angle subtended by the arc BCD at the centre is 140°. BC is produced to P. Determine $\angle BAD$.



9. In a cyclic quadrilateral $ABCD$, if $\angle A = 3\angle C$. Find $\angle A$.

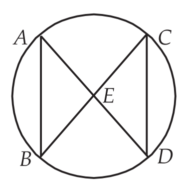


10. In the given figure, O is the centre of the circle. $\angle CAB = 40°$, $\angle CBA = 110°$, then find value of x.

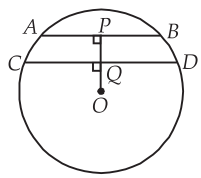


Short Answer Type Questions (SA-I)

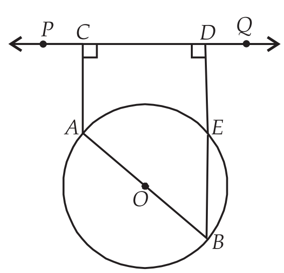
11. In the given figure, $AB = CD$. Prove that $BE = DE$ and $AE = CE$, where E is the point of intersection of AD and BC.



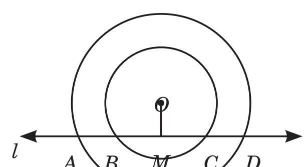
12. In the given figure, AB and CD are two parallel chords of a circle with centre O and radius 13 cm such that $AB = 10$ cm and $CD = 24$ cm. If $OP \perp AB$ and $OQ \perp CD$, find the length of PQ.



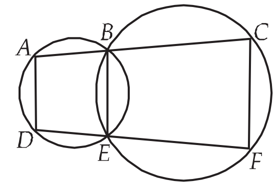
13. In the given figure, AB is a diameter of the circle with centre O. If AC and BD are perpendicular on a line PQ, and BD meets the circle at E, prove that $AC = ED$.



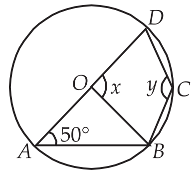
14. Two concentric circles with centre O have A, B, C, D as the points of intersection with the line l as shown in the figure. If $AD = 12$ cm and $BC = 8$ cm, then find the lengths of AB, CD, AC and BD.



15. In the given figure, A, B, C and D, E, F are two sets of collinear points. Prove that $AD \parallel CF$.

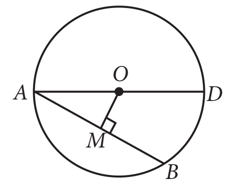


16. In the given figure, O is the centre of the circle and $\angle DAB = 50$. Find the values of x and y.



17. A chord of circle of radius 12.5 cm with centre O is of length 15 cm. Find its distance (in cm) from the centre.

18. AD is a diameter of a circle and AB is a chord. If $AD = 58$ cm, $AB = 40$ cm, then find the distance of AB (in cm) from the centre of the circle.



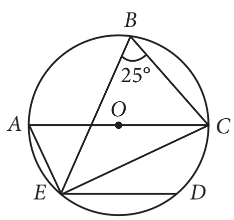
19. Find the length of a chord which is at a distance of 8 cm from the centre of a circle of radius 17 cm.

20. In a ΔABC, if $\angle A = 60°$ and the altitudes from B and C meet AC and AB at P and Q, respectively and intersect each other at I. Then, Prove that $APIQ$ is a cyclic quadrilaterals.

Short Answer Type Questions (SA-II)

21. Prove that, among any two chords of a circle, the larger chord is nearer to the centre.

22. In the given figure, chord ED is parallel to the diameter AC of the circle. Given $\angle CBE = 25°$. If $\angle DEC = \dfrac{1}{k} \times 455°$, then find the value of k.

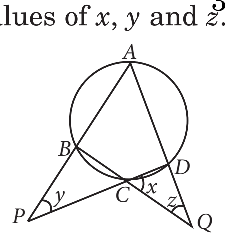


23. Prove that any angle subtended by a minor arc in the alternate segment is acute and any angle subtended by a major arc in the alternate segment is obtuse.

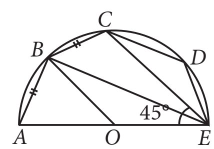
24. Prove that the mid-point of the hypotenuse of a right triangle is equidistant from its vertices.

25. D and E are points on equal sides AB and AC of isosceles ΔABC such that $AD = AE$. Prove that $\angle DBC + \angle CED = 180°$.

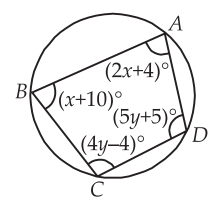
26. In the given figure, if $\dfrac{x}{3} = \dfrac{y}{4} = \dfrac{z}{5}$, then calculate the values of x, y and z.



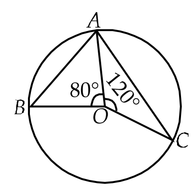
27. In the given figure, O is the centre and AE is the diameter of the semi circle $ABCDE$. If $AB = BC$ and $\angle AEC = 45°$, then find (i) $\angle CBE$ (ii) $\angle CDE$ (iii) $\angle AOB$. And prove that $BO \parallel CE$.



28. In the given figure, find out the value of $(x - y)$ when $\angle A = (2x + 4)°$, $\angle B = (x + 10)°$, $\angle C = (4y - 4)°$ and $\angle D = (5y + 5)°$.

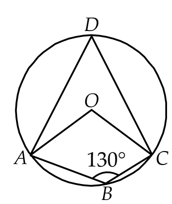


29. In the given figure, A, B, C are three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 80° and 120° respectively.



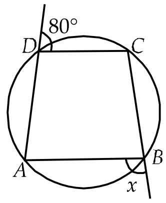
If $\angle BAC$ is $k°$, then find the value of k.

30. In the given figure, O is the centre of the circle and the measure of $\angle ABC$ is 130°.

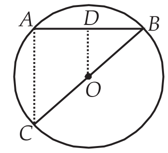


Find the value of $\dfrac{(\angle AOC - \angle ADC)}{25°}$.

31. In the given figure, $ABCD$ is a cyclic quadrilateral. Find the value of $\frac{x}{50°}$.



32. In the given figure, AB is a chord of a circle with centre O and BOC is a diameter. If $OD \perp AB$ such that $OD = 6$ cm, then the value of AC is $k \times 3$. Find the value of k.



Long Answer Type Questions (LA)

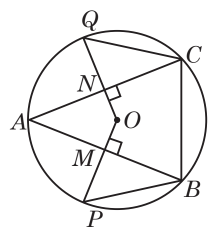
33. PQ and PR are the two chords of a circle of radius r. If the perpendiculars drawn from the centre of the circle to these chords are of lengths a and b, respectively and $PQ = 2PR$, then prove that $b^2 = \frac{a^2}{4} + \frac{3}{4}r^2$.

