Differential Equation

Q.1. Solve the differential equation : $\tan y\, dx + \sec^2 y \tan x\, dy = 0$.

Solution : 1

We have , $\tan y\, dx + \sec^2 y \tan x\, dy = 0$

Or, $\tan y\, dx = -\sec^2 y \tan x\, dy$

Or, $(1/\tan x)\, dx = -(\sec^2 y/\tan y)\, dy$

Integrating both sides , we get

$\log \sin x = -\int(\sec^2 y/\tan y)\, dy$,

Put $\tan y = t \Rightarrow \sec^2 y\, dy = dt$

Therefore , $\log \sin x = -\int dt/t = -\log t$

$= c - \log \tan y$

Or, $\log \sin x + \log \tan y = \log c$

Or , $\sin x . \tan y = c$.

Q.2. Solve the following differential equation : $x (x^2 - x^2y^2)\, dy + y (y^2 + x^2y^2)\, dx = 0$

Solution : 2

We have , $x (x^2 - x^2y^2)\, dy + y (x^2 + x^2y^2)\, dx = 0$

Or, $x3(1 - y^2)\, dy = -y^3(1 + x^2)\, dx$

Or, $\int[(1 - y^2)/y^3]dy = -\int[(1 + x^2)/x^3]\, dx$

Or, $\int[1/y^3 - 1/y]\, dy = -\int[1/x^3 + 1/x]\, dx$

Or, $[y -3 +1]/[-3 +1] - \log y = -[x -3 +1]/[-3 +1] - \log x + c$

Or, $c + 1/2x^2 + 1/2y^2 = \log x - \log y$

Or, $c + 1/2[1/x^2 + 1/y^2] = \log(x/y)$.

Q.3. Solve thee differential equation : $dy/dx - e^{x+y} = e^{x-y}$.

Solution : 3

We have $dy/dx - e^{x+y} = {}^{x-y}$

Or, $dy/dx = e^x e^{-y} + ex\, e^y = e^x (ey + e^{-y})$

Integrating , we get

$\int 1/[e^y + 1/e^y]\, d^y = \int e^x\, dx$

Or, $\int ey/[e2^y + 1]\, dy = \int e^x\, dx$

Putting $e^y = t \Rightarrow e^y dy = dt$ and we get

$\int dt/[t2 + 1] = e^x + c$

Or, $\tan -1\ (t) = e^x + c$

Or, $\tan -1\ (e^x) = e^x + c$.

Q.4. Solve the following : $dy/dx = [(1 + \cos^2 x) \sin^2 y]/[(1 + \sin^2 y)\cos^2 x]$

Solution : 4

We have $dy/dx = [(1 + \cos^2 x)\sin^2 y]/[(1 + \sin^2 y)\cos^2 x]$

Or, $\int[(1 + \sin^2 y)/\sin^2 y]dy = \int[(1 + \cos^2 x)/\cos^2 x]dx$

Or, $\int(\mathrm{cosec}^2\, y + 1)dy = \int(\sec^2 x + 1)dx$

Or, $-\cot y + y = \tan x + x + c$

Or, $y - x = \tan x + \cot y + c$.

Q.5. Solve : $e^y (1 + x^2)\, dy - x/y\, dx = 0$.

