Chapter 3

Analysis of Trusses

CHAPTER HIGHLIGHTS

- *Introduction*
- *Assumptions*
- *Zero–force members*
- *Methods of analysis*
- *Method of joints*
- *Method of sections*
- *Deflection of truss joints*
- *Redundant trusses*

Introduction

In this chapter the procedures for analyzing statically determinate trusses using the method of joints and the method of sections are discussed. The deflection of truss joints due to external load as well as lack of fit and temperature effects are also discussed. This chapter also outlines the concept of redundant trusses.

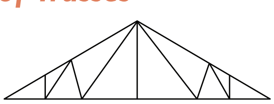
Assumptions

The following assumptions are made:

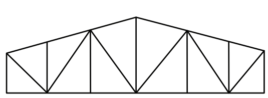
1. Members are joined together by smooth frictionless pins.
2. The loads lie in plane of truss and are applied only at the joints. Therefore members are subjected to axial force only. *BM*, *SF* are neglected.
3. The centroidal axes of various members framing into a joint will intersect at a common point.
4. Self weight of members is neglected.

Common Type of Trusses

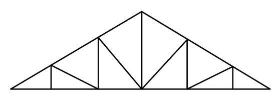
Roof Trusses



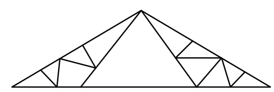
(a) Howe



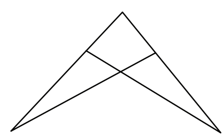
(b) Warren



(c) Pratt

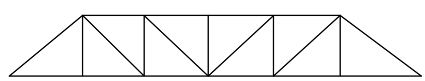


(d) Fink

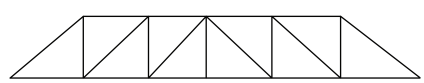


(e) Scissors

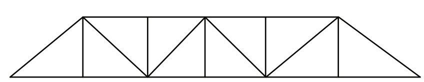
Bridge Trusses



(a) Pratt



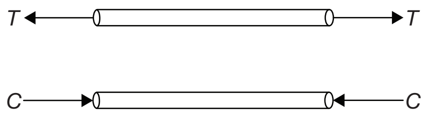
(b) Howe



(c) Warren (with verticals)

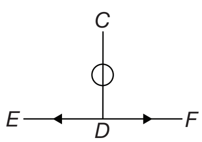
Sign Convention

1. If a force tends to elongate the member or acting away from joints then it is known as tensile force. Generally taken as positive.
2. If a force tends to shorten the member or acting towards the joint then it is known as compressive force. Generally taken as negative.

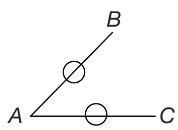


Zero–Force Members

1. If three members form a truss joint for which two of the members are collinear, the third member is a zero force member, provided no external force or support reaction is applied to the joint.
2. If two members act at a joint and if two of them are not along the straight line, then from the equilibrium of joint the force in each member to be zero.



Case 1

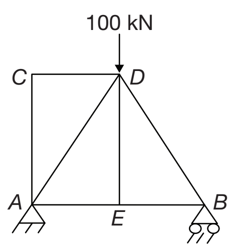


Case 2

SOLVED EXAMPLES

Example 1

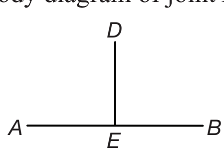
The force in member *DE* of the truss shown in the figure is ______. **[GATE, 1997]**



(A) 100.0 kN (B) zero
(C) 35.5 kN (D) 25.0 kN

Solution

Considering free body diagram of joint *E*,



From the principle of zero–force members, if three members meeting at a joint for which two of them are along the same straight line, the force in third member need to be zero. Hence, the correct answer is option (B).

Methods of Analysis

- Method of joints, method of sections and graphical method (Williot–Mohr diagram) is used to analyse the statically determinate trusses [i.e., $D_s = 0$].
- Unit load method, Maxwells method and graphical method are used to analyse the redundant trusses, [i.e., $D_s > 0$].

Method of Joints

This method is effective when forces in all members are to be required.