34. PQ and RS are two parallel chords of a circle whose centre is O and radius is 10 cm. If $PQ = 16$ cm and $RS = 12$ cm, find the distance between PQ and RS, if they lie

(i) on the same side of the centre O.

(ii) on the opposite side of the centre O.

35. In the adjoining figure, O is the centre of a circle. If AB and AC are chords of the circle such that $AB = AC$, $OP \perp AB$ and $OQ \perp AC$, then prove that $PB = QC$.



ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(c) :** The collection of all points in a plane which are at a fixed distance from a given point of a plane is called "circle".

2. **(c) :** Reflex angle is the angle type in major sector of circle.

3. **(b)**

4. **(c) :** End points of a chord of a circle lie on the circle.

5. **(a) :** The points lying in exterior of the circle don't belong to the circular region of a circle.

6. **(b)**

7. **(a) :** There are 12 divisions of a clock.

$\therefore$ Required angle $= \frac{4}{12} \times 360° = 120°$

8. **(c) :** Each angle of an equilateral triangle is 60°. Equal chords subtend equal angles at the centre.

$\therefore$ $\angle AOB = \angle COD = 60°$

9. **(a) :** Since angle measure in a circle is 360°.

$\therefore$ $\angle AOB + \angle BOC + \angle COD + \angle DOA = 360°$

$\Rightarrow$ $50° + 150° + 85° + \angle DOA = 360°$

$\therefore$ $\angle DOA = 360° - 285° = 75°$

Since, the chord that makes larger angle at the centre is longer.

Here, $\angle BOC$ is larger angle.

$\therefore$ Chord BC is longest.

10. **(b) :** A diameter is the longest chord.

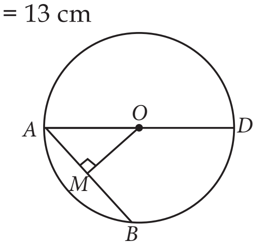
Also, radius $= \frac{1}{2}$ diameter $= \frac{1}{2} \times 12 = 6$ cm

11. **(c) :** $AO = OD = \frac{1}{2}AD = \frac{36}{2} = 18$ cm

Draw $OM \perp AB$.

$\therefore$ $AM = MB = \frac{1}{2}AB = \frac{26}{2}$ cm $= 13$ cm

In ΔAOM,



$OA^2 = OM^2 + AM^2$

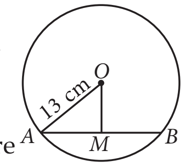
$\Rightarrow$ $18^2 = OM^2 + 13^2$

$\Rightarrow$ $324 = OM^2 + 169$

$\Rightarrow$ $OM^2 = 155$

$\Rightarrow$ $OM = 12.45$ cm

12. **(b) :** Let AB be the chord of a circle with centre O and radius 13 cm such that AB = 24 cm.



From O, draw $OM \perp AB$. Join OA.

Since the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AM = MB = \dfrac{1}{2} AB = \dfrac{1}{2} \times 24 = 12$ cm

In right triangle OAM, we have

$OA^2 = AM^2 + OM^2$

$\Rightarrow \quad 13^2 = 12^2 + OM^2$

$\Rightarrow \quad OM^2 = 169 - 144 = 25$

$\Rightarrow \quad OM = \sqrt{25} = 5$ cm.

13. **(c) :** Infinitely many circles can be drawn passing through two given points.

14. **(a) :** Equal chords of a circle are equidistant from the centre.

15. **(c) :** Angle in a semi-circle is a right angle. So, the centre of circle lies on the hypotenuse.

16. **(b) :** $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\therefore \quad 4x + 2x = 180°$ and $3y + 15y = 180°$

$\Rightarrow \quad 6x = 180°$ and $18y = 180° \Rightarrow x = 30°$ and $y = 10°$

17. **(a) :** In ΔAOC, we have

$AC = \sqrt{OA^2 - OC^2} = \sqrt{25 - 9} = 4$ cm

$\because$ Perpendicular from the centre to a chord bisects the chord

$\therefore \quad AB = 2AC = 2 \times 4 = 8$ cm

18. **(c) :** The angle in a semicircle measures 90°.

19. **(b) :** $AC = CB = \dfrac{1}{2} AB = \dfrac{24}{2}$ cm = 12 cm

[$\because OD \perp AB$]

$OA^2 = OC^2 + AC^2 \Rightarrow 13^2 = OC^2 + 12^2$

$\Rightarrow \quad OC^2 = 169 - 144 = 25 \Rightarrow OC = 5$ cm

Now, $CD = OD - OC = 13 - 5 = 8$ cm

[$\because OD = OA$]

20. **(c) :** If two chords of a circle are equidistant from the centre, then they are equal.

21. **(d) :** $AQ = QB = \dfrac{1}{2} AB = \dfrac{10}{2} = 5$ cm

[$\because OQ \perp AB$]

In right triangle AOQ, $OA^2 = OQ^2 + AQ^2$

$\Rightarrow \quad 13^2 = OQ^2 + 5^2 \Rightarrow OQ^2 = 144 \Rightarrow OQ = 12$ cm

Now, $CP = PD = \dfrac{1}{2} CD = \dfrac{24}{2}$ cm = 12 cm

[$\because OP \perp CD$]

In right triangle COP, $OC^2 = CP^2 + OP^2$

$\Rightarrow \quad 13^2 = 12^2 + OP^2 \Rightarrow OP^2 = 25 \Rightarrow OP = 5$ cm

Now, $PQ = OP + OQ = (5 + 12)$ cm = 17 cm

22. **(b) :** Here, $\angle ACB = 50°$

Since angle subtended by an arc at the centre of the circle is double the angle subtended by the arc at any point on the remaining part of circle.

$\therefore \quad \angle AOB = 2\angle ACB \Rightarrow \angle AOB = 2 \times 50° = 100°$

23. **(c) :** $\angle AOC = 2\angle ADC$

$\Rightarrow \quad 130° = 2\angle ADC \Rightarrow \angle ADC = 65°$

Now, $\angle ADC + \angle ABC = 180°$

[Opposite angles of a cyclic quadrilateral]

$\Rightarrow \quad 65° + \angle ABC = 180° \Rightarrow \angle ABC = 180° - 65° = 115°$

24. **(d) :** $\because$ $ABCD$ is a cyclic quadrilateral

$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow \quad (2x + 4)° + (4x - 4)° = 180°$

and $(y + 10)° + (5y + 20)° = 180°$

$\Rightarrow \quad 6x = 180°$ and $6y = 150°$

Solving above two equations for x and y, we get

$x = 30°$ and $y = 25°$

$\therefore \quad \angle A = 64°$ and $\angle B = 35°$, $\angle C = 116°$ and $\angle D = 145°$

25. **(c) :** In ΔOCD, $OC = OD$

And $OC = CD$ [Given]

$\therefore \quad OC = CD = OD$

So, ΔOCD is an equilateral triangle.