Solution : 5

We are given $e^{y} (1 + x^{2})\, dy - x/y\, dx = 0$

Multiplying by y and dividing by $(1 + x^{2})$, we get

$ye^{y}\, dy - x/(1 + x^{2})\, dx = 0$

Or, $ye^{y}\, dy = x/(1 + x^{2})\, dx$

Integrating both sides, we get

$\int y.e^{y}.dy = \int x/(1 + x^{2}).dx$

Or, $y \int e^{y}.dy - \int\{d/dy(y)\}.e^{y}.dy = 1/2 \int 2x/(1 + x^{2}).dx$

Or, $y.e^{y} - \int e^{y}.dy = 1/2 \log (1 + x^{2}) + c$

Or, $y.e^{y} - e^{y} = 1/2 \log (1 + x^{2}) + c$

Or, $e^{y}.(y - 1) = \log \{\sqrt{(1 + x^{2})}\} + c$

Therefore, $e^{y}.(y - 1) - \log \{\sqrt{(1 + x^{2})}\}= c$

Q.6. Solve : $(x^{2} - yx^{2})dy + (y^{2} + xy^{2})dx = 0$

Solution : 6

We are given,

$(x^{2} - yx^{2})dy + (y^{2} + xy^{2})dx = 0$

Or, $x^{2}(1 - y)dy + y^{2} (1 + x)dx = 0$

Or, $[(1 - y)/y^{2}]dy + [(1 + x)/x^{2}]dx = 0$

Or, $\int(1/y^{2})dy - \int(1/y)dy + \int(1/x^{2})dx + \int(1/x)dx = c$

Or, $- 1/y - \log y - 1/x + \log x = c$

Or, $(\log x - \log y) - (1/x + 1/y) = c$

Or, $(\log x - \log y) - (x + y)/xy = c$.

Q.7. Solve : $(x - y - 2)dx - (2x - 2y - 3)\, dy = 0$

Solution : 7

We are given ,

$(x - y - 2)dx - (2x - 2y - 3)dy = 0$

Put $x - y - 2 = z$, then $1 - dy/dx = dz/dx$ [by differentiation w.r.t. x]

Or, $dy/dx = 1 - dz/dx$

The given differential equation reduces to

$z - (2z + 1)(1 - dz/dx) = 0$

Or, $z - (2z + 1) + (2z + 1)dz/dx = 0$

Or, $(2z + 1)dz/dx = z + 1$

Or, $(2z + 1)dz = (z + 1)dx$

Or, $[(2z + 1)/(z + 1)]dz = dx$

Integrating, we get $\int dx = \int[(2z + 1)/(z + 1)]dz = \int[\{2(z + 1) - 1 \}/(z + 1)]dz$

Or, $x = 2\int dz - \int dz/(z + 1) + c$

Or, $x = 2z - \log (z + 1) + c$

Or, $x = 2(x - y - 2) - \log (x - y - 2 + 1) + c$

Or, $x - 2(x - y - 2) + \log (x - y - 1) = c$.

Q.8. Solve : $dy/dx = e^{x-y} + x^2 e^{-y}$.

Solution : 8

We are given $dy/dx = e^{x-y} + x^2 e^{-y}$

Or, $dy/dx = (ex + x^2)/e^y$

Or, $e^y\, dy = (ex + x^2)\, dx$

Integrating, we get

$\int e^{y}\, dy = \int ex\, dx + \int x^{2}\, dx + c$

Or, $e^{y} = ex + (x3)/3 + c$.

Q.9. Solve the following differential equation : dy/dx + sin (x + y) = sin (x – y).

Solution : 9

We are given , dy/dx + sin (x + y) = sin (x – y)

Or, dy/dx = sin (x – y) – sin (x + y)

= –2 cos [{(x – y) + (x + y)}/2].sin [{(x – y) – (x + y)}/2]

= –2 cos x sin y

Or, dy/sin y = –2 cos x dx

Integrating , we get

∫cosec y dy = –2∫cos x dx + c

Or, log tan (y/2) = –2 sin x + c

Or, log tan (y/2) + 2 sin x = c.

Q.10. Solve the following differential equation : (x cos y) dy = e^{x} (x log x + 1) dx.

Solution : 10

We have, (x cos y) dy = e^{x} (x log x + 1) dx

Or, cos y dy = [e^{x} (x log x + 1)]/x dx

Integrating we get, ∫cos y dy = ∫[e^{x} (x log x + 1)]/x dx

Or, sin y = ∫ e^{x}(log x + 1/x) dx

Or, sin y = e^{x} log x + c.