Procedure:

1. Start the analysis at a joint having atleast one known force and at most two unknown forces.
2. Apply the two force equilibrium equations [$\Sigma F_x = 0$ and $\Sigma F_y = 0$] for a joint and solve for the two unknown member forces with the correct directional sense.
3. The above procedure is repeated for other joints also where again it is necessary to choose a joint having two unknown member forces only.

Method of Sections

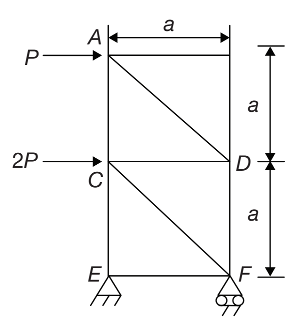
This method is effective when forces in limited members are to be determined.

Procedure:

1. Determine support reactions.
2. Force in required member is to be determined by drawing an imaginary section through the member such that the number of unknown forces through the section should not exceed three.
3. Consider the equilibrium of any one cut part. Equations of equilibrium is to be selected in such a way that moment of two unknown forces acting about a point should be zero so that the unknown force in third member can be easily determined (or) if two of the unknown forces are parallel, forces may be summed perpendicular to the direction of these unknowns to determine directly the third unknown force.

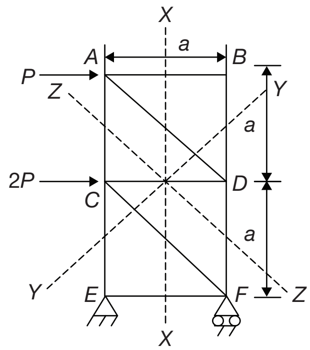
Example 2

The force in member '*CD*' of the truss in the figure is ______. **[GATE, 1988]**

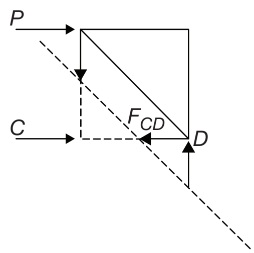


(A) zero (B) $2P$ (compression)
(C) P (compression) (D) P (tensile)

Solution



Out of the three sections X-X, Y-Y, Z-Z, section Z–Z is effective since the number of unknown forces are only three. Considering the equilibrium of above cut part.



Applying force equilibrium to above section $\Sigma F_X = 0$;

$$\boxed{F_{CD} = P(\text{tension})}$$

Hence, the correct answer is option (D).

Deflection of Truss Joints

Due to External Load System

- Maxwell's method or unit load method is used to determine the deflection.
- Unit load method is derived from Castigliano's theorem.

Deflection at any point is given by

$$\boxed{\delta = \sum \frac{PKL}{AE}}$$

Where

P = Forces in various members of structure for the given load system.

K = Forces in various members of structure caused by the unit load applied at the joint where deflection is desired. [The external load system need to be removed]

L = Length of member.

A = Cross-sectional area of member.

Procedure:

1. Find out the forces in various members due to the given external loads system and let it be $P_1, P_2, P_3, \ldots$.
2. Remove the external load system and apply an unit load in a direction in which deflection is desired and then find out the forces in various members due to the unit load only. Let it be $K_1, K_2, K_3, \ldots$.
3. The deflection at the joint is given by $\sum \frac{P_1 K_1}{AE}$. The typical tabular form shall be as follows:

Member	P	K	L	A	PKL/AE
					$\sum \frac{PKL}{AE}$

Due to Lack of Fit or Temperature Effect

$$\delta = \sum \frac{PKL}{AE}, \delta = \sum K\left(\frac{PL}{AE}\right)$$

$$\boxed{\delta = \sum K\delta'}$$

Where

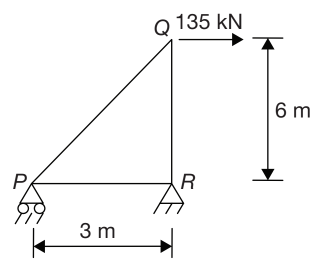
δ' = Deflection due to lack of fit (or) Deflection due to temperature effect

$\delta' = \alpha\,(\Delta T)L$

In case of deflection due to temperature effect.

