

Binomial Theorem

Question1

If $\sum_{r=0}^5 \frac{{}^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$, $\gcd(m, n) = 1$, then $m - n$ is equal to _____.

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Answer: 2035

Solution:

$$(1+x)^{11} = {}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + \dots + {}^{11}C_{11}x^{11}$$
$$\int_0^1 (1+x)^{11} dx = \int_0^1 ({}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + \dots + {}^{11}C_{11}x^{11}) dx$$
$$\left[\frac{(1+x)^{12}}{12} \right]_0^1 = {}^{11}C_0x + \frac{{}^{11}C_1x^2}{2} + \frac{{}^{11}C_2x^3}{3} + \dots + \frac{{}^{11}C_{11}x^{12}}{12} \Bigg|_0^1$$
$$\frac{2^{12}-1}{12} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_{11}}{12} \dots \dots \dots (1)$$

Now,

$$\int_{-1}^0 (1+x)^{11} dx = \int_{-1}^0 ({}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + \dots + {}^{11}C_{11}x^{11}) dx$$
$$\left[\frac{(1+x)^{12}}{12} \right]_{-1}^0 = {}^{11}C_0x + \frac{{}^{11}C_1x^2}{2} + \frac{{}^{11}C_2x^3}{3} + \dots + \frac{{}^{11}C_{11}x^{12}}{12} \Bigg|_{-1}^0$$
$$\frac{1}{12} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} \dots \dots \dots (2)$$

(1) - (2)

$$= \frac{2^{12}-2}{12} = 2 \left[\frac{C_1}{2} + \frac{C_3}{4} + \dots \right]$$
$$\Rightarrow \sum_{r=0}^5 \frac{C_{2r+1}}{2r+2} = \frac{2^{11}-1}{12} = \frac{2047}{12} = \frac{m}{n}$$
$$= 2047 - 12 = 2035$$

Question2

If $\sum_{r=1}^{30} \frac{r^2 ({}^{30}C_r)^2}{{}^{30}C_{r-1}} = \alpha \times 2^{29}$, then α is equal to _____.

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Answer: 465

Solution:

$$\begin{aligned} & \sum_{r=1}^{30} \frac{r^2 ({}^{30}C_r)^2}{{}^{30}C_{r-1}} \\ &= \sum_{r=1}^{30} r^2 \left(\frac{31-r}{r} \right) \cdot \frac{30!}{r!(30-r)!} \\ & \left(\because \frac{{}^{30}C_r}{{}^{30}C_{r-1}} = \frac{30-r+1}{r} = \frac{31-r}{r} \right) \\ &= \sum_{r=1}^{30} \frac{(31-r)30!}{(r-1)!(30-r)!} \\ &= 30 \sum_{r=1}^{30} \frac{(31-r)29!}{(r-1)!(30-r)!} \\ &= 30 \sum_{r=1}^{30} (30-r+1)^{29} C_{30-r} \\ &= 30 \left(\sum_{r=1}^{30} (31-r)^{29} C_{30-r} + \sum_{r=1}^{30} {}^{29}C_{30-r} \right) \\ &= 30 (29 \times 2^{28} + 2^{29}) = 30(29+2)2^{28} \\ &= 15 \times 31 \times 2^{29} \\ &= 465 (2^{29}) \\ \alpha &= 465 \end{aligned}$$

Question3

The sum of all rational terms in the expansion of $(1 + 2^{1/3} + 3^{1/2})^6$ is equal to _____.

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Answer: 612

Solution:

$$\begin{aligned} & \left(1 + 2^{1/3} + 3^{1/2} \right)^6 \\ &= \frac{6}{r_1 \ r_2 \ r_3} (1)^{r_1} (2)^{r_2} (3)^{r_3} \end{aligned}$$

r ₁	r ₂	r ₃
6	0	0
4	0	2
2	0	4
0	0	6
3	3	0
1	3	2
0	6	0

$$= \frac{6}{6 \cdot 0 \cdot 0} + \frac{6}{4 \cdot 0 \cdot 2} (3) + \frac{6}{2 \cdot 0 \cdot 4} (3)^2 + \frac{6}{0 \cdot 0 \cdot 6} (3)^3 + \frac{6}{3 \cdot 3 \cdot 0} (2) + \frac{6}{1 \cdot 3 \cdot 2} (2)^1 (3)^1 + \frac{6}{0 \cdot 6 \cdot 0} (2)^2$$

$$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$$

Question4

If $\alpha = 1 + \sum_{r=1}^6 (-3)^{r-1} {}^{12}C_{2r-1}$, then the distance of the point $(12, \sqrt{3})$ from the line $\alpha x - \sqrt{3}y + 1 = 0$ is _____.

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Answer: 5

Solution:

$$\alpha = 1 + \sum_{r=1}^6 (-1)^{r-1} {}^{12}C_{2r-1} 3^{r-1}$$

$$\alpha = 1 + \sum_{r=1}^6 {}^{12}C_{2r-1} \frac{(\sqrt{3}i)^{2r-1}}{\sqrt{3}i} \quad i = \text{iota, let } \sqrt{3}i = x$$

$$\alpha = 1 + \frac{1}{\sqrt{3}i} ({}^{12}C_1 x + {}^{12}C_3 x^3 + \dots + {}^{12}C_{11} x^{11})$$

$$= 1 + \frac{1}{\sqrt{3}i} \left(\frac{(1 + \sqrt{3}i)^{12} - (1 - \sqrt{3}i)^{12}}{2} \right)$$

$$= 1 + \frac{1}{\sqrt{3}i} \left(\frac{(-2\omega^2)^{12} - (2\omega)^{12}}{2} \right) = 1$$

so distance of $(12, \sqrt{3})$ from $x - \sqrt{3}y + 1 = 0$ is

$$\frac{12 - 3 + 1}{2} = 5$$

Question5

Let $(1 + x + x^2)^{10} = a_0 + a_1 x + a_2 x^2 + \dots + a_{20} x^{20}$. If $(a_1 + a_3 + a_5 + \dots + a_{19}) - 11a_2 = 121k$, then k is equal to _____.

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Answer: 239

Solution:

$$\text{Let } f(x) = (1 + x + x^2)^{10} = \sum_{r=0}^{20} a_r x^r$$

$$\text{The sum of odd coefficients: } S_{\text{odd}} = a_1 + a_3 + a_5 + \dots + a_{19}$$

Subtracting $11a_2$ from above will give the answer

$$S_{\text{odd}} = \frac{f(1) - f(-1)}{2}$$

$$f(1) = (1 + 1 + 1)^{10} = 3^{10}$$

$$f(-1) = (1 - 1 + 1)^{10} = (1)^{10} = 1$$

$$S_{\text{odd}} = \sum_{\text{odd } r} a_r = \frac{3^{10} - 1}{2}$$

Now for a_2

$$1 + x + x^2 = \frac{1-x^3}{1-x} \Rightarrow f(x) = \left(\frac{1-x^3}{1-x}\right)^{10} = \frac{(1-x^3)^{10}}{(1-x)^{10}}$$

Now use:

$$(1 - x^3)^{10} = \sum_{k=0}^{10} (-1)^k \binom{10}{k} x^{3k}$$

$$(1 - x)^{-10} = \sum_{r=0}^{\infty} \binom{r+9}{9} x^r$$

So

$$f(x) = \left(\sum_{k=0}^{10} (-1)^k \binom{10}{k} x^{3k}\right) \cdot \left(\sum_{r=0}^{\infty} \binom{r+9}{9} x^r\right)$$

Only the term with x^0 from the first sum (i.e., $k = 0$) can contribute to x^2 , since all other $k \geq 1$ gives $x^{3k} \geq x^3$

From $(1 - x^3)^{10}$: the x^0 term is $\binom{10}{0} = 1$

From $(1 - x)^{-10}$: the coefficient of x^2 is

$$\binom{2+9}{9} = \binom{11}{9} = 55$$

Hence, $a_2 = 1 \cdot 55 = 55$

$$\text{Now, } S_{\text{odd}} - 11a_2 = \frac{3^{10} - 1}{2} - 11 \cdot 55 = 121k$$

$$3^{10} = 59049$$

So:

$$\begin{aligned} S &= \frac{59049 - 1}{2} - 605 = \frac{59048}{2} - 605 \\ &= 29524 - 605 = 28919 \end{aligned}$$

So:

$$121k = 28919 \Rightarrow k = \frac{28919}{121} = 239$$

Question 6

The sum of the series

$2 \times 1 \times {}^{20}C_4 - 3 \times 2 \times {}^{20}C_5 + 4 \times 3 \times {}^{20}C_6 - 5 \times 4 \times {}^{20}C_7 + \dots + 18 \times 17 \times {}^{20}C_{20}$, is equal to _____.

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Answer: 34

Solution:

$$\begin{aligned}(1-x)^{20} &= {}^{20}C_0 - {}^{20}C_1x + {}^{20}C_2x^2 - \dots + {}^{20}C_{20}x^{20} \\ \frac{(1-x)^{20}}{x^2} &= \frac{{}^{20}C_0}{x^2} - \frac{{}^{20}C_1}{x} + {}^{20}C_2 - {}^{20}C_3x + {}^{20}C_4x^2 - \dots \\ \text{Diff twice and put } x &= 1 \\ &= 6 - {}^{20}C_1(2) + A \\ A &= 40 - 6 = 34\end{aligned}$$

Question7

The product of the last two digits of $(1919)^{1919}$ is

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Answer: 63

Solution:

$$\begin{aligned}(1919)^{1919} &= (1920 - 1)^{1919} \\ &= {}^{1919}C_0(1920)^{1919} - {}^{1919}C_1(1920)^{1918} + \dots \\ &\quad + {}^{1919}C_{1918}(1920)^1 - {}^{1919}C_{1919} \\ &= 100\lambda + 1919 \times 1920 - 1 \\ &= 100\lambda + 3684480 - 1 \\ &= 100\lambda + \dots\dots\dots79 \text{ (last two digit)} \\ &\Rightarrow \text{Number having last two digit } 79 \\ \therefore \text{Product of last two digit } &63\end{aligned}$$

Question8

Let α, β, γ and δ be the coefficients of x^7, x^5, x^3 and x respectively in the expansion of

$$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, x > 1. \text{ If } u \text{ and } v \text{ satisfy the equations}$$

$$\alpha u + \beta v = 18,$$

$$\gamma u + \delta v = 20,$$

then $u + v$ equals :

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Options:

A. 4

B. 3

C. 5

D. 8

Answer: C

Solution:

To find the sum of u and v , we first need to expand the expression:

$$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$$

Using the Binomial Theorem, the expansion yields:

$$= 2 \left({}^5C_0 \cdot x^5 + {}^5C_2 \cdot x^3(x^3 - 1) + {}^5C_4 \cdot x(x^3 - 1)^2 \right)$$

Simplifying this, we obtain:

$$= 2 (5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x)$$

From this expansion, we can identify the coefficients:

The coefficient of x^7 is $\alpha = 10$

The coefficient of x^5 is $\beta = 2$

The coefficient of x^3 is $\gamma = -20$

The coefficient of x is $\delta = 10$

Given the equations:

$$\alpha u + \beta v = 18$$

$$\gamma u + \delta v = 20$$

Substituting in the coefficients:

$$10u + 2v = 18$$

$$-20u + 10v = 20$$

By solving these equations, we find:

From $10u + 2v = 18$, simplify to $5u + v = 9$.

From $-20u + 10v = 20$, simplify to $-2u + v = 2$.

Solving these linear equations simultaneously, we find:

Subtracting equation 2 from equation 1:

$$5u + v = 9$$

$$-(-2u + v = 2)$$

This yields:

$$7u = 7 \Rightarrow u = 1$$

Substitute $u = 1$ back into $5u + v = 9$:

$$5(1) + v = 9 \Rightarrow v = 4$$

Thus, the sum $u + v$ is:

$$u + v = 1 + 4 = 5$$

Question9

If in the expansion of $(1 + x)^p(1 - x)^q$, the coefficients of x and x^2 are 1 and -2, respectively, then $p^2 + q^2$ is equal to :

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Options:

- A. 8
- B. 20
- C. 13
- D. 18

Answer: C

Solution:

$$(1 + x)^p(1 - x)^q = ({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + \dots) ({}^qC_0 - {}^qC_1x + {}^qC_2x^2 + \dots)$$

$$\text{coeff of } x \equiv {}^pC_0{}^qC_1 - {}^pC_1{}^qC_0 = 1$$

$$p - q = 1$$

$$\text{coeff of } x^2 \equiv {}^pC_0{}^qC_2 - {}^pC_1{}^qC_1 + {}^pC_2{}^qC_0 = -2$$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$$

$$q^2 - q - 2pq + p^2 - p = -4$$

$$(p-q)^2 - (p+q) = -4$$

$$p+q = 5$$

$$p = 3$$

$$q = 2$$

$$\text{so } p^2 + q^2 = 13$$

Question10

For some $n \neq 10$, let the coefficients of the 5th, 6th and 7th terms in the binomial expansion of $(1 + x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1 + x)^{n+4}$ is:

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Options:

- A. 10
- B. 35
- C. 70
- D. 20

Answer: B

Solution:

$$\begin{aligned} & (1+x)^{n+4} \\ & {}^{n+4}C_4 {}^{n+4}C_5 {}^{n+4}C_6, \rightarrow \text{A.P.} \\ & \Rightarrow 2 \times {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6 \\ & \Rightarrow 4 \times {}^{n+4}C_5 = ({}^{n+4}C_4 + {}^{n+4}C_5) + ({}^{n+4}C_5 + {}^{n+4}C_6) \\ & \Rightarrow 4 \times {}^{n+4}C_5 = {}^{n+5}C_5 + {}^{n+5}C_6 \\ & \Rightarrow 4 \times \frac{(n+4)!}{5!(n-1)!} = \frac{(n+6)!}{6!n!} \\ & \Rightarrow 4 = \frac{(n+6)(n+5)}{6n} \\ & \Rightarrow n^2 + 11n + 30 = 24n \\ & \Rightarrow n^2 - 13n + 30 = 0 \\ & \Rightarrow n = 3, 10 \text{ (rejected)} \\ & \therefore n \neq 10 \end{aligned}$$

\therefore Largest binomial coefficient in expansion of

$$\begin{aligned} & (1+x)^7 \\ & (\because n+4=7) \end{aligned}$$

is coeff. of middle term

$$\Rightarrow {}^7C_4 = {}^7C_3 = 35$$

N.T.A. Ans Option (2)

Question 11

Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2A = 5B$, then n is equal to:

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Options:

- A. 20
- B. 19
- C. 22

D. 21

Answer: D

Solution:

$$\begin{aligned}A &= {}^{2n-1}C_{29} \quad B = {}^{2n-1}C_{11} \\2^{2n-1}C_{29} &= 5^{2n-1}C_{11} \\2 \frac{(2n-1)!}{29!(2n-30)!} &= 5 \frac{(2n-1)!}{(2n-12)!11!} \\ \frac{1}{29 \dots 12 \cdot 5} &= \frac{1}{(2n-12)(2n-13) \dots (2n-29)2} \\ \frac{1}{30 \cdot 29 \dots 12} &= \frac{1}{(2n-12)(2n-13) \dots (2n-29)12} \\2n-12 &= 30 \\n &= 21\end{aligned}$$

Question12

Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r . Let q be the sum of all rational terms in the binomial expansion of $(\sqrt[4]{3} + \sqrt[3]{4})^{12}$. Then $p+q$ is equal to:

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Options:

A.

295

B.

283

C.

299

D.

287

Answer: B

Solution:

$$(a + b)^{\frac{1}{2}}$$

$$T_r, T_{r+1}, T_{r+2} \rightarrow \text{GP}$$

$$\text{So, } \frac{T_{r+1}}{T_r} = \frac{T_{r+2}}{T_{r+1}}$$

$$\frac{{}^{12}C_r}{{}^{12}C_{r-1}} = \frac{{}^{12}C_{r+1}}{{}^{12}C_r}$$

$$\frac{12 - r + 1}{r} = \frac{12 - (r + 1) + 1}{r + 1}$$

$$(13 - r)(r + 1) = (12 - r)(r)$$

$$-r + 12r + 13 = 12r - r^2$$

$$13 = 0$$

No value of r possible

So $P = 0$

$$\left(3^{\frac{1}{4}} + 4^{\frac{1}{3}}\right)^{12} = \sum {}^{12}C_r \left(3^{\frac{1}{4}}\right)^{12-r} \left(4^{\frac{1}{3}}\right)^r$$

Exponent of $\left(3^{\frac{1}{4}}\right)$ exponent of $\left(4^{\frac{1}{3}}\right)$ term

12	0	27
0	12	256

$$q = 27 + 256 = 283$$

$$p + q = 0 + 283 = 283$$

Question 13

The least value of n for which the number of integral terms in the Binomial expansion of $(\sqrt[3]{7} + \sqrt[12]{11})^n$ is 183, is :

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Options:

A.

2184

B.

2172

C.

2196

D.

2148

Answer: A

Solution:

$$\begin{aligned} \text{General term} &= {}^n C_r \left(7^{1/3}\right)^{n-r} \left(11^{1/12}\right)^r \\ &= {}^n C_r (7)^{\frac{n-r}{3}} (11)^{r/12} \end{aligned}$$

For integral terms, r must be multiple of 12

$$\therefore r = 12k, k \in \mathbb{W}$$

Total values of $r = 183$

Hence $\max r = 12(182)$

$$= 2184$$

Min value of $n = 2184$

Question14

The remainder, when 7^{103} is divided by 23, is equal to:

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Options:

A.

9

B.

6

C.

14

D.

17

Answer: C

Solution:

$$7^{103} = 7(7^{102}) = 7(343)^{34} = 7(345 - 2)^{34}$$

$$7^{103} = 23 K_1 + 7 \cdot 2^{34}$$

$$\text{Now } 7 \cdot 2^{34} = 7 \cdot 2^2 \cdot 2^{32}$$

$$= 28 \cdot (256)^4$$

$$= 28(253 + 3)^4$$

$$\therefore 28 \times 81 \Rightarrow (23 + 5)(69 + 12)$$

$$23 K_2 + 60$$

$$\therefore \text{Remainder} = 14$$

Question15

The term independent of x in the expansion of $\left(\frac{(x+1)}{(x^{2/3}+1-x^{1/3})} - \frac{(x-1)}{(x-x^{1/2})}\right)^{10}$, $x > 1$, is :

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Options:

A. 240

B. 120

C. 150

D. 210

Answer: D

Solution:

$$\begin{aligned} & \left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}} \right)^{10} \\ &= \left(\frac{(x^{1/3})^3+(1^{1/3})^3}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x})^2-(1)^2}{x-x^{1/2}} \right)^{10} \\ &= \left(\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)} \right)^{10} \\ &= \left((x^{1/3}+1) - \frac{(\sqrt{x}+1)}{\sqrt{x}} \right)^{10} \\ &= \left((x^{1/3}+1) - \left(1 + \frac{1}{\sqrt{x}} \right) \right)^{10} \\ &= \left(x^{1/3} - \frac{1}{x^{1/2}} \right)^{10} \end{aligned}$$

[Note:

For $(x^\alpha \pm \frac{1}{x^\beta})^n$ the $(r+1)^{\text{th}}$ term with power m of x is

$$r = \left[\frac{n\alpha - m}{\alpha + \beta} \right]$$

Here $\alpha = \frac{1}{3}$, $\beta = \frac{1}{2}$ and $m = 0$

$$\text{then } r = \frac{10 \times \frac{1}{3} - 0}{\frac{1}{3} + \frac{1}{2}} = \frac{10}{3} \times \frac{6}{5} = 4$$

$\therefore T_5$ is the term independent of x .

$$\therefore T_5 = {}^{10}C_4 = 210$$

Question16

The largest $n \in \mathbf{N}$ such that 3^n divides $50!$ is :

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Options:

- A. 22
- B. 20
- C. 21
- D. 23

Answer: A

Solution:

$$\begin{aligned} V_3(50!) &= \left[\frac{50}{3} \right] + \left[\frac{50}{9} \right] + \left[\frac{50}{27} \right] + \left[\frac{50}{81} \right] + \dots + \left[\frac{50}{3^n} \right] \\ &= 16 + 5 + 1 + 0 + 0 + 0 \dots = 22 \end{aligned}$$

Question17

If $\sum_{r=0}^{10} \left(\frac{10^{r+1}-1}{10^r} \right) \cdot {}^{11}C_{r+1} = \frac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :

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Options:

- A. 11

B. 20

C. 24

D. 15

Answer: B

Solution:

$$\begin{aligned} & \sum_{r=0}^{10} \left(\frac{10^{r-1} - 1}{10^r} \right)^{11} C_{r+1} \\ &= \sum_{r=0}^{10} \left(10 - \frac{1}{10^r} \right)^{11} C_{r+1} \\ &= 10 \sum_{r=0}^{10} {}^{11}C_{r+1} - 10 \sum_{r=0}^{10} \left({}^{11}C_{r+1} \left(\frac{1}{10} \right)^{r+1} \right) \\ &= 10 [{}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11}] \\ &\quad - 10 \left[{}^{11}C_1 \left(\frac{1}{10} \right)^1 + {}^{11}C_2 \left(\frac{1}{10} \right)^2 + \dots + {}^{11}C_{11} \left(\frac{1}{10} \right)^{11} \right] \\ &= 10 [2^{11} - 1] - 10 \left[\left(1 + \frac{1}{10} \right)^{11} - 1 \right] \\ &= 10(2)^{11} - 10 - \frac{11^{11}}{10^{10}} + 10 \\ &= \frac{(20)^{11} - 11^{11}}{10^{10}} \\ \therefore \alpha &= 20 \end{aligned}$$

Question 18

If $\sum_{r=1}^9 \left(\frac{r+3}{2^r} \right) \cdot {}^9C_r = \alpha \left(\frac{3}{2} \right)^9 - \beta$, $\alpha, \beta \in \mathbb{N}$, then $(\alpha + \beta)^2$ is equal to

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Options:

A. 27

B. 81

C. 18

D. 9

Answer: B

Solution:

$$\begin{aligned} \sum_{r=1}^9 \binom{r+3}{2^r} \cdot {}^9C_r &= \sum_{r=1}^9 \frac{r}{2^r} \cdot \frac{9}{r} \cdot {}^8C_{r-1} + \sum_{r=1}^9 3 \cdot {}^9C_r \left(\frac{1}{2}\right)^r \\ &= \frac{9}{2} \sum_{r=1}^9 {}^8C_{r-1} \left(\frac{1}{2}\right)^{r-1} + 3 \sum_{r=1}^9 {}^9C_r \left(\frac{1}{2}\right)^r \\ &= \frac{9}{2} \sum_{r=0}^8 {}^8C_{r-1} \left(\frac{1}{2}\right)^r + 3 \sum_{r=1}^9 {}^9C_r \left(\frac{1}{2}\right)^r \\ &= \frac{9}{2} \left(1 + \frac{1}{2}\right)^8 + 3 \left[\left(1 + \frac{1}{2}\right)^9 - {}^9C_0 \left(\frac{1}{2}\right)^0 \right] \\ &= \frac{9}{2} \cdot \frac{3^8}{2^8} + 3 \left[\frac{3^9}{2^9} - 1 \right] \\ &= \frac{3^{10}}{2^9} + \frac{3^{10}}{2^9} - 3 = 4 \cdot \frac{3^{10}}{2^{10}} - 3 \\ &= 4 \left(\frac{3}{2}\right)^{10} - 3 \\ &= 6 \left(\frac{3}{2}\right)^9 - 3 \\ \alpha = 6, \beta = 3 &\Rightarrow (\alpha + \beta)^2 = 81 \end{aligned}$$

Question 19

The sum of all rational terms in the expansion of $(2 + \sqrt{3})^8$ is :

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Options:

A. 16923

B. 18817

C. 3763

D. 33845

Answer: B

Solution:

To find the sum of all rational terms in the expansion of $(2 + \sqrt{3})^8$, we consider the binomial expansion:

$$S = {}^8C_0(2)^8 + {}^8C_1(2)^7(\sqrt{3}) + \dots + {}^8C_8(\sqrt{3})^8$$

We need to identify and sum only the rational terms. In the binomial expansion, a term is rational if the exponent of $\sqrt{3}$ is even.

Thus, the rational terms are:

$${}^8C_0(2)^8 + {}^8C_2(2)^6(\sqrt{3})^2 + {}^8C_4(2)^4(\sqrt{3})^4 + {}^8C_6(2)^2(\sqrt{3})^6 + {}^8C_8(\sqrt{3})^8$$

Evaluating these terms:

$${}^8C_0(2)^8 = 256$$

$${}^8C_2(2)^6(3) = {}^8C_2 \times 64 \times 3 = 1344$$

$${}^8C_4(2)^4(3)^2 = {}^8C_4 \times 16 \times 9 = 3024$$

$${}^8C_6(2)^2(3)^3 = {}^8C_6 \times 4 \times 27 = 4032$$

$${}^8C_8(3)^4 = 1 \times 81 = 81$$

Summing these rational terms gives:

$$256 + 1344 + 3024 + 4032 + 81 = 18817$$

Therefore, the sum of all rational terms in the expansion is 18,817.

Question20

In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, $n \in \mathbb{N}$, if the ratio of 15th term from the beginning to the 15th term from the end is $\frac{1}{6}$, then the value of nC_3 is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. 4960

B. 2300

C. 1040

D. 4060

Answer: B

Solution:

In the expansion of $(a + b)^n$

$$15^{\text{th}} \text{ term from beginning: } T_{15} = {}^nC_{14}a^{n-14}b^{14}$$

$$15^{\text{th}} \text{ term from end: } T'_{15} = {}^nC_{14}b^{n-14}a^{14}$$

$$\therefore \frac{T_{15}}{T'_{15}} = \frac{1}{6}$$

$$\left(\frac{a}{b}\right)^{n-28} = \frac{1}{6}$$

$$\left(6^{\frac{1}{3}}\right)^{n-28} = 6^{-1}$$

$$\Rightarrow \frac{n-28}{3} = -1$$

$$n = 25$$

$$\therefore {}^{25}C_3 = 2300$$

Question21

For an integer $n \geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x + y)^{2n-3}$ is 16, then the distance of the point P $(2n - 1, n^2 - 4n)$ from the line $x + y = 8$ is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

- A. $\sqrt{2}$
- B. $2\sqrt{2}$
- C. $5\sqrt{2}$
- D. $3\sqrt{2}$

Answer: D

Solution:

$$\begin{aligned}\text{Mean} &= \frac{{}^{2n-3}C_0 + {}^{2n-3}C_1 + {}^{2n-3}C_2 + \dots + {}^{2n-3}C_{2n-3}}{2n-2} = 16 \\ &= 2^{2n-3} = 16(2n-2) \\ &= 2^{2n-3} = 2^5(n-1) \\ &\Rightarrow n = 5 \\ \therefore P(9, 5) \\ d &= \left| \frac{9+5-8}{\sqrt{2}} \right| = \frac{6}{\sqrt{2}} = 3\sqrt{2}\end{aligned}$$

Question22

If $1^2 \cdot ({}^{15}C_1) + 2^2 \cdot ({}^{15}C_2) + 3^2 \cdot ({}^{15}C_3) + \dots + 15^2 \cdot ({}^{15}C_{15}) = 2^m \cdot 3^n \cdot 5^k$, where $m, n, k \in \mathbf{N}$, then $m + n + k$ is equal to :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

- A. 20
- B. 19
- C. 18
- D. 21

Answer: B

Solution:

$$\begin{aligned} & \sum_{r=1}^{15} r^2 \cdot {}^{15}C_r \quad (r^n C_r = n^{n-1} C_{r-1}) \\ &= 15 \sum_{r=1}^{15} r \cdot {}^{14}C_{r-1} \\ &= 15 \sum_{r=1}^{15} (r-1+1) {}^{14}C_{r-1} \\ &= 15 \cdot \sum_{r=1}^{15} (r-1) {}^{14}C_{r-1} + 15 \cdot \sum_{r=1}^{15} {}^{14}C_{r-1} \\ &= 15 \cdot 14 \cdot 2^{13} + 15 \cdot 2^{14} \\ &= 15 \cdot 2^{14} (7+1) \\ &= 5 \cdot 3 \cdot 2^{17} \\ n+m+k &= 17+1+1=19 \end{aligned}$$

Question23

The remainder when $((64)^{(64)})^{(64)}$ is divided by 7 is equal to

JEE Main 2025 (Online) 7th April Morning Shift

Options:

- A. 4
- B. 6
- C. 3
- D. 1

Answer: D

Solution:

$$64^{64} \Rightarrow (63+1)^{64} = 63\lambda + 1$$

$$64^{64^{64}} \Rightarrow (63+1)^{64^{64}}$$

$$63\lambda_1 + 1$$

Required remainder when divided 7 is 1.

Question24

The number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1016}$ is:

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

127

B.

128

C.

130

D.

129

Answer: B

Solution:

$$\begin{aligned}T_r &= {}^{1016}C_r (5)^{\frac{1016-r}{2}} 7^{\frac{r}{8}} \\ \Rightarrow r &= 0, 8, 16, 24, \dots, 1016 \\ 1016 &= 0 + (n-1)8 \\ \Rightarrow n-1 &= \frac{1016}{8} = 127 \\ \text{So, } n &= 128.\end{aligned}$$

Question25

$${}^{n-1}C_r = (k^2 - 8)^n {}^nC_{r+1} \text{ if and only if :}$$

[27-Jan-2024 Shift 1]

Options:

A.

$$2\sqrt{2} < k \leq 3$$

B.

$$2\sqrt{3} < k \leq 3\sqrt{2}$$

C.

$$2\sqrt{3} < k < 3\sqrt{3}$$

D.

$$2\sqrt{2} < k < 2\sqrt{3}$$

Answer: A

Solution:

$${}^{n-1}C_r = (k^2 - 8) {}^n C_{r+1}$$

$$r+1 \geq 0, r \geq 0$$

$\underbrace{\hspace{10em}}_{r \geq 0}$

$$\frac{{}^{n-1}C_r}{{}^n C_{r+1}} = k^2 - 8$$

$$\frac{r+1}{n} = k^2 - 8$$

$$\Rightarrow k^2 - 8 > 0$$

$$(k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$$

$$k \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty) \dots\dots\dots (i)$$

$$\therefore n \geq r+1, \frac{r+1}{n} \leq 1$$

$$\Rightarrow k^2 - 8 \leq 1$$

$$k^2 - 9 \leq 0$$

$$-3 \leq k \leq 3 \dots\dots\dots (ii)$$

From equation (I) and (II) we get

$$k \in [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$$

Question26

If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

[27-Jan-2024 Shift 1]

Options:

A.

$$A = B^3$$

B.

$$3A = B$$

C.

$$B = A^3$$

D.

$$A = 3B$$

Answer: A

Solution:

Sum of coefficients in the expansion of

$$(1 - 3x + 10x^2)^n = A$$

$$\text{then } A = (1 - 3 + 10)^n = 8^n \text{ (put } x = 1)$$

and sum of coefficients in the expansion of

$$(1 + x^2)^n = B$$

$$\text{then } B = (1 + 1)^n = 2^n$$

Question27

The coefficient of x^{2012} in the expansion of $(1 - x)^{2008} (1 + x + x^2)^{2007}$ is equal to

[27-Jan-2024 Shift 2]

Answer: 0

Solution:

$$(1 - x)(1 - x)^{2007} (1 + x + x^2)^{2007}$$

$$(1 - x)(1 - x^3)^{2007}$$

$$(1 - x)({}^{2007}C_0 - {}^{2007}C_1(x^3) + \dots)$$

General term

$$(1 - x)((-1)^r {}^{2007}C_r x^{3r})$$

$$(-1)^r {}^{2007}C_r x^{3r} - (-1)^r {}^{2007}C_r x^{3r+1}$$

$$3r = 2012$$

$$r \neq \frac{2012}{3}$$

$$3r + 1 = 2012$$

$$3r = 2011$$

$$r \neq \frac{2011}{3}$$

Hence there is no term containing x^{2012} .

