7 Straight Lines and Pair of Straight Lines

Section-A : JEE Advanced/ IIT-JEE

A
1. 2 sq. units **2.** $y=x$ **3.** $\left(\frac{3}{4},\frac{1}{2}\right)$ **4.** $\frac{y^2}{9}-\frac{x^2}{7}=1$ **5.** $(1,-2)$ **6.** first quadrant
7. (1, 1) **8.** $x-7y+2=0$

B
1. T **2.** T

C
1. (a) **2.** (c) **3.** (a) **4.** (d) **5.** (b) **6.** (a)
7. (a) **8.** (c) **9.** (c) **10.** (b) **11.** (a) **12.** (d)
13. (d) **14.** (a) **15.** (d) **16.** (d) **17.** (c) **18.** (b)
19. (b) **20.** (c) **21.** (a) **22.** (c) **23.** (b)

D
1. (a, b, c) **2.** (e) **3.** (a, c) **4.** (b) **5.** (c) **6.** (d)
7. (a,c, d) **8.** (b, c) **9.** (a)

E
1. $9x^2+36y^2=4\ell^2$
2. $\left(\frac{-3}{2},\frac{3}{2}\right)$ or $\left(\frac{7}{2},\frac{13}{2}\right)$
3. $4x+7y-11=0,\ 7x-4y-3=0;\ 7x-4y+25=0$
4. **(a)** $(-4,-7)$ **(b)** $(4-\sqrt{5})x+(2\sqrt{5}-3)y-(4\sqrt{5}-2)=0$
5. $x+5y-5\sqrt{2}=0$ or $x+5y+5\sqrt{2}=0$
7. $(-a,\ a(t_1+t_2+t_3)+at_1t_2t_3)$
9. $x-3y-31=0$ or $3x+y+7=0$
10. 32 sq. units
11. (0, 0) or (0, 5/2)
12. $(a^2+b^2)(lx+my+n)-2(al+bm)(ax+by+c)=0$
14. $x-7y+13=0$ or $7x+y-9=0$
15. $x^2+y^2-7x+5y=0$
16. $3x+4y-18=0$ or $x-2=0$
17. $(1,-2)$
18. $\alpha\in\left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{2},1\right)$
19. $\frac{1}{64}$ sq units
20. $2x+3y+22=0$
21. $x(m^2-1)-ym+(m^2+1)b+am=0$
27. 18
28. $y=2x+1$ or $y=-2x+1$

H
1. (c)

I
1. 6

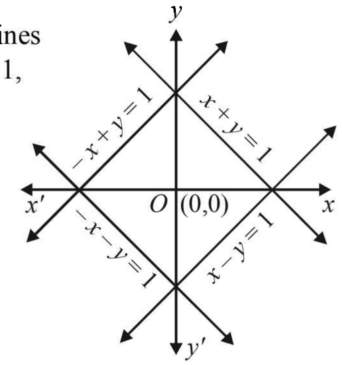
Section-B : JEE Main/ AIEEE

1. (a) **2.** (d) **3.** (a) **4.** (a) **5.** (a) **6.** (a) **7.** (c) **8.** (b) **9.** (b) **10.** (d) **11.** (a) **12.** (c)
13. (a) **14.** (a) **15.** (c) **16.** (c) **17.** (c) **18.** (a) **19.** (c) **20.** (a) **21.** (d) **22.** (d) **23.** (a) **24.** (a)
25. (c) **26.** (b) **27.** (c) **28.** (b) **29.** (b) **30.** (d) **31.** (a) **32.** (b) **33.** (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. $|x|+|y|=1$
The curve represents four lines $x+y=1, x-y=1, -x+y=1, -x-y=1$ which enclose a square of side = distance between opp. sides $x+y=1$ and $x+y=-1$



Side $=\frac{1+1}{\sqrt{1+1}}=\sqrt{2}$

$\therefore$ Req. area = $(\text{side})^2 = 2$ sq. units.

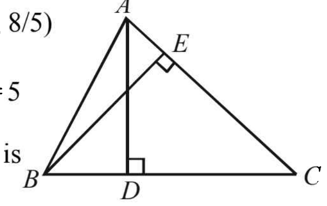
2. As $y=\log_{10}x$ can be obtained by replacing x by y and y by x in $y=10^x$
$\therefore$ The line of reflection is $y=x$.

3. Given that $3a+2b+4c=0 \Rightarrow \frac{3}{4}a+\frac{1}{2}b+c=0$
$\Rightarrow$ The set of lines $ax+by+c=0$ passes through the point (3/4, 1/2).

4. $|AP-BP|=6$
We know that locus of a point, difference of whose distances from two fixed points is constant, is hyperbola with the fixed points as focii and the difference of distances as length of transverse axis.
Thus, $ae=4$ and $2a=6 \Rightarrow a=3, e=4/3$
$\Rightarrow b^2=9\left(\frac{16}{9}-1\right)=7 \quad \therefore$ Equation is $\frac{y^2}{9}-\frac{x^2}{7}=1$
(foci being on y-axis, it is vertical hyperbola)

5. If a, b, c are in A.P. then
$a+c=2b \Rightarrow a-2b+c=0$
$\Rightarrow ax+by+c=0$ passes through (1,– 2).

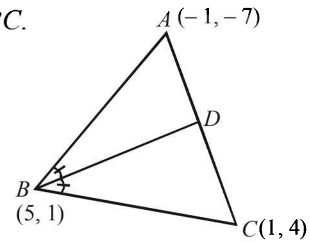
6. **First quadrant.**
The equations of sides of triangle ABC are
$AB \;: x+y=1$
$BC \;: 2x+3y=6$
$CA \;: 4x-y=-4$
Solving these pairwise we get the vertices of Δ as follows $A(-3/5, 8/5)$ $B(-3, 4)$ $C(-3/7, 16/7)$
Now AD is line $\perp^{\text{lar}}$ to BC and passes through A. Any line perpendicular to BC is $3x-2y+\lambda=0$
As it passes through $A(-3/5, 8/5)$



$\therefore \quad \frac{-9}{5}-\frac{16}{5}+\lambda=0 \Rightarrow \lambda=5$
$\therefore$ Equation of altitude AD is $3x-2y+5=0$ ...(1)
Any line perpendicular to side AC is $x+4y+\mu=0$
As it passes through point B (– 3, 4)
$\therefore \quad -3+16+\mu=0 \Rightarrow \mu=-13$
$\therefore$ Equation of altitude BE is $x+4y-13=0$ ...(2)
Now orthocentre is the point of intersection of equations (1) and (2) (AD and BE)
Solving (1) and (2), we get $x=3/7, y=22/7$
As both the co-ordinates are positive, orthocentre lies in first quadrant.

7. Let the variable line be $ax+by+c=0$ .... (1)
Then $\perp^{\text{lar}}$ distance of line from (0, 2) $=\frac{2a+c}{\sqrt{a^2+b^2}}=p_1$
$\perp^{\text{lar}}$ distance of line from (0, 2) $=\frac{2b+c}{\sqrt{a^2+b^2}}=p_2$
$\perp^{\text{lar}}$ distance of line from (1, 1) $=\frac{a+b+c}{\sqrt{a^2+b^2}}=p_3$
ATQ $p_1+p_2+p_3=0$
$\Rightarrow \frac{2a+c+2b+c+a+b+c}{\sqrt{a^2+b^2}}=0$
$\Rightarrow 3a+3b+3c=0$
$\Rightarrow a+b+c=0$ .... (2)
From (1) and (2), we can say variable line (1) passes through the fixed point (1, 1).

8. Let BD be the bisector of $\angle ABC$.
NOTE THIS STEP:
Then $AD : DC = AB : BC$
And



$AB=\sqrt{(5+1)^2+(1+7)^2}=10$
$BC=\sqrt{(5-1)^2+(1-4)^2}=5$
$\therefore \quad AD : DC = 2 : 1$
$\therefore$ By section formula $D\left(\frac{1}{3},\frac{1}{3}\right)$
Therefore equation of BD is
$y-1=\frac{1/3-1}{1/3-5}(x-5) \Rightarrow y-1=\frac{-2/3}{-14/3}(x-5)$
$\Rightarrow 7y-7=x-5 \Rightarrow x-7y+2=0$

B. True / False

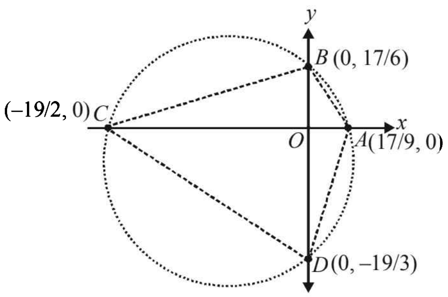
1. Intersection point of $x+2y-10=0$ and $2x+y+5=0$ is $\left(\frac{-20}{3},\frac{25}{3}\right)$ which clearly satisfies the line $5x+4y=0$. Hence the given statement is true.

2. The given lines cut x-axis at

$$A\left(\frac{17}{9},0\right), C\left(\frac{-19}{2},0\right)$$

and y-axis at $B\left(0,\frac{17}{6}\right)$ and $D\left(0,\frac{-19}{3}\right)$.

Now A, B, C, D are concyclic if for AC and BD intersecting at O we have $AO \times OC = BO \times OD$



or, $\frac{AO}{BO} = \frac{OD}{OC}$ if $\frac{17/9}{17/6} = \frac{-19/3}{-19/2}$ i.e. $\frac{2}{3} = \frac{2}{3}$ which is true.

$\therefore$ The given statement is true.

C. MCQs with ONE Correct Answer

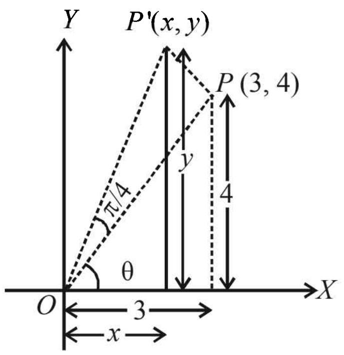
1. (a) The given points are $A(-a, -b)$, $B(0, 0)$, $C(a, b)$ and $D(a^2, ab)$.

Slope of $AB = \frac{b}{a}$ = slope of BC = slope of BD

$\therefore$ A, B, C, D are collinear.

2. (c) Reflection about the line $y = x$, changes the point (4, 1) to (1, 4).

On translation of (1, 4) through a distance of 2 units along +ve direction of x-axis the point becomes $(1 + 2, 4)$, i.e., (3, 4).



On rotation about origin through an angle $\pi/4$ the point P takes the position P' such that

$OP = OP'$

Also $OP = 5 = OP'$ and $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$

Now, $x = OP'\cos\left(\frac{\pi}{4}+\theta\right)$

$$= 5\left(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta\right) = 5\left(\frac{3}{5\sqrt{2}} - \frac{4}{5\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

$$y = OP'\sin\left(\frac{\pi}{4}+\theta\right) = 5\left(\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta\right)$$

$$= 5\left(\frac{3}{5\sqrt{2}} + \frac{4}{5\sqrt{2}}\right) = \frac{7}{\sqrt{2}} \quad \therefore \quad P' = \left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$

3. (a) Solving the given equations of lines pairwise, we get the vertices of Δ as

$A(-2, 2)$ $B(2, -2)$, $C(1, 1)$

Then $AB = \sqrt{16+16} = 4\sqrt{2}$

$BC = \sqrt{1+9} = \sqrt{10}$

$CA = \sqrt{9+1} = \sqrt{10}$ $\quad\therefore$ Δ is isosceles.

4. (a) We have

$P = (1, 0)$, $Q = (-1, 0)$, $R = (2, 0)$

Let $S = (x, y)$

ATQ $SQ^2 + SR^2 = 2SP^2$

$\Rightarrow$ $(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

$\Rightarrow$ $2x^2 + 2y^2 - 2x + 5 = 2x^2 + 2y^2 - 4x + 2$

$\Rightarrow$ $2x + 3 = 0 \Rightarrow x = -3/2$

Which is a straight line parallel to y-axis.