Example 3

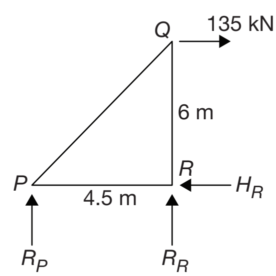
The right triangular truss is made of members having equal cross sectional area of 1550 mm^2 and young's modules of 2×10^5 MPa. The horizontal deflection of the joint Q is



(A) 2.47 mm (B) 10.25 mm
(C) 14.31 mm (D) 15.68 mm

Solution

Step 1: Calculation of forces in all member due to external load system.



$\Sigma F_X = 0$

$135 - H_R = 0$

$$\boxed{H_R = 135 \text{ kN}}$$

$$\Sigma F_y = 0$$

$$R_P + R_R = 0$$

$\curvearrowright$ +ve

$$\Sigma M_R = 0$$

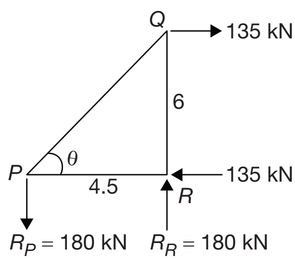
$$R_P \times 4.5 + 135 \times 6 = 0$$

$$R_P = \frac{-135 \times 6}{4.5} = -180 \text{ kN.}$$

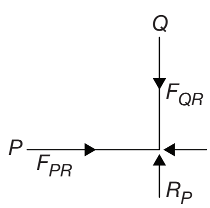
The negative indicates that R_P is acting downwards.

$$R_P = 180 \text{ kN}\downarrow$$

$$R_R = 180 \text{ kN}\uparrow$$



At joint *R*:



+ve

$$\uparrow \Sigma F_y = 0$$

$$F_{PR} - 135 = 0$$

$$\boxed{F_{PR} = 135 \text{kN}(C)}$$

+ve

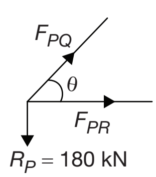
$$\uparrow \Sigma F_Y = 0$$

$$R_P - F_{QR} = 0$$

$$F_{QR} = R_P = 180 \text{ kN}$$

$$\boxed{F_{QR} = 180 \text{ kN}(C)}$$

At joint *P*:



$$F_{PQ} \sin\theta = R_P$$

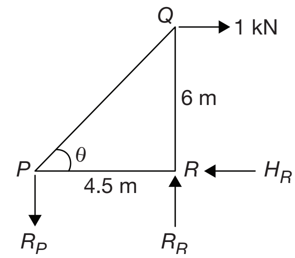
$$\tan\theta = \frac{6}{4.5} \Rightarrow \theta = 53.13°$$

$$F_{PQ} = \frac{R}{\sin(53.13°)}$$

$$= 180 \times \frac{1}{\sin 53.13}$$

$$\boxed{F_{PQ} = 225 \text{ kN}(T)}$$

Step 2: Calculation of forces in members due to unit load remove the external load system and apply a unit load at joint θ in horizontal direction.



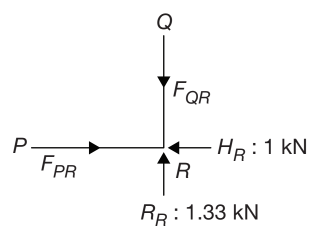
$$HR = 1 \text{ kN}$$

$$1 \times 6 = R_P \times 4.5$$

$$R_P = 1 \times \frac{6}{4.5} = 1.33 \text{ kN}$$

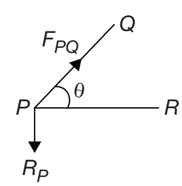
$$R_Q = 1.33 \text{ kN}$$

At joint *R*:



$$\boxed{F_{PR} = 1 \text{ kN(C)}} \quad \boxed{F_{QR} = 1.33 \text{ kN(C)}}$$

At joint *P*:



$$F_{PQ} \sin\theta = 1.33$$

$$F_{PQ} \times \sin(53.13°) = 1.33$$

$$F_{PQ} = \frac{1.33}{\sin(53.13°)}$$

$$\boxed{F_{PQ} = 1.66 \text{ kN}(T)}$$

Step 3: Tabular form

Member	P	K	L	PKL
PQ	225	1.66	7.5	2801.25
QR	–180	–1.32	6	1425.6
PR	–135	–1	4.5	607.5

$$\Sigma PKL = 4834.35$$

$$\therefore \delta = \frac{\sum PKL}{AE}$$

$$= \frac{4834.35}{1550 \times 220} = 0.01568 \text{ m}$$

$$\delta = 15.68 \text{ mm}$$

Hence, the correct answer is option (D).