So coefficient of $x^{2012} = 0$

Question28

If $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_9}{10} = \frac{n}{m}$ with $\gcd(n, m) = 1$, then $n + m$ is equal to

[29-Jan-2024 Shift 1]

Answer: 2041

Solution:

$$\begin{aligned} & \sum_{r=1}^9 \frac{{}^{11}C_r}{r+1} \\ &= \frac{1}{12} \sum_{r=1}^9 {}^{12}C_{r+1} \\ &= \frac{1}{12} [2^{12} - 26] = \frac{2035}{6} \end{aligned}$$

$$\therefore m + n = 2041$$

Question29

Remainder when $64^{32^{32}}$ is divided by 9 is equal to__

[29-Jan-2024 Shift 2]

Answer: 1

Solution:

$$\text{Let } 32^{32} = t$$

$$64^{32^{32}} = 64^t = 8^{2t} = (9-1)^{2t}$$

$$= 9k + 1$$

Hence remainder = 1

Question30

Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to

[30-Jan-2024 Shift 1]

Answer: 138

Solution:

General term in expansion of $\left((7)^{1/2} + (11)^{1/6}\right)^{824}$ is $t_{r+1} = {}^{824}C_r (7)^{\frac{824-r}{2}} (11)^{r/6}$

For integral term, r must be multiple of 6 .

Hence $r = 0, 6, 12, \dots, 822$

Question31

Suppose $2 - p, p, 2 - \alpha, \alpha$ are the coefficient of four consecutive terms in the expansion of $(1 + x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals

[30-Jan-2024 Shift 2]

Options:

A.

4

B.

10

C.

8

D.

2

Answer: D

Solution:

$$2 - p, p, 2 - \alpha, \alpha$$

Binomial coefficients are

$$\Rightarrow {}^n C_r, {}^n C_{r+1}, {}^n C_{r+2}, {}^n C_{r+3} \text{ respectively}$$

$$\Rightarrow {}^n C_r + {}^n C_{r+1} = 2$$

$${}^{n+1} C_{r+1} = 2 \dots\dots (1)$$

$$\text{Also } {}^n C_{r+2} + {}^n C_{r+3} = 2$$

$$\Rightarrow {}^{n+1} C_{r+3} = 2 \dots\dots (2)$$

From (1) and (2)

$${}^{n+1} C_{r+1} = {}^{n+1} C_{r+3}$$

$$2r + 4 = n + 1$$

$$n = 2r + 3$$

$${}^{2r+4} C_{r+1} = 2$$

Data Inconsistent

Question32

$$\text{Let } \alpha = \sum_{k=0}^n \left(\frac{{}^n C_k}{k+1} \right)^2 \text{ and } \beta = \sum_{k=0}^{n-1} \left(\frac{{}^n C_k \cdot {}^n C_{k+1}}{k+2} \right) \text{ If } 5\alpha = 6\beta, \text{ then } n \text{ equals}$$

[30-Jan-2024 Shift 2]

Answer: 10

Solution:

$$\alpha = \sum_{k=0}^n \frac{{}^n C_k \cdot {}^n C_k}{k+1} \cdot \frac{n+1}{n+1}$$

$$= \frac{1}{n+1} \sum_{k=0}^n {}^n C_{k+1} \cdot {}^n C_{n-k}$$

$$\alpha = \frac{1}{n+1} \cdot {}^{2n+1} C_{n+1}$$

$$\beta = \sum_{k=0}^{n-1} {}^n C_k \cdot \frac{{}^n C_{k+1}}{k+2} \cdot \frac{n+1}{n+1}$$

$$\frac{1}{n+1} \sum_{k=0}^{n-1} {}^n C_{n-k} \cdot {}^{n+1} C_{k+2}$$

$$= \frac{1}{n+1} \cdot {}^{2n+1} C_{n+2}$$

$$\frac{\beta}{\alpha} = \frac{{}^{2n+1} C_{n+2}}{{}^{2n+1} C_{n+1}} = \frac{2n+1-(n+2)+1}{n+2}$$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$$n = 10$$

Question33

In the expansion of $(1+x)(1-x^2) \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)^5$, $x \neq 0$, the sum of the coefficient of x^3 and x^{-13} is equal to

[31-Jan-2024 Shift 1]

Answer: 118

Solution:

$$\begin{aligned}
& (1+x)(1-x^2) \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} \right)^5 \\
&= (1+x)(1-x^2) \left(\left(1 + \frac{1}{x} \right)^3 \right)^5 \\
&= \frac{(1+x)^2(1-x)(1+x)^{15}}{x^{15}} \\
&= \frac{(1+x)^{17} - x(1+x)^{17}}{x^{15}} \\
&= \text{coeff}(x^3) \text{ in the expansion} \approx \text{coeff}(x^{18}) \text{ in } (1+x)^{17} - x(1+x)^{17} \\
&= 0 - 1 \\
&= -1
\end{aligned}$$

$$\text{coeff}(x^{-13}) \text{ in the expansion} \approx \text{coeff}(x^2) \text{ in } (1+x)^{17} - x(1+x)^{17}$$

$$\begin{aligned}
&= \binom{17}{2} - \binom{17}{1} \\
&= 17 \times 8 - 17 \\
&= 17 \times 7 \\
&= 119
\end{aligned}$$

$$\text{Hence Answer} = 119 - 1 = 118$$

Question34

If for some m, n ; ${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

[31-Jan-2024 Shift 2]

Options:

- A.
- 380
- B.
- 376
- C.
- 384
- D.
- 372

Answer: D

Solution:

$${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$$

$${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$$

$${}^8C_{m+2} > {}^8C_3$$

$$\therefore m = 2$$

$$\text{And } {}^{n-1}P_3 : {}^nP_4 = 1 : 8$$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

$$\therefore n = 8$$

$$\therefore {}^nP_{m+1} + {}^{n+1}C_m = {}^8P_3 + {}^9C_2$$

$$= 8 \times 7 \times 6 + \frac{9 \times 8}{2}$$

$$= 372$$

Question 35

Let the coefficient of x^r in the expansion of

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) +$$

$$(x+3)^{n-3}(x+2)^2 + \dots + (x+2)^{n-1}$$

be α_r . If $\sum_{r=0}^n \alpha_r = \beta^n - \gamma^n$, $\beta, \gamma \in N$, then the value of $\beta^2 + \gamma^2$ equals

[31-Jan-2024 Shift 2]

Answer: 25

Solution:

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \dots + (x+2)^{n-1}$$

$$\sum \alpha_r = 4^{n-1} + 4^{n-2} \times 3 + 4^{n-3} \times 3^2 + \dots + 3^{n-1}$$

$$= 4^{n-1} \left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \dots + \left(\frac{3}{4}\right)^{n-1} \right]$$

$$= 4^{n-1} \times \frac{1 - \left(\frac{3}{4}\right)^n}{1 - \frac{3}{4}}$$

$$= 4^n - 3^n = \beta^n - \gamma^n$$

$$\beta = 4, \gamma = 3$$

$$\beta^2 + \gamma^2 = 16 + 9 = 25$$

Question36

If the Coefficient of x^{30} in the expansion of $(1 + 1/x)^6(1 + x^2)^7(1 - x^3)^8$; $x \neq 0$ is α , then $|\alpha|$ equals_____

[1-Feb-2024 Shift 1]

Answer: 678

Solution:

coeff of x^{30} in $\frac{(x+1)^6(1+x^2)^7(1-x^3)^8}{x^6}$

coeff. of x^{36} in $(1+x)^6(1+x^2)^7(1-x^3)^8$

General term

$${}^6C_{r_1} {}^7C_{r_2} {}^8C_{r_3} (-1)^{r_3} x^{r_1+2r_2+3r_3}$$

$$r_1 + 2r_2 + 3r_3 = 36$$

Case - I :

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
6	3	8

$$r_1 + 2r_2 = 12 \quad (\text{Taking } r_3 = 8)$$

Case - II :

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

$$r_1 + 2r_2 = 15 \text{ (Taking } r_3 = 7 \text{)}$$

Case - III :

r_1	r_2	r_3
4	7	6
6	6	6

$$r_1 + 2r_2 = 18 \text{ (Taking } r_3 = 6 \text{)}$$

$$\text{Coeff.} = 7 + (15 \times 21) + (15 \times 35) + (35) - (6 \times 8) - (20 \times 7 \times 8) - (6 \times 21 \times 8) + (15 \times 28) + (7 \times 28) = -678 = \alpha$$

$$|\alpha| = 678$$

Question37

Let m and n be the coefficients of seventh and thirteenth terms respectively in the

expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}} \right)^{18}$. Then $(n/m)^{1/3}$ is:

[1-Feb-2024 Shift 2]

Options:

A.

4/9

B.

1/9

C.

1/4

D.

9/4

Answer: D

Solution:

$$\left(\frac{x^{\frac{1}{3}}}{3} + \frac{x^{\frac{-2}{3}}}{2} \right)^{18}$$

$$t_7 = {}^{18}C_6 \left(\frac{x^{\frac{1}{3}}}{3} \right)^{12} \left(\frac{x^{\frac{-2}{3}}}{2} \right)^6 = {}^{18}C_6 \frac{1}{(3)^{12}} \cdot \frac{1}{2^6}$$

$$t_{13} = {}^{18}C_{12} \left(\frac{x^{\frac{1}{3}}}{3} \right)^6 \left(\frac{x^{\frac{-2}{3}}}{2} \right)^{12} = {}^{18}C_{12} \frac{1}{(3)^6} \cdot \frac{1}{2^{12}} \cdot x^{-6}$$

$$m = {}^{18}C_6 \cdot 3^{-12} \cdot 2^{-6} ; n = {}^{18}C_{12} \cdot 2^{-12} \cdot 3^{-6}$$

$$\left(\frac{n}{m} \right)^{\frac{1}{3}} = \left(\frac{2^{-12} \cdot 3^{-6}}{3^{-12} \cdot 2^{-6}} \right)^{\frac{1}{3}} = \left(\frac{3}{2} \right)^2 = \frac{9}{4}$$

Question38

The value $\sum_{r=0}^{22} {}^{22}C_r {}^{23}C_r$ is
[24-Jan-2023 Shift 1]

Options:

A. ${}^{45}C_{23}$

B. ${}^{44}C_{23}$

C. ${}^{45}C_{24}$

D. ${}^{44}C_{22}$

Answer: A

Solution:

Solution:

$$\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_{23-r} \\ = {}^{45}C_{23}$$

Question39

Suppose $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is
[24-Jan-2023 Shift 1]

Answer: 1012

Solution:

Solution:
using result

$$\sum_{r=0}^n r^{2n} C_r = n(n+1) \cdot 2^{n-2}$$

Then $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$

$= 2023 \times \alpha \times 2^{2022}$ So,
 $\alpha = 1012$

Question40

If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \dots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to
[24-Jan-2023 Shift 2]

Options:

A. 30

B. 60

C. 15

D. 10

Answer: C

Solution:

Solution:

$$S = 0 \cdot ({}^{30}C_0)^2 + 1 \cdot ({}^{30}C_1)^2 + 2 \cdot ({}^{30}C_2)^2 + \dots + 30 \cdot ({}^{30}C_{30})^2$$

$$S = 30 \cdot ({}^{30}C_0)^2 + 29 \cdot ({}^{30}C_1)^2 + 28 \cdot ({}^{30}C_2)^2 + \dots + 0 \cdot ({}^{30}C_0)^2$$

$$2S = 30 \cdot ({}^{30}C_0^2 + {}^{30}C_1^2 + \dots + {}^{30}C_{30}^2)$$

$$S = 15 \cdot {}^{60}C_{30} = 15 \cdot \frac{60!}{(30!)^2}$$

$$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$$

$$\Rightarrow \alpha = 15$$

Question41

Let the sum of the coefficients of the first three terms in the expansion of

$\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$, $n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is ____

[24-Jan-2023 Shift 2]

Answer: 405

Solution:

Solution:

Given Binomial $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$, $n \in \mathbb{N}$,

Sum of coefficients of first three terms

$${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 3^2 = 376$$

$$\Rightarrow 3n^2 - 5n - 250 = 0$$

$$\Rightarrow (n - 10)(3n + 25) = 0$$

$$\Rightarrow n = 10$$

Now general term ${}^{10}C_r x^{10-r} \left(\frac{-3}{x^2}\right)^r$

$$= {}^{10}C_r x^{10-r} (-3)^r \cdot x^{-2r}$$

$$= {}^{10}C_r (-3)^r \cdot x^{10-3r}$$

Coefficient of $x^4 \Rightarrow 10 - 3r = 4$

$$\Rightarrow r = 2$$

$${}^{10}C_2 (-3)^2 = 405$$

Question42

If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$

is equal to

[25-Jan-2023 Shift 1]

Options:

A. 4895

B. 1210

C. 5445

D. 3025

Answer: B

Solution:

Solution:

$$\begin{aligned} a_r &= {}^{10}C_{10-r} = {}^{10}C_r \\ &\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}} \right)^2 \\ &= \sum_{r=1}^{10} r^3 \left(\frac{11-r}{r} \right)^2 \\ &= \sum_{r=1}^{10} r(11-r)^2 \\ &= \sum_{r=1}^{10} (121r + r^3 - 22r^2) = 1210 \end{aligned}$$

Question43

The constant term in the expansion of

$$\left(2x + \frac{1}{x^7} + 3x^2 \right)^5 \text{ is } \underline{\hspace{2cm}}.$$

[25-Jan-2023 Shift 1]

Answer: 1080

Solution:

Solution:

$$\text{General term is } \sum \frac{5!(2x)^{n_1}(x^{-7})^{n_2}(3x^2)^{n_3}}{n_1!n_2!n_3!}$$

For constant term,

$$n_1 + 2n_3 = 7n_2$$

$$\&n_1 + n_2 + n_3 = 5$$

Only possibility $n_1 = 1, n_2 = 1, n_3 = 3$

$$\Rightarrow \text{constant term} = 1080$$

Question44

$\sum_{k=0}^6 51-k C_3$ is equal to
[25-Jan-2023 Shift 2]

Options:

A. ${}^{51}C_4 - {}^{45}C_4$

B. ${}^{51}C_3 - {}^{45}C_3$

C. ${}^{52}C_4 - {}^{45}C_4$

D. ${}^{52}C_3 - {}^{45}C_3$

Answer: C

Solution:

Solution:

$$\begin{aligned} & \sum_{k=0}^6 {}^{51-k}C_3 \\ &= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \dots + {}^{45}C_3 \\ &= {}^{45}C_3 + {}^{46}C_3 + \dots + {}^{51}C_3 \\ &= {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + \dots + {}^{51}C_3 - {}^{45}C_4 \\ & \left({}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \right) \\ &= {}^{52}C_4 - {}^{45}C_4 \end{aligned}$$

Question45

The remainder when $(2023)^{2023}$ is divided by 35 is _____.
[25-Jan-2023 Shift 2]

Answer: 7

Solution:

Solution:

$$\begin{aligned} & (2023)^{2023} \\ &= (2030 - 7)^{2023} \\ &= (35K - 7)^{2023} \\ &= {}^{2023}C_0 (35K)^{2023} (-7)^0 + {}^{2023}C_1 (35K)^{2022} (-7)^1 + \\ & \dots + \dots + {}^{2023}C_{2023} (-7)^{2023} \\ &= 35N - 7^{2023} \\ & \text{Now, } -7^{2023} = -7 \times 7^{2022} = -7(7^2)^{1011} \\ &= -7(50 - 1)^{1011} \\ &= -7({}^{1011}C_0 50^{1011} - {}^{1011}C_1 (50)^{1010} + \dots + {}^{1011}C_{1011}) \\ &= -7(5\lambda - 1) \\ &= -35\lambda + 7 \end{aligned}$$

\therefore when $(2023)^{2023}$ is divided by 35 remainder is 7

Question46

If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to _____.
[29-Jan-2023 Shift 1]

Answer: 1

Solution:

Solution:

$$\text{Coefficient of } x^9 \text{ in } \left(\alpha x^3 + \frac{1}{\beta x}\right)^{11} = {}^{11}C_6 \cdot \frac{\alpha^5}{\beta^6}$$

\therefore Both are equal

$$\therefore \frac{11}{C_6} \cdot \frac{\alpha^5}{\beta^6} = -\frac{11}{C_5} \cdot \frac{\alpha^6}{\beta^5}$$

$$\Rightarrow \frac{1}{\beta} = -\alpha$$

$$\Rightarrow \alpha\beta = -1$$

$$\Rightarrow (\alpha\beta)^2 = 1$$

Question47

Let the coefficients of three consecutive terms in the binomial expansion of $(1 + 2x)^n$ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is _____.
[29-Jan-2023 Shift 1]

Answer: 1120

Solution:

Solution:

$$t_{r+1} = {}^nC_r (2x)^r$$

$$\Rightarrow \frac{{}^nC_{r-1} (2)^{r-1}}{{}^nC_r (2)^r} = \frac{2}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!(2)}{r!(n-r)!}} = \frac{2}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{4}{5} \Rightarrow 5r = 4n - 4r + 4$$

$$\Rightarrow 9r = 4(n+1) \dots (1)$$

$$\Rightarrow \frac{{}^n C_r (2)^r}{{}^n C_{r+1} (2)^{r+1}} = \frac{5}{8}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{4} \Rightarrow \frac{r+1}{n-r} = \frac{5}{4}$$

$$\Rightarrow 4r+4 = 5n-5r \Rightarrow 5n-4 = 9r \dots (2)$$

From (1) and (2)

$$\Rightarrow 4n+4 = 5n-4 \Rightarrow n = 8$$

$$(1) \Rightarrow r = 4$$

so, coefficient of middle term is

$${}^8 C_4 2^4 = 16 \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 16 \times 70 = 1120$$

Question48

Let K be the sum of the coefficients of the odd powers of x in the expansion of

$(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a} = \frac{2^l m}{n}$,

where m and n are odd numbers, then the ordered pair (l, n) is equal to :

[29-Jan-2023 Shift 2]

Options:

A. (50, 51)

B. (51, 99)

C. (50, 101)

D. (51, 101)

Answer: C

Solution:

Solution:

In the expansion of

$$(1+x)^{99} = C_0 + C_1x + C_2x^2 + \dots + C_{99}x^{99}$$

$$K = C_1 + C_3 + \dots + C_{99} = 2^{98}$$

$a \Rightarrow$ Middle in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$

$$T_{\frac{200}{2}+1} = {}^{200}C_{100}(2)^{100} \left(\frac{1}{\sqrt{2}}\right)^{100}$$

$$= {}^{200}C_{100} \cdot 2^{50}$$

$$\text{So, } \frac{{}^{200}C_{99} \times 2^{98}}{{}^{200}C_{100} \times 2^{50}} = \frac{100}{101} \times 2^{48}$$

$$\text{So, } \frac{25}{101} \times 2^{50} = \frac{m \cdot 2^l}{n}$$

$\therefore m, n$ are odd so

(l, n) become (50, 101) Ans.

Question49

If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to the coefficient of

x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then

for each such ordered pair (a, b) :

[30-Jan-2023 Shift 1]

Options:

A. $a = b$

B. $ab = 1$

C. $a = 3b$

D. $ab = 3$

Answer: B

Solution:

Solution:

Option (2)

$$\text{Coefficient of } x^{15} \text{ in } \left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r (ax^3)^{15-r} \left(\frac{1}{bx^{1/3}}\right)^r$$

$$45 - 3r - \frac{r}{3} = 15$$

$$30 = \frac{10r}{3}$$

$$r = 9$$

$$\text{Coefficient of } x^{15} = {}^{15}C_9 a^6 b^{-9}$$

$$\text{Coefficient of } x^{-15} \text{ in } \left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r (ax^{1/3})^{15-r} \left(-\frac{1}{bx^3}\right)^r$$

$$5 - \frac{r}{3} - 3r = -15$$

$$\frac{10r}{3} = 20$$

$$r = 6$$

$$\text{Coefficient} = {}^{15}C_6 a^9 \times b^{-6}$$

$$\Rightarrow \frac{a^9}{b^6} = \frac{a^6}{b^9}$$

$$\Rightarrow a^3 b^3 = 1 \Rightarrow ab = 1$$

Question50

The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \dots + x^{500}$ is:
[30-Jan-2023 Shift 1]

Options:

A. ${}^{501}C_{302}$

B. ${}^{500}C_{301}$

C. ${}^{500}C_{300}$

D. ${}^{501}C_{200}$

Answer: D

Solution:

Solution:

$$(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \dots + x^{500}$$

$$= (1+x)^{500} \cdot \left\{ \frac{1 - \left(\frac{x}{1+x}\right)^{501}}{1 - \frac{x}{1+x}} \right\}$$

$$= (1+x)^{500} \frac{((1+x)^{501} - x^{501})}{(1+x)^{501}} \cdot (1+x)$$

$$= (1+x)^{501} - x^{501}$$

Coefficient of x^{301} in $(1+x)^{501} - x^{501}$ is given by

$${}^{501}C_{301} = {}^{501}C_{200}$$

Question 51

Let $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^9$. If $[t]$ denotes the greatest integer $\leq t$, then
[30-Jan-2023 Shift 2]

Options:

A. $[x] + [y]$ is even

B. $[x]$ is odd but $[y]$ is even

C. $[x]$ is even but $[y]$ is odd

D. $[x]$ and $[y]$ are both odd

Answer: A

Solution:

Solution:

$$x = (8\sqrt{3} + 13) = {}^{13}C_0 \cdot (8\sqrt{3})^{13} + {}^{13}C_1(8\sqrt{3})^{12}(13)^1 + \dots$$

$$x' = (8\sqrt{3} - 13)^{13} = {}^{13}C_0(8\sqrt{3})^{13} - {}^{13}C_1(8\sqrt{3})^{12}(13)^1 + \dots$$

$$x - x' = 2[{}^{13}C_1 \cdot (8\sqrt{3})^{12}(13)^1 + {}^{13}C_3(8\sqrt{3})^{10} \cdot (13)^3 \dots]$$

therefore, $x - x'$ is even integer, hence $[x]$ is even

$$\text{Now, } y = (7\sqrt{2} + 9)^9 = {}^9C_0(7\sqrt{2})^9 + {}^9C_1(7\sqrt{2})^8(9)^1$$

$$+ {}^9C_2(7\sqrt{2})^7(9)^2 \dots\dots$$

$$y' = (7\sqrt{2} - 9)^9 = {}^9C_0(7\sqrt{2})^9 - {}^9C_1(7\sqrt{2})^8(9)^1$$

$$+ {}^9C_2(7\sqrt{2})^7(9)^2 \dots\dots$$

$$y - y' = 2[{}^9C_1(7\sqrt{2})^8(9)^1 + {}^9C_3(7\sqrt{2})^6(9)^3 + \dots]$$

$$y - y' = \text{Even integer, hence } [y] \text{ is even}$$

Question52

50^{th} root of a number x is 12 and 50^{th} root of another number y is 18 . Then the remainder obtained on dividing $(x + y)$ by 25 is _____.

[30-Jan-2023 Shift 2]

Answer: 23

Solution:

Solution:

$$\begin{aligned} x + y &= 12^{50} + 18^{50} = (150 - 6)^{25} + (325 - 1)^{25} \\ &= 25K - (6^{25} + 1) = 25K - ((5 + 1)^{25} + 1) \\ &= 25K_1 - 2 \quad \text{Remainder} = 23 \end{aligned}$$

Question53

Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term

$\beta x^{-\alpha}$, $\beta \in \mathbb{N}$.

Then α is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 2

Solution:

Solution:

$$\begin{aligned} T_{r+1} &= {}^{30}C_r (x^{2/3})^{30-r} \left(\frac{2}{x^3}\right)^r \\ &= {}^{30}C_r \cdot 2 \cdot x^{\frac{60-11r}{3}} \\ \frac{60-11r}{3} < 0 &\Rightarrow 11r > 60 \Rightarrow r > \frac{60}{11} \Rightarrow r = 6 \\ T_7 &= {}^{30}C_6 \cdot 2^6 x^{-2} \end{aligned}$$

We have also observed $\beta = {}^{30}C_6(2)^6$ is a natural number.

$\therefore \alpha = 2$

Question54

The remainder on dividing 5^{99} by 11 is _____.
[31-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:

$$\begin{aligned}5^{99} &= 5^4 \cdot 5^{95} \\ &= 625[5^5]^{19} \\ &= 625[3125]^{19} \\ &= 625[3124 + 1]^{19} \\ &= 625[11k \times 19 + 1] \\ &= 625 \times 11k \times 19 + 625 \\ &= 11k_1 + 616 + 9 \\ &= 11(k_2) + 9 \\ \text{Remainder} &= 9\end{aligned}$$

Question55

The Coefficient of x^{-6} , in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is _____
[31-Jan-2023 Shift 2]

Answer: 5040

Solution:

Solution:

$$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$$

$$\text{Now, } T_{r+1} = {}^9C_r \cdot \left(\frac{4x}{5}\right)^{9-r} \left(\frac{5}{2x^2}\right)^r$$

$$= {}^9C_r \cdot \left(\frac{4}{5}\right)^{9-r} \left(\frac{5}{2}\right)^r \cdot x^{9-3r}$$

$$\text{Coefficient of } x^{-6} \text{ i.e. } 9 - 3r = -6 \Rightarrow r = 5$$

Question56

If the constant term in the binomial expansion of $\left(\frac{5}{x^2} - \frac{4}{x^l}\right)^9$ is -84 and the Coefficient of x^{-3l} is $2^\alpha\beta$, where $\beta < 0$ is an odd number, Then $|\alpha l - \beta|$ is equal to

[31-Jan-2023 Shift 2]

Answer: 98

Solution:

Solution:

$$\begin{aligned} & \text{In, } \left(\frac{5}{x^2} - \frac{4}{x^l}\right)^9 \\ T_{r+1} &= {}^9C_r \frac{(x^{5/2})^{9-r}}{2^{9-r}} \left(\frac{-4}{x^l}\right)^r \\ &= (-1)^r \frac{{}^9C_r 4^r x^{\frac{45}{2} - 5r - lr}}{2^{9-r}} \\ &= 45 - 5r - 2lr = 0 \\ r &= \frac{45}{5+2l} \dots (1) \end{aligned}$$

$$\text{Now, according to the question, } (-1)^r \frac{{}^9C_r 4^r}{2^{9-r}} = -84$$

$$= (-1)^r {}^9C_r 2^{3r-9} = 21 \times 4$$

Only natural value of r possible if $3r - 9 = 0$ $r = 3$ and ${}^9C_3 = 84$

$\therefore l = 5$ from equation (1)

$$\text{Now, coefficient of } x^{-3l} = x^{\frac{45}{2} - 5r - lr} \text{ at } l = 5, \text{ gives}$$

$$\therefore {}^9C_5 (-1)^3 \frac{4^5}{2^4} = 2^\alpha \times \beta$$

$$= -63 \times 2^7$$

$$\Rightarrow \alpha = 7, \beta = -63$$

$$\therefore \text{value of } |\alpha l - \beta| = 98$$

Question 57

The value of $\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \dots + \frac{1}{49!2!} + \frac{1}{51!1!}$ is

[1-Feb-2023 Shift 1]

Options:

A. $\frac{2^{50}}{50!}$

B. $\frac{2^{50}}{51!}$

C. $\frac{2^{51}}{51!}$

D. $\frac{2^{51}}{50!}$

Answer: B

Solution:

Solution:

$$\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)} \frac{1}{51!}$$

$$= \frac{1}{51!} \{ {}^{51}C_1 + {}^{51}C_3 + \dots + {}^{51}C_{51} \} = \frac{1}{51!} (2^{50})$$

Question58

The remainder when $19^{200} + 23^{200}$ is divided by 49 , is _____ .
[1-Feb-2023 Shift 1]

Answer: 29

Solution:

Solution:

$$(21+2)^{200} + (21-2)^{200}$$

$$\Rightarrow 2[{}^{100}C_0 21^{200} + {}^{100}C_2 21^{198} \cdot 2^2 + \dots + {}^{100}C_{198} \cdot 2^{198} + 2^{200}]$$

$$\Rightarrow 2[49I_1 + 2^{200}] = 49I_1 + 2^{201}$$

Now , $2^{201} = (8)^{67} = (1+7)^{67} = 49I_2 + {}^{67}C_0 {}^{67}C_1 \cdot 7 =$
 $49I_2 + 470 = 49I_2 + 49 \times 9 + 29$

Question59

Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + {}^5\sqrt{2^{(x-2)\log_2 3}} \right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21 . If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is _____ .
[1-Feb-2023 Shift 2]

Answer: 4

Solution:

Solution:

$$T_6 = {}^m C_5 (10 - 3^x)^{\frac{m-5}{2}} \cdot (3^{x-2}) = 21$$

${}^m C_1, {}^m C_2, {}^m C_3$ are in A.P.

$$2 \cdot {}^m C_2 = {}^m C_1 + {}^m C_3$$

Solving for m, we get

$m = 2$ (rejected), 7

Put in equation (1)

$$21 \cdot (10 - 3^x) \frac{3^x}{9} = 21$$

$$3^x = 3^0, 3^2$$

$$x = 0, 2$$

Sum of the squares of all possible values of $x = 4$

Question60

If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to

[1-Feb-2023 Shift 2]

Answer: 1

Solution:

Solution:

$$T_{r+1} = {}^{22} C_r \cdot \left(x^{\frac{2}{3}}\right)^{22-r} \cdot (\alpha)^r \cdot x^{-3r}$$

$$= {}^{22} C_r \cdot x^{\frac{44}{3} - \frac{2r}{3} - 3r} (\alpha)^r$$

$$\frac{44}{3} = \frac{11r}{3}$$

$$r = 4$$

$${}^{22} C_4 \cdot \alpha^4 = 7315$$

$$\frac{22 \times 21 \times 20 \times 19}{24} \cdot \alpha^4 = 7315$$

$$\alpha = 1$$

Question61

If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left({}^4\sqrt{2} + \frac{1}{{}^4\sqrt{3}}\right)^a$ is $\sqrt{6} : 1$, then the third term from the beginning is :

[6-Apr-2023 shift 1]

Options:

A. $30\sqrt{2}$

B. $60\sqrt{2}$

C. $30\sqrt{3}$

D. $60\sqrt{3}$

Answer: D

Solution:

Solution:

$$\frac{T_5}{T_5} = \frac{{}^nC_4 \cdot \left(\frac{1}{2}\right)^{n-4} \left(\frac{1}{3}\right)^4}{{}^nC_4 \left(\frac{1}{3}\right)^{n-4} \left(\frac{1}{2}\right)^4} = \frac{\sqrt{6}}{1}$$

$$2^{\frac{n-8}{4}} \cdot \left(\frac{1}{3}\right)^{-4-4+n} = \sqrt{6}$$

$$2^{\frac{n-8}{4}} \cdot 3^{\frac{n-8}{4}} = \sqrt{6}$$

$$\frac{n-8}{4} = \frac{1}{2} \Rightarrow n-8 = 2 \Rightarrow n = 10$$

$$T_3 = {}^{10}C_2 \left(\frac{1}{2}\right)^8 \left(\frac{1}{3}\right)^2$$

$$= {}^{10}C_2 \cdot 2^2 \cdot 3^{-\frac{1}{2}} = \frac{10 \cdot 9}{2} \cdot 4 \cdot \frac{1}{\sqrt{3}} = 60\sqrt{3}$$

Question62

If ${}^{2n}C_3 : {}^nC_3 : 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is :
[6-Apr-2023 shift 1]

Options:

A. 27 : 11

B. 35 : 16

C. 2 : 1

D. 65 : 37

Answer: C

Solution:

Solution:

$$\frac{{}^{2n}C_3}{{}^nC_3} = 10 \Rightarrow \frac{2n!(n-3)!}{(2n-3)!n!} = 10$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$$

$$\frac{4(2n-1)}{n-2} = 10 \Rightarrow 8n-4 = 10n-20$$

$$2n = 16$$

$$\begin{aligned} \text{Now } & \frac{n^2 + 3n}{n^2 - 3n + 4} \\ &= \frac{64 + 24}{64 - 24 + 4} = \frac{88}{44} = 2 \end{aligned}$$

Question63

The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is _____.

