The Straight Lines Exercise 1 :
Single Option Correct Type Questions

- This section contains **30 multiple choice questions**. Each question has four choices (a), (b), (c), (d) out of which **ONLY ONE** is correct.

1. The straight line $y = x - 2$ rotates about a point where it cuts X-axis and becomes perpendicular on the straight line $ax + by + c = 0$, then its equation is

(a) $ax + by + 2a = 0$ (b) $ay - bx + 2b = 0$
(c) $ax + by + 2b = 0$ (d) None of these

2. If $\frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^m}{n!}$, then orthocentre of the triangle having sides $x - y + 1 = 0$, $x + y + 3 = 0$ and $2x + 5y - 2 = 0$ is

(a) $(2m - 2n, m - n)$ (b) $(2m - 2n, n - m)$
(c) $(2m - n, m + n)$ (d) $(2m - n, m - n)$

3. If $f(x + y) = f(x)f(y) \forall x, y \in R$ and $f(1) = 2$, then area enclosed by $3|x| + 2|y| \le 8$ is

(a) $f(4)$ sq units (b) $\frac{1}{2} f(6)$ sq units
(c) $\frac{1}{3} f(6)$ sq units (d) $\frac{1}{3} f(5)$ sq units

4. The graph of the function $y = \cos x \cos(x + 2) - \cos^2(x + 1)$ is

(a) a straight line passing through $(0, -\sin^2 1)$ with slope 2
(b) a straight line passing through (0, 0)
(c) a parabola with vertex $(1, -\sin^2 1)$
(d) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ are parallel to the X-axis.

5. A line passing through the point (2, 2) and the axes enclose an area λ. The intercepts on the axes made by the line are given by the two roots of

(a) $x^2 - 2|\lambda|x + |\lambda| = 0$ (b) $x^2 + |\lambda|x + 2|\lambda| = 0$
(c) $x^2 - |\lambda|x + 2|\lambda| = 0$ (d) None of these

6. The set of value of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line $a^2x + aby + 1 = 0 \forall a \in R, b > 0$ are

(a) $b \in (2, 4)$ (b) $b \in (0, 2)$
(c) $b \in [0, 2]$ (d) None of these

7. Line L has intercepts a and b on the co-ordinates axes, when the axes are rotated through a given angle; keeping the origin fixed, the same line has intercepts p and q, then

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$
(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

8. If the distance of any point (x, y) from the origin is defined as $d(x, y) = \max\{|x|, |y|\}$, $d(x, y) = a$ non-zero constant, then the locus is

(a) a circle (b) a straight line
(c) a square (d) a triangle

9. If p_1, p_2, p_3 be the perpendiculars from the points $(m^2, 2m)$, $(mm', m + m')$ and $(m'^2, 2m')$ respectively on the line $x \cos\alpha + y \sin\alpha + \frac{\sin^2\alpha}{\cos\alpha} = 0$, then p_1, p_2, p_3 are in

(a) AP (b) GP
(c) HP (d) None of these

10. $ABCD$ is a square whose vertices A, B, C and D are (0, 0), (2, 0), (2, 2) and (0, 2) respectively. This square is rotated in the xy plane with an angle of 30° in anti-clockwise direction about an axis passing through the vertex A the equation of the diagonal BD of this rotated square is .... If E is the centre of the square, the equation of the circumcircle of the triangle ABE is

(a) $\sqrt{3}x + (1 - \sqrt{3})y = \sqrt{3}$, $x^2 + y^2 = 4$
(b) $(1 + \sqrt{3})x - (1 - \sqrt{2})y = 2$, $x^2 + y^2 = 9$
(c) $(2 - \sqrt{3})x + y = 2(\sqrt{3} - 1)$, $x^2 + y^2 - x\sqrt{3} - y = 0$
(d) None of the above

11. The point (4, 1) undergoes the following three successive transformations

(i) reflection about the line $y = x - 1$.
(ii) translation through a distance 1 unit along the positive direction of X-axis.
(iii) rotation through an angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction

Then, the coordinates of the final point are

(a) (4, 3) (b) $\left(\frac{7}{2}, \frac{7}{2}\right)$
(c) $(0, 3\sqrt{2})$ (d) (3, 4)

12. If the square $ABCD$, where $A(0, 0)$, $B(2, 0)$, $C(2, 2)$ and $D(0, 2)$ undergoes the following three transformations successively

(i) $f_1(x, y) \to (y, x)$
(ii) $f_2(x, y) \to (x + 3y, y)$
(iii) $f_3(x, y) \to \left(\frac{x - y}{2}, \frac{x + y}{2}\right)$

then the final figure is a

(a) square (b) parallelogram
(c) rhombus (d) None of these

13. The line $x + y = a$ meets the axes of x and y at A and B respectively. A triangle AMN is inscribed in the triangle OAB, O being the origin, with right angle at N, M and N lie respectively on OB and AB. If the area of the triangle AMN is $\frac{3}{8}$ of the area of the triangle OAB, then $\frac{AN}{BN}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

14. If $P(1, 0)$, $Q(-1, 0)$ and $R(2, 0)$ are three given points, then the locus of point S satisfying the relation $(SQ)^2 + (SR)^2 = 2(SP)^2$ is

(a) a straight line parallel to X-axis
(b) a circle through the origin
(c) a circle with centre at the origin
(d) a straight line parallel to Y-axis

15. If $A\left(\frac{\sin\alpha}{3} - 1, \frac{\cos\alpha}{2} - 1\right)$ and $B(1, 1)$, $\alpha \in [-\pi, \pi]$ are two points on the same side of the line $3x - 2y + 1 = 0$, then α belongs to the interval

(a) $\left(-\pi, -\frac{3\pi}{4}\right] \cup \left(\frac{\pi}{4}, \pi\right)$ (b) $[-\pi, \pi]$
(c) ϕ (d) None of these

16. The line $x + y = 1$ meets X-axis at A and Y-axis at B, P is the mid-point of AB. P_1 is the foot of the perpendicular from P to OA; M_1 is that of P_1 from OP; P_2 is that of M_1 from OA; M_2 is that of P_2 from OP; P_3 is that of M_2 from OA and so on. If P_n denotes the nth foot of the perpendicular on OA form M_{n-1}, then OP_n is equal to

(a) $\frac{1}{2n}$ (b) $\frac{1}{2^n}$
(c) $2^n - 1$ (d) $2^n + 3$

17. The line $x = c$ cuts the triangle with corners (0, 0); (1, 1) and (9, 1) into two regions. For the area of the two regions to be the same, then c must be equal to

(a) $\frac{5}{2}$ (b) 3
(c) $\frac{7}{2}$ (d) 3 or 15

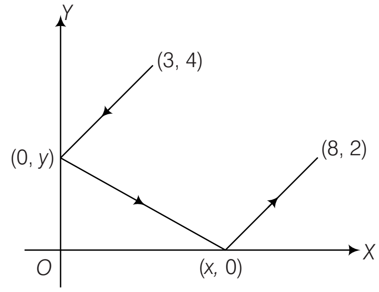
18. If the straight lines $x + 2y = 9$, $3x - 5y = 5$ and $ax + by = 1$ are concurrent, then the straight line $5x + 2y = 1$, passes through the point

(a) $(a, -b)$ (b) $(-a, b)$
(c) (a, b) (d) $(-a, -b)$

19. The ends of the base of the isosceles triangle are at $(2, 0)$ and $(0, 1)$ and the equation of one side is $x = 2$, then the orthocentre of the triangle is

(a) $\left(\frac{3}{4}, \frac{3}{2}\right)$ (b) $\left(\frac{5}{4}, 1\right)$
(c) $\left(\frac{3}{4}, 1\right)$ (d) $\left(\frac{4}{3}, \frac{7}{12}\right)$

20. Suppose that a ray of light leaves the point (3, 4), reflects off the Y-axis towards the X-axis, reflects off the X-axis, and finally arrives at the point (8, 2). The value of x is



(a) $4\frac{1}{2}$ (b) $4\frac{1}{3}$ (c) $4\frac{2}{3}$ (d) $5\frac{1}{3}$

21. m, n are two integers with $0 < n < m$. A is the point (m, n) on the cartesian plane. B is the reflection of A in the line $y = x$. C is the reflection of B in the Y-axis, D is the reflection of C in the X-axis and E is the reflection of D in the Y-axis. The area of the pentagon $ABCDE$ is

(a) $2m(m + n)$ (b) $m(m + 3n)$
(c) $m(2m + 3n)$ (d) $2m(m + 3n)$

22. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinates axes at points P and Q. As L varies, the absolute minimum value of $OP + OQ$ is (O is origin)

(a) 10 (b) 18 (c) 16 (d) 12

23. Drawn from origin are two mutually perpendicular lines forming an isosceles triangle together with the straight line $2x + y = a$, then the area of this triangle is

(a) $\frac{a^2}{2}$ sq units (b) $\frac{a^2}{3}$ sq units
(c) $\frac{a^2}{5}$ sq units (d) None of these

24. The number of integral values of m for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer is

(a) 2 (b) 0 (c) 4 (d) 1

25. A ray of light coming from the point (1, 2) is reflected at a point A on the X-axis and then passes through the point (5, 3). The coordinates of the point A are

(a) $\left(\frac{13}{5}, 0\right)$ (b) $\left(\frac{5}{13}, 0\right)$
(c) $(-7, 0)$ (d) None of these

26. Consider the family of lines $5x + 3y - 2 + \lambda(3x - y - 4) = 0$ and $x - y + 1 + \mu(2x - y - 2) = 0$. Equation of straight line that belong to both families is $ax + by - 7 = 0$, then $a + b$ is

(a) 1 (b) 3 (c) 5 (d) 7

27. In ΔABC equation of the right bisectors of the sides AB and AC are $x + y = 0$ and $x - y = 0$ respectively. If $A \equiv (5, 7)$, then equation of side BC is

(a) $7y = 5x$ (b) $5x = y$
(c) $5y = 7x$ (d) $5y = x$

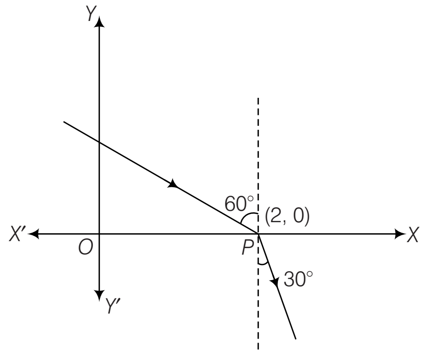
28. Two particles start from the point $(2, -1)$, one moving 2 units along the line $x + y = 1$ and the other 5 units along the line $x - 2y = 4$. If the particles move towards increasing y, then their new positions are

(a) $(2-\sqrt{2}, \sqrt{2}-1); (2\sqrt{2}+2, \sqrt{5}-1)$
(b) $(2\sqrt{2}+2, \sqrt{5}-1); (2\sqrt{2}, \sqrt{2}+1)$
(c) $(2+\sqrt{2}, \sqrt{2}+1); (2\sqrt{2}+2, \sqrt{5}+1)$
(d) $(2-\sqrt{2}, \sqrt{5}-1); (\sqrt{2}-1, 2\sqrt{2}+2)$

29. Let P be $(5, 3)$ and a point R on $y = x$ and Q on the X-axis be such that $PQ + QR + RP$ is minimum, then the coordinates of Q are

(a) $\left(\frac{17}{8}, 0\right)$ (b) $\left(\frac{17}{4}, 0\right)$
(c) $\left(\frac{17}{2}, 0\right)$ (d) $(17, 0)$

30. In the adjacent figure combined equation of the incident and refracted ray is



(a) $(x-2)^2 + y^2 + \frac{4}{\sqrt{3}}(x-2)y = 0$
(b) $(x-2)^2 + y^2 - \frac{4}{\sqrt{3}}(x-2)y = 0$
(c) $(x-2)^2 + y^2 + \frac{y}{\sqrt{3}}(x-2) = 0$
(d) $(x-2)^2 + y^2 - \frac{y}{\sqrt{3}}(x-2) = 0$

The Straight Lines Exercise 2 :
More than One Correct Option Type Questions

- The section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c), and (d) out of which **MORE THAN ONE** may be correct.

31. The point of intersection of the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ lies on

(a) $x - y = 0$
(b) $(x + y)(a + b) = 2ab$
(c) $(lx + my)(a + b) = (l + m)ab$
(d) $(lx - my)(a + b) = (l - m)ab$

32. The equations $(b - c)x + (c - a)y + a - b = 0$ and $(b^3 - c^3)x + (c^3 - a^3)y + a^3 - b^3 = 0$ will represent the same line, if

(a) $b = c$ (b) $c = a$
(c) $a = b$ (d) $a + b + c = 0$

33. The area of a triangle is 5. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. The coordinates of the third vertex cannot be

(a) $\left(\frac{-3}{2}, \frac{3}{2}\right)$ (b) $\left(\frac{3}{4}, \frac{-3}{2}\right)$
(c) $\left(\frac{7}{2}, \frac{13}{2}\right)$ (d) $\left(\frac{-1}{4}, \frac{11}{4}\right)$

34. If the lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then

(a) $\lambda = 2$ (b) $\lambda = -3$ (c) $\lambda = 4$ (d) $\lambda = -4$

35. Equation of a straight line passing through the point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ are perpendicular to one of them is

(a) $x + y + 3 = 0$ (b) $x + y - 3 = 0$
(c) $x - 3y - 5 = 0$ (d) $x - 3y + 5 = 0$

36. If one vertex of an equilateral triangle of side a lies at the origin and the other lies on the line $x - \sqrt{3}y = 0$, the coordinates of the third vertex are

(a) $(0, a)$ (b) $\left(\frac{\sqrt{3}a}{2}, \frac{-a}{2}\right)$ (c) $(0, -a)$ (d) $\left(\frac{-\sqrt{3}a}{2}, \frac{a}{2}\right)$

37. If the line $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent $(a + b + c \neq 0)$ then

(a) $a^3 + b^3 + c^3 - 3abc = 0$ (b) $a = b$
(c) $a = b = c$ (d) $a^2 + b^2 + c^2 - bc - ca - ab = 0$

38. $A(1, 3)$ and $C(7, 5)$ are two opposite vertices of a square. The equation of a side through A is

(a) $x + 2y - 7 = 0$ (b) $x - 2y + 5 = 0$
(c) $2x + y - 5 = 0$ (d) $2x - y + 1 = 0$

39. If $6a^2 - 3b^2 - c^2 + 7ab - ac + 4bc = 0$, then the family of lines $ax + by + c = 0$ is concurrent at

(a) $(-2, -3)$ (b) $(3, -1)$
(c) $(2, 3)$ (d) $(-3, 1)$

40. Consider the straight lines $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$. The line $6x + 6y + 7 = 0$ is

(a) bisector of the angle including origin
(b) bisector of acute angle
(c) bisector of obtuse angle
(d) None of the above

41. Two roads are represented by the equations $y - x = 6$ and $x + y = 8$. An inspection bungalow has to be so constructed that it is at a distance of 100 from each of the roads. Possible location of the bungalow is given by

(a) $(100\sqrt{2} + 1, 7)$ (b) $(1 - 100\sqrt{2}, 7)$
(c) $(1, 7 + 100\sqrt{2})$ (d) $(1, 7 - 100\sqrt{2})$

42. If (a, b) be an end of a diagonal of a square and the other diagonal has the equation $x - y = a$, then another vertex of the square can be

(a) $(a - b, a)$ (b) $(a, 0)$
(c) $(0, -a)$ (d) $(a + b, b)$

43. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then

(a) the lines will pass through a fixed point
(b) there will be a set of parallel lines
(c) all the lines intersect the line $x = x_1$
(d) all the lines will be parallel to the line $y = x_1$

44. Let $L_1 \equiv ax + by + a\sqrt[3]{b} = 0$ and $L_2 \equiv bx - ay + b\sqrt[3]{a} = 0$ be two straight lines. The equations of the bisectors of the angle formed by the foci whose equations are $\lambda_1 L_1 - \lambda_2 L_2 = 0$ and $\lambda_1 L_1 + \lambda_2 L_2 = 0$, λ_1 and λ_2 being non-zero real numbers, are given by

(a) $L_1 = 0$ (b) $L_2 = 0$
(c) $\lambda_1 L_1 + \lambda_2 L_2 = 0$ (d) $\lambda_2 L_1 - \lambda_1 L_2 = 0$

45. The equation of the bisectors of the angles between the two intersecting lines $\frac{x-3}{\cos\theta} = \frac{y+5}{\sin\theta}$ and $\frac{x-3}{\cos\phi} = \frac{y+5}{\sin\phi}$ are $\frac{x-3}{\cos\alpha} = \frac{y+5}{\sin\alpha}$ and $\frac{x-3}{\beta} = \frac{y+5}{\gamma}$, then

(a) $\alpha = \frac{\theta + \phi}{2}$ (b) $\beta = -\sin\alpha$
(c) $\gamma = \cos\alpha$ (d) $\beta = \sin\alpha$

The Straight Lines Exercise 3 :
Paragraph Based Questions

- The section contains **5 Paragraphs** based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c), and (d) out of which **ONLY ONE** is correct.