Redundant Trusses

Maxwell's method based on minimum potential energy is used.

The following procedure is adopted.

Step 1: Remove the redundant member and find the forces in the members of the structure for a given load system.

Step 2: Remove the given load system and apply unit pulls in place of redundant member and find the forces in members of the structure.

Step 3: Tabulate the above results as follows:

Member	P	K	L	A	$\frac{PKL}{AE}$	$\frac{K^2L}{AE}$
			Total		$\sum\frac{PKL}{AE}$	$\sum\frac{K^2L}{AE}$

Calculate $\sum\frac{PKL}{AE}$ and $\sum\frac{K^2L}{AE}$.

Step 4:

$$\boxed{x = -\frac{\sum L}{\sum \frac{K^2L}{AE}}}$$

Correcting factor,

In this expression, the summation $\sum\frac{PKL}{AE}$ does not consider the redundant member but the summation $\sum\frac{K^2L}{AE}$ involves the redundant member.

Step 5: Force in any member is given by

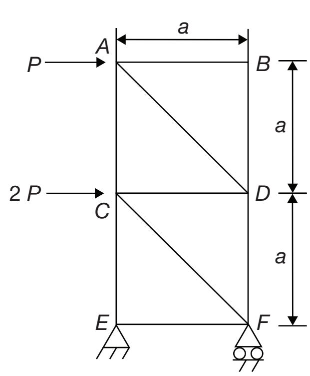
$$\boxed{S = P + KX}$$

Exercises

1. **Assertion (A):** In the analysis of statically determinate planar trusses by the method of joints, not more than two unknown bar forces can be determined.

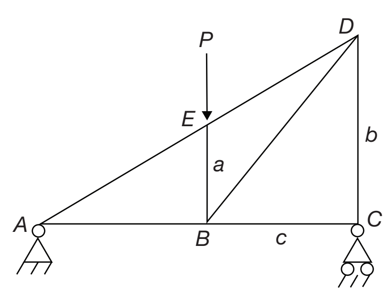
   Reason (R): There are only two equations of force equilibrium available for a co-planar concurrent system.
   (A) Both A and R are true and R is the correct explanation of A.
   (B) Both A and R are true but R is not a correct explanation of A.
   (C) A is true but R is false.
   (D) A is false but R is true.

2. The force in the member 'CD' of the truss in the figure is ______.



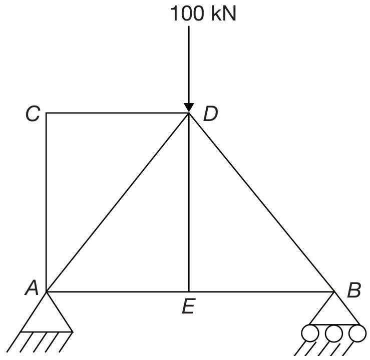
   (A) zero
   (B) $2P$ (Compression)
   (C) P (Compression)
   (D) P (Tensile)

3. The forces in members 'abc' in the truss shown in the following figure, are



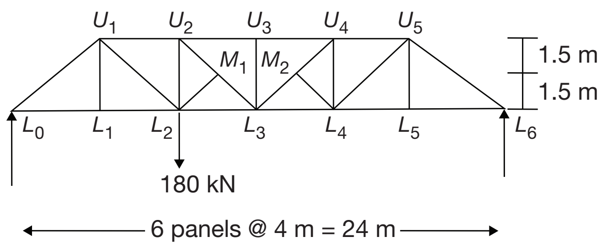
   (A) P, $P/2$, 0
   (B) $P/2$, P, 0
   (C) P, P, P
   (D) $P/2$, $P/2$ 0

