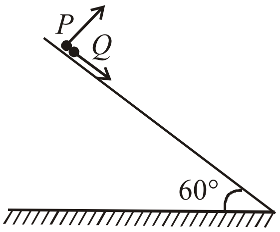
Chapter 2

MOTION IN ONE AND TWO DIMENSIONS

A SINGLE CORRECT CHOICE TYPE

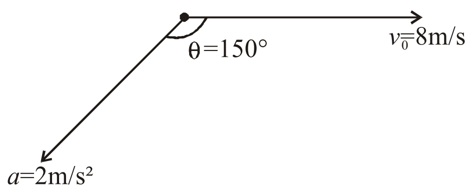
Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide on the inclined plane after t = 4 second. The speed of projection of P is



(a) 5 m/s (b) 10 m/s
(c) 15 m/s (d) 20 m/s

2. The figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The minimum radius of curvature of trajectory of the body is



(a) 2m (b) 4m
(c) 8m (d) 16m

3. A stone projected at an angle θ with horizontal from the roof of a tall building falls on the ground after three second. Two second after the projection it was again at the level of projection. Then the height of the building is

(a) 15m (b) 5m
(c) 25m (d) 20m

4. Two particles A and B separated by a distance $2R$ are moving counter clockwise along the same circular path of radius R each with uniform speed v. At time $t = 0$, A is given a tangential acceleration of magnitude $a = \dfrac{72v^2}{25\pi R}$

(a) the time lapse for the two bodies to collide is $\dfrac{6\pi R}{5v}$

(b) the angle covered by A is $11\pi/6$

(c) angular velocity of A is $\dfrac{11v}{5R}$

(d) radial acceleration of A is $289v^2/5R$

5. A cannon ball has the same range R on a horizontal plane for two angles of projection. If h_1 and h_2 are the greatest heights in the two paths for which this is possible, then

(a) $R = h_1 h_2$ (b) $R = 4\sqrt{h_1 h_2}$

(c) $R = \sqrt[3]{h_1 h_2}$ (d) $R = (h_1 h_2)^{\frac{1}{4}}$

6. A passenger in an open car travelling at 30 m/s throws a ball out over the bonnet. Relative to the car the initial velocity of the ball is 20 m/s at 60^0 to the horizontal. The angle of projection of the ball with respect to the horizontal road will be

(a) $\tan^{-1}\left(\dfrac{2}{3}\right)$ (b) $\tan^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$

(c) $\tan^{-1}\left(\dfrac{4}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\dfrac{3}{4}\right)$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ				

7. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6t$ meter, where t is in second. The velocity with which the projectile is projected is

(a) 8 m/sec
(b) 6 m/sec
(c) 10 m/sec
(d) Not obtainable from the data

8. A particle is projected at an angle of elevation α and after t seconds it appears to have an angle of elevation β as seen from point of projection. The initial velocity will be

(a) $\dfrac{gt}{2\sin(\alpha-\beta)}$ (b) $\dfrac{gt\cos\beta}{2\sin(\alpha-\beta)}$

(c) $\dfrac{\sin(\alpha-\beta)}{2gt}$ (d) $\dfrac{2\sin(\alpha-\beta)}{gt\cos\beta}$

9. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $5\pi/36$ rad should strike a given target. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target

(a) $\dfrac{5\pi}{36}$ rad (b) $\dfrac{11\pi}{36}$ rad

(c) $\dfrac{7\pi}{36}$ rad (d) $\dfrac{13\pi}{36}$ rad

10. A cricket ball is hit at 30° with the horizontal with kinetic energy K. The kinetic energy at the highest point is

(a) 0 (b) $K/4$
(c) $K/2$ (d) $3K/4$

11. A body of mass m is projected at an angle of 45° with the horizontal with velocity u. If air resistance is negligible, then total change in momentum when it strikes the ground is

(a) $2\,mu$ (b) $\sqrt{2}\,mu$
(c) mu (d) $mu/\sqrt{2}$

12. A ball is projected horizontally with a speed v from the top of a plane inclined at an angle 45° with the horizontal. How far from the point of projection will the ball strike the plane

(a) $\dfrac{v^2}{g}$ (b) $\sqrt{2}\dfrac{v^2}{g}$

(c) $\dfrac{2v^2}{g}$ (d) $\sqrt{2}\left[\dfrac{2v^2}{g}\right]$

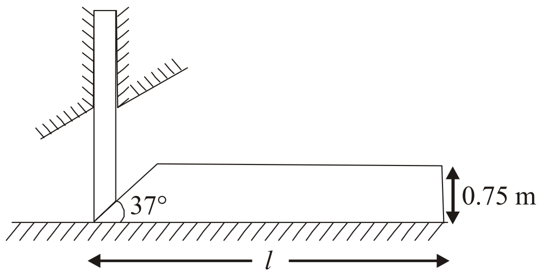
13. A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is

(a) h (b) $h/2$
(c) $2h$ (d) $2h/3$

14. A particle is projected at angle 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane is given 53°. Then the maximum height attained by the particle from the incline plane will be

(a) 3 m (b) 4 m
(c) 5 m (d) zero

15. From the given position, as shown in the figure, the plank starts moving towards left with initial velocity zero and acceleration 8 m/s^2. The rod flies in the air and falls back on the plank. With all surfaces smooth, what should be the least possible length ℓ, so that the rod doesn't fall on the plank?



(a) 4.84 m (b) 3.84 m
(c) 5.62 m (d) 4.62 m

16. For a stone thrown from a lower of unknown height, the maximum range for a projection speed of 10 m/s is obtained for a projection angle of 30°. The corresponding distance between the foot of the lower and the point of landing of the stone is

(a) 10 m (b) 20 m
(c) $(20/\sqrt{3})$ m (d) $(10/\sqrt{3})$ m

17. A particle moves with a constant speed u along the curve $y = \sin x$. The magnitude of its acceleration at the point corresponding to $x = \pi/2$ is

(a) $\dfrac{u^2}{2}$ (b) $\dfrac{u^2}{\sqrt{2}}$

(c) u^2 (d) $\sqrt{2}\,u^2$

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	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ
	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ
	17. ⓐⓑⓒⓓ				

18. A skier travels with a constant speed of 6 m/s along a parabolic path $y = \frac{x^2}{20}$. Find the acceleration of the skier when he is at (10, 5). Neglect the size of skier.

(a) $\frac{9}{10\sqrt{2}}$ units (b) $\frac{9\sqrt{2}}{10}$ units

(c) 1 unit (d) none of these

19. A swimmer can swim in still water with a speed of $\sqrt{5}$ m/s. While crossing a river his average speed is 3 m/s. If he crosses the river in the shortest possible time, what is the speed of flow of water?

(a) 2m/s (b) 4 m/s

(c) 6 m/s (d) 8 m/s

20. A particle starts from rest and moves with an acceleration of $a = \{2 + |t - 2|\}$ m/s^2, the velocity of the particle at $t = 4$ sec is

(a) 2 m/s (b) 4 m/s

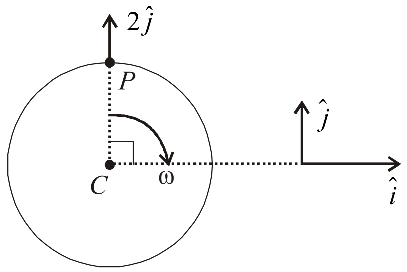
(c) zero (d) 12 m/s

21. A car accelerates from rest with a constant acceleration α on a straight road. After gaining a velocity v_1 the car moves with the same velocity for some-time. Then the car decelerated to rest with a retardation β. If the total distance covered by the car is equal to S, the total time taken for its motion is

(a) $\frac{S}{v} + \frac{v}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ (b) $\frac{S}{v} + \frac{v}{\alpha} + \frac{v}{\beta}$

(c) $\left(\frac{v}{\alpha} + \frac{v}{\beta}\right)$ (d) $\frac{S}{v} - \frac{v}{2}\left(\frac{v}{\alpha} + \frac{v}{\beta}\right)$

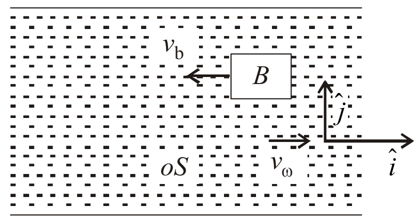
22. A disc having plane parallel to the horizontal is moving such that velocity of point P with respect to ground on its periphery is 2 m/s $\hat{j}$ as shown in the figure. If radius of disc is R = 1 m and angular speed of disc about vertical axis passing through disc is $\omega = 2$ rad/s, the velocity of centre of disc in m/s is



(a) $2\hat{j}$ (b) $2\hat{i} + 2\hat{j}$

(c) $-2\hat{i} + 2\hat{j}$ (d) none of these

23. A boat B is moving upstream with velocity 3 m/s with respect to ground. An observer standing on boat observes that a swimmer S is crossing the river perpendicular to the direction of motion of boat. If river flow velocity is 4 m/s and swimmer crosses the river of width 100 m in 50 sec, then



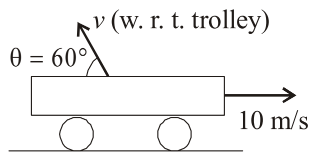
(a) velocity of swimmer w.r.t ground is $\sqrt{13}$ m/s

(b) drift of swimmer along river is zero

(c) drift of swimmer along river will be 50 m

(d) velocity of swimmer w.r.t ground is 2 m/s

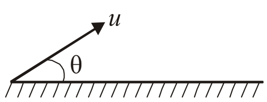
24. For an observer on trolley direction of projection of particle is shown in the figure, while for observer on ground ball rise vertically. The maximum height reached by ball from trolley is



(a) 10m (b) 15m

(c) 20m (d) 5m

25. A particle is projected with a velocity u at an angle θ with the horizontal. After some time velocity of particle becomes perpendicular to initial velocity when the particle is still above the horizontal. The angle of projection may be (choose the most appropriate option)



(a) 30°

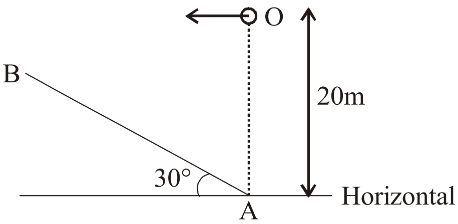
(b) 60°

(c) any value except $\theta = 0°$

(d) possible for no value of θ

MARK YOUR RESPONSE					
	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ
	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ		

26. AB is an inclined plane of inclination 30° with horizontal. Point O is 20 m above point A. A particle is projected horizontally and it collides with the plane AB, perpendicularly. Speed of the particle must be ($g = 10\ m/s^2$)



(a) 13 m/s (b) $8\sqrt{3}\ m/s$

(c) $4\sqrt{5}\ m/s$ (d) $2\sqrt{5}\ m/s$

27. A boy is standing on a cart moving along x-axis with the speed of 10 m/s. When the cart reaches the origin he throws a stone in the horizontal x-y plane with the speed of 5 m/s with respect to himself at an angle θ with the x-axis. It is found that the stone hits a ball lying at rest at a point whose co-ordinates are ($\sqrt{3}$m, 1m). The value of θ is (gravitational effect is to be ignored)

(a) 30° (b) 60°

(c) 90° (d) 120°

28. A particle is moving with a constant speed of π m/s on a circular track of radius 12 m. The magnitude of its average acceleration for the time interval of 6s is

(a) $\frac{\pi^2}{12} m/s^2$ (b) $\frac{\pi}{3\sqrt{2}} m/s^2$

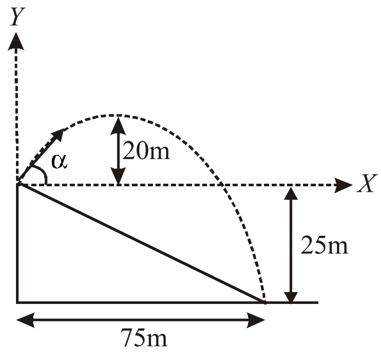
(c) $\frac{\pi}{6} m/s^2$ (d) $0\ m/s^2$

29. A particle, of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, the distance fallen through in time t is

(a) $g\frac{m^2}{\mu^2}\left[e^{-\mu t/m}+1+\frac{\mu t}{m}\right]$ (b) $g\frac{m^2}{\mu^2}\left[e^{-\mu t/m}-1+\frac{\mu t}{m}\right]$

(c) $g\frac{m^2}{\mu^2}\left[e^{-\mu t/m}-1+\frac{2\mu t}{m}\right]$ (d) $g\frac{m^2}{2\mu^2}\left[e^{-\mu t/m}-1+\frac{\mu t}{m}\right]$

30. A ball thrown down the incline strikes at a point on the incline 25m below the horizontal as shown in the figure. If the ball rises to a maximum height of 20m above the point of projection, the angle of projection α (with horizontal x-axis) is



(a) $\tan^{-1}\frac{4}{3}$ (b) $\tan^{-1}\frac{3}{4}$

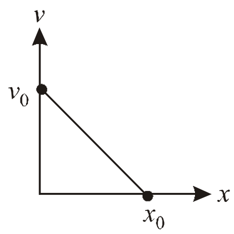
(c) $\tan^{-1}\frac{3}{2}$ (d) $\tan^{-1}\frac{2}{3}$

31. A person throws vertically n balls per second with the same velocity. He throws a ball whenever the previous one is at its highest point. The height to which the balls rise is

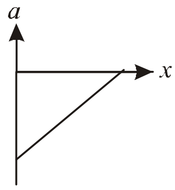
(a) g/n^2 (b) $2gn$

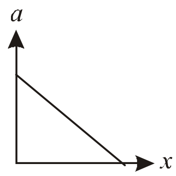
(c) $g/2n^2$ (d) $2gn^2$

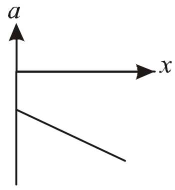
32. The velocity-displacement graph of a particle moving along a straight line is shown

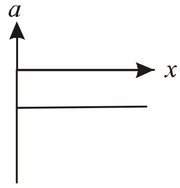


The most suitable acceleration-displacement graph will be

(a)


(b)


(c)


(d)


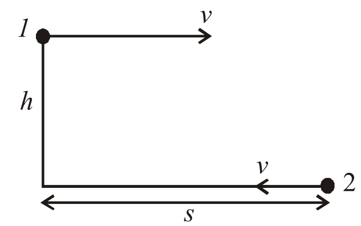
MARK YOUR RESPONSE					
	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ
	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ			

33. A point moves in x-y plane according to the law $x = 3\cos 4t$ and $y = 3(1 - \sin 4t)$. The distance travelled by the particle in 2 sec is (where x and y are in meters)

(a) 48 m (b) 24 m

(c) $48\sqrt{2}$ m (d) $24\sqrt{2}$ m

34. Two particles 1 and 2 are projected with same speed v as shown in figure. Particle 2 is on the ground and particle 1 is at a height h from the ground and at a horizontal distance s from particle 2. If a graph is plotted between v and s for the condition of collision of the two then (v on y-axis and s on x-axis)



(a) It will be a parabola passing through the origin

(b) It will be a straight line passing through the origin and having a slope of $\sqrt{\frac{g}{8h}}$

(c) It will be a straight line passing through the origin and having a slope of $\sqrt{\frac{8h}{g}}$

(d) None of these

35. A particle of unit mass is projected with velocity u at an inclination α above the horizon in a medium whose resistance is k times the velocity. Its direction will again make an angle α with the horizon after a time

(a) $\frac{1}{k}\log\left\{1 - \frac{2ku}{g}\sin\alpha\right\}$ (b) $\frac{1}{k}\log\left\{1 + \frac{2ku}{g}\sin\alpha\right\}$

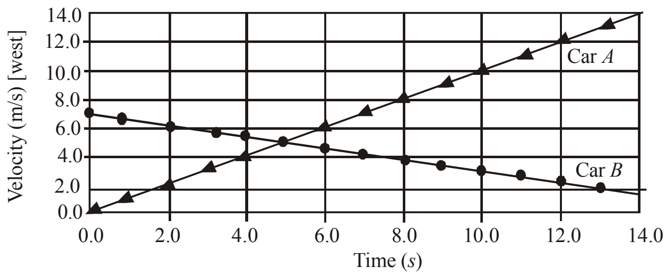
(c) $\frac{1}{k}\log\left\{1 + \frac{ku}{g}\sin\alpha\right\}$ (d) $\frac{1}{k}\log\left\{1 + \frac{2ku}{3g}\sin\alpha\right\}$

36. The graph shown illustrates velocity versus time for two cars A and B constrained to move in a straight line. Both cars were at the same position at t = 0s. Consider the following statements.

(1) Car A is travelling west and Car B is travelling east.

(2) Car A overtakes Car B at t = 5 s.

(3) Car A overtakes Car B at t = 10 s.



Which of the following is correct?

(a) Only statement 1 is true

(b) Only statement 2 is true

(c) Only statement 3 is true

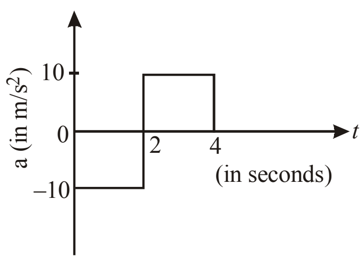
(d) Only statements 1 and 2 are true

37. Two drag racers accelerate from rest down a drag strip. The engine of each car produces a constant forward force of 1200 N on the car. Car A has a mass of 1.25×10^3 kg, while car B has a mass of 1.20×10^3 kg. When car A has gone 1.00×10^2 m, car B will be

(a) 2m behind car A (b) 2m behind car A

(c) 2 m ahead of car A (d) 4 m ahead of car A

38. A particle starts from rest at time t = 0 and moves on a straight line with acceleration as plotted in figure. The speed of the particle will be maximum at time



(a) 1s (b) 2s

(c) 3s (d) 4s

39. A car moves with a speed of 60 km/hr from point A to point B and then with the speed of 40 km/hr from point B to point C. Further it moves to a point D with a speed equal to its average speed between A and C. Points A, B, C and D are collinear and equidistant. The average speed of the car between A and D is

(a) 30 km/hr (b) 50 km/hr

(c) 48 km/hr (d) 60 km/hr

MARK YOUR RESPONSE	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ
	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ			

40. Two particles start moving from rest from the same point along the same straight line. The first moves with constant velocity v and the second with constant acceleration a. During the time that elapse before the second catches the first, the greatest distance between the particles is

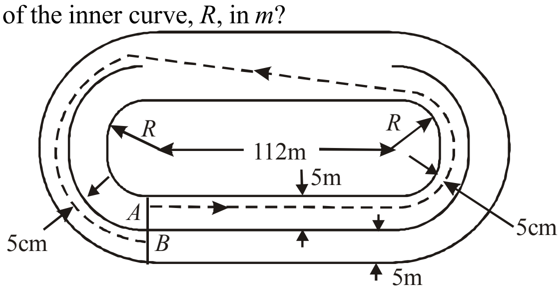
(a) $\frac{v^2}{a}$ (b) $\frac{v^2}{2a}$ (c) $\frac{2v^2}{a}$ (d) $\frac{v^2}{4a}$

41. If a particle is projected with speed u from ground at an angle with horizontal ,then radius of curvature of a point where velocity vector is perpendicular to initial velocity vector is given by

(a) $\frac{u^2 \cos^2\theta}{g}$ (b) $\frac{u^2 \cot^2\theta}{g \sin\theta}$

(c) $\frac{u^2}{g}$ (d) $\frac{u^2 \tan^2\theta}{g\cos\theta}$

42. Rishabh of Raxaul skated the 10,000m race in Salt Lake City in 12min, 58.92 seconds. The oval track is made up of two straight 112.00m sections and two essentially identical semicircular curves. There are two lanes, each 5.00m wide. The 400m lap starts at A on the inner straightway, rounds the inner curve, crosses over in the next straight section in the shortest diagonal path to the outside lane (the other skater crosses over the other way), and rounds the outer curve, ending up on the adjacent lane at B (see dotted line). The measurement is made 5cm out from the inner edge of the lane, and is exactly 400m for one lap. What is the radius of the inner curve, R, in m?



(a) 24.23 (b) 25.47
(c) 25.31 (d) 25.44

43. A particle starts sliding down a frictionless inclined plane. If S_n is the distance travelled by it from time $t = n - 1$ sec to $t = n$ sec, the ratio S_n/S_{n+1} is

(a) $\frac{2n-1}{2n+1}$ (b) $\frac{2n+1}{2n}$

(c) $\frac{2n}{2n+1}$ (d) $\frac{2n+1}{2n-1}$

44. A particle is projected from a horizontal plane (x-z plane) such that its velocity vector at time t is given by $\vec{v} = a\hat{i} + (b - ct)\,\hat{j}$. Its range on the horizontal plane is given by

(a) $\frac{2ab}{c}$ (b) $\frac{ba}{c}$

(c) $\frac{3ba}{c}$ (d) None of these

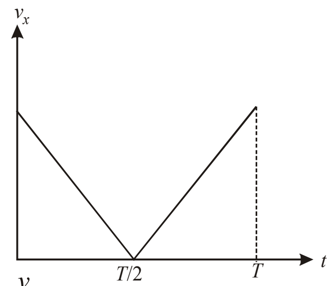
45. The greatest range of a particle, projected with a given velocity on an inclined plane, is x times the greatest vertical altitude above the inclined plane. Find the value of x.
(a) 2 (b) 4 (c) 3 (d) 1/2

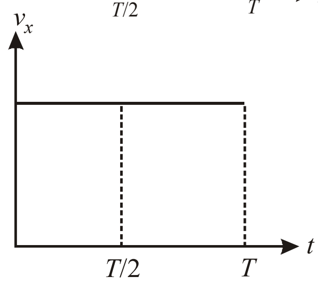
46. An object has velocity $\vec{v}_1$ relative to the ground. An observer moving with a constant velocity $\vec{v}_0$ relative to the ground measures the velocity of the object to $\vec{v}_2$ be (relative to the observer). The magnitudes of these velocities are related by

(a) $v_0 \le v_1 + v_2$ (b) $v_1 \le v_2 + v_0$

(c) $v_2 \le v_0 + v_1$ (d) All the above are true

47. A particle is projected with speed u at an angle θ from horizontal at $t = 0$. Its horizontal component of velocity (v_x) varies with time as following graph :

(a)


(b)


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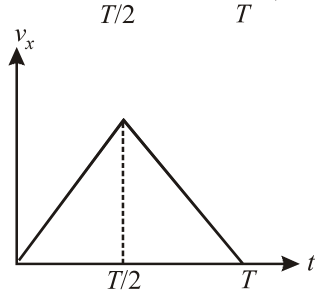

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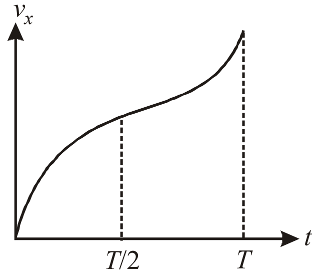
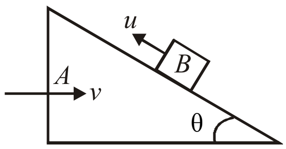


MARK YOUR RESPONSE					
	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ
	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ		

48. An airplane flies from a town A to a town B when there is no wind and takes a total time T_0 for a return trip. When there is a wind blowing in a direction from town A to town B, the plane's time for a similar return trip, T_w, would satisfy
(a) $T_0 < T_w$ (b) $T_0 > T_w$
(c) $T_0 = T_w$
(d) the result depends on the wind velocity between the towns

49. A bird flies with a speed of 10 km/h and a car moves with uniform speed of 8 km/h. Both start from B towards A ($BA = 40$km) at the same instant. The bird having reached A, flies back immediately to meet the approaching car. As soon as it reaches the car, it flies back to A. The bird repeats this till both the car and the bird reach A simultaneously. The total distance flown by the bird is
(a) 80 km (b) 40 km
(c) 50 km (d) cannot be determined.

50. A block B moves with a velocity u relative to the wedge A. If the velocity of the wedge is v as shown in figure, what is the value of θ so that the block B moves vertically as seen from ground ?



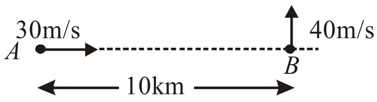
(a) $\cos^{-1}\left(\frac{u}{v}\right)$ (b) $\cos^{-1}\left(\frac{v}{u}\right)$
(c) $\sin^{-1}\left(\frac{u}{v}\right)$ (d) $\sin^{-1}\left(\frac{v}{u}\right)$

51. A hunter tries to hunt a monkey with a small, very poisonous arrow, blown from a pipe with initial speed v_0. The monkey is hanging on a branch of a tree at height H above the ground. The hunter is at a distance L from the bottom of the tree. The monkey sees the arrow leaving the blow pipe and immediately loses the grip on the tree, falling freely down with zero initial velocity. The minimum initial speed v_0 of the arrow for hunter to succeed while monkey is in air is

(a) $\sqrt{\frac{g(H^2+L^2)}{2H}}$ (b) $\sqrt{\frac{gH^2}{\sqrt{H^2+L^2}}}$

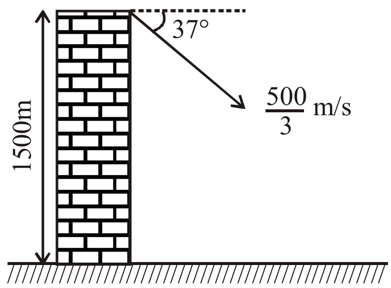
(c) $\sqrt{\frac{g\sqrt{H^2+L^2}}{H}}$ (d) $\sqrt{\frac{2gH^2}{\sqrt{H^2+L^2}}}$

52. Ship A is moving with velocity 30m/s due east and ship B with velocity 40m/s due north. Initial separation between the ships is 10km as shown in figure. After what time ships are closest to each other ?



(a) 80 sec (b) 120 sec
(c) 160 sec (d) None of these

53. A particle is projected from a tower as shown in figure, then the distance from the foot of the tower where it will strike the ground will be



(a) 4000/3 m (b) 2000/m
(c) 1000/3 m (d) 2500/3 m

54. If a boat can travel with a speed of v in still water, which of the following trips will take the least amount of time ?
(a) travelling a distance of $2d$ in still water
(b) travelling a distance of $2d$ across (perpendicular to) the current in a stream
(c) travelling a distance d downstream and returning a distance d upstream
(d) travelling a distance d upstream and returning a distance d downstream

55. A body A is thrown vertically upward with the initial velocity v_1. Another body B is dropped from a height h. Find how the distance x between the bodies depends on the time t if the bodies begin to move simultaneously.
(a) $x = h - v_1 t$ (b) $x = (h - v_1)t$
(c) $x = h - \frac{v_1}{t}$ (d) $x = \frac{h}{t} - v_1$

56. Two swimmers start from point A on one bank of a river to reach a point B on the other bank lying directly opposite to point A. One of them crosses the river along the straight line AB while the other swims at right angles to the stream and then walks the distance which he has been carried away by the stream to get to point B. What was the velocity (assumed uniform) of his walking if both the swimmers reached point B simultaneously ? Velocity of each swimmer in still water is 2.5 km/hr and the stream velocity is 2 km/hr.
(a) 4 km/hr (b) 3 km/hr.
(c) 5 km/hr (d) 8 km/hr

MARK YOUR RESPONSE					
	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ	50. ⓐⓑⓒⓓ	51. ⓐⓑⓒⓓ	52. ⓐⓑⓒⓓ
	53. ⓐⓑⓒⓓ	54. ⓐⓑⓒⓓ	55. ⓐⓑⓒⓓ	56. ⓐⓑⓒⓓ	

57. A body A begins to move with initial velocity 2 m/sec and continues to move at a constant acceleration a. $\Delta t = 10$ seconds after the body A begins to move a body B departs from the same point with an initial velocity 12 m/sec and moves with the same acceleration a. What is the maximum acceleration a at which the body B can overtake A ?

(a) 1 m/s^2 (b) 2 m/s^2
(c) 1/2 m/s^2 (d) 3 m/s^2

58. A person travelling eastward finds the wind to blow from north. On doubling his speed he finds it to come from north-east. If he travel his speed, the wind would appear to him to come from a direction making an angle

(a) $\theta = \tan^{-1}(1/2)$ north of east
(b) $\theta = \tan^{-1}(1/2)$ south of east
(c) $\theta = \tan^{-1}(1/2)$ north of west
(d) None of these

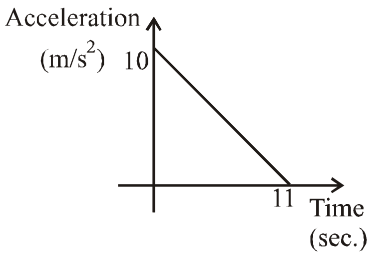
59. A train moving at 30 m/sec reduces its speed to 10 m/sec in a distance of 240m. At what distance will the train come to a stop? If the brake power is increased by 12½%, the train will stop in a total distance of

(a) 120 m (b) 240 m
(c) 360 m (d) 420 m

60. A truck has to carry a load in the shortest time from one station to another station situated at a distance L from the first. It can start up or slowdown at the same acceleration or deceleration a. What maximum velocity must the truck attain to satisfy this condition?

(a) $\sqrt{La}$ (b) $\sqrt{2La}$
(c) $\sqrt{3La}$ (d) none of these

61. A body starts from rest at time $t = 0$, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be

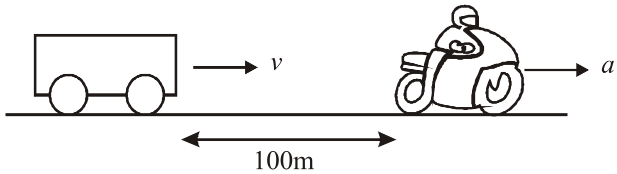


(a) 110 m/s (b) 55 m/s
(c) 650 m/s (d) 550 m/s

62. The velocity of an object moving rectilinearly is given as a function of time by $v = 4t - 3t^2$, where v is in m/s and t is in seconds. The average velocity of particle between $t = 0$ to $t = 2$ seconds is

(a) 0 (b) –2m/s
(c) – 4m/s (d) None of these

63. A man travelling in a car with a maximum constant speed of 20m/s watches the friend start off at a distance 100m ahead on a motor cycle with constant acceleration 'a'. The maximum value of 'a' for which the man in the car can reach his friend is

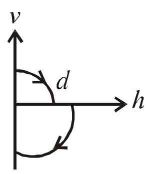


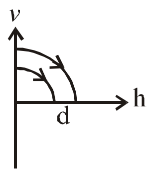
(a) 2 m/s^2 (b) 1 m/s^2
(c) 4 m/s^2 (d) None of these

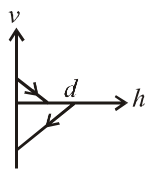
64. Three ships A, B and C are sailing at constant speeds on steady courses. B is sailing due east at 12 km/hr and C is sailing in a direction $N\,30°\,E$ at 8 km/hr. To an observer on C, the direction of A appears to be $S\,30°\,E$ and to an observer on B, A appears to be sailing due south. Find the speed of A.