$\therefore \quad \angle ODC = 60°$

$ABCD$ is a cyclic quadrilateral

$\therefore \quad \angle ADC + \angle ABC = 180°$

$\Rightarrow \quad \angle ODA + \angle ODC + 70° = 180°$

$\Rightarrow \quad \angle ODA + 60° + 70° = 180°$

$\Rightarrow \quad \angle ODA = 180° - 130° = 50°$

26. **(b) :** Since, the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of circle.

$\therefore \quad \angle AOC = 2\angle ABC = 2 \times 20° = 40°$

27. **(d) :** Since angle in a semi circle is a right angle.

$\therefore \quad \angle BCA = 90°$ ...(i)

We have given, $AC = BC$

$\Rightarrow \quad \angle ABC = \angle CAB$ ... (ii)

[Angles opposite to equal sides are equal]

In ΔABC, $\angle CAB + \angle ABC + \angle BCA = 180°$

[Sum of angles of a triangle is 180°]

$\Rightarrow \quad \angle CAB + \angle CAB + 90° = 180°$ [From (i) and (ii)]

$\Rightarrow \quad 2\angle CAB = 90°$

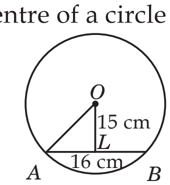
$\therefore \quad \angle CAB = 45°$

28. **(b) :** Given, length of chord AB = 16 cm,

$OL \perp AB$ such that OL = 15 cm.

We know that perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AL = LB = \dfrac{AB}{2} = \dfrac{16}{2} = 8$ cm



In ΔOAL,

$OA^2 = OL^2 + AL^2$ [By Pythagoras theorem]

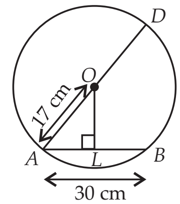
$\Rightarrow \quad OA^2 = (15)^2 + (8)^2 = 289$

$\Rightarrow \quad OA = 17$ cm

[$\because OA \neq -17$, as length can't be negative]

29. **(d) :** Draw $OL \perp AB$.
We know that perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AL = LB = \frac{1}{2}AB = \frac{1}{2} \times 30 = 15$ cm



Now, in right angled ΔOLA,
$OA^2 = OL^2 + AL^2$
[By Pythagoras theorem]
$\therefore \quad (17)^2 = OL^2 + (15)^2$
$\Rightarrow \quad 289 = OL^2 + 225$
$\Rightarrow \quad OL^2 = 64$
$\therefore \quad OL = 8$ cm
[$\because OL \neq -8$, as length can't be negative]
Hence, the distance of the chord AB from the centre is 8 cm.

30. **(a) :** The line drawn through the centre of the circle to bisect a chord is perpendicular to the chord.

31. **(b) :** Since, equal chords subtend equal angles at the centre.

$\therefore \quad \angle AOE = \frac{360°}{5} = 72°$
Now, $OE = OA \Rightarrow \angle OEA = \angle OAE = x$
In ΔOAE, $x + x + \angle AOE = 180°$
$\Rightarrow \quad 2x + 72° = 180° \Rightarrow x = \frac{108°}{2} = 54°$.

32. **(c) :** $\angle BCA = 90°$ [$\because AB$ is diameter]
Also, $\angle ABC = \frac{1}{2} \times \angle AOC = 20°$
[Angle subtended by an arc at centre is double the angle subtended by it at remaining part of the circle]
$\therefore \quad$ In ΔABC, $20° + x + 90° = 180° \Rightarrow x = 70°$

33. **(a) :** $\angle ADC = 180° - \angle CBA = \angle CBE = 130°$
[$\because ABC$ is a cyclic quadrilateral]
$\therefore \quad x = \angle ADC = 130°$

34. **(d) :** $\angle B - \angle C = 44°$ ...(i)
and $\angle C + \angle B = 90°$ ...(ii)
[BC is diameter of circle; $\angle A = 90°$]
From (i) & (ii), we get
$\angle B = 67°$ and $\angle C = 23°$
$\Rightarrow 10x + 17 = 67$ and $15y - 7 = 23 \Rightarrow x = 5$ and $y = 2$

35. **(b) :** $\angle CDB = \angle CAB = 49°$
(Angle in the same segment)
$\Rightarrow \quad \angle ODB = 49°$.

36. **(a) :** We have, $\angle CDA = 180° - 60° = 120°$
and $\angle ABC = 180° - x$
Now, $\angle CDA + \angle ABC = 180°$
$\Rightarrow \quad 120° + 180° - x = 180° \Rightarrow x = 120°$

37. **(b)**

38. **(b) :** In equilateral triangle, all angles are equal and of measure 60°.
$\therefore \quad \angle ABC = 60°$.

39. **(a) :** In equilateral triangle all sides are equal *i.e.*, $AB = BC = CA$.
Given, $AB = 6$ cm $\Rightarrow BC = AC = 6$ cm
$\therefore \quad BC + AC = 12$ cm

40. **(c) :** We know, $\angle BOC = 2\angle BAC$ ($\because$ Angle subtended by an arc at centre of a circle is double the angle subtended by same arc at any point on the circle)
$\therefore \quad \angle BOC = 2 \times 60° = 120°$

41. **(d) :** In ΔOBC, $\angle OBC + \angle OCB + \angle BOC = 180°$
(By angle sum property of a triangle)
$\Rightarrow \quad \angle OBC + \angle OCB = 180° - 120° = 60°$

42. **(a) :** Since QR is the diameter and its length is 10 m
$\therefore \quad$ Radius $= \frac{10}{2} = 5$ m

43. **(b) :** $\angle QPR = 90°$ (Angle in semi-circle)

44. **(d) :** Area of $\Delta PQR = \frac{1}{2} \times$ base $\times$ height
$= \frac{1}{2} \times PQ \times PR = \frac{1}{2} \times 8 \times 6 = 24$ m^2

45. **(c) :** Since, longest chord of the circle is its diameter.
$\therefore \quad$ Length of the longest chord $= QR = 10$ m

46. **(b)**

47. **(d)**

48. **(a)**

49. **(d)**

50. **(b)**

51. **(b)**

52. **(d) :** Join OA and OC.
Since the perpendicular from the centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively. Consequently,
$AP = PB = \frac{1}{2} AB = 3$ cm
and, $CQ = QD = \frac{1}{2} CD = 4$ cm.
In right triangles OAP and OCQ, we have
$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$
$\Rightarrow \quad 5^2 = OP^2 + 3^2$ and $5^2 = OQ^2 + 4^2$
$OP^2 = 5^2 - 3^2$ and $OQ^2 = 5^2 - 4^2$
$\Rightarrow \quad OP^2 = 16$ and $OQ^2 = 9$
$\Rightarrow \quad OP = 4$ and $OQ = 3$
$\therefore \quad PQ = OP + OQ = (4 + 3)$ cm $= 7$ cm.
$\therefore \quad$ Assertion is wrong but Reason is correct.