Paragraph I

(Q. Nos. 46 to 48)

For points $P \equiv (x_1, y_1)$ and $Q = (x_2, y_2)$ of the coordinate plane, a new distance $d(P, Q)$ is defined by
$d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$
Let $O \equiv (0, 0)$, $A \equiv (1, 2)$, $B \equiv (2, 3)$ and $C \equiv (4, 3)$ are four fixed points on x-y plane.

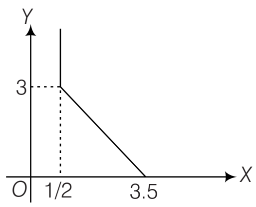
46. Let $R(x, y)$, such that R is equidistant from the point O and A with respect to new distance and if $0 \le x < 1$ and $0 \le y < 2$, then R lie on a line segment whose equation is

(a) $x + y = 3$ (b) $x + 2y = 3$
(c) $2x + y = 3$ (d) $2x + 2y = 3$

47. Let $S(x, y)$, such that S is equidistant from points O and B with respect to new distance and if $x \ge 2$ and $0 \le y < 3$, then locus of S is

(a) a line segment of finite length
(b) a line of infinite length
(c) a ray of finite length
(d) a ray of infinite length

48. Let $T(x, y)$, such that T is equidistant from point O and C with respect to new distance and if T lie in first quadrant, then T consists of the union of a line segment of finite length and an infinite ray whose labelled diagram is

(a)


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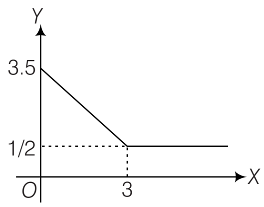

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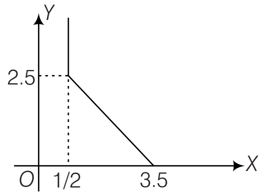

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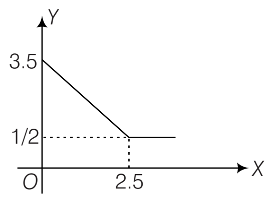


Paragraph II

(Q. Nos. 49 to 51)

In a triangle ABC, if the equation of sides AB, BC and CA are $2x - y + 4 = 0$, $x - 2y - 1 = 0$ *and* $x + 3y - 3 = 0$ *respectively.*

49. Tangent of internal angle A is equal to

(a) -7 (b) -3

(c) $\frac{1}{2}$ (d) 7

50. The equation of external bisector of angle B is

(a) $x - y - 1 = 0$ (b) $x - y + 1 = 0$

(c) $x + y - 5 = 0$ (d) $x + y + 5 = 0$

51. The image of point B w.r.t the side CA is

(a) $\left(-\frac{3}{5}, \frac{26}{5}\right)$ (b) $\left(-\frac{3}{5}, -\frac{26}{5}\right)$

(c) $\left(\frac{3}{5}, -\frac{26}{5}\right)$ (d) $\left(\frac{3}{5}, \frac{26}{5}\right)$

Paragraph III

(Q. Nos. 52 to 54)

$A\ (1, 3)$ *and* $C\left(-\frac{2}{5}, \frac{-2}{5}\right)$ *are the vertices of a triangle ABC and the equation of the angle bisector of* $\angle ABC$ *is* $x + y = 2$.

52. Equation of BC is

(a) $7x + 3y - 4 = 0$ (b) $7x + 3y + 4 = 0$

(c) $7x - 3y + 4 = 0$ (d) $7x - 3y - 4 = 0$

53. Coordinates of vertex B are

(a) $\left(\frac{3}{10}, \frac{17}{10}\right)$ (b) $\left(\frac{17}{10}, \frac{3}{10}\right)$

(c) $\left(-\frac{5}{2}, \frac{9}{2}\right)$ (d) $\left(\frac{9}{2}, -\frac{5}{2}\right)$

54. Equation of AB is

(a) $3x + 7y = 24$

(b) $3x + 7y + 24 = 0$

(c) $13x + 7y + 8 = 0$

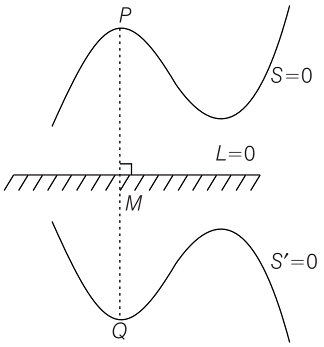
(d) $13x - 7y + 8 = 0$

Paragraph IV

(Q. Nos. 55 to 57)

Let $S' = 0$ *be the image or reflection of the curve* $S = 0$ *about line mirror* $L = 0$. *Suppose P be any point on the curve* $S = 0$ *and Q be the image or reflection about the line mirror* $L = 0$, *then Q will lie on* $S' = 0$.

How to find the image or reflection of a curve?



Let the given curve be $S : f(x, y) = 0$ *and line mirror* $L : ax + by + c = 0$. *We take a point P on the given curve in parametric form. Suppose Q be the image or reflection of point P about line mirror* $L = 0$, *which again contains the same parameter. Let* $Q \equiv (\phi(t), \psi(t))$, *where t is parameter. Now let* $x = \phi(t)$ *and* $y = \psi(t)$

Eliminating t, we get the equation of the reflected curve S′.

55. The image of the line $3x - y = 2$ in the line $y = x - 1$ is

(a) $x + 3y = 2$ (b) $3x + y = 2$

(c) $x - 3y = 2$ (d) $x + y = 2$

56. The image of the circle $x^2 + y^2 = 4$ in the line $x + y = 2$ is

(a) $x^2 + y^2 - 2x - 2y = 0$ (b) $x^2 + y^2 - 4x - 4y + 6 = 0$

(c) $x^2 + y^2 - 2x - 2y + 2 = 0$ (d) $x^2 + y^2 - 4x - 4y + 4 = 0$

57. The image of the parabola $x^2 = 4y$ in the line $x + y = a$ is

(a) $(x - a)^2 = 4(a - y)$ (b) $(y - a)^2 = 4(a - x)$

(c) $(x - a)^2 = 4(a + y)$ (d) $(y - a)^2 = 4(a + x)$

Paragraph V

(Q. Nos. 58 to 60)

In a ΔABC, the equation of the side BC is $2x - y = 3$ and its circumcentre and orhtocentre are (2, 4) and (1, 2) respectively.

58. Circumradius of ΔABC is

(a) $\sqrt{\frac{61}{5}}$ (b) $\sqrt{\frac{51}{5}}$ (c) $\sqrt{\frac{41}{5}}$ (d) $\sqrt{\frac{43}{5}}$

59. $\sin B \cdot \sin C =$

(a) $\frac{9}{2\sqrt{61}}$ (b) $\frac{9}{4\sqrt{61}}$ (c) $\frac{9}{\sqrt{61}}$ (d) $\frac{9}{5\sqrt{61}}$

60. The distance of orthocentre from vertex A is

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{6}{\sqrt{5}}$ (c) $\frac{3}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{5}}$

The Straight Lines Exercise 4 :
Single Integer Answer Type Questions

- The section contains **10 questions**. The answer to eaeh question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

61. The number of possible straight lines passing through (2, 3) and forming a triangle with the coordinate axes, whose area is 12 sq units, is

62. The portion of the line $ax + 3y - 1 = 0$, intercepted between the lines $ax + y + 1 = 0$ and $x + 3y = 0$ subtend a right angle at origin, then the value of $|a|$ is

63. Let ABC be a triangle and $A \equiv (1, 2)$, $y = x$ be the perpendicular bisector of AB and $x - 2y + 1 = 0$ be the angle bisector of $\angle C$. If the equation of BC is given by $ax + by - 5 = 0$, then the value of $a - 2b$ is

64. A lattice point in a plane is a point for which both coordinates are integers. If n be the number of lattice points inside the triangle whose sides are $x = 0$, $y = 0$ and $9x + 223y = 2007$, then tens place digit in n is

65. The number of triangles that the four lines $y = x + 3$, $y = 2x + 3$, $y = 3x + 2$ and $y + x = 3$ form is

66. In a plane there are two families of lines : $y = x + n$, $y = -x + n$, where $n \in \{0, 1, 2, 3, 4\}$. The number of squares of the diagonal of length 2 formed by these lines is

67. Given $A(0, 0)$ and $B(x, y)$ with $x \in (0, 1)$ and $y > 0$. Let the slope of line AB be m_1. Point C lies on line $x = 1$ such that the slope of BC is equal to m_2, where $0 < m_2 < m_1$. If the area of triangle ABC can be expressed as $(m_1 - m_2) f(x)$ and the largest possible value of $f(x)$ is λ, then the value of $\frac{1}{\lambda}$ is

68. If $(\lambda, \lambda + 1)$ is an interior point of ΔABC, where $A \equiv (0, 3)$, $B \equiv (-2, 0)$ and $C \equiv (6, 1)$, then the number of integral values of λ is

69. For all real values of a and b, lines $(2a + b)x + (a + 3b)y + (b - 3a) = 0$ and $\lambda x + 2y + 6 = 0$ and $\lambda x + 2y + 6 = 0$ are concurrent, then the value of $|\lambda|$ is

70. If from point (4, 4) perpendiculars to the straight lines $3x + 4y + 5 = 0$ and $y = mx + 7$ meet at Q and R and area of triangle PQR is maximum, then the value of $3m$ is

The Straight Lines Exercise 5 :
Matching Type Questions

- The section contains **5 questions**. Questions 1, 2 and 3 have four statement (A, B, C and D) given in **Column I** and four statements (p, q, r and s) in **Column II** and questions 74 and 75 have three statements (A, B and C) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement (s) given in **Column II**.

71. Let L_1, L_2, L_3 be three straight lines a plane and n be the number of circles touching all the lines.

	Column I		**Column II**
(A)	The lines are concurrent, then $n + 1$ is a	(p)	natural number
(B)	The lines are parallel, then $2n + 3$ is a	(q)	prime number
(C)	Two lines are parallel, then $n + 2$ is a	(r)	composite number
(D)	The lines are neither concurrent nor parallel, then $n + 2$ is a	(s)	perfect number

72. Match the Columns

	Column I		**Column II**
(A)	Lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then the value of $|\lambda|$ is	(p)	2
(B)	The variable straight lines $3x(a + 1) + 4y(a - 1) - 3(a - 1) = 0$ for different value of 'a' passes through a fixed point (p, q) if $\lambda = p - q$, then the value of $4|\lambda|$	(q)	3
(C)	If the line $x + y - 1 - \left|\frac{\lambda}{2}\right| = 0$ passing through the intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$, is perpendicular to one of them, then the value of $|\lambda + 1|$ is	(r)	4
(D)	If the line $y - x - 1 + \lambda = 0$ is equidistant from the points $(1, -2)$ and $(3, 4)$, then the value of $|\lambda|$ is	(s)	5

73. Consider the triangle formed by the lines $y + 3x + 2 = 0$, $3y - 2x - 5 = 0$ and $4y + x - 14 = 0$

	Column I		Column II
(A)	If $(0, \lambda)$ lies inside the triangle, then integral values are less than $\|3\lambda\|$	(p)	4
(B)	If $(1, \lambda)$ lies inside the triangle, then integral values are less than $\|3\lambda\|$	(q)	5
(C)	If $(\lambda, 2)$ lies inside the triangle, then integral values of $\|6\lambda\|$ are	(r)	6
(D)	If $(\lambda, 7/2)$ lies inside the triangle, then integral value of $\|6\lambda\|$ are	(s)	7

74. Match the following

	Column I		Column II
(A)	The area bounded by the curve max. $\{\|x\|, \|y\|\} = 1$ is	(p)	0
(B)	If the point (a, a) lies between the lines $\|x + y\| = 6$, then $[\|a\|]$ is (where [.] denotes the greatest integer function)	(q)	1
(C)	Number of integral values of b for which the origin and the point (1, 1) lie on the same side of the st. line $a^2x + aby + 1 = 0$ for all $a \in R \sim \{0\}$ is	(r)	2
		(s)	3
		(t)	4

75. Match the following

	Column I		Column II
(A)	If the distance of any point (x, y) from origin is defined as $d(x, y) = 2\|x\| + 3\|y\|$. If perimeter and area of figure bounded by $d(x, y) = 6$ are λ unit and μ sq units respectively, then	(p)	(λ, μ) lies on $x = 3y$
(B)	If the vertices of a triangle are (6, 0), (0, 6) and (6, 6). If distance between circumcentre and orthocentre and distance between circumcentre and centroid are λ unit and μ unit respectively, then	(q)	(λ, μ) lies on $x^2 - y^2 = 64$
(C)	The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6). If the third vertex is (λ, μ), then	(r)	(λ, μ) lies on $x^2 + y^2 - 6x - 6y = 0$
		(s)	(λ, μ) lies on $x^2 - 16y = 16$
		(t)	(λ, μ) lies on $x^2 - y^2 = 16$

The Straight Lines Exercise 6 :
Statement I and II Type Questions

- **Directions** (Q. Nos 76 to 83) are Assertion-Reason type questions. Each of these question contains two statements.

Statement I (Assertion) and
Statement II (Reason)

Each of these questions has four alternative choices, only one of which is the correct answer.

You have to select the correct choice.

