IAT 2023

Biology

1. Match the entries in Column I with their functions described in Column II.

	Column I		Column II
P.	Squamous epithelium	(i)	The nucleus is at the basal side of the cell; also helps in movement of particles and mucous.
Q.	Cuboidal epithelium	(ii)	The nucleus is at the basal side of the cell; also helps in secretion and absorption.
R.	Columnar epithelium	(iii)	The nucleus is at the center of the cell; also helps in secretion and absorption.
S.	Ciliated epithelium	(iv)	It serves as a diffusion barrier.

A. P-(i); Q-(ii); R-(iii); S-(iv)

B. P-(ii); Q-(iv); R-(i); S-(iii)

C. P-(iii); Q-(i); R-(iv); S-(ii)

D. P-(iv); Q-(iii); R-(ii); S-(i)

2. Which one of the following best describes peptones?

A. Zymogen form of pepsin

B. Partially digested proteins

C. Activated form of pepsin

D. An intestinal mixture of proteins, mucous and HCO_3^-

3. Match the biomolecules given in Column I with their corresponding chemical nature given in Column II.

	Column I		Column II
P.	Insulin	(i)	Secondary metabolite
Q.	Inulin	(ii)	Homopolymer
R.	Lectin	(iii)	Quaternary ammonium derivative
S.	Lecithin	(iv)	Hetropolymer

A. P-(i); Q-(iii); R-(ii); S-(iv)

B. P-(ii); Q-(iv); R-(iii); S-(i)

C. P-(iii); Q-(i); R-(iv); S-(ii)

D. P-(iv); Q-(ii); R-(i); S-(iii)

4. A mitotic drug inhibits microtubule formation. Which one of the following stages of karyokinesis will be the first to get affected by the drug?

A. Prophase B. Anaphase C. Metaphase D. Telophase

5. Which one of the following statements regarding seed structure is INCORRECT?

A. In dicots, the outer layer of the seed coat is called testa.

B. The endosperm is not present in some of the mature dicot seeds.

C. Coleoptile and coleorhiza are found in monocotyledonous seeds.

D. In monocot seeds, the membranous seed coat that is fused with the fruit wall is called the aleurone layer.

6. Which one of the following anatomical features of wood can be used to estimate the age of a tree growing in a temperate climate?

A. Spring wood and late wood.

B. Heart wood and sap wood.

C. Spring wood and heart wood.

D. Autumn wood and sap wood.

7. Which one of these statements is CORRECT about biological nitrogen fixation in plants?

A. Atmospheric nitrogen is fixed by Nitrogenase by converting N_2 to NO_3^-.

B. The catalytic redox center of Nitrogenase contains Mo and Fe as cofactors.

C. Nitrogenase can function optimally only in the presence of molecular oxygen.

D. The transport of important amides, like asparagine and glutamine, produced by transamination, to different parts of the plant body is mediated by phloem.

8. Which one of the following is an example of genetic diversity?

A. Greater variation of ecosystems found in India than in Scandinavia.

B. Higher diversity of amphibians in the Western Ghats than in the Eastern Ghats.

C. Variation in potency and concentration of reserpine produced by *Rauwolfia vomitoria*.

D. The greater diversity of plant species found in India compared to Central Asia.

9. When the ribosome encounters a stop codon in the mRNA, during translation, which one of the following binds to the stop codon?

A. Release factor

B. Rho factor

C. Termination factor

D. Sigma factor

10. Match the terms in column I with their physiological roles given in Column II.

	Column I		Column II
P.	Sertoli cells	(i)	Secretion of chorionic gonadotropin
Q.	Follicle stimulating hormone	(ii)	Carries urine away from bladder
R.	Placenta	(iii)	Carries urine away from kidney
S.	Urethra	(iv)	Provides nutrition to developing spermatozoa
		(v)	Triggers ovulation

Which one of the following combinations is correct?

A. P-(i); Q-(iv); R-(iii); S-(v)

B. P-(iii); Q-(ii); R-(v); S-(iv)

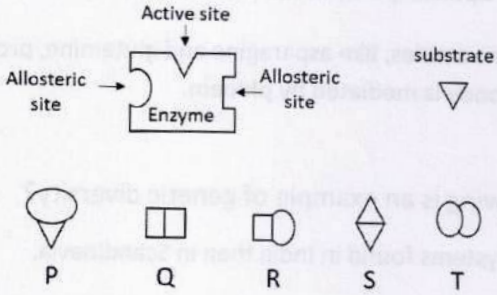
C. P-(iv); Q-(v); R-(i); S-(ii)

D. P-(v); Q-(i); R-(iv); S-(iii)

11. For which one of the following diseases does the causative agent require the splicing of their hnRNA to generate mature mRNA?

A. Tuberculosis B. Pertussis C. Typhoid D. Malaria

12. The diagram represents an enzyme, its substrate and potential inhibitors (P, Q, R, S, T).



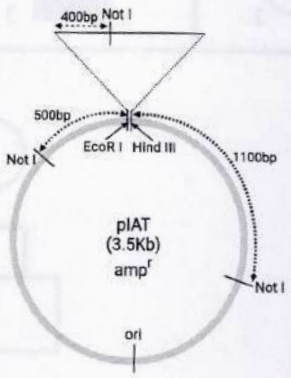
Which one of the following combinations is the best pair of competitive inhibitors for the enzyme?

A. P, S B. Q, R C. S, T D. R, T

13. In an island with 10,000 individuals, four have sickle cell anemia, a recessive autosomal disease. Assuming that the locus is in Hardy-Weinberg equilibrium, how many individuals in that island are expected to be heterozygous for the disease allele?

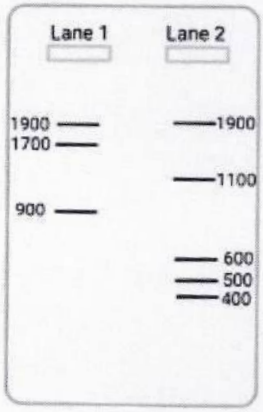
A. 4 B. 9608 C. 9996 D. 392

14. A 1000 base-pair DNA fragment was cloned between Hind III and EcoRI sites of the plasmid vector (pIAT) of size 3500 base-pair. The cloned fragment had a Not I site as shown in the figure. In order to confirm the presence of the insert, the recombinant plasmid was digested completely with (a) Not I and EcoR I, and (b) Not I and Hind III.

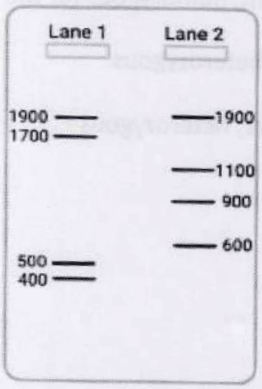


In lane 1 the products of the digestion by Not I and EcoRI was loaded. In lane 2 the products of the digestion by Not I and Hind III was loaded. Which one of the following correctly represents the agarose gel electrophoresis profile of the digested recombinant plasmid for (a) and (b), respectively?

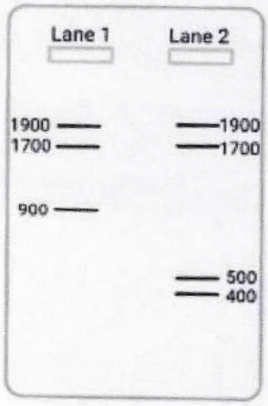
A.



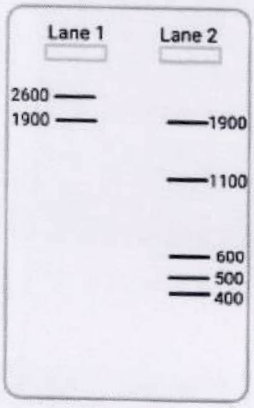
B.



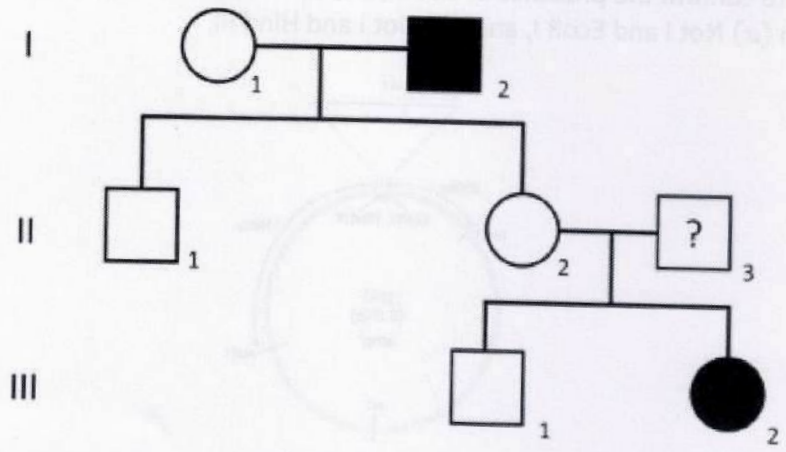
C.



D.



15. The following pedigree chart shows the inheritance of a genetic disorder. I-2 and III-2 are the only affected individuals.



Which one of the following is the correct pattern of inheritance of the disorder, and the genotype of the II- 3 individual?

A. Autosomal recessive, homozygous for the normal allele

B. Autosomal dominant, homozygous for the normal allele

C. X-linked recessive, heterozygous

D. Autosomal recessive, heterozygous

Chemistry

16. How many radial nodes does Ca^{+} have in its 4s orbital?

A. 0 B. 1 C. 2 D. 3

17. Amongst O_2, N_2, F_2, and B_2, which molecules will be attracted to an external magnetic field?

A. O_2 and B_2 B. O_2 and F_2 C. O_2, B_2, and N_2 D. F_2, N_2, and B_2

18. What is the smallest P-P-P bond angle in the highly reactive allotrope of phosphorus?

A. 45° B. 60° C. 109° D. 120°

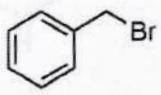
19. Which of the following is an ore of iron?

A. Malachite B. Bauxite C. Siderite D. Quartz

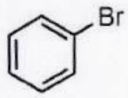
20. Which parameters are plotted in the Ellingham diagram?

A. $\Delta_r H^\circ \ vs \ T$ B. $\Delta_r G^\circ \ vs \ T$ C. $\Delta_r S^\circ \ vs \ T$ D. $\Delta_r S^\circ \ vs \ P$

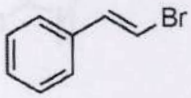
21. Which of the following compounds will NOT undergo the Finkelstein reaction with NaI via S_N2 pathway?