53. **(a) :** In ΔABC, we have

$\angle ABC + \angle BCA + \angle BAC = 180°$

$\Rightarrow \quad 70° + 30° + \angle BAC = 180°$

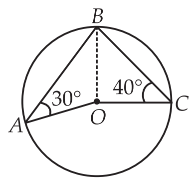
$\Rightarrow \quad \angle BAC = 80°$

$\therefore \quad \angle BDC = \angle BAC = 80°$

(Angles in the same segment are equal)

$\therefore$ Both Assertion are Reason are correct and Reason is correct explanation of Assertion.

54. **(d) :** Join BO.



In ΔAOB, we have

$OA = OB$ [Radii are a circle]

$\Rightarrow \quad \angle OBA = \angle OAB$

[Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \angle OBA = 30°$ ...(i)

Similarly, in ΔBOC, we have $OB = OC$

$\Rightarrow \quad \angle OCB = \angle OBC$

$\Rightarrow \quad \angle OBC = 40°$ ... (ii)

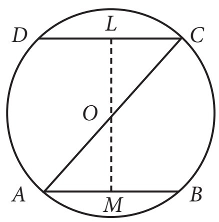
$\therefore \quad \angle ABC = \angle OBA + \angle OBC = 30° + 40° = 70°$

[Using (i) and (ii)]

Since angle subtended by an arc of a circle at the centre of the circle is twice the angle subtended by that arc on the remaining part of the circle.

$\therefore$ Assertion is wrong but Reason is correct.

55. **(d) :** Let O be the centre of the circle.



Draw $OM \perp AB$ and $OL \perp CD$.

Since $AB \;||\; CD$, therefore LOM is a straight line.

In ΔAOM and ΔCOL,

$OA = OC$ [Radii a circle]

$\angle OAM = \angle OCL$

[Alternate angles]

$\angle AOM = \angle COL$ [Vertically opposite angles]

$\therefore \quad \Delta AOM \cong \Delta COL$ [By ASA Congruency]

$\Rightarrow \quad AM = CL$ [By CPCT]

$\Rightarrow \quad 2AM = 2CL$

Since the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AB = CD$

$\therefore$ Assertion is wrong but Reason is correct.

56. **(a) :** Since angle subtended by an arc of a circle at the centre of the circle is twice the angle subtended by that arc on the remaining part of the circle.

$\therefore \quad \angle AOC = 2\angle ADC$

$\Rightarrow \quad \angle ADC = \dfrac{1}{2} \angle AOC = \dfrac{1}{2} \times 140° = 70°$

Now, $\angle ADC + \angle ABC = 180°$

[Opposite angles of cyclic quadrilateral]

$\Rightarrow \quad 70° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 180° - 70° = 110°$

$\therefore$ Both Assertion are Reason are correct and Reason is correct explanation of Assertion.

57. **(a) :** Sum of opposite angles of a cyclic quadrilateral is 180°.

$\therefore \quad \angle DAB + \angle BCD = 180° \Rightarrow 95° + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 180° - 95° = 85°$

$\because \quad BE \parallel DC$

$\therefore \quad \angle CBE = \angle BCD = 85°$

[Alternate interior angles]

$\therefore \quad \angle CBF = \angle CBE + \angle FBE = 85° + 20° = 105°$

Now, $\angle ABC + \angle CBF = 180°$ [Linear pair]

and $\angle ABC + \angle ADC = 180°$

[Opposite angles of cyclic quadrilateral]

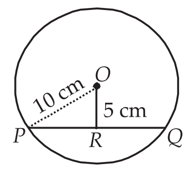
Thus, $\angle ABC + \angle ADC = \angle ABC + \angle CBF$

$\Rightarrow \quad \angle ADC = \angle CBF$

$\Rightarrow \quad \angle ADC = 105°$ [$\because \angle CBF = 105°$]

$\therefore$ Both Assertion are Reason are correct and Reason is the correct explanation of Assertion.

58. **(a) :** Let PQ be a chord of a circle with centre O and radius 10 cm. Draw $OR \perp PQ$.



Now, $OP = 10$ cm and $OR = 5$ cm

In right triangle ORP, we get

$OP^2 = PR^2 + OR^2$

$\Rightarrow \quad PR^2 = OP^2 - OR^2$

$\Rightarrow \quad PR^2 = 10^2 - 5^2 = 75$

$\Rightarrow \quad PR = \sqrt{75} = 8.66$

Since the perpendicular from the centre to a chord bisects the chord.

Therefore, $PQ = 2 \times PR = 2 \times 8.66 = 17.32$ cm

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

59. **(a) :** Since $ABCD$ is a cyclic quadrilateral, so, its opposite angles are supplementary

$\therefore \quad \angle A + \angle C = 180°$ ...(1)

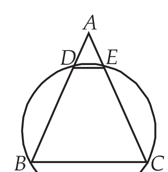
Also, $\angle A - \angle C = 60°$ ...(2)

On solving (1) and (2), we get

$\angle A = 120°, \angle C = 60°$

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

60. **(a) :** To prove $DE \,||\, BC$,



i.e., $\angle B = \angle ADE$.

In ΔABC, we have

$AB = AC \Rightarrow \angle B = \angle C$ ...(1)

In the cyclic quadrilateral $CBDE$, side BD is produced to A. We know that an exterior angle of cyclic quadrilateral is equal to interior opposite angle of cyclic quadrilateral.