(a) Statement I is true, statement II is true; statement II is a correct explanation for statement I
(b) Statement I is true, statement II is true; statement II is not a correct explanation for statement I
(c) Statement I is true, statement II is false
(d) Statement I is false, statement II is true

76. Statement I The lines $x(a + 2b) + y(a + 3b) = a + b$ are concurrent at the point $(2, -1)$

Statement II The lines $x + y - 1 = 0$ and $2x + 3y - 1 = 0$ intersect at the point $(2, -1)$

77. Statement I The points (3, 2) and (1, 4) lie on opposite side of the line $3x - 2y - 1 = 0$

Statement II The algebraic perpendicular distance from the given point to the line have opposite sign.

78. Statement I If sum of algebraic distances from points $A(1, 2)$, $B(2, 3)$, $C(6, 1)$ is zero on the line $ax + by + c = 0$, then $2a + 3b + c = 0$

Statement II The centroid of the triangle is (3, 2)

79. Statement I Let $A \equiv (0, 1)$ and $B \equiv (2, 0)$ and P be a point on the line $4x + 3y + 9 = 0$, then the co-ordinates of P such that $|PA - PB|$ is maximum is $\left(-\frac{12}{5}, \frac{17}{5}\right)$.

Statement II $|PA - PB| \le |AB|$

80. Statement I The incentre of a triangle formed by the line $x \cos\left(\frac{\pi}{9}\right) + y \sin\left(\frac{\pi}{9}\right) = \pi$,

$x \cos\left(\frac{8\pi}{9}\right) + y \sin\left(\frac{8\pi}{9}\right) = \pi$ and $x \cos\left(\frac{13\pi}{9}\right) + y \sin\left(\frac{13\pi}{9}\right) = \pi$ is (0, 0).

Statement II Any point equidistant from the given three non-concurrent straight lines in the plane is the incentre of the triangle.

81. **Statement I** Reflection of the point (5, 1) in the line $x + y = 0$ is $(-1, -5)$.

Statement II Reflection of a point $P(\alpha, \beta)$ in the line $ax + by + c = 0$ is $Q(\alpha', \beta')$, if $\left(\frac{\alpha + \alpha'}{2}, \frac{\beta + \beta'}{2}\right)$ lies on the line.

82. **Statement I** The internal angle bisector of angle C of a triangle ABC with sides AB, AC and BC as $y = 0$, $3x + 2y = 0$, and $2x + 3y + 6 = 0$, respectively, is $5x + 5y + 6 = 0$.

Statement II The image of point A with respect to $5x + 5y + 6 = 0$ lies on the side BC of the triangle.

83. **Statement I** If the point $(2a - 5, a^2)$ is on the same side of the line $x + y - 3 = 0$ as that of the origin, then $a \in (2, 4)$.

Statement II The point (x_1, y_1) and (x_2, y_2) lie on the same or opposite sides of the line $ax + by + c = 0$, as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same or opposite signs.

The Straight Lines Exercise 7 :
Subjective Type Questions

- In this section, there are **15 subjective questions**.

84. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then show that the equation of the line joining A and the circumcentre is given by

$$(\sin 2B)\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + (\sin 2C)\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

85. Find the coordinates of the point at unit distance from the lines

$3x - 4y + 1 = 0, 8x + 6y + 1 = 0$.

86. A variable line makes intercepts on the coordinate axes, the sum of whose squares is constant and equal to k^2. Show that the locus of the foot of the perpendicular from the origin to this line is $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = k^2$.

87. A variable line intersects n lines $y = mx, (m = 1, 2, 3, \dots, n)$ in the points $A_1, A_2, A_3, \dots, A_n$ respectively.

If $\sum_{p=1}^{n} \frac{1}{OA_p} = c$ (constant). Show that line passes through a fixed point. Find the coordinates of this fixed point (O being origin).

88. Given n straight lines and a fixed point O. A straight line is drawn through O meeting these lines in the points $R_1, R_2, R_3, \dots, R_n$ and a point R is taken on it such that

$$\frac{n}{OR} = \sum_{r=1}^{n} \frac{1}{OR_r}$$

Prove that the locus of R is a straight line.

89. Prove that all lines represented by the equation

$$(2\cos\theta + 3\sin\theta)\, x + (3\cos\theta - 5\sin\theta)\, y = 5\cos\theta - 2\sin\theta$$

pass through a fixed point for all θ. What are the coordinates of this fixed point and its reflection in the line $x + y = \sqrt{2}$? Prove that all lines through reflection point can be represented by equation

$$(2\cos\theta + 3\sin\theta)\, x + (3\cos\theta - 5\sin\theta)\, y = (\sqrt{2} - 1)(5\cos\theta - 2\sin\theta)$$

90. P is any point on the line $x - a = 0$. If A is the point $(a, 0)$ and PQ, the bisector of the angle OPA, meets the X-axis in Q. Prove that the locus of the foot of the perpendicular from Q on OP is

$(x - a)^2 (x^2 + y^2) = a^2 y^2$.

91. Having given the bases and the sum of the areas of a number of triangles is constant, which have a common vertex. Show that the locus of this vertex is a straight line.

92. $A(3, 0)$ and $B(6, 0)$ are two fixed points and $U(\alpha, \beta)$ is a variable point on the plane. AU and BU meet the y-axis at C and D respectively and AD meets OU at V. Prove that CV passes through (2, 0) for any position of U in the plane.

93. A variable line is drawn through O to cut two fixed straight lines L_1 and L_2 in R and S. A point P is chosen on the variable line such that $\frac{m+n}{OP} = \frac{m}{OR} + \frac{n}{OS}$. Show that the locus of P is a straight line passing through the point of intersection of L_1 and L_2.

94. A line through $A(-5, -4)$ meets the lines $x + 3y + 2 = 0, 2x + y + 4 = 0$ and $x - y - 5 = 0$ at the points B, C and D respectively, if

$$\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$$

find the equation of the line.

95. Two fixed straight lines X-axis and $y = mx$ are cut by a variable line in the points $A(a, 0)$ and $B(b, mb)$ respectively. P and Q are the feet of the perpendiculars drawn from A and B upon the lines $y = mx$ and X-axis. Show that, if AB passes through a fixed point (h, k), then PQ will also pass through a fixed point. Find the fixed point.

96. Find the equation of straight lines passing through point $(2, 3)$ and having an intercept of length 2 units between the straight lines $2x + y = 3$, $2x + y = 5$.

97. Let $O(0, 0)$, $A(2, 0)$ and $B\left(1, \frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside ΔOAB which satisfy $d(P, OA) \le \min\{d(P, OB), d(P, AB)\}$ where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area.

98. Two triangles ABC and PQR are such that the perpendiculars from A to QR, B to RP and C to PQ are concurrent. Show that the perpendicular from P to BC, Q to CA and R to AB are also concurrent.

The Straight Lines Exercise 8 :
Questions Asked in Previous 13 Year's Exams

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main** & **JEE Advanced** from year **2005** to **2017**.

99. The line parallel to the X-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \ne (0, 0)$ is **[AIEEE 2005, 3M]**

(a) below the X-axis at a distance of $\frac{3}{2}$ from it

(b) below the X-axis at a distance of $\frac{2}{3}$ from it

(c) above the X-axis at a distance of $\frac{3}{2}$ from it

(d) above the X-axis at a distance of $\frac{2}{3}$ from it

100. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is **[AIEEE 2006, 4.5M]**

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$
(c) $4x + 3y = 24$ (d) $3x + 4y = 25$

101. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belong to **[AIEEE 2006, 6M]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$
(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$

102. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT-JEE 2007, 3M]**

Statement I The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$ because

Statement II In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement I is true, statement II is true; statement II is not a correct explanation for statement I
(b) Statement I is true, statement II is true; statement II is not a correct explanation for statement I
(c) Statement I is true, statement II is false
(d) Statement I is false, statement II is true

103. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is **[AIEEE 2007, 3M]**

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

104. Consider the lines given by

$$L_1 : x + 3y - 5 = 0$$
$$L_2 : 3x - ky - 1 = 0$$
$$L_3 : 5x + 2y - 12 = 0$$

Match the statements/Expressions in **Column I** with the statements/Expressions in **Column II**

	Column I		Column II
(A)	L_1, L_2, L_3 are concurrent, if	(p)	$k = -9$
(B)	one of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q)	$k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(r)	$k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(s)	$k = 5$

[IIT-JEE 2008, 6M]

105. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4. Then a possible value of k is **[AIEEE 2008, 3M]**

(a) 1 (b) 2 (c) -2 (d) -4

106. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for **[AIEEE 2009, 4M]**

(a) exactly one values of p (b) exactly two values of p
(c) more than two values of p (d) no value of p

107. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is **[AIEEE 2010, 4M]**

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$
(c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

108. A straight line L through the point $(3,-2)$ is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If L also intersects the X-axis, then the equation of L is **[IIT-JEE 2011, 3M]**

(a) $y+\sqrt{3}x+2-3\sqrt{3}=0$ (b) $y-\sqrt{3}x+2+3\sqrt{3}=0$
(c) $\sqrt{3}y-x+3+2\sqrt{3}=0$ (d) $\sqrt{3}y+x-3+2\sqrt{3}=0$

109. The lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y+2=0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE 2011, 4M]**

Statement I : The ratio $PR:RQ$ equals $2\sqrt{2}:\sqrt{5}$

Statement II : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement I is true, statement II is true; statement II is not a correct explanation for statement I.
(b) Statement I is true, statement II is false.
(c) Statement I is false, statement II is true.
(d) Statement I is true, statement II is true; statement II is a correct explanation for statement I

110. If the line $2x+y=k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals **[AIEEE 2012, 4M]**

(a) $\frac{29}{5}$ (b) 5
(c) 6 (d) $\frac{11}{5}$

111. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching X-axis, the equation of the reflected ray is **[JEE Main 2013, 4M]**

(a) $y=x+\sqrt{3}$ (b) $\sqrt{3}y=x-\sqrt{3}$
(c) $y=\sqrt{3}x-\sqrt{3}$ (d) $\sqrt{3}y=x-1$

112. For $a>b>c>0$, the distance between (1, 1) and the point of intersection of the lines $ax+by+c=0$ and $bx+ay+c=0$ is less than $2\sqrt{2}$. Then **[JEE Advanced 2013, 3M]**

(a) $a+b-c>0$ (b) $a-b+c<0$
(c) $a-b+c>0$ (d) $a+b-c<0$

113. Let PS be the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is **[JEE Main 2014, 4M]**

(a) $4x+7y+3=0$ (b) $2x-9y-11=0$
(c) $4x-7y-11=0$ (d) $2x+9y+7=0$

114. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes, then **[JEE Main 2014, 4M]**

(a) $3bc-2ad=0$ (b) $3bc+2ad=0$
(c) $2bc-3ad=0$ (d) $2bc+3ad=0$

115. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x-y=0$ and $x+y=0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2\le d_1(P)+d_2(P)\le 4$, is **[JEE Advanced 2014, 3M]**

116. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is **[JEE Advanced 2015, 4M]**

(a) 820 (b) 780
(c) 901 (d) 861

117. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus? **[JEE Main 2016, 4M]**

(a) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (b) $\left(-\frac{10}{3},-\frac{7}{3}\right)$
(c) $(-3,-9)$ (d) $(-3,-8)$

Answers

Chapter Exercises

1. (b)	**2.** (a)	**3.** (c)	**4**. (d)	**5.** (c)	**6**. (b)
7. (b)	**8.** (b)	**9**. (b)	**10.** (c)	**11.** (c)	**12.** (b)
13. (c)	**14.** (d)	**15.** (a)	**16.** (b)	**17**. (b)	**18.** (c)
19. (b)	**20.** (b)	**21.** (b)	**22.** (b)	**23.** (c)	**24.** (a)
25. (a)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (a)

31. (a,b,c,d) **32.** (a,b,c,d) **33.** (a,c) **34.** (a,d)

35. (b,d) **36.** (a,b,c,d) **37.** (a,c,d) **38.** (a,d) **39.** (a,b) **40**. (a,b)

41. (a,b,c,d) **42.** (b,d) **43.** (a,b,c) **44.** (a,b) **45.** (a,b,c)

46. (d)	**47.** (d)	**48.** (a)	**49.** (a)	**50.** (d)	**51.** (a)
52. (b)	**53.** (c)	**54.** (a)	**55.** (c)	**56.** (d)	**57.** (b)
58. (a)	**59.** (a)	**60.** (b)	**61.** (3)	**62.** (6)	**63.** (5)
64. (8)	**65.** (3)	**66.** (9)	**67.** (8)	**68.** (2)	**69.** (2)

70. (4) **71.** (A) $\to$ (p); (B) $\to$ (p, q); (C) $\to$ (p, r) (D) $\to$ (p, r, s)

72. (A) $\to$ (p, r); (B) $\to$ (q); (C) $\to$ (q, s) (D) $\to$ (p)

73. (A) $\to$ (p, q); (B) $\to$ (p, q, r, s); (C) $\to$ (p, q, r, s); (D) $\to$ (p, q, r, s)

74. (A) $\to$ (t); (B) $\to$ (p, q, r); (C) $\to$ (s)

75. (A) $\to$ (q, s); (B) $\to$ (p, t); (C) $\to$ (r) **76.** (a) **77.** (a)

78. (d) **79.** (d) **80.** (c) **81**. (b) **82.** (b) **83.** (d)

85. $\left(\frac{6}{5}, \frac{-1}{10}\right), \left(-\frac{2}{5}, \frac{-13}{10}\right), \left(0, \frac{3}{2}\right), \left(\frac{-8}{5}, \frac{3}{10}\right)$

87. $\left(\frac{\pm\sum_{p=1}^{n}\frac{1}{\sqrt{(1+p^2)}}}{c}, \frac{\pm\sum_{p=1}^{n}\frac{p}{\sqrt{(1+p^2)}}}{c}\right)$ **94.** $2x + 3y + 22 = 0$

95. $\left(\frac{h+mk}{1+m^2}, \frac{mh-k}{1+m^2}\right)$ **96.** $3x + 4y - 18 = 0$ and $x - 2 = 0$

97. $(2-\sqrt{3})$ sq units. **99.** (a) **100.** (c) **101.** (c) **102.** (c) **103.** (c)

103. (c) **104.** (A) $\to$ (s); (B) $\to$ (p, q); (C) $\to$ (r); (D) $\to$ (p, q, s)

105.(a) **106.** (a) **107.** (c) **108.** (b) **109.** (b) **110.** (c) **111.** (b)

112.(a) **113.** (d) **114.** (a) **115.** (6) **116.** (b) **117.** (a)

Solutions

1. Equation of line passing through (2, 0) and perpendicular to $ax + by + c = 0$

Then, required equation is

$$y - 0 = \frac{b}{a}(x - 2)$$

$$ay = bx - 2b$$

$$\Rightarrow \quad ay - bx + 2b = 0$$

2. $\because \frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^m}{n!}$

$$\frac{1}{10!}\left\{\frac{2 \times 10!}{1!9!} + \frac{2 \times 10!}{3!7!} + \frac{10!}{5!5!}\right\} = \frac{2^m}{n!}$$

$$\Rightarrow \quad \frac{1}{10!}\{2\,{}^{10}C_1 + 2\,{}^{10}C_3 + {}^{10}C_5\} = \frac{2^m}{n!}$$

$$\frac{1}{10!}\{{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\} = \frac{2^m}{n!}$$

$$\Rightarrow \quad \frac{1}{10!}(2)^{10-1} = \frac{2^m}{n!}$$

$$\therefore \quad m = 9 \text{ and } n = 10$$

Hence, $x - y + 1 = 0$ and $x + y + 3 = 0$ are perpendicular to each other, then orthocentre is the point of intersection which is $(-2, -1)$

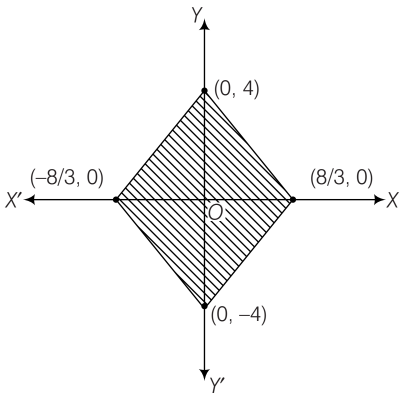
$\therefore \quad -2 = 2m - 2n$ and $-1 = m - n$

$\therefore$ Point is $(2m - 2n, m - n)$.