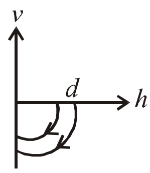
(a) $8\sqrt{3}$ km / hr (b) $4\sqrt{3}$ km / hr
(c) $2\sqrt{3}$ km / hr (d) $3\sqrt{3}$ km / hr

65. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

(a)


(b)


(c)


(d)


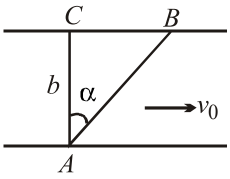
66. A body is projected vertically upwards with a velocity u, after time t another body is projected vertically upwards from the same point with a velocity v, where $v < u$. If they meet as soon as possible, then choose the correct option

(a) $t \quad \dfrac{u - v \quad \sqrt{u^2 \quad v^2}}{g}$ (b) $t \quad \dfrac{u - v + \sqrt{u^2 - v^2}}{g}$

(c) $t \quad \dfrac{u \quad v \quad \sqrt{u^2 - v^2}}{g}$ (d) $t \quad \dfrac{u - v + \sqrt{u^2 - v^2}}{2g}$

MARK YOUR RESPONSE	57. ⓐⓑⓒⓓ	58. ⓐⓑⓒⓓ	59. ⓐⓑⓒⓓ	60. ⓐⓑⓒⓓ	61. ⓐⓑⓒⓓ
	62. ⓐⓑⓒⓓ	63. ⓐⓑⓒⓓ	64. ⓐⓑⓒⓓ	65. ⓐⓑⓒⓓ	66. ⓐⓑⓒⓓ

67. A man in a row boat must get from point A to point B on the opposite bank of the river (figure). The distance $BC = a$. The width of the river $AC = b$. At what minimum speed u relative to still water should the boat travel to reach the point B ? The velocity of flow of the river is v_0.



(a) $\frac{v_0 b}{\sqrt{a^2+b^2}}$ (b) $\frac{2v_0 b}{\sqrt{a^2+b^2}}$

(c) $\frac{v_0 b}{\sqrt{a^2-b^2}}$ (d) $\frac{v_0 b}{\sqrt{a^2+2b^2}}$

68. A particle has initial velocity 10m/s. It moves due to a constant force along the line of velocity which initially produces retardation of $5 m/s^2$. Then

(a) the distance travelled in first 3 seconds is 10.0m

(b) the distance travelled in first 3 seconds is 7.5m

(c) the distance travelled in first 3 seconds is 12.5m

(d) the distance travelled in first 3 seconds is 17.5m

69. A particle is moving in a plane with velocity given by $\vec{u} = u_0\hat{i} + (\omega a \cos\omega t)\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis respectively. If the particle is at the origin at $t = 0$, find its distance from the origin at time $\frac{3\pi}{2\omega}$.

(a) $\sqrt{\frac{9\pi^2 u_0^2}{4\omega^2}+a^2}$ (b) $\sqrt{\frac{9\pi^2 u_0^2}{2\omega^2}+a^2}$

(c) $\sqrt{\frac{3\pi^2 u_0^2}{4\omega^2}+a^2}$ (d) $\sqrt{\frac{7\pi^2 u_0^2}{4\omega^2}+a^2}$

70. A shot is fired with velocity $\sqrt{2gh}$ from the top of a mountain which is in the form of hemisphere of radius r. The farthest points which can be reached by the shot are at a distance (measured in a straight line) from the point of projection is

(a) $(r-\sqrt{r^2-4rh})$ (b) $(r+\sqrt{r^2-4rh})$

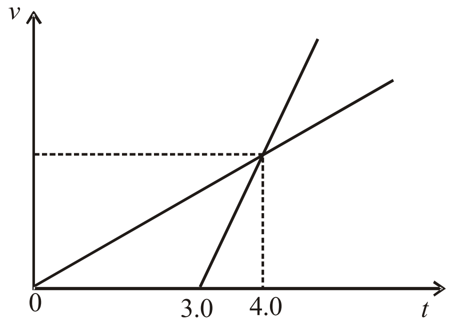
(c) $(r-\sqrt{r^2+4rh})$ (d) $(r-\sqrt{r^2-2rh})$

71. When a shot is projected from a gun at any angle of elevation, the shot as seen from the point of projection will appear to descend past a vertical target with

(a) uniform velocity (b) uniform acceleration

(c) non-uniform velocity (d) None of these

72. The drawing shows velocity (v) versus time (t) graphs for two cyclists moving along the same straight segment of a highway from the same point. The first cyclist starts at $t = 0$ min and the second cyclist starts moving at $t = 3.0$ min. The time at which the two cyclists meet is (Both velocity-time curves intersect at $t = 4$ min)



(a) 4.0 min (b) 6.0 min

(c) 8 min (d) 12 min

73. A sandbag ballast is dropped from a balloon that is ascending with a velocity of 40 ft/s. If the sandbag reaches the ground in 20s, how high was the balloon when the bag was dropped? Neglect air resistance.

(a) 5200 ft (b) 6000 ft

(c) 5000 ft (d) 5600 ft

74. A particle is projected with speed 10m/s at an angle 60° with the horizontal. Then the time after which its speed becomes half of initial is

(a) $\frac{1}{2}$sec (b) $\frac{\sqrt{3}}{2}$sec

(c) 1 sec (d) $\sqrt{\frac{3}{2}}$sec

75. An arrow is shot into the air on a parabolic path to a target. Neglecting air resistance, at its highest point

(a) both velocity and acceleration vectors are horizontal

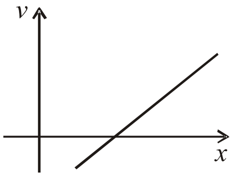
(b) the acceleration vector is zero but not the velocity

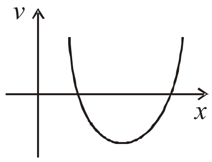
(c) the velocity and acceleration vectors are both zero

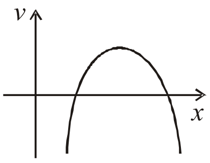
(d) the upward component of velocity is zero but not the acceleration

MARK YOUR RESPONSE	67. ⓐⓑⓒⓓ	68. ⓐⓑⓒⓓ	69. ⓐⓑⓒⓓ	70. ⓐⓑⓒⓓ	71. ⓐⓑⓒⓓ
	72. ⓐⓑⓒⓓ	73. ⓐⓑⓒⓓ	74. ⓐⓑⓒⓓ	75. ⓐⓑⓒⓓ	

76. For a particle moving along x-axis, which of the velocity versus position graphs given in options below is possible (position is represented by x-coordinate of the particle)

(a) 

(b) 

(c) 

(d) None of these

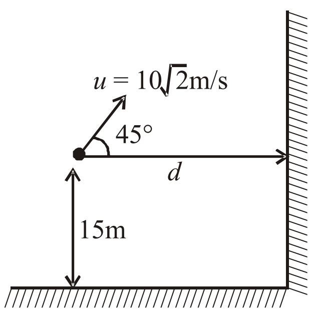
77. Two fixed points A and B are 20 metres apart. At time $t = 0$, the distance between a third point C and A is 20 meters and the distance between C and B is 10 metres. The component of velocity of point C along both CA and CB at any instant is 5m/s. Then the distance between A and C at the instant all the three points are collinear will be

(a) 5 m (b) 15 m
(c) 10 m (d) None of these

78. A ball is thrown upward with initial velocity $v_0 = 15.0$ m/s at an angle of 30° with the horizontal. The thrower stands near the top of a long hill which slopes downward at an angle of 20°. When does the ball strike the slope ?

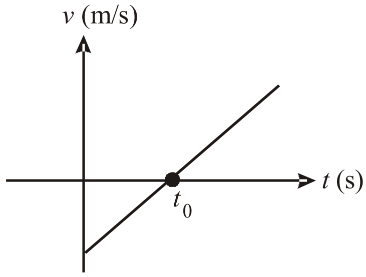
(a) 2.49s (b) 1.13s
(c) 2.12s (d) 5.12s

79. A small ball is thrown from a height of 15m above the ground and at a horizontal distance d from a vertical wall. The ball first hits the wall and then strikes the ground and then it flies back to its initial position of throwing. Take both collisions to be perfectly elastic and neglect friction. The initial speed of the ball is $10\sqrt{2}$ m/s and angle of projection is 45° with the horizontal as shown. Find the horizontal distance of point of throwing from the wall 'd' in meters. (Neglect air resistance and take $g = 10$ m/s^2)



(a) 20 m (b) 10 m
(c) 40 m (d) 30 m

80. The velocity-time graph of a particle moving along a straight line in a given time interval is as shown in figure. Then the particle (with increase in time starting from $t = 0$ sec)

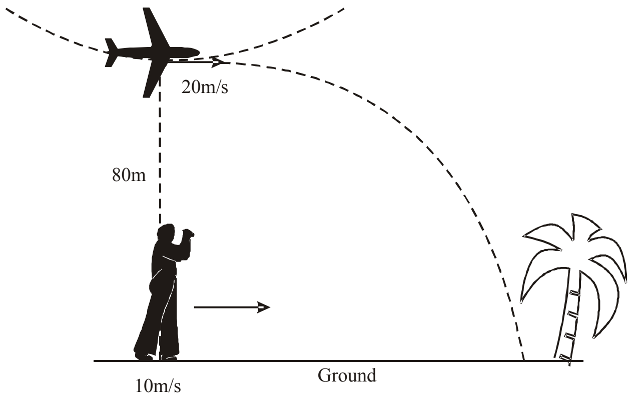


(a) speeds up continuously
(b) first speeds up and then speeds down
(c) speeds down continuously
(d) first speeds down and then speeds up

81. A body is projected horizontally from the top of a tower with initial velocity 18m/s. It hits the ground at angle 45°. What is the vertical component of velocity when it strikes the ground ?

(a) $18\sqrt{2}$ m/s (b) 18 m/s
(c) $9\sqrt{2}$ m/s (d) 9 m/s

82. A bomber plane moving at a horizontal speed of 20 m/s releases a bomb at a height of 80m above ground as shown. At the same instant a Hunter starts running from a point below it, to catch the bomb at 10 m/s. After two seconds he realized that he cannot make it, he stops running and immediately holds his gun and fires in such direction so that just before bomb hits the ground, bullet will hit it. What should be the firing speed of bullet.
(Take $g = 10$ m/s^2)



(a) 10 m/s (b) $20\sqrt{10}$ m/s
(c) $10\sqrt{10}$ m/s (d) None of these

MARK YOUR RESPONSE					
	76. ⓐⓑⓒⓓ	77. ⓐⓑⓒⓓ	78. ⓐⓑⓒⓓ	79. ⓐⓑⓒⓓ	80. ⓐⓑⓒⓓ
	81. ⓐⓑⓒⓓ	82. ⓐⓑⓒⓓ			

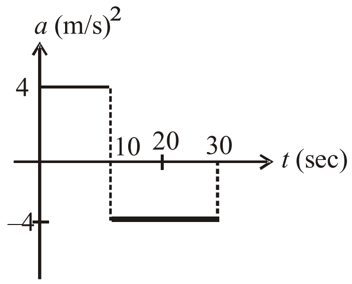
83. A particle is projected from the ground with an initial velocity of 20m/s at an angle of 30° with horizontal. The magnitude of change in velocity in time interval of 0.5 sec starting from instant of projection is
(Neglect air friction and take g = 10 m/s^2)
(a) 5 m/s (b) 2.5 m/s
(c) 2 m/s (d) 4 m/s

84. A bullet is fired from horizontal ground at some angle passes through the point $\left(\frac{3R}{4}, \frac{R}{4}\right)$, where R is the range of the bullet. Assume point of the fire to be origin and the bullet moves in x-y plane with x-axis horizontal and y-axis vertically upwards. Then angle of projection is
(a) 30° (b) 53°
(c) 37° (d) None of these

85. A swimmer crosses a river with minimum possible time 10 second. And when he reaches the other end, he starts swimming in the direction towards the point from where he started swimming. Keeping the direction fixed the swimmer crosses the river in 15sec. The ratio of speed of swimmer with respect to water and the speed of river flow is (Assume constant speed of river and swimmer)
(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{3}{2}$
(c) $\frac{9}{4}$ (d) $\frac{\sqrt{5}}{2}$

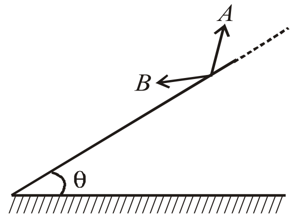
86. A man throws a ball making an angle of 60° with the horizontal. He runs on a level ground and catches the ball. If he had thrown the ball with speed v, then his average velocity must be
(a) v (b) $2v$
(c) $\sqrt{v}$ (d) $v/2$

87. The acceleration versus time graph for a particle moving along a straight line is shown in the figure. If the particle starts from rest at $t = 0$, then its speed at $t = 30$ sec will be



(a) 20 m/sec (b) 0 m/sec
(c) – 40 m/sec. (d) 40 m/sec.

88. Two stones A and B are projected from an inclined plane such that A has range up the incline and B has range down the incline. For range of both stones on the incline to be equal in magnitude, pick up the correct condition. (Neglect air friction).



(a) Component of initial velocity of both stones along the incline should be equal and also component of initial velocity of both stones perpendicular to the incline should be equal.
(b) Component of initial velocity of both stones perpendicular to the incline should be equal and also horizontal component of initial velocity of both stones should be equal.
(c) Horizontal component of initial velocity of both stones should be equal and also vertical component of initial velocity of both stones should be equal.
(d) None of these

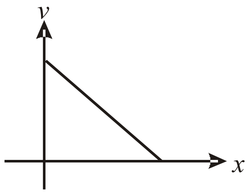
89. An engineer works at a factory out of town. A car is sent for him from the factory every day and arrive at the railway station at the same time as the train. One day the engineer arrived at the station one hour before his usual time and without waiting for the car, started walking towards factory. On his way he met the car and reached his factory 10 minutes before the usual time. For how much time (in minutes) did the engineer walk before he met the car ? The car moves with the same speed everyday.
(a) 55 min (b) 35 min
(c) 45 min (d) 60 min

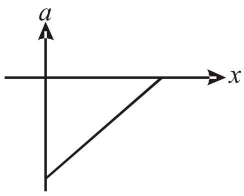
90. A particle is thrown up inside a stationary lift of sufficient height. The time of flight is T. Now it is thrown again with same initial speed v_0 with respect to lift. At the time of second throw, lift is moving up with speed v_0 and uniform acceleration g upward. The new time of flight is
(a) $T/4$ (b) T (c) $T/2$ (d) $2T$

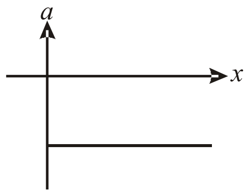
91. A stone is projected horizontally with speed v from a height h above ground. A horizontal wind is blowing in direction opposite to velocity of projection and gives the stone a constant horizontal acceleration f (in direction opposite to initial velocity). As a result the stone falls on ground at a point vertically below the point of projection. Then the value of height h in terms of f, g, v is (g is acceleration due to gravity)
(a) $\frac{2gv^2}{f^2}$ (b) $\frac{gv^2}{2f^2}$
(c) $\frac{gv^2}{f^2}$ (d) $\frac{\sqrt{2}gv^2}{f^2}$

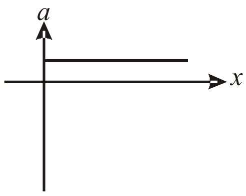
MARK YOUR RESPONSE					
	83. ⓐⓑⓒⓓ	84. ⓐⓑⓒⓓ	85. ⓐⓑⓒⓓ	86. ⓐⓑⓒⓓ	87. ⓐⓑⓒⓓ
	88. ⓐⓑⓒⓓ	89. ⓐⓑⓒⓓ	90. ⓐⓑⓒⓓ	91. ⓐⓑⓒⓓ	

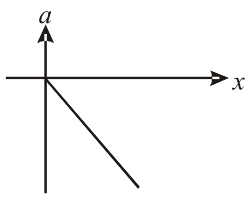
92. A particle moves along x-axis with initial position $x = 0$. Its velocity varies with x-coordinate as shown in graph. The acceleration 'a' of this particle varies with x as –



(a) 

(b) 

(c) 

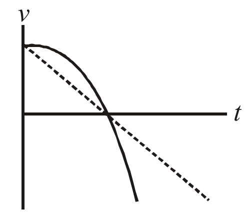
(d) 

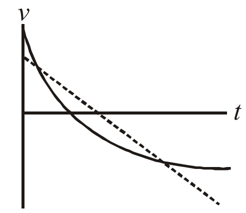
93. Velocity of a stone projected, 2 second before it reaches the maximum height, makes angle 53° with the horizontal, then the velocity at highest point will be (Neglect air friction and take $g = 10$ m/s^2)

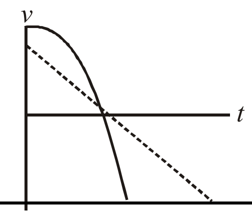
(a) 20 m/s (b) 15 m/s

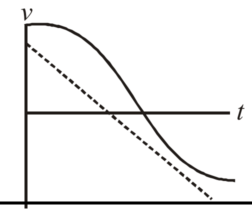
(c) 25 m/s (d) 80/3 m/s

94. You calculate that to throw an object vertically to a height h it needs to be launched with an initial upward velocity v_0, assuming no air resistance. The dashed lines in figure show the motion according to this calculation. Which of the velocity-time graphs shows the motion of an object tossed with initial upward velocity v_1 that will also rise to height h, but this time with air resistance ?

(a) 

(b) 

(c) 

(d) 

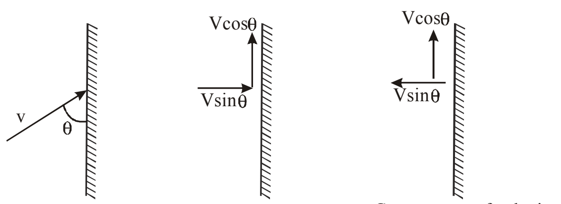
MARK YOUR RESPONSE	92. ⓐⓑⓒⓓ	93. ⓐⓑⓒⓓ	94. ⓐⓑⓒⓓ		

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

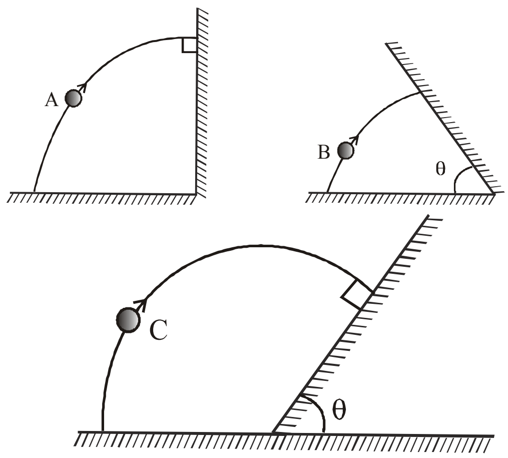
We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic then the perpendicular component of velocity of the ball gets reversed with the same magnitude.



Velocity just before collision — Components of velocity just before collision — Components of velocity just after collision

The other parallel component of velocity will remain constant if wall is given smooth.

Now let us take a problem. Three balls A, B & C are projected from ground with same speed at same angle with the horizontal. The balls A, B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



1. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct
(a) $H_A = H_C > H_B$ (b) $H_A > H_B = H_C$
(c) $H_A > H_C > H_B$ (d) $HA = H_B = H_C$

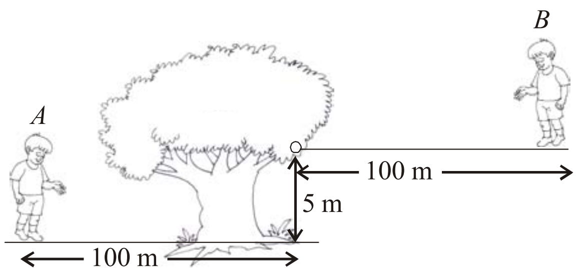
2. If the time taken by the ball A to fall back on ground is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be
(a) 6 sec. (b) 4 sec.
(c) 3 sec. (d) 5 sec.

3. The maximum height attained by ball A from the ground is
(a) 10m (b) 15m
(c) 20m (d) Insufficient information

PASSAGE-2

There is an orange tree at which there is an orange hanging at a height 5 m from ground. Two persons A and B each standing at a horizontal distance of 100 m from the orange starts running towards the orange. B is standing on a platform at the same level as that of orange. B runs with a constant speed of 20 m/s towards orange. A starts running first horizontally with a constant acceleration and then gets a vertical impulse so as to reach the orange. If the maximum height that A can jump due to this impulse is just equal to the height of orange from the ground then.

(Take $g = 10$ m/s^2)



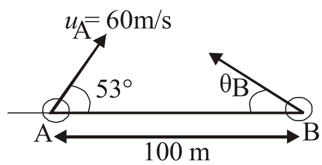
4. The vertical component of velocity of A at the time when he got the impulse is
(a) 10 m/s (b) 5 m/s
(c) 1 m/s (d) None of these

5. What should be the minimum constant acceleration A should have to catch the orange before B catches it
(a) 8 m/s^2
(b) 26/3 m/s^2
(c) 25/3 m/s^2
(d) none of these

6. The horizontal distance from the base of orange tree from where A has to gain impulse to just catch the orange before B is
(a) $\frac{200}{3}$ m (b) $\frac{100}{3}$ m
(c) $\frac{400}{9}$ m (d) None of these

PASSAGE-3

Two particles A and B are projected in same vertical plane as shown in the figure. Their initial positions ($t = 0$), initial speed and angle of projection are indicated in the diagram.



7. If initial angle of projection $\theta_B = 37°$. What should be initial speed of projection of particle B, so that it hits particle A
(a) 80 m/s (b) 75 m/s
(c) 40 m/s (d) 45 m/s

8. Suppose that the speed of B is such that an observer on particle A observes that particle B is moving at angle 8° with horizontal in downward direction. Initial angles of projection of each particle remain same as previous question. Speed of particle B may be
(a) 80 m/s (b) 75 m/s
(c) 60 m/s (d) 40 m/s

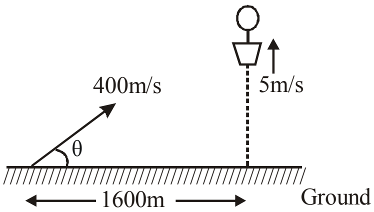
9. In the situation of previous question, when observer on particle A, observes that particle B appears to be moving at 8° below horizontal ($\theta_B = 37°$). After how much time (in second) distance between A and B is minimum? (t = 0 situation is shown in figure)
(a) $\frac{3\sqrt{2}}{5}\sin 8°$ (b) $\frac{3\sqrt{2}}{5}\cos 8°$
(c) $\frac{5}{3\sqrt{2}}\sin 8°$ (d) $\frac{5}{3\sqrt{2}}\cos 8°$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

PASSAGE-4

An observer having a gun observes a remotely controlled balloon. When he first notices the balloon, it was at an altitude of 800m and moving vertically upward at a constant velocity of 5 m/s. The horizontal displacement of balloon from the observer is 1600m. Shells fired from the gun have an initial velocity of 400m/s at a fixed angle θ.

($\sin\theta = 3/5$ and $\cos\theta = 4/5$). The observer having gun waits (for some time after observing balloon) and fires so as to destroy the balloon. Assume $g = 10$ m/s^2. Neglect air resistance.



10. The time of flight of the shell before it strikes the balloon is
(a) 2 sec (b) 5 sec
(c) 10 sec (d) 15 sec

11. The altitude of the collision above ground level is
(a) 1075 m (b) 1200 m
(c) 1250 m (d) 1325 m

12. After noticing the balloon, the time for which observer having gun waits before firing the shell is
(a) 45 sec (b) 50 sec
(c) 55 sec (d) 60 sec

PASSAGE-5

A ball is thrown at an angle of 53° with the horizontal from the centre of a well with the speed of 50 m/s. The ball collides elastically with the vertical wall of the well. The total number of collision on both walls is 12.

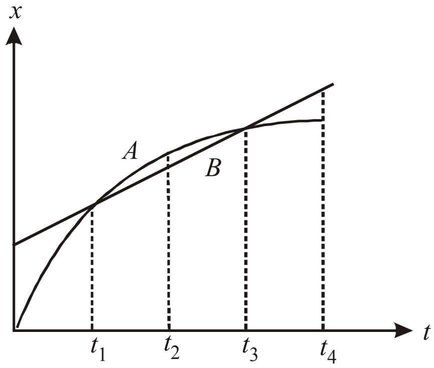
13. The minimum height of the wall so that the ball may not come out of the well (Take $g = 10$ m/s^2), is
(a) 80 m
(b) 79.45 m
(c) 40.25 m
(d) 81.5 m

14. The diameter of the well so that the particle fall back to the initial point of projection
(a) 10 m (b) 20 m
(c) 30 m (d) 4 m

15. The impulse provided by the wall during the 3rd collision with the wall if the mass of the ball is 1 kg is
(a) 60 N-s (b) 70 N-s
(c) 80 N-s (d) 90 N-s

PASSAGE-6

The graph given shows the position of two cars, *A* and *B*, as a function of time. The cars move along the *x*-axis on parallel but separate tracks, so that they can pass each other position without colliding.



16. At which instant of time is car-*A* overtaking the car-*B* ?
(a) t_1 (b) t_2
(c) t_3 (d) t_4

17. At time t_3, which car is moving faster ?
(a) car *A* (b) car *B*
(c) same speed (d) None of these

18. At which instant do the two cars have the same velocity ?
(a) t_1 (b) t_2
(c) t_3 (d) t_4

PASSAGE-7

Two particles are simultaneously projected in the same vertical plane from the same point with velocities *u* and *v* at angles α and β to the horizontal.

19. The line joining them moves
(a) parallel to itself (b) perpendicular to itself
(c) 30° to itself (d) none of these

20. The time that elapses when their velocities are parallel is

(a) $\dfrac{uv\sin(\alpha+\beta)}{g\,(v\cos\beta - u\cos\alpha)}$ (b) $\dfrac{uv\sin(\alpha-\beta)}{g\,(v\cos\beta - u\cos\alpha)}$

(c) $\dfrac{uv\sin(\alpha-\beta)}{g\,(v\cos\beta + u\cos\alpha)}$ (d) $\dfrac{2uv\sin(\alpha-\beta)}{g\,(v\cos\beta - u\cos\alpha)}$

MARK YOUR RESPONSE					
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ				

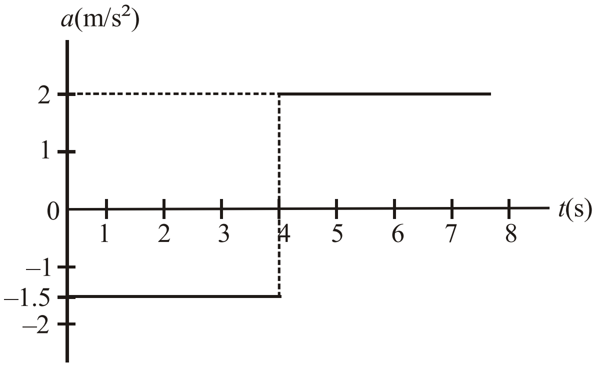
21. The time that elapses between their transits through the other common point is

(a) $\dfrac{2uv\sin(\alpha-\beta)}{g\,(u\cos\alpha+v\cos\beta)}$ (b) $\dfrac{2uv\sin(\alpha+\beta)}{g\,(u\cos\alpha+v\cos\beta)}$

(c) $\dfrac{2uv\sin(\alpha-\beta)}{g\,(u\cos\alpha-v\cos\beta)}$ (d) $\dfrac{uv\sin(\alpha-\beta)}{g\,(u\cos\alpha+v\cos\beta)}$

PASSAGE-8

An object has the acceleration v/s time graph as shown.

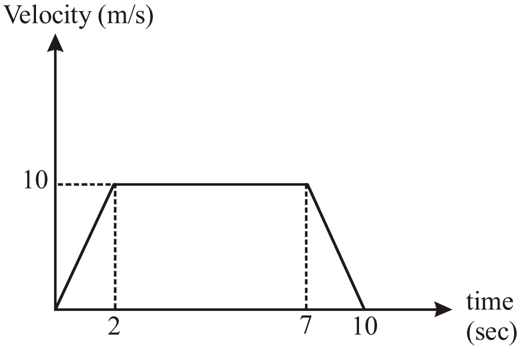


22. When does the object return to it's initial velocity ?
(a) At $t = 4$s (b) At $t = 7$s
(c) At $t = 8$s
(d) Impossible to determine from the given information

23. When is the object at rest ?
(a) At $t = 0$s (b) At $t = 4$s
(c) At $t = 4$s and $t = 8$s
(d) Impossible to determine from the given information

PASSAGE-9

The velocity-time graph of a car moving on a straight track is given below. The car weighs 1000 kg.



