

Chapter 9. Algebraic Expressions and Identities

Question 1

Add: $11x^2 - 8x + 4$ and $6x^2 + 7x - 10$.

Solution:

$$\begin{array}{r} 11x^2 - 8x + 4 \\ 6x^2 + 7x - 10 \\ \hline 17x^2 - x - 6 \\ \hline \end{array}$$

Question 2

Find the sum of $5 + x - x^2 - x^3$, $3x - 5 + 2x^2 - x^3$, $x + 5 + 2x^3 - 5x^2$.

Solution:

$$\begin{array}{r} 5 + x - x^2 - x^3 \\ -5 + 3x + 2x^2 - x^3 \\ 5 + x - 5x^2 + 2x^3 \\ \hline 5 + 5x - 4x^2 + 0x^3 \\ \hline \end{array}$$

Question 3

Add: $3 + 2y - 5y^2 + 6y^3$, $-8 + 3y + 7y^3$ and $5 - 6y - 8y^3 + y^2$.

Solution:

$$\begin{array}{r} 3 + 2y - 5y^2 + 6y^3 \\ -8 + 3y \quad \quad + 7y^3 \\ 5 - 6y + y^2 - 8y^3 \\ \hline 0 - y - 4y^2 + 5y^3 \\ \hline \end{array}$$

Question 4

Subtract $2x^3 - x^2 + 4x - 6$ from $x^3 + 5x^2 - 4x + 6$.

Solution:

$$\begin{array}{r} x^3 + 5x^2 - 4x + 6 \\ 2x^3 - x^2 + 4x - 6 \\ (-) \quad (+) \quad (-) \quad (+) \\ \hline -x^3 + 6x^2 - 8x + 12 \\ \hline \end{array}$$

Question 5

Subtract $20 - 3x + 4x^2 - x^3$ from $x^3 - x^2 + 2x - 19$.

Solution:

$$\begin{array}{r} x^3 - x^2 + 2x - 19 \\ -x^3 + 4x^2 - 3x + 20 \\ (+) \quad (-) \quad (+) \quad (-) \\ \hline 2x^3 - 5x^2 + 5x - 39 \\ \hline \end{array}$$

Question 6

Subtract $8 - 6x + x^2 - 7x^3 + x^5$ from $x^4 - 6x^3 + x^2 - 3x + 1$.

Solution:

$$\begin{array}{r} x^4 - 6x^3 + x^2 - 3x + 1 \\ -x^5 + 0 - 7x^3 + x^2 - 6x + 8 \\ (-) \quad (+) \quad (-) \quad (+) \quad (-) \\ \hline -x^5 + x^4 + x^3 \quad + 3x - 7 \\ \hline \end{array}$$

Question 7

What must be added to $x^3 + 3x - 8$ to get $3x^3 + x^2 + 6$?

Solution:

$$\begin{array}{r} 3x^3 + x^2 + 0 + 6 \\ x^3 \quad \quad + 3x - 8 \\ (-) \quad \quad (-) \quad (+) \\ \hline 2x^3 + x^2 - 3x + 14 \\ \hline \end{array}$$

Question 8

what must be added to $y^2 + 5y - 6$ to get $y^3 - y^2 + 3y - 2$?

Solution:

$$\begin{array}{r} y^3 - y^2 + 3y - 2 \\ y^2 + 5y - 6 \\ (-) \quad (-) \quad (+) \\ \hline y^3 - 2y^2 - 2y + 4 \\ \hline \end{array}$$

Question 9

What must be subtracted from $x^3 - 3x^2 + 5x - 1$ to get $2x^3 + x^2 - 4x + 2$?

Solution:

$$\begin{array}{r} x^3 - 3x^2 + 5x - 1 \\ 2x^3 + x^2 - 4x + 2 \\ (-) \quad (-) \quad (+) \quad (-) \\ \hline -x^3 - 4x^2 + 9x - 3 \\ \hline \end{array}$$

Question 10

What must be subtracted from $y^4 + 2y^2 - 3y + 7$ to get $y^3 + y^2 + y - 1$?

Solution:

$$\begin{array}{r} y^4 + 2y^2 - 3y + 7 \\ y^3 + y^2 + y - 1 \\ (-) \quad (-) \quad (-) \quad (+) \\ \hline y^4 - y^3 + y^2 - 4y + 8 \\ \hline \end{array}$$

Question 11

Multiply

(i) $11xy^2$ by $-8xy^3$

(ii) $-\frac{5}{4}xy^2$ with $\frac{2}{3}zxy$

Solution:

(i) $(11xy^2) \times (-8xy^3) = -88x^2y^5$

(ii) $\left(-\frac{5}{4}xy^2\right) \times \left(\frac{2}{3}zxy\right) = \frac{-10}{12}x^2y^3z$

Question 12

Multiply $(7x - 4x^2 + 2x^3 - 5)$ by $(3x - 2)$.

Solution:

$$\begin{array}{r} 2x^3 - 4x^2 + 7x - 5 \\ \times (3x - 2) \\ \hline 6x^4 - 12x^3 + 21x^2 - 15x \\ \quad - 4x^3 + 8x^2 + 14x + 10 \\ \hline 6x^4 - 16x^3 + 29x^2 - x + 10 \\ \hline \end{array}$$

Question 13

Multiply $(3y^5 - 7y^3 + 2y^2 - y + 4)$ by $(y^3 - 2y^2 + 3y - 1)$.