3. $\therefore$ Required area

$$= 4 \times \frac{1}{2}\left(\frac{8}{3} \times 4\right) = \frac{64}{3} = \frac{2^6}{3} \qquad \text{...(i)}$$

$$\because \quad f(x + y) = f(x)f(y)$$



$$\therefore \quad f(2) = f(1)f(1) = 2^2$$

$$f(3) = f(1 + 2) = f(1)f(2) = 2^3$$

....

....

$$\therefore \quad f(n) = 2^n$$

$$\therefore \quad \text{Area} = \frac{2^6}{3} = \frac{f(6)}{3} \text{ sq units}$$

4. We have, $y = \cos x \cos(x + 2) - \cos^2(x + 1)$

$$y = \frac{1}{2}\{2\cos x \cos(x + 2) - 2\cos^2(x + 1)\}$$

$$= \frac{1}{2}\{\cos(2x + 2) + \cos 2 - 1 - \cos(2x + 2)\}$$

$$= \frac{1}{2}(\cos 2 - 1)$$

$$= \frac{1}{2}(1 - 2\sin^2 1 - 1)$$

$$= -\sin^2 1$$

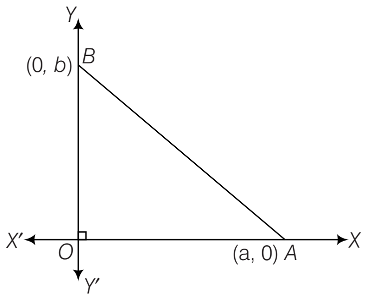
which is a straight line passing through $(\lambda, -\sin^2 1)$; $\forall\ \lambda \in R$ and parallel to the X-axis.

5. Let line $\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

Its passes through (2, 2), then

$$\frac{2}{a} + \frac{2}{b} = 1$$

$$\Rightarrow \quad 2(a + b) = ab \qquad \text{...(ii)}$$



$\because$ Area of $\Delta AOB = \frac{1}{2}ab = |\lambda|$ (given)

$$\therefore \quad ab = 2|\lambda|$$

from Eq. (ii), $a + b = |\lambda|$

Hence, required equation is

$$x^2 - (a + b)x + ab = 0$$

$$\text{or} \quad x^2 - |\lambda|x + 2|\lambda| = 0$$

6. $\frac{\text{Value of } (a^2x + aby + 1) \text{ at } (1, 1)}{\text{Value of } (a^2x + aby + 1) \text{ at } (0, 0)} > 0$

$$\text{or} \quad \frac{a^2 + ab + 1}{1} > 0;\ \forall\ a \in R$$

$$\text{or} \quad a^2 + ab + 1 > 0;\ \forall\ a \in R$$

$$\therefore \quad D < 0$$

$$\Rightarrow \quad b^2 - 4 < 0$$

$$\Rightarrow \quad -2 < b < 2 \text{ but } b > 0$$

$$\therefore \quad 0 < b < 2$$

$$\text{i.e.} \quad b \in (0, 2)$$

7. Equation of L is $\frac{x}{a}+\frac{y}{b}=1$ and let the axis be rotated through an angle θ and let (X, Y) be the new coordinates of any point $P(x, y)$ in the plane, then

$x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$, the equation of the line with reference to original coordinates is

$$\frac{x}{a}+\frac{y}{b}=1$$

i.e. $$\frac{X\cos\theta - Y\sin\theta}{a}+\frac{X\sin\theta + Y\cos\theta}{b}=1 \quad \text{...(i)}$$

and with reference to new coordinates is

$$\frac{X}{p}+\frac{Y}{q}=1 \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{\cos\theta}{a}+\frac{\sin\theta}{b}=\frac{1}{p} \quad \text{...(iii)}$$

and $$-\frac{\sin\theta}{a}+\frac{\cos\theta}{b}=\frac{1}{q} \quad \text{...(iv)}$$

Squaring and adding Eqs. (iii) and (iv), we get

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

8. $d(x, y) = \max\{|x|, |y|\}$ ...(i)

but $d(x, y) = a$ ...(ii)

From Eqs. (i) and (ii), we get

$a = \max\{|x|, |y|\}$

if $|x| > |y|$, then $a = |x|$

$\therefore$ $x = \pm a$

and if $|y| > |x|$, then $a = |y|$

$\therefore$ $y = \pm a$

Therefore locus represents a straight line.

9. $P_1 = |m^2\cos\alpha + 2m\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}|$

$$= \frac{(m\cos\alpha + \sin\alpha)^2}{|\cos\alpha|}$$

$$p_2 = \left|mm'\cos\alpha + (m+m')\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right|$$

$$= \frac{|(m\cos\alpha + \sin\alpha)|\,|m'\cos\alpha + \sin\alpha|}{|\cos\alpha|}$$

and $$p_3 = |m'^2\cos\alpha + 2m'\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}|$$

$$= \frac{(m'\cos\alpha + \sin\alpha)^2}{|\cos\alpha|}$$

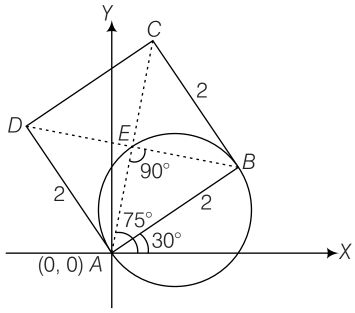
$\because$ $p_2^2 = p_1 p_3$

Hence, p_1, p_2, p_3 are in GP.

10. Side of the square = 2 unit

Coordinates of B, C and D are $(\sqrt{3}, 1)$, $(\sqrt{3}-1, \sqrt{3}+1)$ and $(-1, \sqrt{3})$ respectively.

Slope of $BD = \frac{\sqrt{3}-1}{-1-\sqrt{3}} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{-2} = \sqrt{3}-2$



$\therefore$ Equation of BD is

$$y - 1 = (\sqrt{3}-2)(x-\sqrt{3})$$

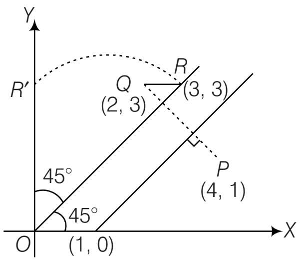
$\Rightarrow$ $$(2-\sqrt{3})x + y = 2(\sqrt{3}-1)$$

and equation of the circumcircle of the triangle ABE (Apply diametric form as AB is diameter)

$$(x-0)(x-\sqrt{3}) + (y-0)(y-1) = 0$$

$\Rightarrow$ $$x^2 + y^2 - x\sqrt{3} - y = 0$$

11. If (α, β) be the image of (4, 1) w.r.t $y = x - 1$, then $(\alpha, \beta) = (2, 3)$, say point Q



After translation through a distance 1 unit along the positive direction of X-axis at the point whose coordinate are $R \equiv (3, 3)$.

After rotation through are angle $\frac{\pi}{4}$ about the origin in the anticlockwise direction, then R goes to R' such that

$$OR = OR' = 3\sqrt{2}$$

$\therefore$ The coordinates of the final point are $(0, 3\sqrt{2})$.

12. $\because$ $A \equiv (0, 0); B \equiv (2, 0); C \equiv (2, 2); D \equiv (0, 2)$

(i) $f_1(x, y) \to (y, x)$, then

$A \equiv (0, 0); B \equiv (0, 2); C \equiv (2, 2), D \equiv (2, 0)$

(ii) $f_2(x, y) \to (x + 3y, y)$, then

$A \equiv (0, 0); B \equiv (6, 2); C \equiv (8, 2), D \equiv (2, 0)$

(iii) $f_3(x, y) \to \left(\frac{x-y}{2}, \frac{x+y}{2}\right)$, then

$A \equiv (0, 0); B \equiv (2, 4); C \equiv (3, 5), D \equiv (1, 1)$

Now, $AB = DC = 2\sqrt{5}$, $AD = BC = \sqrt{2}$

and $AC = \sqrt{34}$, $BD = \sqrt{10}$

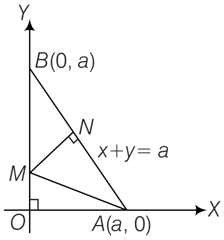
i.e. $AC \neq BD$

$\therefore$ Final figure is a parallelogram.

13. Let $\frac{AN}{BN} = \lambda$

Then, coordinate of N are $\left(\frac{a}{1+\lambda}, \frac{a\lambda}{1+\lambda}\right)$

$\because$ Slope of $AB = -1$



$\therefore$ Slope of $MN = 1$

$\therefore$ Equation on MN is

$$y - \frac{a\lambda}{1+\lambda} = x - \frac{a}{1+\lambda} \Rightarrow x - y = a\left(\frac{1-\lambda}{\lambda+1}\right)$$

So, the coordinates of M are $\left(0, a\left(\frac{\lambda-1}{\lambda+1}\right)\right)$

Therefore, area of $\Delta AMN = \frac{3}{8}$ area of ΔOAB

$$\Rightarrow \quad \frac{1}{2}\cdot AN\cdot MN = \frac{3}{8}\cdot\frac{1}{2}a\cdot a$$

$$\Rightarrow \quad \frac{1}{2}\cdot\left|\frac{a\lambda\sqrt{2}}{1+\lambda}\cdot\frac{a\sqrt{2}}{1+\lambda}\right| = \frac{3}{8}\cdot\frac{1}{2}a\cdot a$$

$$\Rightarrow \quad \frac{a^2\lambda}{(1+\lambda)^2} = \frac{3}{8}\cdot\frac{1}{2}a^2$$

$$\therefore \quad \lambda = 3 \quad \text{or} \quad \lambda = \frac{1}{3}$$

For $\lambda = \frac{1}{3}$, then M lies outside the segment OB and hence the required value of $\lambda = 3$.

14. Let $S = (x, y)$, given $(SQ)^2 + (SR)^2 = 2(SP)^2$

$$\Rightarrow \quad (x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1^2) + y^2]$$

$$\Rightarrow \quad 2x^2 + 2y^2 - 2x + 5 = 2(x^2 + y^2 - 2x + 1)$$

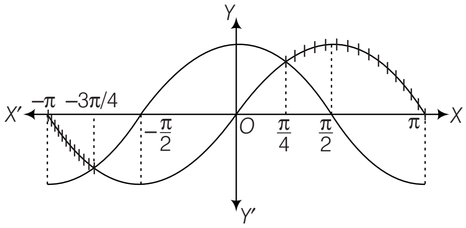
$$\Rightarrow \quad 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$

A straight line parallel to Y-axis.

15. $$\frac{\text{Value of}(3x-2y+1)\text{ at }A}{\text{Value of }(3x-2y+1)\text{ at }B} > 0$$

$$\Rightarrow \frac{(\sin\alpha - 3) - (\cos\alpha - 2) + 1}{(3-2+1)} > 0$$

$$\Rightarrow \quad \sin\alpha - \cos\alpha > 0 \Rightarrow \sin\alpha > \cos\alpha$$



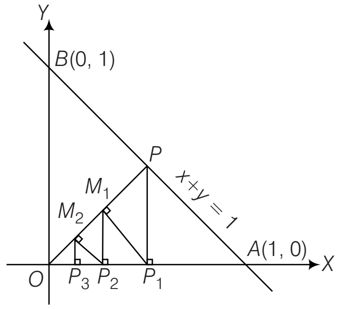
It is clear from the figure

$$\alpha \in \left(-\pi, \frac{-3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right).$$

16. $\because$ Equation of AB is $x + y = 1$, then coordinates of A and B are (1, 0) and (0, 1) respectively.

$\therefore$ Coordinates of P are $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\because$ PP_1 is perpendicular to OA



Equation of OP is $y = x$

Then, $\quad OP_1 = PP_1 = \frac{1}{2}$

We have, $\quad (OM_{n-1})^2 = (OP_n)^2 + (P_nM_{n-1})^2$

$$= 2(OP_n)^2 \qquad \{\because y = x\}$$

$$= 2\alpha_n^2 \text{ (say)}$$

Also, $\quad (OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2$

$$\alpha_{n-1}^2 = 2\alpha_n^2 + \frac{1}{2}\alpha_{n-1}^2$$

$$\Rightarrow \quad \frac{1}{2}\alpha_{n-1}^2 = 2\alpha_n^2$$

$$\Rightarrow \quad \alpha_n = \frac{1}{2}\alpha_{n-1}$$

$$\therefore \quad OP_n = \alpha_n = \frac{1}{2}\alpha_{n-1}$$

$$= \frac{1}{2^2}\alpha_{n-2} = \frac{1}{2^3}\alpha_{n-3}$$

....

....

....

$$= \frac{1}{2^{n-1}}\alpha_1$$

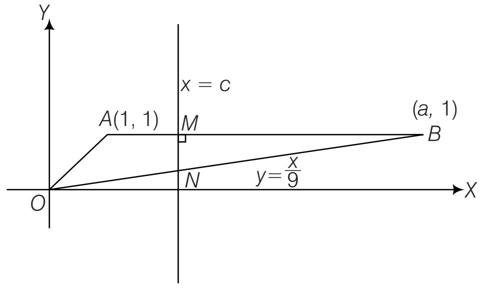
$$= \frac{1}{2^{n-1}}\left(\frac{1}{2}\right) = \frac{1}{2^n}.$$

17. Let $O \equiv (0, 0)$, $A \equiv (1, 1)$ and $B \equiv (9, 1)$

Area of $\Delta OAB = \frac{1}{2}\times 8\times 1 = 4$

It is clear that $1 < c < 9$

and $\quad M \equiv (c, 1)$ and $N \equiv \left(c, \frac{c}{9}\right)$



$\therefore$ Area of $\Delta BMN = 2$ (given)

$$\Rightarrow \quad \frac{1}{2} \times (9-c) \times \left(1 - \frac{c}{9}\right) = 2$$

or $(9-c)^2 = 36$

or $9 - c = \pm 6 \Rightarrow c = 3$ or 15

but $1 < c < 9$

$\therefore$ $c = 3$

18. The three lines are concurrent if

$$\begin{vmatrix} 1 & 2 & -9 \\ 3 & -5 & -5 \\ a & b & -1 \end{vmatrix} = 0$$

or $5a + 2b = 1$

which is three of the line $5x + 2y = 1$ passes through (a, b).