5. (b) As L has intercepts a and b on axes, equation of L is

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad \text{........ (1)}$$

Let x and y axes be rotated through an angle θ in anticlockwise direction.

In new system intercepts are p and q, therefore equation of L becomes

$$\frac{x}{p} + \frac{y}{q} = 1 \qquad \text{........ (2)}$$

KEY CONCEPT : As the origin is fixed in rotation, the distance of line from origin in both the cases should be same.

$\therefore$ We get $d = \left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = \left|\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}\right|$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$$

$\therefore$ (b) is the correct answer.

6. (a) Let the two perpendicular lines be the co-ordinate axes. Let (x, y) be the point sum of whose distances from two axes is 1 then we must have

$|x| + |y| = 1$ or $\pm x \pm y = 1$

These are the four lines

$x + y = 1, x - y = 1, \quad -x + y = 1, -x - y = 1$

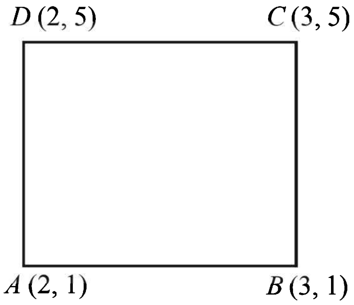
Any two adjacent sides are perpendicular to each other. Also each line is equidistant from origin. Therefore figure formed is a square.

7. **(a)** If variable point is P and $S(-2, 0)$ then $PS = \frac{2}{3}PM$

where PM is the perpendicular distance of point P from given line $x = -9/2$

$\therefore$ By definition P describes an ellipse. $\left(e = \frac{2}{3} < 1\right)$

8. **(c)** The sides of parallelogram are $x = 2, x = 3, y = 1, y = 5$.



$\therefore$ Diagonal AC is $\frac{y-1}{5-1} = \frac{x-2}{3-2}$ or $y = 4x - 7$

Equation diagonal BD is $\frac{x-2}{3-2} = \frac{y-5}{1-5}$ or $4x + y = 13$

9. **(c)** The lines by which Δ is formed are $x = 0, y = 0$ and $x + y = 1$.
Clearly, it is right Δ and we know that in a rightΔ orthocentre coincides with the vertex at which right $\angle$ is formed.
$\therefore$ Orthocentre is (0, 0).

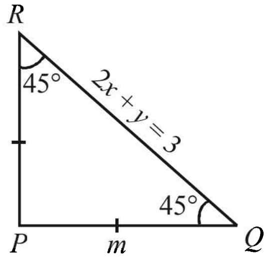
10. **(b)** Let m be the slope of PQ then

$$\tan 45° = \left|\frac{m-(-2)}{1+m(-2)}\right|$$

$\Rightarrow 1 = \left|\frac{m+2}{1-2m}\right| \Rightarrow \pm 1 = \frac{m+2}{1-2m}$

$\Rightarrow m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$

$\Rightarrow m = -1/3$ or $m = 3$



As PR also makes $\angle 45°$ with RQ.
$\therefore$ The above two values of m are for PQ and PR.

$\therefore$ Equation of $PQ, y - 1 = -\frac{1}{3}(x-2)$

$\Rightarrow 3y - 3 = -x + 2 \quad \Rightarrow x + 3y - 5 = 0$

and equation of PR is $\Rightarrow 3x - y - 5 = 0$

$\therefore$ Combined equation of PQ and PR is
$(x - 3y - 5)(3x - y - 5) = 0$
$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

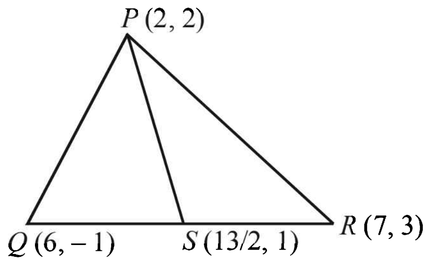
11. **(a)** $x_2 = x_1 r, x_3 = x_1 r^2$ and so is $y_2 = y_1 r, y_3 = y_1 r^2$

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = r.r^2 \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 & y_1 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$$

Hence the points lie on a line, i.e., they are collinear.

12. **(d)** S is the midpoint of Q and R

Therefore, $S \equiv \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$



Now slope of $PS = m = \frac{2-1}{2-13/2} = -\frac{2}{9}$

Now equation of the line passing through (1, – 1) and parallel to PS is

$$y + 1 = -\frac{2}{9}(x-1) \text{ or } 2x + 9y + 7 = 0$$

13. **(d)** Here AB = BC = CA = 2. So, it is an equilaterial triangle and the incentre coincides with centroid.
Therefore,

$$\text{Incentre} = \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$$

14. **(a)** Intersection of $3x + 4y = 9$ and $y = mx + 1$.
For x co-ordinate
$3x + 4(mx + 1) = 9 \Rightarrow (3 + 4m)x = 5$

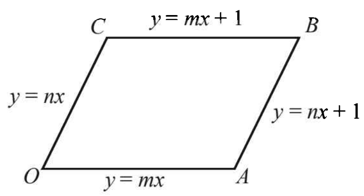
$$x = \frac{5}{3+4m}$$

For x to be an integer $3 + 4m$ should be a divisor of 5 i.e., $1, -1, 5$ or -5.

$3 + 4m = 1 \Rightarrow m = -1/2$ (not integer)
$3 + 4m = -1 \Rightarrow m = -1$ (integer)
$3 + 4m = 5 \Rightarrow m = 1/2$ (not an integer)
$3 + 4m = -5 \Rightarrow m = -2$ (integer)

$\therefore$ There are 2 integral values of m.
$\therefore$ (a) is the correct alternative.

15. **(d)**



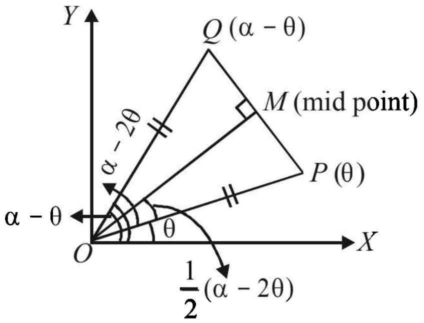
The vertices, $O(0,0), A\left(\frac{1}{m-n}, \frac{m}{m-n}\right), B(0,1)$

$Ar(\|^{gm}\ OABC = 2\,Ar(\Delta OAB)$

$$= 2\frac{1}{2}\left|\left[0\left(\frac{m}{m-n}-1\right)+\frac{1}{m-n}(1-0)+0\left(0-\frac{m}{m-n}\right)\right]\right|$$

$$= \frac{1}{|m-n|}$$

16. **(d)** Clearly $OP = OQ = 1$ and $\angle QOP = \alpha - \theta - \theta = \alpha - 2\theta$.



The bisector of $\angle QOP$ will be a perpendicular to PQ and also bisect it. Hence Q is reflection of P in the line OM which makes an angle $\angle MOP + \angle POX$ with x- axis,

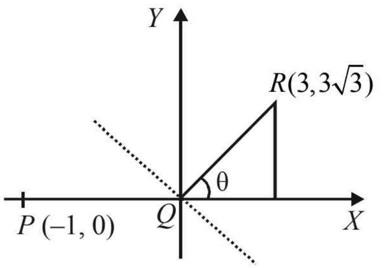
i.e., $\frac{1}{2}(\alpha - 2\theta) + \theta = \alpha/2$.

So that slope of OM is $\tan \alpha/2$.

17. **(c)** $\tan\theta = \sqrt{3} \Rightarrow \theta = 60° \Rightarrow \angle PQR = 120°$

$\Rightarrow$ bisector will have slope $\tan 120°$

$\Rightarrow$ equation of bisector is $\sqrt{3}x + y = 0$

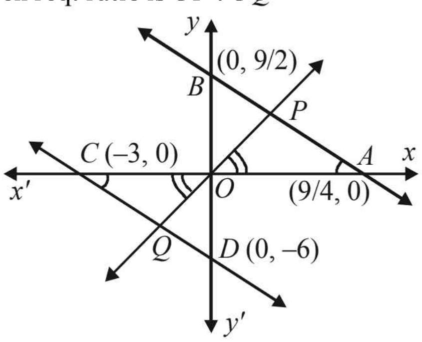


18. **(b)** The given lines are

$2x + y = 9/2$ .... (1)

and $2x + y = -6$ .... (2)

Signs of constants on R.H.S. show that two lines lie on opp. sides of origin. Let any line through origin meets these lines in P and Q respectively then req. ratio is $OP : OQ$



Now in ΔOPA and ΔOQC,

$\angle POA = \angle QOC$ (ver. opp. $\angle$' s)

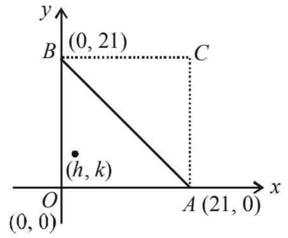
$\angle PAO = \angle OCQ$ (alt. int. $\angle$' s)

$\therefore$ $\Delta OPA \sim \Delta OQC$ (by AA similarly)

$\therefore$ $\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$

$\therefore$ Req. ratio is 3 : 4.

19. **(b)** Total no. of points within the square $OABC$ $= 20 \times 20 = 400$

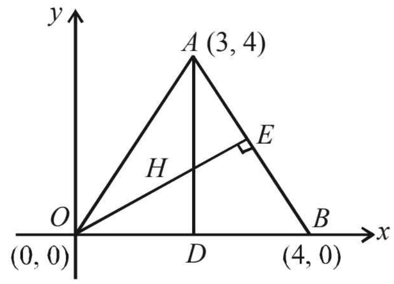


Points on line $AB = 20$ ((1, 1), (2, 2), ... (20, 20))

$\therefore$ Points within ΔOBC and ΔABC = 400 – 20 = 380

By symmetry points within ΔOAB $= \frac{380}{2} = 190$

20. **(c)** We know that orthocentre is the meeting point of altitudes of a Δ.



Equation of alt. AD

$\Rightarrow$ line parallel to y-axis through (3, 4)

$\Rightarrow$ $x = 3$ .... (1)

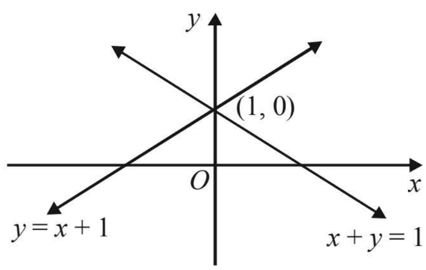
Similarly eq^n of $OE \perp AB$ is

$$y = -\frac{3-4}{4-0}x$$

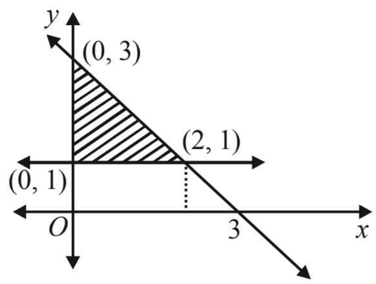
$\Rightarrow$ $y = x/4$ .... (2)

Solving (1) and (2), we get orthocentre as (3, 3/4).

21. **(a)** $x^2 - y^2 + 2y = 1 \Rightarrow x = \pm(y-1)$



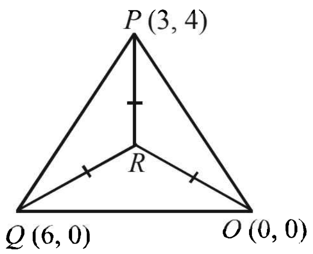
Bisectors of above lines are $x = 0$ and $y = 1$.



So area between x = 0, y = 1 and x + y = 3 is shaded region shown in figure.

Area = $\frac{1}{2} \times 2 \times 2 = 2$ sq. units.