[6-Apr-2023 shift 1]

Answer: 5005

Solution:

Solution:

$$\begin{aligned} & \left(x^4 - \frac{1}{x^3}\right)^{15} \\ T_{r+1} &= {}^{15}C_r (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r \\ 60 - 7r &= 18 \\ r &= 6 \\ \text{Hence coeff. of } x^{18} &= {}^{15}C_6 = 5005 \end{aligned}$$

Question64

If the coefficients of x^7 in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ are equal, then :

[6-Apr-2023 shift 2]

Options:

- A. $64 ab = 243$
- B. $32 ab = 729$
- C. $729ab = 32$
- D. $243 ab = 64$

Answer: C

Solution:

Solution:

$$\left(ax^2 + \frac{1}{2bx}\right)^{11}$$

$$r = \frac{11 \times 2 - 7}{3} = 5$$

Coefficient of x^7 is $= {}^{11}C_5(a^6)\left(\frac{1}{2b}\right)^5$

$$\left(ax - \frac{1}{3bx^2}\right)^{11}$$

$$r = \frac{11 \times 1 - (-7)}{3} = 6$$

Coefficient of x^{-7} is $= {}^{11}C_6 \cdot \frac{a^5}{3^6 b^6}$

$$\therefore {}^{11}C_5(a^6)\left(\frac{1}{2^5 b^5}\right) = {}^{11}C_6 \cdot \frac{a^5}{3^6 b^6}$$

$$\Rightarrow ab = \frac{2^5}{3^6}$$

$$\Rightarrow 729 ab = 32$$

Ans. Option 3

Question65

Among the statements :

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

[6-Apr-2023 shift 2]

Options:

- A. only (S2) is correct
- B. only (S1) is correct
- C. both (S1) and (S2) are incorrect
- D. both (S1) and (S2) are correct

Answer: D

Solution:

Solution:

$$\because x^n - y^n = (x - y)[x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \dots + y^{n-1}]$$

$x^n - y^n$ is divisible by $x - y$

$$\text{Stat 1} \rightarrow (2023)^{2022} - (1999)^{2022}$$

$$(2023) - (1999) = 24$$

$$\text{Stat 2} \rightarrow (2023)^{2022} - (1999)^{2022}$$

is divisible by 8

$$13(1 + 12)^n - 11n - 13$$

$$13[1 + {}^nC_1(12) + {}^nC_2(12)^2 + \dots] - 11n - 13$$

$$\Rightarrow (156n - 11n) + 13 \cdot {}^nC_2(12)^2 + 13 \cdot {}^nC_3(12)^3 + \dots$$

$$\Rightarrow 145n + 13 \cdot {}^nC_2(12)^2 + 13 \cdot {}^nC_3(12)^3 + \dots$$

If $(n = 144m, m \in \mathbb{N})$ then it is divisible by 144 for infinite values of n .

Ans. Option 4

Question66

Let (t) denote the greatest integer $\leq t$, If the constant term in the expansion of

$\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α , then $[\alpha]$ is equal to _____.

[8-Apr-2023 shift 1]

Answer: 1275

Solution:

Solution:

$$\left(3x^2 - \frac{1}{2x^5}\right)^7$$

$$T_{r+1} = {}^7C_r (3x^2)^{7-r} \left(-\frac{1}{2x^5}\right)^r$$

$$14 - 2r - 5r = 14 - 7r = 0$$

$$\therefore r = 2$$

$$\therefore T_3 = {}^7C_2 \cdot 3^5 \left(-\frac{1}{2}\right)^2 = \frac{21 \times 243}{4} = 1275.75$$

$$\therefore [\alpha] = 1275$$

Question67

If a_n is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}$, $n = 1, 2, 3, \dots$, then a is equal to

_____.

[8-Apr-2023 shift 1]

Answer: 5

Solution:

Solution:

$$f(x) = \frac{x^3}{x^4 + 147}$$

$$f'(x) = \frac{(x^4 + 147)3x^2 - x^3(4x^3)}{(x^4 + 147)^2}$$

$$= \frac{3x^6 + 147 \times 3x^2 - 4x^6}{+ve} = x^2(44 - x^4)$$

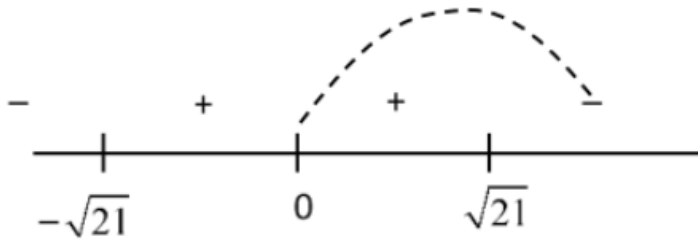
$$f'(x) = 0 \text{ at } x^6 = 147 \times 3x^2$$

$$x^2 = 0, x^4 = 147 \times 3$$

$$x = 0, x^2 = \pm\sqrt{147 \times 3}$$

$$x^2 = \pm 21$$

$$x = \pm\sqrt{21}$$



fmax at f(4) or f(5)

$$f(4) = \frac{64}{403} \approx 0.158 \quad f(5) = \frac{125}{772} \approx 0.161$$

$$\therefore a = 5$$

Question68

The largest natural number n such that 3^n divides $66!$ is _____.
[8-Apr-2023 shift 1]

Answer: 31

Solution:

Solution:

$$\left[\frac{66}{3} \right] + \left[\frac{66}{9} \right] + \left[\frac{66}{27} \right]$$

$$22 + 7 + 2 = 31$$

Question69

The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to
[8-Apr-2023 shift 2]

Options:

A. $10^3 - 10$

B. $11^3 - 11$

C. $12^3 - 12$

D. $13^3 - 13$

Answer: C

Solution:

Solution:

$$\begin{aligned}
T_{r+1} &= {}^{11}C_r (2x^2)^{11-r} \left(\frac{1}{2x}\right)^r \\
&= {}^{11}C_r 2^{11-2r} x^{22-3r} \\
22-3r &= 10 \quad \text{and} \quad 22-3r = 7 \\
r &= 4 \quad \text{and} \quad r = 5 \\
\text{Coefficient of } x^{10} &= {}^{11}C_4 \cdot 2^3 \\
\text{Coefficient of } x^7 &= {}^{11}C_5 \cdot 2^1 \\
\text{difference} &= {}^{11}C_4 \cdot 2^3 - {}^{11}C_5 \cdot 2 \\
&= \frac{11 \times 10 \times 9 \times 8}{24} \times 8 - \frac{11 \times 10 \times 9 \times 8 \times 7}{120} \times 2 \\
&= 11 \times 10 \times 3 \times 8 - 11 \times 3 \times 4 \times 7 \\
&= 11 \times 3 \times 4 \times (20 - 7) \\
&= 11 \times 12 \times 13 \\
&= 12(12-1)(12+1) \\
&= 12(12^2 - 1) \\
&= 12^3 - 12 \quad (\text{Option 3})
\end{aligned}$$

Question70

$25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by
[8-Apr-2023 shift 2]

Options:

- A. 34 but not by 14
- B. 14 but not by 34
- C. Both 14 and 34
- D. Neither 14 nor 34

Answer: A

Solution:

Solution:

$$\begin{aligned}
25^{190} - 8^{190} &\text{ is divisible by } 25 - 8 = 17 \\
19^{190} - 2^{190} &\text{ is divisible by } 19 - 2 = 17 \\
25^{190} - 19^{190} &\text{ is divisible by } 25 - 19 = 6 \\
8^{190} - 2^{190} &\text{ is divisible by } 8 - 2 = 6 \\
\text{L.C.M. of } 1746 &= 34 \\
\therefore &\text{ divisible by } 34 \text{ but not by } 14
\end{aligned}$$

Question71

If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are

equal, then $a^4 b^4$ is equal to :

[10-Apr-2023 shift 1]

Options:

A. 22

B. 44

C. 11

D. 33

Answer: A

Solution:

Solution:

$$\left(ax - \frac{1}{bx^2}\right)^{13}$$

We have,

$$T_{r+1} = {}^n C_r (p)^{n-r} (q)^r$$

$$T_{r+1} = {}^{13} C_r (ax)^{13-r} \left(-\frac{1}{bx^2}\right)^r$$

$$= {}^{13} C_r (a)^{13-r} \left(-\frac{1}{b}\right)^r (x)^{13-r} \cdot (x)^{-2r}$$

$$= {}^{13} C_r (a)^{13-r} \left(-\frac{1}{b}\right)^r (x)^{13-3r} \dots (1)$$

Coefficient of x^7

$$\Rightarrow 13 - 3r = 7$$

$$r = 2$$

r in equation (1)

$$T_3 = {}^{13} C_2 (a)^{13-2} \left(-\frac{1}{b}\right)^2 (x)^{13-6}$$

$$= {}^{13} C_2 (a)^{11} \left(\frac{1}{b}\right)^2 (x)^7$$

Coefficient of x^7 is ${}^{13} C_2 \frac{(a)^{11}}{b^2}$

Now, $\left(ax + \frac{1}{bx^2}\right)^{13}$

$$T_{r+1} = {}^{13} C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$$

$$= {}^{13} C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-r} (x)^{-2r}$$

$$= {}^{13} C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-3r} \dots (2)$$

Coefficient of x^{-5}

$$\Rightarrow 13 - 3r = -5$$

$$r = 6$$

r in equation

$$T_7 = {}^{13} C_6 (a)^{13-6} \left(\frac{1}{b}\right)^6 (x)^{13-18}$$

$$T_7 = {}^{13} C_6 (a)^7 \left(\frac{1}{b}\right)^6 (x)^{-5}$$

Coefficient of x^{-5} is ${}^{13} C_6 (a)^7 \left(\frac{1}{b}\right)^6$

ATQ

Coefficient of $x^7 =$ coefficient of x^{-5}

$$T_3 = T_7$$

$${}^{13} C_2 \left(\frac{a^{11}}{b^2}\right) = {}^{13} C_6 (a)^7 \left(\frac{1}{b}\right)^6$$

$$a^4 \cdot b^4 = \frac{{}^{13} C_6}{{}^{13} C_2}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 1}{13 \times 12 \times 6 \times 5 \times 4 \times 3} = 22$$

Question72

The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is _____.
[10-Apr-2023 shift 1]

Answer: 960

Solution:

Solution:

$$(1 - x + 2x^3)^{10}$$

$$T_n = \frac{10!}{a!b!c!}(-2x)^b(x^3)^c$$

$$= \frac{10!}{a!b!c!}(-2)^b x^{b+3c}$$

$$\Rightarrow b + 3c = 7, a + b + c = 10$$

$$\therefore \text{Coefficient of } x^7 = \frac{10!}{3!7!0!}(-1)^7 + \frac{10!}{5!4!1!}(-1)^4(2)$$

$$+ \frac{10!}{7!1!2!}(-1)^1(2)^2$$

$$= -120 + 2520 - 1440 = 960$$

Question73

Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 13

B. 20

C. 10

D. 5

Answer: D

Solution:

Solution:

$$(22)^{2022} + (2022)^{22}$$

divided by 3

$$(21 + 1)^{2022} + (2022)^{22}$$

$$= 3k + 1$$

$$(\alpha = 1)$$

Divided by 7

$$(21 + 1)^{2022} + (2023 - 1)^{22}$$

$$7k + 1 + 1 \quad (\beta = 2)$$

$$7k+2$$

$$\text{So } \alpha^2 + \beta^2 \Rightarrow 5$$

Question74

If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and -5 respectively, then $2p+3q$ is equal to

[10-Apr-2023 shift 2]

Options:

A. 60

B. 63

C. 66

D. 69

Answer: B

Solution:

Solution:

$$(1+x)^p(1-x)^q$$

$$\left(1+px+\frac{p(p-1)}{2!}x^2+\dots\right)$$

$$\left(1-qx+\frac{q(q-1)}{2!}x^2-\dots\right)$$

$$p-q=4$$

$$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$$

$$p^2+q^2-p-q-2pq=-10$$

$$(q+4)^2+q^2-(q+4)-q-2(4+q)q=-10$$

$$q^2+8q+16-q^2-q-4-q-8q-2q^2=-10$$

$$-2q=-22$$

$$q=11$$

$$p=15$$

$$2(15)+3(11)$$

$$30+33=63$$

Question75

The number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to :

[11-Apr-2023 shift 1]

Answer: 171

Solution:

Solution:

The number of integral term in the expression of $\left(3\frac{1}{2} + 5\frac{1}{4}\right)^{680}$ is equal to

$$\begin{aligned}\text{General term} &= {}^{680}C_r \left(3\frac{1}{2}\right)^{680-r} \left(5\frac{1}{4}\right)^r \\ &= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}\end{aligned}$$

Values' s of r, where $\frac{r}{4}$ goes to integer

$$r = 0, 4, 8, 12, \dots, 680$$

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms = 171.

Question76

The mean of the coefficients of x, x^2, \dots, x^7 in the binomial expansion of $(2+x)^9$ is

[11-Apr-2023 shift 1]

Answer: 2736

Solution:

Solution:

$$\text{Coefficient of } x = {}^9C_1 2^8$$

$$\text{Coef. } x^2 = {}^9C_2 2^7$$

$$\text{Coef. } x^7 = {}^9C_7 \cdot 2^2$$

$$\begin{aligned}\text{Mean} &= \frac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 + \dots + {}^9C_7 \cdot 2^2}{7} \\ &= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7} \\ &= \frac{3^9 - 2^9 - 18 - 1}{7} \\ &= \frac{19152}{7} = 2736\end{aligned}$$

Question77

If the 1011th term from the end in the binominal expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024

times 1011th term from the beginning, the $|x|$ is equal to

[11-Apr-2023 shift 2]

Options:

A. 8

B. 12

C. 10

D. 15

Answer: C

Solution:

Solution:

$$T_{1011} \text{ from beginning} = T_{1010+1} \\ = {}^{2022}C_{1010} \left(\frac{4x}{5}\right)^{1012} \left(\frac{-5}{2x}\right)^{1010}$$

T_{1011} from end

$$= {}^{2022}C_{1010} \left(\frac{-5}{2x}\right)^{1012} \left(\frac{4x}{5}\right)^{1010}$$

$$\text{Given: } = {}^{2022}C_{1010} \left(\frac{-5}{2x}\right)^{1012} \left(\frac{4x}{5}\right)^{1010}$$

$$= 2^{10} \cdot {}^{2022}C_{1010} \left(\frac{-5}{2x}\right)^{1010} \left(\frac{4x}{5}\right)^{1012}$$

$$\left(\frac{-5}{2x}\right)^2 = 2^{10} \left(\frac{4x}{5}\right)^2$$

$$x^4 = \frac{5^4}{2^{16}}$$

$$|x| = \frac{5}{16}$$

Question 78

The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to [11-Apr-2023 shift 2]

Options:

A. 63

B. 92

C. 25

D. 41

Answer: A

Solution:

Solution:

$${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} :: 1 : 3 : 5$$

$$\frac{(n+2)!}{(r-1)!(n-r+3)!} \times \frac{r!(n+2-r)!}{(n+2)!} = \frac{1}{3}$$

$$\frac{r}{(n-r+3)} = \frac{1}{3} \Rightarrow n-r+3 = 3r$$

$$n = 4r - 3 - 0$$

$$\frac{(n+1)!}{r!(n+2-r)!} \times \frac{(r+1)!(n-r+1)!}{(n+2)!} = \frac{3}{5}$$

$$\frac{r+1}{n+2-r} = \frac{3}{5}$$

$$8r - 1 = 3n \dots (2)$$

By equation 1 and 2

$$\frac{8r-1}{3} = 4r-3 \quad n = 4r-3$$

$$r = 2 \quad n = 4 \times 2 - 3$$

$$n = 5$$

$$\text{Sum: } {}^7C_1 + {}^7C_2 + {}^7C_3 = 7 + 21 + 35 = 63$$

Question79

If $\frac{1}{n+1} {}^n C_n + \frac{1}{n} {}^n C_{n-1} + \dots + \frac{1}{2} {}^n C_1 + {}^n C_0 = \frac{1023}{10}$ then n is equal to
[12-Apr-2023 shift 1]

Options:

A. 7

B. 9

C. 6

D. 8

Answer: B

Solution:

Solution:

$$\begin{aligned} \sum_{r=0}^n \frac{{}^n C_r}{r+1} &= \frac{1}{n+1} \sum_{r=0}^{n+1} {}^n C_{r+1} \\ &= \frac{1}{n+1} (2^{n+1} - 1) = \frac{1023}{10} \end{aligned}$$

$$n+1 = 10 \Rightarrow n = 9$$

Question80

The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to
[12-Apr-2023 shift 1]

Options:

A. $-{}^{101}C_{50}$

B. ${}^{99}C_{49}$

C. ${}^{101}C_{50}$

D. $-{}^{99}C_{49}$

Answer: D

Solution:

Solution:

$$(1-x)^{100} = C_0 - C_1x + C_2x^2 - C_3x^3 + \dots + C_{99}x^{99} + C_{100}x^{100}$$
$$\Rightarrow C_0 - C_1 + C_2 - C_3 + \dots - C_{99} + C_{100} = 0$$
$$C_0 - C_1 + C_2 + \dots + C_{99} = -\frac{1}{2}C_{100}$$
$$-\frac{1}{2} \frac{100!}{50!50!} = -\frac{1}{2} \times \frac{100 \times 99!}{50!50!} = -{}^{99}C_{49}$$

Question81

Fractional part of the number is $\frac{4^{2022}}{15}$ equal to
[13-Apr-2023 shift 1]

Options:

- A. $\frac{4}{15}$
- B. $\frac{8}{15}$
- C. $\frac{1}{15}$
- D. $\frac{14}{15}$

Answer: C

Solution:

Solution:

$$\left\{ \frac{4^{2022}}{15} \right\} = \left\{ \frac{2^{4044}}{15} \right\} = \left\{ \frac{(1+15)^{1011}}{15} \right\} = \frac{1}{15}$$

Question82

Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}} \right)^n$, $n \leq 15$. If the

sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to _____.

[13-Apr-2023 shift 1]

Answer: 36

Solution:

Solution:

$$T_{k+1} = {}^n C_k (x)^{\frac{n-k}{2}} (-6)^k (x)^{\frac{-3}{2}k}$$

$$\frac{n-k}{2} - \frac{3}{2}k = 0$$

$$n - 4k = 0$$

$$(-5)^n - \left({}^n C_n (-6)^{\frac{n}{4}} \right) = 649$$

By observation ($625 + 24 = 649$), we get $n = 4$

$\therefore n = 4$ & $k = 1$

Required is coefficient of

$$x^{-4} \text{ is } \left(\sqrt{4} - \frac{6}{x^{\frac{3}{2}}} \right)^4$$

$${}^4 C_1 (-6)^3$$

By calculating we will get $\lambda = 36$

Question 83

Let for $x \in \mathbb{R}$, $S_0(x) = x$, $S_k(x) = C_k x + k \int_0^x S_{k-1}(t) dt$ where

$C_0 = 1$, $C_k = 1 - \int_0^1 S_{k-1}(x) dx$, $k = 1, 2, 3, \dots$ Then $S_2(3) + 6C_3$ is equal to _____.

[13-Apr-2023 shift 1]

Answer: 18

Solution:

Solution:

$$\text{Given, } S_k(x) = C_k x + k \int_0^x S_{k-1}(t) dt$$

put $k = 2$ and $x = 3$

$$S_2(3) = C_2(3) + 2 \int_0^3 S_1(t) dt$$

$$\text{Also, } S_1(x) = C_1(x) + \int_0^x S_0(t) dt \dots \dots (1)$$

$$= C_1 x + \frac{x^2}{2}$$

$$S_2(3) = 3C_2 + 2 \int_0^3 \left(C_1 t + \frac{t^2}{2} \right) dt$$

$$= 3C_2 + 9C_1 + 9$$

Also,

$$C_1 = 1 - \int_0^1 S_0(x) dx = \frac{1}{2}$$

$$C_2 = 1 - \int_0^1 S_1(x) dx = 0$$

$$C_3 = 1 - \int_0^1 S_2(x) dx$$

$$= 1 - \int_0^1 \left(C_2 x + C_1 x^2 + \frac{x^3}{3} \right) dx = \frac{3}{4}$$

$$S_2(x) = C_2 x + 2 \int_0^x S_1(t) dt$$

$$= C_2 x + C_1 x^2 + \frac{x^3}{3}$$

$$\Rightarrow S_2(3) + 6C_3 = 6C_3 + 3C_2 + 9C_1 + 9 = 18$$

Question84

The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

[13-Apr-2023 shift 2]

Options:

A. $\frac{80}{9}$

B. 8

C. 9

D. $\frac{26}{3}$

Answer: A

Solution:

Solution:

general term for $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$T_{r+1} = {}^5C_r \left(-\frac{1}{3x^2}\right)^r (2x^3)^{5-r}$$
$${}^5C_r (-1)^r 3^{-r} 2^{5-r} \cdot x^{15-5r}$$

$$15 - 5r = 5 \Rightarrow r = 2$$

$$\text{Coeff. of } x^5 = {}^5C_2 (-1)^2 3^{-2} 2^3$$

$$= 10 \times \frac{1}{9} \times 8$$

$$= \frac{80}{9}$$

Question85

The remainder, when 7^{103} is divided by 17, is _____.

[13-Apr-2023 shift 2]

Answer: 12

Solution:

Solution:

$$7^{103} = 7 \cdot 7^{102}$$

$$= 7(7^2)^{51}$$

$$= 7(51-2)^{51} \rightarrow \text{remainder} = 7(-2)^{51}$$

$$= 7(2^3)(16)^{12} = -56(17-1)^{12} \rightarrow \text{remainder} = -56(-1)^{12}$$

$$\text{Remainder} = -56 + 17k$$

$$\begin{aligned} &= -56 + 68 \\ &= 12 \end{aligned}$$

Question86

Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$, $a, b, c \in \mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to

[15-Apr-2023 shift 1]

Options:

- A. 8
- B. 12
- C. 6
- D. 15

Answer: B

Solution:

Solution:

$$(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$$

Coefficient of $x^1 = 20$

$$20 = \frac{10!}{9!1!} \times a^9 \times b^1$$

$$a^9 \cdot b = 20$$

$$a = 2, b = 2$$

Coefficient of $x^2 = 210$

$$210 = \frac{10!}{9!1!} \times a^9 \times c^1 + \frac{10!}{8!2!} \times a^8 b^2$$

$$210 = 10 \cdot c + 45 \times 4$$

$$10c = 30$$

$$c = 3$$

$$2(a + b + c) = 12$$

Question87

The remainder when 3^{2022} is divided by 5 is :

[24-Jun-2022-Shift-1]

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

Solution:

$$\begin{aligned} & 3^{2022} \\ &= (3^2)^{1011} \\ &= (9)^{1011} \\ &= (10-1)^{1011} \\ &= {}^{1011}C_0(10)^{1011} + \dots + {}^{1011}C_{1010} \cdot (10)^1 - {}^{1011}C_{1011} \\ &= 10[{}^{1011}C_0(10)^{1010} + \dots + {}^{1011}C_{1010}] - 1 \\ &= 10K - 1 \\ &[\text{As } 10[{}^{1011}C_0 \cdot (10)^{1010} + \dots + {}^{1011}C_{1010}] \text{ is multiple of } 10] \\ &= 10K + 5 - 5 - 1 \\ &= 10K - 5 + 5 - 1 \\ &= 5(2K - 1) + 4 \\ \therefore \text{Unit digit} &= 4 \text{ when divided by } 5. \end{aligned}$$

Question88

The remainder on dividing $1 + 3 + 3^2 + 3^3 + \dots + 3^{2021}$ by 50 is ___
[24-Jun-2022-Shift-2]

Answer: 4

Solution:

Solution:

Given,

$$\begin{aligned} & 1 + 3 + 3^2 + 3^3 + \dots + 3^{2021} \\ &= 3^0 + 3^1 + 3^2 + 3^3 + \dots + 3^{2021} \end{aligned}$$

This is a G.P with common ratio = 3

$$\begin{aligned} \therefore \text{Sum} &= \frac{1(3^{2022} - 1)}{3 - 1} \\ &= \frac{3^{2022} - 1}{2} \\ &= \frac{(3^2)^{1011} - 1}{2} \\ &= \frac{(10-1)^{1011} - 1}{2} \end{aligned}$$

$$\begin{aligned}
&= \frac{[{}^{1011}C_0 \cdot 10^{1011} - {}^{1011}C_1 \cdot 10^{1010} + \dots - {}^{1011}C_{1009} \cdot (10)^2 + {}^{1011}C_{1010} \cdot 10 - {}^{1011}C_{1011}] - 1}{2} \\
&= \frac{10^2[{}^{1011}C_0 \cdot (10)^{1009} - {}^{1011}C_1 \cdot (1008) + \dots + {}^{1011}C_{1009}] + 10110 - 1 - 1}{2} \\
&= \frac{100k + 10110 - 2}{2} \\
&= \frac{100k + 10108}{2} \\
&= 50k + 5054 \\
&= 50k + 50 \times 101 + 4 \\
&= 50[k + 101] + 4 \\
&= 50k \cdot k
\end{aligned}$$

∴ By dividing 50 we get remainder as 4 .

Question89

Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$. If for

$$\alpha, \beta \in \mathbb{R}, C_1 + 3.2C_2 + 5.3C_3 + \dots \text{ upto 10 terms} = \frac{\alpha \times 2^{11}}{2^\beta - 1} \left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots \text{ upto 10 terms} \right)$$

then the value of $\alpha + \beta$ is equal to
[25-Jun-2022-Shift-1]

Answer: 286

Solution:

Solution:

$$(1+x)^{10} = C_0 + C_1x + C_2x^2 + \dots + C_{10}x^{10}$$

Differentiating

$$10(1+x)^9 = C_1 + 2C_2x + 3C_3x^2 + \dots + 10C_{10}x^9$$

replace $x \rightarrow x^2$

$$10(1+x^2)^9 = C_1 + 2C_2x^2 + 3C_3x^4 + \dots + 10C_{10}x^{18}$$

$$10 \cdot x(1+x^2)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + \dots + 10C_{10}x^{19}$$

Differentiating

$$10((1+x^2)^9 \cdot 1 + x \cdot 9(1+x^2)^8 \cdot 2x) \\ = C_1 x + 2C_2 \cdot 3x^3 + 3 \cdot 5 \cdot C_3 x^4 + \dots + 10 \cdot 19C_{10} x^{18}$$

putting $x = 1$

$$10(2^9 + 18 \cdot 2^8) \\ = C_1 + 3 \cdot 2 \cdot C_2 + 5 \cdot 3 \cdot C_3 + \dots + 19 \cdot 10 \cdot C_{10}$$

$$C_1 + 3 \cdot 2 \cdot C_2 + \dots + 19 \cdot 10 \cdot C_{10} \\ = 10 \cdot 2^9 \cdot 10 = 100 \cdot 2^9$$

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_9}{11} + \frac{C_{10}}{11} = \frac{2^{11} - 1}{11}$$

10th term 11th term

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_9}{11} = \frac{2^{11} - 2}{11}$$

$$\text{Now, } 100 \cdot 2^9 = \frac{\alpha \cdot 2^{11}}{2^\beta - 1} \left(\frac{2^{11} - 2}{11} \right)$$

Eqn. of form $y = k(2^x - 1)$

It has infinite solutions even if we take $x, y \in N$.

Question90

The coefficient of x^{101} in the expression $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + \dots + x^{500}$, $x > 0$, is
[25-Jun-2022-Shift-2]

Options:

- A. ${}^{501}C_{101}(5)^{399}$
- B. ${}^{501}C_{101}(5)^{400}$
- C. ${}^{501}C_{100}(5)^{400}$
- D. ${}^{500}C_{101}(5)^{399}$

Answer: A

Solution:

Solution:

Given,

$$(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + \dots + x^{500}$$

This is a G.P. with first term $(5+x)^{500}$

$$\text{Common ratio} = \frac{x(5+x)^{499}}{(5+x)^{500}} = \frac{x}{5+x} \text{ and } 501 \text{ terms present.}$$

$$\therefore \text{Sum} = \frac{(5+x)^{500} \left(\left(\frac{x}{5+x} \right)^{501} - 1 \right)}{\frac{x}{5+x} - 1}$$

$$= \frac{(5+x)^{500} \left(\frac{x^{501} - (5+x)^{501}}{(5+x)^{501}} \right)}{\frac{x-5-x}{5+x}}$$

$$= \frac{x^{501} - (5+x)^{501}}{5+x}$$

$$= \frac{-5}{5+x}$$

$$= \frac{1}{5}((5+x)^{501} - x^{501})$$

Coefficient of x^{101} in $(5+x)^{501}$ is $= {}^{501}C_{101} \cdot 5^{400}$

$$\therefore \text{In } \frac{1}{5}((5+x)^{500} - x^{501}) \text{ coefficient of } x^{101} \text{ is } = \frac{1}{5} \cdot {}^{501}C_{101} \cdot 5^{400}$$

$$= {}^{501}C_{101} \cdot 5^{399}$$

Question91

If the sum of the co-efficient of all the positive even powers of x in the binomial expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta \cdot 3^9$, then β is equal to _____

[25-Jun-2022-Shift-2]

Options:

A.

Answer: 83

Solution:

Solution:

Given, Binomial Expansion

$$\left(2x^3 + \frac{3}{x}\right)^{10}$$

General term

$$T_{r+1} = {}^{10}C_r \cdot (2x^3)^{10-r} \cdot \left(\frac{3}{x}\right)^r$$

$$= {}^{10}C_r \cdot 2^{10-r} \cdot 3^r \cdot x^{30-3r} \cdot x^{-r}$$

$$= {}^{10}C_r \cdot 2^{10-r} \cdot 3^r \cdot x^{30-4r}$$

For positive even power of x, $30 - 4r$ should be even and positive.

For $r = 0$, $30 - 4 \times 0 = 30$ (even and positive)

For $r = 1$, $30 - 4 \times 1 = 26$ (even and positive)

For $r = 2$, $30 - 4 \times 2 = 22$ (even and positive)

For $r = 3$, $30 - 4 \times 3 = 18$ (even and positive)

For $r = 4$, $30 - 4 \times 4 = 14$ (even and positive)

For $r = 5$, $30 - 4 \times 5 = 10$ (even and positive)

For $r = 6$, $30 - 4 \times 6 = 6$ (even and positive)

For $r = 7$, $30 - 4 \times 7 = 2$ (even and positive)

For $r = 8$, $30 - 4 \times 8 = -2$ (even but not positive)

So, for $r = 1, 2, 3, 4, 5, 6$ and 7 we can get positive even power of x .

\therefore Sum of coefficient for positive even power of

$$x = {}^{10}C_0 \cdot 2^{10} \cdot 3^0 + {}^{10}C_{1,2} \cdot 2^9 \cdot 3^1 + {}^{10}C_2 \cdot 2^8 \cdot 3^2 + {}^{10}C_3 \cdot 2^7 \cdot 3^3 + {}^{10}C_4 \cdot 2^6 \cdot 3^4 + {}^{10}C_5 \cdot 2^5 \cdot 3^5 + {}^{10}C_6 \cdot 2^4 \cdot 3^6 + {}^{10}C_7 \cdot 2^3 \cdot 3^7$$

$$= {}^{10}C_{10} \cdot 2^{10} \cdot 3^0 + {}^{10}C_{1,2} \cdot 2^9 \cdot 3^1 + \dots + {}^{10}C_{10} \cdot 2^0 \cdot 3^{10} - [{}^{10}C_8 \cdot 2^2 \cdot 3^8 + {}^{10}C_9 \cdot 2 \cdot 3^9 + {}^{10}C_{10} \cdot 2^0 \cdot 3^{10}]$$

$$= (2+3)^{10} - [45.4.3^8 + 10.2.3^9 + 1.1.3^{10}]$$

$$= 5^{10} - [60 \times 3^9 + 20 \cdot 3^9 + 3 \cdot 3^9]$$

$$= 5^{10} - (60 + 20 + 3)3^9$$

$$= 5^{10} - 83.3^9$$

$$\therefore \beta = 83$$

Question92

The remainder when $(2021)^{2023}$ is divided by 7 is :
[26-Jun-2022-Shift-1]

Options:

A. 1

B. 2

C.