4. The force in the member DE of the truss shown in the figure is ______.



(A) 100.0 kN (B) zero
(C) 35.3 kN (D) 25.0 kN

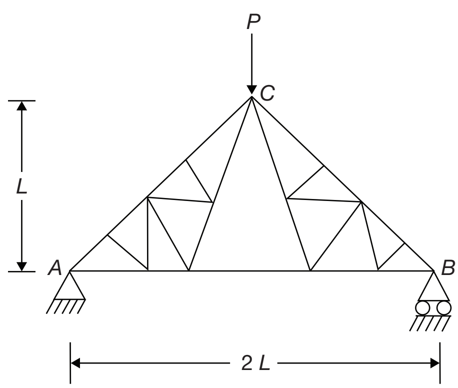
5. A truss, as shown in the figure, is carrying 180 kN load at node L_2. The force in the diagonal member M_2U_4 will be



(A) 100 kN tension.
(B) 100 kN compression.
(C) 80 kN tension.
(D) 80 kN compression.

Direction for questions 6 and 7:

A truss is shown in the figure. Members are of equal cross-section A and same modulus of elasticity E. A vertical force P is applied at point C.



6. Force in the member AB of the truss is _______.

(A) $\frac{P}{\sqrt{2}}$ (B) $\frac{P}{\sqrt{3}}$

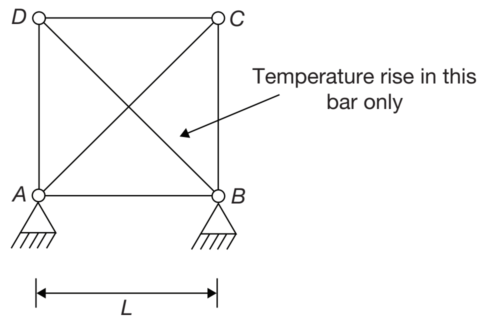
(C) $\frac{P}{2}$ (D) P

7. Deflection of the point C is _______.

(A) $\left(\frac{2\sqrt{2}+1}{2}\right)\frac{PL}{EA}$ (B) $\sqrt{2}\,\frac{PL}{EA}$

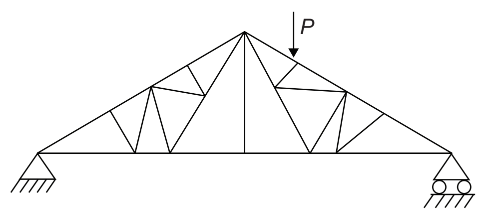
(C) $(2\sqrt{2}+1)\frac{PL}{EA}$ (D) $(\sqrt{2}+1)\frac{PL}{EA}$

8. Identify the FALSE statement from the following, pertaining to the effects due to a temperature rise ΔT of the BD alone in the plane truss shown in the figure:



(A) No reactions develop at supports A and D.
(B) The bar BD will be subject to a tensile force.
(C) The bar AC will be subject to a compressive force.
(D) The BC will be subject to a tensile force.

9. For the plane truss shown in the figure, the number of zero force members for the given loading is _______.



(A) 4 (B) 8
(C) 11 (D) 13

10. Consider the following assumptions:

I. All members have same cross-sectional area.
II. The bending resistance of all the members is small in comparison with their axial force resistance.
III. All the external loads are applied directly or indirectly at the joints.
IV. All joints are idealized to be frictionless hinges.

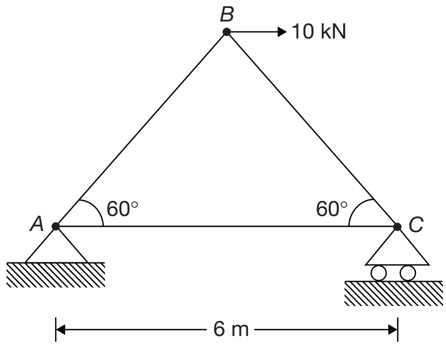
Which of these are the assumptions made in the force analysis of simple trusses?