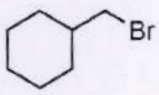
I



II



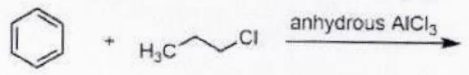
III

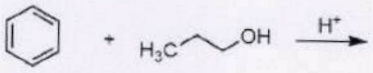


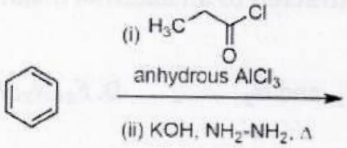
IV

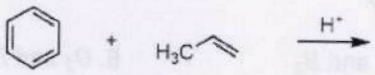
A. I and III B. II and III C. II and IV D. I and IV

22. Which one amongst the following is the most efficient way of synthesizing n-propyl benzene?

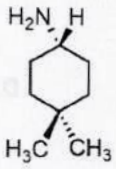
A. 

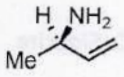
B. 

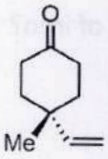
C. 

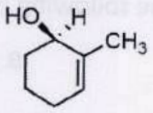
D. 

23. Which amongst the following are chiral compounds?

(I) 

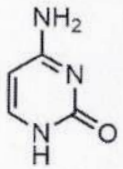
(II) 

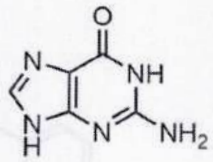
(III) 

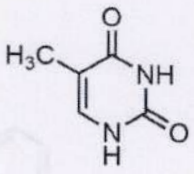
(IV) 

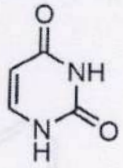
A. II and IV B. I and IV C. II and III D. I and II

24. Which one amongst the following bases is NOT present in RNA?

(I) 

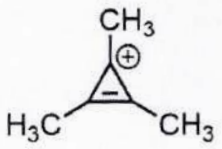
(II) 

(III) 

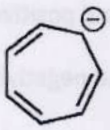
(IV) 

A. I B. II C. III D. IV

25. Which amongst the following are aromatic?

(I) 

(II) 

(III) 

(IV) 

A. *I and II* B. *I and III* C. *II and IV* D. *II and IV*

26. Why is it harder to compress liquids and solids relative to gases?

A. Solids and liquids have definite volume.

B. Due to the presence of electron-nuclear attraction in solids and liquids.

C. Due to the absence of electron-nuclear attraction in solids and liquids

D. Molecules are closer to each other in solids and liquids.

27. Related to the Freundlich adsorption isotherm, which one of the following statements is NOT correct?

A. The value of $\frac{1}{n}$ is between 0 and 1.

B. It holds good over a wide range of pressures.

C. The Freundlich adsorption isotherm equation is an empirical equation.

D. It is used for the adsorption of both gases and solutions.

28. Consider the following reaction: $CH_4(g) \rightarrow C(g)+4H(g); \Delta_a H^0 = 1665 kJmol^{-}1$ Which of the statements is FALSE?

A. $\Delta_a H^0$ is the mean bond enthalpy of a C-H bond.

B. All four C-H bonds in CH_4 are identical in bond length and energy.

C. The energy required to break individual C-H bonds in successive steps is different.

D. Mean C-H bond enthalpies differ slightly from compound to compound

29. In two solutions X (hexane and benzene) and Y (water and HCl), what types of deviations from Raoult's law are observed?

A. Positive deviation and no deviation (ideal behaviour), respectively.

B. Negative deviation and positive deviation, respectively.

C. Positive deviation and negative deviation, respectively

D. No deviation (ideal behaviour) and negative deviation, respectively.

30. In aqueous solution, the hydronium ion gets further hydrated to give which of the following species?

A. $H_3O_2^+$ B. $H_5O_3^+$ C. $H_7O_4^+$ D. $H_9O_4^+$

Mathematics

31. Let $f: \mathbf{R} \to (0, \infty)$ be a continuous decreasing function. Suppose $f(0), \dot{f}(1), \ldots, f(10)$ are in a geometric progression with common ratio $\frac{1}{5}$. In which of the following intervals does the value of $\int_0^{10} f(x)dx$ lie?

A. $(0,2f(0))$

B. $(4f(0), 6f(0))$

C. $(8f(0), 10f(0))$

D. $(12f(0), 14f(0))$

32. Let M be a 3×3 matrix with real entries such that

$$\left\{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} : M\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}\right\} = \left\{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} : x_1 + x_2 = 0 = x_2 + x_3\right\}.$$

What is the value of the determinant of M?

A. 0 B. 1 C. 2 D. 3

33. What is the locus of the center of circles passing through the origin (0,0) with fixed radius?

A. Line B. Hyperbola C. Parabola D. Circle

34. Let α be a real number. What is the total number of distinct point(s) of intersection between the parabola $y = x^2 + 4x\sin\alpha + 6$ and the pair of lines $y^2 = 1$?

A. Zero B. One C. Two D. Four

35. Let L be a straight line passing through the origin, and it makes angles α, β and γ with the positive X, Y and Z-axes, respectively. What is the value of $\cos2\alpha + \cos2\beta + \cos2\gamma$?

A. -3 B. 3 C. -1 D. 1

36. What is the total number of distinct divisors of $2^9 \times 3^{19}$?

A. 30 B. 100 C. 200 D. 435

37. Suppose the mean and the standard deviation of the data $\{x_1, x_2, \cdots, x_9\}$ are μ and $\sigma(\neq 0)$, respectively. After including one more data value x_{10}, the mean of the data $\{x_1, x_2, \cdots, x_9, x_{10}\}$ remains μ. What is the standard deviation of the data $\{x_1, x_2, \cdots, x_9, x_{10}\}$?

A. $\frac{\sqrt{10}}{3}\sigma$ B. $\frac{3}{\sqrt{10}}\sigma$ C. $\frac{10}{9}\sigma$ D. $\frac{9}{10}\sigma$

38. Consider three biased coins. Let the probability of getting head be $\frac{1}{3}$ and the probability of getting tail be $\frac{2}{3}$ in each of the coins. Consider the experiment of tossing the threee coins one by one, and the following events:

E:"At least two heads show up" F:"First coin shows tail".

What is the conditional probability of E given that F has already occurred?

A. 1/9 B. 2/9 C. 1/4 D. 4/9

39. Which of the following differential equations has $y = e^x$ as one of its particular solutions?

A. $y\frac{d^2y}{dx^2} + e^x\frac{dy}{dx} + y^2 = e^{2x}$ B. $y\frac{d^2y}{dx^2} - e^x\frac{dy}{dx} + y^2 = e^{2x}$

C. $y\frac{d^2y}{dx^2} - e^x\frac{dy}{dx} + y^2 = e^x$ D. $y\frac{d^2y}{dx^2} + e^x\frac{dy}{dx} + y^2 = e^x$

40. What is the area of the region $\{(x, y): 0 \leq y \leq xe^{x^2}, 0 \leq x \leq 1\}$?

A. $\frac{1}{2}e$ B. $e - 1$ C. $\frac{1}{2}(e - 1)$ D. $e - 2$

41. Consider the objective function $Z = x - y$ subject to the constraints

$$x + 2y \leq 10$$
$$x + y \geq 2$$
$$x \geq 0, y \geq 0.$$

What is the minimum value of Z subject to the above constraints?

A. 2 B. -2 C. -5 D. -10

42. Let $p(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients b and c. Suppose $p(1) = 5$ and $p(-1) = 3$. What is the product of the roots of $p(x) = 0$?

A. 1 B. -1 C. 2 D. 3

43. Let $f: (-1,2) \to \mathbf{R}$ be a differentiable function such that $f'(x) = \frac{2}{x^2-5}$ and $f(0) = 0$.

Then in which of the following intervals does $f(1)$ lie?

A. $(-\infty, 0)$ B. $(0,2)$ C. $(2,4)$ D. $(4, \infty)$

44. Let $f(x) = \sin(3x)$, $x \in \left[0, \frac{\pi}{2}\right]$. Which of the following statements is true

A. f is increasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

B. f is decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

C. f is increasing on $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$ and decreasing on $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

D. f is decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$ and increasing on $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

45. Which one of the following functions is differentiable at $x = 0$?

A. $|x|$ B. $|x|^{\frac{1}{2}}$ C. $\sin|x|$ D. $\cos|x|$

Physics

46. A ball is thrown vertically upwards with an initial speed u from a height h above the ground. The ball eventually hits the ground with a speed v. The acceleration due to gravity is g and air resistance is negligible. What is the average speed of the ball over its entire trajectory?

A. $\frac{gh}{2(u+v)}$

B. $\frac{u+v}{2}$

C. $\frac{u^2+v^2}{2(u+v)}$

D. $\frac{u^2+gh}{2(u+v)}$

47. Two cubes A and B of same dimensions are made of different materials with densities ρ_A and ρ_B, respectively. Cube A floats in water with a fraction η of its volume immersed. When cube B is placed on top of cube A, it is found that cube A is just entirely immersed, while cube B is entirely above the surface of the water. What is the ratio ρ_B/ρ_A?

A. $(1-\eta)/\eta$ B. $\eta/(1-\eta)$ C. η D. $1/\eta$

48. An object is placed in front of a convex lens. A real inverted image of double its size is formed. When the object is moved closer to the lens by a distance d, the image shifts away from the lens by a distance $8d$ from its previous position. What is the magnitude of the magnification in the final setup?

A. 1/2 B.4 C. 8 D. 1

49. The frequency of the whistle of a train moving with a constant speed is observed by a stationary detector on the platform to be ν_1. The frequency of the same whistle is detected to be ν_2 inside another train moving on a parallel track, at a speed v towards the frst train. If the speed of sound in air is taken to be v_sound, what is the ratio v/v_sound?

A. $\frac{\nu_2-\nu_1}{\nu_1}$ B. $\frac{\nu_2-\nu_1}{\nu_2}$ C. $\frac{\nu_2}{\nu_1}$ D. $\frac{\nu_1}{\nu_2}$

50. A current I flows through a cylindrical cable of length L and uniform cross-sectional area A. The power dissipated due to the current is P_1. The cable is cut into two equal halves. If the cross-sectional area and the current flowing through the two halves remain unchanged and the power dissipated in each half is P_2, which of these options is correct?