5

D. 6

Answer: C

Solution:

Solution:

$$\begin{aligned}
& (2021)^{2023} \\
&= (2016 + 5)^{2023} \text{ [here 2016 is divisible by 7]} \\
&= {}^{2023}C_0(2016)^{2023} + \dots + {}^{2023}C_{2022}(2016)(5)^{2022} + {}^{2023}C_{2023}(5)^{2023} \\
&= 2016[{}^{2023}C_0 \cdot (2016)^{2022} + \dots + {}^{2023}C_{2022} \cdot (5)^{2022}] + (5)^{2023} \\
&= 2016\lambda + (5)^{2023} \\
&= 7 \times 288\lambda + (5)^{2023} \\
&= 7K + (5)^{2023} \dots (1)
\end{aligned}$$

Now, $(5)^{2023}$

$$\begin{aligned}
&= (5)^{2022} \cdot 5 \\
&= (5^3)^{674} \cdot 5 \\
&= (125)^{674} \cdot 5 \\
&= (126 - 1)^{674} \cdot 5 \\
&= (126 - 1)^{674} \cdot 5 \\
&= 5[{}^{674}C_0(126)^{674} + \dots - {}^{674}C_{673}(126) + {}^{674}C_{674}] \\
&= 5 \times 126[{}^{674}C_0(126)^{673} + \dots - {}^{674}C_{673}] + 5 \\
&= 5 \cdot 7 \cdot 18[{}^{674}C_0(126)^{673} + \dots - {}^{674}C_{673}] + 5 \\
&= 7\lambda + 5
\end{aligned}$$

Replacing $(5)^{2023}$ in equation (1) with $7\lambda + 5$, we get,

$$(2021)^{2023} = 7K + 7\lambda + 5$$

$$= 7(K + \lambda) + 5$$

$$\therefore \text{Remainder} = 5$$

Question93

If $({}^{40}C_0) + ({}^{41}C_1) + ({}^{42}C_2) + \dots + ({}^{60}C_{20}) = \frac{m}{n} {}^{60}C_{20}$, m and n are coprime, then $m + n$ is equal to _____
[26-Jun-2022-Shift-2]

Answer: 102

Solution:

Solution:

Here property used is

$${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$$

$$\text{Given, } {}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \dots + {}^{60}C_{20} = \frac{m}{n} {}^{60}C_{20}$$

$$\text{As } {}^{40}C_0 = {}^{41}C_0 = 1$$

So, we replace ${}^{40}C_0$ with ${}^{41}C_0$.

$$\Rightarrow {}^{41}C_0 + {}^{41}C_1 + {}^{42}C_2 + \dots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

$$\Rightarrow {}^{42}C_1 + {}^{42}C_2 + \dots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

$$\Rightarrow {}^{43}C_2 + {}^{43}C_3 + \dots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

$$\Rightarrow {}^{44}C_3 + {}^{44}C_4 + \dots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

$$\Rightarrow {}^{45}C_4 + {}^{45}C_5 + \dots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

⋮

$$\Rightarrow {}^{60}C_{19} + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

$$\Rightarrow {}^{61}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$$

$$\Rightarrow \frac{61!}{20!41!} = \frac{m}{n} \cdot \frac{60!}{20!40!}$$

$$\Rightarrow \frac{61}{41} = \frac{m}{n}$$

$$\therefore m = 61 \text{ and } n = 41$$

$$\therefore m + n = 61 + 41 = 102$$

Question94

If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k \cdot 1$, where I ,

$k \in \mathbb{N}$ and I is co-prime to 5, then k is equal to
[27-Jun-2022-Shift-1]

Answer: 5

Solution:

Solution:

Given Binomial Expansion

$$= \left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$$

\therefore General term

$$\begin{aligned} T_{r+1} &= {}^{60}C_r \cdot \left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r} \cdot \left(\frac{5^{1/2}}{x^{1/3}}\right)^r \\ &= {}^{60}C_r \cdot 5^{\left(\frac{r}{4} - 15 + \frac{r}{2}\right)} \cdot x^{\left(30 - \frac{r}{2} - \frac{r}{3}\right)} \\ &= {}^{60}C_r \cdot 5^{\left(\frac{3r-60}{4}\right)} \cdot x^{\left(\frac{180-5r}{6}\right)} \end{aligned}$$

For x^{10} term,

$$\frac{180-5r}{6} = 10$$

$$\Rightarrow 5r = 120$$

$$\Rightarrow r = 24$$

$$\therefore \text{Coefficient of } x^{10} = {}^{60}C_{24} \cdot 5^{\left(\frac{3 \times 24 - 60}{4}\right)}$$

$$= {}^{60}C_{24} \cdot 5^3$$

$$= \frac{60!}{24!36!} \cdot 5^3$$

It is given that,

$$\frac{60!}{24!36!} \cdot 5^3 = 5^k \cdot 1 \dots \dots (1)$$

Also given that, I is coprime to 5 means I can't be multiple of 5. So we have to find all the factors of 5 in $60!$, $24!$ and $36!$
[Note : Formula for exponent or degree of prime number in $n!$.

$$\text{Exponent of } p \text{ in } n! = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \dots \text{ until } 0 \text{ comes}$$

here p is a prime number.]

\therefore Exponent of 5 in $60!$

$$= \left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right] + \left[\frac{60}{5^3}\right] + \dots$$

$$= 12 + 2 + 0 + \dots$$

$$= 14$$

Exponent of 5 in $24!$

$$= \left[\frac{24}{5}\right] + \left[\frac{24}{5^2}\right] + \left[\frac{24}{5^3}\right] + \dots$$

$$= 4 + 0 + 0 \dots$$

$$= 4$$

Exponent of 5 in $36!$

$$= \left[\frac{36}{5}\right] + \left[\frac{36}{5^2}\right] + \left[\frac{36}{5^3}\right] + \dots$$

$$= 7 + 1 + 0 + \dots$$

$$= 8$$

∴ From equation (1), exponent of 5 overall

$$\frac{5^{14}}{5^4 \cdot 5^8} \cdot 5^3 = 5^k$$

$$\Rightarrow 5^5 = 5^k$$

$$\Rightarrow k = 5$$

Question 95

If the sum of the coefficients of all the positive powers of x , in the Binomial expansion of $\left(x^n + \frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is _____

[27-Jun-2022-Shift-2]

Answer: 57

Solution:

Solution:

Given, Binomial expression is

$$= \left(x^n + \frac{2}{x^5}\right)^7$$

∴ General term

$$T_{r+1} = {}^7C_r \cdot (x^n)^{7-r} \cdot \left(\frac{2}{x^5}\right)^r$$

$$= {}^7C_r \cdot x^{7n-nr-5r} \cdot 2^r$$

For positive power of x ,

$$7n - nr - 5r > 0$$

$$\Rightarrow 7n > r(n+5)$$

$$\Rightarrow r < \frac{7n}{n+5}$$

As r represent term of binomial expression so r is always integer.

Given that sum of coefficient is 939.

When $r = 0$,

$$\text{sum of coefficient} = {}^7C_0 \cdot 2^0 = 1$$

when $r = 1$,

$$\text{sum of coefficient} = {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 = 1 + 14 = 15$$

when $r = 2$,

sum of coefficient

$$= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2$$

$$= 1 + 14 + 84$$

$$= 99$$

when $r = 3$,

sum of coefficient

$$= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2 + {}^7C_3 \cdot 2^3$$

$$= 1 + 14 + 84 + 280$$

$$= 379$$

when $r = 4$,

sum of coefficient

$$= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2 + {}^7C_3 \cdot 2^3 + {}^7C_4 \cdot 2^4$$

$$= 1 + 14 + 84 + 280 + 560$$

$$= 939$$

To get value of $r=4$, value of $\frac{7n}{n+5}$ should be between 4 and 5 .

$$\therefore 4 < \frac{7n}{n+5} < 5$$

$$\Rightarrow 4n + 20 < 7n < 5n + 25$$

$$\therefore 4n + 20 < 7n$$

$$\Rightarrow 3n > 20$$

$$\Rightarrow n > \frac{20}{3}$$

$$\Rightarrow n > 6.66$$

and

$$7n < 5n + 25$$

$$\Rightarrow 2n < 25$$

$$\Rightarrow n < 12.5$$

$$\therefore 6.66 < n < 12.5$$

\therefore Possible integer values of $n = 7, 8, 9, 10, 11, 12$

\therefore Sum of values of $n = 7 + 8 + 9 + 10 + 11 + 12 = 57$

Question96

If

$$\sum_{k=1}^{31} ({}^{31}C_k)({}^{31}C_{k-1}) - \sum_{k=1}^{30} ({}^{30}C_k)({}^{30}C_{k-1}) = \frac{\alpha(60!)}{(30!)(31!)}$$

where $\alpha \in \mathbb{R}$, then the value of 16α is equal to [28-Jun-2022-Shift-1]

Options:

A. 1411

B. 1320

C. 1615

D. 1855

Answer: A

Solution:

Solution:

Given,

$$\sum_{k=1}^{31} ({}^{31}C_k)({}^{31}C_{k-1}) - \sum_{k=1}^{30} ({}^{30}C_k)({}^{30}C_{k-1}) = \frac{\alpha(60!)}{(30!)(31!)}$$

Now,

$$\sum_{k=1}^{31} ({}^{31}C_k)({}^{31}C_{k-1})$$

$$= ({}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + {}^{31}C_3 \cdot {}^{31}C_2 + \dots + {}^{31}C_{31} \cdot {}^{31}C_{30})$$

$$= ({}^{31}C_0 \cdot {}^{31}C_{31-1} + {}^{31}C_1 \cdot {}^{31}C_{31-2} + \dots + {}^{31}C_{30} \cdot {}^{31}C_{31-31})$$

[using ${}^nC_r = {}^nC_{n-r}$]

$$= ({}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + \dots + {}^{31}C_{30} \cdot {}^{31}C_0)$$

$$= {}^{62}C_{30}$$

$$\text{Now, } \sum_{k=1}^{30} ({}^{30}C_k)({}^{30}C_{k-1})$$

$$= ({}^{30}C_1 \cdot {}^{30}C_0 + {}^{30}C_2 \cdot {}^{30}C_1 + \dots + {}^{30}C_{30} \cdot {}^{30}C_{29})$$

$$= ({}^{30}C_0 \cdot {}^{30}C_{29} + {}^{30}C_1 \cdot {}^{30}C_{28} + \dots + {}^{30}C_{29} \cdot {}^{30}C_0)$$

$$= {}^{60}C_{29}$$

$$\begin{aligned}
\therefore {}^{60}C_{30} - {}^{60}C_{29} &= \frac{\alpha(60!)}{30!31!} \\
\Rightarrow \frac{62.61.60!}{30!32!} - \frac{60!}{29!31!} &= \frac{\alpha(60!)}{30!31!} \\
\Rightarrow \frac{62.61.60!}{30!32!} - \frac{60!}{\frac{30!}{30} \cdot 31!} &= \frac{\alpha(60!)}{30!31!} \\
\Rightarrow \frac{60!}{30!31!} \left(\frac{62.61}{32} - 30 \right) &= \frac{\alpha(60!)}{30!31!} \\
\Rightarrow \alpha &= \frac{62.61}{32} - 30 \\
\Rightarrow 16\alpha &= \frac{62.61 - 30 \times 32}{2} \\
\Rightarrow 16\alpha &= \frac{2822}{2} = 1411
\end{aligned}$$

Question97

The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}$, $x \neq 0$ is $2^8 \cdot I$, where I is an odd integer, is ____

[28-Jun-2022-Shift-1]

Answer: 2

Solution:

Solution:

$$\left(2x^3 + \frac{3}{x^k}\right)^{12}$$

$$t_{r+1} = {}^{12}C_r (2x^3)^r \left(\frac{3}{x^k}\right)^{12-r}$$

$$x^{3r - (12-r)k} \rightarrow \text{constant}$$

$$\therefore 3r - 12k + rk = 0$$

$$\Rightarrow k = \frac{3r}{12-r}$$

\therefore possible values of r are 3, 6, 8, 9, 10 and corresponding values of k are 1, 3, 6, 9, 15

Now ${}^{12}C_r = 220, 924, 495, 220, 66$

\therefore possible values of k for which we will get 2^8 are 3, 6

Question98

The term independent of x in the expansion of $(1 - x^2 + 3x^3) \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$, $x \neq 0$ is :

[28-Jun-2022-Shift-2]

Options:

A. $\frac{7}{40}$

B. $\frac{33}{200}$

C. $\frac{39}{200}$

D. $\frac{11}{50}$

Answer: B

Solution:

Solution:

General term of Binomial expansion $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ is

$$T_{r+1} = {}^{11}C_r \cdot \left(\frac{5}{2}x^3\right)^{11-r} \cdot \left(-\frac{1}{5x^2}\right)^r$$
$$= {}^{11}C_r \cdot \left(\frac{5}{2}\right)^{11-r} \cdot \left(-\frac{1}{5}\right)^r \cdot x^{33-5r}$$

In the term,

$$(1-x^2+3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

Term independent of x is when

(1) $33 - 5r = 0$

$$\Rightarrow r = \frac{33}{5} \notin \text{integer}$$

(2) $33 - 5r = -2$

$$\Rightarrow 5r = 35$$

$$\Rightarrow r = 7 \in \text{integer}$$

(3) $33 - 5r = -3$

$$\Rightarrow 5r = 36$$

$$\Rightarrow r = \frac{36}{5} \notin \text{integer}$$

\therefore Only for $r = 7$ independent of x term possible.

\therefore Independent of x term

$$= -\left({}^{11}C_7 \left(\frac{5}{2}\right)^4 \cdot \left(-\frac{1}{5}\right)^7\right)$$

$$= -\left(\frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{5^4}{2^4} \cdot -\frac{1}{5^7}\right)$$

$$= \frac{11 \cdot 10 \cdot 3}{2^4 \cdot 5^3}$$

$$= \frac{11 \cdot 3}{2^3 \cdot 5^2}$$

$$= \frac{33}{200}$$

Question99

If the constant term in the expansion of

$\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot I$, where I is an odd integer, then the value of k is equal to:

[29-Jun-2022-Shift-1]

Options:

A. 6

- B. 7
C. 8
D. 9

Answer: D

Solution:

Solution:

Note : Multinomial Theorem :

The general term of $(x_1 + x_2 + \dots + x_n)^n$ the expansion is

$$\frac{n!}{n_1!n_2!\dots n_n!}x_1^{n_1}x_2^{n_2}\dots x_n^{n_n}$$

where $n_1 + n_2 + \dots + n_n = n$

Given,

$$\left(3x^2 - 2x^2 + \frac{5}{x^5}\right)^{10}$$

$$= \frac{(3x^8 - 2x^7 + 5)^{10}}{x^{50}}$$

Now constant term in $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10} = x^{50}$ term in $(3x^8 - 2x^7 + 5)^{10}$

General term in $(3x^8 - 2x^7 + 5)^{10}$ is

$$= \frac{10!}{n_1!n_2!n_3!}(3x^8)^{n_1}(-2x^7)^{n_2}(5)^{n_3}$$

$$= \frac{10!}{n_1!n_2!n_3!}(3)^{n_1}(-2)^{n_2}(5)^{n_3} \cdot x^{8n_1 + 7n_2}$$

∴ Coefficient of $x^{8n_1 + 7n_2}$ is

$$= \frac{10!}{n_1!n_2!n_3!}(3)^{n_1}(-2)^{n_2}(5)^{n_3}$$

where $n_1 + n_2 + n_3 = 10$

For coefficient of x^{50} :

$$8n_1 + 7n_2 = 50$$

∴ Possible values of n_1, n_2 and n_3 are

n_1	n_2	n_3
1	6	3

∴ Coefficient of x^{50}

$$= \frac{10!}{1!6!3!}(3)^1(-2)^6(5)^3$$

$$= \frac{10 \times 9 \times 8 \times 7}{6} \times 3 \times 5^3 \times 2^6$$

$$= 5 \times 3 \times 8 \times 7 \times 3 \times 5^3 \times 2^6$$

$$= 7 \times 5^4 \times 3^2 \times 2^9$$

$$= 2^k \cdot 1$$

∴ $1 = 7 \times 5^4 \times 3^2 =$ An odd integer

and $2^k = 2^9$

$$\Rightarrow k = 9$$

Question100

Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to
[29-Jun-2022-Shift-2]

Options:

A. $1 + {}^n C_2(8-5) + {}^n C_3(8^2-5^2) + \dots + {}^n C_n(8^{n-1}-5^{n-1})$

B. $1 + {}^n C_3(8-5) + {}^n C_4(8^2-5^2) + \dots + {}^n C_n(8^{n-2}-5^{n-2})$

C. ${}^n C_3(8-5) + {}^n C_4(8^2-5^2) + \dots + {}^n C_n(8^{n-2}-5^{n-2})$

D. ${}^n C_4(8-5) + {}^n C_5(8^2-5^2) + \dots + {}^n C_n(8^{n-3}-5^{n-3})$

Answer: C

Solution:

Solution:

Given,

$$9^n - 8n - 1 = 64\alpha$$

$$\Rightarrow \alpha = \frac{(1+8)^n - 8n - 1}{64}$$

$$= \frac{{}^n C_0 \cdot 1 + {}^n C_1 \cdot 8^1 + {}^n C_2 \cdot 8^2 + \dots + {}^n C_n \cdot 8^n - 8n - 1}{8^2}$$

$$= \frac{1 + 8n + {}^n C_2 \cdot 8^2 + \dots + {}^n C_n \cdot 8^n - 8n - 1}{8^2}$$

$$= \frac{{}^n C_2 \cdot 8^2 + {}^n C_3 \cdot 8^3 + \dots + {}^n C_n \cdot 8^n}{8^2}$$

$$= {}^n C_2 + {}^n C_3 \cdot 8 + {}^n C_4 \cdot 8^2 + \dots + {}^n C_n \cdot 8^{n-2}$$

Also given,

$$6^n - 5n - 1 = 25\beta$$

$$\Rightarrow \beta = \frac{(1+5)^n - 5n - 1}{25}$$

$$= \frac{{}^n C_0 \cdot 1 + {}^n C_1 \cdot 5 + {}^n C_2 \cdot 5^2 + \dots + {}^n C_n \cdot 5^n - 5n - 1}{5^2}$$

$$= \frac{1 + 5n + {}^n C_2 \cdot 5^2 + {}^n C_3 \cdot 5^3 + \dots + {}^n C_n \cdot 5^n - 5n - 1}{5^2}$$

$$= \frac{{}^n C_2 \cdot 5^2 + {}^n C_3 \cdot 5^3 + {}^n C_4 \cdot 5^4 + \dots + {}^n C_n \cdot 5^n}{5^2}$$

$$= {}^n C_2 + {}^n C_3 \cdot 5 + {}^n C_4 \cdot 5^2 + \dots + {}^n C_n \cdot 5^{n-2}$$

$\therefore \alpha - \beta$

$$= ({}^n C_2 + {}^n C_3 \cdot 8 + {}^n C_4 \cdot 8^2 + \dots + {}^n C_n \cdot 8^{n-2}) - ({}^n C_2 + {}^n C_3 \cdot 5 + {}^n C_4 \cdot 5^2 + \dots + {}^n C_n \cdot 5^{n-2})$$

$$= {}^n C_3 \cdot (8-5) + {}^n C_4 \cdot (8^2-5^2) + \dots + {}^n C_n (8^{n-2}-5^{n-2})$$

Question101

Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and

n respectively. If r is a positive integer such that $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to _____

[29-Jun-2022-Shift-2]

Answer: 5

Solution:

Solution:

$$T_{r+1} = (-1)^r \cdot {}^{15}C_r \cdot 2^{15-r} \times \frac{15-2r}{5}$$

$$m = {}^{15}C_{10} 2^5$$

$$n = -1$$

$$\text{so } mn^2 = {}^{15}C_5 2^5$$

Question 102

If the maximum value of the term independent of t in the expansion of

$$\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t} \right)^{15}, \quad x \geq 0, \text{ is } K, \text{ then } 8K \text{ is equal to}$$

[25-Jul-2022-Shift-1]

Answer: 6006

Solution:

Solution:

General term of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t} \right)^{15}$ is

$$T_{r+1} = {}^{15}C_r \cdot \left(t^2 x^{\frac{1}{5}} \right)^{15-r} \cdot \left(\frac{(1-x)^{\frac{1}{10}}}{t} \right)^r$$

$$= {}^{15}C_r \cdot t^{30-2r} \cdot x^{\frac{15-r}{5}} \cdot (1-x)^{\frac{r}{10}} \cdot t^{-r}$$

$$= {}^{15}C_r \cdot t^{30-3r} \cdot x^{\frac{15-r}{5}} \cdot (1-x)^{\frac{r}{10}}$$

Term will be independent of t when $30 - 3r = 0 \Rightarrow r = 10$

$\therefore T_{10+1} = T_{11}$ will be independent of t

$$\therefore T_{11} = {}^{15}C_{10} \cdot x^{\frac{15-10}{5}} \cdot (1-x)^{\frac{10}{10}}$$

$$= {}^{15}C_{10} \cdot x^1 \cdot (1-x)^1$$

T_{11} will be maximum when $x(1-x)$ is maximum.

$$\text{Let } f(x) = x(1-x) = x - x^2$$

$f(x)$ is maximum or minimum when $f'(x) = 0$

$$\therefore f'(x) = 1 - 2x$$

For maximum/minimum $f'(x) = 0$

$$\therefore 1 - 2x = 0$$

$$\Rightarrow x = \frac{1}{2}$$

Now, $f''(x) = -2 < 0$

\therefore At $x = \frac{1}{2}$, $f(x)$ maximum

∴ Maximum value of T_{11} is

$$= {}^{15}C_{10} \cdot \frac{1}{2} \left(1 - \frac{1}{2}\right)$$

$$= {}^{15}C_{10} \cdot \frac{1}{4}$$

$$\text{Given } K = {}^{15}C_{10} \cdot \frac{1}{4}$$

$$\text{Now, } 8K = 2({}^{15}C_{10}) \\ = 6006$$

Question103

The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is
[25-Jul-2022-Shift-2]

Options:

A. 1

B. 4

C. 6

D. 8

Answer: D

Solution:

Solution:

$$\text{Re} \left(\frac{(11)^{1011} + (1011)^{11}}{9} \right) = \text{Re} \left(\frac{2^{1011} + 3^{11}}{9} \right)$$

$$\text{For } \text{Re} \left(\frac{2^{1011}}{9} \right)$$

$$2^{1011} = (9-1)^{337} = {}^{337}C_0 9^{337} (-1)^0 + {}^{337}C_1 9^{336} (-1)^1 + {}^{337}C_2 9^{335} (-1)^2 + \dots + {}^{337}C_{337} 9^0 (-1)^{337}$$

So, remainder is 8

$$\text{and } \text{Re} \left(\frac{3^{11}}{9} \right) = 0$$

So, remainder is 8

Question104

If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q$, $p, q \leq 15$, are -3 and -5 respectively, then the coefficient of x^3 is equal to _____.
[26-Jul-2022-Shift-1]

Answer: 23

Solution:

Solution:

Coefficient of x in $(1+x)^p(1-x)^q$

$${}^p C_0 {}^q C_1 + {}^p C_1 {}^q C_0 = -3 \Rightarrow p - q = -3$$

Coefficient of x^2 in $(1+x)^p(1-x)^q$

$${}^p C_0 {}^q C_2 - {}^p C_1 {}^q C_1 + {}^p C_2 {}^q C_0 = -5$$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -5$$

$$\frac{q^2 - q}{2} - (q-3)q + \frac{(q-3)(q-4)}{2} = -5$$

$$\Rightarrow q = 11, p = 8$$

Coefficient of x^3 in $(1+x)^8(1-x)^{11}$

$$= -{}^{11}C_3 + {}^8C_1 {}^{11}C_2 - {}^8C_2 {}^{11}C_1 + {}^8C_3 = 23$$

Question105

$\sum_{\substack{i,j=0 \\ i \neq j}}^n {}^n C_i {}^n C_j$ is equal to

[26-Jul-2022-Shift-2]

Options:

A. $2^{2n} - 2^n C_n$

B. $2^{2n-1} - 2^{n-1} C_{n-1}$

C. $2^{2n} - \frac{1}{2} 2^n C_n$

D. $2^{2n-1} + 2n - 1 C_n$

Answer: B

Solution:

Solution:

$$\sum_{i,j=0, i \neq j}^n {}^n C_i {}^n C_j = \sum_{i,j=0}^n {}^n C_i {}^n C_j - \sum_{i=j}^n {}^n C_i {}^n C_j$$

$$= \sum_{j=0}^n {}^n C_j \sum_{i=0}^n {}^n C_i - \sum_{i=0}^n {}^n C_i C_i$$

$$= 2^n \cdot 2^n - 2^n C_n$$

$$= 2^{2n} - 2^n C_n$$

Question106

The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is
[27-Jul-2022-Shift-1]

Options:

A. 0

B. 1

C. 2

D. 6

Answer: A

Solution:

Solution:

$$\begin{aligned} & (2021)^{2022} + (2022)^{2021} \\ &= (7k-2)^{2022} + (7k_1-1)^{2021} \\ &= [(7k-2)^3]^{674} + (7k_1)^{2021} - 2021(7k_1)^{2020} + \dots - 1 \\ &= (7k_2-1)^{674} + (7m-1) \\ &= (7n+1) + (7m-1) = 7(m+n) \text{ (multiple of 7)} \\ \therefore \text{Remainder} &= 0 \end{aligned}$$

Question107

Let for the 9th term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0 . If k is the ratio of the coefficient of x^6 to the coefficient of x^3 , then $k + n_0$ is equal to:

[27-Jul-2022-Shift-2]

Answer: 24

Solution:

Solution:

$$(3 + 6x)^n = 3^n(1 + 2x)^n$$

If T_9 is numerically greatest term

$$\therefore T_8 \leq T_9 \leq T_{10}$$

$${}^n C_7 3^{n-7} (6x)^7 \leq {}^n C_8 3^{n-8} (6x)^8 \geq {}^n C_9 3^{n-9} (6x)^9$$

$$\Rightarrow \frac{n!}{(n-7)!7!} 9 \leq \frac{n!}{(n-8)!8!} 3 \cdot (6x) \geq \frac{n!}{(n-9)!9!} (6x)^2$$

$$\Rightarrow \frac{9}{(n-7)(n-8)} \leq \frac{18\left(\frac{3}{2}\right)}{(n-8)8} \geq \frac{36}{9 \cdot 8} \frac{9}{4}$$

$$72 \leq 27(\tilde{n}-7) \text{ and } 27 \geq 9(\tilde{n}-8)$$

$$\frac{29}{3} \leq n \text{ and } n \leq 11$$

$$\therefore n_0 = 10$$

For $(3 + 6x)^{10}$

$$T_{r+1} = {}^{10}C_r$$

$$3^{10-r} (6x)^r$$

For coeff. of x^6

$$r = 6 \Rightarrow {}^{10}C_6 3^4 \cdot 6^6$$

For coeff. of x^3

$$r = 3 \Rightarrow {}^{10}C_3 3^7 \cdot 6^3$$

$$\therefore k = \frac{{}^{10}C_6 \cdot 3^4 \cdot 6^6}{{}^{10}C_3 \cdot 3^7 \cdot 6^3} = \frac{10!7!3!}{6!4!10!} \cdot 8$$

$$\Rightarrow k = 14$$

$$\therefore k + n_0 = 24$$

Question108

The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is :
[28-Jul-2022-Shift-1]

Options:

A. 0

B. 2

C. 3

D. 4

Answer: C

Solution:

Solution:

$$7^{2022} + 3^{2022}$$

$$= (49)^{1011} + (9)^{1011}$$

$$= (50 - 1)^{1011} + (10 - 1)^{1011}$$

$$= 5\lambda - 1 + 5k - 1$$

$$= 5m - 2$$

$$\text{Remainder} = 5 - 2 = 3$$

Question109

Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}} + \beta x\right)^4$, $(1 - 3\beta x)^2$ and $\left(1 - \frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P. , then $50 - \frac{2d}{\beta^2}$ is equal to _____.

[28-Jul-2022-Shift-2]

Answer: 57

Solution:

Solution:

Coefficients of middle terms of given expansions are ${}^4C_2 \frac{1}{6}\beta^2$, ${}^2C_1(-3\beta)$, ${}^6C_3\left(\frac{-\beta}{2}\right)^3$ form an A.P.

$$\therefore 2 \cdot 2(-3\beta) = \beta^2 - \frac{5\beta^3}{2}$$

$$\Rightarrow -24 = 2\beta - 5\beta^2$$

$$\Rightarrow 5\beta^2 - 2\beta - 24 = 0$$

$$\Rightarrow 5\beta^2 - 12\beta + 10\beta - 24 = 0$$

$$\Rightarrow \beta(5\beta - 12) + 2(5\beta - 12) = 0$$

$$\beta = \frac{12}{5}$$

$$d = -6\beta - \beta^2$$

$$\therefore 50 - \frac{2d}{\beta^2} = 50 - 2 \frac{(-6\beta - \beta^2)}{\beta^2} = 50 + \frac{12}{\beta} + 2 = 57$$

Question110

If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \dots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \dots + {}^{50}C_{50})$ is equal to $2^n \cdot m$, where m is odd, then $n + m$ is equal to _____.

[28-Jul-2022-Shift-2]

Answer: 99

Solution:

Solution:

$$1 = 1 + (1 + {}^{49}C_0 + {}^{49}C_1 + \dots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \dots + {}^{50}C_{50})$$

$$\text{As } {}^{49}C_0 + {}^{49}C_1 + \dots + {}^{49}C_{49} = 2^{49}$$

$$\text{and } {}^{50}C_0 + {}^{50}C_2 + \dots + {}^{50}C_{50} = 2^{49}$$

$$\Rightarrow {}^{50}C_2 + {}^{50}C_4 + \dots + {}^{50}C_{50} = 2^{49} - 1$$

$$\therefore 1 = 1 + (2^{49} + 1)(2^{49} - 1)$$

$$= 2^{98}$$

$$\therefore m = 1 \text{ and } n = 98$$

$$m + n = 99$$

Question111

Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(4\sqrt{2} + \frac{1}{4\sqrt{3}}\right)^n$, in the increasing powers of $\frac{1}{4\sqrt{3}}$ be $4\sqrt{6} : 1$. If the sixth term from the beginning is $\frac{\alpha}{4\sqrt{3}}$, then α is equal to _____.

[29-Jul-2022-Shift-1]

Answer: 84

Solution:

Solution:

$$\text{Fifth term from beginning} = {}^n C_4 \left(2 \frac{1}{4}\right)^{n-4} \left(3 \frac{-1}{4}\right)^4$$

$$\text{Fifth term from end} = (n-5+1)^{\text{th}} \text{ term from begin} = {}^n C_{n-4} \left(2 \frac{1}{4}\right)^3 \left(3 \frac{-1}{4}\right)^{n-4}$$

$$\text{Given } \frac{{}^n C_4 2^{\frac{n-4}{4}} \cdot 3^{-1}}{{}^n C_{n-3} 2^{\frac{4}{4}} \cdot 3^{-\left(\frac{n-4}{4}\right)}} = 6 \frac{1}{4}$$

$$\Rightarrow 6 \frac{n-8}{4} = 6 \frac{1}{4}$$

$$\Rightarrow \frac{n-8}{4} = \frac{1}{4} \Rightarrow n = 9$$

$$T_6 = T_{5+1} = {}^9 C_5 \left(2 \frac{1}{4}\right)^4 \left(3 \frac{-1}{4}\right)^5$$

$$= \frac{{}^9 C_5 \cdot 2}{3^{\frac{4}{4}} \cdot 3} = \frac{84}{3^{\frac{4}{4}}} = \frac{\alpha}{3^{\frac{4}{4}}}$$

$$\Rightarrow \alpha = 84$$

Question112

If $\sum_{k=1}^{10} k^2 ({}^{10} C_k)^2 = 22000L$, then L is equal to _____.

[29-Jul-2022-Shift-2]

Answer: 221

Solution:

Solution:

Given,

$$\sum_{k=1}^{10} k^2 ({}^{10} C_k)^2 = 2200L$$

$$\Rightarrow \sum_{k=1}^{10} (k \cdot {}^{10} C_k)^2 = 22000L$$

$$\Rightarrow \sum_{k=1}^{10} \left(k \cdot \frac{10}{k} \cdot {}^9 C_{k-1}\right)^2 = 22000L$$

$$\Rightarrow \sum_{k=1}^{10} (10 \cdot {}^9 C_{k-1})^2 = 22000L$$

$$\Rightarrow 100 \cdot \sum_{k=1}^{10} ({}^9 C_{k-1})^2 = 22000L$$

$$\Rightarrow 100({}^9 C_0)^2 + ({}^9 C_1)^2 + \dots + ({}^9 C_9)^2 = 22000L$$

$$\Rightarrow 100({}^{18} C_9) = 22000L$$

$$[\text{Note : } ({}^n C_1)^2 + ({}^n C_2)^2 + \dots + ({}^n C_n)^2 = {}^{2n} C_n]$$

$$\Rightarrow 100 \times \frac{18!}{9!9!} = 22000L$$

$$\Rightarrow L = 221$$

Question113

If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$ is

[2021, 24 Feb. Shift-II]

Options:

A. $\frac{n(n-1)(2n+1)}{6}$

B. $\frac{n(n+1)(2n+1)}{6}$

C. $\frac{n(2n+1)(3n+1)}{6}$

D. $\frac{n(n+1)^2(n+2)}{12}$

Answer: B

Solution:

Solution:

Given, $n \geq 2$

$$\text{Let } S = {}^2C_2 + {}^3C_2 + \dots + {}^nC_2 = {}^{n+1}C_3$$

$$\text{Now, } {}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + \dots + {}^nC_2)$$

$$= {}^{n+1}C_2 + 2 \times {}^{n+1}C_3$$

$$= ({}^{n+1}C_2 + {}^{n+1}C_3) + {}^{n+1}C_3$$

$$= {}^{n+2}C_3 + {}^{n+1}C_3 = \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)n(n-1)!}{3 \times 2 \times 1 \times (n-1)!}$$

$$+ \frac{(n+1) \times n \times (n-1) \times (n-2)!}{3 \times 2 \times 1 \times (n-2)!}$$

$$= \frac{n(n+1)}{6} [n+2+n-1]$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Question114

For integers n and r , let $\binom{n}{r} = \begin{cases} {}^nC_r & \text{if } n \geq r \geq 0 \\ 0 & \text{otherwise} \end{cases}$ The maximum value of k for which

the sum,

$$\sum_{i=0}^k \binom{10}{i} \binom{5}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i}$$

exists, is equal to

[2021, 24 Feb Shift-II]

Answer: 1

Solution:

Solution:

$$\text{Given, } \binom{n}{r} = \begin{cases} {}^n C_r & \text{if } n \geq r \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{and } \sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i}$$

$$\therefore (1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \dots$$

$$+ {}^{10}C_{10}x^{10}$$

$$\text{and } (1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + {}^{15}C_2x^2$$

$$+ \dots + {}^{15}C_{15}x^{15}$$

$$\therefore \sum_{i=0}^k ({}^{10}C_i)({}^{15}C_{k-i}) = {}^{10}C_0 \cdot {}^{15}C_k$$

$$+ {}^{15}C_1 \cdot {}^{15}C_{k-1} + \dots + {}^{10}C_k \cdot {}^{15}C_0$$

$$\Rightarrow \text{Coefficient of } x^k \text{ in } (1+x)^{25} = {}^{25}C_k$$

$$\text{Also, } \sum_{i=0}^{k+1} ({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0 \cdot {}^{13}C_{k+1}$$

$$+ {}^{12}C_1 \cdot {}^{13}C_k + \dots + {}^{12}C_{k+1} \cdot {}^{13}C_0$$

$$\Rightarrow \text{Coefficient of } x^{k+1} \text{ in } (1+x)^{25} = {}^{25}C_{k+1}$$

$$\Rightarrow {}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$$

As, we know by the definition of ${}^n C_r$, the maximum value of ${}^{26}C_{k+1}$ is possible for any possible large value of k .