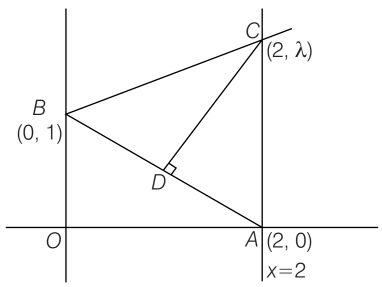
19. $\because BC = AC$

$$\Rightarrow \quad 2^2 + (\lambda - 1)^2 = \lambda^2$$

$$\Rightarrow \quad 4 = \lambda^2 - (\lambda - 1)^2$$

$$= (2\lambda - 1)(1)$$

$$\therefore \quad \lambda = \frac{5}{2}$$



$\because$ Equation. of AB is $\frac{x}{2} + \frac{y}{1} = 1$, $D \equiv \left(1, \frac{1}{2}\right)$ (mid-point of AB)

$\therefore$ Equation of CD is $2x - y = \mu$

$\because$ CD pass through D, thus

$$2 - \frac{1}{2} = \mu \text{ or } \mu = \frac{3}{2}$$

$\therefore$ Equation of CD is $\quad 2x - y = \frac{3}{2}$ ...(i)

and Eq. (i) of line $\perp$ to AC and pass through B is $y = 1$ ...(ii)

from Eqs. (i) and (ii), we get

Orthocentre $\equiv \left(\frac{5}{4}, 1\right)$

20. Let $A \equiv (3, 4)$, $B \equiv (0, y)$, $C \equiv (x, 0)$, $D \equiv (8, 2)$

$\therefore$ Slope of $AB = -$ Slope of BC

$$\Rightarrow \quad \frac{y-4}{0-3} = -\left(\frac{0-y}{x-0}\right)$$

or $4x - xy = 3y$ ...(i)

and slope of $BC = -$ slope of CD

$$\Rightarrow \quad \left(\frac{0-y}{x-0}\right) = -\left(\frac{2-0}{8-x}\right)$$

or $2x + xy = 8y$ ...(ii)

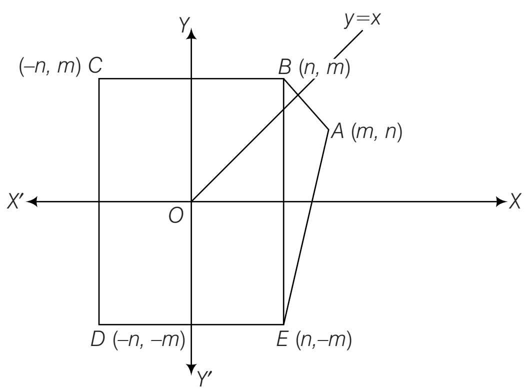
adding Eqs. (i) and (ii), we get

$6x = 11y$ ...(iii)

from Eqs. (ii) and (iii), we get

$$x = \frac{13}{3} = 4\frac{1}{3}$$

21.



Area of rectangle $BCDE = (2n)(2m)$

$= 4mn$

and area of $\Delta ABE = \frac{1}{2} \times 2m \times (m - n)$

$= m(m - n)$

$\therefore$ Area of pentagon $= 4mn + m(m - n)$

$= m(m + 3n)$

22. The equation of the line L, be $y - 2 = m(x - 8)$, $m < 0$

coordinates of P and Q are $P\left(8 - \frac{2}{m}, 0\right)$ and $Q(0, 2 - 8m)$.

So, $\quad OP + OQ = 8 - \frac{2}{m} + 2 - 8m$

$$= 10 + \frac{2}{(-m)} + 8(-m) \geq$$

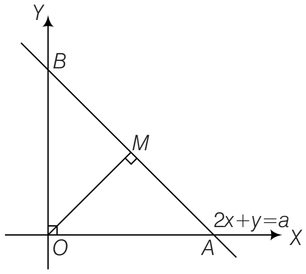
$$10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18$$

So, absolute minimum value of $OP + OQ = 18$

23. Let the two perpendiculars through the origin intersect $2x + y = a$ at A and B so that the triangle OAB is isosceles.

$OM =$ length of perpendicular from O to

AB, $OM = \frac{a}{\sqrt{5}}$.



Also, $AM = MB = OM$

$\Rightarrow \quad AB = \frac{2a}{\sqrt{5}}$

Area of $\Delta OAB = \frac{1}{2} \cdot AB \cdot OM$

$$= \frac{1}{2} \cdot \frac{2a}{\sqrt{5}} \cdot \frac{a}{\sqrt{5}} = \frac{a^2}{5} \text{ sq units}$$

24. Solving given equations, we get

$$x = \frac{5}{3 + 4m}$$

x is an integer, if $3 + 4m = 1, -1, 5, -5$

or $\quad m = \frac{-2}{4}, \frac{-4}{4}, \frac{2}{4}, \frac{-8}{4}$

or $\quad m = -\frac{1}{2}, -1, \frac{1}{2}, -2$

Hence, m has two integral values.

25. Let the coordinates of A be $(a, 0)$. Then the slope of the reflected ray is

$$\frac{3-0}{5-a} = \tan\theta \quad \text{(say) ...(i)}$$

Then the slope of the incident ray

$$= \frac{2-0}{1-a} = \tan(\pi - \theta)$$

From Eqs. (i) and (ii), we get

$$\tan\theta + \tan(\pi - \theta) = 0$$

$\Rightarrow \quad \frac{3}{5-a} + \frac{2}{1-a} = 0$

$\Rightarrow \quad 3 - 3a + 10 - 2a = 0$

$$a = \frac{13}{5}$$

Thus, the coordinate of A is $\left(\frac{13}{5}, 0\right)$

26. Lines $5x + 3y - 2 + \lambda(3x - y - 4) = 0$ are concurrent at $(1, -1)$ and lines

$x - y + 1 + \mu(2x - y - 2) = 0$ are concurrent at $(3, 4)$.

Thus equation of line common to both family is

$$y + 1 = \frac{4+1}{3-1}(x - 1)$$

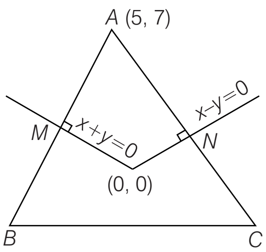
or $\quad 5x - 2y - 7 = 0$

$\therefore \quad a = 5, b = -2 \Rightarrow a + b = 3$

27. $\because$ B is the reflection of $A(5, 7)$ w.r.t the line $x + y = 0$

$\therefore \quad B \equiv (-7, -5)$

and C is the reflection of $A(5, 7)$ w.r.t the line $x - y = 0$



$\therefore \quad C \equiv (7, 5)$

$\therefore$ Equation of BC is $y + 5 = \frac{5+5}{7+7}(x + 7)$ or $7y = 5x$

28. Let $P \equiv (2, -1)$

$P(2, -1)$ goes 2 units along $x + y = 1$ upto A and 5 units along $x - 2y = 4$ upto B.

Now, slope of $x + y = -1$ is $-1 = \tan\theta$ (say)

$\therefore \quad \theta = 135°$

and slope $x - 2y = 4$ is $\frac{1}{2} = \tan\phi$ (say)

$\therefore \quad \sin\phi = \frac{1}{\sqrt{5}}, \cos\phi = \frac{2}{\sqrt{5}}$

The coordinates of A

i.e. $(2 + 2\cos 135°, -1 + 2\sin 135°)$

or $(2 - \sqrt{2}, \sqrt{2} - 1)$

The coordinates of B

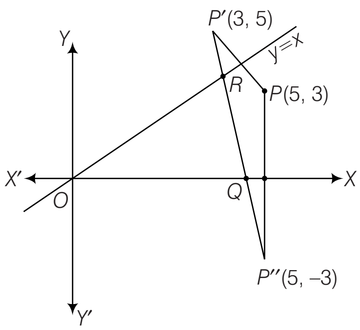
i.e. $(2 + 5\cos\phi, -1 + 5\sin\phi)$ or $(2 + 2\sqrt{5}, \sqrt{5} - 1)$

29. $\because \quad P \equiv (5, 3)$

Let P' and P'' be the images of P w.r.t $y = x$ and $y = 0$ (X-axis) respectively, then $P' \equiv (3, 5)$ and $P'' \equiv (5, -3)$

$\because \quad PQ + QR + RP$ is minimum

$\therefore \quad P', R, Q, P''$ are collinear.



$\therefore$ Equation of $P'P''$ is

$$y + 3 = \left(\frac{5+3}{3-5}\right)(x - 5)$$

or $\quad 4x + y = 17$

$\therefore \quad Q \equiv \left(\frac{17}{4}, 0\right) \quad (\because Q \text{ on } Y\text{-axis})$

30. Equation of incident ray is

$$y - 0 = \tan(90° + 60°)(x - 2)$$

or $$y = -\frac{1}{\sqrt{3}}(x - 2)$$

or $$(x - 2) + y\sqrt{3} = 0$$

and equation of refracted ray is

$$y - 0 = -\tan 60°(x - 2)$$

or $$y = -\sqrt{3}(x - 2)$$

or $$(x - 2) + \frac{y}{\sqrt{3}} = 0$$

$\therefore$ Combined equation is

$$[(x - 2) + y\sqrt{3}]\left((x - 2) + \frac{y}{\sqrt{3}}\right) = 0$$

i.e. $$(x - 2)^2 + y^2 + \frac{4}{\sqrt{3}}(x - 2)y = 0$$

31. Point of intersection of $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ is $P\left(\frac{ab}{a + b}, \frac{ab}{a + b}\right)$, this point P satisfies alternates (a), (b), (c) and (d).

32. The two lines will be identical if their exists some real number k such that

$b^3 - c^3 = k(b - c)$, $c^3 - a^3 = k(c - a)$ and $a^3 - b^3 = k(a - b)$

$\Rightarrow$ $b - c = 0$ or $b^2 + c^2 + bc = k$

$c - a = 0$ or $c^2 + a^2 + ca = k$

and $a - b = 0$ or $a^2 + b^2 + ab = k$

$\Rightarrow$ $a = b$ or $b = c$ or $c = a$

or $b^2 + c^2 + bc = c^2 + a^2 + ca$

$\Rightarrow$ $b = c$ or $c = a$

or $a = b$ or $a + b + c = 0$

33. As the third vertex lies on the line $y = x + 3$, its coordinates are of the form $(x, x + 3)$. The area of the triangle with vertices $(2, 1)$, $(3, -2)$ and $(x, x + 3)$ is given by

$$\frac{1}{2}\left|\begin{vmatrix} x & x+3 & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{vmatrix}\right| = |2x - 2| = 5 \quad \text{(given)}$$

$\therefore$ $2x - 2 = \pm 5 \Rightarrow x = \frac{-3}{2}, \frac{7}{2}$

Thus, the coordinates of the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$.

34. $$\begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$$

$\Rightarrow$ $\lambda^2 + 2\lambda - 8 = 0$

$\therefore$ $(\lambda + 4)(\lambda - 2) = 0$

$\Rightarrow$ $\lambda = -4, 2$

35. Equation of any line through the point of intersection of the given lines is $(3x + y - 5) + \lambda(x - y + 1) = 0$.

Since this line is perpendicular to one of the given lines

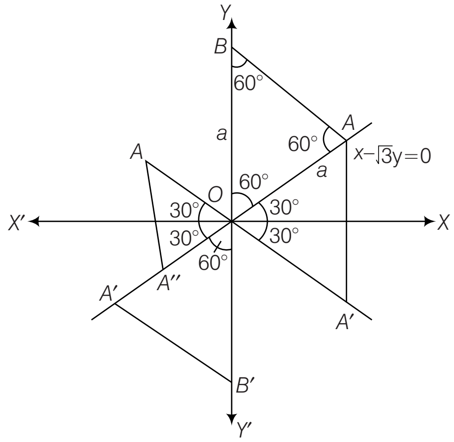
$$\frac{3 + \lambda}{\lambda - 1} = -1 \text{ or } \frac{1}{3}$$

$\Rightarrow \lambda = -1$ or -5, therefore the required straight line is

$x + y - 3 = 0$

or $x - 3y + 5 = 0$

36. If B lies on Y-axis, then coordinates of B are $(0, a)$ or $(0, -a)$



If third vertex in IV quadrant or in II quadrant, then its coordinates are $(a\cos 30°, -a\sin 30°)$ and $(-a\cos 30°, a\sin 30°)$

i.e. $\left(\frac{a\sqrt{3}}{2}, -\frac{a}{2}\right)$ and $\left(-\frac{a\sqrt{3}}{2}, \frac{a}{2}\right)$,

37. Since, $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent

$\therefore$ $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$\Rightarrow$ $3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow$ $-(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$a + b + c \neq 0$

$\therefore$ $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$$\frac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\} = 0$$

As a, b, c are real numbers

$\therefore$ $b - c = 0, c - a = 0, a - b = 0$

$\Rightarrow$ $a = b = c$

38. $\because$ $E \equiv (4, 4)$

$\therefore z_C = 7 + 5i, z_E = 4 + 4i$

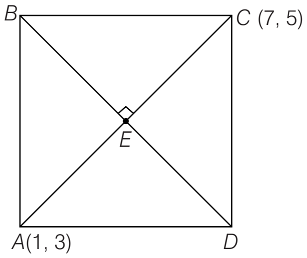
Now, (in ΔBEC)

$$\frac{z_B - z_E}{z_C - z_E} = e^{i\frac{\pi}{2}} = i$$

$\Rightarrow$ $z_B - 4 - 4i = i(7 + 5i - 4 - 4i)$

or $z_B = 3 + 7i$

$\therefore \quad B \equiv (3, 7)$, then $D \equiv (5, 1)$



Equation of AB is

$$y - 3 = \frac{7-3}{3-1}(x-1) \text{ or } 2x - y + 1 = 0$$

and equation of AD is

$$y - 3 = \frac{1-3}{5-1}(x-1) \text{ or } x + 2y - 7 = 0$$

39. Given,

$$6a^2 - 3b^2 - c^2 + 7ab - ac + 4bc = 0$$

$$\Rightarrow 6a^2 + (7b - c)a - (3b^2 - 4bc + c^2) = 0$$

$$\Rightarrow a = \frac{-(7b-c) \pm \sqrt{(7b-c)^2 + 24(3b^2 - 4bc + c^2)}}{12}$$

$$\Rightarrow \quad 12a + 7b - c = \pm(11b - 5c)$$

$$\Rightarrow \quad 12a - 4b + 4c = 0$$

or $\quad 12a + 18b - 6c = 0$

$$\Rightarrow \quad 3a - b + c = 0$$

or $\quad -2a - 3b + c = 0$

Hence $(3, -1)$ or $(-2, -3)$ lies on the line $ax + by + c = 0$,

40. $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$

$$\Rightarrow \quad x + 2y + 4 = 0$$

and $\quad -4x - 2y + 1 = 0$

Here, $\quad (1)(-4) + (2)(-2) = -8 < 0$

$\therefore$Bisector of the angle including the acute angle bisectors and origin is