22. **(c)** $\because$ $Ar(\Delta OPR) = Ar(\Delta PQR) = Ar(\Delta OQR)$



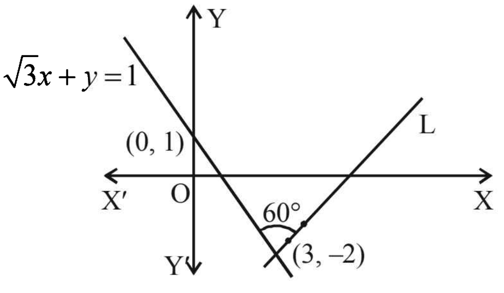
$\therefore$ By simply geometry

R should be the centroid of ΔPQO

$$\Rightarrow R\left(\frac{3+6+0}{3}, \frac{4+0+0}{3}\right) = \left(3, \frac{4}{3}\right)$$

23. **(b)** Let the slope of line L be m.

Then $\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$



$\Rightarrow m + \sqrt{3} = \pm(\sqrt{3} - 3m)$

$\Rightarrow 4m = 0$ or $2m = 2\sqrt{3} \Rightarrow m = 0$ or $m = \sqrt{3}$

$\because$ L intersects x-axis, $\quad \therefore m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}\,(x - 3)$

or $\sqrt{3}\,x - y - (2 + 3\sqrt{3}) = 0$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(a, b, c)**

For concurrency of three lines

$px + qy + r = 0; qx + ry + p = 0;$

$rx + py + q = 0$

We must have,

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$\Rightarrow C_1 + C_2 + C_3, \begin{vmatrix} p+q+r & q & r \\ p+q+r & r & p \\ p+q+r & p & q \end{vmatrix} = 0$$

$$\Rightarrow (p+q+r)\begin{vmatrix} 1 & q & r \\ 1 & r & p \\ 1 & p & q \end{vmatrix} = 0$$

$\Rightarrow C_1 - C_2, C_2 - C_3,$

$$\Rightarrow (p+q+r)\begin{vmatrix} 0 & q-r & r-p \\ 0 & r-p & p-q \\ 1 & p & q \end{vmatrix} = 0$$

$\Rightarrow (p+q+r)(pq - q^2 - rp + rq - r^2 + pr + pr - p^2) = 0$

$\Rightarrow (p+q+r)(p^2 + q^2 + r^2 - pq - pr - rq) = 0$

$\Rightarrow p^3 + q^3 + r^3 - 3pqr = 0$

It is clear that a, b, c are correct options.

2. **(e)** Let $A(0, 8/3)$, $B(1, 3)$ and $C(82, 30)$.

Now, slope of line $AB = \frac{3 - 8/3}{1 - 0} = \frac{1}{3}$

Slope of line $BC = \frac{30 - 3}{82 - 1} = \frac{27}{81} = \frac{1}{3}$

$\Rightarrow AB \parallel BC$ and B is common point.

$\Rightarrow A, B, C$ are collinear.

3. **(a, c)** Substituting the co-ordinates of the points (1, 3), (5, 0) and (– 1, 2) in $3x + 2y$, we obtain the value 8, 15 and 1 which are all +ve. Therefore, all the points lying inside the triangle formed by given points satisfy $3x + 2y \geq 0$.
Hence (a) is correct answer.
Substituting the co-ordinates of the given points in $2x + y - 13$, we find the values – 8, – 3 and – 13 which are all –ve.
So, (b) is not correct.
Again substituting the given points in $2x - 3y - 12$ we get – 19, – 2, – 20 which are all –ve.
It follows that all points lying inside the triangle formed by given points satisfy $2x - 3y - 12 \leq 0$.
So, (c) is the correct answer.
Finally substituting the co-ordinates of the given points in $-2x + y$, we get 1, – 10 and 4 which are not all +ve.
So, (d) is not correct.

Hence, (a) and (c) are the correct answers.

4. **(b)** Consider $\vec{a} = 2p\hat{i} + \hat{j}$ with respect to original axes and $a = (p+1)i + j$ with respect to new axes.
Now, as length of vector will remain the same

$\therefore \quad |\vec{a}| = \sqrt{(2p)^2 + 1} = \sqrt{(p+1)^2 + 1^2}$

$\Rightarrow p^2 + 2p + 2 = 4p^2 + 1 \Rightarrow 3p^2 - 2p - 1 = 0$

$\Rightarrow p = 1$ or $-1/3$

$\therefore$ (b) is the correct answer.

5. **(c)** *PQRS* will represent a parallelogram if and only if the mid-point of *PR* is same as that of the mid-point of *QS*. That is, if and only if

$$\frac{1+5}{2} = \frac{4+a}{2} \text{ and } \frac{2+7}{2} = \frac{6+b}{2}$$

$\Rightarrow a = 2$ and $b = 3$.

6. **(d)** Slope of $x + 3y = 4$ is $-1/3$ and slope of $6x - 2y = 7$ is 3. Therefore, these two lines are perpendicular which shows that both diagonals are perpendicular. Hence *PQRS* must be a rhombus.

7. **(a, c, d)** Since the co-ordinates of in the centre depend on lengths of side of Δ. $\therefore$ it can have irrational coordinates

8. **(b, c)** We know that length of intercept made by a circle on a line is given by $= 2\sqrt{r^2 - p^2}$

where $p = \perp$ distance of line from the centre of the circle.

Here circle is $x^2 + y^2 - x + 3y = 0$ with centre $\left(\frac{1}{2}, \frac{-3}{2}\right)$

and radius $= \frac{\sqrt{10}}{2}$

$L_1: \; y = mx$ (any line through origin)
$L_2: \; x + y - 1 = 0$ (given line)
ATQ circle makes equal intercepts on L_1 and L_2

$$\Rightarrow 2\sqrt{\frac{10}{4} - \frac{\left(\frac{m}{2} + \frac{3}{2}\right)^2}{m^2 + 1}} = 2\sqrt{\frac{10}{4} - \frac{\left(\frac{1}{2} - \frac{3}{2} - 1\right)^2}{2}}$$

$$\Rightarrow \frac{\left(\frac{m+3}{2}\right)^2}{m^2 + 1} = 2$$

$\Rightarrow m^2 + 6m + 9 = 8m^2 + 8 \Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow 7m^2 - 7m + m - 1 = 0 \Rightarrow (7m + 1)(m - 1) = 0$

$\Rightarrow m = 1, -1/7$

$\therefore$ The required line L_1 is $y = x$ or $y = -\frac{x}{7}$,

i.e., $x - y = 0$ or $x + 7y = 0$.

9. **(a)** The intersection point of two lines is $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

Distance between (1, 1) and $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right) < 2\sqrt{2}$

$$\Rightarrow 2\left(1 + \frac{c}{a+b}\right)^2 < 8 \Rightarrow 1 + \frac{c}{a+b} < 2$$

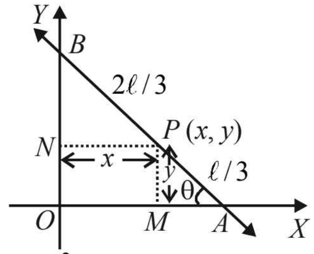
$\Rightarrow a + b - c > 0$

E. Subjective Problems

1. Let P (x, y) divides line segment AB in the ratio 1 : 2, so that AP = $\ell/3$ and BP = $2\ell/3$ where AB = ℓ.
Then PN = x and PM = y
Let $\angle$PAM = θ = $\angle$BPN

In ΔPMA, $\sin\theta = \frac{y}{\ell/3} = \frac{3y}{\ell}$

In ΔPNB, $\cos\theta = \frac{x}{2\ell/3} = \frac{3x}{2\ell}$



Now, $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{9y^2}{\ell^2} + \frac{9x^2}{4\ell^2} = 1 \Rightarrow 9x^2 + 36y^2 = 4\ell^2$$

2. As *C* lies on the line $y = x + 3$, let the co-ordinates of *C* be $(\lambda, \lambda + 3)$. Also *A* (2, 1), *B* (3, –2).
Then area of ΔABC is given by

$$\frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ \lambda & \lambda+3 & 1 \end{vmatrix} = \pm 5$$

$\Rightarrow |2(-2 - \lambda - 3) - 1(3 - \lambda)(3\lambda + 9 + 2\lambda)| = 10$

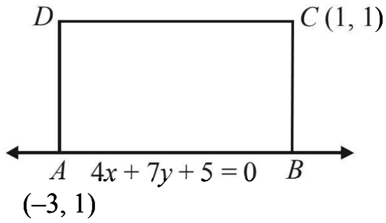
$\Rightarrow |-2\lambda - 10 - 3 + \lambda + 5\lambda + 9| = 10 \Rightarrow |4\lambda - 4| = 10$

$\Rightarrow 4\lambda - 4 = 10$ or $4\lambda - 4 = -10$

$\Rightarrow \lambda = 7/2$ or $\lambda = -3/2$

$\therefore$ Coordinates of *C* are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

3. Let side *AB* of rectangle *ABCD* lies along
$4x + 7y + 5 = 0$.
As (– 3, 1) lies on the line, let it be vertex *A*.
Now (1, 1) is either vertex *C* or *D*.



If (1, 1) is vertex *D* then slope of $AD = 0$

$\Rightarrow$ *AD* is not perpendicular to *AB*.

But it is a contradiction as $ABCD$ is a rectangle.

$\therefore$ (1, 1) are the co-ordinates of vertex C.

CD is a line parallel to AB and passing through C, therefore equation of CD is

$$y-1=-\frac{4}{7}(x-1)\Rightarrow 4x+7y-11=0$$

Also BC is a line perpendicular to AB and passing through C, therefore equation of BC is

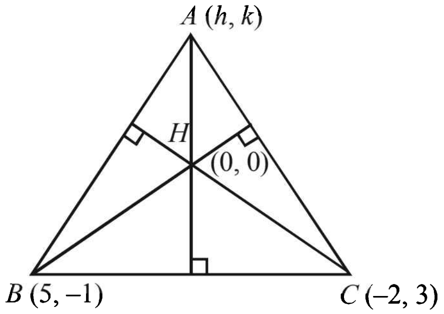
$$y-1=\frac{7}{4}(x-1)\Rightarrow 7x-4y-3=0$$

Similarly, AD is a line perpendicular to AB and passing through $A(-3, 1)$, therefore equation of line AD is

$$y-1=\frac{7}{4}(x+3)\Rightarrow 7x-4y+25=0$$

4. (a) $AH \perp BC \Rightarrow m_{AH}\times m_{BC}=-1$

$$\Rightarrow \frac{k}{h}\times\frac{3+1}{-2-5}=-1$$



$\Rightarrow 4k-7h=0$ ....(1)

Also, $BH \perp AC$

$$\Rightarrow \frac{-1}{5}\times\frac{3-k}{-2-h}=-1 \Rightarrow 3-k=-10-5h$$

$\Rightarrow 5h-k+13=0$ ....(2)

Solving (1) and (2), we get $h=-4$, $k=-7$

$\therefore$ Third vertex is $(-4, -7)$.

(b) The given lines are $x-2y+4=0$ ....(1)

and $4x-3y+2=0$ ....(2)

Both the lines have constant terms of same sign.

$\therefore$ The equation of bisectors of the angles between the given lines are

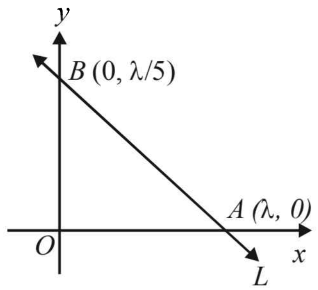
$$\frac{x-2y+4}{\sqrt{1+4}}=\pm\frac{4x-3y+2}{\sqrt{16+9}}$$

Here $a_1a_2+b_1b_2>0$ therefore, taking +ve sign on RHS, we get obtuse angle bisector as

$(4-\sqrt{5})x+(2\sqrt{5}-3)y-(4\sqrt{5}-2)=0$ ....(3)

5. The given line is $5x-y=1$

$\therefore$ The equation of line L which is perpendicular to the given line is $x+5y=\lambda$. This line meets co-ordinate axes at $A(\lambda, 0)$ and $B(0, \lambda/5)$.



