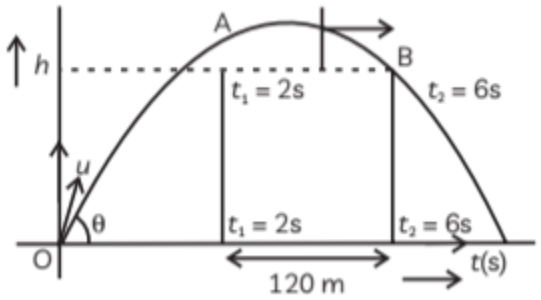
# Motion in a Plane

## Case Study Based Questions

**Read the following passages and answer the questions that fallow:**

**1.** A projectile is projected from a point O on the ground with an initial velocity u at an elevation angle from the horizontal direction as shown in the figure. It just crosses two walls A and B of same height h situated symmetrically at times $t_1$= 2s and $t_2$ = 6s respectively. The horizontal distance between the two walls is d = 120 m. (take g = 10 $m/s^2$)




**(A) The projectile motion is an example of:**

(a) one dimensional motion

(b) two dimensional motion

(c) three dimensional motion

(d) cannot say, precisely.

**(B) The total time of flight of the projectile:**

(a) 8 s

(b) 10 s

(c) 4 s

(d) 12 s

**(C) The value of angle of projection of the projectile is:**

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{5}\right)$

(c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\tan^{-1}\left(\frac{5}{4}\right)$

**(D) The projectile velocity u of the projectile is:**

(a) 30 m/s

(b) 40 m/s
(c) 50 m/s
(d) 20√3 m/s

**(E) The height h of either of two walls is:**

(a) 120 m
(b) 30 m
(c) 15 m
(d) 60 m

**Ans. (A)** (b) two dimensional motion

**(B)** (a) 8 s

**Explanation:** If a projectile passes a certain height h at two times $t_1$ and $t_2$ then,

$t1+t^2$= total time of flight = T

$T = t_1+t^2$

=2+6=8 sec.

**(C)**

(c) $\tan^{-1}\left(\frac{4}{3}\right)$

**Explanation:** We know that

$$T = \frac{2u\sin\theta}{g}$$

$$\Rightarrow \quad u\sin\theta = \frac{gT}{2}$$

$$= \frac{10\times 8}{2} = 40$$

horizontal distance between the walls d

Then, $d = (u\cos\theta)(t1 - t_2)$

$$u\cos\theta = \frac{d}{t_2 - t_1}$$

$$= \frac{120}{6-2} = 30$$

Now, $\frac{u\sin\theta}{u\cos\theta} = \frac{40}{30}$

$\Rightarrow \quad \tan\theta = \frac{4}{3}$

$\therefore \quad \theta = \tan^{-1}\frac{4}{3}$

**(D)** (c) 50 m/s

**Explanation:**

$\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{\sqrt{4^2+3^2}} = \frac{4}{5}$

$u\sin\theta = 40$

$u \times \frac{4}{5} = 40 \Rightarrow 50$ m/s

**(E)** (d) 60 m

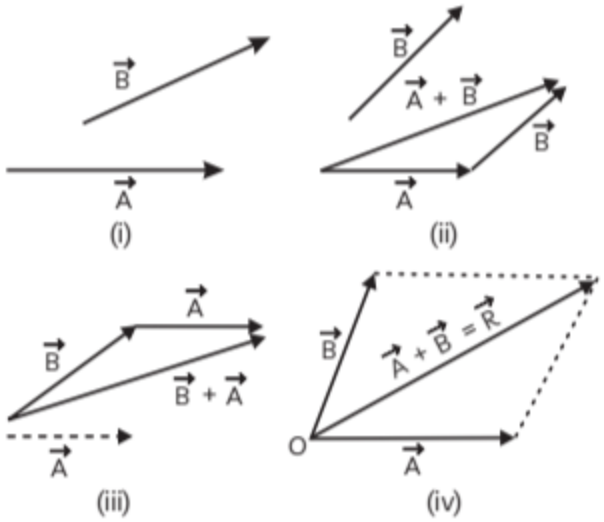
**Explanation:**

$h = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$

$h = 40 \times 2 - \frac{1}{2} \times 10(2)^2$

$= 60$ m

**2.** Parallelogram law of vector addition- states that if two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point then their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.