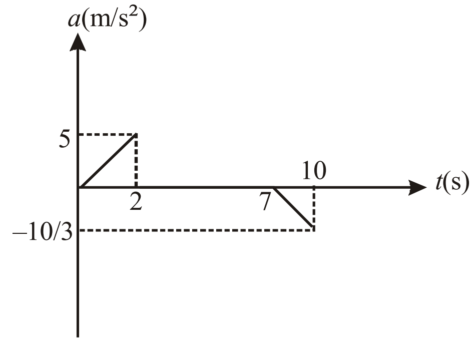
24. The distance travelled by the car during the whole motion is
(a) 50 m (b) 75 m
(c) 100 m (d) 150 m

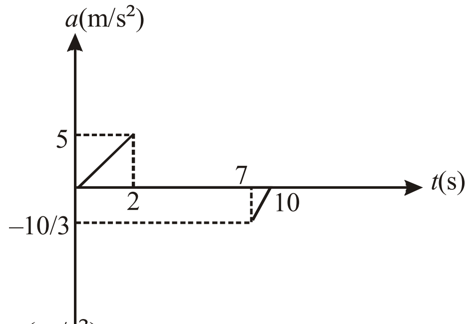
25. The braking force required to bring the car to a stop within one second from the maximum speed is

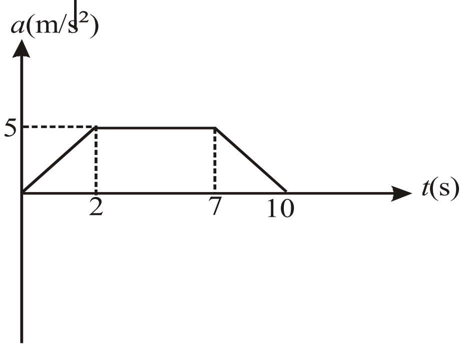
(a) $\dfrac{10000}{3}$ N (b) 5000 N

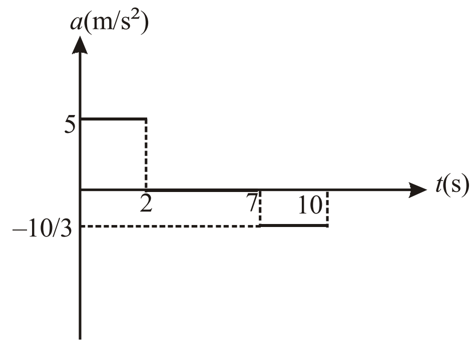
(c) 10000 N (d) $\dfrac{5000}{3}$ N

26. Correct acceleration-time graph representing the motion of car is

(a)


(b)


(c)


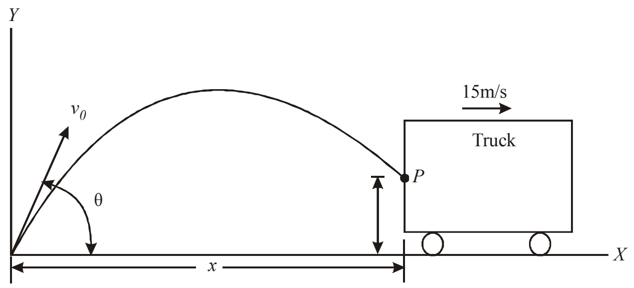
(d)


MARK YOUR RESPONSE					
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ
	26. ⓐⓑⓒⓓ				

PASSAGE-10

With reference to figure, the projectile is fired with an initial velocity $v_0 = 35$ m/s at an angle $\theta = 23°$. The truck is moving along X with a constant speed of 15m/s. At the instant the projectile is fired, the back of the truck is at

$x = 45$m.



27. Find the time for the projectile to strike the back of the truck, if the truck is very tall.

(a) 0.614s (b) 3.614s
(c) 1.614s (d) 2.614s

28. Find the total time if the truck is only 2.0m tall

(a) 1.123s (b) 3.614s
(c) 2.635s (d) 2.614s

29. Find a value of v_0, all other conditions remaining the same, for which the projectile his the truck at $y = 3$m.

(a) 35.3 m/s (b) 50.3 m/s
(c) 25.3 m/s (d) 27.2 m/s

PASSAGE-11

A physics tutor launches a home-built model rocket straight up into the air. At $t = 0$, the rocket is at $y = 0$ with

$v_y\ (t = 0) = 0$. The acceleration of the rocket is given by

$$a_y = \begin{cases} -g + \alpha g - \beta t^4, & 0 < t < t_b \\ -g & , t > t_b \end{cases} \quad \text{where } t_b = \left(\frac{\alpha g}{\beta}\right)^{1/4}$$

is the time at which fuel burns out. α is a positive dimensionless number (> 1).

30. The expression for the velocity $v_y(t)$ valid at all times in the interval $0 < t < t_b$ is –

(a) $v_y = (\alpha - 1)\, gt + \frac{1}{5}\beta t^5$ (b) $v_y = (\alpha - 1)\, gt - \frac{1}{5}\beta t^5$

(c) $v_y = (\alpha + 1)\, gt + \frac{1}{5}\beta t^5$ (d) $v_y = (\alpha + 1)\, gt - \frac{1}{5}\beta t^5$

31. The expression for the velocity $v_y\ (t)$ valid for the time interval $t > t_b$ is –

(a) $v_y = \frac{1}{5}\alpha g t_b + gt$ (b) $v_y = -g\,(t - t_b)$

(c) $v_y = g\,(t - t_b)$ (d) $v_y = \frac{4}{5}\alpha g t_b - gt$

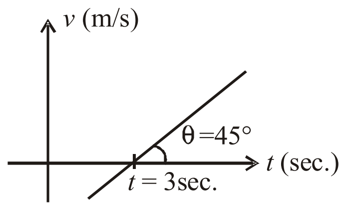
32. The time taken for rocket to reach its maximum height is –

(a) $\frac{3}{5}\alpha t_b$ (b) $\frac{4}{5}\alpha t_b$

(c) $\frac{1}{5}\alpha t_b$ (d) $\frac{2}{5}\alpha t_b$

PASSAGE-12

The plot of velocity versus time graph for a particle moving along a straight line is shown. Answer the following three questions based on given information.



33. If v is velocity at any time t, then the value of $\frac{dv}{dt}$ at $t = 2$ sec is

(a) 1 m/s^2 (b) – 1 m/s^2
(c) 2 1 m/s^2 (d) – 2 m/s^2

34. The value of dot product of velocity and acceleration of particle at $t = 2$ sec is

(a) 1 m^2/s^3 (b) – 1 m^2/s^3
(c) 2 m^2/s^3 (d) – 2 m^2/s^3

35. If v is velocity at any time t, then the value of $\int_{t=2}^{t=5} v\, dt$ is

(where v is in m/s and t is in seconds)

(a) zero (b) 2.5m
(c) 1.5m (d) 2m

PASSAGE-13

For a particle moving along x-axis, the acceleration 'a' of the particle in terms of its x-coordinate x is given by

$a = -9x$, where x is in meters and 'a' is in m/s^2. Take acceleration, velocity and displacement in positive x-direction as positive. The initial velocity of particle at $x = 0$ is $u = +6$m/s.

MARK YOUR RESPONSE					
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ
	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	

36. The velocity of particle at $x = 2$m will be

(a) $+6\sqrt{2}$ m/s (b) $-6\sqrt{2}$ m/s

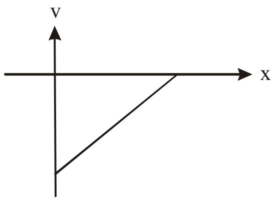
(c) 72 m/s (d) 0

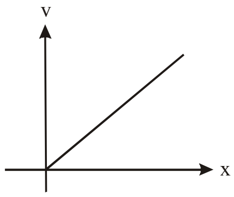
37. The maximum distance of particle from origin will be

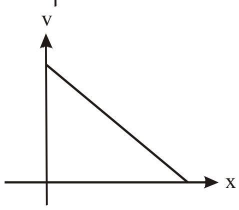
(a) 1m (b) 2m

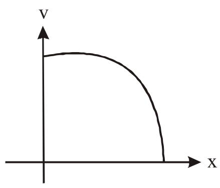
(c) 3m (d) 4m

38. The plot of velocity versus x-coordinate of particle in the duration it moves from origin towards the positive x-direction, is best represented by –

(a)


(b)


(c)


(d)


PASSAGE-14

A particle moves in x-y plane with constant acceleration $\vec{a} = 6\hat{i} - 8\hat{j}$ (in m/s^2). At time $t = 0$, the particle is at point having coordinates (0, 20 m) and its initial velocity is $\vec{u} = -12\hat{i} + 16\hat{j}$ (in m/s)

39. The instant of time when speed of the particle is zero will be–

(a) 1 sec (b) 3 sec

(c) 4 sec (d) 2 sec

40. The speed of the particle at the instant position vector and velocity vector of the particle are mutually perpendicular will be

(a) $12\sqrt{5}$ m/s (b) $4\sqrt{5}$ m/s

(c) 20 m/s (d) None of these

41. The instant of time when the particle crosses x-axis is

(a) 5 sec (b) 3 sec

(c) 4 sec (d) 6 sec

MARK YOUR RESPONSE	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ
	41. ⓐⓑⓒⓓ				

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is true but Statement-2 is false.

(d) Statement-1 is false but Statement-2 is true.

1. **Statement - 1** : A body can have acceleration even if its velocity is zero at a given instant of time.

Statement - 2 : A body is numerically at rest when it reverses its direction.

2. **Statement - 1** : Retardation is directly opposite to the velocity.

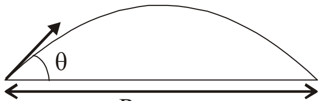
Statement - 2 : Retardation is equal to the time rate of decrease of speed.

3. **Statement - 1** : If a body is thrown upwards, the distance covered by it in the last second of upward motion is about 5 m irrespective of its initial speed.

Statement - 2 : The distance covered in the last second of upward motion is equal to that covered in the first second of downward motion when the particle is dropped.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

4. A particle is projected in vertical plane from horizontal surface, with speed of projection u and angle of projection θ above the horizontal.



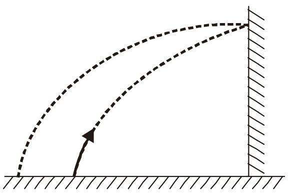
Statement - 1 : The horizontal range must be the farthest displacement of the particle from point of projection along the horizontal.

Statement - 2 : The projectile is farthest distance from the point of projection, when its instantaneous velocity vector and position vector (w.r.t. point of projection) are mutually perpendicular.

5. **Statement - 1** : Two projectiles are launched from the top of a cliff with same initial speed with different angles of projection. They reach the ground with the same speed.

Statement - 2 : The work done by gravity is same in both the cases.

6. **Statement - 1** : A projectile, launched from ground, collides with a smooth vertical wall and returns to the ground. The total time of flight is the same had there been no collision.



Statement - 2 : The collision changes only the horizontal component of velocity .

7. **Statement - 1** : Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.

Statement - 2 : For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.

8. **Statement - 1** : A body is thrown with a velocity u inclined to the horizontal at some angle. It moves along a parabolic path and falls to the ground. Linear momentum of the body, during its motion, will remain conserve.

Statement - 2 : Throughout the motion of the body, a constant force acts on it.

9. **Statement - 1** : Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

Statement - 2 : Relative acceleration between any of the pair of projectiles is zero.

10. **Statement - 1** : The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on the bank. They may reach opposite bank simultaneously moving along different straight line paths.

Statement - 2 : For above boats to cross the river in same time, the component of their velocity relative to river in direction normal to flow should be same.

11. **Statement - 1** : A particle having zero acceleration must have constant speed.

Statement - 2 : A particle having constant speed must have zero acceleration.

12. **Statement - 1** : Two projectiles having same range must have the same time of flight.

Statement - 2 : Horizontal component of velocity is constant in projectile motion under gravity.

13. **Statement - 1** : During a train journey, tree outside appear to move backwards (opposite to the train's motion)

Statement - 2 : Rest or motion depends on choice of frame.

14. **Statement - 1** : For a particle undergoing rectilinear motion with constant acceleration, the average velocity cannot be equal for two different time intervals.

Statement - 2 : The average velocity of particle moving with constant acceleration in a time interval is $\frac{\vec{u}+\vec{v}}{2}$, where $\vec{u}$ is the initial velocity and $\vec{v}$ is the final velocity.

15. **Statement - 1** : Acceleration of a particle in motion always depends on its velocity.

Statement - 2 : Acceleration of a particle is $\frac{d\vec{v}}{dt}$, where $\vec{v}$ is velocity of particle at any time t.

MARK YOUR RESPONSE					
	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ			

16. **Statement-1** : Let v be the speed of particle at any time t. If $\frac{dv}{dt}$ is a non-zero constant then speed of particle may be minimum at some time.

Statement-2 : The speed v of a moving particle may be minimum at time $t = t_0$ when $v = 0$. The speed v of a moving particle may also be minimum at time $t = t_0$ when $\frac{dv}{dt}$ 0 and $\frac{d^2v}{dt^2}$ ve.

17. **Statement-1** : The velocity of a point sized observer in motion as measured by himself is always zero.

Statement-2 : The velocity of a particle A as measured by another particle B is $\vec{v}_A - \vec{v}_B$, where $\vec{v}_A$ and $\vec{v}_B$ are velocities of A and B respectively.

18. **Statement-1** : A stone is projected from ground and moves on a parabolic path. Then the magnitude of displacement of particle from point of projection may increase with time over the entire time of flight. (Neglect air friction)

Statement-2 : The horizontal component of velocity of projectile remains constant for the duration of time of flight. Hence the horizontal component of displacement of particle from point of projection keeps on increasing with time. (Neglect air friction)

19. **Statement-1** : A man projects a stone with speed u at some angle. He again projects a stone with same speed such that time of flight now is different. The horizontal ranges in both the cases may be same. (Neglect air friction)

Statement-2 : The horizontal range is same for two projectiles projected with same speed if one is projected at an angle θ with the horizontal and other is projected at an angle $(90° - \theta)$ with the horizontal. (Neglect air friction)

20. **Statement-1** : A particle starts from rest under action of (nonzero) variable acceleration such that direction of acceleration is always fixed (or constant). Then this particle moves in a straight line.

Statement 2 : The motion of particle starting from rest is always along a straight line.

21. **Statement-1** : The rate of change of velocity with time for a projectile in motion depends on direction of its velocity. (Neglect air friction)

Statement-2 : Acceleration $\vec{a}$ of a particle is defined as rate of change of velocity $\vec{v}$ with time t, that is, $\vec{a}$ $\frac{d\vec{v}}{dt}$.

22. **Statement-1** : Two stones are projected with different velocities from ground from same point and at same instant of time. Then these stones cannot collide in mid air. (Neglect air friction)

Statement 2 : If relative acceleration of two particles initially at same position is always zero, then the distance between the particles either remains constant or increases continuously with time.

23. **Statement-1** : In a given time interval, the average speed of a particle may be equal to magnitude of average velocity.

Statement-2 : The magnitude of instantaneous velocity is equal to instantaneous speed of a particle in motion.

MARK YOUR RESPONSE	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ		

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

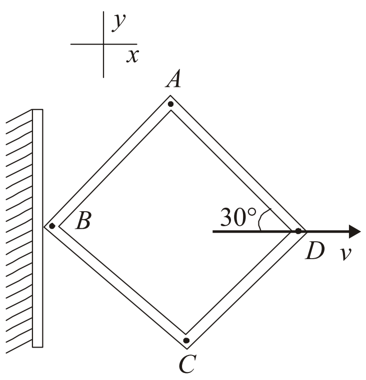
1. ABCD is a wire frame of identical wires in which point D is given velocity v as shown in figure. Choose the correct statement (s)

(a) velocity of point A along x-axis will be $v/2$

(b) speed of point A will be v

(c) speed of point A along y-axis will be $v/2$

(d) velocity of point A will be equal to velocity of point C



2. A particle moves uniformly with speed v along the parabolic path $y = kx^2$. Taking k as a positive constant, the magnitude of acceleration of the particle at $x = 0$ is given by

(a) $2kv^2$ in magnitude (b) along x-axis

(c) $\frac{kv^2}{2}$ in magnitude (d) along y-axis

3. A man can swim with a velocity v relative to water. He has to cross a river of width d flowing with a velocity u ($u > v$). The distance through which he is carried downstream by the river is x. Which of the following statements are correct?

(a) If he crosses the river in minimum time, $x = du/v$

(b) x cannot be less than du/v

(c) For x to be minimum, he has to swim in a direction making an angle of $\pi/2 + \sin^{-1}(v/u)$ with the direction of the flow of water.

(d) x will be maximum if he swims in a direction making an angle of $\pi/2 - \sin^{-1}(v/u)$ with the direction of the flow of water.

4. A particle has an initial velocity of 60m/s up to the right at a slope of 0.75. The components of acceleration are constant at $a_x = -3.6$ m/s^2 and $a_y = -6$ m/s^2. Choose the correct option(s).

(a) The radius of curvature at the start of the path is 1363.64m

(b) The radius of curvature at the top of the path is 384m

(c) The radius of curvature at the start of the path is 323.64m

(d) The radius of curvature at the top of the path is 184m

5. A particle is moving along X-axis whose position is given by $x = 4 - 9t + \frac{t^3}{3}$. Mark the correct statement(s) in relation to its motion.

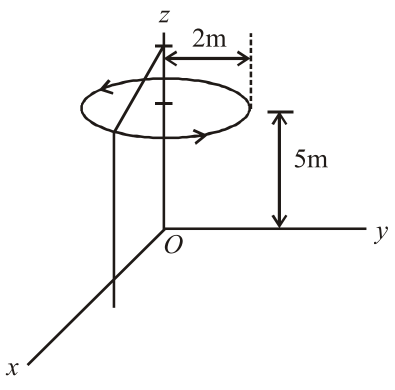
(a) Direction of motion is not changing at any of the instants

(b) Direction of motion is changing at $t = 3$s

(c) For $0 < t < 3$s, the particle is slowing down

(d) For $0 < t < 3$s, the particle is speeding up

6. The upper end of the string of a simple pendulum is fixed to a vertical z-axis, and set in motion such that the bob moves along a horizontal circular path of radius 2m, parallel to the x-y plane, 5m above the origin. The bob has a speed of 3m/s. The string breaks when the bob is vertically above the x-axis, and it lands on the xy plane at a point (x, y). Then

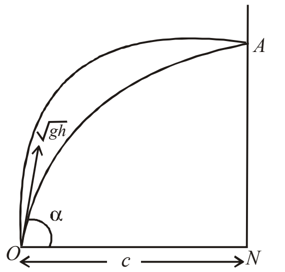


(a) $x = 2$m (b) $x > 2$m

(c) $y = 3$m (d) $y = 5$m

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ				

7. An imperfectly elastic ball is projected with velocity $\sqrt{gh}$ at an angle α with the horizontal, so that it strikes a vertical wall distant c from the point of projection, and returns to the point of projection. If e is the coefficient of restitution, then



(a) Time from A to $O = \dfrac{c}{\sqrt{gh}\cos\alpha}$

(b) Time from A to $O = \dfrac{c}{e\sqrt{gh}\cos\alpha}$

(c) Time from O to $A = \dfrac{c}{\sqrt{gh}\cos\alpha}$

(d) The coefficient of restitution $= \dfrac{c}{(h\sin 2\alpha - c)}$

8. A particle is projected with velocity V along a smooth horizontal plane in a medium whose resistance per unit of mass is μ times the cube of the velocity. Choose the correct options(s).

(a) The distance it has described in time t is

$$\frac{1}{\mu V}[\sqrt{(1+2\mu V^2 t)} - 1]$$

(b) Velocity at time t is $\dfrac{V}{\sqrt{(1+2\mu V^2 t)}}$.

(c) The distance it has described in time t is

$$\frac{1}{\mu V}[\sqrt{(1+2\mu V^2 t)} + 1]$$

(d) Velocity at time t is $\dfrac{V}{\sqrt{(1-2\mu V^2 t)}}$

9. Two shells are fired from a cannon with speed u each, at angles of α and β respectively with the horizontal. The time interval between the shots is T. They collide in mid air after time t from the first shot. Which of the following condition(s) must be satisfied ?

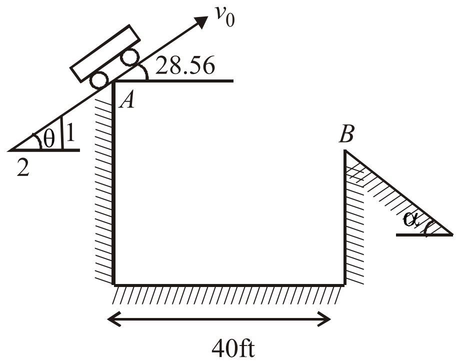
(a) $>$

(b) $t\cos\alpha = (t - T)\cos\beta$

(c) $(t - T)\cos\alpha = t\cos\beta$

(d) $(u\sin\alpha)\,t - \frac{1}{2}gt^2 = (u\sin\beta)\,(t - T) - \frac{1}{2}g\,(t - T)^2$

10. A stunt man is to drive an auto across the water-filled gap shown in figure. Choose the correct option(s).
(In FPS $g = 32.2$ ft/s^2)



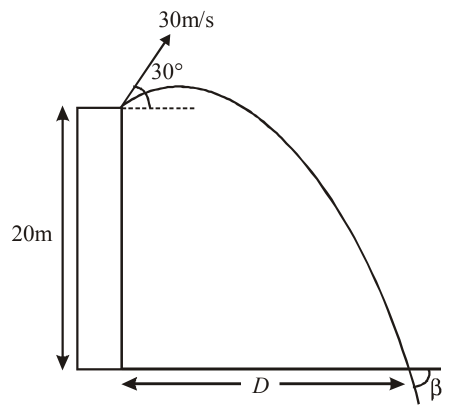
(a) The auto's minimum take-off velocity is 16.76 ft/sec

(b) The angle α of the landing ramp is 45°

(c) The angle α of the landing ramp is 30°

(d) The auto's minimum take-off velocity is 32.76 ft/sec

11. A ball is projected from a building of height 20m at a speed of 30m/sec making an angle of 30° with the horizontal. Then



(a) time after which ball strikes the ground is 4sec

(b) ball comes to a height of 20m again after 3 sec

(c) value of β is $\tan^{-1}\left(\dfrac{5\sqrt{3}}{9}\right)$

(d) Value of D is $60\sqrt{3}$m

MARK YOUR RESPONSE	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ

12. A trapezium with parallel sides of length $2a$ and $4a$ and non-parallel sides each of length $2a$ is placed on the ground with its plane vertical and with the longest side in contact with the ground. A ball is projected from the corner lying on the ground so as just to graze the other three corners of the trapezium. Choose the correct options

(a) The greatest height reached by the ball is $2a/\sqrt{3}$

(b) The total time of flight is $\sqrt{\dfrac{32a}{g\sqrt{3}}}$.

(c) The greatest height reached by the ball is $4a/\sqrt{3}$

(d) The total time of flight is $\dfrac{\sqrt{8a}}{g\sqrt{3}}$.

13. A battle ship is streaming ahead with velocity u. A gun is mounted on the ship so as to point straight backward and is set at an angle of elevation α. If v be the velocity of projection relative to the gun, choose the correct option(s)

(a) The range is $(2v/g) \sin \alpha\, (v \cos \alpha - u)$.

(b) The angle for maximum range is

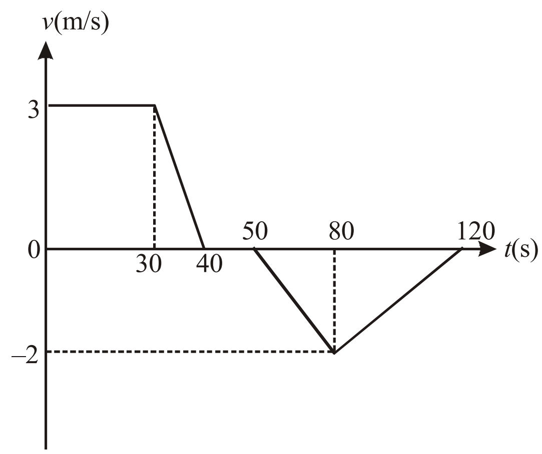
$$\cos^{-1}\left[\frac{u+\sqrt{u^2+8v^2}}{4v}\right].$$

(c) The range is $(2v/g) \sin \alpha\, (v \cos \alpha + u)$

(d) The angle for maximum range is

$$\cos^{-1}\left[\frac{u+\sqrt{u^2+8v^2}}{2v}\right].$$

14. The figure shows the (v, t) graph for a miniature train as it moves along a straight track. At time $t = 0$ the train passes a point A and is moving at 3m/s. The farthest point from A reached by the train in the 120 second period is P. Choose the correct option(s)



(a) The value of t at the instant when the train reaches P is 40 sec.

(b) The magnitude of the acceleration of the train in the time interval $50 < t < 80$ (t in sec) is $\dfrac{1}{15}\,\text{m/s}^2$.

(c) The distance of the train from A at the end of the 120 second period is 35m.

(d) The distance of the train from A at the end of the 120 second period is 105m.

15. A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu\,(a^5/x^2)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance 'a' from O. Choose the correct option(s)

(a) Particle will arrive at O with a velocity $a\sqrt{6\mu}$

(b) Time in which particle will arrive at O is $(8/15)\sqrt{6\mu}$

(c) Particle will arrive at O with a velocity $a\sqrt{3\mu}$

(d) Time in which particle will arrive at O is $(4/15)\sqrt{6\mu}$

16. Which of the following statement(s) is/are true

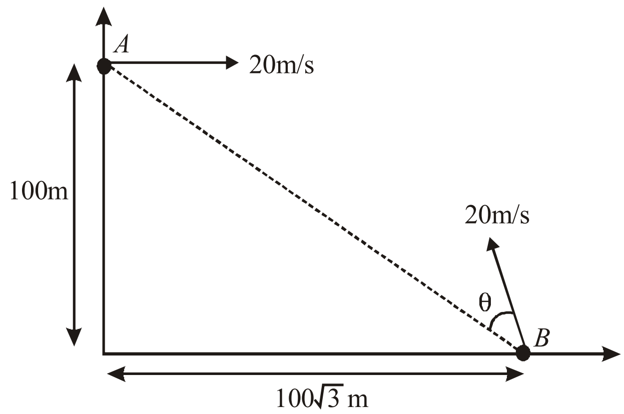
(a) The magnitude of the sum of two vectors must be greater than the magnitude of either vector

(b) If the speed of a moving particle is constant, the acceleration must be zero

(c) Distance travelled by an object must be greater than or equal to magnitude of displacement vector

(d) If a particle is moving along a circle it must be accelerating

17. Two small balls A and B are launched in the same vertical plane simultaneously, with same speed of 20 m/s at $t = 0$. Ball A has an initial horizontal velocity and ball B has initial velocity at an angle θ above the line joining A and B as shown. If the projectiles collide in mid-air at time t, then



(a) $\theta = 30°$ (b) $\theta = 60°$

(c) $t = \dfrac{20}{\sqrt{3}}\,s$ (d) $t = \dfrac{10}{\sqrt{3}}\,s$

MARK YOUR RESPONSE					
	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ
	17. ⓐⓑⓒⓓ				

18. A car starts from rest at $t = 0$ and for the first 4 second of its rectilinear motion the acceleration 'a' (ms^{-2}) at time t (sec) after starting is given by $a = 6 - 2t$. Choose the correct option(s).

(a) The maximum velocity of the car is 6 m/s

(b) The velocity of the car after 4 seconds is 8 m/s

(c) The distance travelled up to 4 seconds is 80/3 m

(d) The maximum velocity of the car is 9 m/s

19. A velocity-time graph of a particle moving rectilinearly can give which of the following quantities with time

(a) Change in speed (b) Change in velocity

(c) Distance covered (d) Change in position

20. A row-boat is observed travelling downstream on a flowing river at a speed of 8 m/s with respect to the shore. A motor-boat comes from the opposite direction at a speed of 10m/s with respect to the water. 10 seconds after meeting each other they are at 160 meters from each other. Choose the correct option(s)

(a) Speed of the river flow is 2 m/sec

(b) The speed of row-boat in still water is 4 m/sec

(c) Speed of the river flow is 3 m/sec

(d) The speed of row-boat in still water is 6 m/sec

21. As a car passes the point A on a straight road, its speed is 10 m/s. The car moves with constant acceleration a m/s^2 along the road for T seconds until it reaches the point B, where its speed is V m/s. The car travels at this speed for a further 10 seconds, when it reaches the point C. From C it travels for a further T seconds with constant acceleration $3a$ m/s^2 until it reaches a speed of 20 m/s at the point D. Choose the correct option(s).

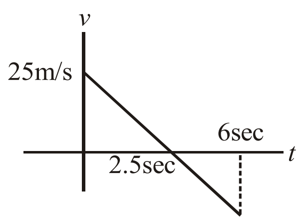
[Given that the distance between A and D is 675m]

(a) Value of V is 12.5 m/sec (b) Value of a is 1/8 m/sec^2

(c) Value of T is 10 sec (d) Value of T is 20 sec

22. A stone is thrown vertically upwards with an initial velocity of 25 m/s from a point 30 meters above the ground.

Choose the correct option(s). [Given $g = 10$ m/s^2]

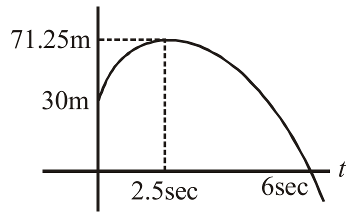


(a) The maximum height of the stone (above the ground) is 71.25m

(b) The time when the stone hits the ground is 6 sec

(c) The (v, t) graph for the motion of the stone up to the time when it hits the ground is

(d) The (h, t) graph showing the height of the stone as a function of time is



23. If a point moves with constant acceleration and u_1 and u_2 are the initial and final velocities then choose the correct option(s).