A. $P_2 = P_1/4$ B. $P_2 = 2P_1$ C. $P_2 = P_1$ D. $P_2 = P_1/2$

51. Particle A with charge Q and particle B with charge 2Q are fixed at positions $\vec{r}_{\mathrm{A}}$ and $\vec{r}_{\mathrm{B}}$, respectively. The force on A due to B is $\vec{F}_{\mathrm{BA}}$, and that on B due to A is $\vec{F}_{\mathrm{AB}}$. Which of the following options is correct?

A. $\vec{F}_{\mathrm{AB}} = \vec{F}_{\mathrm{BA}}$ B. $\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}$ C. $\vec{F}_{\mathrm{AB}} = 2\vec{F}_{\mathrm{BA}}$ D. $\vec{F}_{\mathrm{AB}} = -2\vec{F}_{\mathrm{BA}}$

52. The magnetic fux $\phi_B(t)$ through a coil at a time t is given by $\phi_B(t) = \phi_0 \cos\omega t$, where $0 \leq \omega t \leq \pi$ and ϕ_0 is a non-zero constant. At what time is the magnitude of the induced emf a maximum?

A. 0 B. $\frac{\pi}{\omega}$ C. $\frac{\pi}{2\omega}$ D. $\frac{\pi}{4\omega}$

53. The intrinsic electron and hole concentrations of a Si-based intrinsic semiconductor are $n_e^{(0)}$ and $n_h^{(0)}$, respectively. Upon doping with trivalent impurities the respective carrier concentrations become n_e and n_h. Which of the following options is true?

A. $n_e = n_h$ B. $n_e > n_e^{(0)}$

C. $n_h > n_h^{(0)}$ D. $n_e^{(0)}$ and $n_h^{(0)}$ are independent of temperature

54. The velocity $v(t)$ of a particle moving in one dimension is given by:

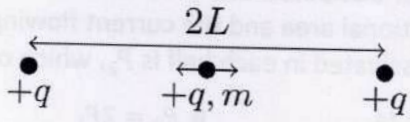
$$v(t) = \begin{cases} \alpha t, & 0 \leq t \leq T/3 \\ \alpha T/3, & T/3 \leq t \leq 2T/3 \\ \alpha(T-t), & 2T/3 \leq t \leq T, \end{cases}$$

where $\alpha(\neq 0)$ is a constant. What is the displacement of the particle from time $t = 0$ to T?

A. $\frac{2\alpha T^2}{9}$ B. $\frac{4\alpha T^2}{9}$

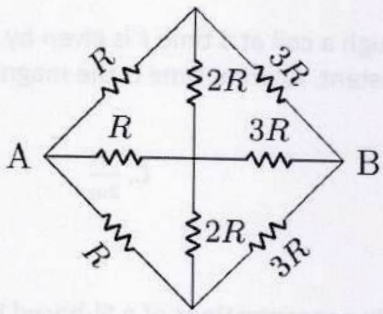
C. $\frac{8\alpha T^2}{9}$ D. $\frac{7\alpha T^2}{9}$

55. Two fixed point particles, each of charge $+q$, are separated by a distance $2L$. Another point charge $+q$ of mass m is oscillating about its equilibrium position as indicated in the figure below. The time period of oscillation is given by $T = 2\pi^{3/2}\alpha\sqrt{m}/q$. Given that ϵ_0 is the permittivity of free space, which of the following options is the dimensionally correct expression for α in SI units?



A. $\epsilon_0^{3/2}L$ B. $\epsilon_0^{1/2}L$ C. $\epsilon_0^{3/2}L^{1/2}$ D. $\epsilon_0^{1/2}L^{3/2}$

56. What is the effective resistance between A and B in the circuit shown below?

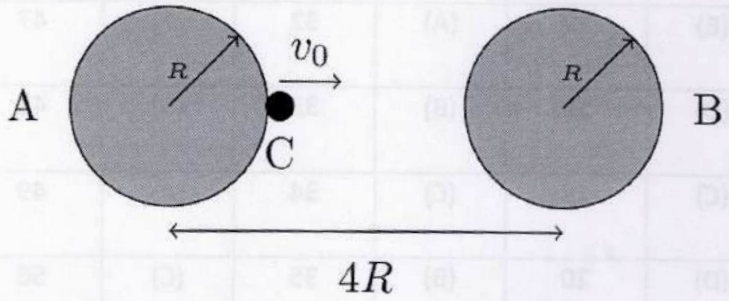


A. R/3 B. 2R/3 C. 4R/3 D. R/6

57. Consider a Carnot heat engine operating between a heat reservoir at temperature 600 K, and an external atmosphere at temperature 300 K. In one cycle, 1000 kJ of heat is extracted from the heat reservoir, and the associated work is input to a reversible refrigerator that operates between 200 K and the same external atmosphere at 300 K. The refrigerator completes one cycle and releases heat into the atmosphere. How much heat is released into the atmosphere at the end of one cycle of the combined system?

A. 2500 Kj B. 2000 kJ C. 1000 kJ D. 500 kJ

58. Two spherical bodies A and B each of mass M and radius R are located such that their centers are apart by a distance of $4R$ as shown in the figure. An object C of mass m is thrown from the surface of A directly towards the center of B with a speed $v_0 = 2v_{min}$, where v_min is the minimum speed needed for C to reach the surface of B. Given that G is the universal gravitational constant, how does the speed $v(x)$ of C change as a function of its distance from the center of A?



$A.\ v(x) = \dfrac{2\sqrt{2GMR}}{x^{\frac{1}{2}}(4R-x)^{\frac{1}{2}}}$

$B.\ v(x) = \dfrac{2R\sqrt{2GMR}}{x(4R-x)}$

$C.\ v(x) = \dfrac{\sqrt{2GMR}}{x^{\frac{1}{2}}(4R-x)^{\frac{1}{2}}}$

$D.\ v(x) = \dfrac{6R^2\sqrt{2GMR}}{x^{\frac{3}{2}}(4R-x)^{\frac{3}{2}}}$

59. Consider the Bohr model of an atom where an electron of charge $-e$ revolves around a nucleus of charge $+e$ in an orbit of radius r. The electron has an orbital angular momentum $2h$. If the nucleus had charge $+2e$, what would have been the radius of the orbit of the electron with the same principal quantum number?

A. $2r$ B. $r/2$ C. r D. $\sqrt{2}r$

60. A quantity has been measured to have a value of 0.00230 in some units. How many significant figures does the measured value have?

A. 2 B. 3 C. 4 D. 5

ANSWER KEY 2023

Question Number	Answer	Question Number	Answer	Question Number	Answer	Question Number	Answer
1	(D)	16	(D)	31	(A)	46	(C)
2	(B)	17	(A)	32	(A)	47	(A)
3	(D)	18	(B)	33	(D)	48	(B)
4	(C)	19	(C)	34	(A)	49	(A)
5	(D)	20	(B)	35	(C)	50	(D)
6	(A)	21	(B)	36	(C)	51	(B)
7	(B)	22	(C)	37	(B)	52	(C)
8	(C)	23	(A)	38	(A)	53	(C)
9	(A)	24	(C)	39	(B)	54	(A)
10	(C)	25	(A)	40	(C)	55	(D)
11	(D)	26	(D)	41	(C)	56	(C)
12	(B)	27	(B)	42	(D)	57	(B)
13	(D)	28	(A)	43	(A)	58	(A)
14	(B)	29	(D)	44	(B)	59	(B)
15	(D)	30	(D)	45	(D)	60	(B)

SOLUTIONS

1. **Squamous epithelium** is made of a single thin layer of flattened cells with irregular boundaries. They are involved in functions like forming a diffusion boundary.
Cuboidal epithelium is composed of a single layer of cube-like cells with nuclei at the centre. Its main functions are secretion and absorption.
Columnar epithelium is composed of a single layer of tall and slender cells. Their nuclei are located at the base. They help in secretion and absorption.
Ciliated epithelium bear cilia on their surfaces. Nuclei are located at the base. Their function is to move particles or mucous in a specific direction over the epithelium.

2. **Peptones** are formed when proteins are partially digested by proteolytic enzymes like pepsin in the stomach. They are intermediate products of protein digestion.

3. **Insulin** is a heteropolymer as it consists of two polypeptide chains (A and B) linked by disulphide bonds.
Inulin is a homopolymer made of fructose units linked together by β (2→1) bonds.
Lectins are carbohydrate-binding proteins that act as secondary metabolites in plants, often involved in defence mechanisms.
Lecithin is a phospholipid that contains a choline group, which is a quaternary ammonium derivative.

4. In **Metaphase**, chromosomes align at the metaphase plate in the middle of the cell. This alignment is orchestrated by microtubules, which form the mitotic spindle. A drug that inhibits microtubule formation would interfere with the spindle fibres' ability to pull chromosomes into proper alignment at the metaphase plate. Hence metaphase is affected.

5. The **aleurone layer** is a proteinaceous outer covering of the endosperm which separates it from the embryo, not the seed coat.

6. **Spring wood** (also called early wood) and **late wood** (also called autumn wood) form distinct annual rings in trees that grow in temperate climates. They are produced because the growth rate changes with the seasons. These alternating rings can be counted to estimate the age of the tree.

7. The enzyme Nitrogenase, which is responsible for biological nitrogen fixation, has a catalytic redox centre that contains molybdenum (Mo) and iron (Fe) as cofactors. These metals are crucial for the enzyme's function in reducing atmospheric nitrogen (N_2) to ammonia (NH_3).

8. Genetic diversity refers to the variation in the genetic makeup within a species. Variation in potency and concentration of reserpine produced by different populations or individuals of *Rauwolfia vomitoria* is an example of genetic diversity, as it shows differences in traits caused by underlying genetic variation.

9. When the ribosome encounters a stop codon during translation, a **release factor** binds to the stop codon. This causes the ribosome to release the newly synthesized polypeptide chain from the tRNA and the ribosome disassembles, ending translation.

10. **Sertoli cells** are nurse cells in the testes that provide support and nutrition to developing sperm cells (spermatogenesis).
Follicle-stimulating hormone (FSH) is essential in stimulating the growth of ovarian follicles in females. While **luteinizing hormone (LH)** directly triggers ovulation, FSH plays a critical role in follicular development leading up to ovulation.
The **placenta** produces human chorionic gonadotropin (hCG), a hormone important for maintaining pregnancy.
The **urethra** is the tube that carries urine from the bladder to the outside of the body.

11. Eukaryotes require the splicing of their hnRNA to generate mature mRNA for translation. **Malaria** is caused by a protozoan known as Plasmodium, which is a eukaryote (protist). Hence its hnRNA undergoes splicing. The other diseases are all caused by bacteria, which are prokaryotes.