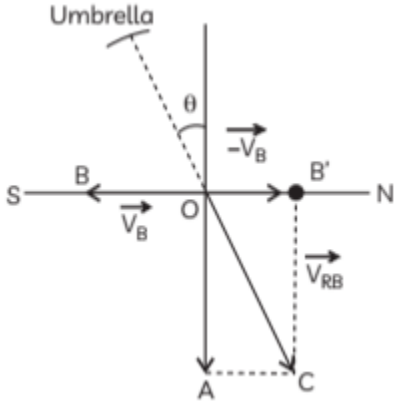

**(A)** State whether the statements are true or false.

(i) If the angle between two-unit vectors is 120° then, their resultant is another unit vector.

(ii) A scalar quantity is one that has values for observers with different orientation of the axis.

**(B)** Four forces are acting on a point. The first force is 200 N acting due north, the second is 100 N acting due south, the third force is 500 N acting due east and the fourth force is 300 N acting due west. What is the magnitude and direction of the resultant force?

**(C)** Rain in falling vertically with a speed of 30 $ms^1$. A woman on a bicycle is travelling with a speed of in the north to south direction. In what direction should she hold an umbrella in order to protect herself from rain?



**Ans.** (A) (i) True

**Explanation:** let $|\vec{A}| = x$ and $|\vec{B}| = x$

$$\therefore |\vec{R}| = x$$

Using, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$x^2 = x^2 + x^2 + 2(x)(x)\cos\theta$$

$$\cos\theta = -\frac{1}{2}$$

So, $\theta = +120°$

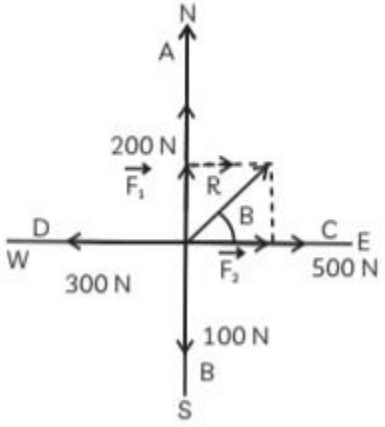
(ii) False,

**Explanation:** The value of scalar quantities (for example-body of a man) measured by observers with different orientation of axes remain unchanged.

(B) Resultant of $\vec{OA}$ and $\vec{OB}$,

$\vec{F_1} = 200 - 100$

$= 100$ N, along the north.



Resultant of $\vec{OC}$ and $\vec{OD}$,

$\vec{F_2}$ = 500 – 300

= 200 N due east

Now $\vec{F_1}$ and $\vec{F_2}$ are at right angles to each other and its resultant $\vec{F}$ is,

$$\vec{F} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (200)^2 + 2(100)(200)\cos 90^\circ}$$

$\vec{F}$ = 253 N

Angle made by F with $F_2$

$$\tan\beta = \frac{F_1}{F_2} = \frac{100}{100} = 0.5$$

$$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26°34',$$

the direction is north of east.

**(C)**

Velocity of rain $\left(\vec{v_R}\right)$ = 30 $ms^{-1}$

Velocity of bicycle $\left(\vec{v_B}\right)$ = 10 $ms^{-1}$

To protect herself, the direction of umbrella should be in direction to relative velocity

*i.e.*, $\vec{v_{RB}} = \vec{v_R} + \left(-\vec{v_B}\right)$

Then angle that $\vec{v_{RB}}$ makes with the vertical

Then $\tan\theta = \frac{AC}{OA} = \frac{\left|-\vec{v_B}\right|}{\left|\vec{v_R}\right|} = \frac{10}{30} = \frac{1}{3}$

$$\theta = \tan^{-1}\frac{1}{3} = 18^\circ$$

Then women hold her umbrellas in a vertical plane containing N-S direction at an angle of about 18° with the vertical towards south.