Solution:

$$\begin{array}{r} 3y^5 - 7y^3 + 2y^2 - y + 4 \\ y^3 - 2y^2 + 3y - 1 \\ \hline 3y^8 - 7y^6 + 2y^5 - y^4 + 4y^3 \\ - 6y^7 + 14y^5 - 4y^4 + 2y^3 - 8y^2 \\ \quad + 9y^6 - 21y^4 + 6y^3 - 3y^2 + 12y \\ \quad \quad - 3y^5 + 7y^3 - 2y^2 + y - 4 \\ \hline 3y^8 - 6y^7 + 2y^6 + 13y^5 - 26y^4 + 19y^3 - 13y^2 + 13y - 4 \\ \hline \end{array}$$

Question 14

Expand $(\sqrt{3}x - 2y)^2$.

Solution:

$$\begin{aligned} (\sqrt{3}x - 2y)^2 &= (\sqrt{3}x)^2 + (2y)^2 - 2(\sqrt{3}x)(2y) \\ &= 3x^2 + 4y^2 - 4\sqrt{3}xy \end{aligned}$$

Question 15

Expand $\left(\frac{x}{6} - \frac{y}{5}\right)^2$.

Solution:

$$\left(\frac{x}{6} - \frac{y}{5}\right)^2 = \frac{x^2}{36} - 2 \times \frac{x}{6} \times \frac{y}{5} + \frac{y^2}{25} = \frac{x^2}{36} - \frac{xy}{15} + \frac{y^2}{25}$$

Question 16

Find the value of $49x^2 + 36y^2 + 84xy$ when $x = 3$, $y = 6$.

Solution:

$$\begin{aligned}49x^2 + 36y^2 + 84xy &= (7x)^2 + (6y)^2 + 2 \times (7x)(6y) \\&= (7x + 6y)^2 \\&= (7(3) + 6(6))^2 = (21 + 36)^2 \\&= (57)^2 \\&= 3249\end{aligned}$$

Question 17

Using the identity find the product of $(x - 2y)(x + 2y)(x^2 + 4y^2)$.

Solution:

$$\begin{aligned}(x - 2y)(x + 2y)(x^2 + 4y^2) &= [(x - 2y)(x + 2y)](x^2 + 4y^2) \\&= [x^2 - (2y)^2](x^2 + 4y^2) \\&= (x^2 - 4y^2)(x^2 + 4y^2) \\&= (x^2)^2 - (4y^2)^2 \\&= x^4 - 16y^4\end{aligned}$$

Question 18

Evaluate $(0.99)^2$.

Solution:

$$\begin{aligned}(0.99)^2 &= (1 - 0.01)^2 = 1^2 + (0.01)^2 - 2 \times 1 \times 0.01 \\&= 1 + 0.0001 - 0.02 \\&= 0.9801\end{aligned}$$

Question 19

Find the product $(4x + 2y)(16x - 4y)$.

Solution:

$$\begin{aligned}(4x + 2y)(16x - 4y) &= (4x + 2y)[4(4x - 2y)] \\&= 4(4x + 2y)[4x - 2y] \\&= 4[(4x)^2 - (2y)^2] \\&= 16x^2 - 4y^2\end{aligned}$$

Question 20

Expand $\left(\sqrt{3}x - \frac{\sqrt{2}}{5}y\right)^2$.

Solution:

$$\begin{aligned}\left(\sqrt{3}x - \frac{\sqrt{2}}{5}y\right)^2 &= (\sqrt{3}x)^2 - 2 \times \sqrt{3} \times \frac{\sqrt{2}x}{5} + \left(\frac{\sqrt{2}}{5}\right)^2 \\ &= 3x^2 - \frac{2}{5}\sqrt{6}x + \frac{4}{25}\end{aligned}$$

Question 21

Expand $\left(\sqrt{ax} + \frac{\sqrt{by}}{5}\right)^2$.

Solution:

$$\begin{aligned}\left(\sqrt{ax} + \frac{\sqrt{by}}{5}\right)^2 &= (\sqrt{ax})^2 + \frac{2}{5}\sqrt{a}\sqrt{b}xy + \left(\frac{\sqrt{b}}{5}\right)^2 \\ &= ax^2 + \frac{2}{5}\sqrt{ab}xy + \frac{b}{25}\end{aligned}$$

Question 22

Simplify : $(\sqrt{5}x - \sqrt{2}y)(\sqrt{5}x + \sqrt{2}y)$

Solution:

$$\begin{aligned}(\sqrt{5}x - \sqrt{2}y)(\sqrt{5}x + \sqrt{2}y) &= (\sqrt{5}x)^2 - (\sqrt{2}y)^2 \\ &= 5x^2 - 2y^2\end{aligned}$$

Question 23

Evaluate 136×107 .

Solution:

$$\begin{aligned}136 \times 107 &= (100 + 36)(100 + 7) \\ &= (100)^2 + (36 + 7) \times 100 + 36 \times 7 \\ &= 10000 + 4300 + 252 = 14552\end{aligned}$$

Question 24

Evaluate $176 \times 176 - 124 \times 124$.

Solution:

$$\begin{aligned} 176 \times 176 - 124 \times 124 &= (176)^2 - (124)^2 \\ &= (176 + 124) \times (176 - 124) \\ &= 300 \times 52 = 15600 \end{aligned}$$