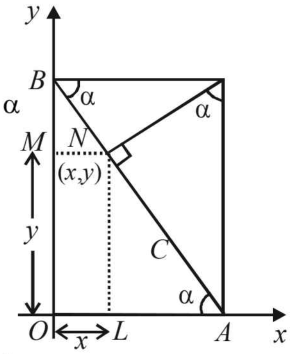
$\therefore$ Area of $\Delta OAB=\frac{1}{2}\times OA\times OB$

$$\Rightarrow 5=\frac{1}{2}\times\lambda\times\frac{\lambda}{5} \Rightarrow \lambda^2=5^2\times 2\Rightarrow \lambda=\pm 5\sqrt{2}$$

$\therefore$ The equation of line L is $x+5y-5\sqrt{2}=0$

or $x+5y+5\sqrt{2}=0$.

6. From figure,

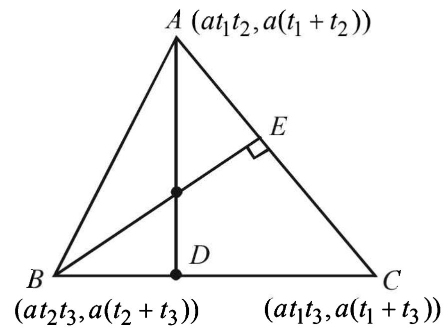


$x = OA - AL$
$= c\cos\alpha - AN\cos\alpha$
$= c\cos\alpha - (AP\sin\alpha.)\cos\alpha$
$= c\cos\alpha - c\sin\alpha\,.\sin\alpha\cos\alpha$
$= c\cos\alpha\,(1-\sin^2\alpha)$
$= c\cos^3\alpha$

$y = OB - MB$
$= c\sin\alpha - BN\sin\alpha$
$= c\sin\alpha - BP\cos\alpha\sin\alpha$
$= c\sin\alpha - c\cos\alpha\,.\cos\alpha\sin\alpha$
$= c\sin\alpha\,(1-\cos^2\alpha) = c\sin^3\alpha$

$\therefore$ Locus of (x, y) is $\left(\frac{x}{c}\right)^{\frac{2}{3}}+\left(\frac{y}{c}\right)^{\frac{2}{3}}=1$ or $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$

7.



$$\text{Slope of } BC=\frac{a(t_1+t_3)-a(t_2+t_3)}{at_1t_3-at_2t_3}$$

$$=\frac{a(t_1+t_3-t_2-t_3)}{at_3(t_1-t_2)}=\frac{1}{t_3}$$

$\therefore$ Slope of $AD=-t_3$

$\therefore$ Eq. of AD,

$y-a(t_1+t_2)=-t_3(x-at_1t_2)$

or $xt_3+y=at_1t_2t_3+a(t_1+t_2)$ ....(1)

Similarly, by symm. equation of BE is

$\Rightarrow \; xt_1 + y = at_1t_2t_3 + a(t_2 + t_3)$ .... (2)

Solving (1) and (2), we get $x = -a$

$y = a(t_1 + t_2 + t_3) + at_1t_2t_3)$

$\therefore$ Orthocentre $H(-a, a(t_1 + t_2 + t_3) + at_1t_2t_3)$

8. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= \frac{1}{2}[6(7) + 3(5) + 4(-2)] = \frac{49}{2}$

Area of $\Delta PBC = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= \frac{1}{2}(7x + 7y - 14) - \frac{7}{2}|x + y - 2|$

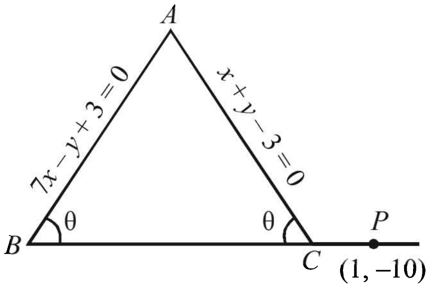
$ATQ, \frac{Ar(\Delta PBC)}{Ar(\Delta ABC)} = \frac{\frac{7}{2}|x + y - 2|}{\frac{49}{2}} = \left|\frac{x + y - 2}{7}\right|$

9. Let equations of equal sides AB and AC of isosceles ΔABC are

$7x - y + 3 = 0$ .... (1)

and $x + y - 3 = 0$ .... (2)

The third side BC of Δ passes through the point $(1, -10)$. Let its slope be m.



As $AB = AC$

$\therefore \; \angle B = \angle C$

$\Rightarrow \; \tan B = \tan C$ .... (3)

Now slope of $AB = 7$ and slope of $AC = -1$

Using $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$, we get

$\tan B = \left|\frac{7 - m}{1 + 7m}\right|$ and $\tan C = \left|\frac{-1 - m}{1 - m}\right|$

From eq. (3), we get

$\left|\frac{7 - m}{1 + 7m}\right| = \left|\frac{-1 - m}{1 - m}\right|$

$\Rightarrow \; \frac{7 - m}{1 + 7m} = \pm\left(\frac{-1 - m}{1 - m}\right)$

Taking '+' sign, we get

$(7 - m)(1 - m) = -(1 + m)(1 + 7m)$

$\Rightarrow \; 7 - 8m + m^2 + 7m^2 + 8m + 1 = 0$

$\Rightarrow \; 8m^2 + 8 = 0 \Rightarrow m^2 + 1 = 0$

It has no real solution.

Taking '–' sign, we get

$(7 - m)(1 - m) = (1 + m)(1 + 7m)$

$\Rightarrow \; 7 - 8m + m^2 - 7m^2 - 8m - 1 = 0$

$\Rightarrow \; -6m^2 - 16m + 6 = 0 \Rightarrow 3m^2 + 8m - 3 = 0$

$\Rightarrow \; (3m - 1)(m + 3) = 0 \Rightarrow m = 1/3, -3$

$\therefore$ The required line is

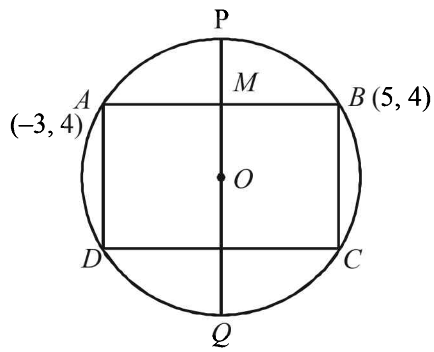
$y + 10 = \frac{1}{3}(x - 1)$ or $y + 10 = -3(x - 1)$

i.e. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$.

10. Let O be the centre of the circle. M is the mid point of AB. Then

$OM \perp AB$

Let OM when produced meets the circle at P and Q.



$\therefore$ PQ is a diameter perpendicular to AB and passing through M.

$M = \left(\frac{-3 + 5}{2}, \frac{4 + 4}{2}\right) = (1, 4)$

Slope of $AB = \frac{4 - 4}{5 + 3} = 0$

$\therefore$ PQ, being perpendicular to AB, is a line parallel to y-axis passing through $(1, 4)$.

$\therefore$ Its equation is

$x = 1$ .... (1)

Also eq. of one of the diameter given is

$4y = x + 7$ .... (2)

Solving (1) and (2), we get co-ordinates of centre O

$O(1, 2)$

Also let co-ordinates of D be (α, β)

Then O is mid point of BD, therefore

$\left(\frac{\alpha + 5}{2}, \frac{\beta + 4}{2}\right) = (1, 2) \Rightarrow \alpha = -3, \beta = 0$

$\therefore$ $D(-3, 0)$

Using the distance formula we get

$AD = \sqrt{(-3 + 3)^2 + (4 - 0)^2} = 4$

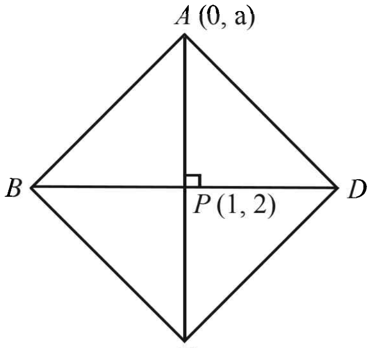
$AB = \sqrt{(5 + 3)^2 + (4 - 4)^2} = 8$

$\therefore$ Area of rectangle $ABCD = AB \times AD = 8 \times 4$

$= 32$ square units.

11. A being on y-axis, may be chosen as $(0, a)$.

The diagonals intersect at $P(1, 2)$.



Again we know that diagonals will be parallel to the angle bisectors of the two sides $y = x + 2$ and $y = 7x + 3$

i.e., $\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}}$

$\Rightarrow\ 5x - 5y + 10 = \pm(7x - y + 3)$

$\Rightarrow\ 2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$m_1 = -1/2 \qquad m_2 = 2$

Let diagonal d_1 be parallel to $2x + 4y - 7 = 0$ and diagonal d_2 be parallel to $12x - 6y + 13 = 0$. The vertex A could be on any of the two diagonals. Hence slope of AP is either $-1/2$ or 2.

$\Rightarrow\ \frac{2-a}{1-0} = 2 \quad$ or $\quad \frac{-1}{2}$

$\Rightarrow\ a = 0 \quad$ or $\quad \frac{5}{2}$

$\therefore\ A$ is (0, 0) or (0, 5/2)

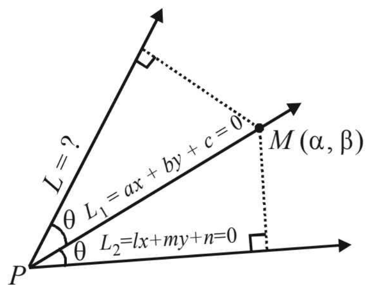
12. Let the equation of other line L, which passes through the point of intersection P of lines

$L_1 \equiv ax + by + c = 0 \qquad$ ....(1)

and $L_2 \equiv \ell x + my + n = 0 \qquad$ ....(2)

be $L_1 + \lambda L_2 = 0$

i.e. $(ax + by + c) + \lambda(\ell x + my + n) = 0$ ....(3)



From figure it is clear that L_1 is the bisector of the angle between the lines given by (2) and (3) [i.e. L_2 and L]

Let $M(\alpha, \beta)$ be any point on L_1 then

$a\alpha + b\beta + c = 0 \qquad$ ......(4)

Also from M, lengths of perpendiculars to lines L and L_2 given by equations (3) and (4), are equal

$$\frac{\ell\alpha + m\beta + n}{\sqrt{\ell^2 + m^2}} = \pm\frac{(a\alpha + b\beta + c) + \lambda(l\alpha + m\beta + n)}{\sqrt{(a+\lambda)^2 + (b+\lambda m)^2}}$$

$$\Rightarrow\ \frac{1}{\sqrt{\ell^2 + m^2}} = \pm\frac{\lambda}{\sqrt{(\ell^2 + m^2)\lambda^2 + 2(a\ell + bm)\lambda + (a^2 + b^2)}}$$

[Using 4]

$\Rightarrow\ (\ell^2 + m^2)\lambda^2 + 2(a\ell + bm)\lambda + (a^2 + b^2) = \lambda^2(\ell^2 + m^2)$

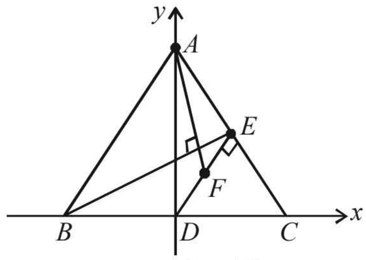
$\Rightarrow\ \lambda = -\frac{a^2 + b^2}{2(a\ell + bm)}$

Substituting this value of λ in eq. (3), we get L as

$$(ax + by + c) - \frac{(a^2 + b^2)}{2(a\ell + bm)}(\ell x + my + n) = 0$$

$\Rightarrow\ (a^2 + b^2)(\ell x + my + n) - 2(a\ell + bm)(ax + by + c) = 0$

13. Let BC be taken as x-axis with origin at D, the mid-point of BC, and DA will be y-axis.



$AB = AC$.

Let $BC = 2a$, then the co-ordinates of B and C are $(-a, 0)$ and $(a, 0)$.

Let $DA = h$, so that co-ordinates of A are $(0, h)$.