$\therefore \quad \angle ADE = \angle C$ ... (2)

From (1) and (2), we get

$\angle B = \angle ADE$. Hence, $DE \,||\, BC$

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

SUBJECTIVE TYPE QUESTIONS

1. We know that, the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle PAQ = \angle PCQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 80° = 40°$

2. In ΔACE,

$\angle ACE = 90°$ (Angle in a semi circle)

Now, $\angle CAE + \angle ACE + \angle AEC = 180°$

$\Rightarrow \quad \angle CAE + 90° + 45° = 180°$

$\Rightarrow \quad \angle CAE = 180° - 135° = 45°$

Also, $\angle CDE = \angle CAE = 45°$

(Angles in same segment are equal)

$\therefore \quad x = 45°$

3. $\angle AOB = \angle AOC + \angle COB$

$= 45° + 35° = 80°$

Now, $2\angle ADB = \angle AOB$

($\because$ Angle subtended at centre is double the angle subtended at remaining part of the circle)

$\Rightarrow \quad 2\angle ADB = 80° \Rightarrow \angle ADB = 40°$

4. Since OM bisects $\angle QOR$

$\therefore \quad \angle QOR = 2 \times \angle QOM = 2y$ ...(i)

Since, angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad 2\angle QPR = \angle QOR$

$\Rightarrow \quad 2x = 2y$ [From (i)]

$\Rightarrow \quad x = y$

5. $\angle BAD = \frac{1}{2}\angle BOD = \frac{1}{2} \times 160° = 80°$

Now, $\angle BCD + \angle BAD = 180°$ (Opposite angles of a cyclic quadrilateral are equal)

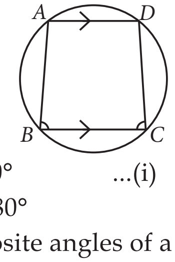
$\Rightarrow \quad \angle BCD + 80° = 180°$

$\Rightarrow \quad \angle BCD = 100°$

Now, $\angle BPD = \angle BCD = 100°$

(Angles in same segment are equal)

6. We have, $ABCD$ is a cyclic quadrilateral and $AD \parallel BC$.



$\therefore \quad \angle A + \angle B = 180°$ ...(i) (Co-interior angles)

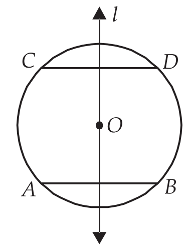
Also, $\angle A + \angle C = 180°$ ...(ii)

(Opposite angles of a cyclic quadrilateral)

From (i) and (ii), we get $\angle A + \angle B = \angle A + \angle C$

$\therefore \quad \angle B = \angle C$

7. We know that the perpendicular bisector of any chord of a circle always passes through the centre of the circle. Since l is the perpendicular bisector of AB. Therefore, l passes through the centre of the circle. But $l \perp AB$ and $AB \parallel CD \Rightarrow l \perp CD$.

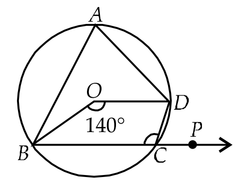


Thus $l \perp CD$ and passes through the centre of the circle. So, l is the perpendicular bisector of CD also.

8. Since the angle subtended by an arc BCD at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore \quad \angle BAD = \frac{1}{2}\angle BOD$$

$$= \frac{1}{2} \times (140°) = 70°$$



9. Since $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180°$

$\Rightarrow \quad 3\angle C + \angle C = 180° \Rightarrow \angle C = 45°$

$\therefore \quad \angle A = 3 \times 45° = 135°$

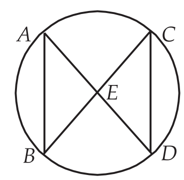
10. In ΔABC, $40° + 110° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 150° = 30°$

Now, $\angle AOB = 2 \times \angle C$

$\Rightarrow \quad x = 2 \times 30° = 60°$.

11. In ΔAEB and ΔCED, we have



$\angle BAE = \angle DCE$

[Angles in the same segment]

and $\angle ABE = \angle CDE$

[Angles in the same segment]

Also, $AB = CD$

$\therefore \Delta AEB \cong \Delta CED$ [By ASA congruence]

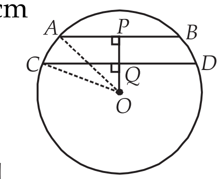
$\Rightarrow AE = CE$ and $BE = DE$ [C.P.C.T.]

12. Since $OP \perp AB$, $OQ \perp CD$ and $AB \parallel CD$, the points O, Q, P are collinear.

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore \quad AP = \frac{1}{2}AB = \left(\frac{1}{2} \times 10\right) \text{cm} = 5 \text{ cm}$$

and $CQ = \frac{1}{2}CD = \frac{1}{2}(24) = 12$ cm



Join OA and OC.

Then, $OA = 13$ cm. [Radius of circle]

From the right angled ΔOPA,

we have

$OP^2 = (OA^2 - AP^2) = [(13)^2 - (5)^2] = 144$

$\Rightarrow \quad OP = 12$ cm

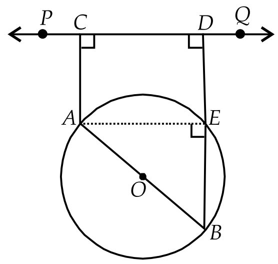
Now, from right angled ΔOQC, we have

$(OQ)^2 = (OC)^2 - (CQ)^2 = 13^2 - 12^2 = 25$

$\Rightarrow \quad OQ = 5$ cm

$\therefore \quad PQ = OP - OQ = 12 - 5 = 7$ cm

13. Join AE.



Now, $\angle AEB = 90°$

[Angle in a semicircle]

But, BED being a straight line, we have,

$\angle AEB + \angle AED = 180°$

$\Rightarrow \quad 90° + \angle AED = 180°$

$\Rightarrow \quad \angle AED = 90°$

Now, $\angle EAC + \angle ACD + \angle CDE + \angle AED = 360°$

[Sum of the angles of quadrilateral $EACD$ is 360°]

$\Rightarrow \quad \angle EAC + 90° + 90° + 90° = 360°$

$\Rightarrow \quad \angle EAC = 90°$

Thus, each angle of quadrilateral $EACD$ is 90°

So, $EACD$ is a rectangle.

$\therefore \quad AC = ED$ [Opposite sides of a rectangle].