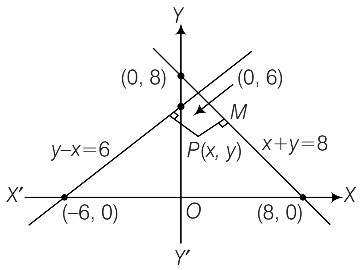
$$\frac{x + 2y + 4}{\sqrt{5}} = \frac{(-4x - 2y + 1)}{2\sqrt{5}}$$

$$\Rightarrow \quad 6x + 6y + 7 = 0$$

41. Let position of bunglow is $P(x_1, y_1)$, then $PM = 100$ and $PN = 100$

$$\therefore \quad \frac{x_1 + y_1 - 8}{\sqrt{2}} = \pm 100$$

and $\quad \dfrac{x_1 - y_1 + 6}{\sqrt{2}} = \pm 100$



After solving, we get

$$x_1 = 1 \pm 100\sqrt{2}, 1$$

and $\quad y_1 = 7, 7 \pm 100\sqrt{2}$

Hence, $\quad (1 + 100\sqrt{2}, 7), (1 - 100\sqrt{2}, 7),$
$(1, 7 + 100\sqrt{2}), (1, 7 - 100\sqrt{2})$

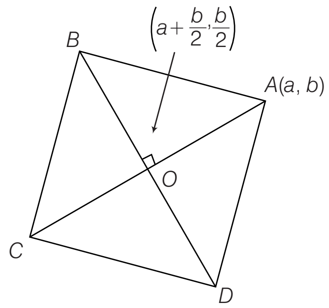
42. Equation of the other diagonal is $x + y = \lambda$ which pass through (a, b), then

$$a + b = \lambda$$

$\therefore$ Equation of other diagonal is

$$x + y = a + b$$

i.e. then centre of the square is the point of intersection of $x - y = a$ and $x + y = a + b$ is $\left(a + \frac{b}{2}, \frac{b}{2}\right)$, then vertex



$$C \equiv (2a + b - a, b - b)$$

$\therefore \quad C \equiv (a + b, 0)$

If $\quad B \equiv z$

Then, $\quad \dfrac{z - \left(a + \frac{b}{2} + \frac{ib}{2}\right)}{(a + ib) - \left(a + \frac{b}{2} + \frac{ib}{2}\right)} = \dfrac{BO}{AO} e^{i\frac{\pi}{2}} = i \quad (\because BO = AO)$

$$\Rightarrow \quad z - \left(a + \frac{b}{2} + \frac{ib}{2}\right) = i\left(-\frac{b}{2} + \frac{ib}{2}\right) = -\frac{ib}{2} - \frac{b}{2}$$

$\therefore \quad z = a$

$\therefore \quad B \equiv (a, 0)$

then, $\quad D \equiv (a + b, b)$

Hence, other vertices are $(a + b, 0), (a, 0)$ and $(a + b, b)$.

43. $(y - y_1) - m(x - x_1) = 0$ is family of lines

$\therefore \quad y - y_1 = 0, x - x_1 = 0$

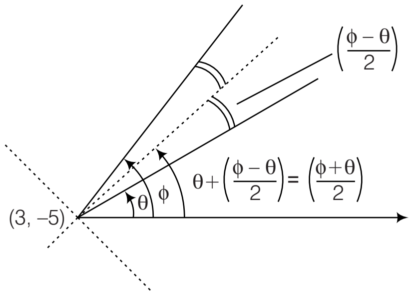
Then, $\quad y = y_1$ and $x = x_1$

44. Given lines $L_1 = 0$ and $L_2 = 0$ are perpendicular and given bisectors are $\lambda_1 L_1 - \lambda_2 L_2 = 0$ and $\lambda_1 L_1 + \lambda_2 L_2 = 0$

$\therefore$bisectors are perpendicular to each other.

Hence, bisectors of $\lambda_1 L_1 - \lambda_2 L_2 = 0$ and $\lambda_1 L_1 + \lambda_2 L_2 = 0$ are $L_1 = 0$ and $L_2 = 0$.

45. $\therefore$ One bisector makes an angle $\left(\dfrac{\theta + \phi}{2}\right)$ with X-axis, then other bisector makes an angle $90° + \left(\dfrac{\theta + \phi}{2}\right)$ with X-axis.



∴ Equations of bisectors are

$$\frac{x-3}{\cos\left(\frac{\theta+\phi}{2}\right)}=\frac{y+5}{\sin\left(\frac{\theta+\phi}{2}\right)} \quad \text{...(i)}$$

and $$\frac{x-3}{\cos\left(\frac{\pi}{2}+\frac{\theta+\phi}{2}\right)}=\frac{y+5}{\sin\left(\frac{\pi}{2}+\frac{\theta+\phi}{2}\right)}$$

$$\Rightarrow \quad \frac{x-3}{-\sin\left(\frac{\theta+\phi}{2}\right)}=\frac{y+5}{\cos\left(\frac{\theta+\phi}{2}\right)} \quad \text{...(ii)}$$

But given bisector are $\frac{x-3}{\cos\alpha}=\frac{y+5}{\sin\alpha}$

$$\therefore \quad \alpha=\frac{\theta+\phi}{2} \text{ and } \frac{x-3}{\beta}=\frac{y+5}{\gamma} \quad \text{[from Eq. (i)]...(iii)}$$

$$\therefore \quad \beta=-\sin\left(\frac{\theta+\phi}{2}\right)=-\sin\alpha \quad \text{[from Eq. (ii)]}$$

and $$\gamma=\cos\left(\frac{\theta+\phi}{2}\right)=\cos\alpha$$

46. $\because OR = AR$

$\Rightarrow \quad |x-0|+|y-0|=|x-1|+|y-2|$

$\Rightarrow \quad |x|+|y|=|x-1|+|y-2|$

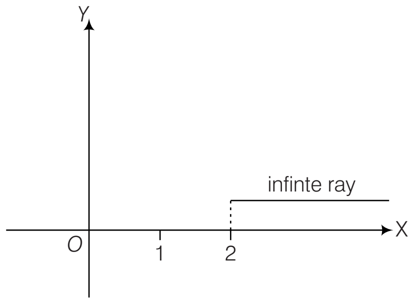
$\because \quad 0 \le x < 1$ and $0 \le y < 2$

$\therefore \quad x+y=-(x-1)-(y-2)$

$\Rightarrow \quad 2x+2y=3$

47. $OS = BS$

$\Rightarrow |x-0|+|y-0|=|x-2|+|y-3|$



$\Rightarrow |x|+|y|=|x-2|+|y-3|$

$\because \quad x \ge 2$ and $0 \le y < 3$

$\therefore \quad x+y=x-2+3-y$

$\Rightarrow \quad 2y=1$

$\therefore \quad y=\frac{1}{2}$

48. $\because OT = CT$

$\Rightarrow \quad |x-0|+|y-0|=|x-4|+|y-3|$

$\because \quad x \ge 0, y \ge 0$

$\Rightarrow \quad x+y=|x-4|+|y-3|$

Case I : If $0 \le x \le 4$ and $0 \le y \le 3$

$x+y=4-x+3-y$

$\Rightarrow \quad x+y=\frac{7}{2}$

Case II : If $0 \le x \le 4$ and $y \ge 3$

$x+y=4-x+y-3$

$\Rightarrow \quad x=\frac{1}{2}$

Case III : If $x \ge 4$ and $0 \le y \le 3$

$x+y=x-4+3-y$

$y=-1/2$ (impossible)

Case IV : If $x \ge 4$ and $y \ge 3$

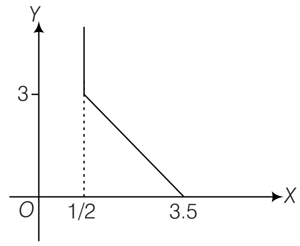
$x+y=x-4+y-3$

$\Rightarrow \quad 0=-7$ (impossible)

Combining all cases, we get

$$x+y=\frac{7}{2}, \forall\ 0 \le x \le 4 \text{ and } 0 \le y \le 3$$

and $$x=\frac{1}{2}, \forall\ 0 \le x \le 4 \text{ and } y \ge 3$$

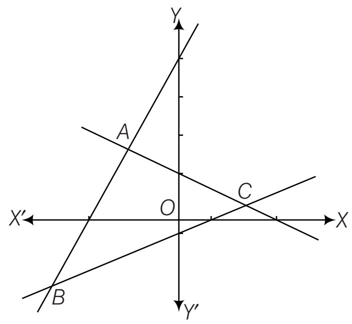


Sol. (Q. Nos. 49 to 51)

$AB: 2x-y+4=0,$

$BC: x-2y-1=0$

and $CA: x+3y-3=0$



$\therefore \quad m_{AB}=m_1=2$

$m_{BC}=m_2=\frac{1}{2}$

and $m_{CA}=m_3=-\frac{1}{3}$

$\because \quad m_1 > m_2 > m_3$

49. $\because \angle A$ is obtuse

$$\therefore \quad \tan A = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{-\frac{1}{3} - 2}{1 - \frac{2}{3}} = -7$$

50. For external bisector of B

$AB : 2x - y + 4 = 0$

$BC : -x + 2y + 1 = 0$

$\because \quad (2)(-1) + (-1)(2) = -4 < 0$

$\therefore$ External bisector of B is

$$\left(\frac{2x - y + 4}{\sqrt{5}}\right) = -\frac{(-x + 2y + 1)}{\sqrt{5}}$$

or $\quad x + y + 5 = 0$

51. Let (α, β) be the image of $B(-3, -2)$ w.r.t. the line $x + 3y - 3 = 0$, then

$$\frac{\alpha + 3}{1} = \frac{\beta + 2}{3} = \frac{-2(-3 - 6 - 3)}{1 + 9}$$

or $$\frac{\alpha + 3}{1} = \frac{\beta + 2}{3} = \frac{12}{5}$$

or $$\alpha = -\frac{3}{5} \text{ and } \beta = \frac{26}{5}$$

$\therefore$ Required image is $\left(-\frac{3}{5}, \frac{26}{5}\right)$,

Sol. (Q. Nos. 52 to 54)

Let $\quad B \equiv (\lambda, 2 - \lambda) \quad (\because B$ lies on $x + y = 2)$

Slope of line $AB = m_1 = \dfrac{1 + \lambda}{1 - \lambda}$

and Slope of line $BC = m_2 = \dfrac{5\lambda - 12}{-5\lambda - 2} = \dfrac{12 - 5\lambda}{2 + 5\lambda}$

Let slope of bisector $(x + y = 2) = m_3 = -1$

Now, $$\frac{m_3 - m_1}{1 + m_3 m_1} = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$\Rightarrow \quad \frac{-1 - \frac{1 + \lambda}{1 - \lambda}}{1 - \frac{1 + \lambda}{1 - \lambda}} = \frac{\frac{12 - 5\lambda}{2 + 5\lambda} + 1}{1 - \frac{12 - 5\lambda}{2 + 5\lambda}}$$

or $$\frac{-2}{-2\lambda} = \frac{14}{-10 + 10\lambda}$$

or $\quad 14\lambda = -10 + 10\lambda$

$\therefore \quad \lambda = \dfrac{-5}{2} \quad$ ...(i)

52. Equation of BC is

$$y - (2 - \lambda) = \frac{-\frac{2}{5} - (2 - \lambda)}{-\frac{2}{5} - \lambda}(x - \lambda)$$

or $$y - 2 - \frac{5}{2} = \frac{-\frac{2}{5} - \frac{9}{2}}{-\frac{2}{5} + \frac{5}{2}}\left(x + \frac{5}{2}\right)$$

or $\quad 7x + 3y + 4 = 0$

53. Coordinates of vertex B are $(\lambda, 2 - \lambda)$

i.e. $\quad \left(-\frac{5}{2}, \frac{9}{2}\right) \quad$ [from Eq. (i)]

54. $A \equiv (1, 3)$ and $B \equiv \left(-\frac{5}{2}, \frac{9}{2}\right)$

$\therefore$ Equation of AB is

$$y - 3 = \frac{\frac{9}{2} - 3}{-\frac{5}{2} - 1}(x - 1)$$

or $\quad 3x + 7y = 24$

55. Any point on the line $3x - y = 2$ is $(t, 3t - 2)$, t being parameter.

If (x, y) be image of the point $(t, 3t - 2)$ in the line $y = x - 1$ or $x - y - 1 = 0$, then

$$\frac{x - t}{1} = \frac{y - (3t - 2)}{-1} = -\frac{2(t - 3t + 2 - 1)}{1 + 1}$$

$$\Rightarrow \quad \frac{x - t}{1} = \frac{y - 3t + 2}{-1} = 2t - 1$$

or $\quad x - t = 2t - 1$

$\Rightarrow \quad x + 1 = 3t \quad$ ...(i)

and $\quad y - 3t + 2 = -2t + 1$

$\Rightarrow \quad y + 1 = t \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$x + 1 = 3(y + 1)$

$\Rightarrow \quad x - 3y = 2$

56. Any point on the circle $x^2 + y^2 = 4$ is $(2\cos\theta, 2\sin\theta)$, θ being parameter.

If (x, y) be image of the point $(2\cos\theta, 2\sin\theta)$, in the line $x + y = 2$, then

$$\frac{x - 2\cos\theta}{1} = \frac{y - 2\sin\theta}{1} = \frac{-2(2\cos\theta + 2\sin\theta - 2)}{1 + 1}$$

or $\quad x - 2\cos\theta = y - 2\sin\theta$

$\quad = -2\cos\theta - 2\sin\theta + 2 \quad$ ...(i)

or $\quad x - 2\cos\theta = -2\cos\theta - 2\sin\theta + 2$

$\Rightarrow \quad x - 2 = -2\sin\theta$

and $\quad y - 2\sin\theta = -2\cos\theta - 2\sin\theta + 2$

$\Rightarrow \quad y - 2 = -2\cos\theta \quad$ ...(ii)

From Eqs. (i) and (ii),

$(x - 2)^2 + (y - 2)^2 = 4$

$\Rightarrow \quad x^2 + y^2 - 4x - 4y + 4 = 0$

57. Any point on the parabola $x^2 = 4y$ is $(2t, t^2)$, t being parameter.

If (x, y) be image of the point $(2t, t^2)$ in the $x + y = a$, then

$$\frac{x-2t}{1} + \frac{y-t^2}{1}$$
$$= \frac{-2(2t + t^2 - a)}{1+1}$$
$$= -2t - t^2 + a$$

or $\quad x - 2t = -2t - t^2 + a$

$\Rightarrow \quad x - a = -t^2 \quad$ ...(i)

and $\quad y - t^2 = -2t - t^2 + a$

$\Rightarrow \quad y - a = -2t \quad$ ...(ii)

From Eqs. (i) and (ii) we get

$$(y-a)^2 = 4t^2 = -4(x-a)$$

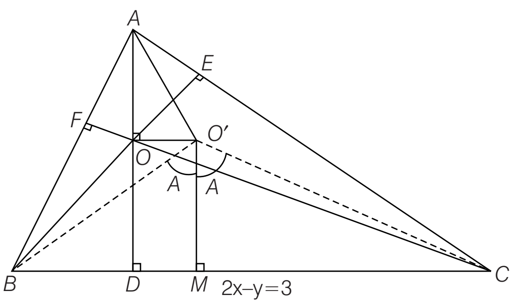
or $\quad (y-a)^2 = 4(a-x)$

Sol. (Q. Nos. 58 to 60)