Hence, k can have any large value.

Question115

The value of $-{}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \dots - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5$
 $+ \dots + {}^{14}C_{11}$ is

[2021, 24 Feb. Shift-I]

Options:

A. $2^{16} - 1$

B. $2^{13} - 14$

C. $2^{13} - 13$

D. 2^{14}

Answer: B

Solution:

Solution:

Given,

$$(-^{15}C_1 + 2 \cdot ^{15}C_2 - 3 \cdot ^{15}C_3 + \dots - 15 \cdot ^{15}C_{15}) + (^{14}C_1 + ^{14}C_3 + \dots + ^{14}C_{11})$$

Let

$$S_1 = -^{15}C_1 + 2 \cdot ^{15}C_2 - 3 \cdot ^{15}C_3 + \dots - 15 \cdot ^{15}C_{15}$$

$$= \sum_{r=1}^{15} (-1)^r \cdot r \cdot ^{15}C_r = 15 \sum_{r=1}^{15} (-1)^r \cdot ^{14}C_{r-1}$$

$$= 15(-^{14}C_0 + ^{14}C_1 - ^{14}C_2 + \dots - ^{14}C_{14})$$

$$= 15(0) = 0$$

$$S_2 = ^{14}C_1 + ^{14}C_3 + ^{14}C_5 + \dots + ^{14}C_{11}$$

$$= (^{14}C_1 + ^{14}C_3 + \dots + ^{14}C_{13}) - ^{14}C_{13}$$

$$= 2^{13} - 14$$

Now, the required value is

$$(-^{15}C_1 + 2 \cdot ^{15}C_2 - 3 \cdot ^{15}C_3 + \dots - 15 \cdot ^{15}C_{15})$$

$$+ (^{14}C_1 + ^{14}C_3 + \dots + ^{14}C_{11})$$

$$= S_1 + S_2$$

$$= 0 + 2^{13} - 14$$

$$= 2^{13} - 14$$

Question116

If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is
[2021, 25 Feb. Shift-II]

Answer: 1

Solution:

Solution:

Given, when x is divided by 4, the remainder is 3.

Let $x = 4p + 3$, then

$$(2020 + x)^{2022} = (2020 + 4p + 3)^{2022}$$

$$= (2024 + 4p - 1)^{2022}$$

$$= (4k - 1)^{2022}$$

$$(\because 2024 \text{ is divisible by } 4) = ^{2022}C_0(4K)^{2022}(-1)^0 + ^{2022}C_1(4K)^{2021}(-1)^1 + \dots + ^{2022}C_{2022}(4A)^0(-1)^{2022}$$

On expansion $(2020 + x)^{2022}$, we get the form of $8\lambda + 1$. Since, each terms have 2022 and $4k_1$ so if we take 2 common from 2022 we get 8. Thus, each term have 8 in common.

Hence, remainder is 1.

Question117

Let $m, n \in \mathbb{N}$ and $\gcd(2, n) = 1$. If

$$30 \binom{30}{0} + 29 \binom{30}{28} + \dots + 2 \binom{30}{28} + 1 \binom{30}{29} = n \cdot 2^m, \text{ then } n + m \text{ is } \dots \text{ (Here,}$$

$$\binom{n}{k} = {}^n C_k$$

[2021, 26 Feb. Shift-I]

Answer: 45

Solution:

Solution:

Given,

$$30 \cdot {}^{30}C_0 + 29 \cdot {}^{30}C_1 + \dots + 2 \cdot {}^{30}C_{28} + {}^{30}C_{29}$$

$$= n \cdot 2^m$$

This can be written as,

$$\sum_{r=0}^{29} (30-r) {}^{30}C_r = n \cdot 2^m$$

$$\text{or } \sum_{r=0}^{30} (30-r) \cdot {}^{30}C_r = n \cdot 2^m$$

$$\Rightarrow \sum_{r=0}^{30} 30 \cdot {}^{30}C_r - \sum_{r=0}^{30} r \cdot {}^{30}C_r = n \cdot 2^m$$

$$\Rightarrow 30 \sum_{r=0}^{30} {}^{30}C_r - \sum_{r=0}^{30} r \cdot {}^{30}C_r = n \cdot 2^m \text{ Using combination properties,}$$

$$\Rightarrow 30 \cdot (2)^{30} - 30 \cdot (2)^{29} = n \cdot 2^m$$

$$\Rightarrow 30 \cdot (2)^{29} (2-1) = n \cdot 2^m$$

$$\Rightarrow 2 \cdot 15 \cdot (2)^{29} = n \cdot 2^m$$

$$\Rightarrow 15 \cdot (2)^{30} = n \cdot 2^m \text{ Comparing both sides,}$$

$$n = 15 \text{ and } m = 30$$

$$\Rightarrow n + m = 15 + 30 = 45$$

Question 118

The maximum value of the term independent of 't' in the expansion of

$$\left(tx^{1/5} + \frac{(1-x)^{1/10}}{t} \right)^{10}, \text{ where } x \in (0, 1) \text{ is}$$

[2021, 26 Feb. Shift-I]

Options:

A. $\frac{10!}{\sqrt{3}(5!)^2}$

B. $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$

C. $\frac{2 \cdot 10!}{3(5!)^2}$

D. $\frac{10!}{3(5!)^2}$

Answer: B

Solution:

Solution:

Using Binomial expansion, its $(r + 1)$ th term be,

$$T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r} \left\{ \frac{(1-x)^{1/10}}{t} \right\}^r$$
$$= {}^{10}C_r \frac{(t)^{10-r}}{(t)^r} (x^{1/5})^{10-r} (1-x)^{r/10}$$

$$= {}^{10}C_r (t)^{10-2r} (x)^{\frac{10-r}{5}} (1-x)^{r/10}$$

If this term is independent of 't', then we have $10 - 2r = 0$ gives, $r = 5$

$$\therefore T_6 = {}^{10}C_5 (x)^1 (1-x)^{1/2}$$

Let $f(x) = x(1-x)^{1/2}$, to obtain its maximum value, we have to differentiate it and equate it to 0.

$$\text{i.e. } f'(x) = 0 \Rightarrow \frac{x}{2\sqrt{1-x}}(-1) + \sqrt{1-x} = 0$$

$$\Rightarrow -x + 2(1-x) = 0$$

$$\Rightarrow -3x + 2 = 0$$

$$\Rightarrow x = 2/3$$

(Maximum value)

Thus, greatest term will be

$$T_6 = {}^{10}C_5 \left(\frac{2}{3}\right) \left(1 - \frac{2}{3}\right)^{1/2}$$
$$= {}^{10}C_5 \frac{2}{3\sqrt{3}} = \frac{10! \cdot 2}{(5!)^2 (3\sqrt{3})}$$

Question 119

The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x-x^{1/2}} \right]^{10}$, $x \neq 1$, is equal

to

[2021, 18 March Shift-II]

Answer: 210

Solution:

Solution:

$$\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x-x^{1/2}} \right]^{10}$$
$$= \left[\frac{(x^{1/3})^3 + (1)^3}{x^{2/3} - x^{1/3} + 1} - \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)} \right]^{10}$$
$$= \left[(x^{1/3} + 1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}} \right) \right]^{10}$$

[use $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$]

$$= (x^{1/3} - x^{-1/2})^{10}$$

General term, $T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{1/2})^r$

$$= {}^{10}C_r (x)^{\frac{10-r}{3}} \cdot (-x)^{-\frac{r}{2}}$$

For term independent of x , we must put

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$$\Rightarrow 20 - 2r - 3r = 0$$

$$\Rightarrow r = 4$$

\therefore

$$T_{4+1} = T_5 = {}^{10}C_4 = \frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{4 \times 3 \times 2 \times 1 \times 6!} = 210$$

Question 120

Let nC_r denote the binomial coefficient of x^r in the expansion of $(1+x)^n$.

If $\sum_{k=0}^{10} (2^2 + 3k) {}^nC_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$

$\alpha, \beta \in \mathbf{R}$, then $\alpha + \beta$ is equal to
[2021, 18 March Shift-II]

Answer: 19

Solution:

Solution:

$$\begin{aligned} \sum_{k=0}^{10} (2^2 + 3k) \cdot {}^nC_k &= 4 \sum_{k=0}^{10} {}^nC_k + 3 \cdot \sum_{k=0}^{10} k \cdot {}^nC_k \\ &= 4 \times 2^n + 3 \times n \sum_{k=1}^{n-1} C_{k-1} \\ &= 4 \times 2^n + 3n \times 2^{n-1} \left[\because {}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1} \right] \\ &= 2^n \left(4 + \frac{3n}{2} \right) \\ &= \left(4 + \frac{3n}{2} \right) \cdot 2^n + 0 \times 3^n \sum_{k=0}^{10} (2^2 + 3k) \cdot {}^nC_k = 4 \sum_{k=0}^{10} {}^nC_k + 3 \cdot \sum_{k=0}^{10} k \cdot {}^nC_k \\ &= 4 \times 2^n + 3 \times n \sum_{k=1}^{n-1} C_{k-1} \\ &= 4 \times 2^n + 3n \times 2^{n-1} \left[\because {}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1} \right] \\ &= 2^n \left(4 + \frac{3n}{2} \right) \\ &= \left(4 + \frac{3n}{2} \right) \cdot 2^n + 0 \times 3^n \end{aligned}$$

On comparing, $\left[0 \times 3^n + \left(4 + \frac{3n}{2} \right) \cdot 2^n \right] + 0$

$[\alpha \cdot 3^{10} + \beta \cdot 2^{10}]$,

we get $n = 10, \alpha = 0, \beta = 19$

$\therefore \alpha + \beta = 0 + 19 = 19$

Question 121

If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x , where $x \in \mathbf{N}$ is equal to

[2021, 17 March Shift-I]

Options:

- A. 2
- B. 4
- C. 3
- D. 1

Answer: A

Solution:

Solution:

$$(x + x^{\log_2 x})^7$$

$$T_4 = 4480$$

$$T_r = {}^nC_r (x^{\log_2 x})^r x^{(n-r)}$$

$$T_4 = {}^7C_4 (x^{\log_2 x})^4 x^3 = 35x^{4\log_2 x} x^3$$

$$T_4 = 4480$$

$$35x^{3+4\log_2 x} = 4480$$

$$x^{3+4\log_2 x} = 128 = 2^7$$

Taking log on both sides,

$$\log_2 x^{3+4\log_2 x} + 4\log_2 x = \log_2 2^7$$

$$\Rightarrow (3 + 4\log_2 x)(\log_2 x) = 7$$

$$\Rightarrow 4(\log_2 x)^2 + 3\log_2 x - 7 = 0$$

$$\Rightarrow (\log_2 x - 1)(4\log_2 x + 7) = 0$$

$$\Rightarrow \log_2 x = 1 \quad (\because x \in \mathbb{N})$$

$$\therefore x = 2$$

Question 122

If $(2021)^{3762}$ is divided by 17, then the remainder is
[2021, 17 March Shift-I]

Answer: 4

Solution:

Solution:

$$(2021)^{3762}$$

$$2021 = (17 \times 119 - 2) \Rightarrow (17\lambda - 2)$$

$$(2021)^{3762} = (17\lambda - 2)^{3762} = C_0(17\lambda)^{3762}$$

$$-C_1(17\lambda)^{3761}2^1 + \dots - C_n 2^{3762}$$

Now, $(2021)^{3762}$ will be divisible by 17 all the terms except the last one for last one.

$$\therefore (2021)^{3762} = 17\mu - 2^{3762}$$

$$= 17\mu - 2^2(2^{3760})$$

$$= 17\mu - 4(16)^{235}$$

$$= 17\mu - 4 \cdot (17 - 1)^{235}$$

$$(17 - 1)^{235} = (-1)(1 - 17)^{235}$$

$$= -(C_0 - C_1 17 + C_2 17^2 - \dots)$$

$$\begin{aligned}
 &= -C_0 + 17\gamma = -1 + 17\gamma \\
 17\mu - 4(17-1)^{235} &= 17\mu - 4[-1 + 17\gamma] \\
 &= 17(\mu - 4\gamma) + 4 \\
 \therefore (2021)^{3762} &= 17k + 4 \\
 \text{Hence, 4 is the remainder.}
 \end{aligned}$$

Question 123

Let the coefficients of third, fourth and fifth terms in the expansion of

$\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then, the term independent of x in the expansion, is equal to
[2021, 17 March Shift-II]

Answer: 4

Solution:

Solution:

$$\begin{aligned}
 \text{General term, } T_{r+1} &= {}^nC_r \cdot x^{n-r} \cdot \left(\frac{a}{x^2}\right)^r \\
 &= {}^nC_r \cdot a^r \cdot x^{n-3r} \\
 \therefore T_3 &= {}^nC_2 \cdot a^2 \cdot x^{n-6} \\
 T_4 &= {}^nC_3 a^3 \cdot x^{n-9} \\
 T_5 &= {}^nC_4 \cdot a^4 \cdot x^{n-12}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } \frac{\text{coefficient of } T_3}{\text{coefficient of } T_4} &= \frac{{}^nC_2 \cdot a^2}{{}^nC_3 \cdot a^3} \\
 &= \frac{3}{a(n-2)} = \frac{3}{2} \\
 \Rightarrow a(n-2) &= 2 \dots\dots\dots (i)
 \end{aligned}$$

$$\begin{aligned}
 \text{Also, } \frac{\text{coefficient of } T_4}{\text{coefficient of } T_5} &= \frac{{}^nC_3 \cdot a^2}{{}^nC_4 \cdot a^3} \\
 &= \frac{4}{a(n-3)} = \frac{8}{3} \\
 \Rightarrow a(n-3) &= \frac{3}{2} \dots\dots\dots (ii)
 \end{aligned}$$

$$\text{From Eqs. (i) and (ii), } n = 6, a = \frac{1}{2}$$

For the term independent of 'x'

$$\begin{aligned}
 n - 3r &= 0 \\
 \Rightarrow r &= \frac{n}{3} \\
 \Rightarrow r &= \frac{6}{3} \\
 \Rightarrow r &= 2
 \end{aligned}$$

\therefore Independent term is T_3 .

$$\begin{aligned}
 \text{Now, } T_3 &= {}^6C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot (x)^0 \\
 &= \frac{15}{4} = 3.75 \approx 4
 \end{aligned}$$

Question124

If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then $(n - 1)$ is divisible by
[2021, 16 Mar Shift-I]

Options:

- A. 26
- B. 30
- C. 8
- D. 7

Answer: A

Solution:

Solution:

$$(3^{1/4} + 5^{1/8})^{60}$$

By using Binomial expansion, $(r + 1)$ th term,

$$T_{r+1} = {}^{60}C_r (3^{1/4})^r (5^{1/8})^{60-r}$$

$$= {}^{60}C_r 3^{r/4} 5^{(60-r)/8}$$

For this term to be a rational number, r should be a multiple of 4 and $(60 - r)$ should be a multiple of 8.

Let A be a set when r is the multiple of 4.

$$A = \{4, 8, 12, \dots, 56, 60\}$$

$$n(A) = 15$$

Let B be a set of r , when $(60 - r)$ is the multiple of 8.

$$B = \{4, 12, 20, 28, 36, 44, 52, 60\}$$

$$n(B) = 8$$

$$\text{Now, } n(A \cap B) = 8$$

So, there are only 8 terms out of 61 terms which will be rational numbers.

53 terms will be irrational.

$$\text{So, } n = 53$$

and $n - 1 = 52$ which is divisible only by 26 among the given options.

Question125

Let $[x]$ denote greatest integer less than or equal to x . If for $n \in \mathbb{N}$,

$$(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j, \text{ then } \sum_{j=0}^{\lfloor \frac{3n}{2} \rfloor} a_{2j} + 4 \sum_{j=0}^{\lfloor \frac{3n-1}{2} \rfloor} a_{2j} + 1 \text{ is equal to}$$

[2021, 16 March Shift-I]

Options:

- A. 2
- B. 2^{n-1}
- C. 1

D. n

Answer: C

Solution:

Solution:

$$\text{Given, } (1 - x + x^3)^n = \sum_{j=0}^{3n} a_j \cdot x^j$$

$$(1 - x + x^3)^n = a_0 + a_1x + a_2x^2 + \dots + a_{3n}x^{3n}$$

Putting $x = 1$,

$$(1 - 1 + 1)^n = a_0 + a_1 + a_2 + \dots + a_{3n}$$

$$1 = a_0 + a_1 + a_2 + \dots + a_{3n} \dots \text{ (i)}$$

Putting $x = -1$,

$$(1 + 1 - 1)^n = a_0 - a_1 + a_2 - a_3 + \dots + (-1)^{3n} a_{3n}$$

$$1 = a_0 - a_1 + a_2 - a_3 + \dots + (-1)^{3n} a_{3n} \dots \text{ (ii)}$$

Adding Eqs. (i) and (ii), we get

$$2 = 2(a_0 + a_2 + a_4 + a_6 \dots)$$

$$a_0 + a_2 + a_4 + \dots = 1$$

On subtracting Eq. (ii) from Eq. (i), we get

$$0 = 2(a_1 + a_3 + a_5 + \dots)$$

$$a_1 + a_3 + a_5 + \dots = 0$$

$$\left[\frac{3n}{2} \right] \left[\frac{3n-1}{2} \right]$$

$$\begin{aligned} \text{Now, } \sum_{j=0}^{3n} a_{2j} + 4 \sum_{j=0}^{3n} a_{2j} + 1 &= [a_0 + a_2 + a_4 + \dots] + 4 \\ &= 1 + 4 \times 0 \\ &= 1 \end{aligned}$$

Question126

The value of $\sum_{r=0}^6 ({}^6C_r \cdot {}^6C_{6-r})$ is equal to
[2021, 17 March Shift-II]

Options:

A. 1124

B. 1324

C. 1024

D. 924

Answer: D

Solution:

Solution:

Method (1) (Proper Method)

$$\begin{aligned} \sum_{r=0}^6 {}^6C_r \cdot {}^6C_{6-r} \\ = {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + {}^6C_2 \cdot {}^6C_4 + \dots \end{aligned}$$

$${}^6C_3 \cdot {}^6C_3 + {}^6C_4 \cdot {}^6C_2 + {}^6C_5 \cdot {}^6C_1 + {}^6C_6 \cdot {}^6C_0$$

Now,

$$(1+x)^6 \cdot (1+x)^6 = ({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + \dots + {}^6C_6x^6)({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + \dots + {}^6C_6x^6)$$

On comparing the coefficients of x^6 from both sides, we have

$$\begin{aligned} & {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + {}^6C_2 \cdot {}^6C_4 + \dots + \\ & = \frac{{}^6C_6 \cdot {}^6C_0 = {}^{12}C_6}{6!(12-6)!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6! \times (6 \times 5 \times 4 \times 3 \times 2 \times 1)} \\ & = 924 \end{aligned}$$

Method (2) (Short-cut Method)

As, we know that,

$$r {}^nC_0^2 + {}^nC_1^2 + {}^nC_2^2 + {}^nC_3^2 + \dots + {}^nC_n^2 = 2^n {}^nC_n$$

$$\Rightarrow {}^nC_0 \cdot {}^nC_0 + {}^nC_1 \cdot {}^nC_1 + {}^nC_2 \cdot {}^nC_2 + \dots +$$

$${}^nC_n \cdot {}^nC_n = 2^n {}^nC_n$$

$$\Rightarrow {}^nC_0 \cdot {}^nC_n + {}^nC_1 \cdot {}^nC_{n-1} + {}^nC_2 \cdot {}^nC_{n-2}$$

$$+ \dots + {}^nC_n \cdot {}^nC_0 = 2^n {}^nC_n (\because {}^nC_r = {}^nC_{n-r})$$

Putting $n = 6$, we get,

$${}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + {}^6C_2 \cdot {}^6C_4 + \dots +$$

$${}^6C_6 \cdot {}^6C_0 = {}^{12}C_6$$

Question 127

Let n be a positive integer. Let

$$A = \sum_{k=0}^n (-1)^k {}^nC_k \left[\begin{array}{l} \left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k \\ + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \end{array} \right]$$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to

[2021, 16 March Shift-II]

Answer: 6

Solution:

Solution:

$$\text{Given, } A = \sum_{k=0}^n (-1)^k \cdot {}^nC_k$$

$$\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$$

$$\Rightarrow A = \sum_{k=0}^n (-1)^k$$

$$\left[\begin{array}{l} {}^nC_k \left(\frac{1}{2}\right)^k + {}^nC_k \left(\frac{3}{4}\right)^k + {}^nC_k \left(\frac{7}{8}\right)^k \\ + {}^nC_k \left(\frac{15}{16}\right)^k + {}^nC_k \left(\frac{31}{32}\right)^k \end{array} \right]$$

$$\Rightarrow (1-x)^n = \sum_{r=0}^n (-1)^r \cdot C_r x^r$$

$$x = \frac{1}{2}$$

$$\Rightarrow \left(1 - \frac{1}{2}\right)^n = C_0 - C_1 \left(\frac{1}{2}\right) + C_2 \left(\frac{1}{2}\right)^2 \dots$$

$$\Rightarrow x = \frac{3}{4}$$

$$\left(1 - \frac{3}{4}\right)^n = C_0 - C_1 \left(\frac{3}{4}\right) + C_2 \left(\frac{3}{4}\right)^2 \dots$$

Similarly, we will get

$$A = \sum_{k=0}^n (-1)^k \left[\begin{aligned} & {}^n C_k \left(\frac{1}{2}\right)^k + {}^n C_k \left(\frac{3}{4}\right)^k + {}^n C_k \left(\frac{7}{8}\right)^k \\ & + {}^n C_k \left(\frac{15}{16}\right)^k + {}^n C_k \left(\frac{31}{32}\right)^k \end{aligned} \right]$$

$$\Rightarrow A = \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n$$

$$+ \left(1 - \frac{15}{16}\right)^n + \left(1 - \frac{31}{32}\right)^n$$

$$\Rightarrow A = \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \frac{1}{2^{4n}} + \frac{1}{2^{5n}} \Rightarrow A = \frac{1}{2^n} \cdot \left[\frac{1 - \frac{1}{(2^n)^5}}{1 - \frac{1}{2^n}} \right]$$

$$\Rightarrow A = \left(\frac{1}{2^n}\right) \left(\frac{2^{5n}-1}{2^n-1}\right) \left(\frac{2^n}{2^{5n}}\right)$$

$$\Rightarrow A = \left(\frac{2^{5n}-1}{2^n-1}\right) \left(\frac{1}{2^{5n}}\right)$$

$$\therefore 63A = 1 - \frac{1}{2^{30}} \Rightarrow 63 \frac{(2^{5n}-1)}{2^{5n}(2^n-1)} = \frac{2^{30}-1}{2^{30}}$$

$$\Rightarrow \left(\frac{63}{2^n-1}\right) \left(1 - \frac{1}{2^{5n}}\right) = \left(1 - \frac{1}{2^{30}}\right)$$

$n = 6$, satisfies the equation.

Question 128

The term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is

equal to

[2021, 2 July Shift I]

Answer: 210

Solution:

Solution:

$$\begin{aligned} & \left[\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} \right) - \left(\frac{x-1}{x-x^{1/2}} \right) \right]^{10} \\ & = \left(x^{1/3} + 1 - \frac{x^{1/2}+1}{x^{1/2}} \right)^{10} \\ & = \left(x^{1/3} - \frac{1}{x^{1/2}} \right)^{10} \end{aligned}$$

General term,

$$\Rightarrow T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} \left(\frac{-1}{x^{1/2}} \right)^r$$

For independent term,

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$$\Rightarrow r = 4$$

$$\therefore T_5 = {}^{10}C_4 = 210$$

Question 129

If b is very small as compared to the value of a , so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$, then the value of γ is
[2021, 25 July Shift-I]

Options:

A. $\frac{a^2+b}{3a^3}$

B. $\frac{a+b}{3a^2}$

C. $\frac{b^2}{3a^3}$

D. $\frac{a+b^2}{3a^3}$

Answer: C

Solution:

Solution:

$$(a-b)^{-n} = a^{-n} \left(1 - \frac{b}{a} \right)^{-n}$$

$$\left(1 - \frac{b}{a} \right)^{-n} = \left[1 + n \left(\frac{b}{a} \right) + \frac{n(n+1)}{2} \left(\frac{b}{a} \right)^2 \right]$$

As, we can ignore the powers greater than or equal to 3.

$$a^{-n} \left(1 - \frac{b}{a} \right)^{-n}$$

$$= \frac{1}{a^n} + \frac{n \cdot b}{a^{n+1}} + \frac{n(n+1)}{2} \cdot \frac{b^2}{a^{n+2}}$$

$$= \frac{1}{a^n} + \frac{(a-b)^{-n}}{a^{n+1}} + \frac{n(n+1)}{2} \cdot \frac{b^2}{a^{n+2}}$$

When $n = 1$

$$c(a-b)^{-1} = \frac{1}{a} + \frac{b}{a^2} + \frac{b^2}{a^3} b \rightarrow rb$$

$$\sum_{r=b}^n (a-rb)^{-1} = \sum \left[\frac{1}{a} + r \left(\frac{b}{a^2} \right) + r^2 \left(\frac{b^2}{a^3} \right) \right]$$

$$S = \frac{n}{a} + \frac{n(n+1)b}{2a^2} + \frac{n(n+1)(2n+1)b^2}{6a^3}$$

$$\text{Coefficient of } n^3 = \frac{2b^2}{6a^3} = \frac{b^2}{3a^3}$$

Question130

The sum of all those terms which are rational numbers in the expansion of $(2^{1/3} + 3^{1/4})^{12}$ is
[2021, 25 July Shift-II]

Options:

- A. 89
- B. 27
- C. 35
- D. 43

Answer: D

Solution:

Solution:
In the given expansion of $(2^{1/3} + 3^{1/4})^{12}$

General term $\Rightarrow T_{r+1} = {}^{12}C_r (2^{1/3})^r (3^{1/4})^{12-r} \left(2^{1/3}\right)^r$

will be a rational number when $r = 0, 3, 6, 9, 12$ and $\left(3^{1/4}\right)^{12-r}$

will be rational number when $r = 0, 4, 8, 12 \Rightarrow r = 0, 12$

If $r = 0$, then

$$T_1 = {}^{12}C_0 \left(2^{1/3}\right)^0 \left(3^{1/4}\right)^{12} = 27$$

$$\text{If } r = 12, \text{ then } T_{13} = {}^{12}C_{12} \left(2^{1/3}\right)^{12} \left(3^{1/4}\right)^0 = 16$$

$$\text{So, } T_1 + T_{13} = 27 + 16 = 43$$

Question131

If the greatest value of the term independent of x in the expansion of $\left(x \sin \alpha + a \frac{\cos \alpha}{x}\right)$

10 is $\frac{10!}{(5!)^2}$, then the value of a is equal to

[2021, 25 July Shift-II]

Options:

- A. -1

B. 1

C. -2

D. 2

Answer: D

Solution:

Solution:

In the expansion of $\left(x \sin \alpha + a \frac{\cos \alpha}{x}\right)^{10}$

$$T_{r+1} = {}^{10}C_r (x \sin \alpha)^{10-r} \left(\frac{a \cos \alpha}{x}\right)^r$$

$$= {}^{10}C_r (x)^{10-2r} (\sin \alpha)^{10-r} (a \cos \alpha)^r$$

T_{r+1} is independent of x , when $10 - 2r = 0$

$$\therefore 2r = 10$$

$$T_6 = {}^{10}C_5 (\sin \alpha)^5 (a^5) (\cos \alpha)^5$$

$$= {}^{10}C_5 a^5 \cdot \frac{1}{2^5} (\sin 2\alpha)^5$$

For greatest value, $\sin 2\alpha = 1$

$$= {}^{10}C_5 (a)^5 \cdot \frac{1}{2^5}$$

Given, that the greatest value is $\frac{10!}{(5!)^2}$.

$$\text{So, } {}^{10}C_5 \frac{a^5}{2^5} = \frac{10!}{(5!)^2}$$

$$\Rightarrow \frac{10!}{(5!)^2} \cdot \frac{a^5}{2^5} = \frac{10!}{(5!)^2}$$

$$\Rightarrow a = 2$$

Question 132

For the natural numbers m, n , if $(1 - y)^m (1 + y)^n$

$$= 1 + a_1 y + a_2 y^2 + \dots + a_{m+n} y^{m+n} \text{ and}$$

$a_1 = a_2 = 10$, then the value of

$(m + n)$ is equal to

[2021, 20 July Shift-II]

Options:

A. 88

B. 64

C. 100

D. 80

Answer: D

Solution:

Solution:

$$\text{Given, } (1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \dots + a_{m+n}y^{m+n}$$

Now,

$$(1-y)^m(1+y)^n = (1 - my + {}^mC_2y^2 + \dots +$$

$$(-1)^m \cdot {}^mC_my^m)$$

$$\times (1 + ny + {}^nC_2y^2 + \dots + {}^nC_ny^n)$$

On expanding,

$$(1-y)^m(1+y)^n = 1 + (n-m)y$$

$$+ ({}^nC_2 - mn + {}^mC_2)y^2 + \dots$$

$$\therefore \text{Coefficient of } y = n - m$$

$$\text{Coefficient of } y^2 = {}^nC_2 - mn + {}^mC_2$$

Given expression have

$$\text{Coefficient of } y = a_1 = 10$$

$$\text{Coefficient of } y^2 = a_2 = 10$$

$$\therefore n - m = 10 \quad \dots\dots\dots (i)$$

$$\text{and } {}^nC_2 + {}^mC_2 - mn = 10 \quad \dots\dots\dots (ii)$$

From Eq. (ii),

$$\frac{n!}{2!(n-2)!} + \frac{m!}{2!(m-2)!} - mn = 10$$

$$\frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn = 10$$

$$\Rightarrow n(n-1) + m(m-1) - 2mn = 20$$

$$\Rightarrow n^2 - n + m^2 - m - 2mn = 20$$

$$\Rightarrow (m^2 + n^2 - 2mn) - (m+n) = 20$$

$$\Rightarrow (m-n)^2 - (m+n) = 20$$

$$\Rightarrow (-10)^2 - (m+n) = 20 \quad [\text{using Eq. (i)}]$$

$$\Rightarrow 100 - (m+n) = 20$$

$$\Rightarrow m+n = 100 - 20 = 80$$

Question133

The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is
[2021, 20 July Shift-I]

Options:

A. ${}^{100}C_{16}$

B. ${}^{100}C_{15}$

C. $-{}^{100}C_{16}$

D. $-{}^{100}C_{15}$

Answer: B

Solution:

Solution:

$$(1-x)^{101}(x^2+x+1)^{100}$$

Coefficient of x^{256}

$$= [(1-x)(1+x+x^2)]^{100}(1-x) =$$

$$(1-x^3)^{100}(1-x)$$

$$\Rightarrow ({}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2x^6 - {}^{100}C_3x^9 \dots)$$

$$(1-x) \sum (-1)^r {}^{100}C_r x^{3r} (1-x)$$

$$\Rightarrow 3r = 256 \text{ or } 255 \Rightarrow r = \frac{256}{3} \text{ (Reject)}$$

$$r = 85$$

$$\text{Coefficient} = {}^{100}C_{85} = {}^{100}C_{15}$$

Question 134

The number of rational terms in the binomial expansion of $(4^{1/4} + 5^{1/6})^{120}$ is
[2021, 20 July Shift-I]

Answer: 21

Solution:

Solution:

$$\left(4^{1/4} + 5^{1/6}\right)^{120}$$

$$\text{General term} = {}^{120}C_r \left(4^{1/4}\right)^r \left(5^{1/6}\right)^{120-r}$$

$$= {}^{120}C_r 4^{r/4} 5^{20 - r/6}$$

$$= {}^{120}C_r 2^{r/2} 5^{20 - r/6}$$

For this term to be rational, r should be a multiple of 2 and 6 i.e. r should be a multiple of 6 .

$$r = \{0, 6, 12, 18, \dots, 120\}$$

$$\text{Number of terms} = 21$$

Question 135

If the constant term, in Binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180 , then r is equal to
[2021, 22 July Shift-II]

Answer: 8

Solution:

Solution:

$$\left(2x^r + \frac{1}{x^2}\right)^{10}$$

$$\text{General term} = {}^{10}C_k (2x^r)^{10-k} x^{-2k}$$

$${}^{10}C_k (2^{10-k}) (x^{10r-rk-2k})$$

$$10r - rk - 2k = 0$$

$$\Rightarrow k = \frac{10r}{r+2} \Rightarrow r = \frac{2k}{10-k}$$

$$\Rightarrow r = -2 + \frac{20}{10-k} \Rightarrow k < 10$$

If $k = 8$ and $r = -2 + 10 = 8$

$$rk = 5 \text{ and } r = 2$$

$${}^{10}C_k 2^{10-k} = 180$$

At, $r = 8$ only this is satisfied.