12. Competitive inhibitors closely resemble the substrate in its molecular structure and inhibit the activity of the enzyme. **P and S** both can fit in the active site of the enzyme in the same way as the substrate would, hence inhibiting enzymatic activity.

13. Let the allelic frequency of the disease-free dominant gene be denoted by p and that of the affected one by q. In a Hardy-Weinberg Equilibrium, sum of allelic frequencies is 1, i.e.

$$p + q = (p + q)^2 = p^2 + 2pq + q^2 = 1$$

As the gene is recessive, the only affected individuals will be homozygous recessive.

$$q^2 = \frac{4}{10000}$$

$$q = \sqrt{\frac{4}{10000}} = 0.02.$$

$$p = 1 - q = 1 - 0.02 = 0.98$$

Heterozygous frequency $= 2pq = 2 \times 0.98 \times 0.02 = 0.0392$
Heterozygous individuals $= 0.0392 \times 10000 =$ **392**.

14. In case (a), the plasmid is cut at the three NotI sites and one EcoRI site, producing 4 fragments of length 1900, 1700, 500 and 400 bp. In case (b), the plasmid is cut at the three NotI sites and one HindIII site, again producing 4 fragments, this time of lengths 1900, 1100, 900 and 600 bp.

15. Disorder is **recessive** as it skips a generation. It is **autosomal** as it equally affects males and females. Individual III-2 is affected, which is only possible in a homozygous condition for a recessive disorder. Hence she receives the recessive gene from both parents, making individual II-3 **heterozygous**.

16. To determine the number of radial nodes in the 4s orbital of Ca^+, we can use the formula for calculating the number of radial nodes in an orbital:

$$\text{number of radial nodes} = n - l - 1$$

For the 4s orbital:

$$n = 4,\ l = 0 \text{ (since it is an s orbital)}$$

$$\text{Using the formula: number of radial nodes} = 4 - 0 - 1 = 3$$

So, the number of radial nodes in the 4s orbital of Ca^++ is 3.

17. To determine which molecules will be attracted to an external magnetic field, we need to consider their magnetic properties, specifically whether they are paramagnetic or diamagnetic. Paramagnetic substances are attracted to an external magnetic field, while diamagnetic substances are not.

O_2 (Oxygen molecule): O_2, is paramagnetic because it has two unpaired electrons in its molecular orbital configuration.

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2(\pi_{2p_x})^2\left(\pi_{2p_y}\right)^2(\pi_{2p_x}^*)^1\left(\pi_{2p_y}^*\right)^1$$

N_2 (Nitrogen molecule): N_2 is diamagnetic because it has all paired electrons in its molecular orbital configuration.

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\pi_{2p_x})^2\left(\pi_{2p_y}\right)^2(\sigma_{2p_z})^2$$

F_2 (Fluorine molecule): F_2 is diamagnetic because it has all paired electrons in its molecular orbital configuration.

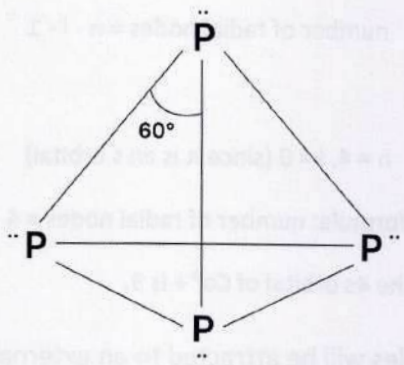
$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2(\pi_{2p_x})^2\left(\pi_{2p_y}\right)^2(\pi_{2p_x}^*)^2\left(\pi_{2p_y}^*\right)^2$$

B_2 (Boron molecule): B_2 is paramagnetic because it has two unpaired electrons in its molecular orbital configuration.

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\pi_{2p_x})^1\left(\pi_{2p_y}\right)^1$$

Therefore, the molecules that will be attracted to an external magnetic field are O_2 and B_2.

18. The smallest P-P-P bond angle in the highly reactive allotrope of phosphorus, which is white phosphorus (P_4), can be determined by looking at the structure of the P_4 molecule.



In white phosphorus, the molecule consists of four phosphorus atoms arranged in a tetrahedral shape, forming a P_4 tetrahedron. Each phosphorus atom is bonded to three other phosphorus atoms, creating a strained ring structure. The bond angles in a tetrahedron are 60° as all sides are equilateral triangles.

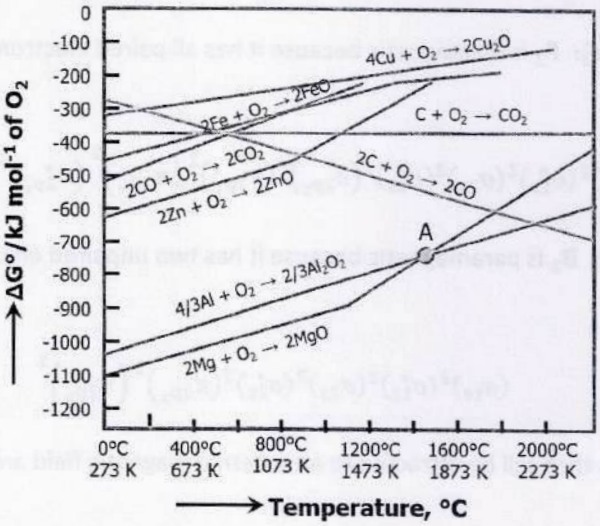
Therefore, the smallest P-P-P bond angle in the highly reactive allotrope of phosphorus (white phosphorus) is 60°.

19.

Siderite: Ore of iron. **Bauxite:** Ore of aluminum. **Malachite:** Ore of copper

Quartz: Not an ore of any metal but a common mineral consisting of silicon dioxide.

20.



$\Delta_r G^\circ$ *vs* T *is the correct answer*

21. The Finkelstein reaction is a nucleophilic substitution reaction where an alkyl halide reacts with sodium iodide (NaI) in an aprotic solvent like acetone, leading to the replacement of the halide (usually Cl or Br) with iodine. This reaction typically occurs via the S_N2 pathway, which involves a backside attack by the nucleophile and requires a relatively unhindered carbon center.

Let's evaluate each compound:

***I*-Bromophenyl methane:** This is a benzylic bromide. The benzylic position is quite reactive in S_N2 reactions due to the stability provided by resonance with the aromatic ring. This compound will undergo the Finkelstein reaction.

II*-Bromobenzene:** In this compound, the bromine is directly attached to the benzene ring. Aryl halides do not undergo S_N2 reactions because the carbon attached to the halide is part of an sp^2-hybridized aromatic system, which is not susceptible to backside attack by a nucleophile. Therefore, this compound will ***not undergo the Finkelstein reaction.

III*-1-phenyl 2-bromo ethene:** This compound is a vinyl bromide, where the bromine is attached to a carbon-carbon double bond. Vinyl halides also do not undergo S_N2 reactions due to the double bond, which prevents the necessary backside attack. Thus, this compound will ***not undergo the Finkelstein reaction.

***IV*-1-cyclohexane 1-bromo methane:** This is a primary alkyl bromide, and primary alkyl halides are usually good substrates for S_N2 reactions. Hence, this compound will undergo the Finkelstein reaction.

Conclusion: The two compounds that will **not** undergo the Finkelstein reaction via the S_N2 pathway are:

- Bromobenzene (II)
- 1-phenyl 2-bromo ethene (III)

22. To determine the most efficient way to synthesize n-propylbenzene from the given options, we need to consider the methods shown

Analysis of the Options:

Option I: Step 1: Friedel-Crafts acylation of benzene with CH_3CH_2COCl in the presence of anhydrous $AlCl_3$ to give $C_6H_5CH_2COCH_3$.

Step 2: Wolff-Kishner reduction (using hydrazine and KOH, heat) to reduce the carbonyl group to a methylene group, forming n-propylbenzene ($C_6H_5CH_2CH_2CH_3$).

Option II: Direct Friedel-Crafts alkylation of benzene with n-propyl chloride ($CH_3CH_2CH_2Cl$) in the presence of anhydrous $AlCl_3$. However, this reaction often leads to rearrangement of the propyl group, forming isopropylbenzene instead of n-propylbenzene due to the formation of a more stable secondary carbocation intermediate.

Option III: This method involves the formation of phenol ($Ph-OH$) followed by treatment with propanol ($CH_3CH_2CH_2OH$) in acidic conditions. This will likely lead to ether formation rather than forming n-propylbenzene.

Option IV: Similar to Option III, this involves phenol followed by treatment with propene ($CH_3CH=CH_2$) in acidic conditions. This will likely lead to ether formation or other side reactions rather than forming n-propylbenzene.

Conclusion: Among the given options, Option I is the most efficient way to synthesize n-propylbenzene as it involves the Friedel-Crafts acylation followed by reduction, which ensures the formation of the desired n-propyl group without rearrangement issues.

Thus, the correct answer is Option I.

23. Chiral compounds have non-superimposable mirror images (enantiomers) and typically feature a carbon atom bonded to four different groups.

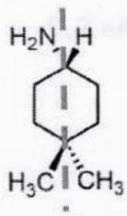
Rules to Identify Chiral Compounds

1. Chiral Center: Carbon atom bonded to four different groups.

2. Non-Superimposable: Mirror images that cannot be superimposed.

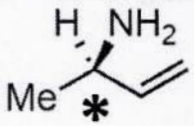
3. No Internal Plane of Symmetry(POS): No plane dividing the molecule into two identical halves.

4. Optical Activity: Rotates plane-polarized light.



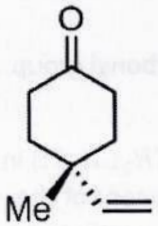
The given structure does not have chiral center and for any confusion for the carbon at the top, since there is a plane of symmetry, the carbon is not a chiral center.

Therefore, Compound I is not chiral.



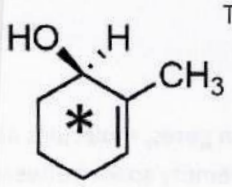
II has a chiral center and also does not have POS

Therefore, Compound II is chiral.



The carbonyl carbon is not a chiral center, as it is bonded to two identical groups (part of the ring).

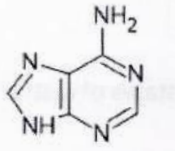
Therefore, Compound III is not chiral.

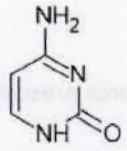


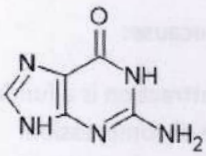
The carbon bonded to -OH, -H, -CH3, and the ring is a chiral center since it has four different groups attached.