14. Since, $OM \perp BC$

$\therefore \quad BM = CM = \frac{1}{2} BC = 4$ cm $\quad$ [$\because$ Perpendicular from the centre of a circle to a chord bisects the chord]

Similarly, $OM \perp AD$

$\Rightarrow \quad AM = DM = \frac{1}{2} AD = 6$ cm

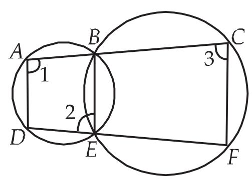
Now, $AB = AM - BM = (6 - 4)$ cm $= 2$ cm

Also, $CD = DM - CM = (6 - 4)$ cm $= 2$ cm

$AC = AB + BC = (2 + 8)$ cm $= 10$ cm

and $BD = BC + CD = (8 + 2)$ cm $= 10$ cm

15. In order to prove that $AD \parallel CF$, it is sufficient to prove that $\angle 1 + \angle 3 = 180°$



Since $ABED$ is a cyclic quadrilateral.

$\therefore \quad \angle 1 + \angle 2 = 180° \quad$ ...(i)

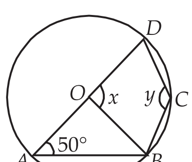
Now, $BCFE$ is a cyclic quadrilateral and in a cyclic quadrilateral an exterior angle is equal to the interior opposite angle.

$\therefore \quad \angle 2 = \angle 3 \quad$ ...(ii)

From (i) and (ii), we get $\angle 1 + \angle 3 = 180°$

Hence, $AD \parallel CF$

16. $ABCD$ is a cyclic quadrilateral.



$\therefore \quad 50° + y = 180°$

$\Rightarrow \quad y = 130°$

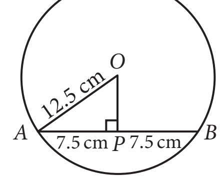
Clearly, ΔOAB is an isosceles triangle with $OA = OB$.

$\therefore \quad \angle OBA = \angle OAB = 50°$

$\Rightarrow \quad \angle AOB = 180° - (50° + 50°) = 80°$

Hence, $x = 180° - 80° = 100°$

17. Let AB be the chord of length 15 cm.



Draw $OP \perp AB$

$\therefore \quad P$ is the mid-point of chord AB

$\Rightarrow \quad AP = \frac{1}{2} \times 15 = 7.5$ cm

Now, in right angled ΔAPO,

$OP^2 = AO^2 - AP^2$

$= (12.5)^2 - (7.5)^2 = 156.25 - 56.25 = 100$

$\therefore \quad OP = \sqrt{100} = 10$ cm

18. $\because$ The perpendicular drawn from centre to the chord bisects it.

$\therefore \quad AM = \frac{1}{2} AB = \frac{1}{2} \times 40$ cm $= 20$ cm

Also, $OA = \frac{1}{2} AD = \frac{1}{2} \times 58$ cm $= 29$ cm

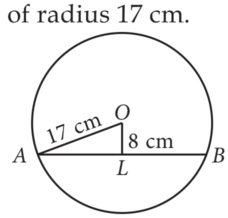
In right ΔOAM, we have $\quad OA^2 = OM^2 + AM^2$

$\Rightarrow \quad 29^2 = OM^2 + 20^2 \Rightarrow 841 = OM^2 + 400$

$\Rightarrow \quad OM^2 = 841 - 400 \Rightarrow OM^2 = 441 \Rightarrow OM = 21$ cm.

19. Let AB be the chord of a circle of radius 17 cm.

Draw $OL \perp AB$, join OA.



In right angled ΔALO

$AL = \sqrt{OA^2 - OL^2}$

$= \sqrt{17^2 - 8^2}$

$= \sqrt{289 - 64} = \sqrt{225} = 15$ cm

Since perpendicular from the centre to the chord bisects the chord.

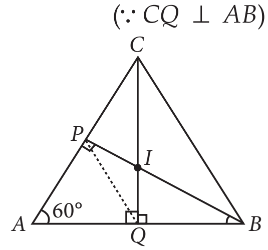
$\therefore \quad AB = 2AL = 2 \times 15$ cm $= 30$ cm

20. $\angle A = 60° \quad$ (Given)

$\angle APB = 90° \quad$ ($\because PB \perp AC$)

and $\angle AQC = 90° \quad$ ($\because CQ \perp AB$)

In quadrilateral $APIQ$,



$\angle P = 90°, \angle Q = 90°,$

$\angle A = 60°$

$\therefore \quad \angle I = 360° - (90° + 90° + 60°)$

$= 120° \quad$ ...(i)

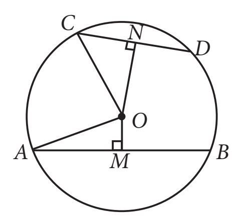
Now, we get $\angle P + \angle Q = 180°$

and $\angle A + \angle I = 180°$

Hence, $APIQ$ is a cyclic quadrilateral.

21. Let AB be the larger chord and CD be the shorter one.

Draw $OM \perp AB$



$\Rightarrow \quad AM = \frac{1}{2} AB$

Draw $ON \perp CD$

$\Rightarrow \quad CN = \frac{1}{2} CD$

In ΔOAM and ΔOCN

$OA^2 = OM^2 + AM^2$ and $OC^2 = ON^2 + CN^2$

But $OA = OC \Rightarrow OA^2 = OC^2$ (Radii of same circle)

$\Rightarrow \quad OM^2 + AM^2 = ON^2 + CN^2$

$\Rightarrow \quad AM^2 - CN^2 = ON^2 - OM^2$ …(i)

Since $AB > CD \Rightarrow \frac{1}{2}AB > \frac{1}{2}CD \Rightarrow AM > CN$

$\Rightarrow \quad AM^2 > CN^2 \Rightarrow AM^2 - CN^2 > 0$

$\Rightarrow \quad ON^2 - OM^2 > 0$ [Using (i)]

$\Rightarrow \quad ON^2 > OM^2 \Rightarrow ON > OM$ or $OM < ON$

Thus, AB is nearer to the centre.

22. Consider the arc CDE. We find that $\angle CBE$ and $\angle CAE$ are the angles in the same segment of arc CDE.

$\therefore \quad \angle CAE = \angle CBE$

$\Rightarrow \quad \angle CAE = 25°$ [$\because \angle CBE = 25°$]

Since AC is the diameter of the circle and the angle in a semi-circle is a right angle. Therefore, $\angle AEC = 90°$.

Now, in ΔACE, we have

$\angle ACE + \angle AEC + \angle CAE = 180°$

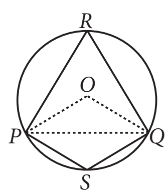
$\Rightarrow \quad \angle ACE + 90° + 25° = 180° \Rightarrow \angle ACE = 65°$

But $\angle DEC$ and $\angle ACE$ are alternate angles, because $AC \parallel DE$.

