JEE 2023

Relations & Functions

DPP-09

1. If $-1 \le \left[2x^2-3\right] < 2$, then x belongs to: (where [.] is G.I.F.)

(A) $-\sqrt{\frac{5}{2}} < x \le -1$ only

(B) $1 \le x \le \sqrt{\frac{5}{2}}$ only

(C) $-\sqrt{\frac{5}{2}} < x \le -1$ or $1 \le x < \sqrt{\frac{5}{2}}$

(D) $-1 \le x \le 1$

2. Which of the following is true?

(A) $|x| = x\,\text{sgn}(x)$

(B) sgn (sgn(x)) = sgn(x)

(C) $x = |x|$ sgn (x)

(D) All of these

3. Which of the following is correct? ([.] represents greatest integer function, {.} represents fractional function)

(A) $[\{x\}] + \{[x]\} + \left[\left\{x^2+x+2\right\}\right] + \left\{\left[x^2+x+2\right]\right\} = 0$

(B) $\left(\left[\frac{200+1}{2}\right] + \left[\frac{200+2}{2^2}\right] + \left[\frac{200+2^2}{2^3}\right] + \cdots\infty\right) + \left([10] + \left[10+\frac{1}{10}\right] + \left[10+\frac{2}{10}\right] + \cdots\left[10+\frac{9}{10}\right]\right)$ = 300

(C) $[[[x]]] = [x]$

(D) $[|x|] = |[x]|$

4. The solution set of the inequality $\log_{0.8}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$ containing set(s)

(A) (–4, –3)
(B) (–3, 8)
(C) (8, ∞)
(D) (–∞, –4)

5. The range of the function $f(x) = x\{x\} - x[-x]$ (where [.] and { } denote the greatest integer function and fractional part function respectively) does not contain

(A) –1
(B) –2
(C) 1
(D) 2

6. Function $f : N \to N, f(x) = 2x+3$ is:

(A) One-one onto
(B) One-one into
(C) Many-one onto
(D) Many –one into

7. The function $f : R \to R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is:

(A) One-one but not onto
(B) Onto but not one-one
(C) Both one-one and onto
(D) Neither one-one nor onto

8. If $f_1(x) = 2x+3$, $f_2(x) = 3x^3+5$, $f_3(x) = x+\cos x$ are defined from $R \to R$, then f_1, f_2 and f_3 are:

(A) One-one-onto
(B) Many one into
(C) One-one-into
(D) Many one onto

9. If $f : R \to R$, then $f(x) = |x|$ is:

(A) One-one but not onto
(B) Onto but not one-one
(C) One-one and onto
(D) None of these

10. The function $f : R \to R$ defined by $f(x) = e^x$ is:

(A) Onto
(B) Many-one
(C) One-one and into
(D) Many one and onto

Answer Key

1.	**(C)**	**6.**	**(B)**
2.	**(D)**	**7.**	**(B)**
3.	**(A, B, C)**	**8.**	**(A)**
4.	**(A, C)**	**9.**	**(D)**
5.	**(A, B)**	**10.**	**(C)**

Hint & Solutions

1. **(C)**

$-1 \le [2x^2 - 3] < 2$

$-1 \le 2x^2 - 3 < 2$ [because $n_1 \le [x] < n_2, n_1 \le x < n_2$]

$2 \le 2x^2 < 5$; $1 \le x^2 < \frac{5}{2}$

On solving, $x \in \left(-\frac{\sqrt{5}}{2}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right)$

2. **(D)**

$y = \text{sgn}(x)$

$|x| = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$, $\text{sgn}(x) = \begin{cases} 1 \text{ if } x > 0 \\ -1 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \end{cases}$ and

also $|x| = x \text{ sgn}(x)$

(B) & (C) also true

3. **(A, B, C)**

(i) Fractional part of any integer is zero and integral part of any fraction is zero

(ii) $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \ldots + \left[x + \frac{n-1}{n}\right]$

$= [nx].x \in N$

Hence = 200 + 100 = 300

4. **(A, C)**

$\frac{x^2 + x}{x + 4} > 6 \Rightarrow \frac{(x-8)(x+3)}{x+4} > 0$

5. **(A, B)**

$f(x) = \begin{cases} x^2 & , x \in I \\ x^2 + x & , x \notin I \end{cases}$

So, $f(x) \ne -1$ or –2 for any $x \in R$

6. **(B)**

f is one-one because $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$

Further $f^{-1}(x) = \frac{x-3}{2} \notin N$ (domain)

when x = 1, 2, 3 etc.

$\therefore$ f is into which shows that f is one-one into.

7. **(B)**

We have $f(x) = (x-1)(x-2)(x-3)$

$\Rightarrow f(1) = f(2) = f(3) = 0 \Rightarrow f(x)$ is not one-one

For each $y \in R$, there exists $x \in R$ such that $f(x) = y$.

Therefore, f is onto.

Hence, $f: R \to R$ is onto but not one-one.

8. **(A)**

$f_3(x) = x + \cos x$

$f_3'(x) = 1 - \sin x \ge 0$

$\therefore$ $f_3'(x) = 0$ hold for only point $x = (2n+1)\frac{\pi}{2}, n \in 1$

i.e. at discrete points not in interval hence function strictly increasing. Hence function is one-one and onto.

Similarly we can prove for $f_1 \cdot f_2$

9. **(D)**

$f(-1) = f(1) = 1$ $\therefore$ function is many-one function.

Obviously, f is not onto so f is neither one-one nor onto.

10. **(C)**

Function $f : R \to R$ is defined by $f(x) = e^x$. Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$ or $e^{x_1} = e^{x_2}$ or $x_1 = x_2$

Therefore, f is one-one.

Let $f(x) = e^x = y$.

Taking log on both sides, we get $x = \log y$.

We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number. Therefore, function f is into.