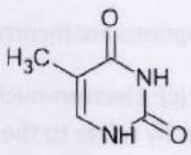
Therefore, Compound IV is chiral.

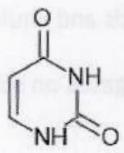
24.

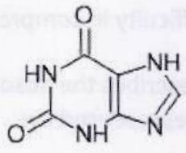

Adenine

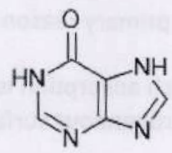

Cytosine


Guanine


Thymine


Uracil


Xanthine


Hypoxanthine

I- Cytosine II-Guanine III-Thymine IV-Uracil

The nitrogenous bases in RNA are adenine, guanine, uracil, and cytosine. III (Thymine) is NOT present in RNA.

25. Huckel's rule of aromaticity:

I-All carbons are sp^2 hybridised. There are 3 carbon atoms which form a conjugated system and moreover it has a positive charge therefore it has 3-1=2 pi electrons. If we put in Huckel's expression (4n+2)pi (n=0).

Therefore, Compound I is aromatic.

II- This is the furan cation. It is cyclic and planar. It has 6 π electrons (4 from the double bonds and 2 from the lone pair on oxygen).According to Huckel's rule, for n=1, $4n+2$=6.

Therefore, Compound II is aromatic.

26. The correct answer is: Molecules are closer to each other in solids and liquids.

Explanation:

Solids and liquids have their molecules much closer together compared to gases. In gases, molecules are far apart and move freely, making it easier to compress the gas as there is a lot of empty space between the molecules. In solids and liquids, the molecules are already packed closely together, leaving little space for compression. Therefore, it requires significantly more force to bring these molecules even closer.

Other options are incorrect because:

(b) and (c): Electron-nuclear attraction is a fundamental force present in all states of matter and does not directly relate to the ease of compression.

(a): Having a definite volume does not directly explain why it is harder to compress. The closeness of molecules is the primary reason for the difficulty in compressing solids and liquids.

27. The Freundlich adsorption isotherm describes the adsorption of gases on solid surfaces and is suitable for heterogeneous surfaces. It is represented by:

$$\frac{x}{m} = kP^{1/n}$$

where $\frac{x}{m}$ is the amount of adsorbate per unit mass of adsorbent, P is the pressure of the adsorbate, and k and n are empirical constants that depend on temperature and the nature of the adsorbent and adsorbate. The constant $1/n$ typically ranges between 0 and 1, indicating the adsorption intensity. This model is effective over a range of pressures but less accurate at very high pressures

An empirical equation is derived from experimental data and observations rather than theory. It provides a mathematical relationship that fits the observed data. The Freundlich adsorption isotherm is an empirical equation used for describing the adsorption of gases and liquids on solid surfaces.

It applies to both gases and solutions due to its versatility in fitting experimental adsorption data across various systems.

28. **1. $\Delta_a H^0$ is the mean bond enthalpy of a C-H bond.**

False. $\Delta_a H^0$ represents the total enthalpy change for breaking all four C-H bonds in one mole of CH_4 to form carbon and hydrogen atoms in the gas phase. The mean bond enthalpy would be

$$\frac{1665}{4} = 416.25\ kJ/mol.$$

2. All four C-H bonds in CH_4 are identical in bond length and energy.

True. Methane (CH_4) has a tetrahedral structure with four equivalent C-H bonds that are identical in length and energy due to its symmetrical structure.

3.The energy required to break individual C-H bonds in successive steps is different

True. In a polyatomic molecule like CH_4,the energy required to break each successive C-H bond can differ slightly due to changes in the electronic environment as each bond is broken.

4. Mean C-H bond enthalpies differ slightly from compound to compound.

True. The mean bond enthalpy can vary depending on the specific chemical environment and molecular structure in different compounds

29.

1. Solution X (Hexane and Benzene):

Nature of Interaction: Both hexane and benzene are non-polar, and their intermolecular forces are primarily London dispersion forces (van der Waals forces).

Deviation Type: **Ideal Behavior**

Since the nature and strength of intermolecular forces between hexane and benzene are similar, the solution tends to obey Raoult's law quite well. Therefore, it exhibits either ideal behavior or very slight deviations, which are usually negligible.

2. Solution Y (Water and HCl):

Nature of Interaction: Water is polar with strong hydrogen bonding, while HCl dissociates into H^+ and Cl^- ions in water, leading to strong ion-dipole interactions.

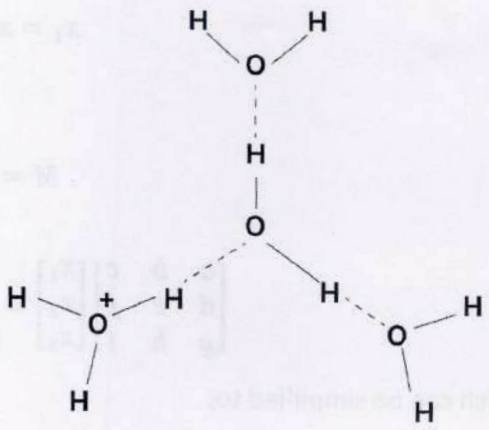
Deviation Type: **Negative Deviation.**

The ion-dipole interactions between water and HCl are much stronger than the hydrogen bonding in pure water or the relatively weaker dipole-dipole interactions in pure HCl. As a result, the vapor pressure of the solution is lower than what Raoult's law predicts, leading to a negative deviation.

30.In aqueous solution, the hydronium ion (H_3O+) can get further hydrated to form various hydrated species. Among the options provided, the species that forms is:

$$H_9O_4^+$$

This species, $H_9O_4^+$+, represents a hydronium ion that is surrounded by three water molecules, forming a more complex hydrated structure often referred to as the "Eigen cation".



31. Since it is given that $f(0), f(1) \cdots f(10)$is in a GP, we can write the general form of $f(x)$ as $f(x) = f(0)\left(\frac{1}{5}\right)^x$

$$\int a^x dx = \frac{a^x}{ln(a)} + c$$

integrating $f(x)$, we get,

$$\int_0^{10} f(x) = f(0)\left[\left(\frac{1}{5}\right)^x\right]_0^{10} \times \frac{[}{ln\left(\frac{1}{5}\right)} = f(0)\left[\frac{1}{5}^{10} - 1\right] \times \frac{1}{-ln(5)}$$

$$= \frac{f(0)\left[1 - \frac{1}{5}^{10}\right]}{ln(5)}$$

Use calculator for this part,

$$= f(0) \times 0.62$$

Hence,

$$\boxed{\int_0^{10} f(x) \in (0, 2f(0))}$$

32. Firstly,

$$x_1 + x_2 = x_2 + x_3 \qquad x_1 + x_2 = 0$$

$$\Rightarrow x_1 = x_3 \qquad \Rightarrow x_1 = -x_2$$

So let,

$$x_1 = x_3 = -x_2 = x$$

Let,

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 + cx_3 \\ dx_1 + ex_2 + fx_3 \\ gx_1 + hx_2 + ix_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Which can be simplified to,

$$\begin{bmatrix} x(a - b + c) \\ x(d - e + f) \\ x(g - h + i) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + c = b$ $\quad d + f = e$ $\quad g + i = f$

Now you can see,

$$C_1 + C_3 = C_2$$

Now using the properties of determinants,

$C_2 \to C_2 - C_1 - C_3$ will not change the value of the determinant:

$$C_2 \to C_2 - (C_1 + C_3) = 0$$

Now upon expanding along C_2:

$$|M| = \begin{vmatrix} a & \downarrow 0 & c \\ d & 0 & f \\ g & 0 & i \end{vmatrix} = 0$$

$$\boxed{\det(M) = 0}$$

33. Analysing the general equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Here, (h, k) is the center of the circle and (r) is the radius.

Since the circle passes through the origin $(0,0)$, we can substitute $x = 0$ and $y = 0$ into the equation:

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$h^2 + k^2 = r^2$$

The equation $(h^2 + k^2 = r^2)$ represents a circle with radius (r) and center at the origin $(0,0)$.

34. The lines represented by $y^2 = 1$ are $y = \pm 1$

Let,

$$y = x^2 + 4xsin(\alpha) + 6$$

First, let's find the critical points by differentiating y with respect to x:

$$\frac{dy}{dx} = 2x + 4sin(\alpha) = 0$$

$$\Rightarrow x = -2sin(\alpha)$$

$$\frac{d^2y}{dx^2} = 2 \to Minima$$

Minimum value of $f(x)$ is at $x = -2sin(\alpha)$

$$\big(-2sin(\alpha)\big)^2 - 8sin^2(\alpha) + 6 = 6 - 4sin^2(\alpha)$$

Therefore, The lowest value of of $f(x)$ is 2, which will never intersect with the pair of lines $y^2 = 1$

35. The fundamental relationship between these direction cosines is given by:

$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$$

using the identity, $cos2x = 2cos^2x - 1$

$$cos2\alpha + cos2\beta + cos2\gamma = (2cos^2\alpha - 1) + (2cos^2\beta - 1) + (2cos^2\gamma - 1)$$

$$= 2(cos^2\alpha + cos^2\beta + cos^2\gamma) - 3 = 2 - 3 = \boxed{-1}$$

36.

$$N = 2^9 \times 3^{19}$$

Divisors of N:

Any divisor of N can be written in the form $2^a \times 3^b$, where a and b are non-negative integers such that $0 \leq a \leq 9$ and $0 \leq b \leq 19$.

-As, a can take any integer value from 0 to 9. This gives us 10 possible values (0 through 9 inclusive).