(a) The space average of the velocity over any distance is

$$\frac{2}{3} \cdot \frac{u_1^2 + u_1 u_2 + u_2^2}{u_1 + u_2}$$

(b) The time-average is $\frac{u_1 + u_2}{2}$

(c) The space average of the velocity over any distance is

$$\frac{1}{3} \cdot \frac{u_1^2 + u_1 u_2 + u_2^2}{u_1 + u_2}$$

(d) The time-average is $\frac{2u_1 u_2}{u_1 + u_2}$

24. Two particles are projected from a horizontal plane with the same initial velocity v_0 at two different angles of projection θ_1 and θ_2, such that their ranges are the same. The ratio of their maximum heights reached is/are

(a) $\tan^2 \theta_1$

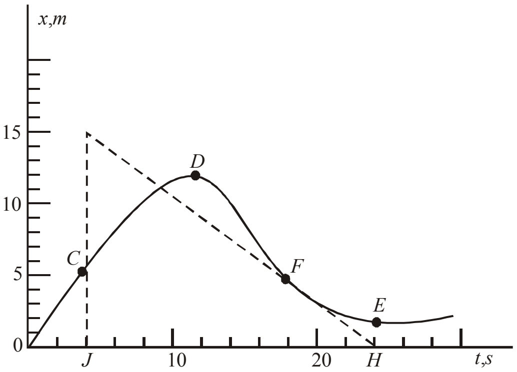
(b) $\cot^2 \theta_2$

(c) $\sin^2 \theta_1 \operatorname{cosec}^2 \theta_2$

(d) $\sin^2 \theta_1 \cos^2 \theta_2$

MARK YOUR RESPONSE					
	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ
	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ			

25. In figure for the motion of an object along the x-axis, choose the correct options (approximately)



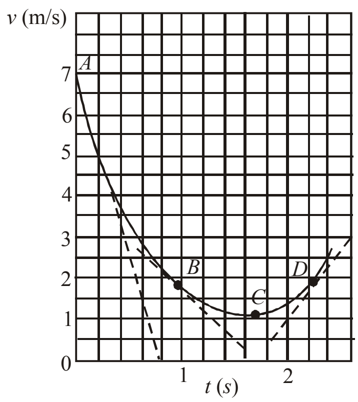
(a) The instantaneous velocity of the object at point D is zero

(b) The instantaneous velocity of the object at point C is 1.3 m/s

(c) The instantaneous velocity of the object at point E is –0.13m/s

(d) The instantaneous velocity of the object at point E is – 1.3m/s

26. Figure shows the velocity of a particle as it moves along the x-axis.



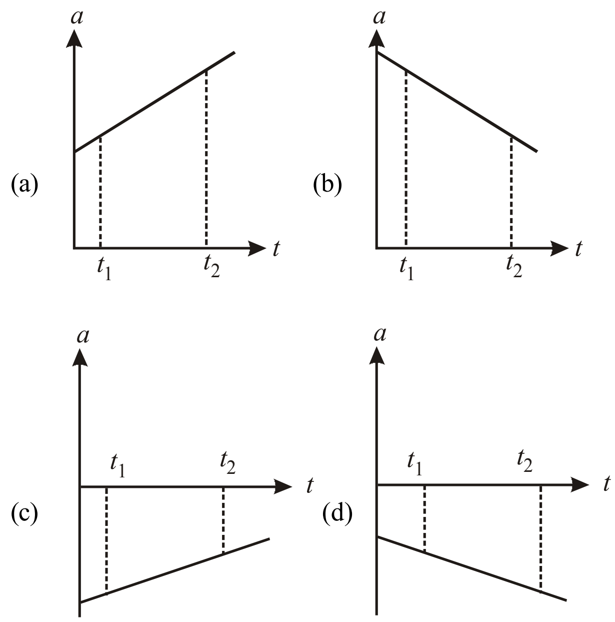
(a) The acceleration at A is -9.6 m/s^2

(b) The acceleration at C is zero

(c) The acceleration at A is zero

(d) The acceleration at C is -9.6 m/s^2

27. Each of the three graphs represents acceleration versus time for an object that already has a non-zero positive velocity at time t_1. Which graph shows an object whose speed is increasing for the entire time interval between t_1 and t_2 ?



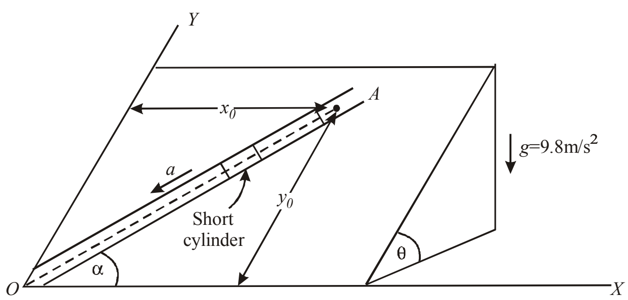
28. Two balls are projected with same speed from a point on ground at same moment. For the horizontal range of both balls to be same, the angles of projection for both balls respectively is/are

(a) 40°, 60° (b) 57°, 33°

(c) 53°, 37° (d) 45°, 45°

29. An inclined plane as in the figure makes an angle θ with the horizontal. A groove OA cut in the plane makes an angle α with OX. A short smooth cylinder is free to slide down the groove under the influence of gravity, starting from rest at the point (x_0, y_0). Let $\theta = 30°$, $x_0 = 3$m, $y_0 = 4$m.

Choose the correct options –



(a) Downward acceleration along the groove is 3.92 m/s^2

(b) The time to reach O is 1.597s

(c) Velocity at O is 6.26 m/s

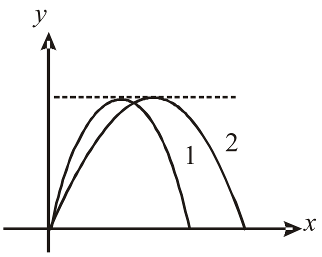
(d) Velocity at O is 3.12 m/s

MARK YOUR RESPONSE	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ

30. A stone is projected from level ground at time $t = 0$. Let v_x and v_y are the horizontal and vertical components of velocity at any time t ; x and y are displacements along horizontal and vertical from the point of projection at any time t. Then

(a) $v_y - t$ graph is a straight line
(b) $y - t$ graph is a straight line passing through origin
(c) $x - t$ graph is a straight line passing through origin
(d) $v_x - t$ graph is a straight line

31. Two stones are projected from level ground. Trajectories of two stones are shown in figure. Both stones haves same maximum heights above level ground as shown. Let T_1 and T_2 be their time of flights and u_1 and u_2 be their speeds of projection respectively (neglect air resistance). Then



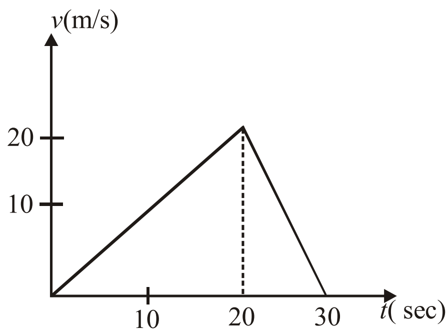
(a) $T_2 > T_1$ (b) $T_1 = T_2$
(c) $u_1 > u_2$ (d) $u_1 < u_2$

32. If a particle moving along a straight line under uniform acceleration covers successive equal distances (s each) in time intervals t_1, t_2 and t_3 respectively. Then the expression for average speed of particle in covering the given distance $3s$ is (Assume the particle is speeding up for the entire journey)

(a) $\frac{s}{t_1}+\frac{s}{t_2}+\frac{s}{t_3}$ (b) $\frac{s}{t_1}-\frac{s}{t_2}+\frac{s}{t_3}$

(c) $\frac{3s}{t_1+t_2+t_3}$ (d) $\frac{s}{t_1}+\frac{s}{t_2}-\frac{s}{t_3}$

33. v-t graph of an object of mass 1 kg is shown



(a) net work done on the object in 30 sec is zero
(b) the average acceleration from 0 to 30 sec of the object is zero
(c) the average velocity from 0 to 30 sec of the object is zero
(d) the average force from 0 to 30 sec on the object is zero.

34. A ball is released from rest at the edge of a deep ravine. Assume that air resistance gives it an acceleration of $-b\frac{dy}{dt}$, where y is measured positive downward. (This negative acceleration is proportional to its speed, $\frac{dy}{dt}$, the positive constant b can be found by experiment.)

The ball has a total acceleration of $-b\frac{dy}{dt}+g$, and so

$$\frac{d^2y}{dt^2} = -b\frac{dy}{dt}+g \qquad \text{............ (1)}$$

is the differential equation of motion. Choose the correct options

(a) Solution to differential equation (1) is $y = k\,(e^{-bt} - 1) + (g/b)\,t$
(b) At $t = 0$, $k = g/b^2$
(c) Assuming $b = 0.1\ s^{-1}$, the distance fallen and the speed reached after 10s are 360m and 62 m/s.
(d) After 1 min the ball will have essentially reached its terminal velocity of 9.8 m/s.

35. A person, standing on the roof of a 40m high tower, throws a ball vertically upwards with speed 10m/s. Two seconds later, he throws another ball again in vertical direction (use $g = 10$ m/s^2). Both the balls hit the ground simultaneously.

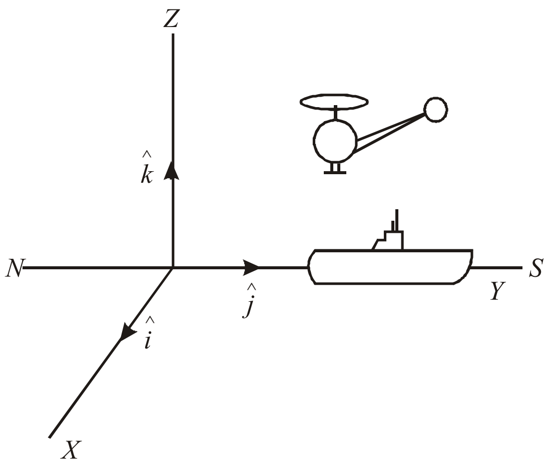
(a) The first stone hits the ground after 4 seconds
(b) The second ball was projected vertically downwards with speed 5m/s
(c) The distance travelled by the first ball is 10m greater than the distance travelled by the second ball.
(d) Both balls hit the ground with same velocities.

36. A particle is projected vertically upwards in absence of air resistance with a velocity u from a point O. When it returns to the point of projection

(a) its average velocity is zero
(b) its displacement is zero
(c) its average speed is $u/2$
(d) its average speed is u

MARK YOUR RESPONSE	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ
	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ			

37. A helicopter is trying to land on a submarine deck which is moving south at 17 m/s. A 12 m/s wind is blowing into the west. If to the submarine crew the helicopter is descending vertically at 5 m/s, choose the correct options



(a) Speed of helicopter relative to the water is $(17\hat{j} - 5\hat{k})$ m/s

(b) Speed of helicopter relative to the air is $(-12\hat{i} + 17\hat{j} - 5\hat{k})$ m/s

(a) Speed of helicopter relative to the water is $(-12\hat{i} + 17\hat{j} - 5\hat{k})$ m/s

(b) Speed of helicopter relative to the air is $(17\hat{j} - 5\hat{k})$ m/s

38. A particle is moving rectilinearly so that its acceleration is given as $a = 3t^2 + 1$ m/s^2. Its initial velocity is zero –

(a) The displacement of the particle in 1 sec will be 2m

(b) The velocity of the particle at t = 1 sec will be 2m/s

(c) The particle will continue to move in positive direction

(d) The particle will come back to its starting point after some time

39. A ball of small size is rolled off the edge of a horizontal table. The ball has an initial speed v_0 and lands on the floor at some distance from the base of the table. Which of the following statements concerning the fall of the ball is/are true ?

(a) The time of flight depends on the height of the table

(b) The horizontal component of average velocity for the duration of flight will be v_0.

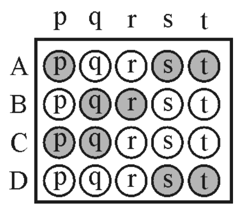
(c) The bat will accelerate for the duration of flight

(d) The ball will have a longer flight time if v_0 is increased.

MARK YOUR RESPONSE	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ		

E MATRIX-MATCH TYPE

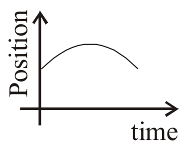
Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.



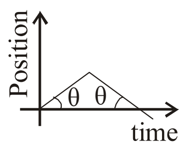
1. Match the situation given in column I with the possible curves in column II

Column I | **Column II**

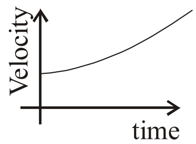
(A) Particle moving with constant speed

(p) 

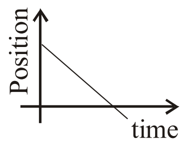
(B) Particle moving with increasing acceleration

(q) 

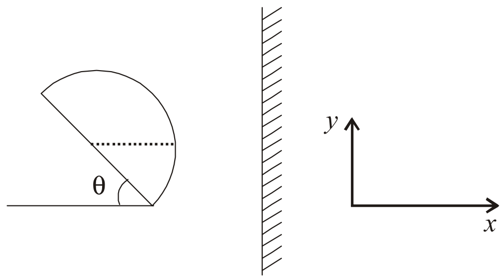
(C) Particle moving with constant negative acceleration

(r) 

(D) Particle moving with zero acceleration

(s) 

2. Figure shows a rigid solid hemisphere of mass m and radius R and a wall in y-z plane in a gravity free space. The velocity of centre of mass of body is $u_x\hat{i} + u_y\hat{j}$ before collision and $v_x\hat{i} + v_y\hat{j}$ after an elastic collision with the wall. Consider that the wall is frictionless and the body had zero angular velocity before the collision.



Column I	Column II
(A) If $\theta = 45°, u_x \neq 0, u_y \neq 0$	(p) $u_x \neq v_x$
(B) If $\theta = 45°, u_x \neq 0, u_y = 0$	(q) $u_y \neq v_y$
(C) If $\theta = 90°, u_x \neq 0, u_y \neq 0$	(r) Body will rotate after collision
(D) If $\theta = 90°, u_x \neq 0, u_y = 0$	(s) None of these

3. A body is projected with velocity v and angle of projection is θ.

Column I	Column II
(A) Variation of kinetic energy (K) with respect to horizontal position (x) is	(p) Linear
(B) Variation of potential energy (U) with respect to horizontal position (x) is	(q) Parabolic
(C) Kinetic energy at the highest point is	(r) Minimum
	(s) Maximum

MARK YOUR RESPONSE

1.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

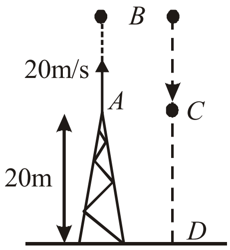
2.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

3.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

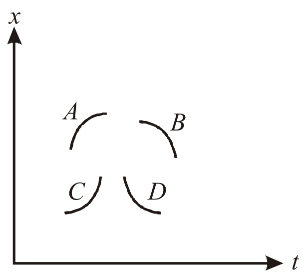
4. A particle is projected vertically upwards with speed 20 m/s from the top of a tower of height 20m as shown in figure. Given B is top most point of trajectory and C is at same height as A.



Column I	Column II
(A) Ratio of maximum height from ground (BD) to the initial height from ground (AD) is	(p) $1/\sqrt{2}$
(B) Ratio of distance travelled in 1^{st} second to the distance travelled in 2nd second is	(q) 1
(C) Ratio of initial speed at A to the final just before reaching to ground (D) is	(r) 2
(D) ratio of time taken from A to C and time taken from A to B is	(s) 3

5. Column I shows the position-time graph of particles moving along a straight line.

Column I



Column II

(p) Acceleration $a > 0$
(q) Acceleration $a < 0$
(r) Speeding up
(s) Speeding down

6. Match the columns.

Column I	Column II
(A) Particles are projected at same speed from same point on level ground such that their ranges are same	(p) Time of flight will be same
(B) Particles are projected from same point on ground such that maximum heights reached are same.	(q) Speed just before reaching ground will be same
(C) Particles are projected horizontally from same point at a height with different initial velocities	(r) Vertical component of velocity just before reaching ground will be same
(D) Particles are projected from the same point at a height with same initial speed, direction of velocity makes equal angle with horizontal one below and the other above horizontal	(s) Minimum kinetic energy during the flight will be equal.

MARK YOUR RESPONSE

4.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

5.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

6.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

7. For a particle moving in x-y plane initial velocity of particle is $\vec{u} = u_1\hat{i} + u_2\hat{j}$ and acceleration of particle is always $\vec{a} = a_1\hat{i} + a_2\hat{j}$ where u_1, u_2, a_1, a_2 are constants. Some parameters of motion is given in column I, match the corresponding path given in column II.

Column I	**Column II**
(A) If $u_1 \neq 0, u_2 = 0, a_1 \neq 0, a_2 \neq 0$	(p) path of particle must be parabolic
(B) If $u_1 = 0, u_2 \neq 0, a_1 \neq 0, a_2 \neq 0$	(q) path of particle must be straight line
(C) If $u_1 = 0, u_2 = 0, a_1 \neq 0, a_2 \neq 0$	(r) path of particle may be parabolic
(D) If $u_1 \neq 0, u_2 \neq 0, a_1 \neq 0, a_2 \neq 0$	(s) path of particle may be straight line

8. A truck is moving on a straight horizontal road towards east with a constant speed 10m/s. In each situation of column I, a ball is thrown in different ways. Match each situation of column I with the nature of trajectory of ball as seen by different observers as given in column II.

Column I	**Column II**
(A) Ball is thrown vertically upward from the truck	(p) Motion of ball as seen from ground is straight line.
(B) Ball is thrown at 60° to horizontal from ground with a speed of 20m/s in east direction	(q) Motion of ball as seen from truck is straight line.
(C) Ball is thrown vertically upward from ground	(r) Motion of ball as seen from ground is parabola
(D) Ball is thrown at 60° with horizontal towards west at a speed of 20m/s from the truck	(s) Motion of ball as seen from truck is parabola.

MARK YOUR RESPONSE

7.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

8.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

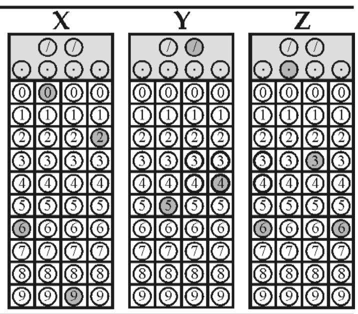
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010, 3 etc.) or a fraction (2/3, 23/7) or a decimal (2.35, 0.546).

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to question X, Y & Z are 6092, 5/4 & 6.36 respectively then the correct darkening of bubbles will look like the following.

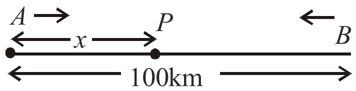
For single digit integer answer darken the extreme right bubble only.



1. Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side $d = 50$ cm at $t = 0$. Each of the particles moves with constant speed $v = 2$ m/s. A always has its velocity along AB, B always along BC and C along CA. At what time(in sec), will all the particles meet each other?

2. Two cars started simultaneously towards each other from towns A and B which are 480 km apart. It took first car travelling from A to B 8 hours to cover the distance and second car travelling from B to A 12 hours. Determine the distance (in km) from town A where the cars meet. Assuming that both the cars travelled with constant speed.

3. Two trains A and B, 100 km. apart, are travelling towards each other with starting speeds of 50 km/hr. for both. The train A is accelerating at 18 km/hr² and B is decelerating at 18 km/hr². Find the distance (in km) from the initial position of A of the point when the engines cross each other.



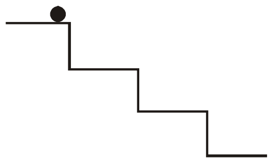
4. A ball is thrown upwards from the ground with an initial speed of u. The ball is at a height of 80m at two times, the time interval being 6s. Find u (in m/s). Take $g = 10$ m/s^2.

5. Two swimmers leave point A on one bank of the river to reach point B lying right across on the other bank. One of them crosses the river along the straight line AB while the other swims at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point B. What was the velocity u (in kph) of her walking if both swimmers reached the destination simultaneously ? The stream velocity is 2km/hr and the velocity of each swimmer with respect to water is 2.5 km/hr.

6. A body is projected downwards at an angle of 30º to the horizontal with a velocity of 9.8 m/s from the top of a tower 29.4 m high. How long will it take before striking the ground?

7. A particle having a mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 60º. What is the magnitude of the change in momentum in kg m/s of the particle after 10 s ?

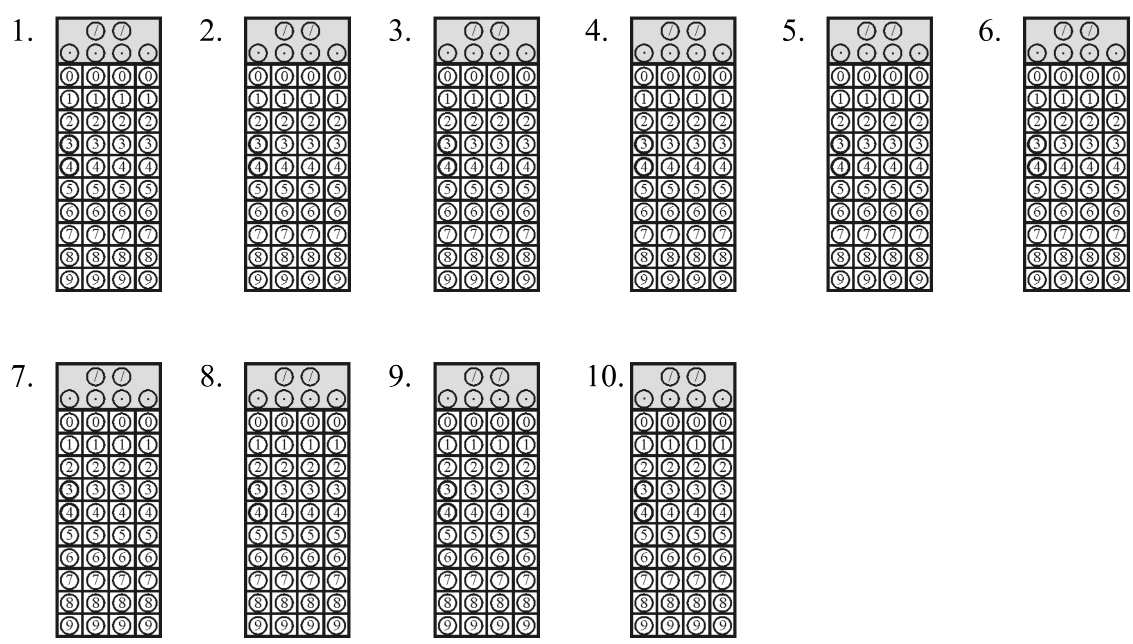
8. A stair case contains three steps, each 10 cm high and 20 cm wide. What should be the minimum horizontal velocity (in m/s) of a ball rolling off the uppermost plane so as to hit directly the lowest plane (take g = 10m/s).



9. A particle is projected from a point O with an initial speed of 30 m/s to pass through a point which is 40m from O horizontally and 10m above O. There are two angles of projection for which this is possible. If these angles are α and β then find the value of $\tan[-(\alpha + \beta)]$.

10. Two boys stationed at A and B fire bullets simultaneously at a bird stationed at C. The bullets are fired from A and B at angles of 53° and 37° with the vertical. Both the bullets fire the bird simultaneously. What is the value of v_A if $v_B = 60$ units? Given : $\tan 37° = 3/4$

MARK YOUR RESPONSE



Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(b)	**11**	(b)	**21**	(a)	**31**	(c)	**41**	(b)	**51**	(a)	**61**	(b)	**71**	(a)	**81**	(b)	**91**	(a)
2	(c)	**12**	(d)	**22**	(c)	**32**	(a)	**42**	(d)	**52**	(b)	**62**	(a)	**72**	(b)	**82**	(c)	**92**	(a)
3	(a)	**13**	(c)	**23**	(a)	**33**	(b)	**43**	(a)	**53**	(a)	**63**	(a)	**73**	(d)	**83**	(a)	**93**	(b)
4	(b)	**14**	(a)	**24**	(b)	**34**	(b)	**44**	(a)	**54**	(a)	**64**	(a)	**74**	(b)	**84**	(b)	**94**	(b)
5	(b)	**15**	(a)	**25**	(b)	**35**	(b)	**45**	(b)	**55**	(a)	**65**	(a)	**75**	(d)	**85**	(a)		
6	(b)	**16**	(d)	**26**	(c)	**36**	(c)	**46**	(d)	**56**	(b)	**66**	(b)	**76**	(d)	**86**	(d)		
7	(c)	**17**	(c)	**27**	(a)	**37**	(d)	**47**	(b)	**57**	(a)	**67**	(a)	**77**	(b)	**87**	(c)		
8	(b)	**18**	(b)	**28**	(b)	**38**	(b)	**48**	(a)	**58**	(a)	**68**	(c)	**78**	(a)	**88**	(b)		
9	(d)	**19**	(a)	**29**	(b)	**39**	(c)	**49**	(c)	**59**	(b)	**69**	(a)	**79**	(a)	**89**	(a)		
10	(d)	**20**	(d)	**30**	(a)	**40**	(b)	**50**	(a)	**60**	(a)	**70**	(a)	**80**	(b)	**90**	(c)		

B COMPREHENSION TYPE

1	(a)	**6**	(b)	**11**	(a)	**16**	(a)	**21**	(a)	**26**	(d)	**31**	(d)	**36**	(d)	**41**	(a)
2	(c)	**7**	(a)	**12**	(b)	**17**	(b)	**22**	(b)	**27**	(d)	**32**	(b)	**37**	(b)		
3	(c)	**8**	(c)	**13**	(b)	**18**	(b)	**23**	(d)	**28**	(c)	**33**	(a)	**38**	(d)		
4	(a)	**9**	(d)	**14**	(b)	**19**	(a)	**24**	(b)	**29**	(a)	**34**	(b)	**39**	(d)		
5	(c)	**10**	(b)	**15**	(a)	**20**	(b)	**25**	(c)	**30**	(b)	**35**	(c)	**40**	(a)		

C REASONING TYPE

1	(a)	**4**	(c)	**7**	(d)	**10**	(a)	**13**	(a)	**16**	(a)	**19**	(a)	**22**	(a)
2	(a)	**5**	(a)	**8**	(d)	**11**	(d)	**14**	(d)	**17**	(a)	**20**	(d)	**23**	(b)
3	(a)	**6**	(a)	**9**	(a)	**12**	(c)	**15**	(d)	**18**	(b)	**21**	(d)		

D MULTIPLE CORRECT CHOICE TYPE

1	(a, b)	**6**	(a, c)	**11**	(a, b, c, d)	**16**	(c, d)	**21**	(a, b, d)	**26**	(a, b)	**31**	(b, d)	**36**	(a, b, c)
2	(a, d)	**7**	(b, c, d)	**12**	(b, c)	**17**	(a, d)	**22**	(a, b, c, d)	**27**	(a, b)	**32**	(b, c)	**37**	(a, b)
3	(a, c)	**8**	(a, b)	**13**	(a, b)	**18**	(b, c, d)	**23**	(a, b)	**28**	(b, c, d)	**33**	(a, b, d)	**38**	(b, c)
4	(a, b)	**9**	(a, b, d)	**14**	(a, b, c)	**19**	(a, b, c, d)	**24**	(a, b, c)	**29**	(a, b, c)	**34**	(a, b, c, d)	**39**	(a, b, c)
5	(b, c)	**10**	(b, d)	**15**	(a, b)	**20**	(a, d)	**25**	(a, b, c)	**30**	(a, c, d)	**35**	(a, c, d)		

E MATRIX-MATCH TYPE

1. **A - q, s; B - r; C - p; D - s**
2. **A - p, r; B - p, r; C - p; D - p**
3. **A - q; B - q; C - r**
4. **A - r; B - s; C - p; D - r**
5. **A - q, s; B - q, r; C - p, r; D - p, s**
6. **A - q; B - p, r; C - p, r; D - q, r**
7. **A - p, B - p, C - q, s ; D - r, s**
8. **A - q, r ; B - q, r; C - p, s; D - p, s**

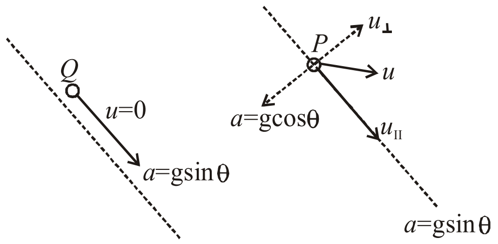
F NUMERIC/INTEGER ANSWER TYPE

1	0.83	**2**	288	**3**	59	**4**	50	**5**	3.0
6	2	**7**	49	**8**	2	**9**	4	**10**	80

Solutions

A SINGLE CORRECT CHOICE TYPE

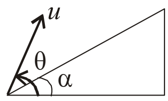
1. **(b)**



It can be observed from figure that P and Q shall collide if the initial component of velocity of P on inclined plane i.e., along incline $u_{\parallel} = 0$ that is particle is projected perpendicular to incline.

Time of flight on an inclined plane of inclination α is given by

$$T = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha},$$

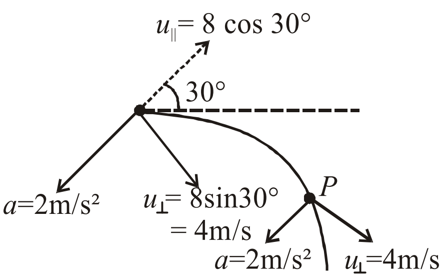


$$\Rightarrow \quad 4 = \frac{2u\sin 90^\circ}{10 \times \frac{1}{2}} \Rightarrow u = 10 \text{ m/s}$$

2. **(c)** The acceleration vector shall change the component of velocity $u_{\parallel}$ along the acceleration vector.

$$r = \frac{v^2}{a_n}$$

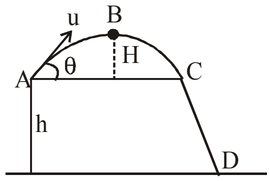
Radius of curvature r_{min} means v is minimum and a_n is maximum. Thus at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_{\parallel} = 0$



$$\therefore \; R = \frac{u_{\perp}^2}{a} = \frac{4^2}{2} = 8 \text{ metres.}$$

3. **(a)** $T_{AC} = 2$ sec.