Question136

The number of elements in the set $\{n \in \{1, 2, 3, \dots, 100\} : (11)^n > (10)^n + (9)^n\}$ is
[2021, 22 July Shift-II]

Answer: 96

Solution:

Solution:

$$11^n > 10^n + 9^n$$

$$\Rightarrow 11^n - 9^n > 10^n$$

$$\Rightarrow (10+1)^n - (10-1)^n > 10^n$$

$$\{ {}^nC_0 10^n + {}^nC_1 10^{n-1} + {}^nC_2 10^{n-2} + \dots \}$$

$$- \{ {}^nC_0 10^n - {}^nC_1 10^{n-1} \}$$

$$\Rightarrow 2({}^nC_1 10^{n-1} + {}^nC_3 10^{n-3}) > 10^n$$

For, $n = 1 \Rightarrow 2 \cdot 1 \times 10$
 For, $n = 2 \Rightarrow 2(2) \times 100$
 For, $n = 3 \Rightarrow 2(3 \cdot 10^2 + 1) \times 1000$
 For, $n = 5 \Rightarrow$
 $2({}^5C_1 10^4 + {}^5C_3 10^2 + {}^5C_5) \times 10^5$

Hence, $n \in \{5, 6, 7, \dots, 100\}$
 \therefore Number of elements = 96

Question137

The lowest integer which is greater than $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$ is

[2021, 25 July Shift-11]

Options:

- A. 3
- B. 4
- C. 2
- D. 1

Answer: A

Solution:

Solution:

$$\text{Let } P = \left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$$

$$\text{Let } x = 10^{100} \Rightarrow P = \left(1 + \frac{1}{x}\right)^x$$

$$P = {}^x C_0 + {}^x C_1 + {}^x C_1 \left(\frac{1}{x}\right) + {}^x C_2 \left(\frac{1}{x}\right)^2$$

$$+ {}^x C_3 \left(\frac{1}{x}\right)^3 + \dots \text{ up to } (10^{100} + 1) \text{ term}$$

$$\Rightarrow P = 1 + x \left(\frac{1}{x}\right) + \frac{x(x-1)}{2!} \cdot \frac{1}{x^2}$$

$$+ \frac{x(x-1)(x-2)}{3!} \cdot \frac{1}{x^3}$$

$$+ \dots \text{ upto } (10^{100} + 1) \text{ terms}$$

$$\Rightarrow P = 1 + 1 + \left[\left(\frac{1}{2!} - \frac{1}{2!x^2}\right) + \left(\frac{1}{3!} + \dots\right) + \dots \right]$$

$$\Rightarrow P = 2 +$$

$$\left(\text{Positive value less than } \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \right)$$

Now,

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

$$\Rightarrow \frac{1}{2!} + \frac{1}{3!} + \dots = e - 2$$

$$\Rightarrow P = 2 + [\text{Positive value less than } (e-2)]$$

$$\Rightarrow P \in (2, 3)$$

So, lowest integer which is greater than P is 3.

Question 138

If the co-efficient of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to

[2021, 25 July Shift-II]

Answer: 55

Solution:

Solution:

$$\text{The coefficient of } x^7 \text{ in the expansion of } \left(2 + \frac{x}{3}\right)^n = {}^n C_7 2^{n-7} \frac{1}{3^7}$$

$$\text{and the coefficient of } x^8 = {}^n C_8 2^{n-8} \frac{1}{3^8}$$

According to the question,

$${}^n C_7 2^{n-7} \cdot \frac{1}{3^7} = {}^n C_8 2^{n-8} \frac{1}{3^8}$$

$$\frac{n!}{7!(n-7)!} \cdot 2 \cdot 2^{n-8} \frac{1}{3^7} = \frac{n!}{8!(n-8)!} 2^{n-8} \times \frac{1}{3^8}$$

$$\Rightarrow \frac{1}{7!(n-7)(n-8)!} \times 2 = \frac{1}{8 \cdot 7!(n-8)!} \times \frac{1}{3}$$

$$\Rightarrow \frac{2}{n-7} = \frac{1}{24}$$

$$\Rightarrow n-7 = 48 \Rightarrow n = 55$$

Question139

$\sum_{k=0}^{20} ({}^{20}C_k)^2$ is equal to

[2021, 27 Aug. Shift-I]

Options:

A. ${}^{40}C_{21}$

B. ${}^{40}C_{19}$

C. ${}^{40}C_{20}$

D. ${}^{41}C_{20}$

Answer: C

Solution:

Solution:

$$\sum_{k=0}^{20} ({}^{20}C_k)^2 = ({}^{20}C_0)^2 + ({}^{20}C_1)^2 + ({}^{20}C_2)^2 + \dots + ({}^{20}C_{20})^2$$

$$\left[\because C_0^2 + C_1^2 + \dots + C_n^2 = \frac{(2n)!}{(n!)^2} \right]$$

$$= \frac{40!}{(20!)^2} = {}^{40}C_{20}$$

Question140

If ${}^{20}C_r$ is the coefficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r \cdot {}^{20}C_r$ is equal to

[2021, 26 Aug. Shift-I]

Options:

A. 420×2^{19}

B. 380×2^{18}

C. 380×2^{19}

D. 420×2^{18}

Answer: D

Solution:

Solution:

$${}^n C_r = \left(\frac{n}{r}\right)^{n-1} C_{r-1}$$

$$r^n C_r = n^{n-1} C_{r-1}$$

$$\text{Similarly, } (r-1)^{n-1} C_{r-1} = (n-1)^{n-2} C_{r-2} \dots\dots\dots (i)$$

Multiplying Eq. (i) with $(r-1)$

$$r(r-1)^n C_r = n(r-1)^{n-1} C_{r-1}$$

$$\Rightarrow r(r-1)^n C_r = n \cdot (n-1)^{n-2} C_{r-2}$$

$$r^{2n} C_r = [r(r-1) + r]^n C_r$$

$$= r(r-1)^n C_r + r^n C_r$$

$$= n(n-1)^{n-2} C_{r-2} + n \cdot {}^{n-1} C_{r-1}$$

$$\sum r^{2n} C_r = n(n-1) \sum_{r=2}^{n-2} {}^{n-2} C_{r-2} + n \sum_{r=1}^{n-1} C_{r-1}$$

Now, when $n = 20$

$$\sum r^{2n} C_r = (20 \times 19) \sum {}^{18} C_r + 20 \sum {}^{19} C_r$$

$$= (20 \times 19) 2^{18} + 20 \cdot 2^{19} = 420 \cdot 2^{18}$$

Question141

Let $\binom{n}{k}$ denotes ${}^n C_k$ and

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k} & \text{if } 0 \leq k \leq n \\ 0 & \text{otherwise .} \end{cases}$$

$$\text{If } A_k = \sum_{i=0}^9 \binom{9}{i} \begin{bmatrix} 12 \\ 12-k+i \end{bmatrix}$$

$$+ \sum_{i=0}^8 \binom{8}{i} \begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$$

and $A_4 - A_3 = 190p$, then p is equal to

[2021, 26 Aug. Shift-II]

Answer: 49

Solution:

Solution:

$$\text{Given, } a_k = \sum_{i=0}^9 ({}^9C_i \times {}^{12}C_{12-k+i}) + \sum_{i=0}^8 ({}^8C_j \times {}^{13}C_{13-k+i})$$

$$\Rightarrow A_k = \sum_{i=0}^9 {}^9C_i {}^{12}C_{k-i} + \sum_{i=0}^8 {}^8C_i {}^{13}C_{k-i}$$

$${}^9C_0 {}^{12}C_k + {}^9C_1 {}^{12}C_{k-1} + {}^9C_2 {}^{12}C_{k-2} + \dots +$$

$${}^9C_9 {}^{12}C_{k-9} = {}^{21}C_k \left[\because \sum_{r=0}^a {}^nC_r \times {}^mC_{a-r} = {}^{m+n}C_a \right]$$

Similarly,

$$\sum_{i=0}^8 {}^8C_i {}^{13}C_{k-i} = {}^{21}C_k$$

$$A_k = {}^{21}C_k + {}^{21}C_k 2^{21}C_k$$

$$A_4 - A_3 = 2 \cdot ({}^{21}C_4 - {}^{21}C_3)$$

$$= 2(5985 - 1330)$$

$$\Rightarrow 190p = 9310$$

$$p = 49$$

Question 142

If the coefficients of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to
[2021, 27 July Shift-1]

Options:

A. 2

B. -1

C. 1

D. -2

Answer: C

Solution:

Solution:

$$\text{Coefficient of } x^7 \text{ in } \left(x^2 + \frac{1}{bx}\right)^{11}$$

$$\text{General term} = {}^{11}C_r (x^2)^r \left(\frac{1}{bx}\right)^{11-r}$$

$$= {}^{11}C_r x^{3r-11} b^{r-11}$$

$$\Rightarrow 3r - 11 = 7 \Rightarrow r = 6$$

$$\therefore \text{Coefficient of } x^7 = {}^{11}C_6 b^{-5}$$

$$\text{Now, coefficient of } x^{-7} \text{ in } \left(x - \frac{1}{bx^2}\right)^{11}$$

$$\text{General term} = {}^{11}C_r x^r \left(\frac{-1}{bx^2}\right)^{11-r}$$

$$= {}^{11}C_r \left(\frac{-1}{b}\right)^{11-r} x^r \cdot \frac{1}{x^{22-2r}}$$

$$= {}^{11}C_r \left(\frac{-1}{b}\right)^{11-r} x^{3r-22}$$

$$|r| \Rightarrow 3r - 22 = -7$$

$$\Rightarrow r = 5$$

$$\text{Coefficient} = {}^{11}C_5 \left(\frac{-1}{b} \right) = {}^{11}C_5 b^{-6}$$

Now, according to the question,

$${}^{11}C_6 b^{-5} = {}^{11}C_5 b^{-6}$$

$$b = 1$$

Question 143

A possible value of 'x', for which the ninth term in the expansion of

$$\left\{ 3^{\log_3 \sqrt{25^{x-1} + 7}} + 3 \left(-\frac{1}{8} \right)^{\log_3 (5^{x-1} + 1)} \right\}^{10}$$

is equal to 180, is

[2021, 27 July Shift-II]

Options:

A. 0

B. -1

C. 2

D. 1

Answer: D

Solution:

Solution:

We have,

$$\begin{aligned} & \left\{ 3^{\log_3 \sqrt{25^{x-1} + 7}} + 3 \left(-\frac{1}{8} \right)^{\log_3 (5^{x-1} + 1)} \right\}^{10} \\ &= \left\{ \sqrt{25^{x-1} + 7} + (5^{x-1} + 1) \frac{-1}{8} \right\}^{10} \end{aligned}$$

Ninth term in the expansion is 180.

$$\text{So, } {}^{10}C_8 \left(\sqrt{25^{x-1} + 7} \right)^{10-8} \left[(5^{x-1} + 1) \frac{-1}{8} \right]^8$$

$$= 180 \{ \because (r+1) \text{th term of expansion } (x+a)^n, \}$$

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\Rightarrow {}^{10}C_8 (25^{x-1} + 7) (5^{x-1} + 1)^{-1} = 180$$

$$\Rightarrow 45 (25^{x-1} + 7) (5^{x-1} + 1)^{-1} = 180$$

$$\Rightarrow \frac{25^{x-1} + 7}{5^{x-1} + 1} = 4$$

$$\text{Let } 5^{x-1} = t$$

$$\Rightarrow \frac{t^2 + 7}{t + 1} = 4$$

$$\Rightarrow t^2 + 7 = 4t + 4$$

$$\Rightarrow t^2 - 4t + 3 = 0$$

$$\Rightarrow (t-3)(t-1) = 0$$

$$\Rightarrow t = 3 \text{ or } t = 1$$

$$\text{When } t = 3$$

$$5^{x-1} = 3$$

$$5^x = 15$$

$$x = \log_5 15$$

When, $t = 1$

$$r r 5^{x-1} = 1 [t = 5^{x-1}]$$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

Question 144

The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is

[2021, 25 July Shift-I]

Answer: 1

Solution:

Solution:

Coefficient of middle term of $(1+x)^{20}$ is ${}^{20}C_{10}$.

Coefficient of middle term of $(1+x)^{19}$ is ${}^{19}C_9$ and ${}^{19}C_{10}$

According to the question

$$\left(\frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} \right) = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$$

Question 145

If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x , in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, then k is equal to

[2021, 31 Aug. Shift-I]

Answer: 55

Solution:

Solution:

$$\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$$

$$T_{r+1} = (-1)^r {}^{12}C_r \left(\frac{x}{4}\right)^{12-r} \left(\frac{12}{x^2}\right)^r$$

$$= (-1)^r \left(\frac{{}^{12}C_r \cdot 12^r}{4^{12-r}}\right) x^{(12-r-2r)}$$

Term independent of r

$$12 - 3r = 0$$

$$\Rightarrow r = 4$$

$$T_5 = (-1)^4 \left(\frac{{}^{12}C_4 \cdot 12^4}{4^8} \right) = \frac{3^6}{4^4} k$$

$$k = 55$$

Question 146

$3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder
[2021, 27 Aug. Shift-II]

Answer: 15

Solution:

Solution:

$$3 \times 7^{22} + 2 \times 10^{22} - 44$$
$$= 3 \times (6+1)^{22} + 2 \times (9+1)^{22} - 44$$

Now,

$$(1+6)^{22} = 1 + {}^{22}C_1 6 + {}^{22}C_2 \cdot 6^2 + \dots + {}^{22}C_{22} 6^{22}$$
$$= (1 + 6\lambda)$$

$$(1+9)^{22} = 1 + {}^{22}C_1 9 + {}^{22}C_2 \cdot 9^2 + \dots + {}^{22}C_{22} 9^{22}$$
$$= (1 + 9\mu)$$

$$\therefore = 3(1 + 6\lambda) + 2(1 + 9\mu) - 44$$
$$= 18\lambda + 3 + 18\mu + 2 - 44$$
$$= 18\delta - 39 = 18\alpha + 15$$

$3 \times 7^{22} + 2 \times 10^{22} - 44$, when divided by 18
leaves remainder 15 .

Question 147

If the coefficient of $a^7 b^8$ in the expansion of $(a + 2b + 4ab)^{10}$ is $k \cdot 2^{16}$, then k is equal to
[2021, 31 Aug. Shift-II]

Answer: 315

Solution:

Solution:

$$(a + 2b + 4ab)^{10} = a^{10} b^{10} \left(\frac{1}{b} + \frac{2}{a} + 4 \right)^{10}$$

General term

$$= a^{10} b^{10} \frac{10! \left(\frac{1}{b} \right)^{r_1} \left(\frac{2}{a} \right)^{r_2} 4^{10-r_1-r_2}}{r_1! \cdot r_2! (10-r_1-r_2)!}$$

So, $r_1 = 2, r_2 = 3$

$$\begin{aligned} \text{Coefficient of } a^7 b^8 &= \frac{10! \cdot 2^3 \cdot 4^{10-2-3}}{2!3!(10-2-3)!} \\ &= \frac{2^{13} \cdot 10!}{2!3!5!} = 2^{16} \cdot 315 \\ \therefore k &= 315 \end{aligned}$$

Question148

If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \dots + x^{2n})(1 - x + x^2 - x^3 + \dots + x^{2n})$ is 61, then n is equal to _____.
[NA Jan. 7, 2020 (I)]

Answer: 30

Solution:

Solution:

$$\begin{aligned} \text{Let } (1 - x + x^2 - \dots - x^{2n})(1 + x + x^2 - \dots - x^{2n}) \\ = a_0 + a_1 x + a_2 x^2 + \dots \end{aligned}$$

put $x = 1$

$$1(2n+1) = a_0 + a_1 + a_2 + \dots + a_{2n} \dots (i)$$

put $x = -1$

$$(2n+1) \times 1 = a_0 - a_1 + a_2 - \dots - a_{2n} \dots (ii)$$

Adding (i) and (ii), we get,

$$4n + 2 = 2(a_0 + a_2 + \dots) = 2 \times 61$$

$$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30$$

Question149

The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is _____.
[NA Jan. 9, 2020 (I)]

Answer: 615

Solution:

Solution:

$$\text{General term of the expansion} = \frac{10!}{\alpha! \beta! \gamma!} x^{\beta+2\gamma}$$

For coefficient of x^4 ; $\beta + 2\gamma = 4$

Here, three cases arise

Case-1 : When $\gamma = 0$, $\beta = 4$, $\alpha = 6$

$$\Rightarrow \frac{10!}{\alpha! \beta! \gamma!} x^{\beta+2\gamma}$$

Case-2 : When $\gamma = 1$, $\beta = 2$, $\alpha = 7$

$$\Rightarrow \frac{10!}{7!2!1!} = 360$$

Case-3 : When $\gamma = 2, \beta = 0, \alpha = 8$

$$\Rightarrow \frac{10!}{8!0!2!} = 45$$

Hence, total = 615

Question150

If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $(x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6$, then:

[Jan. 8, 2020 (II)]

Options:

- A. $\alpha + \beta = 60$
- B. $\alpha + \beta = -30$
- C. $\alpha - \beta = 60$
- D. $\alpha - \beta = -132$

Answer: D

Solution:

Solution:

Using Binomial expansion

$$(x + a)^n + (x - a)^n = 2(T_1 + T_3 + T_5 + T_7 \dots)$$

$$\therefore (x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6 = 2(T_1 + T_3 + T_5 + T_7)$$

$$2[{}^6C_0x^5 + {}^6C_2x^4(x^2 - 1) + {}^6C_4x^2(x^2 - 1)^2 + {}^6C_6(x^2 - 1)^3]$$

$$= 2[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^4 + x^6)]$$

$$= 2(32x^6 - 48x^4 + 18x^2 - 1)$$

$$\alpha = -96 \text{ and } \beta = 36$$

$$\therefore \alpha - \beta = -132$$

Question151

In the expansion of $\left(\frac{x}{\cos \theta} + \frac{1}{x \sin \theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8} \leq \theta \leq \frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16} \leq \theta \leq \frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to :

[Jan. 9, 2020 (II)]

Options:

- A. 1: 8
- B. 16: 1
- C. 8: 1

D. 1: 16

Answer: B

Solution:

Solution:

General term of the given expansion

$$T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin \theta} \right)^{16-r} \left(\frac{1}{x \cos \theta} \right)^r$$

For $r = 8$ term is free from 'x'

$$T_9 = {}^{16}C_8 \frac{1}{\sin^8 \theta \cos^8 \theta}$$

$$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$$

When $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4} \right]$, then least value of the term independent of x

$$l_1 = {}^{16}C_8 2^8 \quad [\because \text{min. value of } l_1 \text{ at } \theta = \pi/4]$$

When $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8} \right]$, then least value of the term independent of x,

$$l_2 = {}^{16}C_8 = \frac{2^8}{\left(\frac{1}{\sqrt{2}} \right)^8} = {}^{16}C_8 \cdot 2^8 \cdot 2^4$$

[\because min. value of l_2 at $\theta = \pi/8$]

$$\text{Now, } \frac{l_2}{l_1} = \frac{{}^{16}C_8 \cdot 2^8 \cdot 2^4}{{}^{16}C_8 \cdot 2^8} = 16$$

Question152

If $\{p\}$ denotes the fractional part of the number p, then $\left\{ \frac{3^{200}}{8} \right\}$, is equal to :

[Sep. 06, 2020 (I)]

Options:

A. $\frac{5}{8}$

B. $\frac{7}{8}$

C. $\frac{3}{8}$

D. $\frac{1}{8}$

Answer: D

Solution:

Solution:

$$\frac{3^{200}}{8} = \frac{1}{8}(9^{100})$$

$$= \frac{1}{8}(1+8)^{100} = \frac{1}{8} \left[1 + n \cdot 8 + \frac{n(n-1)}{2} \cdot 8^2 + \dots \right]$$

$$= \frac{1}{8} + \text{Integer}$$

$$\therefore \left\{ \frac{3^{200}}{8} \right\} = \left\{ \frac{1}{8} + \text{integer} \right\} = \frac{1}{8}$$

Question153

The natural number m , for which the coefficient of x in the binomial expansion of

$$\left(x^m + \frac{1}{x^2} \right)^{22} \text{ is } 1540, \text{ is } \underline{\hspace{2cm}}.$$

[NA Sep. 05, 2020 (I)]

Answer: 13

Solution:

Solution:

$$T_{r+1} = {}^{22}C_r \cdot (x^m)^{22-r} \cdot \left(\frac{1}{x^2} \right)^r$$

$$T_{r+1} = {}^{22}C_r \cdot x^{22m - mr - 2r}$$

$$\therefore 22m - mr - 2r = 1$$

$$\Rightarrow r = \frac{22m - 1}{m + 2} \Rightarrow r = 22 - \frac{3 \cdot 3 \cdot 5}{m + 2}$$

So, possible value of $m = 1, 3, 7, 13, 43$

$$\text{But } {}^{22}C_r = 1540$$

\therefore Only possible value of $m = 13$.

Question154

The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x , is $\underline{\hspace{2cm}}$.

[NA Sep. 04, 2020 (I)]

Answer: 120

Solution:

Solution:

$$\text{Coefficient of } x^4 \text{ in } \left(\frac{1-x^4}{1-x} \right)^6 = \text{coefficient of } x^4 \text{ in } (1-6x^4)(1-x)^{-6}$$

$$= \text{coefficient of } x^4 \text{ in } (1-6x^4)[1 + {}^6C_1x + {}^7C_2x^2 + \dots]$$

$$= {}^9C_4 - 6 \cdot 1 = 126 - 6 = 120$$

Question 155

Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{12}}$ is equal to _____.

[NA Sep. 04, 2020 (I)]

Answer: 8

Solution:

Solution:

The given expression is $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$

General term = $\frac{10!}{r_1! r_2! r_3!} (2x^2)^{r_1} (3x)^{r_2} (4)^{r_3}$

Since, $a_7 =$ Coeff. of x^7

$2r_1 + r_2 = 7$ and $r_1 + r_2 + r_3 = 10$

Possibilities are

r_1	r_2	r_3
0	7	3
1	5	4
2	3	5
3	1	6

$$a_7 = \frac{10!3^7 4^3}{7!3!} + \frac{10!(2)(3)^5(4)^4}{5!4!} + \frac{10!(2)^2(3)^3(4)^5}{2!3!5!} + \frac{10!(2)^3(3)(4)^6}{3!6!}$$

$a_{13} =$ Coeff. of x^{13}

$2r_1 + r_2 = 13$ and $r_1 + r_2 + r_3 = 10$

Possibilities are

r_1	r_2	r_3
3	7	0
4	5	1
5	3	2
6	1	3

$$a_{13} = \frac{10!(2^3)(3^7)}{3!7!} + \frac{10!(2^4)(3^5)(4)}{4!5!}$$

$$+ \frac{10!(2^5)(3^3)(4^2)}{5!3!2!} + \frac{10!(2^6)(3)(4^3)}{6!1!3!}$$

$$\therefore \frac{a_7}{a_{13}} = 2^3 = 8$$

Question156

If the constant term in the binomial expansion of $\left(\sqrt{x} \frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals:

[Sep. 06, 2020 (II)]

Options:

- A. 9
- B. 1
- C. 3
- D. 2

Answer: C

Solution:

Solution:

$$\text{General term} = T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \cdot \left(-\frac{k}{x^2}\right)^r$$

$$= {}^{10}C_r (-k)^r \cdot x^{\frac{10-r}{2} - 2r}$$

$$= {}^{10}C_r (-k)^r \cdot x^{\frac{10-5r}{2}}$$

Since, it is constant term, then

$$\frac{10-5r}{2} = 0 \Rightarrow r = 2$$

$$\therefore {}^{10}C_2 (-k)^2 = 405$$

$$\Rightarrow k^2 = \frac{405 \times 2}{10 \times 9} = \frac{81}{9} = 9$$

$$\therefore |k| = 3$$

Question157

If for some positive integer n , the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is :

[Sep. 04, 2020 (II)]

Options:

- A. 462
- B. 330
- C. 792

D. 252

Answer: A

Solution:

Solution:

Consider the three consecutive coefficients of $(1+x)^{n+5}$ be ${}^{n+5}C_r$, ${}^{n+5}C_{r+1}$, ${}^{n+5}C_{r+2}$

$$\therefore \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{1}{2} \text{ (Given)}$$

$$\Rightarrow \frac{r+1}{n+5-r} = \frac{1}{2} \Rightarrow 3r = n+3 \dots (i)$$

$$\text{and } \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}} = \frac{5}{7}$$

$$\Rightarrow \frac{r+2}{n+4-r} = \frac{5}{7} \Rightarrow 12r = 5n+6 \dots (ii)$$

Solving (i) and (ii) we get $r = 4$ and $n = 6$

\therefore Largest coefficient in the expansion is ${}^{11}C_6 = 462$.

Question158

If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :

[Sep. 03, 2020 (I)]

Options:

A. 264

B. 128

C. 256

D. 248

Answer: C

Solution:

Solution:

$$\text{Here } \left(3^{1/2} + 5^{1/8} \right)^n$$

$$T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}}$$

$$\therefore \frac{n-r}{2} \text{ and } \frac{r}{8} \text{ are integer}$$

So, r must be 0, 8, 16, 24,.....

$$\text{Now } n = t_{33} = a + (n-1)d = 0 + 32 \times 8 = 256$$

$$\Rightarrow n = 256$$

Question159

If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k , then $18k$ is equal to :

[Sep. 03, 2020 (II)]

Options:

- A. 5
- B. 9
- C. 7
- D. 11

Answer: C

Solution:

Solution:

$$\text{General term} = T_{r+1} = {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r$$

$$= {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$$

The term is independent of x , then

$$18 - 3r = 0 \Rightarrow r = 6$$

$$\therefore T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = {}^9C_3 \left(\frac{1}{6}\right)^3$$

$$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$$

$$\therefore 18k = 18 \times \frac{7}{18} = 7$$

Question 160

Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent

of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is $10k$, then k is equal to :

[Sep. 02, 2020 (I)]

Options:

- A. 336
- B. 352
- C. 84
- D. 176

Answer: A

Solution:

Solution:

General term of

$$\left(\alpha x \frac{1}{9} + \beta x \frac{-1}{6}\right)^{10} = {}^{10}C_r \left(\alpha x \frac{1}{9}\right)^{10-r} \left(\beta x \frac{-1}{6}\right)^r$$
$$= {}^{10}C_r \alpha^{10-r} \beta^r (x)^{\frac{10-r}{9} - \frac{r}{6}}$$

Term independent of x if $\frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r = 4$

\therefore Term independent of $x = {}^{10}C_4 \alpha^6 \beta^4$

Since $\alpha^3 + \beta^2 = 4$

Then, by AM-GM inequality

$$\frac{\alpha^3 + \beta^2}{2} \geq (\alpha^3 \beta^2)^{\frac{1}{2}}$$

$$\Rightarrow (2)^2 \geq \alpha^3 \beta^2 \Rightarrow \alpha^6 \beta^4 \leq 16$$

\therefore The maximum value of the term independent of $x = 10k$

$$\therefore 10k = {}^{10}C_4 \cdot 16 \Rightarrow k = 336$$

Question161

For a positive integer n , $\left(1 + \frac{1}{x}\right)^n$ is expanded in increasing powers of x . If three consecutive coefficients in this expansion are in the ratio, $2 : 5 : 12$, then n is equal to

[NA Sep. 02, 2020 (II)]

Answer: 118

Solution:

Solution:

According to the question,

$${}^n C_{r-1} : {}^n C_r : {}^n C_{r+1} = 2 : 5 : 12$$

$$\Rightarrow \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{5}{2} \Rightarrow \frac{n-r+1}{r} = \frac{5}{2}$$

$$\Rightarrow 2n - 7r + 2 = 0 \dots (i)$$

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{12}{5} \Rightarrow \frac{n-r}{r+1} = \frac{12}{5}$$

$$\Rightarrow 5n - 17r - 12 = 0 \dots (ii)$$

Solving eqns. (i) and (ii),

$$n = 118, r = 34$$

Question162

The value of $\sum_{r=0}^{20} 50^{-r} C_6$ is equal to:

[Sep. 04, 2020 (I)]

Options:

A. ${}^{51}C_7 - {}^{30}C_7$

B. ${}^{50}C_7 - {}^{30}C_7$

C. ${}^{50}C_6 - {}^{30}C_6$

D. ${}^{51}C_7 + {}^{30}C_7$

Answer: A

Solution:

Solution:

The given series, $\sum_{r=0}^{20} {}^{50-r}C_6$

$$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + \dots + {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$$

$$= ({}^{30}C_7 + {}^{30}C_6) + {}^{31}C_6 + {}^{32}C_6 + \dots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$$

$$= ({}^{31}C_7 + {}^{31}C_6) + {}^{32}C_6 + \dots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$$

$$= ({}^{32}C_7 + {}^{32}C_6) + \dots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$$

.....

.....