-As, b can take any integer value from 0 to 19. This gives us 20 possible values (0 through 19 inclusive).

$$10 \times 20 = 200$$

37. **Mean of the Original Data Set:**

$$\mu = \frac{1}{9}\sum_{i=1}^{9} x_i \Rightarrow \sum_{i=1}^{9} x_i = 9\mu$$

Mean of the New Data Set:

Since the mean remains the same after including x_{10}

$$\mu = \frac{1}{10}\sum_{i=1}^{10} x_i = \frac{1}{10}\left(\sum_{i=1}^{9} x_i + x_{10}\right)$$

$$= \frac{1}{10}(9\mu + x_{10})$$

Solving for x_{10},

$$10\mu = 9\mu + x_{10} \Rightarrow x_{10} = \mu$$

Standard Deviation of the Original Data Set:

The standard deviation σ of the original data set is given by:

$$\sigma = \sqrt{\frac{1}{9}\sum_{i=1}^{9}(x_i - \mu)^2}$$

The variance of the new data set is:

$$\text{Var} = \frac{1}{10}\sum_{i=1}^{10}(x_i - \mu)^2$$

Breaking this sum into two parts, we have:

$$\sum_{i=1}^{10}(x_i - \mu)^2 = \sum_{i=1}^{9}(x_i - \mu)^2 + (x_{10} - \mu)^2$$

Since $x_{10} = \mu$,

$$(x_{10} - \mu)^2 = (\mu - \mu)^2 = 0$$

Therefore, $\sum_{i=1}^{10}(x_i - \mu)^2 = \sum_{i=1}^{9}(x_i - \mu)^2$

The variance of the new data set becomes: $\text{Var} = \frac{1}{10}\sum_{i=1}^{9}(x_i - \mu)^2$

Using the fact that the variance of the original data set is σ^2,

$$\sigma^2 = \frac{1}{9}\sum_{i=1}^{9}(x_i - \mu)^2$$

Therefore,

$$\sum_{i=1}^{9}(x_i - \mu)^2 = 9\sigma^2$$

Substituting this into the variance expression for the new data set, we get:

$$\text{Var} = \frac{1}{10} \cdot 9\sigma^2 = \frac{9}{10}\sigma^2$$

The standard deviation of the new data set is then:

$$\sqrt{\text{Var}} = \sqrt{\frac{9}{10}\sigma^2} = \frac{3}{\sqrt{10}}\sigma = \frac{3\sigma}{\sqrt{10}}$$

Thus, the standard deviation of the new data set $x_1, x_2, \cdots, x_9, x_{10} = \mu$ is $\frac{3\sigma}{\sqrt{10}}$.

38. Given:

$$P(\mathrm{H}) = \frac{1}{3}$$

$$P(\mathrm{T}) = \frac{2}{3}$$

Let,

$$P(F) = P(\text{Tail on first coin}) = \frac{2}{3}$$

The conditional probability $P(E|F)$ is given by:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Where:

$$P(F) = \frac{2}{3}$$

-$P(E \cap F)$ is the probability that the first coin is a tail and at least two heads show up in the three coin tosses.

Given that the first coin is a tail, we only consider the outcomes of the last two coins where at least two heads can occur: $P(\mathrm{HH}) = \frac{1}{9}$.

So,

$$P(E \cap F) = P(F) \cdot P(\mathrm{HH}) = \frac{2}{3} \cdot \frac{1}{9} = \frac{2}{27}$$

Therefore,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{27}}{\frac{2}{3}} = \frac{2}{27} \cdot \frac{3}{2} = \frac{1}{9}$$

Thus, the conditional probability $P(E|F)$ is:

$$\boxed{\frac{1}{9}}$$

39.

$$y = e^x \qquad \frac{dy}{dx} = e^x \qquad \frac{d^2y}{dx^2} = e^x$$

$$y\frac{d^2y}{dx^2} - e^x\frac{dy}{dx} + y^2 = e^{2x} - e^{2x} + e^{2x} = e^{2x}$$

$$y\frac{d^2y}{dx^2} + e^x\frac{dy}{dx} + y^2 = e^{2x} + e^{2x} + e^{2x} = 3e^{2x} \neq e^{2x}$$

$$y\frac{d^2y}{dx^2} - e^x\frac{dy}{dx} + y^2 = e^{2x} - e^{2x} + e^{2x} = e^{2x} \neq e^x$$

$$y\frac{d^2y}{dx^2} + e^x\frac{dy}{dx} + y^2 = e^{2x} + e^{2x} + e^{2x} = 3e^{2x} \neq e^x$$

40. For the given area, we have to integrate the function xe^{x^2} with the limits 0 and 1

For this we can use method of substitution,

Let $t = x^2$,

$$\frac{dt}{dx} = 2x \rightarrow x = \frac{1}{2}\frac{dt}{dx}$$

Then

$$\int_0^1 x\, e^{x^2} dx = \int_0^1 \frac{1}{2}\frac{dt}{dx} e^t dx$$

$$= \frac{1}{2}[e^t]_0^1 = \boxed{\frac{1}{2}(e-1)}$$

41. The constraints: $\begin{cases} x + 2y \leq 10 \\ x + y \geq 2 \\ x \geq 0 \\ y \geq 0 \end{cases}$

Identify the corner points (vertices) of the feasible region.

Determine the minimum value of Z.

First, convert the inequalities to equations to find the boundary lines.

$$x + 2y = 10$$
$$x + y = 2$$

Intersection Points with Axes

1. For $x + 2y = 10$:

When $x = 0$: $2y = 10 \Rightarrow y = 5$ (Point:(0,5))

When $y = 0$: $x = 10$ (Point:(10,0))

2. For $x + y = 2$:

When $x = 0 : y = 2$ (Point:(0,2))

When $y = 0$: $x = 2$ (Point:(2,0))

1. (0,5) 2. (2,0)

(x, y)	$Z = x - y$
$(0, 5)$	$Z = 0 - 5 = -5$
$(2, 0)$	$Z = 2 - 0 = 2$

42.

$$p(x) = x^2 + bx + c$$

$$p(1) = 5$$

$$\rightarrow 1 + b + c = 5$$

$$p(-1) = 3$$

$$\rightarrow 1 - b + c = 3$$

Adding both equations,

$$2c + 2 = 8 \Rightarrow c = 3$$

Multiple of the roots of $p(x) = 0$

$$= \alpha\beta = \frac{c}{a} = \boxed{3}$$

43.

$$f'(x) = \frac{2}{x^2 - 5}$$

On integrating both side wrt x,

$$f(x) = \int \frac{2}{x^2 - 5} dx$$

Using the identity,

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} ln \left| \frac{x - a}{x + a} \right| + c$$

Here, $a = \sqrt{5}$

$$f(x) = 2\int \frac{1}{x^2 - 5} dx = \frac{2}{2\sqrt{5}} ln \left| \frac{x - \sqrt{5}}{x + \sqrt{5}} \right| + c$$

It is given that, $f(0) = 0$

$$\rightarrow \frac{1}{\sqrt{5}} ln|-1| + c = 0$$

$$c = 0 \; as \; log|-1| = log1 = 0$$

$$f(1) = \frac{1}{\sqrt{5}} ln \left| \frac{1 - \sqrt{5}}{1 + \sqrt{5}} \right| = \frac{1}{\sqrt{5}} \ln \left| \frac{(1 - \sqrt{5})^2}{4} \right|$$

Using a calculator, we get $f(0) \approx -.43$ which lies in $(-\infty, 0)$

44. You can simply see this as oscillatory motion as $\sin(\omega x)$, with $\omega = 3$ and as $T = \frac{2\pi}{\omega}$, Time period of $\sin(3x)$ will be a third of $\sin(x)$

$sin(x)$ is increasing in $\left[0, \frac{\pi}{2}\right)$ and decreasing in $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$

For the function $sin(3x)$ we can simply divide thetime periods by 3 to understand the curve:

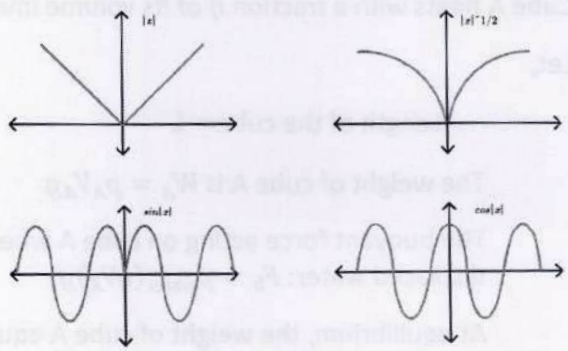
$$sin(3 \cdot 0) = sin(0)$$

$$sin\left(3 \cdot \frac{\pi}{2 \times 3}\right) = sin\left(3 \cdot \frac{\pi}{6}\right) = sin\left(\frac{\pi}{2}\right)$$

$$sin\left(3 \cdot \frac{\pi}{3/2 \times 3}\right) = sin\left(3 \cdot \frac{\pi}{2}\right) = sin\left(\frac{3\pi}{2}\right)$$

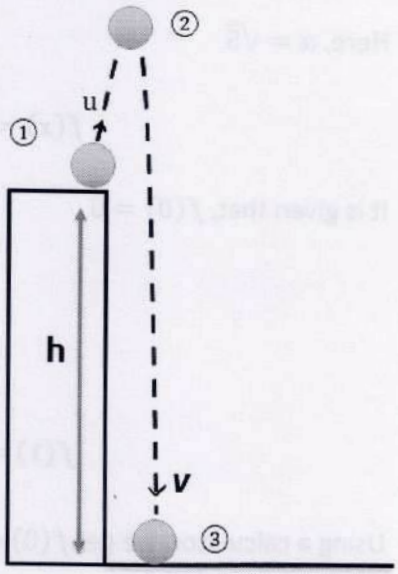
⇒ The function is increasing in the interval $\left[0, \frac{\pi}{6}\right)$ and is decreasing in the interval $\left(\frac{\pi}{6}, \frac{\pi}{2}\right]$

.45. If a curve is smooth at a point, it has no sharp turns or discontinuities, meaning a consistent tangent can be drawn. This ensures continuity and a unique slope at the point, fulfilling the conditions for differentiability. Thus, smoothness at a point guarantees the curve is differentiable there.Shortcut/advice for imagining graph: Here, The |x| makes the graph mirror itself about the y-axis(. As cos(x) is the same around y-axis, it behaves the same as cos|x|.



46. Let's break the motion down into two parts ($u \to 0, 0 \to v$)and then evaluate using:

$$s_{avg} = \frac{\text{total distance}}{\text{total time}}$$



The total time in the ascent of the ball from the point ① → ② can be calculated by using $v = u + at$:

$$t_1 = \frac{u}{g}$$

similarly, ② → ③ $t_2 = \frac{v}{g}$

$$t_{net} = t_1 + t_2 = \frac{u}{g} + \frac{v}{g} = \frac{u+v}{g}$$

And the distance can be calculated using $v^2 - u^2 = 2as$

$d_1 = \frac{u^2}{2g}$ with velocity u

$d_2 = \frac{v^2}{2g}$ with velocity v

$$d_{total} = d_1 + d_2 = \frac{u^2}{2g} + \frac{v^2}{2g} = \frac{u^2+v^2}{2g}$$

Now, we can find the average speed by dividing the total distance by the total time:

$$\text{Average speed} = \frac{d_{\text{total}}}{t_{\text{total}}} = \frac{\frac{u^2+v^2}{2g}}{\frac{u+v}{g}}$$

Therefore, the average speed over the entire path is: $\boxed{\frac{u^2+v^2}{2(u+v)}}$

47. **1. Cube A Floating Alone**:

Cube A floats with a fraction η of its volume immersed in water.