So, $\dfrac{2u\sin\theta}{g} = 2 \Rightarrow u\sin\theta = g = 10\text{m/s}$



Now $y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow -h = u\sin\theta \times 3 - \frac{1}{2}g \times 3^2$

$\Rightarrow \; -h = 10 \times 3 - \frac{1}{2} \times 10 \times 9 \Rightarrow -h = 30 - 45$

$\Rightarrow h = 45 - 30 = 15\text{m}$

4. **(b)** As when they collide $vt + \frac{1}{2}\left(\frac{72v^2}{25\pi R}\right)t^2 - \pi R = vt$

$$\therefore \; t = \frac{5\pi R}{6v}$$

Now, angle covered by $A = \pi + \dfrac{vt}{R}$

Put t, $\therefore$ angle covered by $A = \dfrac{11\pi}{6}$

5. **(b)** $h_1 = \dfrac{u^2\sin^2\alpha}{2g}$

$$h_2 = \frac{u^2\sin^2(90^\circ-\alpha)}{2g} = \frac{u^2\cos^2\alpha}{2g}$$

$$h_1 h_2 = \frac{1}{4}\left(\frac{u^2\sin\alpha\cos\alpha}{g}\right)^2 = \frac{1}{4}\left(\frac{R}{2}\right)^2$$

$$R = 4\sqrt{h_1 h_2}$$

6. **(b)** $(\vec{v}_{bc})_x = (\vec{v}_b)_x - (\vec{v}_c)_x$

$20\cos 60^\circ = (\vec{v}_b)_x - 30$

$(\vec{v}_b)_x = 40$

$(\vec{v}_{bc})_y = (\vec{v}_b)_y - (\vec{v}_c)_y$

$20\sin 60^\circ = (\vec{v}_b)_y - 0$

$(\vec{v}_b)_y = 10\sqrt{3}$

$$\tan\theta = \frac{(\vec{v}_b)_y}{(\vec{v}_b)_x} = \frac{10\sqrt{3}}{40} = \frac{\sqrt{3}}{4}$$

7. **(c)** $x = 6t,\quad v_x = \dfrac{dx}{dt} = 6$

$y = 8t - 5t^2$,

$v_y = \dfrac{dy}{dt} = 8 - 10t$

Initial $(t = 0)$ $v_y = 8$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10$ m/s

8. **(b)** $t = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$. ; $u = \dfrac{gt\cos\beta}{2\sin(\alpha - \beta)}$

9. **(d)** Ranges for complementary angles are same

$\therefore$ Required angle $= \dfrac{\pi}{2} - \dfrac{5\pi}{36} = \dfrac{13\pi}{36}$

10. **(d)** $KE_{highestpoint} = \dfrac{1}{2}mu^2\cos^2\theta = \dfrac{1}{2}mu^2\left(\dfrac{3}{4}\right)$

$= \dfrac{3}{4}K$

11. **(b)** $\vec{p}_i = mu\cos 45°\hat{i} + mu\sin 45°\hat{j}$

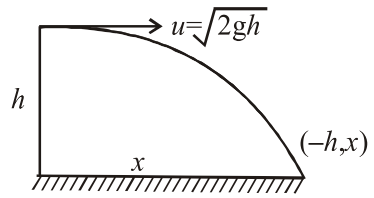
$= \dfrac{mu}{\sqrt{2}}\hat{i} + \dfrac{mu}{\sqrt{2}}\hat{j}$

$\vec{p}_f = \dfrac{mu}{\sqrt{2}}\hat{i} - \dfrac{mu}{\sqrt{2}}\hat{j}$

$\vec{p}_f - \vec{p}_i = -\sqrt{2}mu\ \hat{j}$

12. **(d)** Use $\alpha = \beta = 45°$ in the formula for range down the incline plane.

13. **(c)**



Using equation to trajectory

$$-h = x\tan(0°) - \frac{gx^2}{2(2gh)(\cos^2 0°)} \Rightarrow x = 2h$$

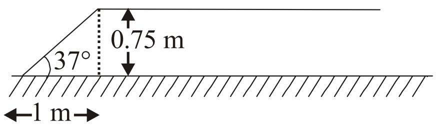
14. **(a)** $H = \dfrac{u_\perp^2}{2g_\perp} \Rightarrow H = \dfrac{10\sin 37° \times 10\sin 37°}{2 \times g\cos 53°}$

$\Rightarrow H = \dfrac{36}{2 \times 10 \times \frac{3}{5}} = 3\text{m}$

15. **(a)** Speed of plank when the rod leaves the plank
$v^2 = 2 \times 8 \times 1 = 16 \Rightarrow v = 4$ m/s
By constraint relation speed of the rod.
$u/4 = \tan 37° \Rightarrow u = 3$ m/s (vertically upwards)

Time period of its flight $= \dfrac{2 \times 3}{10} = 0.6$s

$\ell = 1 + 4 \times (0.6) + \dfrac{1}{2} \times 8 \times (0.6)^2 = 4.84$ m



16. **(d)** $\tan\theta = \dfrac{u^2}{Rg} \Rightarrow R = \dfrac{u^2}{g\tan\theta} = \dfrac{100}{10 \times \sqrt{3}} = \dfrac{10}{\sqrt{3}}$ m

17. **(c)** Say at any instant, the velocity makes an angle θ with the x-axis.

$\Rightarrow \vec{u} = u(\cos\theta\hat{i} + \sin\theta\hat{j})$

$\Rightarrow \vec{a} = \dfrac{d\vec{v}}{dt} = u\left[-\sin\theta\dfrac{d\theta}{dt}\hat{i} + \cos\theta\dfrac{d\theta}{dt}\hat{j}\right]$ ...(1)

Now, $\tan\theta = \dfrac{dy}{dx} = \cos x$

$\Rightarrow \sec^2\theta\dfrac{d\theta}{dt} = -\sin x\left(\dfrac{dx}{dt}\right)$

$\Rightarrow \dfrac{d\theta}{dt} = -\cos^2\theta\sin x\left(\dfrac{dx}{dt}\right)$

Now, at $x = \pi/2$, $\theta = 0°$, $\dfrac{dx}{dt} = u$

$\Rightarrow \dfrac{d\theta}{dt} = -u$

Putting this in (1), we get

$(\vec{a}) = u^2$

18. **(b)** $a_T = 0,\quad a_R = \dfrac{v^2}{R} = \dfrac{36}{R}$

$\dfrac{1}{R} = \dfrac{d^2y/dx^2}{[1 + (dy/dx)^2]^{\frac{3}{2}}} = 10 \times (2)^{3/2}$

$\left[\because \dfrac{dy}{dx} = \dfrac{x}{10} \text{ and } \dfrac{d^2y}{dx^2} = \dfrac{1}{10}\right]$

$\therefore a_R = \dfrac{9\sqrt{2}}{10}$

19. **(a)** Avg. speed $= 3 = \dfrac{\sqrt{(v_r t)^2 + (v_{mr}t)^2}}{t}$

$\Rightarrow v_r^2 + 5 = 9 \Rightarrow v_r = 2m/s$

20. **(d)** $a = 2 + |t - 2|$
For $t \le 2$
$a = 2 - t + 2$
$a = 4 - t$

$dv = (4 - t)\, dt$
$v = 4t - t^2/2$
at $t = 2$, $v = 6$ m/s
For $t > 2$
$a = 2 + t - 2 = t$

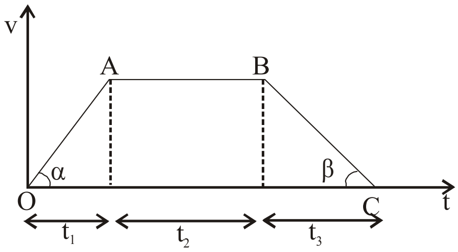
$$\int_6^v dv = \int_2^t t\,dt$$

$$v - 6 = \left[t^2/2\right]_2^t$$

$$v = \frac{t^2}{2} + 4$$

at $t = 4$, $v = 12$ m/s

21. **(a)** From $v = u + at$, we have



$$v = 0 + \alpha t_1 \Rightarrow t_1 = v/\alpha$$

$$0 = v - \beta t_3 \Rightarrow t_3 = v/\beta$$

Now, $S = \frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3 = \frac{v^2}{2\alpha} + vt_2 + \frac{v^2}{2\beta}$

$$\Rightarrow t_2 = \frac{S}{v} - \frac{v}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

Hence, $t_1 + t_2 + t_3 = \frac{S}{v} + \frac{v}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

22. **(c)** $\vec{v}_{cg} = \vec{v}_{CP} + \vec{v}_{Pg}$

$$= -R\omega\hat{i} + 2\hat{j}$$

$$= -2\hat{i} + 2\hat{j}$$

23. **(a)** $\vec{v}_{SB} = v\hat{j} = \vec{v}_s + 3\hat{i}$

$\vec{v}_S = v\hat{j} - 3\hat{i}$ and $v = \frac{100}{50} = 2\text{m/s}$

$$\therefore \quad \left|\vec{v}_S\right| = \sqrt{v^2 + (3)^2}$$

$$= \sqrt{2^2 + 9} = \sqrt{13} \text{ m/s}$$

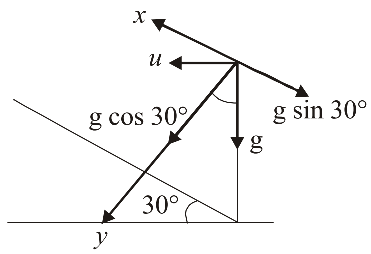
Drift $= 50 \times 3 = 150$ m

24. **(b)** $10 - v\cos 60° = 0$
$v = 20$ m/s

$$H = \frac{v^2 \sin^2 60°}{2g} = 15m$$

25. **(b)** It is possible only when $\theta > 45°$

26. **(c)**



X-direction
$\Rightarrow v_x = u_x + a_x t$
$0 = u\cos 30° - g\sin 30°\,.t$

$\Rightarrow u\sqrt{3} = gt$ ... (i)

Y-direction

$$\Rightarrow x_y = u_y t + \frac{1}{2}a_y t^2$$

$$20\cos 30° = u\cos 60°\, t + \frac{1}{2}g\cos 30°\, t^2$$

$$20\sqrt{3} = ut + \frac{1}{2}gt^2\sqrt{3}$$

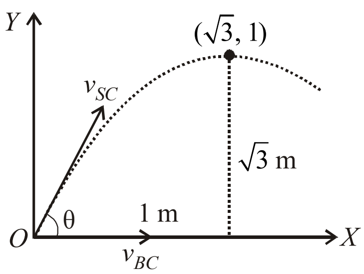
$$20\sqrt{3} = \frac{u^2\sqrt{3}}{g} + \frac{\sqrt{3}}{2}g.\frac{3u^2}{g^2}$$

$$20 = \frac{5u^2}{2g}$$

$u = \sqrt{80}$ m/s $= 4\sqrt{5}$ m/s.

27. **(a)** From figure,

$$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \theta = 30°$$



28. **(b)** $\Delta v = \sqrt{2}\,\pi$
$\Delta t = 6$ s.

$$\therefore \; a = \frac{\Delta v}{\Delta t} = \frac{\pi}{3\sqrt{2}} \text{m/s}^2.$$

29. **(b)** Resistance $= \mu v$
$\therefore$ Equation of motion is

$$m\frac{d^2x}{dt^2} = mg - \mu v$$

i.e. $\frac{d^2x}{dt^2} = g - \frac{\mu}{m}v$

This may be written as

$\frac{dv}{dt} = g - \frac{\mu}{m}v$ or $\frac{dv}{dt} + \frac{\mu}{m}v = g$

This is a linear differential equation in v and t, so the integrating factor is $e^{\mu t/m}$, hence its solution is

$$ve^{\mu t/m} = \int ge^{\mu t/m}dt + C$$

i.e. $ve^{\mu t/m} = \frac{m}{\mu}ge^{\mu t/m} + C$ .... (1)

Initially, when $t = 0$, $v = 0$, $\therefore$ $C = -\frac{m}{\mu}g$.

Hence eq. (1) becomes

$$ve^{\frac{\mu}{m}t} = \frac{m}{\mu}g\left(e^{\frac{\mu}{m}t} - 1\right)$$

or $\frac{dx}{dt} = \frac{m}{\mu}g\left(1 - e^{-\frac{\mu}{m}t}\right)$

Integrating it, we have

$x = \frac{m}{\mu}g\left(t + \frac{m}{\mu}e^{-\frac{\mu}{m}t}\right) + D$ .... (2)

Initially, when $t = 0, x = 0$, $D = -\frac{m^2}{\mu^2}g$

Hence eq. (2) becomes

$$x = \frac{m}{\mu}g\left(t + \frac{m}{\mu}e^{-\frac{\mu}{m}t}\right) - \frac{m^2}{\mu^2}g$$

$$= g\frac{m^2}{\mu^2}\left(e^{-\frac{\mu}{m}t} - 1 + \frac{\mu}{m}t\right)$$

30. **(a)** $20 = \frac{u^2\sin^2\alpha}{2g} \Rightarrow u\sin\alpha = 20$ .... (1)

For total time of light

$-25 = u\sin\alpha t - \frac{1}{2}gt^2$

$5t^2 - 20t - 25 = 0$

$t^2 - 4t - 5 = 0$

$t = \frac{4 \pm \sqrt{16 + 20}}{2} = 5$ sec.

In 5 sec, horizontal displacement = 75m.

Now $u\cos\alpha \times 5 = 75 \Rightarrow u\cos\alpha = 15$ .... (2)

From (1) and (2)

$\tan\alpha = \frac{20}{15} = \frac{4}{3} \Rightarrow \alpha = \tan^{-1}\left(\frac{4}{3}\right)$

31. **(c)** n balls are thrown per second.

Time interval between two balls thrown $= \frac{1}{n}$ sec

In this time it reaches highest point

$\Rightarrow$ $v = u + at$

$0 = u - g\left(\frac{1}{n}\right) \Rightarrow u = \frac{g}{n}$

$v^2 - u^2 = 2as \Rightarrow 0^2 - \left(\frac{g}{n}\right)^2 = 2 \times (-g)\,h$

$\Rightarrow h = \frac{g}{2n^2}$

32. **(a)** The equation for the given v-x graph is

$v = -\frac{v_0}{x_0}x + v_0$ ... (i)

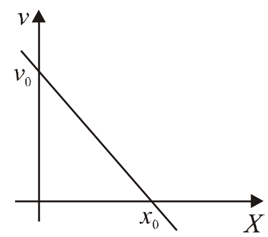
Differentiating the above equation w.r.t x, we get

$\frac{dv}{dx} = -\frac{v_0}{x_0}$

Multiplying the above equation both sides by v, we get

$v\frac{dv}{dx} = -\frac{v_0}{x_0} \times v = -\frac{v_0}{x_0}\left[-\frac{v_0}{x_0}x + v_0\right]$ From (i)

$\therefore$ $a = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}$ .... (ii) $\left[\because a = v\frac{dv}{dx}\right]$



On comparing the equation (ii) with equation of a straight line

$y = mx + c$

we get $m = \frac{v_0^2}{x_0^2} = +$ve, i.e. $\tan\theta = +$ ve, i.e., θ is acute.

Also $c = -\frac{v_0^2}{x_0}$, i.e., the y-intercept is negative

The above conditions are satisfied in graph (a).

33. **(b)** $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-12\sin 4t)^2 + (-12\cos 4t)^2}$

$\Rightarrow v = 12$ m/s

Speed is constant.

Distance = speed × time = 12 × 2 = 24m

34. **(b)** Assuming particle '2' be at rest

Substituting in $y = x \tan - \dfrac{gx^2}{2u^2 \cos\theta}$

$\Rightarrow \; -h = -\dfrac{gs^2}{2\times 4v^2} \Rightarrow v = \sqrt{\dfrac{g}{8h}}.s$,

which is a straight line passing through the origin with slope $\sqrt{\dfrac{g}{8h}}$.

35. **(b)** Resistance = $kv \quad \left(= k\dfrac{ds}{dt}\right)$

Equations of motion are

$$\frac{d^2x}{dt^2} = -k\frac{dx}{dt} \quad \text{............ (1)}$$

$$\frac{d^2y}{dt^2} = -k\frac{dy}{dt} - g \quad \text{............ (2)}$$

Integrating (1) and (2) and using the initial conditions, we get

$$\frac{dx}{dt} = u\cos\alpha . e^{-kt} \quad \text{............ (3)}$$

and $k\dfrac{dy}{dt} + g = (ku\sin\alpha + g).e^{-kt}$

i.e., $\dfrac{dy}{dt} = \dfrac{1}{k}[(ku\sin\alpha + g).e^{-kt} - g]$ .... (4)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{[(ku\sin\alpha + g).e^{-kt} - g]}{ku\cos\alpha . e^{-kt}} \quad \text{.... (5)}$$

Direction of projection was α with the horizontal, when the direction of motion again makes the angle α with the horizontal, it really makes the angle (π – α) with the horizontal in the sense of the direction of projection. If this happens after the time t, we have from (5),

$$\tan(\pi - \alpha) = \frac{(ku\sin\alpha + g).e^{-kt} - g}{ku\cos\alpha . e^{-kt}}$$

i.e., $-\tan\alpha = \dfrac{(ku\sin\alpha + g) - ge^{-kt}}{ku\cos\alpha}$

i.e., $-ku\sin\alpha = ku\sin\alpha + g - g.e^{-kt}$

or $e^{kt} = 1 + \dfrac{2ku}{g}\sin\alpha$

or $t = \dfrac{1}{k}\log\left(1 + \dfrac{2ku}{g}\sin\alpha\right)$

36. **(c)** The distance travelled by each car is equal to the area under the graph. These areas are equal at 10.0 s.

37. **(d)** $a_A = \dfrac{1200}{1.25\times 10^3} = 0.960 \text{ m/s}^2$,

$a_B = \dfrac{1200}{1.20\times 10^3} = 1.00 \text{ m/s}^2$

$1.00\times 10^2 = \dfrac{1}{2}(0.960)t^2 \Rightarrow t = 14.4\text{s}$

$d_B = \dfrac{1}{2}(1.00)\,(14.4)^2 = 104\text{m}$

∴ Car B will be 4 m ahead of Car A.

38. **(b)** Area under a-t curve gives change in velocity. Between 0 to 2 second speed will increase and between 2 to 4 second speed will decreases.

39. **(c)** Let the points A, B, C and D be separated by 1 km. Then

$t_{AB} = \dfrac{1}{60}hr,\; t_{BC} = \dfrac{1}{40}hr$

$\therefore \langle v_{AC}\rangle = \dfrac{1+1}{\dfrac{1}{60}+\dfrac{1}{40}} = 48 \text{ km/hr} \;\; \Rightarrow t_{CD}\,\dfrac{1}{48}hr$

Now $\langle v_{AD}\rangle = \dfrac{1+1+1}{\dfrac{1}{60}+\dfrac{1}{40}+\dfrac{1}{48}} = 48 \text{ km/hr}$

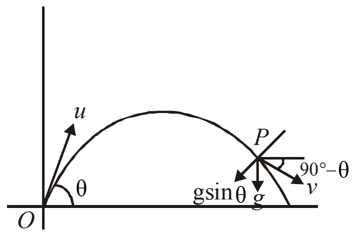
40. **(b)** Initial relative velocity, $v_{AB} = v - 0 = v$

Acceleration $a_{AB} = 0 - a = -a$

For max. separation $v_{AB} = 0$

$0 = v^2 - 2as \Rightarrow s = \dfrac{v^2}{2a}$

41. **(b)**



Horizontal components of velocity at O and P are equal.

$\therefore v\cos(90° - \theta) = u\cos\theta$

or $v\sin\theta = u\cos\theta$

or $v = u\cot\theta$

At P, $\dfrac{v_T^2}{R} = a_c$; $\dfrac{u^2\cot^2\theta}{g\sin\theta} = R$

42. **(d)** This is a straight forward problem in calculating the total distance. The only trick is to make sure that in the cross-over lap you use the hypotenuse. The total distance must be precisely 400 m, so we must have $400 = 112 + \pi(R + 0.05) + \pi(R + 5.05) + (112^2 + 5^2)^{1/2}$. Solving for R gives 25.44 m.

43. **(a)** $s_n = \frac{a}{2}(2n-1);$

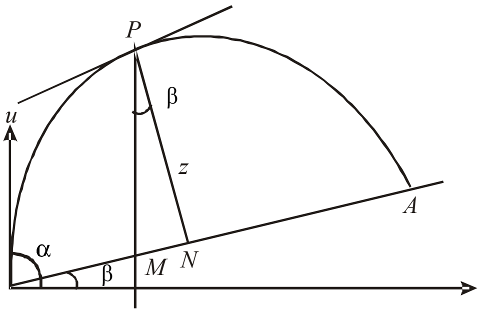
$$s_{n+1} = \frac{a}{2}[2(n+1)-1] = \frac{a}{2}(2n+1)$$

$$\frac{s_n}{s_{n+1}} = \frac{2n-1}{2n+1}$$

44. **(a)** $v_y = 0$ at $t = \frac{b}{c}$, $T = \frac{2b}{c}$,

Range $= aT = \frac{2ab}{c}$

45. **(b)** P be the point where the tangent is parallel to the inclined plane. If $PN = z$ be perpendicular from P on the inclined plane and PM the vertical altitude of P then evidently for all points on the path, P is the point where z is the greatest and consequently PM is greatest.



Now for the point P, velocity perpendicular to the inclined plane is zero. Now the velocity and acceleration perp. to the plane at O is u sin $(\alpha-\beta)$ and $g\cos\beta$ and this velocity becomes zero at P.

$\therefore \quad 0 = u^2\sin^2(\alpha-\beta) - 2g\cos\beta . z$

$$z = \frac{u^2\sin^2(\alpha-\beta)}{2g\cos\beta}$$

For max. range $\alpha = \frac{\pi}{4}+\frac{\beta}{2}$ or $\alpha-\beta = \frac{\pi}{4}-\frac{\beta}{2}$

Hence, $z = \frac{u^2}{2g\cos\beta}\sin^2\left(\frac{\pi}{4}-\frac{\beta}{2}\right)$

$$= \frac{u^2}{4g\cos\beta}\left[1-\cos\left(\frac{\pi}{2}-\beta\right)\right]$$

$= \frac{u^2}{4g\cos\beta}(1-\sin\beta)$ or $PM = z\sec\beta$

$= \frac{u^2}{4g\cos^2\beta}(1-\sin\beta) = \frac{u^2}{4g(1+\sin\beta)} = \frac{1}{4}$ (max. range)

$\Rightarrow$ Maximum range $= 4 \times PM$

46. **(d)** By definition of relative velocity

$$\vec{v}_1 = \vec{v}_0 + \vec{v}_2$$

$$\Rightarrow \vec{v}_0 + \vec{v}_2 + (-\vec{v}_1) = 0$$

$\Rightarrow v_0, v_1$ and v_2 will be sides of a triangle and we know that the sum of any two sides is greater than third side of the triangle.

47. **(b)** $v_x =$ const. No acceleration.

48. **(a)** $\frac{2s}{v} = T_0$

$$T_w = \frac{s}{v+v_w}+\frac{s}{v+v_w} = s\left[\frac{2v}{v^2-v_w^2}\right] = \frac{2s}{v}\left[\frac{v^2}{v^2-v_w^2}\right]$$

$$T_w = T_0\left[\frac{1}{1-(v_w/v)^2}\right]$$

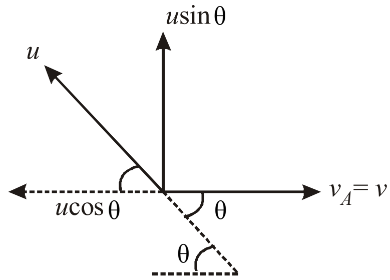
49. **(c)**

$v_L = 8$ km/h, $s = v_0 \times t$

$t = \frac{40}{8} = 5\text{h}$,

Total distance flown by the bird $= 10 \times 5 = 50$ km.

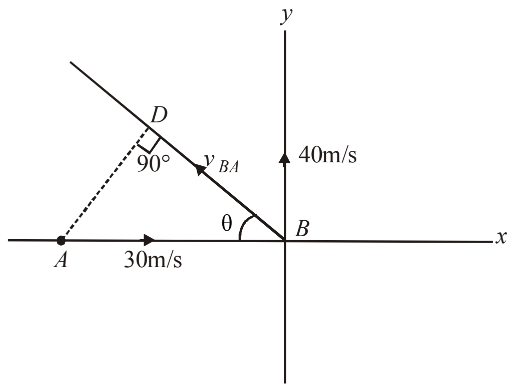
50. **(a)** $\vec{v}_B = \vec{v}_{BA} + \vec{v}_A$



$$u\cos\theta = v \Rightarrow \theta = \cos^{-1}\left(\frac{v}{u}\right)$$

51. **(a)** $t = \frac{s_{rel}}{v_{rel}} = \sqrt{H^2+L^2} \div \sqrt{\frac{2H}{g}}$

52. **(b)**



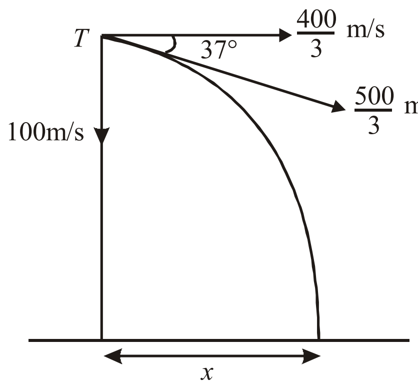
$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 40\hat{j} - 30\hat{i}$; $\tan\theta = \frac{40}{30}$

$|\vec{v}_{BA}| = 50\text{m/s}$

Required time $= \frac{BD}{v_{BA}} = \frac{AB\cos\theta}{v_{BA}}$

$$= \frac{10^4\,\text{m}}{50\text{ms}^{-1}}\frac{3}{5} = 120\,\text{sec}.$$

53. (a)



$1500 = 100t + \frac{1}{2}10t^2$; $t = 10$ sec.

$x = \frac{400}{3}t = \frac{4000}{3}$ m

54. (a) In a, b, c and d time taken are respectively

$$\frac{2d}{v}, \frac{2d}{\sqrt{v^2-u^2}}, \frac{d}{u+v} - \frac{d}{u-v} = \frac{2du}{u^2-v^2}, \frac{2du}{u^2-v^2}$$

[u = stream speed]

55. (a) The distance travelled by the body A is h_1 given by $v_1t - \frac{gt^2}{2}$ and that travelled by the body B is $h_2 = \frac{gt^2}{2}$

The distance between the bodies = $x = h - (h_1 + h_2)$.

Since $h_1 + h_2 = v_1t$, the relation sought is $x = h - v_1t$

56. (b) The swimmer who wants to cross the river along the line AB should swim in such a direction such that his velocity along the direction of stream is zero.

His velocity along the direction $AB = v\cos\theta$

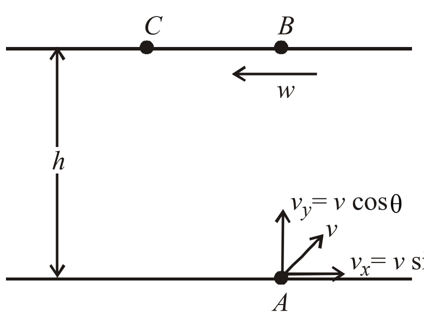
Time taken by the swimmer to reach $B = \frac{\ell}{v\cos\theta}$ where ℓ is the width of river.

Now the second swimmer crosses the river in a direction perpendicular to the stream.

Time taken to reach the opposite bank = $\frac{\ell}{v}$.

But during this time the swimmer is carried away by stream with velocity w and he reaches the point C.

Distance CB = velocity × time = $\frac{w\ell}{v}$.



If the swimmer walks this distance with uniform velocity w', then time taken = $\frac{w\ell}{vw'}$.

Since both the swimmers reach B simultaneously, we have

$$\frac{\ell}{v\cos\theta} = \frac{\ell}{v} + \frac{w\ell}{vw'}$$

$$\therefore \frac{1}{\cos\theta} = 1 + \frac{w}{w'}$$

We have $w = 2$ km/hr and $v = 2.5$ km/hr.

$$\cos\theta = \frac{v_y}{v} = \frac{\sqrt{v^2 - w^2}}{v} = \frac{\sqrt{2.5^2 - 2^2}}{2.5} = \frac{1.5}{2.5}$$

$$\therefore \frac{1.5}{2.5} = \frac{w'}{w'+2}$$

$\therefore$ Velocity of walking of the second swimmer $w' = 3$ km/hr.

57. (a) $a = \frac{v_0'' - v_0'}{\Delta t}$

On substituting

$v'_0 = 2$ m/s, $v''_0 = 12$ m/s and $\Delta t = 10$ sec.

we get, $a = 1$ m/s^2.

58. (a) Let the unit vectors towards east and north be $\hat{i}, \hat{j}$ and the true velocity of the wind be $a\hat{i} + b\hat{j}$ and the man's velocity be $u\hat{i}$. Now the relative velocities of the wind in the 3 cases are :

$(a\hat{i} + b\hat{j}) - u\hat{i}, (a\hat{i} + b\hat{j}) - 2u\hat{i}, (a\hat{i} + b\hat{j}) - 3u\hat{i}$

From the given conditions they are of the form

$$a\hat{i} + b\hat{j} - u\hat{i} = -\lambda\hat{j} \quad \text{........... (1)}$$

$$a\hat{i} + b\hat{j} - 2u\hat{i} = -\mu\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) \quad \text{........... (2)}$$

$$a\hat{i} + b\hat{j} - 3u\hat{i} = -\gamma\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right) \quad \text{........... (3)}$$

On the RHS of (1), the $\hat{i}$ component is 0. So also on the LHS. So, $a - u = 0$ or $a = u$

On the RHS of (2), the $\hat{i}$ and $\hat{j}$ component are equal. So also on the LHS. Therefore $a - 2u = b$ or $b = -u$

From (3), we get

$-\gamma\cos\theta = a - 3u, \theta - \gamma\sin\theta = b$

$$\therefore \tan\theta = \frac{b}{a-3u} = \frac{-u}{u-3u} = \frac{1}{2}$$

59. (b) In the first and the second cases let the retardations be a m/sec^2, a' m/sec^2.

Now, for the first phase of the first case, we have

$u = 30$, $v = 10$ m/s, $s = 240$ m

So, by $v^2 = u^2 - 2as$, we get a from

$100 = 900 - 2a \times 240$ as $a = 5/3$.

Let s_1 be distance travelled by the train in the second phase. In that phase the initial velocity is 10 m/s and the final velocity is 0 and therefore $0 = 10^2 - 2as_1$ or

$$s_1 = \frac{100}{2a} = \frac{100\times 3}{2\times 5} = 30\text{m}.$$

In the second case, the retardation is

$$a' = a + a\times\frac{12\frac{1}{2}}{100} = \frac{9a}{8} = \frac{9}{8}\times\frac{5}{3} = \frac{15}{8}$$

If s_2 is the distance in this case

$$0 = 30^2 - 2\left(\frac{15}{8}\right)s_2 \text{ or } s_2 = 240$$

60. **(a)** Let v be the maximum velocity attained and t the total time of journey. t' is the duration of acceleration and retardation. Then $v = 0 + at'$.

$$\therefore\ L = \frac{1}{2}at'^2 + v(t-2t') + \frac{1}{2}at'^2$$

$$= a\left(\frac{v}{a}\right)^2 + v\left(t - \frac{2v}{a}\right) = \frac{v^2}{a} + vt - \frac{2v^2}{a} = vt - \frac{v^2}{a}$$

$$t = \frac{L}{v} + \frac{v}{a}$$

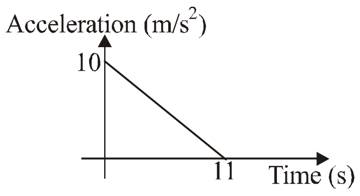
$$\frac{dt}{dv} = -\frac{L}{v^2} + \frac{1}{a}$$

When t is minimum, $\frac{dt}{dv} = 0$

$$\therefore\ v_{\max} = \sqrt{La}$$

61. **(b)** Change in velocity = area under the graph

Area under the graph = $\frac{1}{2}\times 10\times 11 = 55\,\text{m/s}$



Since, initial velocity is zero, final velocity is 55 m/s.