.....

$$= {}^{51}C_7 - {}^{30}C_7$$

Question163

Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + \dots + a_{50}x^{50}$ for all $x \in \mathbb{R}$; then $\frac{a_2}{a_0}$ is equal to:

[Jan. 11, 2019 (II)]

Options:

A. 12.50

B. 12.00

C. 12.25

D. 12.75

Answer: C

Solution:

Solution:

$$(x + 10)^{50} + (x - 10)^{50}$$

$$= a_0 + a_1x + a_2x^2 + \dots + a_{50}x^{50}$$

$$\therefore a_0 + a_1x + a_2x^2 + \dots + a_{50}x^{50}$$

$$= 2({}^{50}C_0x^{50} + {}^{50}C_2x^{48} \cdot 10^2 + {}^{50}C_4x^{46} \cdot 10^4 + \dots)$$

$$\therefore a_0 = 2 \cdot {}^{50}C_0 \cdot 10^{50}$$

$$a_2 = 2 \cdot {}^{50}C_2 \cdot 10^{48}$$

$$\therefore \frac{a_2}{a_0} = \frac{{}^{50}C_2 \times 10^{48}}{{}^{50}C_0 \cdot 10^{50}}$$

$$= \frac{50 \times 49}{2 \times 100} = \frac{49}{4} = 12.25$$

Question164

If the third term in the binomial expansion of $(1 + x^{\log_2 x})^5$ equals 2560, then a possible value of x is:

[Jan. 10, 2019 (I)]

Options:

- A. $\frac{1}{4}$
- B. $4\sqrt{2}$
- C. $\frac{1}{8}$
- D. $2\sqrt{2}$

Answer: A

Solution:

Solution:

$$\begin{aligned}\text{Third term of } (1 + x^{\log_2 x})^5 &= {}^5C_2 (x^{\log_2 x})^{5-3} \\ &= {}^5C_2 (x^{\log_2 x})^2\end{aligned}$$

$$\text{Given, } {}^5C_2 (x^{\log_2 x})^2 = 2560$$

$$\Rightarrow (x^{\log_2 x})^2 = 256 = (\pm 16)^2$$

$$\Rightarrow x^{\log_2 x} = 16 \text{ or } x^{\log_2 x} = -16 \text{ (rejected)}$$

$$\Rightarrow x^{\log_2 x} = 16 \Rightarrow \log_2 x \log_2 x = \log_2 16 = 4$$

$$\Rightarrow \log_2 x = \pm 2 \Rightarrow x = 2^2 \text{ or } 2^{-2}$$

$$\Rightarrow x = 4 \text{ or } \frac{1}{4}$$

Question165

The positive value of λ for which the co-efficient of x^2 in the expression $x^2 \left(\sqrt{x} + \frac{\lambda}{x^2} \right)^{10}$

is 720, is:

[Jan. 10, 2019 (II)]

Options:

- A. 4
- B. $2\sqrt{2}$
- C. $\sqrt{5}$
- D. 3

Answer: A

Solution:

Solution:

Since, coefficient of x^2 in the expression x^2

$\left(\sqrt{x} + \frac{\lambda}{x^2}\right)$ is a constant term, then

Coefficient of x^2 in $x^2 \left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$

= co-efficient of constant term in $\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$

General term in $\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10} = {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{\lambda}{x^2}\right)^r$

$$= {}^{10}C_r (x)^{\frac{10-r}{2}-2r} \cdot \lambda^2$$

Then, for constant term,

$$\frac{10-r}{2} - 2r = 0 \Rightarrow r = 2$$

Co-efficient is x^2 in expression = ${}^{10}C_2 \lambda^2 = 720$

$$\Rightarrow \lambda^2 = \frac{720}{5 \times 9} = 16 \Rightarrow \lambda = 4d$$

Question166

If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:

[Jan. 9, 2019 (I)]

Options:

- A. 6
- B. 8
- C. 4
- D. 14

Answer: B

Solution:

Solution:

$$\begin{aligned} 2^{403} &= 2^{400} \cdot 2^3 \\ &= 2^{4 \times 100} \cdot 2^3 \\ &= (2^4)^{100} \cdot 8 \\ &= 8(2^4)^{100} = 8(16)^{100} \\ &= 8(1+15)^{100} \\ &= 8 + 15\mu \end{aligned}$$

When 2^{403} is divided by 15, then remainder is 8

Hence, fractional part of the number is $\frac{8}{15}$

Therefore value of k is 8

Question167

The total number of irrational terms in the binomial expansion of $\left(7^{\frac{1}{5}} - 3^{\frac{1}{10}}\right)^{60}$ is :
Jan. 12, 2019 (II)]

Options:

- A. 55
- B. 49
- C. 48
- D. 54

Answer: D

Solution:

Solution:

Let the general term of the expansion

$$\begin{aligned}T_{r+1} &= {}^{60}C_r \left(7^{\frac{1}{5}}\right)^{60-r} \left(-3^{\frac{1}{10}}\right)^r \\ &= {}^{60}C_r \cdot (7)^{12-\frac{r}{5}} (-1)^r \cdot (3)^{\frac{r}{10}}\end{aligned}$$

Then, for getting rational terms, r should be multiple of L.C.M. of (5,10)

Then, r can be 0,10,20,30,40,50,60 .

Since, total number of terms = 61

Hence, total irrational terms = 61 - 7 = 54

Question168

A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}} + \frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is:

[Jan. 12, 2019 (I)]

Options:

- A. $1 : 2(6)^{\frac{1}{3}}$
- B. $1 : 4(16)^{\frac{1}{3}}$
- C. $4(36)^{\frac{1}{3}} : 1$
- D. $2(36)^{\frac{1}{3}} : 1$

Answer: C

Solution:

Solution:

$$\left(2\frac{1}{3} + \frac{1}{2(3)\frac{1}{3}}\right)^{10} = {}^{10}C_0 \left(2\frac{1}{3}\right)^0 \left(\frac{1}{2(3)^{1/3}}\right)^{10} +$$

$$\dots + {}^{10}C_{10} \left(2\frac{1}{3}\right)^{10} \left(\frac{1}{2(3)^{1/3}}\right)^0$$

$$\text{5th term from beginning } T_5 = {}^{10}C_4 \left(2\frac{1}{3}\right)^6 \frac{1}{\left(2.3\frac{1}{3}\right)^4}$$

$$\text{and 5th term from end } T_{11-5+1} = {}^{10}C_6 \left(2\frac{1}{3}\right)^4 \left(\frac{1}{2.3\frac{1}{3}}\right)^6$$

$$\therefore T_5 : T_7 = {}^{10}C_4 \left(2\frac{1}{3}\right)^6 \left(\frac{1}{2.3\frac{1}{3}}\right)^4 : {}^{10}C_6 \left(2\frac{1}{3}\right)^4 \left(\frac{1}{2.3\frac{1}{3}}\right)^6$$

$$= \left(2\frac{1}{3}\right)^2 : \left(\frac{1}{2.3\frac{1}{3}}\right)^2$$

$$= \frac{2}{2\frac{1}{3}} \cdot 2^2 \cdot 3\frac{2}{3} = 4(6)\frac{2}{3} : 1 = 4 \cdot (36)\frac{1}{3} : 1$$

Question 169

The sum of the real values of x for which the middle term in the binomial expansion of

$$\left(\frac{x^3}{3} + \frac{3}{x}\right)^8 \text{ equals 5670 is :}$$

[Jan. 11, 2019 (I)]

Options:

- A. 0
- B. 6
- C. 4
- D. 8

Answer: A

Solution:

Solution:
Middle Term, $\left(\frac{n}{2} + 1\right)^{\text{th}}$ term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ is,

$$T_{4+1} = {}^8C_4 \left(\frac{x^3}{3}\right)^4 \left(\frac{3}{x}\right)^4 = 5670$$

$$\Rightarrow \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} \times x^{12-4} = 5670$$

$$\Rightarrow x^8 = 81$$

$$\Rightarrow x^8 - 81 = 0$$

\therefore sum of all values of $x =$ sum of roots of equation $(x^8 - 81 = 0)$.

Question170

The value of r for which ${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$ is maximum, is :
[Jan. 11, 2019 (I)]

Options:

- A. 15
- B. 20
- C. 11
- D. 10

Answer: B

Solution:

Solution:

Consider the expression

$${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$$

For maximum value of above expression r should be equal to 20 .

$$\text{as } {}^{20}C_{20} \cdot {}^{20}C_0 + {}^{20}C_{19} \cdot {}^{20}C_1 + \dots + {}^{20}C_0 \cdot {}^{20}C_{20}$$

$$= ({}^{20}C_0)^2 + ({}^{20}C_1)^2 + \dots + ({}^{20}C_{20})^2 = {}^{40}C_{20}$$

Which is the maximum value of the expression, So, $r = 20$.

Question171

If $\sum_{r=0}^{25} \{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \} = K ({}^{50}C_{25})$, then K is equal to:
[Jan. 10, 2019 (II)]

Options:

- A. $(25)^2$
- B. $2^{25} - 1$
- C. 2^{24}
- D. 2^{25}

Answer: D

Solution:

Solution:

$$\begin{aligned} \sum_{r=0}^{25} ({}^{50}C_r \cdot {}^{50-r}C_{25-r}) &= \sum_{r=0}^{25} \left(\frac{|50}{|50-r|r} \frac{|50-r}{|25|25-r} \right) \\ &= \sum_{r=0}^{25} \left(\frac{|50}{|25} \times \frac{1}{|25} \times \left(\frac{|25}{|25-r|r} \right) \right) \\ &= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = {}^{50}C_{25} (2^{25}) \end{aligned}$$

Then, by comparison, $K = 2^{25}$

Question172

The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t} \right)^3$

[Jan. 09, 2019 (II)]

Options:

- A. 14
- B. 15
- C. 10
- D. 12

Answer: B

Solution:

Solution:

Consider the expression

$$\begin{aligned} \left(\frac{1-t^6}{1-t} \right)^3 &= (1-t^6)^3 (1-t)^{-3} \\ &= (1-3t^6+3t^{12}-t^{18}) \left(1+3t+ \frac{3 \cdot 4}{2!} t^2 \right. \\ &\quad \left. + \frac{3 \cdot 4 \cdot 5}{3!} t^3 + \frac{3 \cdot 4 \cdot 5 \cdot 6}{4!} t^4 + \dots \infty \right) \end{aligned}$$

Hence, the coefficient of $t^4 = 1 \cdot \frac{3 \cdot 4 \cdot 5 \cdot 6}{4!}$

$$= \frac{3 \times 4 \times 5 \times 6}{4 \times 3 \times 2 \times 1} = 15$$

Question173

The smallest natural number n , such that the coefficient of x in the expansion of

$\left(x^2 + \frac{1}{x^3} \right)^n$ is ${}^n C_{23}$, is :

[April 10, 2019 (II)]

Options:

- A. 38

B. 58

C. 23

D. 35

Answer: A

Solution:

Solution:

$$\left(x^2 + \frac{1}{x^2}\right)^n$$

$$\text{General term } T_{r+1} = {}^n C_r (x^2)^{n-r} \left(\frac{1}{x^2}\right)^r = {}^n C_r \cdot x^{2n-5r}$$

To find coefficient of x , $2n - 5r = 1$

$$\text{Given } {}^n C_r = {}^n C_{23} \Rightarrow r = 23 \text{ or } n - r = 23$$

$$\therefore n = 58 \text{ or } n = 38$$

Minimum value is $n = 38$

Question 174

If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ ($x > 0$) is 20×8^7 , then a value of x is:

[April 9, 2019 (I)]

Options:

A. 8^3

B. 8^2

C. 8

D. 8^{-2}

Answer: B

Solution:

Solution:

$$\therefore T_4 = 20 \times 8^7$$

$$\Rightarrow {}^6 C_3 \left(\frac{2}{x}\right)^3 \times (x^{\log_8 x})^3 = 20 \times 8^7$$

$$\Rightarrow 8 \times 20 \times \left(\frac{x^{\log_8 x}}{x}\right)^3 = 20 \times 8^7 \Rightarrow \frac{x^{\log_8 x}}{x} = 64$$

Now, take \log_8 on both sides, then

$$(\log_8 x)^2 - (\log_8 x) = 2$$

$$\Rightarrow \log_8 x = -1 \text{ or } \log_8 x = 2$$

$$\Rightarrow x = \frac{1}{8} \text{ or } x = 8^2$$

Question175

If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2: 15: 70 , then the average of these three coefficients is:
[April 09, 2019 (II)]

Options:

- A. 964
- B. 232
- C. 227
- D. 625

Answer: B

Solution:

Solution:

Given ${}^n C_{r-1} : {}^n C_r : {}^n C_{r+1} = 2 : 15 : 70$

$$\Rightarrow \frac{{}^n C_{r-1}}{{}^n C_r} = \frac{2}{15} \text{ and } \frac{{}^n C_r}{{}^n C_{r+1}} = \frac{15}{70}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{2}{15} \text{ and } \frac{r+1}{n-r} = \frac{3}{14}$$

$$\Rightarrow 17r = 2n + 2 \text{ and } 17r = 3n - 14$$

$$\text{i.e., } 2n + 2 = 3n - 14 \Rightarrow n = 16 \text{ \& } r = 2$$

$$\therefore \text{Average} = \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16 + 120 + 560}{3}$$

$$= \frac{696}{3} = 232$$

Question176

The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$, ($x > 1$) is equal to :
[April 8, 2019 (I)]

Options:

- A. 29
- B. 32
- C. 26
- D. 24

Answer: D

Solution:

Solution:

$$\begin{aligned}
 & \text{(d) } (x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6 \\
 &= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3 - 1) + {}^6C_4x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3] \\
 &= 2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1] \\
 &\text{Hence, the sum of coefficients of even powers of } x = 2[1 - 15 + 15 + 15 - 3 - 1] = 24
 \end{aligned}$$

Question177

If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200,

and $x > 1$, then the value of x is:

[April 08, 2019 (II)]

Options:

A. 100

B. 10

C. 10^3

D. 10^4

Answer: B

Solution:

Solution:

\therefore fourth term is equal to 200 .

$$T_4 = {}^6C_3 \left(\sqrt{\frac{1}{1+\log_{10}x}}\right)^3 \left(x^{\frac{1}{12}}\right)^3 = 200$$

$$\Rightarrow 20x^{\frac{3}{2(1+\log_{10}x)}} \cdot x^{\frac{1}{4}} = 200$$

$$x^{\frac{1}{4} + \frac{3}{2(1+\log_{10}x)}} = 10$$

Taking \log_{10} on both sides and putting $\log_{10}x = t$

$$\left(\frac{1}{4} + \frac{3}{2(1+t)}\right)t = 1 \Rightarrow t^2 + 3t - 4 = 0$$

$$\Rightarrow t^2 + 4t - t - 4 = 0 \Rightarrow t(t+4) - 1(t+4) = 0$$

$$\Rightarrow t = 1 \text{ or } t = -4$$

$$\log_{10}x = 1 \Rightarrow x = 10$$

$$\text{or } \log_{10}x = -4 \Rightarrow x = 10^{-4}$$

According to the question $x > 1$, $\therefore x = 10$

Question178

The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right) \cdot \left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to :

[NA April 12, 2019 (II)]

Options:

- A. -72
- B. 36
- C. -36
- D. -108

Answer: D

Solution:

Solution:

Given expression is,

$$\left(\frac{1}{60} - \frac{x^8}{81}\right) \left(2x^2 - \frac{3}{x^2}\right)^6$$

$$= \frac{1}{60} \left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{x^8}{81} \left(2x^2 - \frac{3}{x^2}\right)^6$$

Term independent of x

$$= \text{Coefficient of } x^0 \text{ in } \frac{1}{60} \left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}$$

$$\text{coefficient of } x^{-8} \text{ in } \left(2x^2 - \frac{3}{x^2}\right)^6$$

$$= \frac{-1}{60} {}^6C_3 (2)^3 (3)^3 + \frac{1}{81} {}^6C_5 (2)(3)^5$$

$$= -72 + 36 = -36$$

Question179

If ${}^{20}C_1 + (2^2) {}^{20}C_2 + (3^2) {}^{20}C_3 + \dots + (20^2) {}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to :
[April 12, 2019 (II)]

Options:

- A. (420,19)
- B. (420,18)
- C. (380,18)
- D. (380,19)

Answer: B

Solution:

Solution:

$$\text{Given, } {}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \dots + 20^2 \cdot {}^{20}C_{20}$$

$$= A(2^\beta)$$

Taking L.H.S.,

$$= \sum_{r=1}^{20} r^2 \cdot {}^{20}C_r = 20 \sum_{r=1}^{20} r \cdot {}^{19}C_{r-1}$$

$$= 20 \left[\sum_{r=1}^{20} (r-1) {}^{19}C_{r-1} + \sum_{r=1}^{20} {}^{19}C_{r-1} \right]$$

$$= 20 \left[19 \sum_{r=2}^{20} {}^{18}C_{r-2} + 2^{19} \right] = 20[19 \cdot 2^{18} + 2^{19}]$$

$$= 420 \times 2^{18}$$

Now, compare it with R.H.S., $A = 420$ and $\beta = 18$

Question180

The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is :
[April 12, 2019 (I)]

Options:

- A. 84
- B. -126
- C. -84
- D. 126

Answer: A

Solution:

Solution:

Given expression,

$$(1-x)^{10}(1+x+x^2)^9(1+x) = (1-x^3)^9(1-x^2)$$

$$= (1-x^3)^9 - x^2(1-x^3)^9$$

$$\Rightarrow \text{Coefficient of } x^{18} \text{ in } (1-x^3)^9 - \text{coeff. of } x^{16} \text{ in } (1-x^3)^9$$

$$= {}^9C_6 - 0 = \frac{9!}{6!3!} = \frac{7 \times 8 \times 9}{6} = 84$$

Question181

If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1+ax+bx^2)(1-3x)^{15}$ in powers of x , then the ordered pair (a, b) is equal to:
[April 10, 2019 (I)]

Options:

- A. (28,861)
- B. (-54,315)
- C. (28,315)
- D. (-21,714)

Answer: C

Solution:

Solution:

Given expression is $(1 + ax + bx^2)(1 - 3x)^{15}$

Co-efficient of $x^2 = 0$

$$\Rightarrow {}^{15}C_2(-3)^2 + a \cdot {}^{15}C_1(-3) + b \cdot {}^{15}C_0 = 0$$

$$\Rightarrow \frac{15 \times 14}{2} \times 9 - 15 \times 3a + b = 0$$

$$\Rightarrow 945 - 45a + b = 0 \dots (i)$$

Now, co-efficient of $x^3 = 0$

$$\Rightarrow {}^{15}C_3(-3)^3 + a \cdot {}^{15}C_2(-3)^2 + b \cdot {}^{15}C_1(-3) = 0$$

$$\Rightarrow \frac{15 \times 14 \times 13}{3 \times 2} \times (-3 \times 3 \times 3) + a \times \frac{15 \times 14 \times 9}{2} - b \times 3 \times 15 = 0$$

$$\Rightarrow 15 \times 3[-3 \times 7 \times 13 + a \times 7 \times 3 - b] = 0$$

$$\Rightarrow 21a - b = 273 \dots (ii)$$

From (i) and (ii), we get,

$$a = 28, b = 315 \Rightarrow (a, b) \equiv (28, 31, 5)$$

Question182

The sum of the series $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + 11 \cdot {}^{20}C_3 + \dots + 62 \cdot {}^{20}C_{20}$ is equal to :

[April 8, 2019 (I)]

Options:

A. 2^{26}

B. 2^{25}

C. 2^{23}

D. 2^{24}

Answer: B

Solution:

Solution:

$$2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20}$$

$$= \sum_{r=0}^{20} (3r+2) {}^{20}C_r = 3 \sum_{r=0}^{20} r \cdot {}^{20}C_r + 2 \sum_{r=0}^{20} {}^{20}C_r$$

$$= 60 \sum_{r=1}^{20} {}^{19}C_{r-1} + 2 \sum_{r=0}^{20} {}^{20}C_r$$

$$= 60 \times 2^{19} + 2 \times 2^{20} = 2^{21}[15 + 1] = 2^{25}$$

Question183

The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to

[Online April 15, 2018]

Options:

A. 52

B. 44

C. 50

D. 56

Answer: A

Solution:

Solution:

$$\because (1+x)^2 = 1 + 2x + x^2,$$

$$(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$$

$$\text{and } (1+x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$$

So, the possible combinations for x^{10} are:

$$x \cdot x^9, x \cdot x^6 \cdot x^3, x^2 \cdot x^2 \cdot x^6, x^4 \cdot x^6$$

Corresponding coefficients are $2 \times 4, 2 \times 1 \times 4, 1 \times 3 \times 6, 3 \times 6$ or $8, 8, 18, 18$

\therefore Sum of the coefficient is $8 + 8 + 18 + 18 = 52$

Therefore, the coefficient of x^{10} in the expansion of

$$(1+x)^2(1+x^2)^3(1+x^3)^4 \text{ is } 52.$$

Question184

If n is the degree of the polynomial, $\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \right]^8$ and m is

the coefficient of x^n in it, then the ordered pair (n, m) is equal to
[Online April 15, 2018]

Options:

A. $(12, (20)^4)$

B. $(8, 5(10)^4)$

C. $(24, (10)^8)$

D. $(12, 8(10)^4)$

Answer: D

Solution:

Solution:

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \right]^8$$

After rationalise the polynomial we get

$$\begin{aligned} & \left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \right]^8 \\ & + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \right]^8 \\ & = \left[\frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)} \right]^8 + \left[\frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)} \right]^8 \\ & = \frac{1}{2^8} [(\sqrt{5x^3+1}+\sqrt{5x^3-1})^8 + (\sqrt{5x^3+1}-\sqrt{5x^3-1})^8]^8 \end{aligned}$$

$$= \frac{1}{2^8} \left[\begin{array}{c} {}^8C_0(\sqrt{5x^3+1})^8 + {}^8C_2(\sqrt{5x^3+1})^6(\sqrt{5x^3-1})^2 \\ + {}^8C_4(\sqrt{5x^3+1})^4(\sqrt{5x^3-1})^4 \\ + {}^8C_6(\sqrt{5x^3+1})^2(\sqrt{5x^3-1})^6 + {}^8C_8(\sqrt{5x^3-1})^8 \end{array} \right]$$

$$= \frac{1}{2^8} \left[\begin{array}{c} {}^8C_0(5x^3+1)^4 + {}^8C_2(5x^3+1)^3(5x^3-1) + {}^8C_4 \\ (5x^3+1)^2(5x^3-1)^2 \\ + {}^8C_6(5x^3+1)(5x^3-1)^3 + {}^8C_8(5x^3-1)^4 \end{array} \right]$$

So, the degree of polynomial is 12 .

Now, coefficient of $x^{12} = [{}^8C_05^4 + {}^8C_25^4 + {}^8C_45^4 + {}^8C_65^4 + {}^8C_85^4]$

$$= 5^4 \times \frac{2^8}{2} = 5^4 \times 2^4 \times \frac{2^4}{2}$$

$$= 10^4 \times 2^3 = 8(10)^4$$

Question185

The coefficient of x^2 in the expansion of the product

$(2 - x^2) \cdot ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is

[Online April 16, 2018]

Options:

A. 106

B. 107

C. 155

D. 108

Answer: A

Solution:

Solution:

$$\text{Let } a = ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$$

\therefore Coefficient of x^2 in the expansion of the product

$$(2 - x^2)((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$$

$$= 2(\text{Coefficient of } x^2 \text{ in } a) - 1(\text{Constant of expansion})$$

In the expansion of $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$.

$$\text{Constant} = 1 + 1 = 2$$

$$\text{Coefficient of } x^2 = [\text{Coefficient of } x^2 \text{ in } ({}^6C_0(1 + 2x)^6(3x^2)^0)]$$

$$+ [\text{Coefficient of } x^2 \text{ in } ({}^6C_1(1 + 2x)^5(3x^2)^1)] - [{}^6C_1(4x^2)]$$

$$= 60 + 6 \times 3 - 24 = 54$$

\therefore The coefficient of x^2 in $(2 - x^2)((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$$= 2 \times 54 - 1(2) = 108 - 2 = 106$$

Question186

The sum of the co-efficients of all odd degree terms in the expansion of

$$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1) \text{ is :}$$

[2018]

Options:

- A. 0
- B. 1
- C. 2
- D. -1

Answer: C

Solution:

Solution:

Since we know that,

$$(x+a)^5 + (x-a)^5 = 2[{}^5C_0x^5 + {}^5C_2x^3 \cdot a^2 + {}^5C_4x \cdot a^4]$$

$$\therefore \left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$$

$$= 2[{}^5C_0x^5 + {}^5C_2x^3(x^3 - 1) + {}^5C_4x(x^3 - 1)^2]$$

$$\Rightarrow 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$$

$$\therefore \text{Sum of coefficients of odd degree terms} = 2$$

Question187

If $(27)^{999}$ is divided by 7, then the remainder is:

[Online April 8, 2017]

Options:

- A. 1
- B. 2
- C. 3
- D. 6

Answer: D

Solution:

Solution:

$$\frac{(28-1)^{999}}{7} = \frac{28\lambda - 1}{7}$$

$$\Rightarrow \frac{28\lambda - 7 + 7 - 1}{7} = \frac{7(4\lambda - 1) + 6}{7}$$

$$\therefore \text{Remainder} = 6$$

Question188

The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}$ where

$x \neq 0, 1$, is

[Online April 9, 2017]

Options:

A. 1

B. 4

C. -4

D. -1

Answer: A

Solution:

Solution:

$$\left[\frac{(x^{1/3} + 1)(x^{2/3} - x^{1/3} + 1)}{(x^{2/3} - x^{1/3} + 1)} - \frac{(\sqrt{x} - 1)(\sqrt{x} + 1)}{\sqrt{x}(\sqrt{x} - 1)} \right]^{10}$$
$$= (x^{1/3} + 1 - 1 - 1/x^{1/2})^{10} = (x^{1/3} - 1/x^{1/2})^{10}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{20-5r}{6}}$$

for $r = 10$

$$T_{11} = {}^{10}C_{10} x^{-5}$$

$$\text{Coefficient of } x^{-5} = {}^{10}C_{10}(1)(-1)^{10} = 1$$

Question189

The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$ is:

[2017]

Options:

A. $2^{20} - 2^{10}$

B. $2^{21} - 2^{11}$

C. $2^{21} - 2^{10}$

D. $2^{20} - 2^9$

Answer: A

Solution:

Solution:

$$\begin{aligned} & \text{We have } ({}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_{10}) \\ &= \frac{1}{2} [({}^{21}C_1 + \dots + {}^{21}C_{10}) + ({}^{21}C_{11} + \dots + {}^{21}C_{20})] - (2^{10} - 1) \\ & (\because {}^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_{10} = 2^{10} - 1) \\ &= \frac{1}{2} [2^{21} - 2] - (2^{10} - 1) \\ &= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10} \end{aligned}$$

Question190

If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}} + \frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, ($x > 0$), are m and n respectively, then $\frac{m}{n}$ is equal to :

[Online April 10, 2016]

Options:

- A. 27
- B. 182
- C. $\frac{5}{4}$
- D. $\frac{4}{5}$

Answer: B

Solution:

Solution:

$$\begin{aligned} T_{r+1} &= {}^{18}C_r \left(x^{\frac{1}{3}}\right)^{18-r} \left(\frac{1}{2x^{\frac{1}{3}}}\right)^r = {}^{18}C_r x^{6 - \frac{2r}{3}} \frac{1}{2^r} \\ & \left\{ \begin{array}{l} 6 - \frac{2r}{3} = -2 \Rightarrow r = 12 \\ 6 - \frac{2r}{3} = -4 \Rightarrow r = 15 \end{array} \right\} \\ \Rightarrow \frac{\text{coefficient of } x^{-2}}{\text{coefficient of } x^{-4}} &= \frac{{}^{18}C_{12} \frac{1}{2^{12}}}{{}^{18}C_{15} \frac{1}{2^{15}}} = 182 \end{aligned}$$

Question191

If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is:
[2016]

Options:

- A. 243
- B. 729
- C. 64
- D. 2187

Answer: B

Solution:

Solution:

Total number of terms = ${}^{n+2}C_2 = 28$

$(n+2)(n+1) = 56$; $n = 6$

\therefore Put $x = 1$ in expansion $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^6$

we get sum of coefficient = $(1 - 2 + 4)^6$
 $= 3^6 = 729$

Question192

If the coefficients of the three successive terms in the binomial expansion of $(1 + x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is:
[Online April 10, 2015]

Options:

- A. 8th
- B. 6th
- C. 7th
- D. 9th

Answer: C

Solution:

Solution:

$$\frac{{}^nC_r}{1} = \frac{{}^nC_{r+1}}{7} = \frac{{}^nC_{r+2}}{42}$$

By solving we get $r = 6$
so, it is 7th term.

Question193

The term independent of x in the binomial expansion of $\left(1 - \frac{1}{x} + 3x^5\right) \left(2x^2 - \frac{1}{x}\right)^8$ is :

[Online April 11, 2015]

Options:

- A. 496
- B. -496
- C. 400
- D. -400

Answer: C

Solution:

Solution:

General term of $\left(2x^2 - \frac{1}{x}\right)^8$ is ${}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r$

∴ Given expression is equal to

$$\left(1 - \frac{1}{x} + 3x^5\right) {}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r = {}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r - \frac{1}{x} {}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r$$

$$+ 3x^5 \cdot {}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r$$

$$= {}^8C_r 2^{8-r} (-1)^r x^{16-3r} - {}^8C_r 2^{8-r} (-1)^r x^{15-3r}$$

$$+ 3 \cdot {}^8C_r 2^{(8-r)} \left(\frac{-1}{x}\right)^r (-1)^r x^{21-3r}$$

For the term independent of x, we should have

$$16 - 3r = 0, 15 - 3r = 0, 21 - 3r = 0$$

From the simplification we get $r = 5$ and $r = 7$

$$\therefore -{}^8C_5(2^3)(-1)^5 - 3 \cdot {}^8C_7 \cdot 2$$

$$+ \left[\frac{8!}{5!3!} \times 8 \right] - 3 \times \left[\frac{8!}{7!1!} \times 2 \right]$$

$$= (56 \times 8) - 48$$

$$= 448 - 6 \times 8 = 448 - 48 = 400$$

Question194

The sum of coefficients of integral power of x in the binomial expansion $(1 - 2\sqrt{x})^{50}$ is :
[2015]

Options:

- A. $\frac{1}{2}(3^{50} - 1)$
- B. $\frac{1}{2}(2^{50} + 1)$
- C. $\frac{1}{2}(3^{50} + 1)$
- D. $\frac{1}{2}(3^{50})$

Answer: C

Solution:

Solution:

$$\text{We know that } (a+b)^n + (a-b)^n \\ = 2[{}^nC_0 a^n b^0 + {}^nC_2 a^{n-2} b^2 + {}^nC_4 a^{n-4} b^4 \dots]$$

$$(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50} \\ 2[{}^{50}C_0 + {}^{50}C_2(2\sqrt{x})^2 + {}^{50}C_4(2\sqrt{x})^4 \dots] \\ = 2[{}^{50}C_0 + {}^{50}C_2 2^2 x + {}^{50}C_4 2^4 x^2 + \dots]$$

Putting $x = 1$, we get

$${}^{50}C_0 + {}^{50}C_2 2^2 + {}^{50}C_4 2^4 \dots = \frac{3^{50} + 1}{2}$$

Question195

If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to:

[2014]

Options:

A. $\left(14, \frac{272}{3}\right)$

B. $\left(16, \frac{272}{3}\right)$

C. $\left(16, \frac{251}{3}\right)$

D. $\left(14, \frac{251}{3}\right)$

Answer: B

Solution:

Solution:

$$\text{Consider } (1 + ax + bx^2)(1 - 2x)^{18} \\ = (1 + ax + bx^2)[{}^{18}C_0 - {}^{18}C_1(2x) \\ + {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 - \dots]$$

$$\text{Coeff. of } x^3 = {}^{18}C_3(-2)^3 + a \cdot (-2)^2 \cdot {}^{18}C_2 + b(-2) \cdot {}^{18}C_1 = 0$$

$$\text{Coeff. of } x^4 = -{}^{18}C_3 \cdot 8 + a \times 4 \cdot {}^{18}C_2 - 2b \times 18 = 0$$

$$= -\frac{18 \times 17 \times 16}{6} \cdot 8 + \frac{4a + 18 \times 17}{2} - 36b = 0$$

$$= -51 \times 16 \times 8 + a \times 36 \times 17 - 36b = 0$$

$$= -34 \times 16 + 51a - 3b = 0 \Rightarrow 51a - 3b = 34 \times 16 = 544$$

$$= 51a - 3b = 544 \dots (i)$$

Only option number (b) satisfies the equation number (i)

Question196

If $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$ and $Y = \{9(n-1) : n \in \mathbb{N}\}$, where \mathbb{N} is the set of natural numbers, then $X \cup Y$ is equal to:

[2014]

Options:

- A. X
- B. Y
- C. N
- D. $Y - X$

Answer: B

Solution:

Solution:

$$\begin{aligned}4^n - 3n - 1 &= (1+3)^n - 3n - 1 \\&= [{}^n C_0 + {}^n C_1 \cdot 3 + {}^n C_2 \cdot 3^2 + \dots + {}^n C_n 3^n] - 3n - 1 \\&= 9[{}^n C_2 + {}^n C_3 \cdot 3 + \dots + {}^n C_n \cdot 3^{n-2}]\end{aligned}$$

$\therefore 4^n - 3n - 1$ is a multiple of 9 for all n

$\therefore X = \{x : x \text{ is a multiple of } 9\}$

Also, $Y = \{9(n-1) : n \in \mathbb{N}\}$

$= \{ \text{All multiples of } 9 \}$

Clearly $X \subset Y \therefore X \cup Y = Y$

Question 197

The number of terms in the expansion of $(1+x)^{101}(1+x^2-x)^{100}$ in powers of x is:
[Online April 9, 2014]

Options:

- A. 302
- B. 301
- C. 202
- D. 101

Answer: C

Solution:

Solution:

Given expansion is

$$\begin{aligned}(1+x)^{101}(1-x+x^2)^{100} \\&= (1+x)(1+x)^{100}(1-x+x^2)^{100} \\&= (1+x)[(1+x)(1-x+x^2)]^{100} \\&= (1+x)[(1-x^3)]^{100}\end{aligned}$$

Expansion $(1-x^3)^{100}$ will have $100+1=101$ terms

So, $(1+x)(1-x^3)^{100}$ will have $2 \times 101 = 202$ terms

Question198

If $1 + x^4 + x^5 = \sum_{i=0}^5 a_i(1+x)^i$, for all x in \mathbb{R} , then a_2 is:
[Online April 12, 2014]

Options:

- A. -4
- B. 6
- C. -8
- D. 10

Answer: A

Solution:

Solution:

$$\begin{aligned}1 + x^4 + x^5 &= \sum_{i=0}^5 a_i(1+x)^i \\&= a_0 + a_1(1+x) + a_2(1+x)^2 + a_3(1+x)^3 \\&\quad + a_4(1+x)^4 + a_5(1+x)^5 \\&\Rightarrow 1 + x^4 + x^5 \\&= a_0 + a_1(1+x) + a_2(1+2x+x^2) + a_3(1+3x+3x^2+x^3) \\&\quad + a_4(1+4x+6x^2+4x^3+x^4) + a_5(1+5x+10x^2+10x^3+5x^4+x^5) \\&\Rightarrow 1 + x^4 + x^5 \\&= a_0 + a_1 + a_1x + a_2 + 2a_2x + a_2x^2 + a_3 + 3a_3x \\&\quad + 3a_3x^2 + a_3x^3 + a_4 + 4a_4x + 6a_4x^2 + 4a_4x^3 + a_4x^4 + a_5 \\&\quad + 5a_5x + 10a_5x^2 + 10a_5x^3 + 5a_5x^4 + a_5x^5 \\&\Rightarrow 1 + x^4 + x^5 \\&= (a_0 + a_1 + a_2 + a_3 + a_4 + a_5) + x(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5) \\&\quad + x^2(a_2 + 3a_3 + 6a_4 + 10a_5) + x^3(a_3 + 4a_4 + 10a_5) \\&\quad + x^4(a_4 + 5a_5) + x^5(a_5)\end{aligned}$$

On comparing the like coefficients, we get

$$a_5 = 1 \dots (i) ; a_4 + 5a_5 = 1 \dots \dots (ii) ;$$

$$a_3 + 4a_4 + 10a_5 = 0 \dots \dots (iii)$$

$$\text{and } a_2 + 3a_3 + 6a_4 + 10a_5 = 0 \dots (iv)$$

from (i) & (ii), we get

$$a_4 = -4 \dots (v)$$

from (i), (iii) & (v), we get

$$a_3 = +6 \dots \dots (vi)$$

Now, from (i), (v) and (vi), we get

$$a_2 = -4$$

Question199

If $\left(2 + \frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are:

[Online April 12, 2014]

Options:

- A. 7th and 8th
- B. 8th and 9th
- C. 28th and 29th
- D. 27th and 28th

Answer: A

Solution:

Solution:

Let r^{th} and $(r+1)^{\text{th}}$ term has equal coefficient

$$\left(2 + \frac{x}{3}\right)^{55} = 2^{55} \left(1 + \frac{x}{6}\right)^{55}$$

$$r^{\text{th}} \text{ term} = 2^{5555} C_r \left(\frac{x}{6}\right)^r$$

$$\text{Coefficient of } x^r \text{ is } 2^{5555} C_r \frac{1}{6^r}$$

$$(r+1)^{\text{th}} \text{ term} = 2^{5555} C_{r+1} \left(\frac{x}{6}\right)^{r+1}$$

$$\text{Coefficient of } x^{r+1} \text{ is } 2^{5555} C_{r+1} \cdot \frac{1}{6^{r+1}}$$

Both coefficients are equal

$$2^{5555} C_r \frac{1}{6^r} = 2^{5555} C_{r+1} \frac{1}{6^{r+1}}$$

$$\frac{1}{|r|55-r} = \frac{1}{|r+1|54-r} \cdot \frac{1}{6}$$

$$6(r+1) = 55 - r$$

$$6r + 6 = 55 - r$$

$$7r = 49$$

$$r = 7 \quad (r+1) = 8$$

Coefficient of 7th and 8th terms are equal.