Then equation of AC is $\frac{x}{a} + \frac{y}{h} = 1 \qquad$ ......(1)

And equation of $DE \perp$ to AC and passing through origin is

$\frac{x}{h} - \frac{y}{a} = 0 \Rightarrow x = \frac{hy}{a} \qquad$ ....(2)

Solving (1) and (2) we get the co-ordinates of pt E as follows

$\frac{hy}{a^2} + \frac{y}{h} = 1 \ \Rightarrow\ h^2y + a^2y = a^2h$

$\Rightarrow\ y = \frac{a^2h}{a^2 + h^2} \ \Rightarrow\ x = \frac{ah^2}{a^2h^2}$

$\therefore\ E\left(\frac{ah^2}{a^2 + h^2}, \frac{a^2h}{a^2 + h^2}\right)$

Since F is mid pt. of DE, therefore, its co-ordinates are

$F\left(\frac{ah^2}{2(a^2 + h^2)}, \frac{a^2h}{2(a^2 + h^2)}\right)$

$$\therefore\ \text{Slope of } AF = \frac{h - \frac{a^2h}{2(a^2 + h^2)}}{0 - \frac{ah^2}{2(a^2 + h^2)}} = \frac{2h(a^2 + h^2) - a^2h}{-ah^2}$$

$\Rightarrow\ m_1 = -\frac{a^2 + 2h^2}{ah} \qquad$ ....(i)

$$\text{And slope of } BE = \frac{\frac{a^2h}{a^2 + h^2} - 0}{\frac{ah^2}{a^2 + h^2} + a} = \frac{a^2h}{ah^2 + a^3 + ah^2}$$

$\Rightarrow\ m_2 = \frac{ah}{a^2 + 2h^2} \qquad$ ....(ii)

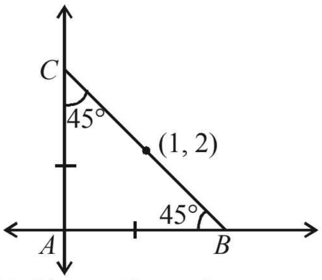
From (i) and (ii), we observe that

$m_1 m_2 = -1 \Rightarrow AF \perp BE.$ **Hence Proved.**

14. The given st. lines are $3x+4y=5$ and $4x-3y=15$. Clearly these st. lines are perpendicular to each other $(m_1 m_2 = -1)$, and intersect at A. Now B and C are pts on these lines such that $AB = AC$ and BC passes through (1, 2)

From fig. it is clear that

$\angle B = \angle C = 45°$



Let slope of BC be m. Then using

$$\tan B = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|, \text{ we get } \tan 45° = \left|\frac{m+3/4}{1-\frac{3}{4}m}\right|$$

$\Rightarrow 4m+3 = \pm(4-3m)$

$\Rightarrow 4m+3 = 4-3m$ or $4m+3 = -4+3m$

$\Rightarrow m = 1/7$ or $m = -7$

$\therefore$ Eq. of BC is, $y-2 = \frac{1}{7}(x-1)$

or $y-2 = -7(x-1)$

$\Rightarrow 7y-14 = x-1$ or $y-2 = -7x+7$

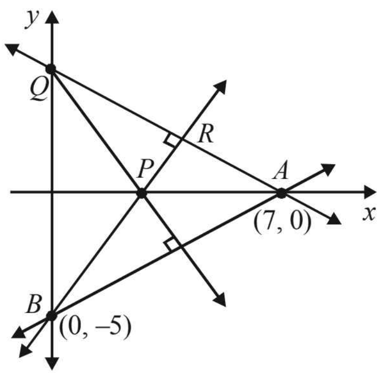
$\Rightarrow x-7y+13 = 0$ or $7x+y-9=0$

15. Eq. of the line AB is

$$\frac{x}{7} - \frac{y}{5} = 1 \qquad [A(7,0), B(0,-5)]$$

$\Rightarrow 5x-7y-35=0$

Eq. of line $PQ \perp AB$ is $7x+5y+\lambda=0$ which meets axes of x and y at pts $P(-\lambda/7, 0)$ and $Q(0, -\lambda/5)$ resp.



Eq. of AQ is,

$$\frac{x}{y} + \frac{y}{-\lambda/5} = 1 \Rightarrow \lambda x - 35y - 7\lambda = 0 \qquad ....(2)$$

Eq. of BP is,

$$\frac{-7x}{\lambda} - \frac{y}{5} = 1 \Rightarrow 35x + \lambda y + 5\lambda = 0 \qquad ....(3)$$

Locus of R the pt. of intersection of (2) and (3) can be obtained by eliminating λ from these eq. ' s, as follows

$$35x + (5+y)\left(\frac{35y}{x-7}\right) = 0$$

$\Rightarrow 35x(x-7) + 35y(5+y) = 0 \Rightarrow x^2+y^2-7x+5y=0$

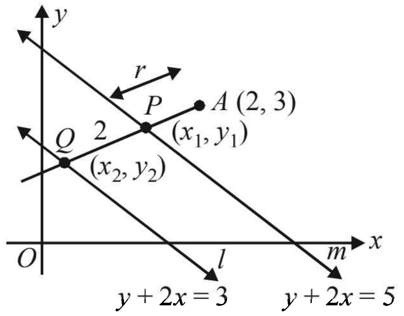
16. Let the equation of line through A which makes an intercept of 2 units between.

$2x+y=3$ .... (1)

and $2x+y=5$ .... (2)

be $\frac{x-2}{\cos\theta} = \frac{y-3}{\sin\theta} = r$

Let $AP = r$ then $AQ = r+2$



Then for pt $P(x_1, y_1)$,

$$\frac{x_1-2}{\cos\theta} = \frac{y_1-3}{\sin\theta} = r \Rightarrow \frac{2(x_1-2)+(y_1-3)}{2\cos\theta+\sin\theta} = r$$

$$\left(\text{Using } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{\lambda a_1 + \mu b_1}{\lambda a_2 + \mu b_2}\right)$$

$$\Rightarrow \frac{(2x_1+y_1)-7}{2\cos\theta+\sin\theta} = r \Rightarrow \frac{5-7}{2\cos\theta+\sin\theta} = r$$

[Using $2x_1 + y_1 = 5$ as $P(x_1, y_1)$ lies on $2x+y=5$]

$$\frac{-2}{2\cos\theta+\sin\theta} = r \qquad \ldots(i)$$

For pt $Q(x_2, y_2)$,

$$\frac{x_2-2}{\cos\theta} = \frac{y_2-3}{\sin\theta} = r+2$$

$$\Rightarrow \frac{2(x_2-2)+(y_2-3)}{2\cos\theta+\sin\theta} = r+2$$

$$\Rightarrow \frac{-4}{2\cos\theta+\sin\theta} = r+2 \qquad \ldots(ii)$$

(ii) – (i)

$$\Rightarrow \frac{-2}{2\cos\theta+\sin\theta} = 2$$

$\Rightarrow 2\cos\theta + \sin\theta = -1$ ...(3)

$\Rightarrow 2\cos\theta = -(1+\sin\theta)$

Squaring on both sides, we get

$\Rightarrow 4\cos^2\theta = 1 + 2\sin\theta + \sin^2\theta$

$\Rightarrow (5\sin\theta - 3)(\sin\theta+1) = 0 \Rightarrow \sin\theta = 3/5, -1$

$\Rightarrow \cos\theta = -4/5, 0$ [Using eq. (3)]

$\therefore$ The required equation is either

$$\frac{x-2}{-4/5}=\frac{y-3}{3/5} \text{ or } \frac{x-2}{0}=\frac{y-3}{-1}$$

$\Rightarrow$ either $3x-6=-4y+12$ or $x-2=0$

$\Rightarrow$ either $3x+4y-18=0$ or $x-2=0$

17. The given curve is

$3x^2-y^2-2x+4y=0 \quad \ldots(1)$

Let $y=mx+c$ be the chord of curve (1) which subtends an $\angle$ of 90° at origin.

Then the combined eq. of lines joining points of intersection of curve (1) and chord $y = mx + c$ to the origin, can be obtained by making the eq. of curve homogeneous with the help of eq. of chord, as follows.

$$3x^2-y^2-2x\left(\frac{y-mx}{c}\right)+4y\left(\frac{y-mx}{c}\right)=0$$

$\Rightarrow$ $(3c+2m)x^2-2(1+2m)xy+(4-c)y^2=0$

As the lines represented by this pair are perpendicular to each other, therefore we must have

coeff. of x^2 + coeff. of $y^2=0$

$\Rightarrow$ $3c+2m+4-c=0$

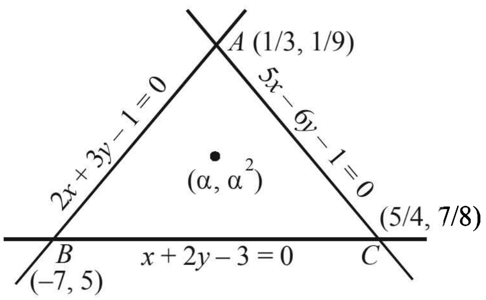
$\Rightarrow$ $-2=m.1+c$

Which on comparision with eq. of chord, implies that $y=mx+c$ passes though $(1,-2)$.

Hence the family of chords must pass through $(1,-2)$.

18. The points of intersection of given lines are

$$A\left(\frac{1}{3},\frac{1}{9}\right), B(-7,5), C\left(\frac{5}{4},\frac{7}{8}\right)$$



If (α, α^2) lies inside the Δ formed by the given lines, then $\left(\frac{1}{3},\frac{1}{9}\right)$ and (α, α^2) lie on the same side of the line $x+2y-3=0$

$$\Rightarrow \frac{\alpha+2\alpha^2-3}{\frac{1}{3}+\frac{2}{9}-3}>0 \Rightarrow 2\alpha^2+\alpha-3<0 \ldots(1)$$

Similarly $\left(\frac{5}{4},\frac{7}{8}\right)$ and (α, α^2) lie on the same side of the line $2x+3y-1=0$.

$$\Rightarrow \frac{2\alpha+3\alpha^2-1}{\frac{10}{4}+\frac{21}{8}-1}>0 \Rightarrow 3\alpha^2+2\alpha-1>0 \ldots(2)$$

$(-7, 5)$ and (α, α^2) lie on the same side of the line $5x-6y-1=0$.

$$\Rightarrow \frac{5\alpha+6\alpha^2-1}{-35-30-1}>0 \Rightarrow 6\alpha^2-5\alpha+1>0 \ldots(3)$$

Now common solution of (1), (2) and (3) can be obtained as in the previous method,

$$\therefore \quad \alpha \in \left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{2},1\right)$$

19. The given curve is

$y=x^3 \quad \ldots(1)$

Let the pt, P_1 be (t, t^3), $t \neq 0$

Then slope of tangent at $P_1 = \frac{dy}{dx}=(3x^2)_{x=t}=3t^2$

$\therefore$ Equation of tangent at P_1 is

$y-t^3=3t^2(x-t) \Rightarrow y=3t^2x-2t^3$

$\Rightarrow$ $3t^2x-y-2t^3=0 \quad \ldots(2)$

Now this tangent meets the curve again at P_2 which can be obtained by solving (1) and (2)

i.e., $3t^2x-x^3-2t^3=0$ or $x^3-3t^2x+2t^3=0$

$(x-t)^2(x+2t)=0 \Rightarrow x=-2t$ as $x=t$ is for P_1

$\therefore$ $y=-8t^3$

Hence pt P_2 is $(-2t,-8t^2)=(t_1,t_1^3)$ say.

Similarly, we can find that tangent at P_2 which meets the curve again at $P_3\,(2t_1,-8t_1^3)$ i.e., $(4t, 64t^3)$.

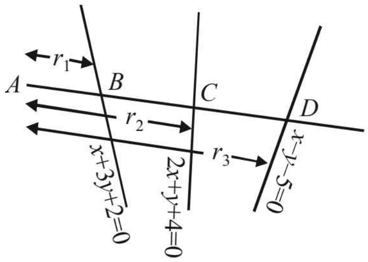
Similarly, $P_4 \equiv (-8t,-512t^3)$ and so on.