(A) I, II and IV (B) II, III and IV
(C) I, II and III (D) III and IV

11. What does the Williot–Mohr diagram yield?

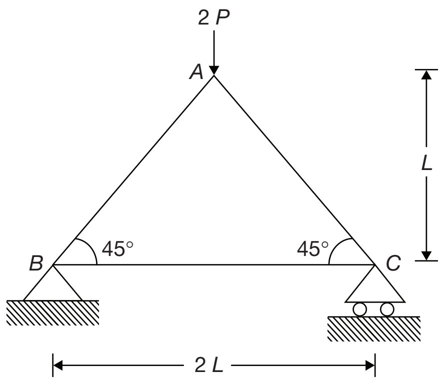
(A) Forces in members of a truss
(B) Moments in a fixed beam
(C) Reactions at the supports
(D) Joint displacement of a pin jointed plane frame

12. What is the force in the member *BC* of the plane frame shown below?



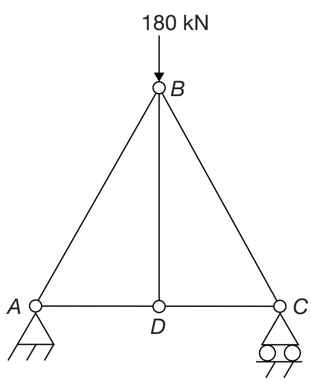
(A) 10 kN tensile
(B) 10 kN compressive
(C) 5.76 kN compressive
(D) Zero

13. A simple plane truss acted upon by a load $2P$ at the apex *A* is shown below. The axial force in the member *AB* is



(A) P
(B) $\sqrt{2}P$
(C) $\frac{\sqrt{3}}{2}P$
(D) $\sqrt{3}P$

14. A truss, as shown in the figure is carrying 180 kN at *B*. The force in member *BD* is



(A) 180 kN (Tensile)
(B) 180 kN (Compressive)
(C) $180\sqrt{2}$ kN (Tensile)
(D) zero

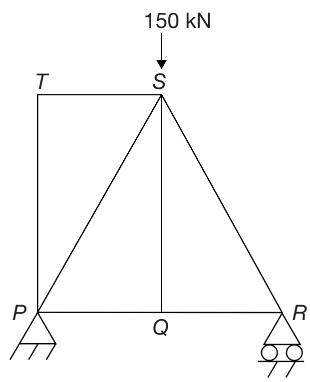
15. Match List I and List II and select the correct answer using the code given below.

	List I		List II
a.	Column analogy method	**1.**	Stiffness
b.	Kanis rotation contribution method	**2.**	Force method
c.	Castiglianos theorem of minimum strain energy		
d.	Moment distribution method		

Codes:

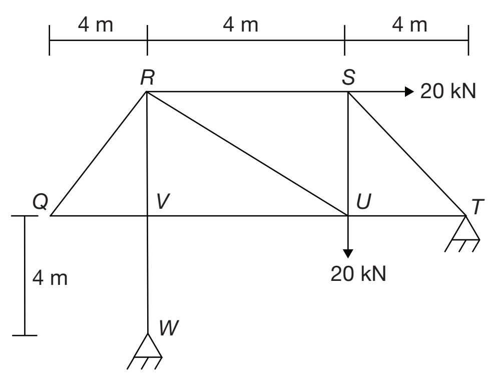
	a	b	c	d
(A)	2	1	2	1
(B)	1	2	1	2
(C)	1	1	2	2
(D)	1	2	2	1

16. The force in the member *QS* of the truss shown in the figure is:



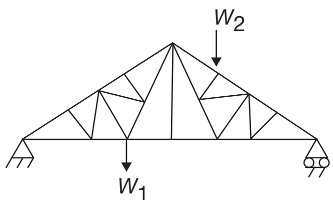
(A) 150 kN
(B) $\frac{150}{\sqrt{2}}$ kN
(C) Zero
(D) 35.5 kN

17. The pin jointed 2-D truss is loaded with a horizontal force of 20 kN at joint '*S*' and another 20 kN vertical force at joint '*U*' as shown. Find the force in the member *RS* (in kN). (Taking tension as positive and compression as negative)



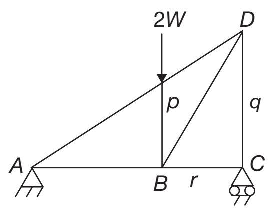
(A) zero
(B) 20
(C) 40
(D) −20

18. For the plane truss shown in the figure, the number of zero force members for the given loading is



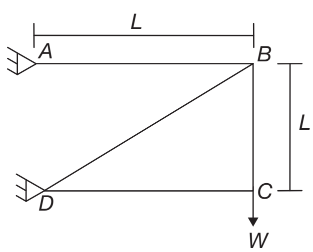
(A) 8 (B) 4
(C) 11 (D) 13

19. The force in members p, q, r in the truss shown in the figure are



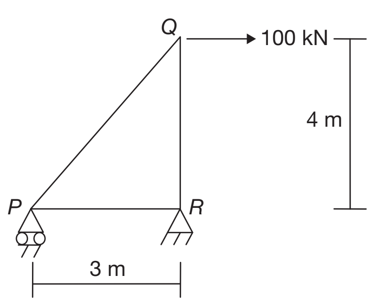
(A) $2W, W, 0$ (B) $2W, W, W$
(C) $\frac{W}{2}, W, 0$ (D) $\frac{W}{2}, \frac{W}{2}, 0$