Alternatively :

From the given graph, $a - t$ equation is

$$a = \left(-\frac{10}{11}\right)t + (10)$$

$$\text{or}\quad \frac{dv}{dt} = \left(-\frac{10}{11}\right)t + (10)$$

$$\text{or}\quad dv = \left(\frac{-10}{11}\right)t\,dt + (10)\,dt$$

Integrating,

$$\int dv = \left(-\frac{10}{11}\right)\cdot\int t\,dt + 10\int dt$$

$$v = \left(-\frac{10}{11}\right)\cdot\frac{t^2}{2} + 10t$$

For v to be maximum,

$$\frac{dv}{dt} = 0$$

$$\Rightarrow\ -10t + 110 = 0$$

$$t = 11\text{s}$$

$$\therefore\ v_{\max} = \left(\frac{-10}{11}\right).\frac{11\times 11}{2} + 110 = (-55+110)\text{ m/s} = 55\text{ m/s}$$

62. **(a)** $v = 4t - 3t^2$

$$\frac{ds}{dt} = 4t - 3t^2\ ;\ \int_0^s ds = \int_0^t (4t - 3t^2)\,dt$$

$$s = \frac{4t^2}{2} - \frac{3t^3}{3}\ ;\ s = 2t^2 - t^3$$

$$v_{avg} = \frac{s_{(2)} - s_{(0)}}{2-0}\quad [s_{(2)} = 8 - 8 = 0]$$

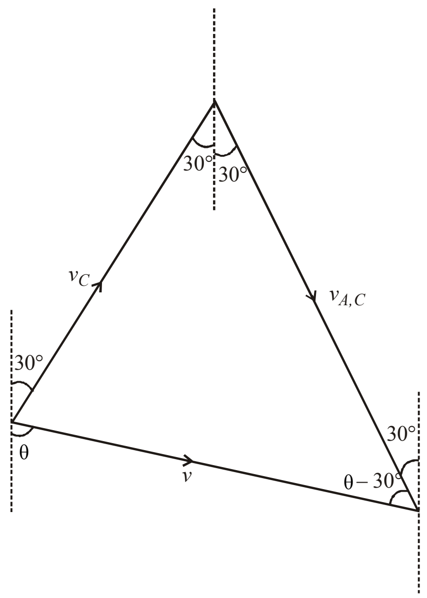
$$v_{avg} = \frac{0-0}{2} = 0$$

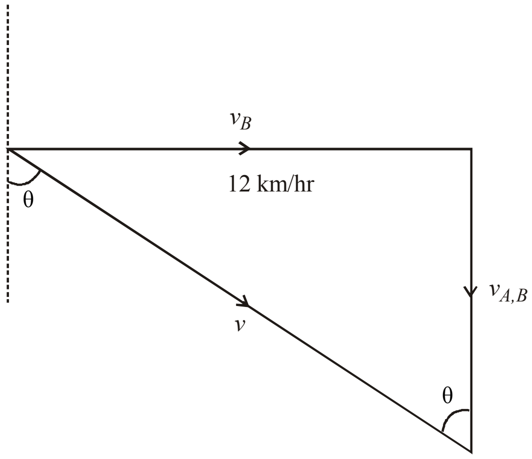
63. **(a)** $v^2 = u^2 + 2as$

$400 > 2a(100)\ ;\ a < 2$

64. **(a)** Let v = velocity of A at $S\ \theta°$ E

The velocity triangles for the two cases are,





By sine rule for triangles,

$$\frac{v}{\sin 60°} = \frac{8}{\sin(\theta - 30°)} \quad \text{........... (1)}$$

$$\frac{v}{\sin 90°} = \frac{12}{\sin\theta} \quad \text{........... (2)}$$

Eq. (1) and Eq. (2) give

$$\frac{\sin(\theta - 30°)}{\sin\theta} = \sin 60° \times \frac{8}{12} = \frac{\sqrt{3}}{2} \times \frac{2}{3} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\sin\theta\cos 30° - \cos\theta\sin 30°}{\sin\theta} = \frac{\sqrt{3}}{2} - \frac{1}{2}\cot\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 3 - \sqrt{3}\cot\theta = 2 \;\Rightarrow\; \cot\theta = \frac{1}{\sqrt{3}} \;\therefore\; \theta = 60°$$

$$\text{From (2)},\; v = \frac{12}{\sin 60°} = 12 \times \frac{2}{\sqrt{3}} = 8\sqrt{3}\ \text{km/hr.}$$

$\therefore$ Speed of $A = 8\sqrt{3}$ km / hr , Direction $= S\,60°\,E$

65. (a) Before hitting the ground, the velocity v is given by $v^2 = 2\,gd$ (quadratic equation and hence parabolic path) Downwards direction means **negative** velocity. After collision, the direction become positive and velocity decreases.

Further, $v'^2 = 2g \times \left(\frac{d}{2}\right) = gd$;

$$\therefore \left(\frac{v}{v'}\right) = \sqrt{2} \text{ or } v = v'\sqrt{2} \Rightarrow v' = \frac{v}{\sqrt{2}}$$

As the direction is reversed and speed is decreased graph (a) represents these conditions correctly.

66. (b) Let the two bodies meet each other at a height h after time T of the projection of second body. Then before meeting, the first body was in motion for time $(t + T)$ whereas the second body was in motion for time T.

The distance moved by the first body in time $(t + T)$

$= u(t+T) - \frac{1}{2}g(t+T)^2$.

And the distance moved by the second body in time T

$= vT - \frac{1}{2}gT^2 = h$ (supposed above). .... (1)

$\because$ The two bodies meet each other,

$\therefore$ They are equidistant from the point of projection.

Hence, $u(t+T) - \frac{1}{2}g(t+T)^2 = vT - \frac{1}{2}gT^2$

or $u(t+T) - \frac{1}{2}g(t^2 + 2tT) = vT$

or $gt^2 + 2t(gT - u) + 2(v - u)T = 0$ .... (2)

Also from (1) we get, $h = vT - \frac{1}{2}gT^2$

$\therefore \frac{dh}{dT} = v - gT$

$\therefore$ h increases as T increases

$\therefore$ T is minimum when h is minimum i.e., when

$\frac{dh}{dT} = 0$, i.e. when $v - gT = 0$ or $T = v/g$.

Substituting this value of T in (2), we get

$gt^2 + 2t(v - u) + 2(v - u)(v/g) = 0$

or $g^2t^2 - 2gt(u - v) + 2v(u - v) = 0$

$$\text{or } t = \frac{2g(u-v) + \sqrt{4g^2(u-v)^2 + 8vg^2(u-v)}}{2g^2}$$

$$\text{or } t = \frac{u - v + \sqrt{u^2 - v^2}}{g}$$

neglecting the negative sign which gives negative value of t.

67. (a) Let $\angle CAB = \alpha$.

Then $\cos\alpha = \frac{b}{\sqrt{a^2 + b^2}}$

To reach B, the resultant velocity must be along AB. Take the river bank as X-axis and line perpendicular to it as Y-axis. Then $\vec{v}_{river} = v_0\hat{i}$

Let the boat start at an angle β with river bank.

Then $\vec{u}_{boat} = u\cos\beta\,\hat{i} + u\sin\beta\,\hat{j}$.

$\therefore$ $\vec{v}$ (resultant velocity of boat)

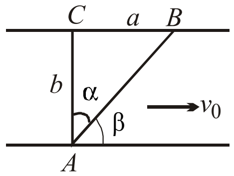
$= (v_0 + u\cos\beta)\,\hat{i} + u\sin\beta\,\hat{j}$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{u\sin\beta}{v_0 + u\cos\beta}$$

$$\cot\alpha = \frac{u\sin\beta}{v_0 + u\cos\beta}$$

$v_0\cot\alpha + u\cos\beta\cot\alpha = u\sin\beta$

Differentiating,



$$0+\cot\alpha\left(\frac{du}{d\beta}\cos\beta-u\sin\beta\right)=\frac{du}{d\beta}\sin\beta+u\cos\beta$$

When u is minimum, $\frac{du}{d\beta}=0$

$-u\cot\alpha\sin\beta=u\cos\beta$ or $\tan\alpha=-\tan\beta$

$\tan(\pi-\alpha)=\tan\beta$

$\alpha+\beta=\pi$

If u_0 be the minimum value of u, then

$$\cot\alpha=\frac{u\sin(\pi-\alpha)}{v_0+u\cos(\pi-\alpha)}$$

$v_0\cos\alpha-u\cos^2\alpha=u\sin^2\alpha$

$$u=v_0\cos\alpha=\frac{v_0 b}{\sqrt{a^2+b^2}}$$

68. (c) $v=0$ at $t=2$ sec.

$$s=10(2)-\frac{1}{2}(5)(2)^2=10\text{m}$$

$$s_{t=3}=\frac{1}{2}(5)(1)^2=2.5\text{m}$$

69. (a) The velocity of the particle is

$$\vec{u}=u_0\hat{i}+(\omega a\cos\omega t)\hat{j} \qquad \text{........... (1)}$$

Writing $\vec{u}$ in terms of Cartesian components in XY plane as $\vec{u}=u_x\hat{i}+u_y\hat{j}$

Comparing coefficients of $\hat{i}$ and $\hat{j}$, the components of velocity along X and Y axes are

$$u_x=\frac{dx}{dt}=u_0 \qquad \text{........ (2)}$$

and $$u_y=\frac{dy}{dt}=\omega a\cos\omega t \qquad \text{........ (3)}$$

Integrating (2) and (3) with respect to t, we get

$x=u_0t+C_1$ .... (4)

and $$y=\frac{\omega a\sin\omega t}{\omega}+C_2=a\sin\omega t+C_2 \qquad \text{........ (5)}$$

where C_1 and C_2 are constants of integration.

At $t=0$, the particle is at origin $x=0, y=0$

So $C_1=0$ and $C_2=0$

Eq. (4) and (5) take the form

$x=u_0t$ and $y=a\sin\omega t$ .... (6)

Substituting $t=\frac{x}{u_0}$, we get $y=a\sin\frac{\omega x}{u_0}$.

This is the required equation of the trajectory.

Substituting $t=\frac{3\pi}{2\omega}$ in eq. (6), we get

$$x=u_0\frac{3\pi}{2\omega} \text{ and } y=a\sin\omega\left(\frac{3\pi}{2\omega}\right)=-a$$

$\therefore$ distance of the particle from the origin at $t=\frac{3\pi}{2\omega}$ is

$$s=\sqrt{x^2+y^2}=\sqrt{\left(u_0\cdot\frac{3\pi}{2\omega}\right)^2+(-a)^2}$$

$$s=\sqrt{\frac{9\pi^2u_0^2}{4\omega^2}+a^2}$$

70. (a) To solve the problem you should have knowledge of conic section-parabola.

The enveloping parabola is $x^2=-4h(y-h)$ where

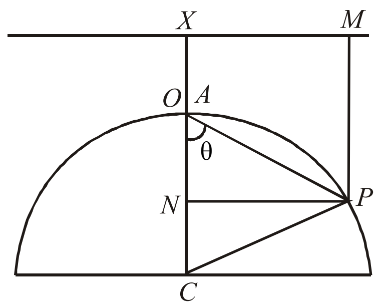
$$h=\frac{u^2}{2g}$$

Its focus is the point of projection O, vertex is A where $OA=h$ and directrix is XM.

Let this enveloping parabola cut the sphere at P so that OP is the max. range. Draw PM perp. to directrix and PN perpendicular to CO, C the centre of the sphere. Since O is the focus, $PO=PM=XN=XO+ON=2h+PO\cos\theta$.

Also, $PO=2r\cos\theta$. Hence, $PO\left(1-\frac{PO}{2r}\right)=2h$

or $R\left(1-\frac{R}{2r}\right)=2h$ where $R=PO$



or $R^2-2r+R+4rh=0$

$R=r\pm\sqrt{r^2-4rh}$

The plus sign corresponds to the second point where the parabola cuts the sphere again hence the minus sign is to be taken

$\therefore\ R=r-\sqrt{r^2-4rh}$

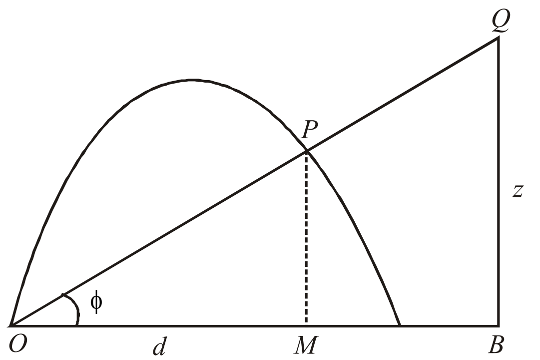
From this result is follows that the max. value of h possible is $r/4$ when the two values of R become same and equal to r. In this case the parabola touches the sphere and the velocity of the projection is $\sqrt{2g.h} = \sqrt{2g.\frac{r}{4}} = \sqrt{\frac{1}{2}gr}$. The particle will then clear the sphere and the least value of the velocity of projection must be $\sqrt{\frac{1}{2}gr}$.

71. **(a)** BQ is the vertical target, $OB = d$. P is the position of the shot at any instant, OP is the line of sight cutting BQ at Q. We have to find the velocity of Q.

Let $BQ = z$ and $\angle QOB = \phi$

Then $z = d \tan\phi = d.\dfrac{PM}{OM}$

$$= d.\frac{y}{x} = d.\frac{u\sin\alpha t - \frac{1}{2}gt^2}{u\cos\alpha . t}$$



$$= d.\frac{u\sin\alpha - \frac{1}{2}gt}{u\cos\alpha} = d\left(\tan\alpha - \frac{1}{2}\frac{g}{u\cos\alpha}.t\right)$$

$$\therefore\ z = -\frac{1}{2}\frac{gd}{u\cos\alpha} = \text{constant since } u\cos\alpha = \text{const.}$$

72. **(b)** From the slopes of the graph, we can obtain the ratio of accelerations of the two cyclists :

$$\frac{a_2}{a_1} = \frac{\left(\frac{v'-0}{4-3}\right)}{\left(\frac{v'-0}{4}\right)} = 4$$

The distances travelled by the two cyclists must be equal

$$\frac{a_1 t^2}{2} = \frac{a_2 (t-3)^2}{2}$$

$$\frac{a_2}{a_1} = \left(\frac{t}{t-3}\right)^2 \Rightarrow \frac{t}{t-3} = 2 \Rightarrow t = 6\,\text{min}.$$

73. **(d)** Take downward as positive. Then $a = g = 32$ ft/s^2 and $v_0 = -40$ ft/s.

$$s = v_0 t + \frac{1}{2}at^2 = -40\,(20) + \frac{1}{2}\,(32)\,(20)^2$$

$$= -800 + 6400 = 5600 \text{ ft.}$$

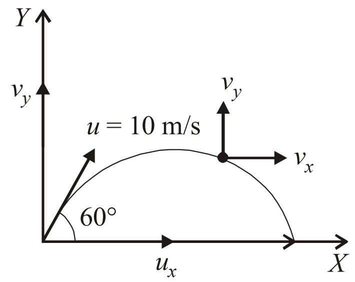
The balloon was 5600 ft above the ground.

74. **(b)** Let the required time be t. Then

$v_x = u\cos 60° = 5$ m/s, $v_y = u\sin 60° - 10t$

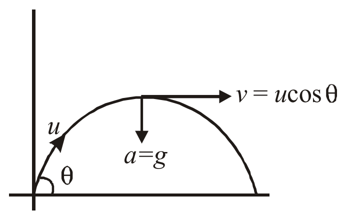
$v^2 = (u\sin 60° - 10t)^2 + (u\cos 60°)^2$

But $v = \dfrac{u}{2}$



$$\therefore\ \frac{u^2}{4} = \left(u\frac{\sqrt{3}}{2} - 10t\right)^2 + \frac{u^2}{4} \Rightarrow 10t = \frac{10\sqrt{3}}{2} \Rightarrow t = \frac{\sqrt{3}}{2}$$

75. **(d)** At the highest point, the vertical component of the velocity is zero but acceleration acts vertically downwards.

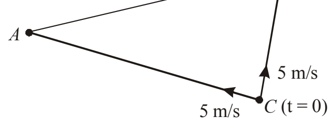


76. **(d)** It is clearly visible from all graphs that as x-increases velocity change sign. Since this is not possible, no graph represents the possible motion.

77. **(b)** The length of side CA at any time t is $= 20 - 5t$

The length of side CB at any time t is $= 10 - 5t$

At the instant A, B and C are collinear



$(20 - 5t) + (10 - 5t) = 20.$

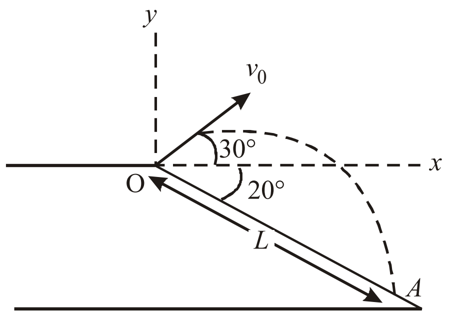
Solving we get $t = 1$.

Therefore, length of CA at $t = 1$ is $20 - 5 = 15$ m.

78. **(a)** We choose the launch point at the origin (figure). The equations of motion of the ball are

$x = v_0 \cos 30° t = (13.0 \text{ m/s})\, t$

$$y = v_0 \sin 30° - \frac{1}{2}gt^2 = (7.5 \text{ m/s})\, t - (4.9 \text{ m/s}^2)\, t^2$$



The equation of the straight line incline is

$y = -x \tan 20° = -0.364x$.

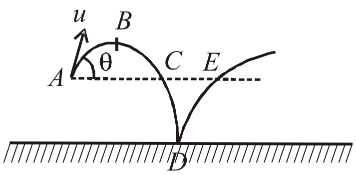
We want the time at which the (x, y) values for the ball satisfy this equation. We thus substitute the time expression for y and x.

$(7.5\text{ m/s})\, t - (4.9\text{ (m/s}^2))\, t^2 = -0.364\, [(13.0\text{ m/s})\, t]$

or $12.2t = 4.9t^2$.

The solutions are $t = 0$ (corresponding to $x = y = 0$) and $t = 2.49$s.

79. (a) Assume the wall to be absent. Let C and E be two points lying on trajectory at same horizontal level as point of projection.



Then the wall must be placed a distance $d = \frac{AE}{2}$ from A.

The maximum height of ball above ground at B is

$$H = 15 + \frac{10^2}{2 \times g} = 20\text{m}$$

$\therefore$ Time taken to fall from B to C is given by

$$5 = \frac{1}{2} g t_1^2 \quad \text{or} \quad t_1 = 1 \text{ sec.}$$

Time taken to fall from B to D is $t_2 = \sqrt{\frac{2 \times 20}{10}} = 2$ sec.

$\therefore$ Time taken by projectile to move from A to $E = 4$ sec.

Hence $2d = u \cos\theta \times 4 = 40$ or $d = 20$m.

80. (b) From $t = 0$ sec to $t = t_0$ second, the speed decreases and after $t = t_0$ second the speed increases. Hence particle first decelerates and then accelerates.

81. (b) Horizontal velocity $v_x = u_x = 18$ m/s

Now, $\tan 45° = \frac{v_y}{v_x}$

$\therefore\ v_y = v_x = 18$ m/s

82. (c) In 2 sec, horizontal distance travelled by bomb

$= 20 \times 2 = 40$m

In 2 sec, vertical distance travelled by bomb

$= \frac{1}{2} \times 10 \times 2^2 = 20$m

In 2 sec, horizontal distance travelled by Hunter

$= 10 \times 2 = 20$m

Time remaining for bomb to hit ground

$$= \sqrt{\frac{2 \times 80}{10}} - 2 = 2 \text{ sec}$$

Let v_x and v_y be the velocity components of bullet along horizontal and vertical direction.

Thus we have,

$$\frac{2v_y}{g} = 2 \Rightarrow v_y = 10\text{m/s and } \frac{20}{v_x - 20} = 2$$

$\Rightarrow\ v_x = 30$ m/s

Thus velocity of firing is $v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{10}$ m/s

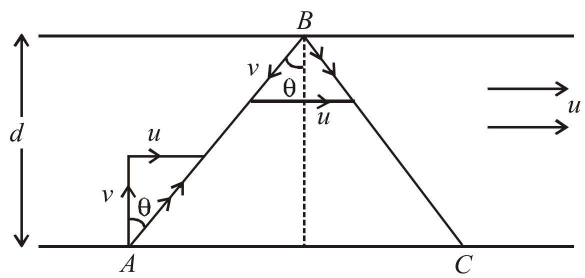
83. (a) As horizontal component of velocity remains constant. Hence, magnitude of change in velocity = magnitude of change in vertical component of velocity

$= |v_y - u_y| = gt = 10 \times 0.5 = 5$ m/s

84. (b) $y = x\tan\theta\left(1 - \frac{x}{R}\right) \Rightarrow \frac{R}{4} = \frac{3R}{4}\tan\theta\left(1 - \frac{3R}{4R}\right)$

$$\Rightarrow 1 = 3\tan\theta\left(\frac{1}{4}\right) \Rightarrow \tan\theta = \frac{4}{3} \Rightarrow \theta = 53°$$

85. (a)



v = velocity of man w.r.t. river

u = velocity of river

$$A \xrightarrow{t} B = \frac{d}{v} \Rightarrow 10 = \frac{d}{v} \Rightarrow d = 10v \qquad \text{........ (1)}$$

$$B \xrightarrow{t} C = \frac{d}{v\cos\theta}$$

$$\Rightarrow 15 = \frac{d}{v\cos\theta} \Rightarrow d = 15v\cos\theta \qquad \text{....... (2)}$$

Eq. (1) and (2) $\Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \sec\theta = \frac{3}{2}$

$\because\ \tan\theta = \frac{u}{v} \qquad \therefore\ \sqrt{\sec^2\theta - 1} = \frac{u}{v}$

$$\Rightarrow \frac{u}{v} = \sqrt{\frac{9}{4} - 1} = \frac{\sqrt{5}}{2} \Rightarrow \frac{v}{u} = \frac{2}{\sqrt{5}}$$

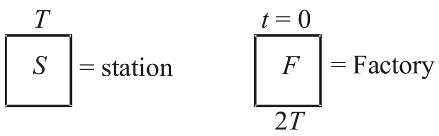
86. **(d)** $v_m = (v_x)_{ball} = v \cos 60° = v/2$

87. **(c)** For $a–t$ curve, area under the graph gives change in velocity at $t = 10$ sec, $v = 40$ m/s
For $10 – 30$ sec, $\Delta v = – 80$, $v_{30sec} – 40 = –80$
Speed at 30 sec $= – 40$ m/s

88. **(b)** If component of velocity normal to incline are equal, time of flight is same. Also if horizontal components are equal, range on inclined plane will be equal for both.

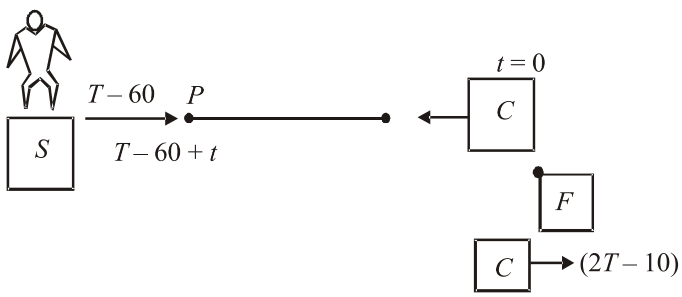
89. **(a)** In the figures $S \to$ Station, $F \to$ Factory and P is the place where he meets the car.

Usual day :



Car starts from F at $t = 0$, reaches station at T and again reaches at the factory at time $2T$.

This day :



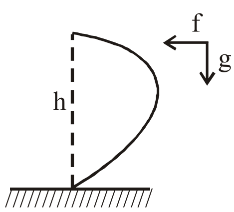
Person reaches S at $T – 60$. Car starts at $t = 0$ from F. Person walks for time t and reaches point P at time $T – 60 + t$. At this time car also reaches P. Car comes back at F at time $(2T – 10)$. That means car takes time $T – 5$ from F to P.
That means car reach at P at time $T – 5$.
Now $T – 5 = T – 60 + t \Rightarrow t = 55$ min

90. **(c)** With respect to lift initial speed $= v_0$
$a = – 2g$, displacement $= 0$

$$\therefore \; s = ut + \frac{1}{2}at^2 \;\; \Rightarrow \; 0 = v_0 T' + \frac{1}{2} \times 2g \times T'^2$$

$$\therefore \; T' = \frac{v_0}{g} = \frac{1}{2} \times \frac{2v_0}{g} = \frac{1}{2}T$$

91. **(a)**



Time taken to reach the ground is given by

$$h = \frac{1}{2}gt^2 \qquad \text{..........(1)}$$

Since horizontal displacement in time t is zero

$$\therefore \; t = \frac{2v}{f} \qquad \text{.......... (2)}$$

From eq. (1) and (2)

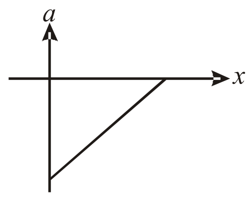
$$h = \frac{2gv^2}{f^2}$$

92. **(a)** The linear relationship between v and x is
$v = – mx + c$ where m and c are positive constants.

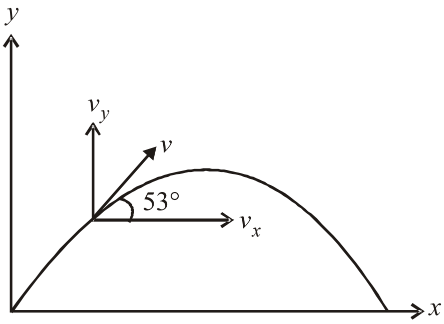
$$\therefore \; \text{Acceleration, } \; a = v\frac{dv}{dx} = -m\,(-mx + c)$$

$$\therefore \; a = m^2x – mc$$

Hence the graph relating a to x is



93. **(b)**



$$\tan 53° = \frac{v_y}{v_x}$$

$$\tan 53° = \frac{u\sin\theta - g\left(\frac{T}{2} - 2\right)}{u\cos\theta}$$

$$\Rightarrow \frac{4}{3} = \frac{u\sin\theta - 10\left(\frac{u\sin\theta}{g} - 2\right)}{u\cos\theta}$$

$$\Rightarrow \frac{4}{3} = \frac{u\sin\theta - u\sin\theta + 20}{u\cos\theta} \Rightarrow 4u\cos\theta = 60$$

$$\Rightarrow u\cos\theta = \frac{60}{4} = 15 \;\Rightarrow u\cos\theta = 15$$

At maximum height vertical component of velocity is zero so velocity at maximum height $= u\cos\theta = 15$m/sec

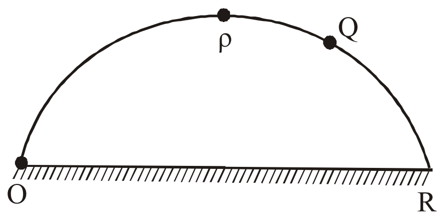
94. **(b)** Initial velocity v_1 must be greater than v_0 if it has to reach same height and finally while coming back it will approach a terminal velocity.

B COMPREHENSION TYPE

1. (a) $H_A = H_C > H_B$

Obviously A just reaches its maximum height and C has crossed its maximum height which is equal to A as u and θ are same. But B is unable to reach its maximum height.

2. (c)



Time of flight of A is 4 seconds which is same as the time of flight if wall was not there.

Time taken by B to reach the inclined roof is 1 sec.

$T_{OR} = 4$

$T_{QR} = 1$

$\therefore T_{OQ} = T_{OR} - T_{QR} = 3$ sec.

3. (c) From above $T = \dfrac{2u\sin\theta}{g} = 4\text{s}$

$\therefore u\sin\theta = 20\text{m/s}$

$\Rightarrow$ vertical component is 20 m/s

for maximum height

$v^2 = u^2 + 2as \Rightarrow 0^2 = 20^2 - 2 \times 10 \times s$

$\Rightarrow s = 20\text{m}$

4. (a) **5. (c)** **6. (b)**

$T = \dfrac{100}{20} = 5$ sec.

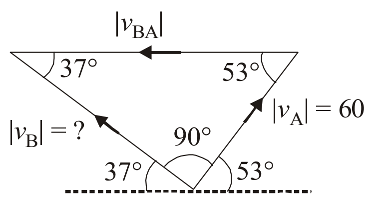
$v = \sqrt{2gH} = \sqrt{2 \times 10 \times 5} = 10$ m/sec.

$\therefore \quad T - t = \sqrt{\dfrac{2H}{g}} = \sqrt{\dfrac{2 \times 5}{10}} = 1$ sec.

$\therefore \quad t = 4$ sec.

$\therefore \quad \dfrac{1}{2}at^2 + at(T - t) = 100 \Rightarrow a = \dfrac{25}{3}$

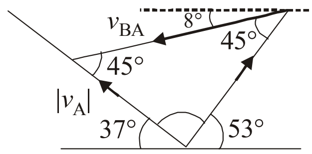
7. (a) Velocity triangle



$$\frac{60}{\sin 37°} = \frac{|v_B|}{\sin 53°}$$

$$\Rightarrow \quad |v_B| = \frac{60 \times \frac{4}{5}}{3/5} = 80 \text{ m/s.}$$

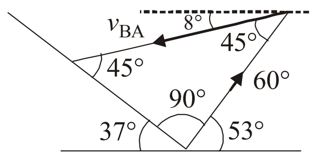
8. (c) Velocity triangles



$$\frac{v_B}{\sin 45°} = \frac{60}{\sin 45°}$$

$v_B = 60$ m/s.