We observe that abscissae of pts. $P_1, P_2, P_3\ldots$ are $t,-2t,4t,\ldots$ which form a GP with common ratio -2. Also ordinates of these pts. $t^3,-8t^3,64t^3,\ldots$ also form a GP with common ratio -8.

$$\text{Now, } \frac{Ar(\Delta P_1P_2P_3)}{Ar(\Delta P_2P_3P_4)}=\frac{\begin{vmatrix}1 & t & t^3\\ 1 & -2t & -8t^3\\ 1 & 4t & 64t^3\end{vmatrix}}{\begin{vmatrix}1 & -2t & -8t^3\\ 1 & 4t & -64t^3\\ 1 & -8t & -512t^3\end{vmatrix}}$$

$$=\frac{t^4\begin{vmatrix}1 & 1 & 1\\ 1 & -2 & -8\\ 1 & 4 & 64\end{vmatrix}}{(-2)(-8)t^4\begin{vmatrix}1 & 1 & 1\\ 1 & -2 & -8\\ 1 & 4 & -64\end{vmatrix}}=\frac{1}{16} \text{ sq. units.}$$

20. Let θ be the inclination of line through $A(-5,-4)$. Therefore equation of this line is



$$\frac{x+5}{\cos\theta}=\frac{y+4}{\sin\theta}=r_1, r_2, r_3$$

$\Rightarrow$ $B(r_1\cos\theta-5,\ r_1\sin\theta-4)$
$C(r_2\cos\theta-5,\ r_2\sin\theta-4)$
$D(r_3\cos\theta-5,\ r_3\sin\theta-4)$

But B lies on $x+3y+2=0$. therefore
$r_1\cos\theta-5+3r_1\sin\theta-12+2=0$

$$\Rightarrow r_1=\frac{15}{\cos\theta+3\sin\theta}=\text{AB}$$

$$\Rightarrow \frac{15}{AB}=\cos\theta+3\sin\theta \quad \dots(1)$$

As C lies on $2x+y+4=0$, therefore
$2(r_2\cos\theta-5)+(r_2\sin\theta-4)+4=0$

$$\Rightarrow r_2=\frac{10}{2\cos\theta+\sin\theta}=AC$$

$$\Rightarrow \frac{10}{AC}=2\cos\theta+\sin\theta \quad \dots(2)$$

Similarly D lines on $x-y-5=0$, therefore
$r_3\cos\theta-5-r_3\sin\theta+4-5=0$

$$\Rightarrow r_3=\frac{6}{\cos\theta-\sin\theta}=AD$$

$$\Rightarrow \frac{6}{AD}=\cos\theta-\sin\theta \quad \dots(3)$$

Now, ATQ, $\left(\frac{15}{AB}\right)^2+\left(\frac{10}{AC}\right)^2=\left(\frac{6}{AD}\right)^2$

$\Rightarrow$ $(\cos\theta+3\sin\theta)^2+(2\cos\theta+\sin\theta)^2$
$=(\cos\theta-\sin\theta)^2$ [Using (1), (2) and (3)]

$\Rightarrow$ $4\cos^2\theta+9\sin^2\theta+12\sin\theta\cos\theta=0$

$\Rightarrow$ $2\cos\theta+3\sin\theta=0$

$\Rightarrow$ $\tan\theta=-\frac{2}{3}$

$\therefore$ Equation of req. line is $y+4=-\frac{2}{3}(x+5)$

$\Rightarrow$ $2x+3y+22=0$

21. Let the co-ordinates of Q be (b, α) and that of S be $(-b, \beta)$. Let PR and SQ intersect each other at G.

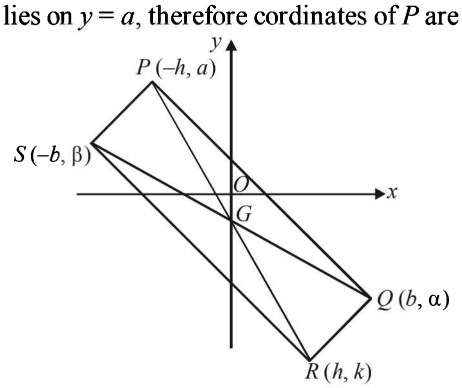
$\because$ G is the mid pt of SQ.
($\because$ diagonals of a rectangle bisect each other)

$\therefore$ x co-ordinates of G must be a.

Let the co-ordinates of R be (h, k).

$\therefore$ The x-coordinates of P is $-h$
($\because$ G is the mid point of PR)

As P lies on $y=a$, therefore cordinates of P are $(-h, a)$.



$\because$ PQ is parallel to $y=mx$,
Slope of $PQ=m$

$$\therefore \frac{\alpha-a}{b+h}=m \Rightarrow \alpha=a+m(b+h) \quad \dots(1)$$

Also $RQ\perp PQ \Rightarrow$

Slope of $RQ=\frac{-1}{m}$

$$\therefore \frac{k-\alpha}{h-b}=\frac{-1}{m}\Rightarrow \alpha=k+\frac{1}{m}(h-b) \quad \dots(2)$$

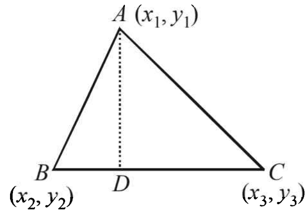
From (1) and (2) we get

$$a+m(b+h)=k+\frac{1}{m}(h-b)$$

$\Rightarrow$ $(m^2-1)h-mk+b(m^2+1)+am=0$

$\therefore$ Locus of vertex $R(h, k)$ is
$(m^2-1)x-my+b(m^2+1)+am=0.$

22. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of ΔABC



Then equation of alt. AD is

$$y-y_1=-\left[\frac{x_2-x_3}{y_2-y_3}\right](x-x_1)$$

or $(x-x_1)(x_2-x_3)+(y-y_1)(y_2-y_3)=0$ ... (1)

Similarly equations of other two attitudes are
$(x-x_2)(x_3-x_1)+(y-y_2)(y_3-y_1)=0$ ...(2)
and $(x-x_3)(x_1-x_2)+(y-y_3)(y_1-y_2)=0$ ...(3)

Now, above three lines will be concurrent if

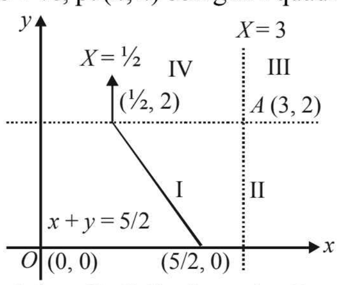
$$\begin{vmatrix} x_2-x_3 & y_2-y_3 & -x_1(x_2-x_3)-y_1(y_2-y_3) \\ x_3-x_1 & y_3-y_1 & -x_2(x_3-x_1)-y_2(y_3-y_1) \\ x_1-x_2 & y_1-y_2 & -x_3(x_1-x_2)-y_3(y_1-y_2) \end{vmatrix}=0$$

On L.H.S.
Operating $R_1+R_2+R_3$, R_1 becomes row of zeros.
$\therefore$ Value of determinant $=0=$ R.H.S.
Hence the altitudes are concurrent.

23. Let $P=(h, k)$ be a general point in the first quadrant such that $d(P, A)=d(P, O)$

$\Rightarrow$ $|h-3|+|k-2|=|h|+|k|=h+k$ ... (1)

[h and k are + ve, pt (h, k) being in I quadrant.]



If $h<3$, $k<2$ then (h, k) lies in region I.
It $h>3$, $k<2$, (h, k) lies in region II.
If $h>3$, $k>2$ (h, k) lies in region III.

If $h<3, k>2$ (h, k) lies in region IV.

In region I, eq. (1)

$\Rightarrow\ 3-h+2-k=h+k \Rightarrow h+k=\frac{5}{2}$

In region II, eq. (1) becomes

$\Rightarrow\ h-3+2-k=h+k \Rightarrow k=-\frac{1}{2}$ not possible.

In region III, eq. (1) becomes

$\Rightarrow\ h-3+k-2=h+k \Rightarrow -5=0$ not possible.

In region IV, eq. (1) becomes

$\Rightarrow\ 3-h+k-2=h+k \Rightarrow h=1/2$

$\Rightarrow$ Hence required set consists of line segment $x+y=5/2$ of finite length as shown in the first region and the ray $x=1/2$ in the fourth region.

24. Let the co-ordinates of the vertices of the ΔABC be $A(a_1, b_1)$, $B(a_2, b_2)$ and $C(a_3, b_3)$ and co-ordinates of the vertices of the ΔPQR be

$P(x_1, y_1), B(x_2, y_2)$ and $R(x_3, y_3)$

Slope of $QR=\frac{y_2-y_3}{x_2-x_3}$

$\Rightarrow$ Slope of straight line perpendicular to

$QR=-\frac{x_2-x_3}{y_2-y_3}$

Equation of straight line passing through A (a_1, b_1) and perpendicular to QR is

$$y-b_1=-\frac{x_2-x_3}{y_2-y_3}(x-a_1)$$

$$\Rightarrow (x_2-x_3)x+(y_2-y_3)y-a_1(x_2-x_3)-b_1(y_2-y_3)=0 \quad \ldots(1)$$

Similarly equation of straight line from B and perpendicular to RP is

$$(x_3-x_1)x+(y_3-y_1)y-a_2(x_3-x_1)-b_2(y_3-y_1)=0 \quad \ldots(2)$$

and eq^n of straight line from C and perpendicular to PQ is

$$(x_1-x_2)x+(y_1-y_2)y-a_3(x_1-x_2)-b_3(y_1-y_2)=0 \quad \ldots(3)$$

As straight lines (1), (2) and (3) are given to be concurrent, we should have

$$\begin{vmatrix} x_2-x_3 & y_2-y_3 & a_1(x_2-x_3)+b_1(y_2-y_3) \\ x_3-x_1 & y_3-y_1 & a_2(x_3-x_1)+b_2(y_3-y_1) \\ x_1-x_2 & y_1-y_2 & a_1(x_1-x_2)+b_3(y_1-y_2) \end{vmatrix}=0 \quad \ldots(4)$$

Operating $R_1 \to R_1+R_2+R_3$, we get

$$\begin{vmatrix} 0 & 0 & S \\ x_3-x_1 & y_3-y_1 & a_2(x_3-x_1)+b_2(y_3-y_1) \\ x_1-x_2 & y_1-y_2 & a_3(x_1-x_2)+b_3(y_1-y_2) \end{vmatrix}=0$$

where

$[S=a_1(x_2-x_3)+b_1(y_2-y_3)+a_2(x_3-x_1)+b_2(y_3-y_1)+a_3(x_1-x_2)+b_3(y_1-y_2)]$

Expanding along R_1

$\Rightarrow [(x_3-x_1)(y_1-y_2)-(x_1-x_2)(y_3-y_1)]S=0$

$\Rightarrow \left[\frac{y_1-y_2}{x_1-x_2}-\frac{y_3-y_1}{x_3-x_1}\right]S=0$

$\Rightarrow [m_{PQ}-m_{PR}]S=0 \Rightarrow S=0$

$[m_{PQ}=m_{PR} \Rightarrow PQ \parallel PR$ which is not possible in $\Delta PQR]$

$$\Rightarrow a_1(x_2-x_3)+b_1(y_2-y_3)+a_2(x_3-x_1)+b_2(y_3-y_1)+a_3(x_1-x_2)+b_3(y_1-y_2)=0 \ldots(5)$$

$$\Rightarrow x_1(a_3-a_2)+y_1(b_3-b_2)+x_2(a_1-a_3)+y_2(b_1-b_3)+x_3(a_2-a_1)+y_3(b_2-b_1)=0 \quad \ldots(6)$$

(Rearranging the equation (5))

But above condition shows

$$\begin{vmatrix} a_3-a_2 & b_3-b_2 & x_1(a_3-a_2)+y_1(b_3-b_2) \\ a_1-a_3 & b_1-b_3 & x_2(a_1-a_3)+y_2(b_1-b_3) \\ a_2-a_1 & b_2-b_1 & x_3(a_2-a_1)+y_3(b_2-b_1) \end{vmatrix}=0 \quad \ldots(7)$$

[Using the fact that as (4) $\Leftrightarrow$ (5) in the same way (6) $\Leftrightarrow$ (7)]

Clearly equation (7) shows that lines through P and perpendicular to BC, through Q and perpendicular to AB are concurrent. **Hence Proved.**

25. $C_1 \to aC_1$

$$\Delta=\frac{1}{a}\begin{vmatrix} a^2x-aby-ac & bx+ay & cx+a \\ abx+a^2y & -ax+by-c & cy+b \\ acx+a^2 & cy+b & -ax-by+c \end{vmatrix}$$

Applying $C_1 \to C_1+bC_2+cC_3$

$$\Delta=\frac{1}{a}\begin{vmatrix} (a^2+b^2+c^2)x & ay+bx & cx+a \\ (a^2+b^2+c^2)y & by-c-ax & cy+b \\ (a^2+b^2+c^2) & b+cy & -ax-by+c \end{vmatrix}$$

$$=\frac{1}{a}\begin{vmatrix} x & ay+bx & cx+a \\ y & by-c-ax & b+cy \\ 1 & b+cy & c-ax-by \end{vmatrix},$$

as $a^2+b^2+c^2=1$

$C_2 \to C_2-bC_1$ and $C_3 \to C_3-cC_1$

then $\Delta=\frac{1}{a}\begin{vmatrix} x & ay & a \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$

$$=\frac{1}{ax}\begin{vmatrix} x^2 & axy & ax \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$$

$R_1 \to R_1+yR_2+R_3$

$$\Delta=\frac{1}{ax}\begin{vmatrix} x^2+y^2+1 & 0 & 0 \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$$

On expanding along R_1

$$\Delta = \frac{(x^2+y^2+1)}{ax} ax(ax+by+c)$$
$$= (x^2+y^2+1)(ax+by+c)$$

Given $\Delta = 0$

$\Rightarrow ax+by+c=0$, which represents a straight line.