20. For the truss shown in the figure, the force in member AB is



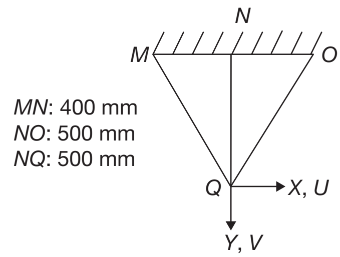
(A) W (B) $\frac{W}{2}$
(C) zero (D) None of these

21. The right triangular truss is made of members having equal cross-sectional area of 1500 mm^2 and Young's of $10^5 \times W^5$ MPa. The horizontal deflection at join Q is



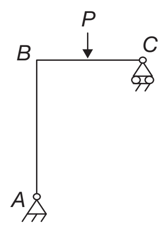
(A) 7.79 mm (B) 4.60 mm
(C) 8.20 mm (D) None of these

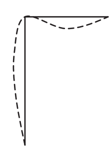
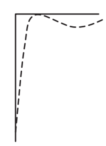
22. In a redundant joint model, three bar members are pin connected at Q as shown in the figure. Under some load placed at Q, the elongation of the members MQ and OQ are found to be 40 mm and 25 mm. then the horizontal displacement 'U' and vertical displacement 'V' of the node Q_1 in mm will be respectively

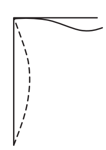


(A) 8.84 mm, 44.20 mm
(B) 5 mm, 20 mm
(C) 0 mm, 44.20 mm
(D) 7 mm, 35 mm

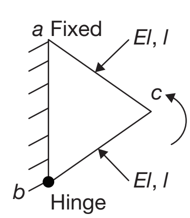
23. Identify the correct deflection diagram corresponding to the loading in the plane frame.



(A)  (B) 

(C)  (D) None of these

24. Rotational stiffness coefficient, K_{11} at joint 'C' for the frame having two members of equal $\frac{EI}{l}$ is given by

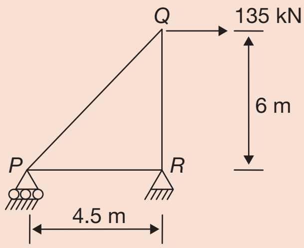


(A) $\frac{7EI}{l}$ (B) $\frac{5EI}{l}$

(C) $\frac{4EI}{l}$ (D) $\frac{6EI}{l}$

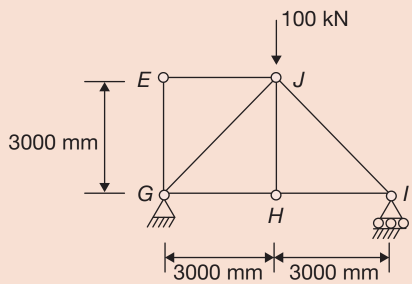
Previous Years' Questions

1. The right triangular truss is made of members having equal cross sectional area of 1550 mm^2 and Young's of 2×10^5 MPa. The horizontal deflection of the joint Q is _______. **[GATE, 2007]**



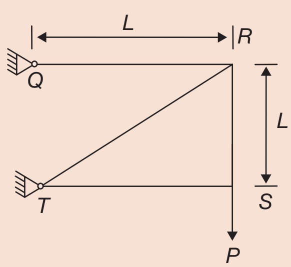
(A) 2.47 mm (B) 10.25 mm
(C) 14.31 mm (D) 15.68 mm

2. The members EJ and IJ of a steel truss shown in the figure are subjected to a temperature rise of 30°C. The coefficient of thermal expansion of steel is 0.000012 per °C per unit length. The displacement (mm) of joint E relative to joint H along the direction HE of the truss, is ________. **[GATE, 2008]**