9. (d)



$$\frac{|v_{BA}|}{\sin 90°} = \frac{60}{\sin 45°}$$

$\Rightarrow \quad |v_{BA}| = 60\sqrt{2},$

Particle B has to travel 100 cos 8° (w.r.t. A)

$$t = \frac{100\cos 8°}{60\sqrt{2}} = \frac{5}{3\sqrt{2}}\cos 8° \text{ second}$$

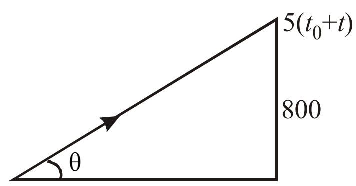
10. (b) $t = \dfrac{s_x}{u_x} = \dfrac{1600}{400\cos\theta} = \dfrac{1600}{400 \times \frac{4}{5}} = 5\,\text{sec}.$

11. (a) $H = (s_y)_{bullet} = v_y t - \dfrac{1}{2}a_y t^2$

$= 400 \times \dfrac{3}{5} \times 5 - \dfrac{1}{2} \times 10 \times 5^2$

$= 1200 - 125 = 1075\text{m}$

12. (b) Let t_0 be the waiting time.



$$t = \frac{1600}{400\cos\theta} = 5 \text{ sec}$$

$800 + 5\,(t_0 + 5)$

$$= 400 \times \frac{3}{5} \times 5 - \frac{1}{2} \times 10 \times 5^2 = 1200 - 125 = 1075\text{m}.$$

$800 + (t_0 + 5) \times 5 = 1075$

$(t_0 + 5)\,5 = 275$;

$t_0 = 55 - 5 = 50$ sec

13. (b) **14. (b)** **15. (a)**

$$d_x = \frac{d}{2} + (d + d + d + d + d) = \frac{11d}{2} \qquad \text{..........(1)}$$

Total $d_x = 12d = \dfrac{u^2 \sin 2\theta}{g}, d = 20m$

Now, $\dfrac{11d}{2} = \dfrac{11 \times 20}{2} = 110m = u\cos\theta.t$

$\Rightarrow t = \dfrac{110}{30} = \dfrac{11}{30}$ sec.

So H of wall

$$= H_{max} = u\sin\theta t - \frac{1}{2}gt^2 = \frac{11}{3}\left[\frac{40 \times 6 - 110}{6}\right]$$

$= 79.44$ m

$$m\,(ucos\theta - (-u\cos\theta)) = 2 \times 50 \times \frac{3}{5} = 60\text{N-s}$$

16. (a) **17. (b)** **18. (b)**

At t_1 car A overtaking car B

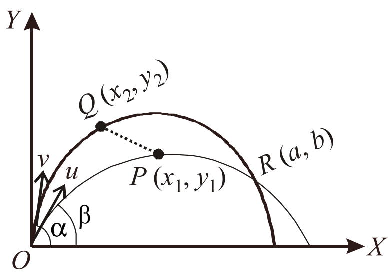
B has more slope than A

Same slope.

19. **(a)** Let O be the point of projection, $P\,(x_1, y_1)$ and $Q\,(x_2, y_2)$ be the two particles at time t.

Then, $x_1 = u\cos\alpha\, t, \quad y_1 = u\sin\alpha\, t - \dfrac{1}{2}gt^2$

$x_2 = v\cos\beta\, t, \quad y_2 = v\sin\beta\, t - \dfrac{1}{2}gt^2$



$\therefore$ Slope of $PQ = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{u\sin\alpha - v\sin\beta}{u\cos\alpha - v\cos\beta}$

independent of t, hence same for all times i.e., PQ remains parallel to itself.

20. **(b)** If θ_1, θ_2 be the angles that directions of motion make at time t,

$\tan\theta_1 = \dfrac{u\sin\alpha - gt}{u\cos\alpha}$ and

$\tan\theta_2 = \dfrac{v\sin\beta - gt}{v\cos\beta}$

Hence, when $\theta_1 = \theta_2$ we have

$$\frac{u\sin\alpha - gt}{u\cos\alpha} = \frac{v\sin\beta - gt}{v\cos\beta}$$

$$\therefore\ t = \frac{uv\sin(\alpha - \beta)}{g\,(v\cos\beta - u\cos\alpha)}$$

21. **(a)** The paths of the two particles are

$$y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$$

and $y = x\tan\beta - \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\beta}$

If $R\,(a, b)$ be the common point then

$b = a\tan\alpha - \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\alpha}$ and

$b = a\tan\beta - \dfrac{1}{2}\dfrac{ga^2}{v^2\cos^2\beta}$

Hence $a\tan\alpha - \dfrac{1}{2}\dfrac{ga^2}{u^2\cos^2\alpha} = a\tan\beta - \dfrac{ga^2}{2v^2\cos^2\beta}$

i.e., $\tan\alpha - \tan\beta = \dfrac{1}{2}ga\left(\dfrac{1}{u^2\cos^2\alpha} - \dfrac{1}{v^2\cos^2\beta}\right)$ ..... (1)

If t_1, t_2 be their times to pass through the common point then a $= u\cos\alpha.t_1$; also $a = v\cos\beta\, t_2$

$$\therefore\ t = t_1 - t_2 = a\left(\frac{1}{u\cos\alpha} - \frac{1}{v\cos\beta}\right) \qquad \text{.........(2)}$$

Hence dividing eq. (1) by eq. (2), we get

$$\frac{\tan\alpha - \tan\beta}{t} = \frac{1}{2}g.\left(\frac{1}{u\cos\alpha} + \frac{1}{v\cos\beta}\right)$$

$$= \frac{g\,(u\cos\alpha + v\cos\beta)}{2uv\cos\alpha\cos\beta}$$

Hence, $t = \dfrac{2uv\sin(\alpha - \beta)}{g\,(u\cos\alpha + v\cos\beta)}$

22. (b) **23. (d)**

$A_1 = 4 \times 1.5 = 6,$

$A_2 = 2 \times t' = 2t'$

$t' = 3$; $t = 3 + 4 = 7s$

24. **(b)** Distance travelled by the car in 10 seconds is equal to displacement in 10 seconds and it is same as area under the *v-t* curve.

$$\therefore \text{ Distance} = \frac{1}{2}\times 2\times 10 + 10\times 5 + \frac{1}{2}\times 3\times 10$$

$$= 10 + 50 + 15 = 75\text{m}.$$

25. **(c)** Retardation $= \dfrac{10\text{ms}^{-1}}{1} = 10\text{ms}^{-2}$

$\therefore$ Braking force $= ma$

$= 1000 \times 10 = 10000$ N

26. **(d)** From *v-t* graph we can analyze in $t = 0$ to $t = 2$ sec slope is positive and constant. Hence acceleration is positive and constant and it is $\dfrac{10}{2}$ = 5 ms^{-2}.

Between $t = 2$ to $t = 7$, slope is zero, so the acceleration is zero. Between $t = 7$ to $t = 10$, slope is negative and constant. Hence acceleration is negative and constant and its value is $-\dfrac{10}{3}\text{m}/\text{s}^2$.

27. **(d)** In this case, the projectile hits the back of the truck at the moment of overtaking it, which is the moment at which the distance of the back of the truck, $x_1 = 45 + 15t$, equals the horizontal distance of the projectile,

$x = (v_0 \cos\theta)\, t = 32.22t.$

Thus $t = \dfrac{45}{32.22 - 15} = 2.614\text{s}.$

28. **(c)** At t = 2.614s, when the projectile overtakes the back of the truck, faster its height is, noting $v_0 \sin\theta = 13.67$ m/s,

$y = (13.67)(2.614) - \frac{1}{2}(9.8)(2.614)^2 = 2.25\text{m}$, i.e. 25cm.

above the top of the truck. Since the projectile travels horizontally than does the truck, it is clear that thereafter the projectile remains ahead of the back of the truck, and so never hits the back.

The projectile will reach (for the second time) a height of 2m in a total time t_2 given by

$2 = (13.67)\, t_2 - \frac{1}{2}(9.8)\, t_2^2$, or $t_2 = 2.635$s, that is

$2.635 - 2.614 = 0.021$s after overtaking the back of the truck. Thus the projectile hits the top of the truck of a distance of $(32.22 - 15)(0.021) = 0.36\text{m} = 36\text{cm}$. in front of the rear edge.

29. **(a)** The time taken to overtake the back of the truck is given by

$45 + 15t = (v_0 \cos\theta)\, t$

or $t = \dfrac{45}{v_0\cos\theta - 15}$

At this time

$$y = 3 = (v_0 \sin\theta)\, t - \frac{1}{2}(9.8)\, t^2$$

$$= (v_0\sin\theta)\left(\frac{45}{v_0\cos\theta - 15}\right) - \frac{1}{2}(9.8)\left(\frac{45}{v_0\cos\theta - 15}\right)^2$$

Inserting the numerical values of sin θ and cos θ we obtain the following quadratic equation for v_0:

$v_0^2(4.55) - v_0(60.3) - 3532 = 0.$

Solving, $v_0 = 35.3$ m/s

30. **(b)** $a_y = \dfrac{dv_y}{dt} \Rightarrow dv_y = a_y dt$

$$\therefore \int_0^{v_y} dv_y = \int_0^{t_b} a_y dt = \int_0^{t_b} (-g + \alpha g - \beta t^4)\, dt$$

$$\therefore v_y - 0 = [\alpha - 1]\, gt - \frac{1}{5}\beta t^5 \qquad [\because v_y(t=0) = 0]$$

$$\therefore v_y = (\alpha - 1)\, gt - \frac{1}{5}\beta t^5 \qquad \text{..........(1)}$$

31. **(d)** $\displaystyle\int_{v_y(t=t_b)}^{v_y} dy = \int_{t_b}^{t} a\, dt$

$$\therefore v_y - v_y(t = t_b) = \int_{t_b}^{t} (-g)\, dt = -g\,(t - t_b)$$

$$\therefore v_y = v_y(t = t_b) - g\,(t - t_b)$$

$$= (\alpha - 1)\, gt_b - \frac{1}{5}\beta t_b^5 - g\,(t - t_b)$$

[Using (1)]

$$= \frac{4}{5}\alpha g t_b - gt$$

32. **(b)** For maximum height, $v_y = 0$ (at $t = t_f > t_b$)

$$\therefore 0 = \frac{4}{5}\alpha g t_b - g t_f$$

$$\Rightarrow t_f = \frac{4}{5}\alpha t_b$$

33. **(a)** From *v-t* graph $\dfrac{dv}{dt}$ = tan 45° = 1

34. **(b)** From *v-t* graph velocity at $t = 2$ sec., $v = -1$m/sec

acceleration at $t = 2$ sec is $a = 1$ m/s^2

so do not product of v and $a = -1\text{m}^2/\text{s}^3$

35. **(c)** From *v-t* graph

$$\int_{t=2}^{t=5} v\, dt = \frac{1}{2}(-1) + \frac{1}{2}\times 2\times 2 = -\frac{1}{2} + 2 = 1.5 \text{ metre}$$

36. (d) $\frac{vdv}{dx} = 9x \Rightarrow \int_6^v vdv = \int_0^2 -9x\,dx$

$\frac{v^2 - 6^2}{2} = -18$ or $v = 0$

37. (b) Since $v = 0$ at $x = 2$

Hence maximum distance of particle from origin will be $x = 2$m.

38. (d) Quadratic relation hence parabola.

39. (d) **40. (a)** **41. (a)**

The velocity vector at time t is

$\vec{v} = \vec{u} + \vec{a}t = -12\hat{i} + 16\hat{j} + (6\hat{i} - 8\hat{j})\,t$ ..... (1)

Solving for $\vec{v} = 0$ we get $t = 2$ sec.

The position vector at any time t is

$\vec{r} = r_0 + \vec{u}t + \frac{1}{2}at^2$

$= (-12\hat{i} + 16\hat{j})\,t + \frac{1}{2}(6\hat{i} - 8\hat{j})\,t^2$ ...... (2)

Solving for $\vec{r}.\vec{v} = 0$ we get $t = \frac{8}{3\sqrt{5} - 5}$ sec.

Putting t in eq. (1) we get

$|\,\vec{v}\,| = 12\sqrt{5}\ m/s$

The x-coordinate as function of time is

$x = x_0 + u_x t + \frac{1}{2}a_x t^2 = -12t + 3t^2$

Solving for $x = 0$ we get $t = 5$ sec.

C REASONING TYPE

1. (a) When a body is thrown upwards vertically, at the highest point its velocity becomes zero but gravitational force continuous to act on it so it has acceleration in downward direction even at the highest point. So statement-1 is true.

A body is numerically at rest but it reverses its direction due to acceleration present in it. Statement-2 is true & it supports statement-1.

2. (a) Retardation $= \frac{\text{decrease in velocity}}{\text{time}}$

It acts opposite to velocity.

3. (a) The distance covered in the last second, final velocity becomes zero. So if we drop an object with zero velocity it will cover the same distance in one second while going downwards.

Now distance travelled in the later case

$S = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times 1$

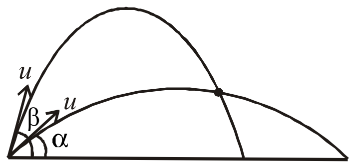
$S = 5$m

4. (c) Statement – 1 is true and Statement – 2 is false because it is also possible that when position vector and velocity vector are perpendicular, the distance from the point of projection not a maximum.

5. (a) Since W = Δ K implies that the final speed will be same.

6. (a) The time of flight depends only on the vertical component of velocity which remains unchanged in collision with a vertical wall.

7. (d)



Trajectories may intersect at some point but not at the same instant

8. (d) Linear momentum during parabolic path changes continuously.

9. (a) Acceleration of all three projectiles = g

Relative acceleration = 0

10. (a) $t = \frac{L}{v\cos\theta}$,

θ angle between boat velocity relative to river and normal.

11. (c) In uniform circular motion speed is constant but acceleration exists due to change in direction of velocity. So that statement-2 is wrong. If acceleration is 0 then velocity is constant. Hence the magnitude of the velocity will be constant. Hence speed will be constant.

12. (d) Statement-1 is false because angles of projection θ and $(90° - \theta)$ give same range but time of flight will be different. Statement-2 is true because in horizontal direction acceleration is zero.

13. (a) Statement-1 is true on the basis of our personal experience, statement-2 is true and explain statement-1 properly.

14. (d) Average velocity of particle moving with constant acceleration in time interval $(t_1 + t_2)$ is equal to the average velocity in time interval $(t_1 + \Delta t)$ to $(t_2 + \Delta t)$.

15. (d) For a projectile, acceleration is constant and hence does not depend on velocity.

16. (a) For a stone projected vertically upwards, the speed v is least at highest point even though $\frac{dv}{dt}$ is constant.

Hence statement-1 is true.

17. **(a)** Both statements are true and statement-2 is a correct explanation for statement-1.

18. **(b)** Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

19. **(a)** In statement-2, if speed of both projectiles are same, horizontal ranges will be same. Hence statement-2 is correct explanation of statement-1.

20. **(c)** The motion of particle starting from rest is always along a straight line if and only if direction of acceleration is fixed (constant).

21. **(d)** Statement-1 is wrong because on earth, the acceleration is g which is downwards. Statement-2 is true.

22. **(a)** Both statements are true and statement-2 is a correct explanation for statement-1.

23. **(b)** Both statements are correct and independent.

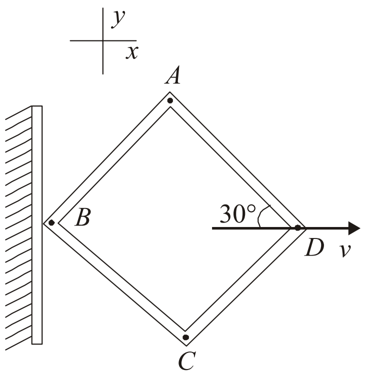
D MULTIPLE CORRECT CHOICE TYPE

1. **(a, b)** $v_x = v/2$

$v_y = v_x \cot 30°$

$v_y = \frac{v}{2}\sqrt{3}$

$v_A = v$



2. **(a, d)** At $x = 0$, $v_y = 0$

$v_x = v$ and $a_x = 0$ [$\because$ particle moves uniformly)

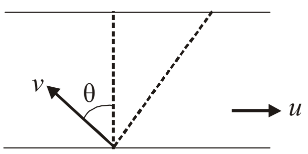
$v_y = 2k\,x\,v_x$

$a_y = 2k\left[v_x^2 + x a_x\right]$

$\therefore\ a_y = 2k\,v^2$

$\hat{a} = 2kv^2\,\hat{j}$

3. **(a, c)** $t = \dfrac{d}{v\cos\theta}$



$x = (u - v\sin\theta)\,t$

$= \dfrac{ud}{v}\sec\theta - d\tan\theta$

For x to minimum

$\dfrac{dx}{d\theta} = 0 \Rightarrow \sin\theta = \dfrac{v}{u}$

4. **(a, b)** $v_0 = 60$ m/s at 36.87°

$\therefore\ v_{ox} = 48$m/s and $v_{oy} = 36$ m/s

$\therefore\ \vec{v} = 48\hat{i} + 36\hat{j} + 0\hat{k}$ and $\vec{a} = -3.6\hat{i} - 6\hat{j} + 0\hat{k}$

$$\therefore\ \vec{v}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 48 & 36 & 0 \\ -3.6 & -6 & 0 \end{vmatrix} = (-158.4)\,\hat{k}$$

$\therefore\ |\vec{v}\times\vec{a}| = 158.4$

$\therefore\ R = \dfrac{v^3}{|\vec{v}\times\vec{a}|} = \dfrac{60^3}{158.4}$

$\therefore\ R = 1363.64$m at the start.

[$R = \dfrac{v^3}{|\vec{v}\times\vec{a}|}$ is basically $R = \dfrac{v^2}{a_c}$; $a_c = \dfrac{v^2}{R}$]

Now at the top, $v_x = v_{ox} = 48$ m/s, $v_y = 0$

$\therefore\ \vec{v} = 48\hat{i} + 0\hat{j} + 0\hat{k}\quad \therefore\ v = 48$ m/s

$$\vec{v}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 48 & 0 & 0 \\ -3.6 & -6 & 0 \end{vmatrix} = (-288)\,\hat{k}$$

$\therefore\ |\vec{v}\times\vec{a}| = 288\quad \therefore\ R = \dfrac{v^3}{|\vec{v}\times\vec{a}|} = \dfrac{48^3}{288} = 384m$

$\therefore\ R = 384$m at the top

5. **(b, c)** $v = 0 - 9 + t^2$

$a = 0 - 0 + 2t$

At $t = 3$, $v = 0$, $a \neq 0$

6. **(a, c)** Time taken to reach x-y plane is given by

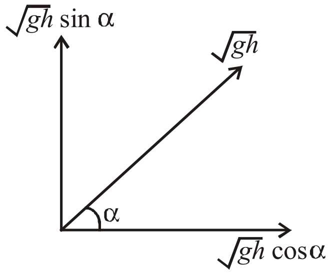
$\frac{1}{2}gt^2 = 5 \Rightarrow t = 1$ sec

Initial horizontal velocity of bob is $3\hat{j}$

$y = 3 \times 1 = 3$m

$\therefore\ x = 2$m, $y = 3$m

7. **(b, c, d)**



$$T = \frac{2\sqrt{gh}\sin\alpha}{g} = \frac{c}{\sqrt{gh}\cos\alpha} + \frac{c}{e\sqrt{gh}\cos\alpha}$$

$$\Rightarrow \frac{2(gh)\sin\alpha\cos\alpha}{gc} - 1 = \frac{1}{e}$$

$$\Rightarrow e = \frac{c}{h\sin 2\alpha - c}$$

8. **(a, b)** Equation of motion is

$$\frac{dv}{dt} = -\mu v^3 \text{ i.e. } -\frac{dv}{v^3} = \mu\, dt$$

Integrating it, we have $\frac{1}{v^2} = 2\mu t + A$ .... (1)

Initially, when $t = 0$, $v = V$

$$\therefore \quad A = \frac{1}{V^2}$$

Hence, equation (1) becomes

$$\frac{1}{v^2} = 2\mu t + \frac{1}{V^2} = \frac{1 + 2\mu V^2 t}{V^2}$$

i.e. $v = \frac{dx}{dt} = \frac{V}{\sqrt{(1 + 2\mu V^2 t)}}$

This proves the second option.
Integrating it, we have

$$x = V\int (1 + 2\mu V^2 t)^{-1/2} dt + D$$

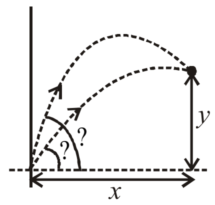
$$= \frac{1}{\mu V}\sqrt{(1 + 2\mu V^2 t)} + D$$

Initially when $t = 0, x = 0$

$$\therefore \quad D = -\frac{1}{\mu V}$$

So that, we have $x = \frac{1}{\mu V}[\sqrt{(1 + 2\mu V^2 t)} - 1]$

9. **(a, b, d)**



$x = u\cos\alpha t = u\cos\beta\,(t - T)$;

$\cos\alpha < \cos\beta \Rightarrow \alpha > \beta$

$$y = u\sin\alpha t - \frac{1}{2}gt^2 = u\sin\beta\,(t - T) - \frac{1}{2}g\,(t - T)^2$$

10. **(b, d)** $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 28.56°$

Let v_0 = initial velocity of auto
$s_x = v_{ox} \times t$
$40 = v_0 \cos 28.56° \times t$

$t = \frac{45.54}{v_0}$ .... (1)

Also $s_y = v_{oy}t - \frac{1}{2}gt^2 = v_0 \sin 28.56° \times t - \frac{1}{2}gt^2$

$$s_y = v_0 \sin 28.5° \times \frac{45.54}{v_0} - \frac{1}{2} \times 32.2 \times \left(\frac{45.54}{v_0}\right)^2$$

= – 10 (given)
Solving this, $v_0 = 32.76$ ft/sec.
$v_{ox} = 32.76 \cos 28.56° = 29.3$ ft/sec
$v_{oy} = 32.76 \sin 28.56° = 15.66$ ft/sec
Also

$$v_y^2 = v_{0y}^2 + 2a_y s_y = (15.66)^2 - 2\,(32.2)\,(-10)$$

$v_y = 29.3$ ft/s, $v_x = 29.3$ ft/s ; $\alpha = \tan^{-1}\frac{v_y}{v_x} = 45°$

11. **(a, b, c, d)**

(a) $-20 = 15t - 5t^2$
$t = -1, t = 4$

(b) $0 = 15t - 5t^2$; $0 = 5t\,(3 - t) = 0$;
$t = 3$

(c) $v_y^2 = (15)^2 + 2 \times 10 \times 20$

$v_y = (625)^{1/2} = 25$; $v_x = 15 \times \sqrt{3}$;

$$\tan\beta = \frac{v_y}{v_x} = \frac{25}{15 \times \sqrt{3}} = \frac{5}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$\tan\beta = \frac{5\sqrt{3}}{9}$; $\beta = \tan^{-1}\left(\frac{5\sqrt{3}}{9}\right)$

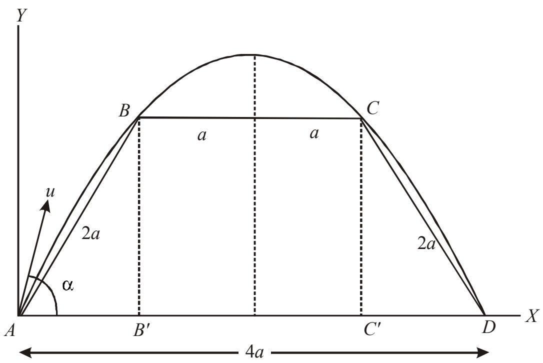
(d) $x = v_x \times t$; $D = 15\sqrt{3} \times 4 = 60\sqrt{3}\ m$

12. (b, c) Let $ABCD$ be the given trapezium whose sides are as follows :
$AD = 4a, AB = BC = CD = 2a$.
From B and C draw BB' and CC' perpendiculars to AD. Then $B'C' = 2a$

$$\therefore AB' = C'D = \frac{1}{2}(2a) = a$$

$$BB' = \sqrt{AB^2 - AB'^2} = \sqrt{4a^2 - a^2} = a\sqrt{3}$$

Let u and α be the velocity and angle of projection of the ball from the point A. Take A as origin and the horizontal line AD and vertical line through A as coordinate axes. Then the coordinates of B are $(a, a\sqrt{3})$.



Also the equation of the trajectory of the ball is

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$\because B = (a, a\sqrt{3})$ lies on it, so we have

$$a\sqrt{3} = a\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha} \quad \text{..........(1)}$$

Also horizontal range $= AD$

$$\text{or } \frac{2u^2\sin\alpha\cos\alpha}{g} = 4a$$

$$\text{or } \frac{u^2\sin\alpha\cos\alpha}{g} = 2a \quad \text{..........(2)}$$

Substituting the values of u^2 from (2) in (1) we get

$$\sqrt{3} = \tan\alpha - \frac{ga\sin\alpha\cos\alpha}{4ag\cos^2\alpha} = \tan\alpha - \frac{1}{4}\tan\alpha$$

$$\text{or } \tan\alpha = \frac{4}{3}\sqrt{3} = \frac{4}{\sqrt{3}} \quad \text{..........(3)}$$

$\therefore$ From (1), we get

$$\sqrt{3} = \frac{4}{\sqrt{3}} - \frac{ga}{2u^2\cos^2\alpha}$$

$$\text{or } \frac{ga}{2u^2\cos^2\alpha} = \frac{4}{\sqrt{3}} - \sqrt{3} = \frac{1}{\sqrt{3}}$$

$$\text{or } u^2\cos^2\alpha = \frac{1}{2}ga\sqrt{3} \quad \text{..........(4)}$$

From (2), we have $(u^2\sin^2\alpha)\dfrac{u^2\cos^2\alpha}{g^2} = 4a^2$

[squaring both sides of (2)]

$$\text{or } u^2\sin^2\alpha = \frac{4a^2g^2}{u^2\cos^2\alpha} = \frac{8a^2g^2}{ag\sqrt{3}} = \frac{8ag}{\sqrt{3}}$$

$$\therefore \text{ Time of flight} = \frac{2u\sin\alpha}{g} = \sqrt{\frac{4u^2\sin^2\alpha}{g^2}}$$

$$= \sqrt{\frac{4\times 8ag}{g^2\sqrt{3}}} = \sqrt{\frac{32a}{g\sqrt{3}}}$$

Maximum height attained by the ball

$$= \frac{u^2\sin^2\alpha}{2g} = \frac{8ga}{2g\sqrt{3}} = \frac{4a}{\sqrt{3}}$$

13. (a, b) As the ship is moving horizontally with a velocity u in a direction opposite to that of the projection from the gun and the horizontal and vertical components of velocity of projection relative to the gun are $v\cos\alpha$ and $v\sin\alpha$ respectively, so initially the actual horizontal components of velocity $= v\cos\alpha - u$ and the vertical component of velocity $= v\sin\alpha$.
Also we know that horizontal range of a particle $= (2/g)$ (horizontal comp. of velocity)
$\times$ (initial vertical comp. of velocity)
$\therefore$ If R be the required range, then
$R = (2/g)(v\cos\alpha - u)(v\sin\alpha)$ .... (1)
Now if R is maximum, then $dR/d\alpha = 0$ and $d^2R/d\alpha^2 = -\text{ve}$.
From (1),

$$\frac{dR}{d\alpha} = (2g)\,[(v\cos\alpha - u)(v\cos\alpha) + (v\sin\alpha)(-v\sin\alpha)]$$

$$= (2/g)\,[v^2(\cos^2\alpha - \sin^2\alpha) - uv\cos\alpha]$$

If $\dfrac{dR}{d\alpha} = 0$, then

$v^2(\cos^2\alpha - \sin^2\alpha) - uv\cos\alpha = 0$
or $v(2\cos^2\alpha - 1) - u\cos\alpha = 0$,

$\because \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$
or $2v\cos^2\alpha - u\cos\alpha - v = 0$

$$\text{or } \cos\alpha = \frac{u \pm \sqrt{u^2 + 8v^2}}{4v}$$

The negative sign before the radical renders the value of cos α negative i.e. α is obtuse which is against the problem.

Hence, $\cos\alpha = \left[\dfrac{u+\sqrt{u^2+8v^2}}{4v}\right]$

14. (a, b, c) Train reaches the farthest point at $t = 40$ sec. Acceleration in the interval $50 < t < 80$ (t in sec) is

$$|a| = \frac{\Delta v}{\Delta t} = \left|\frac{-2}{30}\right| = \frac{1}{15}\text{ m/s}^2$$

Area under curve from $t = 0$ to 40 sec.

$$= 30\times 3 + \frac{1}{2}\times 10\times 3 = 90 + 15 = 105\text{m.}$$

Area from $t = 50$ to 120 sec. $= \dfrac{1}{2}\times 70\times 2 = 70$m.

Distance from $A = [105 - 70] = 35$m.

15. (a, b) The equation of motion is

$$\frac{d^2x}{dt^2} = -\mu\left(\frac{a^5}{x^2}\right)^{1/3}$$

or $v\dfrac{dv}{dx} = -\dfrac{\mu a^{5/3}}{x^{2/3}}$ $\qquad \left[\because \dfrac{d^2x}{dt^2} = v\dfrac{dv}{dx}\right]$

or $v\,dv = -\dfrac{\mu a^{5/3}}{x^{2/3}}$ dx

Integrating, $\dfrac{1}{2}v^2 = -3\mu a^{5/3}.x^{1/3} + C$, where C is constant.