Let,

Length of the cube= L Volume of cube, $V_A = L^3$

The weight of cube A is $W_A = \rho_A V_A g$

The buoyant force acting on cube A when it is floating is equal to the weight of the displaced water: $F_b = \rho_{\text{water}}(\eta V_A)g$.

At equilibrium, the weight of cube A equals the buoyant force:

$$\rho_A V_A g = \rho_{\text{water}}(\eta V_A) g$$

Simplifying, we get:

$$\rho_A = \eta \rho_{\text{water}}$$

2. Cube B Placed on Top of Cube A:

Let the density of cube B be ρ_B.

The total weight of the system (both cubes A and B) must be supported by the buoyant force when cube A is fully submerged.

The total weight is $W_{\text{total}} = W_A + W_B = \rho_A V_A g + \rho_B V_B g$.

Since $V_A = V_B = L^3$, we can write:

$$W_{\text{total}} = \rho_A L^3 g + \rho_B L^3 g$$

The buoyant force when cube A is fully submerged is $F_b = \rho_{\text{water}} V_A g = \rho_{\text{water}} L^3 g$.

At equilibrium, the total weight equals the buoyant force:

$$\rho_A L^3 g + \rho_B L^3 g = \rho_{\text{water}} L^3 g$$

Dividing both sides by $L^3 g$, we get:

$$\rho_A + \rho_B = \rho_{\text{water}}$$

3. Combining the Equations:

From the first equation, we have $\rho_A = \eta \rho_{\text{water}}$.

Substituting ρ_A into the second equation:

$$\eta \rho_{\text{water}} + \rho_B = \rho_{\text{water}}$$

$$\rho_B = \rho_{\text{water}}(1 - \eta)$$

To find the ratio $\frac{\rho_B}{\rho_A}$:

$$\frac{\rho_B}{\rho_A} = \frac{\rho_{\text{water}}(1-\eta)}{\eta \rho_{\text{water}}} = \frac{1-\eta}{\eta}$$

Therefore, the ratio $\frac{\rho_B}{\rho_A}$ is:

$$\boxed{\frac{\rho_B}{\rho_A} = \frac{1-\eta}{\eta}}$$

48. It is given that the image is twice the size of the object $\rightarrow m = 2$:

$$m = -\frac{v}{u} = 2$$

$$v = -2u$$

When the object is moved:

$$u' = u - d$$

$$v' = v + 8d = -2u + 8d$$

Using the formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{v'} - \frac{1}{u'}$$

Now writing all terms in terms of u and d:

$$\frac{1}{-2u} - \frac{1}{u} = \frac{1}{-2u + 8d} - \frac{1}{u - d}$$

$$\frac{-3}{2u} = \frac{u - d + 2u - 8d}{(-2u + 8d)(u - d)} = \frac{3u - 9d}{-2u^2 + 2ud + 8ud - 8d^2}$$

On cross-multiplying,

$$2u^2 + 8d^2 - 10ud = 2u^2 - 6ud$$

$$8d^2 = 4ud$$

$$\Rightarrow d = \frac{u}{2}$$

Now calculating m':

$$m' = \frac{-v'}{u'} = \frac{-(-2u + 4u)}{u - u/2}$$

$$\boxed{|m| = |-4| = 4}$$

49. For the stationary detector:

The frequency observed by the stationary detector (ν_1) when the source is moving towards the detector is given by:

$$\nu_1 = \nu \frac{v_{sound}}{v_{sound} - v_{source}}$$

Where ν is the actual frequency of the train's whistle, v_{sound} is the speed of sound, and v_{source} is the speed of the train (the source).

For the moving detector:

The frequency observed by a detector in another train moving towards the first train (ν_2) is given by:

$$\nu_2 = \nu \frac{v_{sound} + v_{detector}}{v_{sound} - v_{source}}$$

Here, $v_{detector}$ is the speed of the second train, which is the same as v_{source} (i.e., v).

We need to find the ratio $\frac{v}{v_{sound}}$. To do this, we can set up a ratio of ν_2 to ν_1:

$$\frac{\nu_2}{\nu_1} = \frac{\nu\left(\frac{v_{sound} + v}{v_{sound} - v}\right)}{\nu\left(\frac{v_{sound}}{v_{sound} - v}\right)} = \frac{v_{sound} + v}{v_{sound}}$$

So,

$$\frac{\nu_2}{\nu_1} = 1 + \frac{v}{v_{sound}}$$

$$\frac{v}{v_{sound}} = \frac{\nu_2}{\nu_1} - 1$$

Therefore, the ratio $\frac{v}{v_{sound}}$ is:

$$\boxed{\frac{v}{v_{sound}} = \frac{\nu_2 - \nu_1}{\nu_1}}$$

50.

$$= \rho \frac{L}{A}$$

$$P = I^2 R$$

$$\frac{P_1}{P_2} = \frac{I_1^2 R_1}{I_2^2 R_2}$$

It is given that, $I_1 = I_2, A_1 = A_2$ and $L_2 = \frac{L_1}{2}$ and since they are made up of the same materials, $\rho_1 = \rho_2$

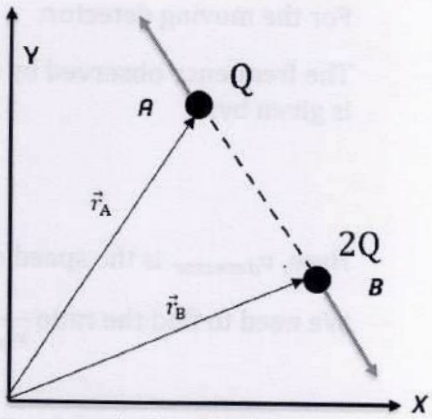
$$\frac{P_1}{P_2} = \frac{L_1}{L_2} = 2$$

$$\boxed{P_2 = \frac{P_1}{2}}$$

51. The force on particle A due to particle B ($\vec{F}_{AB}$) and the force on particle B due to particle A ($\vec{F}_{BA}$) are action-reaction pairs. According to Newton's third law of motion, these forces are equal in magnitude and opposite in direction.

Or

You can also use Coulomb's law and you will find the magnitude to be the same, and for directions, you can see they are both positively charged and will move in the opposite direction from each other.



52. The magnitude of the induced electromotive force (emf) in a coil is given by Faraday's Law of Induction:

$$\mathcal{E} = -\frac{d(\phi_B(t))}{dt}$$

Given that the magnetic flux through the coil is:

$$\phi_B(t) = \phi_0 \cos(\omega t)$$

$$\frac{d\phi_B(t)}{dt} = -\phi_0 \omega \sin(\omega t)$$

The induced emf is:

$$\mathcal{E} = -\frac{d\phi_B(t)}{dt} = \phi_0 \omega \sin(\omega t)$$

We need to find the maximum value of $|\mathcal{E}|$:

$$|\mathcal{E}| = |\phi_0 \omega \sin(\omega t)|$$

The sine function $\sin(\omega t)$ reaches its maximum value of 1

$$\omega t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega}$$

Therefore, the time at which the magnitude of the induced emf is a maximum is:

$$\boxed{t = \frac{\pi}{2\omega}}$$

53. In an intrinsic semiconductor, the intrinsic electron concentration $n_e^{(0)}$ and hole concentration $n_h^{(0)}$ are equal, i.e., $n_e^{(0)} = n_h^{(0)}$.

When the semiconductor is doped with trivalent impurities (which are known as acceptors), these impurities accept electrons, creating more holes. This type of doping is called p-type doping.

Trivalent impurities (acceptors): These impurities have three valence electrons, which means they can accept an electron to form a complete bond with the silicon atoms. This acceptance of electrons leaves behind holes.

Upon doping with trivalent impurities:

The hole concentration n_h will increase significantly because the acceptors create additional holes.

The electron concentration n_e will decrease because the acceptors capture electrons, thereby reducing the number of free electrons.

Therefore, in a Si-based semiconductor doped with trivalent impurities, the hole concentration n_h will be greater than the electron concentration n_e. Thus, $n_h > n_e$.

Note: The intrinsic electron concentration $n_e^{(0)}$ and hole concentration $n_h^{(0)}$ in a semiconductor are highly dependent on temperature. As temperature increases, both $n_e^{(0)}$ and $n_h^{(0)}$ increase.

54. It is given that:

$$v(t) = \begin{cases} \alpha t, & 0 \le t \le T/3 \\ \alpha t/3, & T/3 \le t \le 2T/3 \\ \alpha(T-t), & 2T/3 \le t \le T, \end{cases}$$

Using $s = \int v dt$, we can evaluvate the velocity in three parts for the three intervals and find total displacment:

For $0 \le t \le T/3$,

$$s_1 = \int_0^{T/3} \alpha\, t = \left[\frac{\alpha t^2}{2}\right]_0^{T/3}$$

$$s_1 = \frac{\alpha}{2}\left(\frac{T^2}{9}\right)$$

$$s_1 = \frac{\alpha T^2}{18}$$

For $T/3 \le t \le 2T/3$

$$s_2 = \int_{T/3}^{2T/3} \frac{\alpha T}{3} = \left[\frac{\alpha t}{3}\right]_{T/3}^{2T/3}$$

$$s_2 = \frac{\alpha T}{9}$$

For $2T/3 \leq t \leq T$

$$s_3 = \int_{2T/3}^{T} \alpha\,(T - t) = \alpha\left[Tt - \frac{t^2}{2}\right]_{2T/3}^{T}$$

$$= \alpha\left(T(T/3) - \frac{1}{2}\left(T^2 - \frac{4T^2}{9}\right)\right)$$

$$s_3 = \alpha\frac{T^2}{18}$$

$$s_{net} = s_1 + s_2 + s_3$$

$$\boxed{s_{net} = \frac{2\alpha T^2}{9}}$$

55. It is given that $T = 2\pi^{3/2}\alpha\sqrt{m}/q$,

$$[\alpha] = \frac{[T][q]}{[\sqrt{m}]}$$

$$[\alpha] = \frac{[T][IT]}{[M^{1/2}]}$$

$$[\alpha] = [M^{-1/2}L^0T^2I^1]$$

For the dimensions of ϵ_0, you can use Coulomb's law:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