(A) 0.255 (B) 0.589
(C) 0.764 (D) 1.026

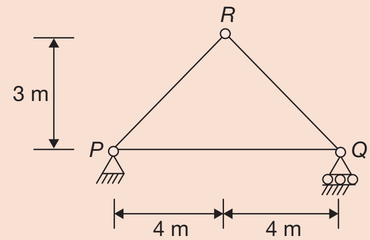
3. For the truss shown in the figure, the force in the member QR is ______. **[GATE, 2010]**



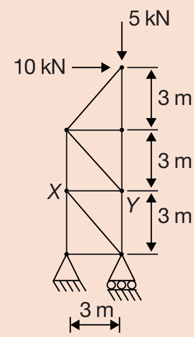
(A) zero (B) $\dfrac{P}{\sqrt{2}}$

(C) P (D) $\sqrt{2}P$

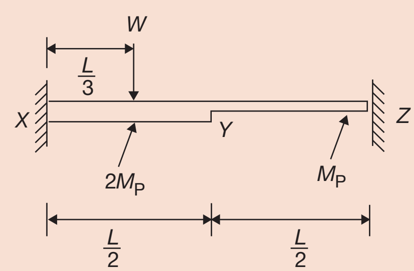
4. For the truss shown below, the member PQ is short by 3 mm. the magnitude of vertical displacement of joint R (in mm) is _______. **[GATE, 2014]**



5. For the 2D truss with the applied loads shown below, the strain energy in the member XY is ________ kN-m. For member XY, assume $AE = 30$ kN, where A is cross-section area and E is the modulus of elasticity. **[GATE, 2015]**

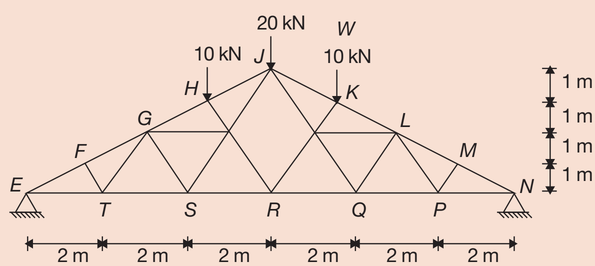


6. A fixed end beam is subjected to a load, W at 1/3 span from the left support as shown in the figure. The collapse load of the beam is **[GATE, 2015]**



(A) 16.5 M_p/L (B) 15.5 M_p/L
(C) 15.0 M_p/L (D) 16.0 M_p/L

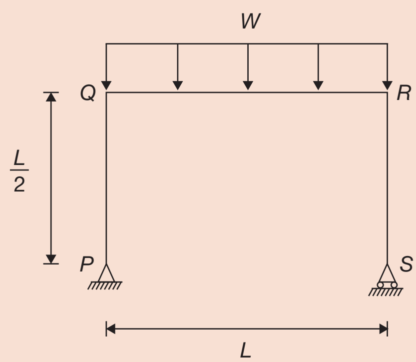
7. A plane truss with applied loads is shown in the figure. **[GATE, 2016]**



The members which do not carry any force are

(A) FT, TG, HU, MP, PL
(B) ET, GS, UR, VR, QL
(C) FT, GS, HU, MP, QL
(D) MP, PL, HU, FT, UR

8. The portal frame shown in the figure is subjected to a uniformly distributed vertical load w (per unit length). **[GATE, 2016]**



The bending moment in the beam at the joint 'Q' is

(A) zero
(B) $\frac{wL^2}{24}$ (hogging)
(C) $\frac{wL^2}{12}$ (hogging)
(D) $\frac{wL^2}{8}$ (sagging)

Answer Keys

Exercises

1. A	**2.** D	**3.** A	**4.** A	**5.** A	**6.** C	**7.** A	**8.** B	**9.** B	**10.** B
11. D	**12.** B	**13.** B	**14.** D	**15.** A	**16.** C	**17.** A	**18.** A	**19.** A	**20.** A
21. A	**22.** A	**23.** A	**24.** A						

Previous Years' Questions

1. D	**2.** C	**3.** C	**4.** 2	**5.** 5	**6.** C	**7.** A	**8.** A