**3.** During a take-off, a plane has to attain a specific speed before it can lift off the runway. Every plane has a different speed to achieve depending on their shape, size and total mass. On smaller airports, the runway is smaller therefore, the acceleration is the key. For theoretical purposes, we just consider the length of the runway, initial and final velocity and acceleration but in real world, there are other factors which affect the take-off of a plane like temperature, wind speed, etc.

**(A) Suppose forces of 12 N and 3 N are exerting on a body and its resultant is 13.74 N, then the angle between two forces to get the resultant of 13.74 N is:**

(a) 30°

(b) 60°

(c) 90°

(d) 120°

**(B) A car moving at the speed of 40 km/h can be stopped by applying brakes after at least 2 m. The same car is moving with the speed of 80 km/h. What is the minimum stopping distance?**

(a) $\frac{20}{3}$ m (b) 20 m

(c) 60 m (d) 180 m

**(C) The initial velocity of a particle is u (at t = 0) and the acceleration is given by f = at, which of the following relations are valid?**

(a) $v = u + at^2$ (b) $v = u + \frac{at^2}{2}$

(c) $v = u + at$ (d) $v = u$

**(D) The velocity of a particle moving with constant acceleration at an instant to is 10 m/s. After 5 seconds of that instant, the velocity of the particle is 20 m/s. The velocity at 3 second before to is:**

(a) 8 m/s

(b) 4 m/s

(c) 6 m/s

(d) 7 m/s

**(E) A helicopter dropped rations, medicines, and other goods in a flood-stricken region. At a height of 98 meters above the surface, the helicopter was flying. Authorities were assisted in evacuating the victims by students from a local school. They noticed a toddler who was drowning. They ran to the boy with the lifeboat and saved him. What is the time taken by the objects dropped from a helicopter to reach the ground?**

(a) 4.5 s

(b) 5.2 s

(c) 2.5 s

(d) 5.0 s

**Ans. (A)** (b) 60°

**Explanation:** We know,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Here, R = 13.74, A = 12 N, B = 3 N, θ = ?

So, $(13.74)^2 = (12)^2 + (3)^2 + 2(12)(3)\cos\theta$

$(13.74)^2 - 153 = 72\cos\theta$

$35.70 = 72\cos\theta$

$35.70 = 72\cos\theta$

$$\cos\theta = \frac{1}{2}$$

Or $$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 60°$$

**(B)** (d) 180 m

**Explanation:** We know that, s $u^2$,

$$\frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$$

$$\frac{20}{s_2} = \frac{10^2}{30^2}$$

$$s_2 = 180 \text{ m}$$

**(C)**

(b) $v = u + \frac{at^2}{2}$

**Explanation:** As given $f = at = \frac{dv}{dt}$

$$\int_u^v dv = \int_0^t atdt$$

$$v - u = \frac{at^2}{2}$$

$$v = u + \frac{at^2}{2}$$

**(D)** (b) 4 m/s

**Explanation:** The acceleration of the particle is,

$$a = \frac{20-10}{5} = 2 \text{ m/s}^2$$

For 3 seconds before $t_0$,

$v = 10 \text{ ms}^{-1}$; $t = 3$ s; $a = 2 \text{ ms}^{-2}$ $u = ?$

$$v = u + at$$

$$10 = u + 2(3)$$

Or $u = 10 - (2)(3) = 4$ m/s

**(E)** (a) 4.5 s

**Explanation:** The time taken by the objects dropped from helicopter to reach the ground is:

$$h = v_i t + \frac{1}{2} gt^2$$

If $v_i = 0$,

then $t = \frac{\sqrt{2h}}{g}$

$$t = \sqrt{\frac{2 \times 98}{9.8}}$$

$$= \sqrt{20} = 4.5 \text{ s}$$