$\therefore \quad \angle DEC = \angle ACE = 65°$

$\Rightarrow \quad (1/k) \times 455° = 65° \Rightarrow k = 7$

23. We known that, the angle subtended by an arc of a circle at its centre is twice the angle subtended by it at any point on the remaining part of the circle.



Since, $\widehat{PSQ}$ is a minor arc and $\angle PRQ$ is the angle formed by it in alternate segment.

$\therefore \quad 2\angle PRQ = \angle POQ$

$\Rightarrow \quad 2\angle PRQ < 180°$

[Since, $\angle POQ$ is an angle of ΔPOQ]

$\Rightarrow \quad \angle PRQ < 90°$

So, $\angle PRQ$ is an acute angle.

Now, $\widehat{QRP}$ is a major arc and $\angle PSQ$ is the angle formed by it in the alternate segment.

$\therefore \quad 2\angle PSQ =$ Reflex $\angle POQ$

$\Rightarrow \quad 2\angle PSQ = 360° - \angle POQ$

$\Rightarrow \quad 2\angle PSQ > 360° - 180°$ [$\because \angle POQ < 180°$]

$\Rightarrow \quad \angle PSQ > 90°$

Hence, $\angle PSQ$ is an obtuse angle.

24. Let ΔABC be a right triangle such that $\angle BAC = 90°$. Let O be the mid-point of the hypotenuse BC.

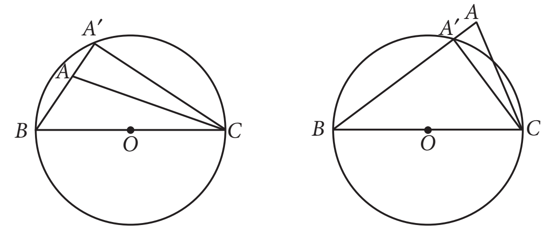
Then, $OB = OC$.

With O as centre and OB as radius, draw a circle. Clearly this circle passes through the points B and C. If possible, suppose this circle does not pass through A. Let it meet BA produced at A'. Then,

$\angle BA'C = 90°$ ($\because$ Angle in a semicircle is 90°)

But, $\angle BAC = 90°$

$\therefore \quad \angle BA'C = \angle BAC$



This is not possible, because an exterior angle of a triangle can never be equal to its interior opposite angle.

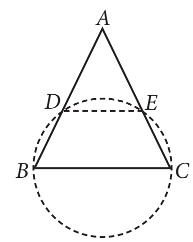
Thus, $\angle BA'C = \angle BAC \Rightarrow A'$ coincides with A

So, the circle which passes through B and C also passes through A.

Consequently, $OA = OB = OC =$ radius of the circle.

Hence, the mid-point O of the hypotenuse BC of right triangle ABC is equidistant from its vertices.

25. To prove $BCED$ a cyclic quadrilateral, it is sufficient to prove $\angle CED + \angle DBC = 180°$



We have, $AB = AC \Rightarrow \angle ABC = \angle ACB$

Now, $AD = AE \Rightarrow \angle ADE = \angle AED$

In ΔABC,

$\angle A + \angle ABC + \angle ACB = 180°$ …(i)

In ΔADE,

$\angle A + \angle ADE + \angle AED = 180°$ …(ii)

From (i) and (ii), we have

$\angle A + \angle ABC + \angle ACB = \angle A + \angle ADE + \angle AED$

$\Rightarrow \quad \angle ABC + \angle ABC = \angle ADE + \angle ADE$

$\Rightarrow \quad 2\angle ABC = 2\angle ADE$

$\angle ABC = \angle ADE$

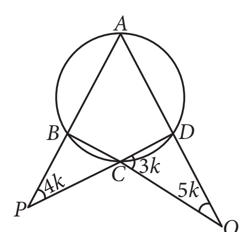
$\therefore \quad DE \parallel BC$

$\therefore \quad \angle ABC + \angle BDE = 180°$ (Co-interior angles)

Now, as $\angle ADE = \angle AED$, $\therefore \angle BDE = \angle CED$

$\Rightarrow \quad \angle DBC + \angle CED = 180°$

26. Let $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$



Then, $x = 3k$, $y = 4k$ and $z = 5k$

$\angle BCP = \angle DCQ = 3k$

[Vertically opposite angles]

In ΔDCQ,

$\angle CDQ = 180° - (3k + 5k) = 180° - 8k$

$\angle ADC + \angle CBA = 180°$

(Opposite angles of cyclic quadrilateral)

$\Rightarrow \quad 180° - (180° - 8k) + \angle CBA = 180°$

$\Rightarrow \quad \angle CBA = 180° - 8k$

$\therefore \quad \angle PBC = 180° - \angle CBA = 180° - (180° - 8k) = 8k$

In ΔPBC, $\angle P + \angle B + \angle C = 180°$

$\Rightarrow \quad 4k + 8k + 3k = 180° \Rightarrow 15k = 180°$

$\Rightarrow \quad k = 12 \therefore x = 36°, y = 48°$ and $z = 60°$

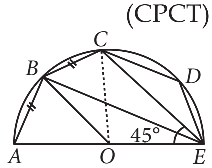
27. Join OC.

$\angle AOC = 2\angle AEC \Rightarrow \angle AOC = 2 \times 45° = 90°$

In ΔAOB and ΔBOC, $AB = BC$, $AO = OC$
and $OB = OB$

$\therefore \quad \Delta BOA \cong \Delta BOC$ (By SSS congruency)

$\therefore \quad \angle BOA = \angle BOC$ (CPCT)



$\angle BOA + \angle BOC = \angle AOC = 90°$

$\Rightarrow \quad 2\angle BOA = 90°$

$\Rightarrow \quad \angle BOA = 45°$ and $\angle BOC = 45°$

$\Rightarrow \quad \angle BOA = \angle CEO$

(Each equal 45°)

But they are corresponding angles.

$\therefore \quad BO \parallel CE$

Now, $\angle AOC + \angle COE = 180°$

$\therefore \quad \angle COE = 90°$ $(\because \angle AOC = 90°)$

Now, $2\angle CBE = \angle COE = 90°$

$\therefore \quad \angle CBE = 45°$

Since $BCDE$ is a cyclic quadrilateral.