Question200

The coefficient of x^{1012} in the expansion of $(1 + x^n + x^{253})^{10}$, (where $n \leq 22$ is any positive integer), is

[Online April 19, 2014]

Options:

- A. 1
- B. $^{10}C_4$
- C. 4n
- D. $^{253}C_4$

Answer: B

Solution:

Solution:

Given expansion $(1 + x^n + x^{253})^{10}$

Let $x^{1012} = (1)^a(x^n)^b \cdot (x^{253})^c$

Here a, b, c, n are all +ve integers and $a \leq 10, b \leq 10, c \leq 4, n \leq 22, a + b + c = 10$

Now $bn + 253c = 1012$

$\Rightarrow bn = 253(4 - c)$

For $c < 4$ and $n \leq 22; b > 10$, which is not possible.

$\therefore c = 4, b = 0, a = 6$

$\therefore x^{1012} = (1)^6 \cdot (x^n)^0 \cdot (x^{253})^4$

Hence the coefficient of $x^{1012} = \frac{10!}{6!0!4!} = {}^{10}C_4$

Question201

If the 7th term in the binomial expansion of $\left(\frac{3}{\sqrt{[3]84}} + \sqrt{3} \ln x \right)^9, x > 0$, is equal to 729,

then x can be

[Online April 22, 2013]

Options:

A. e^2

B. e

C. $\frac{e}{2}$

D. 2e

Answer: B

Solution:

Solution:

Let $r + 1 = 7 \Rightarrow r = 6$

Given expansion is

$\left(\frac{3}{\sqrt{[3]84}} + \sqrt{3} \ln x \right)^9, x > 0$

We have

$T_{r+1} = {}^nC_r(x)^{n-r}a^r$ for $(x+a)^n$

\therefore According to the question

$729 = {}^9C_6 \left(\frac{3}{\sqrt{[3]84}} \right)^3 \cdot (\sqrt{3} \ln x)^6$

$\Rightarrow 3^6 = 84 \times \frac{3^3}{84} \times 3^3 \times (6 \ln x)$

$\Rightarrow (\ln x)^6 = 1 \Rightarrow (\ln x)^6 = (\ln e)^6$

$\Rightarrow x = e$

Question202

If for positive integers $r > 1$, $n > 2$, the coefficients of the $(3r)^{\text{th}}$ and $(r + 2)^{\text{th}}$ powers of x in the expansion of $(1 + x)^{2n}$ are equal, then n is equal to :
 [Online April 25, 2013]

Options:

- A. $2r + 1$
- B. $2r - 1$
- C. $3r$
- D. $r + 1$

Answer: A

Solution:

Solution:

Expansion of $(1 + x)^{2n}$ is $1 + {}^{2n}C_1x + {}^{2n}C_2x^2 + \dots + {}^{2n}C_r x^r + {}^{2n}C_{r+1}x^{r+1} + \dots + {}^{2n}C_{2n}x^{2n}$

As given ${}^{2n}C_{r+2} = {}^{2n}C_3$

$$\Rightarrow \frac{(2n)!}{(r+2)!(2n-r-2)!} = \frac{(2n)!}{(3r)!(2n-3r)!}$$

$$\Rightarrow (3r)!(2n-3r)! = (r+2)!(2n-r-2)! \dots (1)$$

Now, put value of n from the given choices.

Choice (a) put $n = 2r + 1$ in (1)

$$\text{LH S : } (3r)!(4r+2-3r)! = (3r)!(r+2)!$$

$$\text{RH S : } (r+2)!(3r)!$$

$$\Rightarrow \text{LH S} = \text{RH S}$$

Question203

The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}} + 3^{\frac{1}{5}}\right)^{10}$ is:
 [Online April 23, 2013]

Options:

- A. 25
- B. 32
- C. 9
- D. 41

Answer: D

Solution:

Solution:

$$(2^{1/2} + 3^{1/5})^{10} = {}^{10}C_0(2^{1/2})^{10}$$

$$+ {}^{10}C_1(2^{1/2})^9(3^{1/5}) + \dots + {}^{10}C_{10}(3^{1/5})^{10}$$

There are only two rational terms - first term and last term.

Now sum of two rational terms
 $= (2)^5 + (3)^2 = 32 + 9 = 41$

Question204

The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}} \right)^{10}$ is

[2013]

Options:

- A. 4
- B. 120
- C. 210
- D. 310

Answer: C

Solution:

Solution:

Given expression can be written as

$$\left[\frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{(\sqrt{x})^2 - 1^2}{\sqrt{x}(\sqrt{x} - 1)} \right]^{10}$$
$$= \left((x^{1/3} + 1) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}} \right) \right)^{10} = \left(x^{1/3} + 1 - 1 - \frac{1}{\sqrt{x}} \right)^{10}$$
$$= (x^{1/3} - x^{-1/2})^{10}$$

General term = T_{r+1}

$$= {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r = {}^{10}C_r x^{\frac{10-r}{3}} \cdot (-1)^r \cdot x^{-\frac{r}{2}}$$
$$= {}^{10}C_r (-1)^r \cdot x^{\frac{10-r}{3} - \frac{r}{2}}$$

Term will be independent of x when $\frac{10-r}{3} - \frac{r}{2} = 0$

$$\Rightarrow r = 4$$

So, required term = $T_5 = {}^{10}C_4 = 210$

Question205

The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x} \right)^{15}$ is :

[Online April 9, 2013]

Options:

- A. 7: 16
- B. 7: 64

C. 1: 4

D. 1: 32

Answer: D

Solution:

Solution:

$$T_{r+1} = {}^{15}C_r (x^2)^{15-r} \cdot (2x^{-1})^r = {}^{15}C_r \times (2)^r \times x^{30-3r}$$

For independent term, $30 - 3r = 0 \Rightarrow r = 10$

Hence the term independent of x ,

$$T_{11} = {}^{15}C_{10} \times (2)^{10}$$

For term involve x^{15} , $30 - 3r = 15 \Rightarrow r = 5$

Hence coefficient of $x^{15} = {}^{15}C_5 \times (2)^5$

$$\text{Required ratio} = \frac{{}^{15}C_5 \times (2)^5}{{}^{15}C_{10} \times (2)^{10}} = \frac{\frac{15!}{10!5!}}{\frac{15!}{5!10!} \times (2)^5}$$

$$= 1 : 32$$

Question206

If n is a positive integer, then $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$ is
[2012]

Options:

A. an irrational number

B. an odd positive integer

C. an even positive integer

D. a rational number other than positive integers

Answer: A

Solution:

Solution:

Consider $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$

$$= 2 \left[\begin{matrix} 2n \\ C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \dots \end{matrix} \right]$$

$$\because (a+b)^n - (a-b)^n$$

$$= 2[{}^nC_1 a^{n-1} b + {}^nC_3 a^{n-3} b^3 \dots]$$

= which is an irrational number.

Question207

If $f(y) = 1 - (y - 1) + (y - 1)^2 - (y - 1)^3$
 $+ \dots - (y - 1)^{17}$ then the coefficient of y^2 in it is [O

[Online May 7, 2012]

Options:

A. ${}^{17}C_2$

B. ${}^{17}C_3$

C. ${}^{18}C_2$

D. ${}^{18}C_3$

Answer: D

Solution:

Solution:

Given function is

$$f(y) = 1 - (y - 1) + (y - 1)^2 - (y - 1)^3 + \dots - (y - 1)^{17}$$

In the expansion of $(y - 1)^n$

$$T_{r+1} = {}^nC_r y^{n-r} (-1)^r$$

$$\text{coeff of } y^2 \text{ in } (y - 1)^2 = {}^2C_0$$

$$\text{coeff of } y^2 \text{ in } (y - 1)^3 = -{}^3C_1$$

$$\text{coeff of } y^2 \text{ in } (y - 1)^4 = {}^4C_2$$

So, coeff of termwise is

$${}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 + \dots + {}^{17}C_{15}$$

$$= 1 + {}^3C_1 + {}^4C_2 + {}^5C_3 + \dots + {}^{17}C_{15}$$

$$= ({}^3C_0 + {}^3C_1) + {}^4C_2 + {}^5C_3 + \dots + {}^{17}C_{15}$$

$$= {}^4C_1 + {}^4C_2 + {}^5C_3 + \dots + {}^{17}C_{15}$$

$$= {}^5C_2 + {}^5C_3 + \dots + {}^{17}C_{15}$$

$$= {}^{18}C_{15} = {}^{18}C_3$$

Question208

The number of terms in the expansion of $(y^{1/5} + x^{1/10})^{55}$, in which powers of x and y are free from radical signs are

[Online May 12, 2012]

Options:

A. six

B. twelve

C. seven

D. five

Answer: A

Solution:

Solution:

Given expansion is $\left(y\frac{1}{5} + x\frac{1}{10}\right)^{55}$

The general term is

$$T_{r+1} = {}^{55}C_r \left(y\frac{1}{5}\right)^{55-r} \cdot \left(x\frac{1}{10}\right)^r$$

T_{r+1} would be free from radical sign if powers of y and x are integers.

i.e. $\frac{55-r}{5}$ and $\frac{r}{10}$ are integers.

$\Rightarrow r$ is a multiple of 10.

Hence, $r = 0, 10, 20, 30, 40, 50$

It is an A.P.

Thus, $50 = 0 + (k-1)10$

$50 = 10k - 10 \Rightarrow k = 6$

Thus, the six terms of the given expansion in which x and y are free from radical signs.

Question 209

The middle term in the expansion of $\left(1 - \frac{1}{x}\right)^n (1-x)^n$ in powers of x is

[Online May 26, 2012]

Options:

A. $-{}^{2n}C_{n-1}$

B. $-{}^{2n}C_n$

C. ${}^{2n}C_{n-1}$

D. ${}^{2n}C_n$

Answer: D

Solution:

Solution:

Given expansion can be written as

$$\left(\frac{x-1}{x}\right)^n \cdot (1-x)^n = (-1)^n x^{-n} (1-x)^{2n}$$

Total number of terms will be $2n+1$ which is odd ($\because 2n$ is always even)

$$\therefore \text{Middle term} = \frac{2n+1+1}{2} = (n+1)\text{th}$$

$$\text{Now, } T_{r+1} = {}^nC_r (1)^r x^{n-r}$$

$$\text{So, } \frac{{}^{2n}C_n \cdot x^{2n-n}}{x^n \cdot (-1)^n} = {}^{2n}C_n \cdot (-1)^n$$

Middle term is an odd term. So, $n+1$ will be odd. So, n will be even.

$$\therefore \text{Required answer is } {}^{2n}C_n$$

Question 210

Statement -1: For each natural number n , $(n+1)^7 - 1$ is divisible by 7.

Statement -2: For each natural number n , $n^7 - n$ is divisible by 7.

[2011 RS]

Options:

- A. Statement- 1 is true, Statement- 2 is true; Statement- 2 is a correct explanation for Statement-1.
- B. Statement- 1 is true, Statement- 2 is true; Statement-2 is NOT a correct explanation for Statement-1
- C. Statement- 1 is true, Statement-2 is false
- D. Statement-1 is false, Statement- 2 is true

Answer: A

Solution:

Solution:

Statement 2:

$P(n) : n^7 - n$ is divisible by 7

Put $n = 1$, $1 - 1 = 0$ is divisible by 7, which is true

Let $n = k$, $P(k) : k^7 - k$ is divisible by 7, true

Put $n = k + 1$

$\therefore P(k+1) : (k+1)^7 - (k+1)$ is div. by 7 $P(k+1) : k^7 + {}^7C_1k^6 + {}^7C_2k^5 + \dots + {}^7C_6k + 1 - k - 1$, is div. by 7.

$P(k+1) : (k^7 - k) + ({}^7C_1k^6 + {}^7C_2k^5 + \dots + {}^7C_6k)$ is div. by 7

Since 7 is coprime with 1,2,3,4,5,6.

So ${}^7C_1, {}^7C_2, \dots, {}^7C_6$ are all divisible by 7

$\therefore P(k+1)$ is divisible by 7

Hence $P(n) : n^7 - n$ is divisible by 7

Statement 1: $n^7 - n$ is divisible by 7

$\Rightarrow (n+1)^7 - (n+1)$ is divisible by 7

$\Rightarrow (n+1)^7 - n^7 - 1 + (n^7 - n)$ is divisible by 7

$\Rightarrow (n+1)^7 - n^7 - 1$ is divisible by 7

Hence both Statements 1 and 2 are correct and Statement 2 is the correct explanation of Statement -1

Question211

The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

[2011]

Options:

- A. -132
- B. -144
- C. 132
- D. 144

Answer: B

Solution:

Solution:

$$(1 - x - x^2 + x^3)^6 = [(1 - x) - x^2(1 - x)]^6$$

$$= (1 - x)^6(1 - x^2)^6$$

$$= (1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6) \times (1 - 6x^2 + 15x^4 - 20x^6 + 15x^8 - 6x^{10} + x^{12})$$

Coefficient of $x^7 = (-6)(-20) + (-20)(15) + (-6)(-6)$
 $= -144$

Question212

Let $S_1 = \sum_{j=1}^{10} j(j-1)^{10} C_j$, $S_2 = \sum_{j=1}^{10} j^{10} C_j$ and $S_3 = \sum_{j=1}^{10} j^{210} C_j$.

Statement -1: $S_3 = 55 \times 2^9$

Statement -2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

[2010]

Options:

- A. Statement - 1 is true, Statement -2 is true; Statement -2 is not a correct explanation for Statement -1.
- B. Statement -1 is true, Statement -2 is false.
- C. Statement -1 is false, Statement -2 is true.
- D. Statement - 1 is true, Statement 2 is true; Statement -2 is a correct explanation for Statement -1 .

Answer: B

Solution:

Solution:

$$S_2 = \sum_{j=1}^{10} j^{10} C_j = \sum_{j=1}^{10} 10^9 C_{j-1}$$

$$\left[\because {}^n C_r = \frac{n^{n-1}}{r} C_{r-1} \right]$$

$$= 10[{}^9 C_0 + {}^9 C_1 + {}^9 C_2 + \dots + {}^9 C_9] = 10 \cdot 2^9$$

Question213

The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is:

[2009]

Options:

- A. 2
- B. 7
- C. 8
- D. 0

Answer: A

Solution:

Solution:

$$\begin{aligned}
& (8)^{2n} - (62)^{2n+1} \\
&= (64)^n - (62)^{2n+1} \\
&= (63+1)^n - (63-1)^{2n+1} \\
&= [{}^n C_0 (63)^n + {}^n C_1 (63)^{n-1} + {}^n C_2 (63)^{n-2} + \dots + {}^n C_{n-1} (63) + {}^n C_n] + {}^{2n+1} C_2 (63)^{2n-1} - \dots + (-1)^{2n+1} C_{2n+1} \\
&= 63 \times [{}^n C_0 (63)^{n-1} + {}^n C_1 (63)^{n-2} + {}^n C_2 (63)^{n-3} + \dots + {}^n C_{n-1}] + 1 - 63 \times \\
&\quad [{}^{2n+1} C_0 (63)^{2n} - {}^{2n+1} C_1 (63)^{2n-1} + \dots + {}^{2n+1} C_{2n}] + 1 \\
&= 63 \times \text{some integral value} + 2 \\
&\text{Hence, when divided by 9 leaves 2 as the remainder.}
\end{aligned}$$

Question214

Statement -1: $\sum_{r=0}^n (r+1)^n C_r = (n+2)2^{n-1}$

Statement-2: $\sum_{r=0}^n (r+1)^n C_r x^r = (1+x)^n + nx(1+x)^{n-1}$.

[2008]

Options:

- A. Statement -1 is false, Statement- 2 is true
- B. Statement - 1 is true, Statement- 2 is true; Statement -2 is a correct explanation for Statement-1
- C. Statement -1 is true, Statement- 2 is true; Statement -2 is not a correct explanation for Statement-1
- D. Statement -1 is true, Statement- 2 is false

Answer: B

Solution:

Solution:

From statement 2:

$$\begin{aligned}
\sum_{r=0}^n (r+1)^n C_r x^r &= \sum_{r=0}^n r \cdot {}^n C_r x^r + \sum_{r=0}^n {}^n C_r x^r \\
&= \sum_{r=1}^n r \cdot \frac{n}{r} {}^{n-1} C_{r-1} x^r + (1+x)^n = nx \sum_{r=1}^n {}^{n-1} C_{r-1} x^{r-1} + (1+x)^n \\
&= nx(1+x)^{n-1} + (1+x)^n = \text{RH S}
\end{aligned}$$

∴ Statement 2 is correct.

Putting $x = 1$, we get

$$\sum_{r=0}^n (r+1)^n C_r = n \cdot 2^{n-1} + 2^n = (n+2) \cdot 2^{n-1}.$$

∴ Statement 1 is also true and statement 2 is a correct explanation for statement 1

Question215

In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$

Statement -2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6A 's and 4B 's in a row.

[2008]

Options:

- A. Statement -1 is false, Statement-2 is true
- B. Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
- C. Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
- D. Statement -1 is true, Statement-2 is false

Answer: A

Solution:

Solution:

The given situation in statement 1 is equivalent to find the non negative integral solutions of the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 6$$

which is coeff. of x^6 in the expansion of

$$(1 + x + x^2 + x^3 + \dots + \infty)^5 = \text{coeff. of } x^6 \text{ in } (1 - x)^{-5}$$

$$= \text{coeff. of } x^6 \text{ in } 1 + 5x + \frac{5 \cdot 6}{2!} x^2 + \dots$$

$$= \frac{5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{6!} = \frac{10!}{6!4!} = {}^{10}C_6$$

\therefore Statement 1 is wrong.

Number of ways of arranging 6A 's and 4B 's in a row

$$= \frac{10!}{6!4!} = {}^{10}C_6 \text{ which is same as the number of ways the child can buy six icecreams.}$$

\therefore Statement 2 is true.

Question216

In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then a/b equals

[2007]

Options:

A. $\frac{n-5}{6}$

B. $\frac{n-4}{5}$

C. $\frac{5}{n-4}$

D. $\frac{6}{n-5}$

Answer: B

Solution:

Solution:

$T_{r+1} = (-1)^r \cdot {}^nC_r (a)^{n-r} \cdot (b)^r$ is an expansion of $(a - b)^n$

$$\therefore 5 \text{ th term} = t_5 = t_{4+1}$$

$$= (-1)^4 \cdot {}^nC_4 (a)^{n-4} \cdot (b)^4 = {}^nC_4 \cdot a^{n-4} \cdot b^4$$

$$6 \text{ th term} = t_6 = t_{5+1} = (-1)^5 {}^nC_5 (a)^{n-5} (b)^5$$

$$\text{Given } t_5 + t_6 = 0$$

$$\therefore {}^nC_4 \cdot a^{n-4} \cdot b^4 + (-1)^5 {}^nC_5 \cdot a^{n-5} \cdot b^5 = 0$$

$$\begin{aligned} &\Rightarrow \frac{n!}{4!(n-4)!} \cdot \frac{a^n}{a^4} \cdot b^4 - \frac{n!}{5!(n-5)!} \cdot \frac{a^n b^5}{a^5} = 0 \\ &\Rightarrow \frac{n! \cdot a^n b^4}{4!(n-5)! a^4} \left[\frac{1}{(n-4)} - \frac{b}{5 \cdot a} \right] = 0 \quad [\because a \neq 0, b \neq 0] \\ &\Rightarrow \frac{1}{n-4} - \frac{b}{5a} = 0 \Rightarrow \frac{a}{b} = \frac{n-4}{5} \end{aligned}$$

Question217

The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots - \dots + {}^{20}C_{10}$ is
[2007]

Options:

- A. 0
- B. ${}^{20}C_{10}$
- C. $-{}^{20}C_{10}$
- D. $\frac{1}{2^{20}}C_{10}$

Answer: D

Solution:

Solution:

$$\begin{aligned} &\text{We know that, } (1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 \\ &+ \dots + {}^{20}C_{10}x^{10} + \dots + {}^{20}C_{20}x^{20} \\ &\text{Put } x = -1, (0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots \\ &+ {}^{20}C_{10} - {}^{20}C_{11} + \dots + {}^{20}C_{20} \\ &\Rightarrow + \dots - {}^{20}C_9 + {}^{20}C_{10} \\ &\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 \\ &+ \dots - {}^{20}C_9 + {}^{20}C_{10} \end{aligned}$$

Question218

For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \dots$ and $a_1 = a_2 = 10$,
then (m, n) is
[2006]

Options:

- A. (20,45)
- B. (35,20)
- C. (45,35)

D. (35,45)

Answer: D

Solution:

Solution:

$$\begin{aligned} & (1-y)^m(1+y)^n \\ &= [1 - {}^m C_1 y + {}^m C_2 y^2 - \dots][1 + {}^n C_1 y + {}^n C_2 y^2 + \dots] \\ &= 1 + (n-m)y + \left\{ \frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn \right\} y^2 + \dots \end{aligned}$$

$$\therefore a_1 = n - m = 10$$

$$\text{and } a_2 = \frac{m^2 + n^2 - m - n - 2mn}{2} = 10$$

$$(m-n)^2 - (m+n) = 20$$

$$\Rightarrow m+n = 80 \dots \text{(ii) [from (i)]}$$

Solving (i) and (ii), we get

$$\therefore m = 35, n = 45$$

Question219

If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx} \right) \right]^{11}$ equals the coefficient of x^{-7} in

$\left[ax - \left(\frac{1}{bx^2} \right) \right]^{11}$, then a and b satisfy the relation

[2005]

Options:

A. $a - b = 1$

B. $a + b = 1$

C. $\frac{a}{b} = 1$

D. $ab = 1$

Answer: D

Solution:

Solution:

T_{r+1} in the expansion

$$\begin{aligned} \left[ax^2 + \frac{1}{bx} \right]^{11} &= {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx} \right)^r \\ &= {}^{11}C_r (a)^{11-r} (b)^{-r} (x)^{22-2r-r} \end{aligned}$$

For the Coefficient of x^7 , we have

$$22 - 3r = 7 \Rightarrow r = 5$$

\therefore Coefficient of x^7

$$= {}^{11}C_5 (a)^6 (b)^{-5}$$

Again T_{r+1} in the expansion

$$\begin{aligned} \left[ax - \frac{1}{bx^2} \right]^{11} &= {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2} \right)^r \\ &= {}^{11}C_r (a)^{11-r} (-1)^r \times (b)^{-r} (x)^{-2r+11-r} \end{aligned}$$

For the Coefficient of x^{-7} , we have
 Now $11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$
 \therefore Coefficient of x^{-7}
 $= {}^{11}C_6 a^5 \times 1 \times (b)^{-6}$
 \therefore Coefficient of $x^7 =$ Coefficient of x^{-7}
 From (i) and (ii),
 $\therefore {}^{11}C_5 (a)^6 (b)^{-5} = {}^{11}C_6 a^5 \times (b)^{-6}$
 $\Rightarrow ab = 1$

Question220

If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$
 may be approximated as
 [2005]

Options:

A. $1 - \frac{3}{8}x^2$

B. $3x + \frac{3}{8}x^2$

C. $-\frac{3}{8}x^2$

D. $\frac{x}{2} - \frac{3}{8}x^2$

Answer: C

Solution:

Solution:

$\therefore x^3$ and higher powers of x may be neglected

$$\begin{aligned} \therefore & \frac{(1+x)^{\frac{3}{2}} - \left(1 + \frac{x}{2}\right)^3}{\left(1 - x^{\frac{1}{2}}\right)} \\ &= (1-x)^{\frac{-1}{2}} \left[\left(1 + \frac{3x}{2} + \frac{\frac{3}{2} \cdot \frac{1}{2} x^2}{2!}\right) - \left(1 + \frac{3x}{2} + \frac{3 \cdot 2}{2!} \frac{x^2}{4}\right) \right] \\ &= \left[1 + \frac{x}{2} + \frac{\frac{1}{2} \cdot \frac{3}{2} x^2}{2!} \right] \left[\frac{-3}{8} x^2 \right] = \frac{-3}{8} x^2 \end{aligned}$$

Question221

The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is
 [2004]

Options:

- A. $(-1)^{n-1}n$
- B. $(-1)^n(1-n)$
- C. $(-1)^{n-1}(n-1)^2$
- D. $(n-1)$

Answer: B

Solution:

Solution:

Coeff. of x^n in $(1+x)(1-x)^n$

= coeff of x^n in

$$(1+x)(1 - {}^n C_1 x + {}^n C_2 x^2 - \dots + (-1)^n {}^n C_n x^n)$$

$$= (-1)^n {}^n C_n + (-1)^{n-1} n {}^n C_{n-1}$$

$$= (-1)^n + (-1)^{n-1} \cdot n$$

$$= (-1)^n(1-n)$$

Question222

The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals
[2004]

Options:

- A. $\frac{3}{5}$
- B. $\frac{10}{3}$
- C. $\frac{-3}{10}$
- D. $\frac{-5}{3}$

Answer: C

Solution:

Solution:

The middle term in the expansion of

$$(1 + \alpha x)^4 = T_3 = {}^4 C_2 (\alpha x)^2 = 6\alpha^2 x^2$$

The middle term in the expansion of

$$(1 - \alpha x)^6 = T_4 = {}^6 C_3 (-\alpha x)^3 = -20\alpha^3 x^3$$

According to the question

$$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$$

Question223

The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is
[2003]

Options:

- A. 35
- B. 32
- C. 33
- D. 34

Answer: C

Solution:

$$\begin{aligned} T_{r+1} &= {}^{256}C_r (\sqrt{3})^{256-r} (\sqrt[8]{5})^r \\ &= {}^{256}C_r (3)^{\frac{256-r}{2}} (5)^{r/8} \end{aligned}$$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer.

It is possible if r is an integral multiple of 8 and $0 \leq r \leq 256$
 $\therefore r = 33$

Question224

If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is
[2003]

Options:

- A. 6 th term
- B. 7 th term
- C. 5 th term
- D. 8 th term

Answer: D

Solution:

$$T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} (x)^r$$

For first negative term, $n-r+1 < 0 \Rightarrow r > n+1$

$$\Rightarrow r > \frac{32}{5} \therefore r = 7 \cdot \left(\because n = \frac{27}{5} \right)$$

Therefore, first negative term is T_8

Question225

r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r + 2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1 + x)^{2n}$ are equal, then n equals
[2002]

Options:

- A. $3r$
- B. $3r + 1$
- C. $2r$
- D. $2r + 1$

Answer: C

Solution:

Solution:

$$t_{r+2} = {}^{2n}C_{r+1}x^{r+1}; t_{3r} = {}^{2n}C_{3r-1}x^{3r-1}$$

Given that, ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;

$$\Rightarrow r + 1 + 3r - 1 = 2n$$

$$\Rightarrow 2n = 4r \Rightarrow n = 2r$$

Question226

The coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are
[2002]

Options:

- A. equal
- B. equal with opposite signs
- C. reciprocals of each other
- D. none of these

Answer: A

Solution:

Solution:

We know that $t_{p+1} = p+q C_p x^p$ and $t_{q+1} = p+q C_q x^q$

$$\therefore {}^{p+q}C_p = {}^{p+q}C_q \cdot [\text{Remember } {}^nC_r = {}^nC_{n-r}]$$

Question227

The positive integer just greater than $(1 + 0.0001)^{10000}$ is
[2002]

Options:

- A. 4
- B. 5
- C. 2
- D. 3

Answer: D

Solution:

Solution:

$$\begin{aligned}(1 + 0.0001)^{10000} &= \left(1 + \frac{1}{n}\right)^n, n = 10000 \\ &= 1 + n \cdot \frac{1}{n} + \frac{n(n-1)}{2!} \frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!} \frac{1}{n^3} + \dots + \frac{1}{n^n} \\ &= 1 + 1 + \frac{1}{2!} \left(1 - \frac{1}{n}\right) + \frac{1}{3!} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) + \dots + \frac{1}{n^n} \\ &< 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(9999)!} \\ &= 1 + \frac{1}{1!} + \frac{1}{2!} + \dots \infty = e < 3\end{aligned}$$

Question228

If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is [2002]

Options:

- A. 1594
- B. 792
- C. 924
- D. 2924

Answer: C

Solution:

Solution:

Take $a = 1$ and $b = 1$ in $(a + b)^n$.

$$2^n = 4096 = 2^{12} \Rightarrow n = 12$$

The greatest coeff = coeff of middle term.

So middle term = t_7

$$\Rightarrow t_7 = t_{6+1} = {}^{12}C_6 a^6 b^6$$

$$\Rightarrow \text{Coeff of } t_7 = {}^{12}C_6 = \frac{12!}{6!6!} = 924$$