Initially, $x = a$, velocity $= 0$,

$\therefore\ C = 3\mu a^{5/3}\,a^{1/3}$

$\therefore\ v^2 = 6\mu a^{5/3}\,(a^{1/3} - x^{1/3})$

or $(dx/dt)^2 = 6\mu a^{5/3}\,(a^{1/3} - x^{1/3})$ .... (1)

or $\dfrac{dx}{dt} = -\sqrt{6\mu a^{5/3}}.\sqrt{a^{1/3} - x^{1/3}})$

or $dt = -\dfrac{1}{\sqrt{6\mu a^{5/3}}}.\dfrac{dx}{\sqrt{a^{1/3} - x^{1/3}}}$

Required time from $x = a$ to O, where $x = 0$,

$$= -\frac{1}{\sqrt{6\mu a^{5/3}}}.\int_{x=a}^{0}\frac{dx}{\sqrt{a^{1/3} - x^{1/3}}}$$

$$= \frac{1}{\sqrt{6\mu a^{5/3}}}.\int_{x=0}^{a}\frac{dx}{\sqrt{a^{1/3} - x^{1/3}}}$$

$$= \frac{1}{\sqrt{6\mu a^{5/3}}}.\int_{\theta=0}^{\pi/2}\frac{6a\sin^5\theta\cos\theta\,d\theta}{\sqrt{a^{1/3}}\,\cos\theta}$$

putting $x = a\sin^6\theta$

$$= \frac{6a}{\sqrt{6\mu a^2}}\int_0^{\pi/2}\sin^5\theta\,d\theta = \frac{6}{\sqrt{6\mu}}.\left[\frac{4}{5}.\frac{2}{3}\right]$$

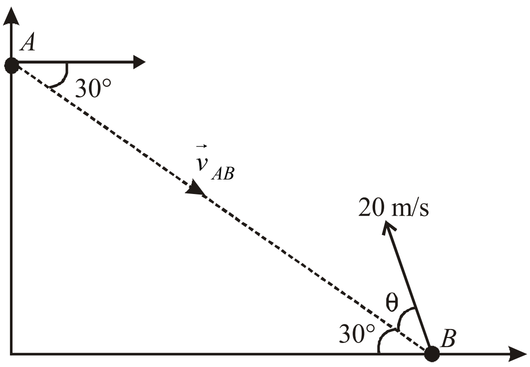
$$= \left[\frac{8}{15}\right]\sqrt{\frac{6}{\mu}}$$

Also v, the velocity at O i.e. at $x = 0$ is given by $v^2 = 6\mu a^{5/3}\,(a^{1/3})$, putting $x = 0$ in (1)

or $v = a\sqrt{6\mu}$

16. (c, d) (a) is wrong because $a - b < |\vec{a} + \vec{b}| < a + b$, (b) is wrong because in uniform circular motion, speed is constant but there exists acceleration, (c) is correct because by definition distance $\geq$ $\left|\overline{\text{displacement}}\right|$; (d) is correct because a_c exists.

17. (a, d)



Condition for collision in mid air

$\vec{a}_{AB} = 0$ and $\vec{v}_{AB}$ should be directed from A to B.

$\therefore\ \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$= 20\hat{i} - [-20\cos(\theta+30°)\,\hat{i} + 20\sin(\theta+30°)\,\hat{j}]$

$= [20 + 20\cos(\theta+30°)]\,\hat{i} - 20\sin(\theta+30°)\,\hat{j}$

$$\tan\theta = \frac{(v_{AB})_y}{(v_{AB})_x} = \frac{20\sin(\theta+30°)}{20+20\cos(\theta+30°)}$$

$1 + \cos(\theta+30°) = \sqrt{3}\sin(\theta+30°)$

$1 = \sqrt{3}\sin(\theta+30°) - \cos(\theta+30°)$

$\dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\sin(\theta+30°) - \dfrac{1}{2}\cos(\theta+30°)$

$\dfrac{1}{2} = \sin(\theta+30-30)$; $\sin\left(\dfrac{\pi}{6}\right) = \sin\theta$;

$\theta = \dfrac{\pi}{6} = 30°$

$\therefore\ \vec{v}_{AB} = (20 + 20\cos 60°)\,\hat{i} - 20\sin 60°\,\hat{j}$

$= 30\hat{i} - 10\sqrt{3}\hat{j}$

$|\vec{v}_{AB}| = \sqrt{(30)^2 + (10\sqrt{3})^2} = 10\sqrt{9+3} = 20\sqrt{3}\ \text{ms}^{-1}$

Time to collide $= \dfrac{\text{relative distance}}{\text{relative speed}}$

Time to collide $= \dfrac{200}{20\sqrt{3}} = \dfrac{10}{\sqrt{3}}$

Hence, answers are (a) and (d).

18. (b, c, d) $a = 6 - 2t = \dfrac{dv}{dt}$

For maximum velocity, $\dfrac{dv}{dt} = 0$

$6 - 2t = 0,\ t = 3$ sec.

$\dfrac{dv}{dt} = 6 - 2t \quad ; \quad \int_0^v dv = \int_0^t (6 - 2t)\, dt$

$v = 6t - t^2 = 18 - 9 = 9$ m/s

After 4 sec, $v = 6t - t^2 = 6 \times 4 - 16 = 24 - 16 = 8$ m/s

$\int_0^v dx = \int_0^t (6t - t^2)\, dt$

$x = 3t^2 - \dfrac{t^3}{3} = \dfrac{80}{3}$ (putting $t = 4$ sec.)

19. **(a, b, c, d)** Area of v-t graph $= \int_{t_i}^{t_f} v\, dt = \int_{\vec{r}_i}^{\vec{r}_f} dr = \vec{r}_f - \vec{r}_i$

Hence area of v-t graph gives change in positive vector. Hence (d) is correct.

$\therefore$ |change in position| = |displacement|

= distance

We can read data v on the v-axis. So we can get change in speed and velocity.

Hence (a) and (b) are correct.

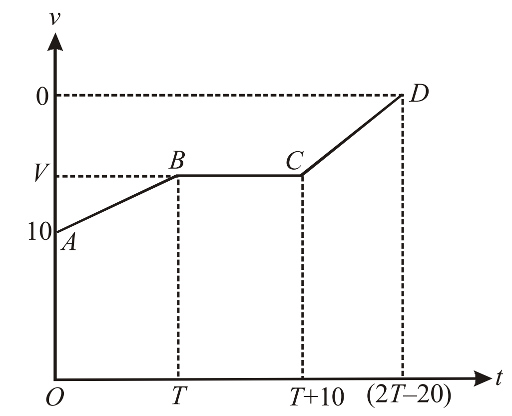
20. **(a, d)** $8 \times 10 + (10 - v_r)\, 10 = 160$

$v_r = 2$ m/sec.

$8 = v_{b/r} + v_r$

$v_{b/r} = 8 - v_r = 8 - 2 = 6$ m/sec.

21. **(a, b, d)**



$10 + aT = V$ .... (1)

$10 + 4aT = 20$ .... (2)

From (1) and (2),

$V = 12.5$ m/sec

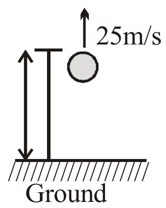
Distance between A and D = 675 m = Area under the curve

$\dfrac{1}{2}T(10 + V) + 10V + \dfrac{1}{2}(V + 20)\,T = 675$

$T = 20$ sec.

From eq. (1), $a = \dfrac{12.5 - 10}{20} = \dfrac{1}{8}\ \text{m/sec}^2$

22. **(a, b, c, d)**



Height above ground

$= 30 + \dfrac{u^2}{2g} = 30 + \dfrac{25 \times 25}{2 \times 10} = 30 + \dfrac{125}{4}$

$= 30 + 41.25$

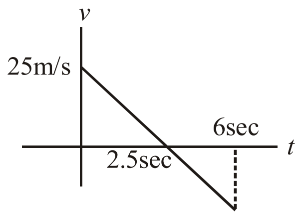
$= 71.25$m.

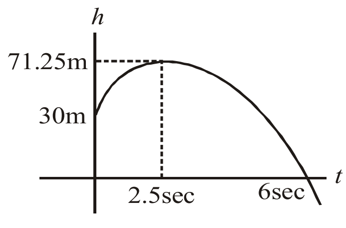
$h = ut - \dfrac{1}{2}gt^2\ ;\ -30 = 25t - \dfrac{1}{2} \times 10t^2$

$5t^2 - 25t - 30 = 0$

$t^2 - 5t - 6 = 0$

$(t - 6)(t + 1) = 0 \Rightarrow t = 6$





23. **(a, b)** If v be the velocity at a distance s from the starting point and S be the total space covered, then

Space-average of velocity $= \dfrac{1}{S}\int_0^S v\, ds$

and if T be the total time,

time average of velocity $= \dfrac{1}{T}\int_0^T v\, dt$

Now if f be the constant acceleration,
$v = u_1 + ft$ and $v^2 = u_1^2 + 2fs$

So that space-average $= \frac{1}{S}\int_0^S \sqrt{u_1^2 + 2fs}\, ds$

$$= \frac{1}{3fS}\left[(u_1^2 + 2fS)^{3/2}\right]_0^S$$

$$= \frac{1}{3fS}\left[(u_1^2 + 2fS)^{3/2} - u_1^3\right], \text{ but } u_2^2 = u_1^2 + 2fS$$

$$= \frac{2}{3}\frac{u_2^3 - u_1^3}{u_2^2 - u_1^2} = \frac{2}{3}\cdot\frac{u_1^2 + u_1u_2 + u_2^2}{u_1 + u_2}$$

and time-average $= \frac{1}{T}\int_0^T (u_1 + ft)\, dt$

$$= \frac{1}{2fT}\left[(u_1 + ft)^2\right]_0^T$$

$$= \frac{1}{2fT}\left[(u_1 + fT)^2 - u_1^2\right],$$

but $u_2 = u_1 + fT$

$$= \frac{1}{2}\frac{u_2^2 - u_1^2}{u_2 - u_1} = \frac{u_2 + u_1}{2}$$

24. **(a, b, c)** $\theta_2 = 90° - \theta_1$

$$\frac{h_1}{h_2} = \frac{\sin^2\theta_1}{\sin^2(90° - \theta_1)} = \tan^2\theta_1 = \cot^2\theta_2$$

25. **(a, b, c)**

(a) Point D is a maximum of the x vs t curve. Therefore $v = dx/dt = 0$.

(b) Without the exact equation for x as function of t one cannot get a precise answer. The best we can do is to draw the tangent line at point C and get the slope. This yields the answer.

$$v_C = \left.\frac{dx}{dt}\right|_C \approx 1.3\ m/s$$

(c) We proceed as in part (B), but here the tangent line has a negative slope and the answer should be

$$v_E = \left.\frac{dx}{dt}\right|_E \approx -0.13\ m/s$$

26. **(a, b)** The acceleration at any time is the slope of the v vs t curve.

(a) At A the slope is, from where the tangent line through A cuts the coordinate axes, $a = -7.0/0.73 = -9.6$ m/s^2.

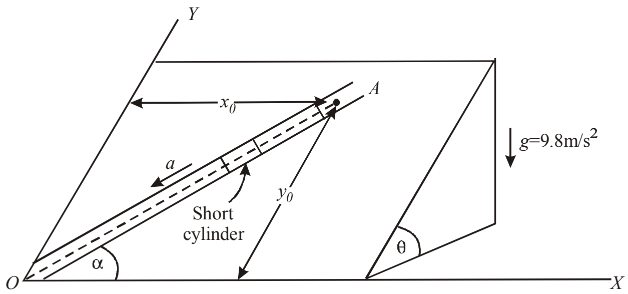
(b) The slope and therefore a is zero.

27. **(a, b)** $v > 0$ initially and $a > 0$ always
$\Rightarrow v > 0$ always and increasing.

28. **(b,c,d)** It is known that range is same for complementary angles of projection
$\therefore\ \theta_1 + \theta_2 = 90°$ is satisfied by (B, C, D)

29. **(a, b, c)**

(a) The downward component of g parallel to OY is $g \sin\theta$, hence the downward component along the groove is $a = g \sin\theta \sin\alpha$. Since



$$\sin\alpha = \frac{y_0}{(x_0^2 + y_0^2)^{1/2}} = 0.8$$

and $\sin\theta = 0.5$.
$a = (9.8)(0.5)(0.8) = 3.92$ m/s^2

(b) $s = v_0 t + \frac{1}{2}at^2$, where

$s = (x_0^2 + y_0^2)^{1/2} = 5m$ and $v_0 = 0$.

Thus, $s = \frac{1}{2}(3.92)t^2$ or $t = 1.597$s

(c) $v = 0 + (3.92)(1.597) = 6.26$ m/s.

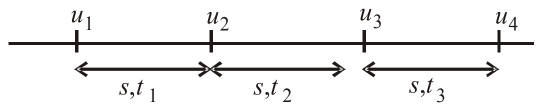
30. **(a, c, d)** Let u_x and u_y be horizontal and vertical components of velocity respectively at $t = 0$.
Then, $v_y = u_y - gt$
Hence, $v_y - t$ graph is straight line.
$x = v_x t$
Hence, x–t graph is straight line passing through origin.

The relation between y and t is $y = u_y t - \frac{1}{2}gt^2$

Hence, y-t graph is parabolic
v_x = constant
Hence v_x-t graph is a straight line.

31. **(b, d)** Since maximum heights are same, their time of flight should be same
$\therefore T_1 = T_2$
Also, vertical components of initial velocity are same
$\therefore$ Since range of 2 is greater than range of 1.
$\therefore$ Horizontal component of velocity of 2 > horizontal component of velocity of 1.
Hence, $u_2 > u_1$.

32. **(b, c)** By definition average speed $= \frac{3s}{t_1 + t_2 + t_3}$



For t_1 : $\dfrac{s}{t_1} = \dfrac{u_1 + u_2}{2}$ .... (1)

For t_2 : $\dfrac{s}{t_2} = \dfrac{u_2 + u_3}{2}$ .... (2)

For t_3 : $\dfrac{s}{t_3} = \dfrac{u_3 + u_4}{2}$ .... (3)

For $t_1 + t_2 + t_3$: $\dfrac{3s}{t_1 + t_2 + t_3} = \dfrac{u_1 + u_4}{2}$ .... (4)

From eq. (1), (2) and (3)

$$\frac{s}{t_1} - \frac{s}{t_2} + \frac{s}{t_3} = \frac{u_1 + u_4}{2}$$

Hence average speed is also $= \dfrac{s}{t_1} - \dfrac{s}{t_2} + \dfrac{s}{t_3}$

33. **(a, b, d)**

Work by F_{net} = Change in KE

In our case it is zero.

$$a_{average} = \frac{v_f - v_i}{\Delta t} = 0 \quad \therefore \; v_i = v_f$$

$$v_{average} = \frac{\text{displacement}}{\text{time}} = \frac{\text{area of } v\text{-}t \text{ curve}}{\text{time}} \neq 0$$

34. **(a, b, c, d)**

(a) $\dfrac{dy}{dt} = -bke^{-bt} + g/b$.

Differentiating once more $\dfrac{d^2y}{dt^2} = b^2ke^{-bt}$

Multiplying our expression for $\dfrac{dy}{dt}$ by $-b$ and adding g yields b^2ke^{-bt}.

Thus (1) is satisfied. Substituting $t = 0$ and recalling that $e^0 = 1$, we get $y = 0$.

(b) Since the ball is released from rest, $y = 0$ at $t = 0$.

Using our expression for $\dfrac{dy}{dt}$ from (a),

we have $0 = -bk + g/b$, which yields $k = g/b^2$.

(c) If $b = 0.1$/s and using $g = 9.8$ m/s^2,

we have $k = g/b^2 = 980$m. Then at $t = 10$s,

$y = (980\text{m})\,(e^{-1} - 1) + (98 \text{ m/s})\,(10\text{s}) = 360$m.

$$v = \frac{dy}{dt} = (-98m/s)\,e^{-1} + (9.8m/s^2)\,(0.1/s) = 62\ m/s$$

(d) At $t = 60$s,

$$\frac{dy}{dt} = (-98m/s)\,e^{-6} + 98m/s$$

$e^{-6} \approx 0.0025$,

so $\dfrac{dy}{dt} \approx 9.8\ m/s$

35. **(a, c, d)**

The first ball is in air for 4 seconds and second ball is in air for 2 seconds.

36. **(a, b, c)** When the particle returns to same point, displacement is zero, therefore average velocity is also zero.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2\,(u^2/2g)}{2\,(u/g)} = \frac{u}{2}$$

37. **(a, b)**

(a) $v_{hel/water} = v_{sub/water} + v_{hel/sub}$

$$= 17\hat{j} + (-5)\hat{k} = (17\hat{j} - 5\hat{k}) \text{ m/s}$$

(b) $v_{hel/air} = v_{hel/water} + v_{water/air} = v_{hel/water} - v_{air/water}$

$$= (17\hat{j} - 5\hat{k}) - 12\hat{i} = (-12\hat{i} + 17\hat{j} - 5\hat{k}) \text{ m/s}$$

38. **(b, c)** $a = 3t^2 + 1$

$$\frac{dv}{dt} = 3t^2 + 1 \Rightarrow \int_0^v dv = \int_0^1 (3t^2 + 1)\,dt$$

$$v = (t^3 + 1)_0^1 = 2m/s \Rightarrow v = t^3 + 1$$

$$\therefore \int_0^s ds = \int_0^1 (t^3 + 1)\,dt \Rightarrow s = \frac{1}{4} + \frac{1}{2} = 0.75$$

39. **(a, b, c)** The time of flight is independent of horizontal component of velocity of sphere. Hence whatever be the value of v_0, the time of flight shall remain same.

E MATRIX-MATCH TYPE

1. **(A) → (q, s); (B) → (r); (C) → (p); (D) → (s)**

2. **(A) → (p, r); (B) → (p, r); (C) → (p); (D) → (p)**

3. **(A) → q; (B) → q; (C) → r**

(A) $K = \dfrac{1}{2}mu^2 - mgy$, $y = Ax - Bx^2 \Rightarrow$ Parabolic

(B) $U = mgy = mg\,(Ax - Bx^2)$

(C) $KE_H = \dfrac{1}{2}mu^2\cos^2\theta = Min.\ KE$

4. **(A) → r; (B) → s; (C) → p; (D) → r**

(A) AB = maximum height $= \dfrac{v^2}{2a} = \dfrac{20 \times 20}{2 \times 10} = 20$ m

$\therefore \quad \frac{BD}{AD} = \frac{20+20}{20} = 2$

(B) $\frac{s\,(1^{st})}{s\,(2^{nd})} = \frac{20\times 1-\frac{1}{2}\times 10\times 1^2}{20-[15]} = \frac{15}{5} = 3$

(C) $\frac{v_A}{v_B} = \frac{20}{\sqrt{20^2+2\times 10\times 20}} = \frac{20}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

(D) $\frac{t_{AC}}{t_{AB}} = \frac{2t_{AB}}{t_{AB}} = 2$

5. (A) $\rightarrow$ q, s; (B) $\rightarrow$ q, r; (C) $\rightarrow$ p, r; (D) $\rightarrow$ p, s

(A) Slope is positive and decreasing $\Rightarrow a < 0$

(B) Slope is negative and its magnitude is increasing

(C) Slope is positive and increasing

(D) Slope is negative and its magnitude is decreasing

6. (A) $\rightarrow$ q; (B) $\rightarrow$ p, r; (C) $\rightarrow$ p, r; (D) $\rightarrow$ q, r

(A) For same range, angles of projection are θ and $(90° - \theta)$

$R_1 = R_2, T_1 \neq T_2, h_1 \neq h_2, v_{y_1} \neq v_{y_2}$, same speed

(B) For same height, $v_{y_1} = v_{y_2}, T_1 = T_2$

(C) $t = \sqrt{\frac{2h}{g}}$, $v_y^2 = 2gh$

(D) $v^2 = u^2 + 2gh$

$v_y^2 = u_y^2 + 2gh$

θ above horizontal

$-h = u \sin\theta\, t - (1/2)\, gt^2$ .... (1)

θ below horizontal

$-h = -u \sin\theta\, t - (1/2)\, gt^2$ .... (2)

From (1) and (2), t will be different.

7. (A) – p, (B) – p, (C) q, s ; (D) r, s

If angle between constant acceleration vector $\vec{a}$ and velocity $\vec{v}$ is zero or 180° then path is straight line otherwise path must be parabolic.

8. (A) – q,r ; (B) – q,r ; (C) – p, s ; (D) – p, s

(A) Since truck is moving, as seen from ground the ball will have a horizontal component of velocity hence the path of the ball seen from ground will be parabola.

(B) In for the truck, the horizontal component of velocity will be zero, hence straight line w.r.t. truck.

(C) Ball will have a horizontal component with truck hence parabola as seen from truck.

(D) As seen from ground, the horizontal component of the velocity will be zero.

F NUMERIC/INTEGER ANSWER TYPE

1. 0.83

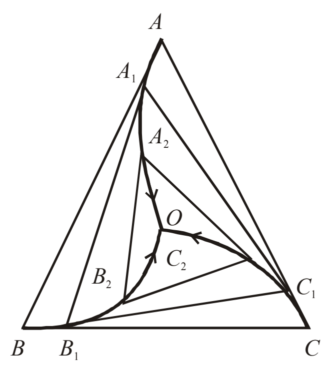


Figure shows the motion of the particles. These particles will meet at the centroid O of the triangle. At any instant the particles will form an equilateral triangle ABC with the same centroid O. Let us fix our attention on the motion of any one particle, say A. At any instant, its velocity makes an angle 30° with AO. The component of this velocity along AO is v cos 30°. This component will represent the rate of decrease of the distance AO.

Initially, $AO = \frac{2}{3}\sqrt{d^2 - \left(\frac{d}{2}\right)^2} = \frac{d}{\sqrt{3}}$

Therefore, the time taken for AO to become zero

$= \frac{\frac{d}{\sqrt{3}}}{v\cos 30°} = \frac{2d}{\sqrt{3}v\times\sqrt{3}} = \frac{2d}{3v} = \frac{2\times 0.5}{3\times 2} = \mathbf{0.83}\,\boldsymbol{s}$

2. 288

Velocity of car from A $= \frac{480}{8} = 60$ km / hour

Velocity of car from B $= \frac{480}{12} = 40$ km / hour

Let the two cars meet at t hour

$\therefore t = \frac{480}{60+40} = 4.8$ hours

The distance $s = v_A \times t = 60 \times 4.8 =$ **288 km.**

3. 59

Such problems can be tackled by keeping in mind, that for any event, to occur, time remains same for both bodies.

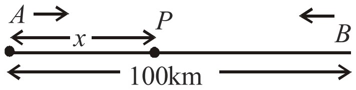
Let P be the point, where the two engines cross each other. If t hr be the time to occur this event, then total distance covered by the two trains should be equal to 100 km.(fig.)

i.e., $AP + BP = 100$

$$\Rightarrow\ 50t + \frac{1}{2}\times 18t^2 + 50t - \frac{1}{2}\times 18t^2 = 100$$

$\Rightarrow\ 100t = 100$

$\Rightarrow\ t = 1$ hr.



$\therefore\ x = AP = 50\,(1) + \dfrac{1}{2}\times 18(1) \Rightarrow x = 50 + 9 =$ **59 km.**

4. 50

Here, $u = u$ m/s, $a = g = -10$ m/s^2 and $s = 80$m.
Substituting the values in

$s = ut + \dfrac{1}{2}at^2$, we have, $80 = ut - 5t^2$

or $5t^2 - ut + 80 = 0$ or $t = \dfrac{u + \sqrt{u^2 - 1600}}{10}$ and $\dfrac{u - \sqrt{u^2 - 1600}}{10}$

Now it is given that,

$$\frac{u + \sqrt{u^2 - 1600}}{10} - \frac{u - \sqrt{u^2 - 1600}}{10} = 6$$

or $\dfrac{\sqrt{u^2 - 1600}}{5} = 6$

or $\sqrt{u^2 - 1600} = 30$

or $u^2 - 1600 = 900$

$\therefore\ u^2 = 2500$

or $u = \pm 50$ m/s

Ignoring the negative sign, we have, $u =$ **50 m/s**

5. 3.0

Let us take velocity of swimmer with respect to water is v and that of river current is v_r. The swimmer which crosses the river along the straight line AB, has to swim in upstream direction such that its resultant velocity becomes toward AB as shown in figure. If the width of river is assumed to be d, then

Resultant velocity of first swimmer is $v_1 = \sqrt{v^2 - v_r^2}$

Time taken by her to cross the river is

$$t = \frac{d}{\sqrt{v^2 - v_r^2}} = \frac{d}{\sqrt{2.5^2 - 2^2}} = \frac{d}{1.5}\ \text{hr.}$$

Second swimmer if swims along AB, she is drifted towards point C, due to river flow as shown in figure and then she has to walk down to reach point B with velocity u.

Here crossing velocity of second swimmer is v, as its is swimming along normal direction.

Time taken to cross the river by her is,

$$t_1 = \frac{d}{v} = \frac{d}{1.5}\ \text{hr}$$

Her drift due to river flow is,

$$x = v_r \times \frac{d}{v}$$

Time taken to reach point B by walking is,

$$t_2 = \frac{x}{u} = \frac{v_r d}{uv} = \frac{2\times d}{u\times 2.5} = \frac{d}{12.5u}\ \text{hr}$$

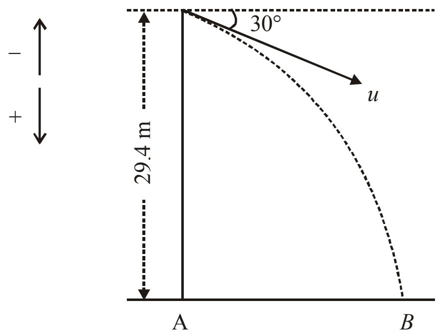
Given that both the swimmers reach the destination simultaneously, so we have

$$t = t_1 + t_2 \ \text{ or }\ \frac{d}{1.5} = \frac{d}{2.5} + \frac{d}{12.5u}$$

or $u =$ **3.0 kph.**

6. 2

The situation is shown in fig.



The time taken by the body is equal to the time taken by the freely falling body from the height 29.4 m with initial velocity $u \sin\theta = 9.8 \sin 30°$. This is given by

$$\frac{u}{2} = \frac{9.8}{2} = 4.9\ \text{m/s}$$

Applying the formula, $s = u\,t + \dfrac{1}{2}\,g\,t^2$, we have

$$29.4 = 4.9\,t + \frac{1}{2}\,(9.8)\,t^2$$

or $4.9\,t^2 + 4.9\,t - 29.4 = 0$

(because s, u & g are all in downward direction)

$t^2 + t - 6 = 0$

or $t = 2$s or -3s

$\therefore$ Time taken to reach ground = **2 second**

7. 49

Vertical velocity after 10s is,

$$v = (98\sin 60°) - (9.8)\times 10 = 98\times\frac{\sqrt{3}}{2} - 98$$

$$= 98\left[\frac{\sqrt{3}}{2} - 1\right] = 98\times(0.866 - 1)$$

$= 98\times(-0.134)$.

Vertical momentum of the ball after $10\text{s} = mv = 0.5\times(-98\times 0.134)$ kg m/s;

Initial vertical momentum of the ball

$$= 0.5\times 98\times\frac{\sqrt{3}}{2} = 0.5\times 98\times 0.866$$

Change in vertical momentum

$= 0.5\times 98\times 0.866 - (-0.5\times 98\times 0.134)$

$= 0.5\times 98\,[0.866 + 0.134] = 0.5\times 98 = 49$ kg m/s;

Horizontal component of velocity remains same, so there is no change in momentum along horizontal direction.

$\therefore$ Change in momentum = **49 kg m/s.**

8. **2**

For body to hit the lowest plane with minimum velocity, it should be able to just clear the second step

Let R = range for minimum velocity

h = height for minimum velocity

u = minimum velocity

t = time of motion

$R = u \times t \Rightarrow t = R/u$ ....(1)

$$h = \frac{1}{2} g t^2 \qquad \text{....(2)}$$

From (1) & (2),

$$h = \frac{1}{2} g \times \frac{R^2}{u^2} \Rightarrow h = \frac{gR^2}{2u^2} \qquad \text{....(3)}$$

$R = 2 \times 20$ cm $= 40$cm $= 0.4$ m

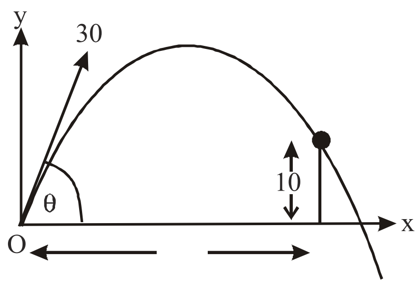
$h = 2 \times 10$cm $= 20$ cm $= 0.2$m

$$\therefore u^2 = \frac{1}{2} g \times \frac{R^2}{h} \Rightarrow u^2 = \frac{1}{2} \times 10 \times \frac{0.4 \times 0.4}{0.2} = 4$$

$$\Rightarrow \boldsymbol{u = 2\text{m/s}}$$

9. **4**

Let the angle of projection of the particle be θ.



The path of the particle has to pass through the point where $x = 40, y = 10$.

Using the equation of the path of the projectile in the form

$$y = x \tan\alpha - \frac{x^2 g}{2v^2}(1 + \tan^2\alpha)$$

gives $$y = x\tan\theta - \frac{x^2}{180}(1 + \tan^2\theta)$$

The point (40, 10) lies on this path so

$$10 = 40\tan\theta - \frac{80}{9}(1 + \tan^2\theta)$$

$$\Rightarrow \quad 8\tan^2\theta - 36\tan\theta + 17 = 0 \qquad \text{....(1)}$$

This is quadratic equation in $\tan\theta$ with two positive roots. Therefore there are two values of θ less than 90°, so there are two possible angles of projection.

Now we are asked to calculate $\tan(\alpha + \beta)$ which is equal to

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

where $\tan\alpha$ and $\tan\beta$ are the roots of equation (1).

Hence $\tan\alpha + \tan\beta = \frac{36}{8} = \frac{9}{2}$ (sum of roots)

and $\tan\alpha\tan\beta = \frac{17}{8}$ (product of roots)

Therefore $$\tan(\alpha + \beta) = \frac{9/2}{1 - (17/8)} = -4$$

$\Rightarrow \tan\left[-(\alpha + \beta)\right]$

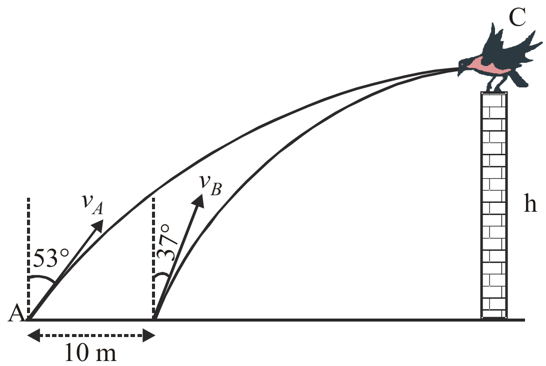
$= -\tan(\alpha + \beta)$

$= -(-4)$

$=$ **4**

10. **80**

The vertical components must be equal.



$$\therefore \quad v_A \cos 53° = v_B \cos 37°$$

$$\text{or} \quad v_A = v_B \frac{\cos 37°}{\cos(90° - 37°)}$$

$$\text{or} \quad v_A = 60 \cot 37° = \frac{60}{\tan 37°}$$

$$= \frac{60 \times 4}{3} = \textbf{80 units}$$