$\therefore \quad \angle CBE + \angle CDE = 180° \Rightarrow 45° + \angle CDE = 180°$

$\Rightarrow \quad \angle CDE = 180° - 45° = 135°$

28. We know that the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow \quad 2x + 4 + 4y - 4 = 180$

$\Rightarrow \quad 2x + 4y = 180 \Rightarrow x + 2y = 90$ ...(i)

And $x + 10 + 5y + 5 = 180$

$\Rightarrow x + 5y = 165$ ...(ii)

Subtracting (i) from (ii), we get

$3y = 75 \Rightarrow y = 25$

Putting the value of y in (i), we get

$x = 40$

$\therefore \quad x - y = 15$

29. Since arc BC makes $\angle$BOC at the centre and $\angle$BAC at a point on the remaining part of the circle.

$$\therefore \quad \angle BAC = \frac{1}{2}\angle BOC$$

Now, $\angle BOC = 360° - (120° + 80°) = 160°$

$$\Rightarrow \quad \angle BAC = \frac{1}{2} \times 160° = 80° \qquad \therefore k = 80$$

30. $\because ABCD$ is a cyclic quadrilateral

$\therefore \quad \angle ABC + \angle ADC = 180°$

$\Rightarrow \quad 130° + \angle ADC = 180° \Rightarrow \angle ADC = 50°$

Since, angle subtended by arc at centre is double the angle subtended by it at any point on the remaining part of circle.

$\therefore \quad \angle AOC = 2\angle ADC = 100°$

$$\text{Now, } \frac{\angle AOC - \angle ADC}{25°} = \frac{(100° - 50°)}{25°} = \frac{50°}{25°} = 2$$

31. We have, $\angle CDA = 180° - 80° = 100°$

and $\angle ABC + x = 180°$

Now, $\angle CDA + \angle ABC = 180°$

$\Rightarrow \quad 100° + 180° - x = 180°$

$\Rightarrow \quad x = 100°$

$$\therefore \quad \frac{x}{50°} = \frac{100°}{50°} = 2.$$

32. $OD \perp AB \Rightarrow D$ is the midpoint of AB.

Also, O is the midpoint of BC.

Now, in ΔBAC, D is the midpoint of AB and O is the midpoint of BC.

$\therefore \quad$ By midpoint theorem,

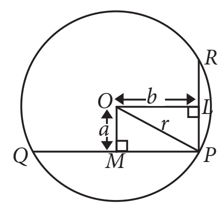
$$OD = \frac{1}{2}AC \Rightarrow AC = 2 \times OD = (2 \times 6) \text{ cm} = 12 \text{ cm}.$$

Now, $AC = k \times 3$

$\Rightarrow 12 = k \times 3 \Rightarrow k = 4$ cm

33. Join OP.

Since the perpendicular from the centre of the circle to the chord bisects the chord.



$\therefore \quad PM = MQ = \frac{1}{2}PQ$

and $PL = LR = \frac{1}{2}PR$

In right angled ΔOMP, we have

$PM^2 = OP^2 - OM^2$

$$\left(\frac{1}{2}PQ\right)^2 = r^2 - a^2 \Rightarrow \frac{PQ^2}{4} = r^2 - a^2$$

$\Rightarrow \quad PQ^2 = 4r^2 - 4a^2$ ...(i)

Again, in right angled ΔOLP, we have

$PL^2 = OP^2 - OL^2$

$$\Rightarrow \quad \left(\frac{1}{2}PR\right)^2 = r^2 - b^2 \Rightarrow PR^2 = 4r^2 - 4b^2 \qquad \text{...(ii)}$$

Also, $PQ = 2PR$ [Given]

$\Rightarrow \quad PQ^2 = 4PR^2$ ...(iii)

From (i), (ii) and (iii), we have

$4r^2 - 4a^2 = 4(4r^2 - 4b^2) \Rightarrow r^2 - a^2 = 4r^2 - 4b^2$

$\Rightarrow \quad 4b^2 = 4r^2 - r^2 + a^2 \Rightarrow 4b^2 = 3r^2 + a^2$

$$\Rightarrow \quad b^2 = \frac{3}{4}r^2 + \frac{1}{4}a^2$$

$$\text{or} \quad b^2 = \frac{a^2}{4} + \frac{3}{4}r^2$$

34. We have, $OP = OR = 10$ cm, $PQ = 16$ cm and $RS = 12$ cm

Now, draw a perpendicular from centre O to chords PQ and RS, meets at L and M respectively *i.e.*, $OL \perp PQ$ and $OM \perp RS$.

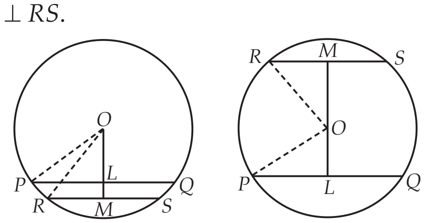


Fig. I

Fig. II

$\therefore \quad PL = 8$ cm and $RM = 6$ cm

[$\because$ Perpendicular drawn from the centre of a circle to a chord bisects the chord]

In right triangles OLP and OMR, we have

$OP^2 = OL^2 + PL^2$ and $OR^2 = OM^2 + RM^2$

[By Pythagoras theorem]

$\Rightarrow \quad 100 = OL^2 + 64$ and $100 = OM^2 + 36$

$\Rightarrow \quad OL^2 = 36$ and $OM^2 = 64$

$\Rightarrow \quad OL = 6$ cm and $OM = 8$ cm

(i) In this case, from fig. I, we have

Distance between PQ and RS

$= LM = OM - OL = (8 - 6)$ cm $= 2$ cm

(ii) In this case, from fig. II, we have

Distance between PQ and $RS = LM$

$= OL + OM = (6 + 8)$ cm $= 14$ cm

35. Given : AB and AC be two equal chords of circle with centre O. Also, $OP \perp AB$ at M and $OQ \perp AC$ at N.

To prove : $PB = QC$.

Proof : We know that, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore \quad AM = MB = \frac{1}{2}AB$ [$\because$ $OP \perp AB$]

and $AN = NC = \frac{1}{2}AC$ [$\because$ $OQ \perp AC$]

Since, $AB = AC$ [Given]

$\therefore \quad \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow \quad AM = MB = AN = NC$ ...(i)

$\Rightarrow \quad OM = ON$ ...(ii)

[$\because$ Equal chords of a circle are equidistant from the centre]

$\Rightarrow \quad OP = OQ$ [Radii of same circle] ...(iii)

$\Rightarrow \quad OP - OM = OQ - ON$ [Subtracting (ii) from (iii)]

$\Rightarrow \quad PM = QN$

Now, in ΔPMB and ΔQNC, we have

$MB = NC$ [From (i)]

$\angle PMB = \angle QNC$ [Each equal to 90°]

$PM = QN$ [Proved above]

$\therefore \quad \Delta PMB \cong \Delta QNC$ [By SAS congruency criteria]

$\Rightarrow \quad PB = QC$ [By C.P.C.T]

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