[$\because x^2+y^2+1 \neq 0$, being + ve].

26. The line $y = mx$ meets the given lines in

$P\left(\frac{1}{m+1}, \frac{m}{m+1}\right)$ and $Q\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$

Hence equation of L_1 is

$$y - \frac{m}{m+1} = 2\left(x - \frac{1}{m+1}\right)$$

$$\Rightarrow y - 2x - 1 = -\frac{3}{m+1} \quad \ldots(1)$$

and that of L_2 is

$$y - \frac{3m}{m+1} = -3\left(x - \frac{3}{m+1}\right)$$

$$\Rightarrow y + 3x - 3 = \frac{6}{m+1} \quad \ldots(2)$$

From (1) and (2)

$$\frac{y-2x-1}{y+3x-3} = -\frac{1}{2}$$

$\Rightarrow x - 3y + 5 = 0$ which is a straight line.

27. Let the equation of the line be

$(y-2) = m(x-8)$ where $m < 0$

$$\Rightarrow P \equiv \left(8 - \frac{2}{m}, 0\right) \text{ and } Q \equiv (0, 2-8m)$$

$$\text{Now, } OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$$

$$= 10 + \frac{2}{-m} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{-m} \times 8(-m)} \geq 18$$

28. A line passing through $P(h, k)$ and parallel to x-axis is

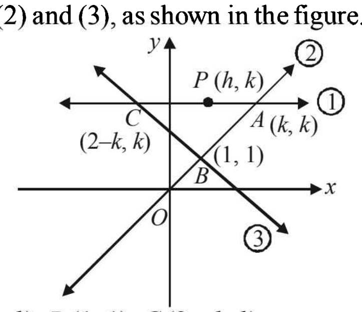
$y = k$. $\ldots(1)$

The other two lines given are

$y = x$ $\ldots(2)$

and $x + y = 2$ $\ldots(3)$

Let ABC be the Δ formed by the points of intersection of the lines (1), (2) and (3), as shown in the figure.



Then $A(k, k)$, $B(1, 1)$, $C(2-k, k)$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$$

Operating $C_1 - C_2$ we get

$$\frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$$

$\Rightarrow \frac{1}{2}|(2-2k)(k-1)| = 4h^2 \Rightarrow (k-1)^2 = 4h^2$

$\Rightarrow k - 1 = 2h$ or $k - 1 = -2h$

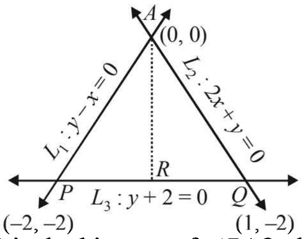
$\Rightarrow k = 2h + 1$ or $k = -2h + 1$

$\therefore$ Locus of (h, k) is, $y = 2x + 1$ or $y = -2x + 1$.

H. Assertion & Reason Type Questions

1. (c) Point of intersection of L_1 and L_2 is $A(0, 0)$.

Also $P(-2, -2)$, $Q(1, -2)$



$\because$ AR is the bisector of $\angle PAQ$, therefore R divides PQ in the same ratio as $AP : AQ$.

Thus $PR : RQ = AP : AQ = 2\sqrt{2} : \sqrt{5}$

$\therefore$ Statement-1 is true.

Statement-2 is clearly false.

I. Integer Value Correct Type

1. (6) Let the point P be (x, y)

Then $d_1(P) = \left|\frac{x-y}{\sqrt{2}}\right|$ and $d_2(P) = \left|\frac{x+y}{\sqrt{2}}\right|$

For P lying in first quadrant $x > 0, y > 0$.

Also $2 \leq d_1(P) + d_2(P) \leq 4$

$$\Rightarrow 2 \leq \left|\frac{x-y}{\sqrt{2}}\right| + \left|\frac{x+y}{\sqrt{2}}\right| \leq 4$$

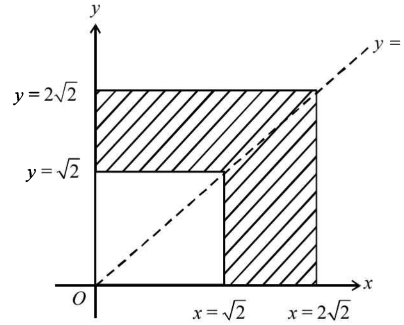
If $x > y$, then $2 \leq \frac{x-y+x+y}{\sqrt{2}} \leq 4$

or $\sqrt{2} \leq x \leq 2\sqrt{2}$

If $x < y$, then

$$2 \leq \frac{y-x+x+y}{\sqrt{2}} \leq 4 \text{ or } \sqrt{2} \leq y \leq 2\sqrt{2}$$

The required region is the shaded region in the figure given below.



$\therefore$ Required area $= \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 = 8 - 2 = 6$ sq units.

Section-B JEE Main/ AIEEE

1. (a) $AB = \sqrt{(4+1)^2+(0+1)^2} = \sqrt{26}$;

$BC = \sqrt{(3+1)^2+(5+1)^2} = \sqrt{52}$

$CA = \sqrt{(4-3)^2+(0-5)^2} = \sqrt{26}$;

In isosceles triangle side $AB = CA$

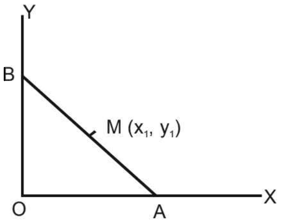
For right angled triangle, $BC^2 = AB^2 + AC^2$

So, here $BC = \sqrt{52}$ or $BC^2 = 52$

or $(\sqrt{26})^2 + (\sqrt{26})^2 = 52$

So, the given triangle is right angled and also isosceles

2. (d) Equation of AB is



$x \cos\alpha + y \sin\alpha = p$;

$\Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$;

$\Rightarrow \frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$

So co-ordinates of A and B are

$\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$;

So coordinates of midpoint of AB are

$\left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right) = (x_1, y_1)(let)$;

$x_1 = \frac{p}{2\cos\alpha}$ & $y_1 = \frac{p}{2\sin\alpha}$;

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$;

$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \frac{p^2}{4}\left(\frac{1}{x_1^2} + \frac{1}{y_1^2}\right) = 1$

Locus of (x_1, y_1) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

3. (a) Put $x = 0$ in the given equation

$\Rightarrow by^2 + 2fy + c = 0$.

For unique point of intersection $f^2 - bc = 0$

$\Rightarrow af^2 - abc = 0$.

Since $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

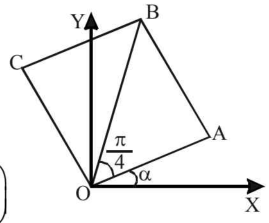
$\Rightarrow 2fgh - bg^2 - ch^2 = 0$

4. (a) $3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$.;

$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

5. (a) Co-ordinates of $A = (a\cos\alpha, a\sin\alpha)$

Equation of OB,



$y = \tan\left(\frac{\pi}{4}+\alpha\right)x$

$CA \perp^r$ to OB

$\therefore$ slope of $CA = -\cot\left(\frac{\pi}{4}+\alpha\right)$

Equation of CA

$y - a\sin\alpha = -\cot\left(\frac{\pi}{4}+\alpha\right)(x - a\cos\alpha)$

$\Rightarrow (y - a\sin\alpha)\left(\tan\left(\frac{\pi}{4}+\alpha\right)\right) = (a\cos\alpha - x)$

$\Rightarrow (y - a\sin\alpha)\left(\frac{\tan\frac{\pi}{4} + \tan\alpha}{1 - \tan\frac{\pi}{4}\tan\alpha}\right)(a\cos\alpha - x)$

$\Rightarrow (y - a\sin\alpha)(1+\tan\alpha) = (a\cos\alpha - x)(1 - \tan\alpha)$

$\Rightarrow (y - a\sin\alpha)(\cos\alpha + \sin\alpha) = (a\cos\alpha - x)(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos + \sin\alpha) - a\sin\alpha\cos\alpha - a\sin^2\alpha$

$= a\cos^2\alpha - a\cos\alpha\sin\alpha - x(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a.$

6. (a) Equation of bisectors of second pair of straight lines

is, $qx^2 + 2xy - qy^2 = 0$ ....(1)

It must be identical to the first pair

$x^2 - 2pxy - y^2 = 0$ ....(2)

from (1) and (2) $\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$.

7. (c) $x = \frac{a\cos t + b\sin t + 1}{3} \Rightarrow a\cos t + b\sin t = 3x - 1$

$y = \frac{a\sin t - b\cos t}{3} \Rightarrow a\sin t - b\cos t = 3y$

Squaring & adding, $(3x-1)^2 + (3y)^2 = a^2 + b^2$

8. (b) Taking co-ordinates as

$\left(\frac{x}{r}, \frac{y}{r}\right); (x, y) \,\&\, (xr, yr)$.

Then slope of line joining

$\left(\frac{x}{r}, \frac{y}{r}\right), (x, y) = \frac{y\left(1 - \frac{1}{r}\right)}{x\left(1 - \frac{1}{r}\right)} = \frac{y}{x}$

and slope of line joining (x, y) and (xr, yr)

$= \frac{y(r-1)}{x(r-1)} = \frac{y}{x}$ $\quad \therefore m_1 = m_2$

$\Rightarrow$ Points lie on the straight line.

9. (b) $(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$

$(a_1 - a_2)x + (b_1 - b_2)y$

$+ \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$

$c = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

10. **(d)** Let the vertex C be (h, k), then the centroid of ΔABC is $\left(\frac{2-2+h}{3}, \frac{-3+1+k}{3}\right)$

or $\left(\frac{h}{3}, \frac{-2+k}{3}\right)$. It lies on $2x + 3y = 1$

$\Rightarrow \frac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$

= Locus of C is $2x + 3y = 9$

11. **(a)** Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(1)

then $a + b = -1$ ....(2)

(1) passes through (4, 3), $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(3)

Eliminating b from (2) and (3), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

12. **(c)** Let the lines be $y = m_1 x$ and $y = m_2 x$ then

$m_1 + m_2 = -\frac{2c}{7}$ and $m_1 m_2 = -\frac{1}{7}$

Given $m_1 + m_2 = 4\, m_1 m_2$

$\Rightarrow \frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$

13. **(a)** $3x + 4y = 0$ is one of the lines of the pair

$6x^2 - xy + 4cy^2 = 0$, Put $y = -\frac{3}{4}x$,

we get $6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$

$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$

14. **(a)** The line passing through the intersection of lines $ax + 2by = 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

As this line is parallel to x-axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a/b$

$\Rightarrow \quad ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$\Rightarrow \quad ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$

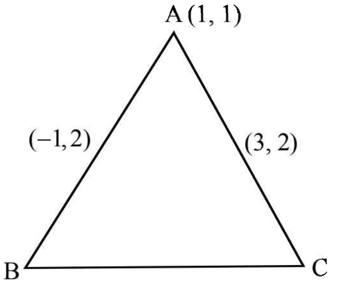
$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$

$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$

So it is 3/2 units below x-axis.