Given orthocentre $O \equiv (1, 2)$

and circumcentre

$$O' = (2, 4)$$



$\because \quad$ Slope of OO' = Slope of $(2x - y = 3)$

and $\quad OD = O'M = \frac{3}{\sqrt{5}}$

Let R be the circumradius

$\therefore \quad O'M = R\cos A$

$\Rightarrow \quad R\cos A = \frac{3}{\sqrt{5}} \quad$ ...(i)

58. $R = AO' = \sqrt{(AO)^2 + (OO')^2}$

$$= \sqrt{(2R\cos A)^2 + 5}$$
$$= \sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 + 5} \quad \text{[from Eq. (i)]}$$
$$= \sqrt{\frac{61}{5}}$$

59. $\because OD = 2R\cos B\cos C$

$\therefore \quad 2R\cos B\cos C = \frac{3}{\sqrt{5}}$

$= R\cos A \quad$ [from Eq. (i)] ...(ii)

$\Rightarrow \quad \cos A = 2\cos B\cos C$

$\Rightarrow \quad -\cos(B + C) = 2\cos B\cos C \quad (\because A + B + C = \pi)$

$\Rightarrow \quad -(\cos B\cos C - \sin B\sin C) = 2\cos B\cos C$

or $\quad \sin B\sin C = 3\cos B\cos C$

$$= 3 \times \frac{3}{2R\sqrt{5}}$$
$$= \frac{9}{2\sqrt{61}} \quad \left(\because R = \sqrt{\frac{61}{5}}\right)$$

60. $\because AO = 2R\cos A$

$$= 2 \times \frac{3}{\sqrt{5}} \quad \text{[from Eq. (i)]}$$
$$= \frac{6}{\sqrt{5}}$$

61. The equation of straight line through (2, 3) with slope m is

$$y - 3 = m(x - 2)$$

or $\quad mx - y = 2m - 3$

or $\quad \dfrac{x}{\left(\dfrac{2m-3}{m}\right)} + \dfrac{y}{(3-2m)} = 1$

Here, $\quad OA = \dfrac{2m-3}{m}$ or $OB = 3 - 2m$

$\because$ The area of $\Delta OAB = 12$

$\Rightarrow \quad \left|\frac{1}{2} \times OA \times OB\right| = 12$

or $\quad \frac{1}{2}\left(\frac{2m-3}{m}\right)(3-2m) = \pm 12$

or $\quad (2m-3)^2 = \pm 24m$

Taking positive sign, we get $4m^2 - 36m + 9 = 0$

Here $D > 0$, This is a quadratic in m which given two value of m, and taking negative sign, we get $(2m+3)^2 = 0$.

This gives one line of m as $\frac{-3}{2}$.

Hence, three straight lines are possible.

62. $\because$ Point of intersection of $ax + 3y - 1 = 0$ and $ax + y + 1 = 0$ is $A\left(-\frac{2}{a}, 1\right)$ and point of intersection of $ax + 3y - 1 = 0$ and $x + 3y = 0$ is $B\left(\frac{1}{a-1}, -\frac{1}{3(a-1)}\right)$

$\Rightarrow \quad$ Slope of OA is $m_{OA} = -\frac{a}{2}$

and $\quad$ Slope of OB is $m_{OB} = -\frac{1}{3}$

$\because \quad m_{OA} \times m_{OB} = -1$

$\therefore \quad -\frac{a}{2} \times -\frac{1}{3} = -1$

or $\quad a = -6$

$\therefore \quad |a| = 6$

63. Here, B is the image of A w.r.t line $y = x$

$\therefore$ $B \equiv (2, 1)$ and C is the image of A w.r.t line $x - 2y + 1 = 0$ if $C \equiv (\alpha, \beta)$, then

$$\frac{\alpha - 1}{1} = \frac{\beta - 2}{-2} = \frac{-2(1 - 4 + 1)}{1 + 4}$$

or $$\alpha = \frac{9}{5} \text{ and } \beta = \frac{2}{5}$$

$\therefore$ $$C \equiv \left(\frac{9}{5}, \frac{2}{5}\right)$$

$\Rightarrow$ Equation of BC is

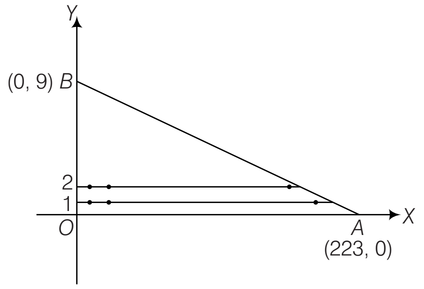
$$y - 1 = \frac{\left(\frac{2}{5} - 1\right)}{\left(\frac{9}{5} - 2\right)}(x - 2)$$

or $3x - y - 5 = 0$ $\quad$ ($\because$ Eq. of BC is $ax + by - 5 = 0$)

Here, $a = 3, b = -1$

$\therefore$ $a - 2b = 5$

64. On the line $y = 1$, the number of lattice points is $\left[\frac{2007 - 223}{9}\right] = 198$



Hence, the total number of points

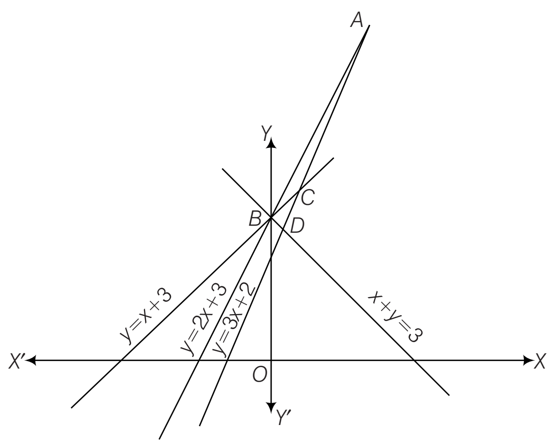
$$= \sum_{y=1}^{8} \left[\frac{2007 - 223y}{9}\right]$$

$$= 198 + 173 + 148 + 123 + 99 + 74 + 49 + 24 = 888$$

Hence, tens place digit is 8.

65. A rough sketch of the lines is given.

There are three triangle namely ABC, BCD and ABD



66. Let a be the length of side of square

$\therefore$ $$a^2 + a^2 = 2^2 \Rightarrow a = \sqrt{2}$$

i.e. distance between parallel lines is $\sqrt{2}$

Now, let two lines of family $y = x + n$ are $y = x + n$, and $y = x + n_2$, where

$$n_1, n_2 \in \{0, 1, 2, 3, 4\}$$

$\therefore$ $$\frac{|n_1 - n_2|}{\sqrt{2}} = \sqrt{2}$$

or $$|n_1 - n_2| = 2$$

$\Rightarrow$ $\{n_1, n_2\}$ are $\{0, 2\}$, $\{1, 3\}$ and $\{2, 4\}$

Hence, both the family have three such pairs. So, the number of squares possible is $3 \times 3 = 9$.

67. Let the coordinate of C be $(1, c)$, then

$$m_2 = \frac{c - y}{1 - x}$$

or $$m_2 = \frac{c - m_1 x}{1 - x} \quad (\because \text{ slope of } AB = m_1)$$

$\Rightarrow$ $$m_2(1 - x) = c - m_1 x$$

or $$c = (m_1 - m_2)x + m_2$$

Now, the area of ΔABC is $\frac{1}{2}|cx - y|$

$$= \frac{1}{2}|((m_1 - m_2)x + m_2)x - m_1 x| \quad (\because y = m_1 x)$$

$$= \frac{1}{2}(m_1 - m_2)(x - x^2) \quad [\because m_1 > m_2 \text{ and } x \in (0, 1)]$$

Hence, $$f(x) = \frac{1}{2}(x - x^2)$$

$\therefore$ $$\frac{df(x)}{dx} = \frac{1}{2}(1 - 2x)$$

and $$\frac{d^2 f(x)}{dx^2} = -1 < 0$$

For maximum of

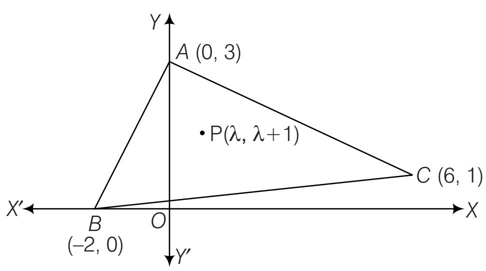
$$f(x), \frac{df(x)}{dx} = 0 \Rightarrow x = \frac{1}{2}$$

$\therefore$ $$f(x)|_{\max} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{1}{8} = \lambda \quad \text{(given)}$$

$\Rightarrow$ $$\frac{1}{\lambda} = 8$$

68. Equation of AB is $3x - 2y + 6 = 0$



Equation of BC is $x - 8y + 2 = 0$,
Equation of CA is $x + 3y - 9 = 0$
Let $\quad P \equiv (\lambda, \lambda + 1)$
$\because$ B and P lie on one side of AC, then

$$\frac{\lambda + 3(\lambda + 1) - 9}{-2 + 0 - 9} > 0$$

or $\quad 4\lambda - 6 < 0$

or $\quad \lambda < \frac{3}{2} \qquad$ (i)

and C and P lie on one side of AB, then

$$\frac{3\lambda - 2(\lambda + 1) + 6}{18 - 2 + 6} > 0$$

or $\quad \lambda + 4 > 0$

or $\quad \lambda > -4 \qquad$ ...(ii)

Finally, A and P lie on one side of BC, then

$$\frac{\lambda - 8(\lambda + 1) + 2}{0 - 24 + 2} > 0$$

or $\quad -7\lambda - 6 < 0$

or $\quad \lambda > -\frac{6}{7} \qquad$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$-\frac{6}{7} < \lambda < \frac{3}{2}$$

Integral values of λ are 0 and 1.
Hence, number of integral values of λ is 2.

69. Lines

$$(2a + b)x + (a + 3b)y + b - 3a = 0$$

or $\quad a(2x + y - 3) + b(x + 3y + 1) = 0$

are concurrent at the point of intersection of lines $2x + y - 3 = 0$ and $x + 3y + 1 = 0$ which is $(2, -1)$.
Now, line $\lambda x + 2y + 6 = 0$ must pass through $(2, -1)$, therefore,

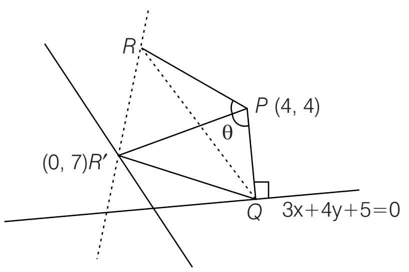
$$2\lambda - 2 + 6 \text{ or } \lambda = -2$$

$\therefore \quad |\lambda| = 2$

70. Since, PQ is of fixed length.

Area of $\Delta PQR = \frac{1}{2}|PQ||RP|\sin\theta$

This will be maximum, if $\sin\theta = 1$ and RP is maximum.

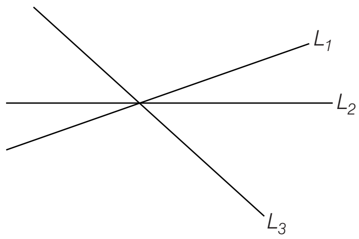


Since, line $y = mx + 7$ rotates about $(0, 7)$, if PR' is perpendicular to the line than PR' is maximum value of PR.

$\therefore \quad m = -\left(\frac{4 - 0}{4 - 7}\right) = \frac{4}{3}$

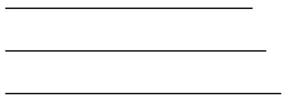
Hence, $\quad 3m = 4$

71. **(A)**



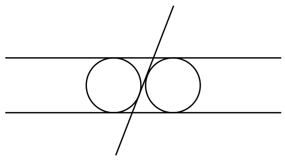
In this case no circle
$\therefore \quad n = 0 \Rightarrow n + 1 = 1$

(B)



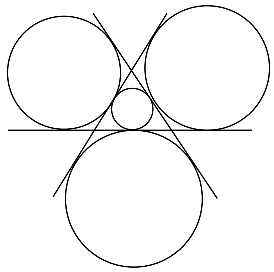
In this case no circle
$\therefore \quad n = 0 \Rightarrow 2n + 3 = 3$

(C)



In this case two circle which are touching all three lines
$\therefore \quad n = 2 \Rightarrow n + 2 = 4$

(D)



In this case four circle which are touching all three lines
$\therefore \quad n = 4 \Rightarrow n + 2 = 6$

72. **(A)** The given lines an concurrent. So,

$$\begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$$

or $\quad \lambda^2 + 2\lambda - 8 = 0$

or $\quad \lambda = 2, -4$

$\therefore \quad |\lambda| = 2, 4$

(B) Given family is

$$3x(a + 1) + 4y(a - 1) - 3(a - 1) = 0$$

or $\quad a(3x + 4y - 3) + (3x - 4y + 3) = 0$

for fixed point=

$$3x + 4y - 3 = 0$$

and $\quad 3x - 4y + 3 = 0$

$\therefore \quad x = 0, y = \frac{3}{4}$

Fixed point is $\left(0, \frac{3}{4}\right)$, Here $p = 0, q = \frac{3}{4}$

$\therefore \quad 4|\lambda| = 4|p - q| = 3$

(C) The point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ is (1, 2). It lies on the line

$$x + y - 1 - \left|\frac{\lambda}{2}\right| = 0$$

$\Rightarrow \quad 1 + 2 - 1 - \left|\frac{\lambda}{2}\right| = 0$

or $\quad |\lambda| = 4$ or $\lambda = -4, 4$

$\therefore \quad \lambda + 1 = -3, 5$ or $|\lambda + 1| = 3, 5$

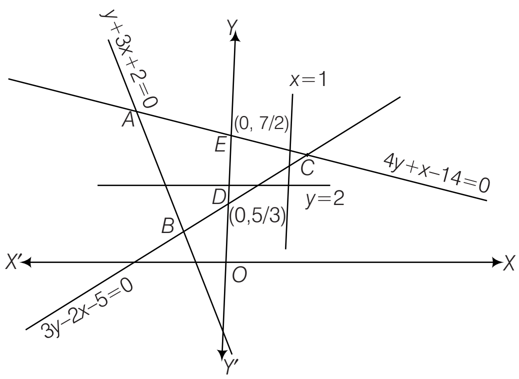
(D) The mid-point of $(1, -2)$ and (3, 4) will satisfy

$$y - x - 1 + \lambda = 0$$

or $\quad 1 - 2 - 1 + \lambda = 0$

$\therefore \quad \lambda = 2$ or $|\lambda| = 2$

73.



(A) The points on the line $x = 0$, whose y-coordinate lies between $\frac{5}{3}$ and $\frac{7}{2}$ inside the triangle ABC.