$$[\epsilon_0] = [M^{-1}L^{-3}T^4I^2]$$

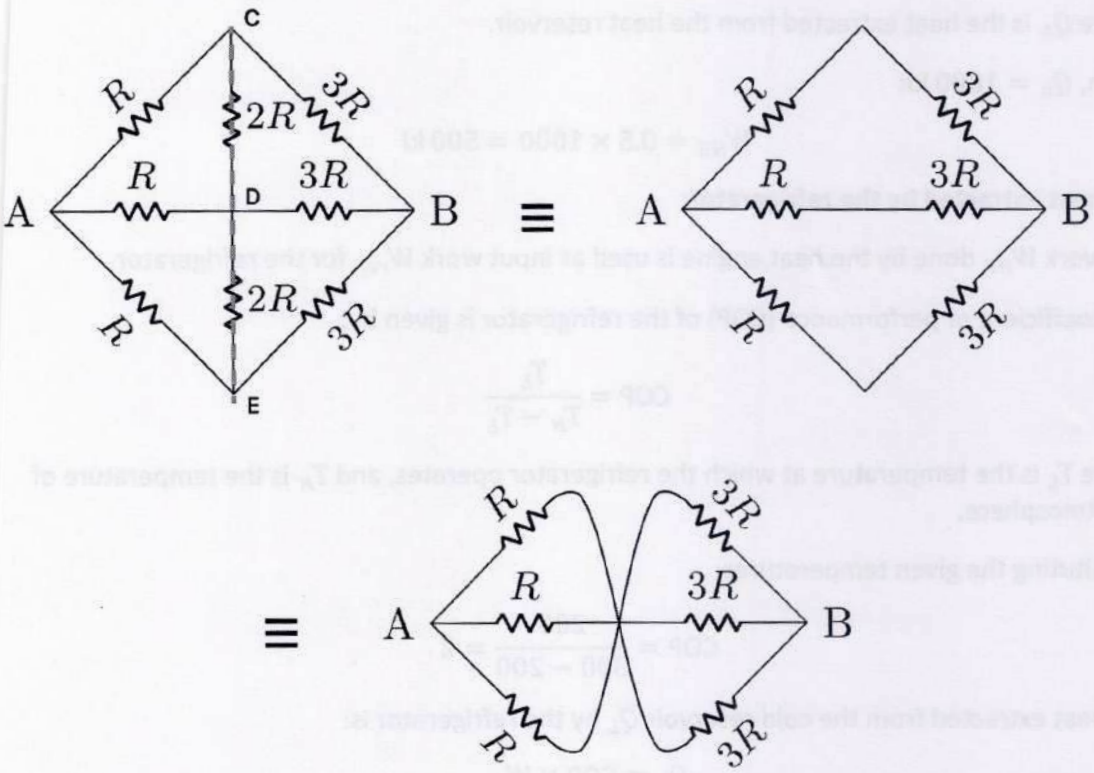
$$[\epsilon_0]^{1/2} = \left[M^{-1/2}L^{-3/2}T^2I^1\right]$$

Now, we can clearly see that $\epsilon_0^{1/2}$ dimensions are very similar to that of α except for a factor of $L^{3/2}$. So, the answer should be

$$\boxed{\epsilon_0^{1/2}L^{3/2}}$$

56. In a symmetrical circuit, equipotential points have the same voltage. Joining these points or removing resistors connecting these points does not change the voltage distribution or the overall behaviour of the circuit because no potential difference exists between them.

They are symmetrically opposite each other across the central axis will have the same potential and can be joined to reduce the complexity of the analysis.



The symmetry across the direct line from C to E shows that points C, D, and E are at the same potential. So, we can ignore the two $2R$ resistances and the circuit becomes as shown above (1st fig), this can be solved by adding in series and then in parallel. Otherwise, if all points are connected (2nd fig), add in parallel and then in series.

Which will give us

$$\boxed{R_{net} = \frac{4R}{3}}$$

57. The Carnot efficiency η is given by:

$$\eta = 1 - \frac{T_c}{T_h}$$

where T_h is the temperature of the heat reservoir and T_c is the temperature of the atmosphere.

$$\eta = 1 - \frac{300}{600} = 0.5$$

The work done by the heat engine:

$$W_{HE} = \eta Q_h$$

where Q_h is the heat extracted from the heat reservoir.

Given, $Q_h = 1000$ kJ:

$$W_{HE} = 0.5 \times 1000 = 500 \text{ kJ}$$

The heat extracted by the refrigerator:

The work W_{HE} done by the heat engine is used as input work W_{ref} for the refrigerator.

The coefficient of performance (COP) of the refrigerator is given by:

$$\text{COP} = \frac{T_L}{T_H - T_L}$$

where T_L is the temperature at which the refrigerator operates, and T_H is the temperature of the atmosphere.

Substituting the given temperatures:

$$\text{COP} = \frac{200}{300 - 200} = 2$$

The heat extracted from the cold reservoir Q_L by the refrigerator is:

$$Q_L = \text{COP} \times W_{ref}$$

Substituting $W_{ref} = 500$ kJ:

$$Q_L = 2 \times 500 = 1000 \text{ kJ}$$

Total heat released into the atmosphere:

The heat released into the atmosphere by the heat engine Q_c is:

$$Q_c = Q_h - W_{HE}$$

Substituting the values:

$$Q_c = 1000 - 500 = 500 \text{ kJ}$$

The heat released into the atmosphere by the refrigerator Q_H is:

$$Q_H = Q_L + W_{ref}$$

Substituting the values:

$$Q_H = 1000 + 500 = 1500 \text{ kJ}$$

The total heat released into the atmosphere by the combined system is:

$$Q_{\text{total}} = Q_c + Q_H$$

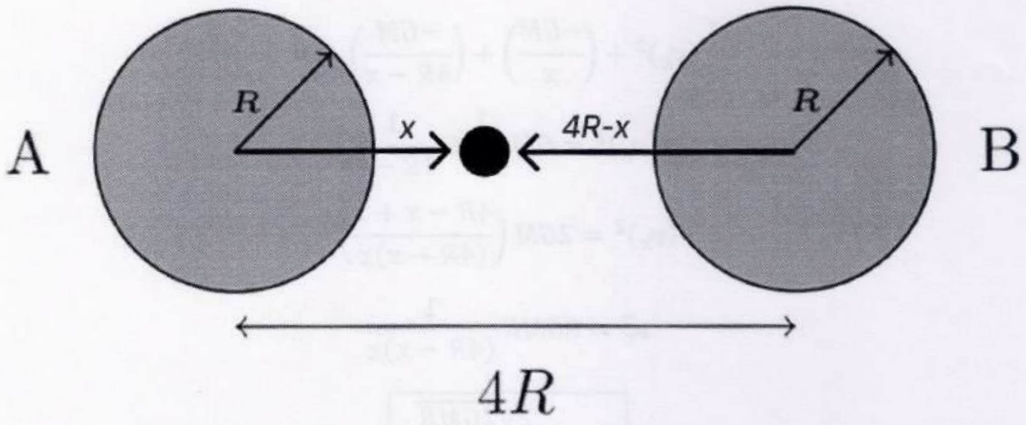
$$Q_{\text{total}} = 500 + 1500$$

$$\boxed{Q_{\text{total}} = 2000 \text{ kJ}}$$

58. Let us first understand all the given information.

$v_0 = 2v_{min}$,

Here v_{min} would mean the velocity to required to reach the midpoint of A and B as then C would have escaped the gravitational field of A.



Now to obtain the expression, we need to use the conservation of energy to find v_x:

$$\frac{1}{2}m(v_0)^2 + \left(\frac{-GMm}{R}\right) + \left(\frac{-GMm}{3R}\right) = \frac{1}{2}m(v_x)^2 + \left(\frac{-GMm}{x}\right) + \left(\frac{-GMm}{4R - x}\right)$$

But before solving:

Notice that v_0 is not present in any of the options so we need to replace v_0 with $2v_{min}$

v_{min} would also be found by conservation of energy (by equation initial energy to energy at midpoint):

$$\frac{1}{2}mv_{min} + \left(\frac{-GMm}{R}\right) + \left(\frac{-GMm}{3R}\right) = 0 + \left(\frac{-GMm}{2R}\right) + \left(\frac{-GMm}{2R}\right)$$

(velocity at mid-point will be 0)

$$\frac{1}{2}mv_{min}^2 = \frac{GMm}{3R}$$

$$\Rightarrow v_{min}^2 = \frac{2GM}{3R}$$

$$\Rightarrow v_0^2 = (2v_{min})^2 = 4 \times v_{min}^2 = \frac{8GM}{3R}$$

Now, on substituting:

$$\frac{1}{2}m\left(\frac{8GM}{3R}\right) + \left(\frac{-GMm}{R}\right) + \left(\frac{-GMm}{3R}\right) = \frac{1}{2}m(v_x)^2 + \left(\frac{-GMm}{x}\right) + \left(\frac{-GMm}{4R-x}\right)$$

Cancelling out m:

$$\left(\frac{4GM}{3R}\right) + \left(\frac{-GM}{R}\right) + \left(\frac{-GM}{3R}\right) = \frac{1}{2}(v_x)^2 + \left(\frac{-GM}{x}\right) + \left(\frac{-GM}{4R-x}\right)$$

LHS=0:

$$\frac{1}{2}(v_x)^2 + \left(\frac{-GM}{x}\right) + \left(\frac{-GM}{4R-x}\right) = 0$$

$$\frac{1}{2}(v_x)^2 = GM\left(\frac{1}{x} + \frac{1}{4R-x}\right)$$

$$(v_x)^2 = 2GM\left(\frac{4R-x+x}{(4R-x)x}\right)$$

$$v_x^2 = 8GMR\frac{1}{(4R-x)x}$$

$$\boxed{v_x = \frac{2\sqrt{2GMR}}{x^{\frac{1}{2}}(4R-x)^{\frac{1}{2}}}}$$

59. In Bohr's atomic model,

$$r_n = \frac{n^2h^2}{4\pi^2me^2Z}$$

or $r_n \propto \frac{n^2}{Z}$

-n is the principal quantum number,

-Z is the atomic number (number of protons in the nucleus, or effective charge).

In the question the only quantities we need to consider n and Z as the rest will be constant regardless of orbital angular momentum.

It is given that the second nucleus has double the charge of the first one, which means $Z_2 = 2Z_1$ and since the principal quantam number is the same $n_2 = n_1$

So,

$$\frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \cdot \frac{z_2}{z_1}$$

$$\boxed{r_2 = \frac{r_1}{2}}$$

60. The measured value (0.00230) has three significant figures.

-The zeros before the 2 are not significant; they are only there to position the decimal point.

-The digits 2 and 3 are significant.

-The trailing zero after 3 is also significant because it comes after the decimal point and a non-zero digit.