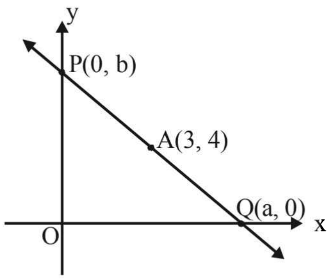
15. **(c)** Vertex of tringle is (1, 1) and midpoint of sides through this vertex is (– 1, 2) and (3, 2)



$\Rightarrow$ vertex B and C come out to be (– 3, 3) and (5, 3)

$\therefore$ Centroid is $\frac{1-3+5}{3}, \frac{1+3+5}{3} \Rightarrow \left(1, \frac{7}{3}\right)$

16. **(c)**

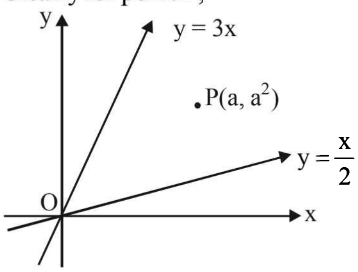


$\because A$ is the mid point of PQ, therefore

$\frac{a+0}{2} = 3, \ \frac{0+b}{2} = 4 \Rightarrow a = 6, b = 8$

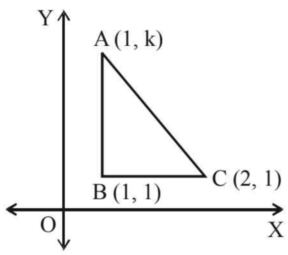
$\therefore$ Equation of line is $\frac{x}{6} + \frac{y}{8} = 1$ or $4x + 3y = 24$

17. **(c)** Clearly for point P,



$a^2 - 3a < 0$ **and** $a^2 - \frac{a}{2} > 0 \Rightarrow \frac{1}{2} < a < 3$

18. **(a)** **Given :** The vertices of a right angled triangle A(l, k), B(1, 1) and C(2, 1) and Area of $\Delta ABC = 1$ square unit

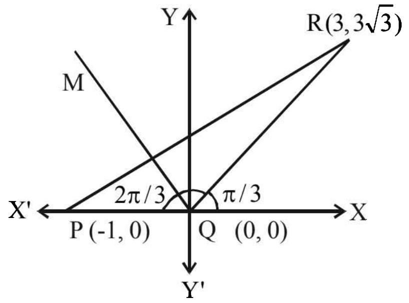


We know that, area of right angled triangle

$= \frac{1}{2} \times BC \times AB = 1 = \frac{1}{2}(1)|(k-1)|$

$\Rightarrow \pm(k-1) = 2 \Rightarrow k = -1, 3$

19. (c) **Given :** The coordinates of points P, Q, R are (–1, 0), (0, 0), $(3, 3\sqrt{3})$ respectively.



Slope of QR $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3\sqrt{3}}{3}$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

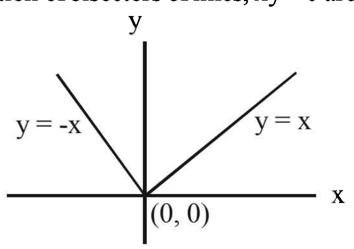
Let QM bisects the $\angle PQR$,

$\therefore$ Slope of the line QM $= \tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}(x - 0)$

$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

20. (a) Equation of bisectors of lines, xy = 0 are $y = \pm x$



$\therefore$ Put $y = \pm x$ in the given equation
$my^2 + (1 - m^2)xy - mx^2 = 0$

$\therefore mx^2 + (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow 1 - m^2 = 0 \Rightarrow m = \pm 1$

21. (d) Slope of $PQ = \frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of PQ = (k –1)

Also mid point of PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow 2y - 7 = 2(k-1)x - (k^2 - 1)$

$\Rightarrow 2(k-1)x - 2y + (8 - k^2) = 0$

$\therefore$ y-intercept $= -\frac{8-k^2}{-2} = -4$

$\Rightarrow 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

22. (d) Let (a^2, a) be the point of shortest distance on $x = y^2$
Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$$D = \frac{a^2 - a + 1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[(a - \frac{1}{2})^2 + \frac{3}{4}\right]$$

It is min when $a = \frac{1}{2}$ and $D_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

23. (a) If the lines $p(p^2 + 1)x - y + q = 0$
and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$
are perpendicular to a common line then these lines must be parallel to each other,

$\therefore m_1 = m_2 \Rightarrow -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2 + 1)(p + 1) = 0$

$\Rightarrow p = -1$

$\therefore$ p can have exactly one value.

24. (a) Given that

$P(1, 0), Q(-1, 0)$ and $\frac{AP}{AQ} = \frac{BP}{BQ} = \frac{CP}{CQ} = \frac{1}{3}$

$\Rightarrow 3AP = AQ$

Let $A = (x, y)$ then $3AP = AQ \Rightarrow 9AP^2 = AQ^2$

$\Rightarrow 9(x-1)^2 + 9y^2 = (x+1)^2 + y^2$

$\Rightarrow 9x^2 - 18x + 9 + 9y^2 = x^2 + 2x + 1 + y^2$

$\Rightarrow 8x^2 - 20x + 8y^2 + 8 = 0$

$\Rightarrow x^2 + y^2 - \frac{5}{3}x + 1 = 0$ ....(1)

$\therefore$ A lies on the circle given by eq (1). As B and C also follow the same condition, they must lie on the same circle.

$\therefore$ Centre of circumcircle of ΔABC

= Centre of circle given by (1) $= \left(\frac{5}{4}, 0\right)$

25. (c) Slope of line L $= -\frac{b}{5}$

Slope of line K $= -\frac{3}{c}$

Line L is parallel to line k.

$\Rightarrow \frac{b}{5} = \frac{3}{c} \Rightarrow bc = 15$

(13, 32) is a point on L.

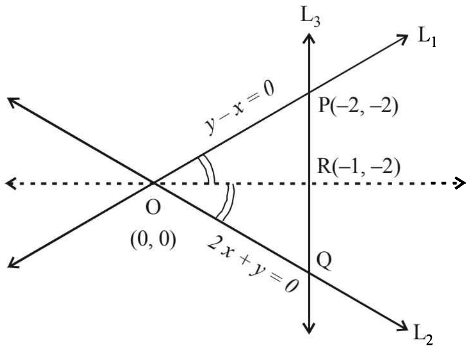
$\therefore \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20 \Rightarrow c = -\frac{3}{4}$

Equation of K : $y - 4x = 3 \Rightarrow 4x - y + 3 = 0$

Distance between L and K $= \frac{|52 - 32 + 3|}{\sqrt{17}} = \frac{23}{\sqrt{17}}$

26. (b)



$L_1 : y - x = 0$
$L_2 : 2x + y = 0$
$L_3 : y + 2 = 0$
On solving the equation of line L_1 and L_2 we get their point of intersection (0, 0) i.e., origin O.
On solving the equation of line L_1 and L_3,
we get $P = (-2, -2)$.
Similarly, we get $Q = (-1, -2)$
We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

27. (c) Let the joining points be A (1,1) and B (2,4).
Let point C divides line AB in the ratio 3 : 2.
So, by section formula we have

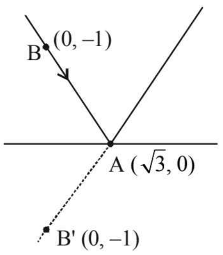
$$C = \left(\frac{3\times2+2\times1}{3+2}, \frac{3\times4+2\times1}{3+2}\right) = \left(\frac{8}{5}, \frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$\therefore$ C satisfies the equation $2x + y = k$.

$$\Rightarrow \frac{2+8}{5} + \frac{14}{5} = k \Rightarrow k = 6$$

28. (b) Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of

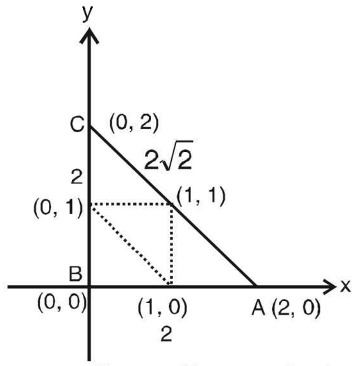


Reflected Ray is $\frac{-1-0}{0-\sqrt{3}} = \frac{y-0}{x-\sqrt{3}}$

$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

29. (b) From the figure, we have

$a = 2, b = 2\sqrt{2},\ c = 2$
$x_1 = 0, x_2 = 0, x_3 = 2$



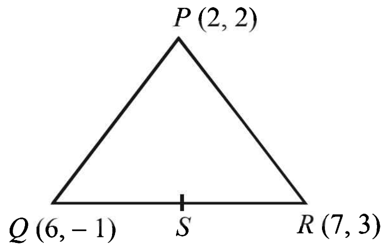
Now, x-co-ordinate of incentre is given as

$$\frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

$$\Rightarrow x\text{-coordinate of incentre} = \frac{2\times0 + 2\sqrt{2}.0 + 2.2}{2+2+2\sqrt{2}}$$

$$= \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$$

30. (d) Let P, Q, R, be the vertices of ΔPQR



Since PS is the median, S is mid-point of QR

$$\text{So, } S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$$

$$\text{Now, slope of } PS = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$$

Since, required line is parallel to PS therefore slope of required line = slope of PS Now, eqn of line passing through $(1, -1)$ and having slope $-\frac{2}{9}$ is

$$y - (-1) = -\frac{2}{9}(x-1)$$

$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$

31. (a) Given lines are
$4ax + 2ay + c = 0$
$5bx + 2by + d = 0$
The point of intersection will be

$$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$$

$$\Rightarrow x = \frac{2(ad-bc)}{-2ab} = \frac{bc-ad}{ab}$$

$$\Rightarrow y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$$

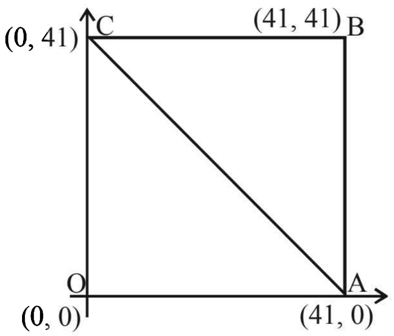
$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.
Also distance from axes is same
So $x = -y$ ($\because$ distance from x-axis is $-y$ as y is negative)

$$\frac{bc-ad}{ab} = \frac{5bc-4ad}{2ab}$$

$\Rightarrow \quad 3bc - 2ad = 0$

32. **(b)** Total number of integral points inside the square OABC $= 40 \times 40 = 1600$

No. of integral points on AC



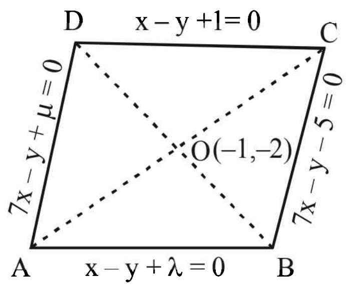
= No. of integral points on OB

$= 40$ [namely (1, 1), (2, 2) ... (40, 40)]

$\therefore$ No. of integral points inside the ΔOAC

$$= \frac{1600-40}{2} = 780$$

33. **(a)**



Let other two sides of rhombus are
$x - y + \lambda = 0$
and $7x - y + \mu = 0$
then O is equidistant from AB and DC and from AD and BC

$\therefore |-1+2+1| = |-1+2+\lambda| \Rightarrow \lambda = -3$

and $|-7+2-5| = |-7+2+\mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$
On solving the eq^{n}s of sides pairwise, we get

the vertices as $\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$