$\therefore \quad \frac{5}{3} < \lambda < \frac{7}{2}$ or $5 < 3\lambda < 10.5$

$\therefore \quad |3\lambda| = 6, 7, 8, 9, 10$

(B) $\because C \equiv (2, 3)$

The points on the line $x = 1$, whose y-coordinate lies between

$\frac{8}{3}$ $\quad$ (put $x = 1$ in $3y - 2x - 5 = 0$)

and $\frac{13}{4}$ $\quad$ (put $x = 1$ in $4y + x - 14 = 0$)

$\therefore \quad \frac{8}{3} < \lambda < \frac{13}{4}$ or $8 < 3\lambda < 9.75$

$\therefore \quad |3\lambda| = 9$

(C) $\because \; B \equiv (-1, 1)$

The point on the line $y = 2$, whose x-coordinate lies between

$\frac{-4}{3}$ $\quad$ (put $y = 2$ in $y + 3x + 2 = 0$)

and $\frac{1}{2}$ $\quad$ (put $y = 2$ in $3y - 2x - 5 = 0$)

$\therefore \quad \frac{-4}{3} < \lambda < \frac{1}{2}$ or $-8 < 6\lambda < 3$

Integral values of 6λ are

$$-7, -6, -5, -4, -3, -2, -1, 0, 1, 2$$

$\therefore \quad |6\lambda| = 7, 6, 5, 4, 3, 2, 1, 0$

(D) $\because A \equiv (-2, 4)$

The points on the line $y = \frac{7}{2}$, whose x-coordinates lies between

0 $\quad$ (put $y = \frac{7}{2}$ in $4y + x - 14 = 0$)

and $\frac{-11}{6}$ $\quad$ (put $y = \frac{7}{2}$ in $y + 3x + 2 = 0$)

$\therefore \quad \frac{-11}{6} < \lambda < 0$

or $\quad -11 < 6\lambda < 0$

Integral value of 6λ are

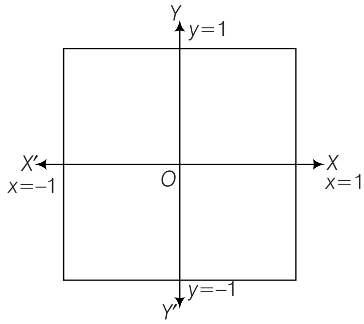
$$-10, -9, -8, -7, -6, -3, -2, -1$$

$\therefore \quad |6\lambda| = 10, 9, 8, 7, 6, 5, 4, 3, 2, 1$

74. **(A)** $\because \max.\{|x|, |y|\} = 1$

If $\quad |x| = 1$ and if $|y| = 1$

then $x = \pm 1$ and $y = \pm 1$



$\therefore \quad$ Required area $= 2 \times 2 = 4$ sq units

(B) The line $y = x$ cuts the lines $|x + y| = 6$

i.e. $\quad x + y = \pm 6$

at $\quad x = \pm 3, y = \pm 3$

or $\quad (-3, -3)$ and (3, 3)

then $\quad -3 < a < 3$

$\therefore \quad 0 \le |a| < 3$

$\therefore \quad [|a|] = 0, 1, 2$

(C) Since (0, 0) and (1, 1) lie on the same side.

So, $\quad a^2 + ab + 1 > 0$

$\because$ Coefficient of a^2 is > 0

$\therefore \quad D < 0$

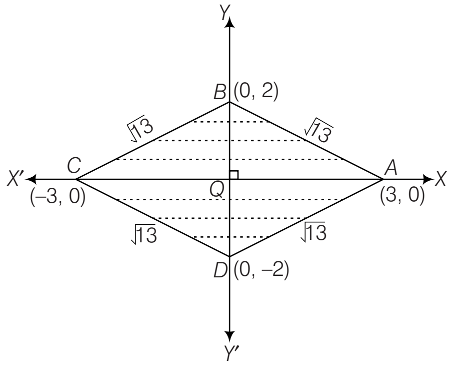
$\quad b^2 - 4 < 0$ or $-2 < b < 2$

$\Rightarrow \quad b = -1, 0, 1$

$\therefore$ Number of values of b is 3.

75. **(A)** $\because\ d(x, y) = 2|x| + 3|y| = 6$ (given)

$\therefore \quad \dfrac{|x|}{3} + \dfrac{|y|}{2} = 1$



$\therefore$Perimeter, $\lambda = 4\sqrt{13}$

and area, $\mu = 4 \times \dfrac{1}{2} \times 3 \times 2 = 12$

then $\dfrac{\lambda^2}{16} - \mu = 1$

and $\lambda^2 - \mu^2 = 64$

Hence, locus of (λ, μ) are

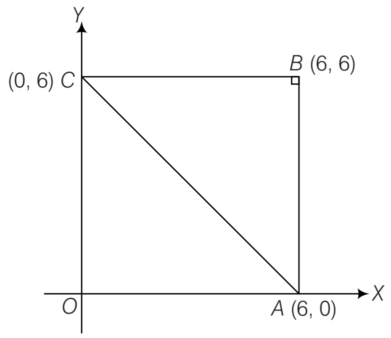
$x^2 - 16y = 16$

and $x^2 - y^2 = 64$

(B) It is clear that orthocentre is (6, 6)

$O' \equiv (6, 6)$,

Circumcentre is $C' \equiv (3, 3)$ and centroid is $G' \equiv (4, 4)$



$\therefore \quad \lambda = O'C' = \sqrt{(0-3)^2 + (6-3)^2}$

$= \sqrt{9+9} = 3\sqrt{2}$

and $\mu = C'G' = \sqrt{(4-3)^2 + (4-3)^2}$

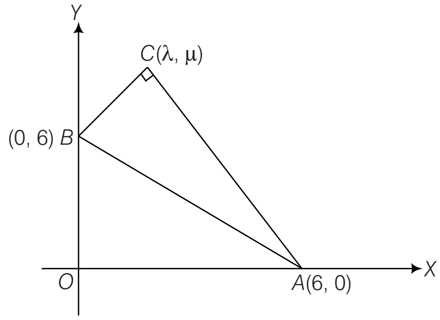
$= \sqrt{1+1} = \sqrt{2}$

$\therefore \quad \lambda^2 - \mu^2 = 16$ and $\lambda = 3\mu$

Hence, locus of (λ, μ) are

$x^2 - y^2 = 16$ and $x = 3y$

(C) $\therefore$ Slope of $AC \times$ slope of $BC = -1$



$\Rightarrow \quad \left(\dfrac{\mu - 0}{\lambda - 6}\right) \times \left(\dfrac{\mu - 6}{\lambda - 0}\right) = -1$

$\Rightarrow \quad \mu^2 - 6\mu = -\lambda^2 + 6\lambda$

or $\quad \lambda^2 + \mu^2 - 6\lambda - 6\mu = 0$

Hence, locus of (λ, μ) is

$x^2 + y^2 - 6x - 6y = 0$

76. $\because \quad x(a + 2b) + y(a + 3b) = a + b$

$\Rightarrow \quad a(x + y - 1) + b(2x + 3y - 1) = 0$

then $\quad x + y - 1 = 0$ and $2x + 3y - 1 = 0$

$\therefore$ point of intersection is $(2, -1)$

Hence, both statement are true and statement II is correct explanation for statement I.

77. $\because$ Algebraic perpendicular from (3, 2) to the line

$3x - 2y + 1 = 0$ is $\dfrac{9 - 4 + 1}{\sqrt{(9+4)}}$ i.e. $\dfrac{6}{\sqrt{13}} = p_1$ (say)

and algebraic perpendicular distance from (1, 4) to the line $3x - 2y + 1 = 0$ is

$\dfrac{3 - 8 + 1}{\sqrt{9+4}}$ i.e. $\dfrac{-4}{\sqrt{13}} = p_2$ (say)

$\therefore \quad p_1 p_2 = \dfrac{6}{\sqrt{13}} \times \dfrac{-4}{\sqrt{13}} = \dfrac{-24}{13} < 0$

Hence, both statements are true and statement II is a correct explanation for statement I.

78. Sum of algebraic distances from points $A(1, 2)$, $B(2, 3)$, $C(6, 1)$ to the line $ax + by + c = 0$ is zero (given), then

$$\frac{a + 2b + c}{\sqrt{(a^2 + b^2)}} + \frac{(2a + 3b + c)}{\sqrt{(a^2 + b^2)}} + \frac{(6a + b + c)}{\sqrt{(a^2 + b^2)}} = 0$$

$\Rightarrow \quad 9a + 6b + 3c = 0$

or $\quad 3a + 2b + c = 0$

$\therefore$ Statement I is false.

Also, centroid of ΔABC is $\left(\dfrac{1 + 2 + 6}{3}, \dfrac{2 + 3 + 1}{3}\right)$

i.e. (3, 2)

$\therefore$ Statement II is true.

79. Equation of AB is

$$y-1=\frac{0-1}{2-0}(x-0) \Rightarrow x+2y-2=0$$

$\because \qquad |PA-PB| \le |AB|$

$\Rightarrow |PA-PB|$ to be maximum, then A, B and P must be collinear.

Solving $\qquad x+2y-2=0$

and $\qquad 4x+3y+9=0,$

we get, $\qquad P \equiv \left(\frac{24}{5}, \frac{17}{5}\right)$

Hence, Statement I is false and Statement II is obviously true.

80. Statement II is false as the point satisfying such a property can be the excentre of the triangle.

Let $\qquad L_1 \equiv x\cos\left(\frac{\pi}{9}\right)+y\sin\left(\frac{\pi}{9}\right)-\pi=0,$

$L_2 \equiv x\cos\left(\frac{8\pi}{9}\right)+y\sin\left(\frac{8\pi}{9}\right)-\pi=0$ and

$L_3 \equiv x\cos\left(\frac{13\pi}{9}\right)+y\sin\left(\frac{8\pi}{9}\right)-\pi=0$

and $\qquad P \equiv (0, 0)$

Length $\perp$ from P to L_1 = Length of $\perp$ from P to L_2 = Length of $\perp$ from P to $L_3=\pi$ and P lies inside the triangle.

$\therefore P(0, 0)$ is incentre of triangle.

Hence, statement I is true and statement II is false.

81. $\because$ Mid-point of $(5, 1)$ and $(-1, -5)$ i.e. $(2, -2)$ lies on $x+y=0$ and (slope of $x+y=0$) $\times$ (slope of line joining $(5, 1)$ and $(-1, -5)$) $=(-1)\times\frac{-6}{-6}$

$\therefore$ Statement I is true.

Statement II is also true.

Hence, both statements are true but statement II is not correct explanation of statement I.

82. Equation of AC and BC are $3x+2y=0$ and $2x+3y+6=0$

$\because (3)(2)+(2)(3)=12>0$

$\therefore$ Internal angle bisector of C is

$$\left(\frac{3x+2y}{\sqrt{13}}\right)=-\left(\frac{2x+3y+6}{\sqrt{13}}\right)$$

or $\qquad 5x+5y+6=0$

$\Rightarrow$ Statement I is true.

Also, the image of A about the angle bisectors of angle B and C lie on the side BC. (by congruence).

$\therefore$ Statement II is true.

Both statements are true and statement II is not correct explanation of statement I.

83. $\because$ Points (x_1, y_1) and (x_2, y_2) lie on the same or opposite sides of the line

$$ax+by+c=0, \text{ as}$$

$$\frac{ax_1+by_1+c}{ax_2+by_2+c}>0 \text{ or } <0$$

$\therefore \quad$ Statement II is true.

Also, $(2a-5, a^2)$ and $(0, 0)$ on the same side of $x+y-3=0$, then

$$\frac{2a-5+a^2-3}{0+0-3}>0$$

$\Rightarrow \qquad a^2+2a-8<0$

or $\qquad (a+4)(a-2)<0$

$\therefore \qquad a \in (-4, 2)$

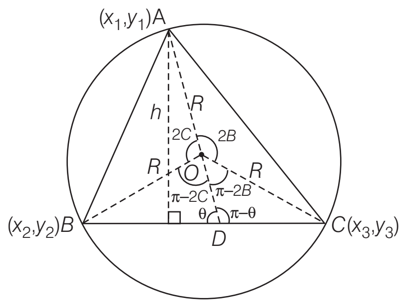
$\Rightarrow$ Statement I is false

Hence, statement I is false and statement II is true.

84. In ΔOBD, $\qquad \frac{BD}{\sin(\pi-2C)}=\frac{R}{\sin\theta}$ $\qquad$ ...(i)

In ΔODC, $\qquad \frac{DC}{\sin(\pi-2B)}=\frac{R}{\sin(\pi-\theta)}$ $\qquad$ ...(ii)

From Eqs. (i) and (ii), $\qquad \frac{BD}{DC}=\frac{\sin 2C}{\sin 2B}$



$\therefore$ Coordinates of D are

$$\left(\frac{x_2\sin 2B+x_3\sin 2C}{\sin 2B+\sin 2C}, \frac{y_2\sin 2B+y_3\sin 2C}{\sin 2B+\sin 2C}\right)$$

Let (x, y) be any point on AD, then equation of AD is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2\sin 2B+x_3\sin 2C}{\sin 2B+\sin 2C} & \frac{y_2\sin 2B+y_3\sin 2C}{\sin 2B+\sin 2C} & 1 \end{vmatrix}=0$$

or $$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2\sin 2B+x_3\sin 2C & y_2\sin 2B+y_3\sin 2C & \sin 2B+\sin 2C \end{vmatrix}=0$$

or $$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2\sin 2B & y_2\sin 2B & \sin 2B \end{vmatrix}+\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3\sin 2C & y_3\sin 2C & \sin 2C \end{vmatrix}=0$$

or $$(\sin 2B)\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}+(\sin 2C)\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0$$

85. Let (x_1, y_1) be the coordinates of a point at unit distance from each of the given lines.

$$\Rightarrow \quad \frac{|3x_1 - 4y_1 + 1|}{\sqrt{3^2 + 4^2}} = 1 \text{ and } \frac{|8x_1 + 6y_1 + 1|}{\sqrt{8^2 + 6^2}} = 1$$

$\Rightarrow \quad 3x_1 - 4y_1 + 1 = \pm 5$ and $8x_1 + 6y_1 + 1 = \pm 10$

$\Rightarrow \quad 3x_1 - 4y_1 - 4 = 0$ ...(i)

or $\quad 3x_1 - 4y_1 + 6 = 0$ ...(ii)

$8x_1 + 6y_1 - 9 = 0$ ...(iii)

or $\quad 8x_1 + 6y_1 + 11 = 0$ ...(iv)

$(1) \cap (3)$

$\Rightarrow \quad x_1 / 60 = y_1 / -5 = 1/50,$

$\therefore \quad (x_1, y_1) = \left(\frac{6}{5}, -\frac{1}{10}\right)$

$(1) \cap (4)$

$\Rightarrow \quad x_1 / -20 = y_1 / -65 = 1/50,$

$\therefore \quad (x_1, y_1) = \left(-\frac{2}{5}, -\frac{13}{10}\right)$

$(2) \cap (3)$

$\Rightarrow \quad x_1 / 0 = y_1 / 75 = 1/50, \therefore (x_1, y_1) = (0, 3/2)$

$(2) \cap (4)$

$\Rightarrow \quad x_1 / -80 = y_1 / 15 = 1/50, \therefore (x_1, y_1) = \left(-\frac{8}{5}, \frac{3}{10}\right)$

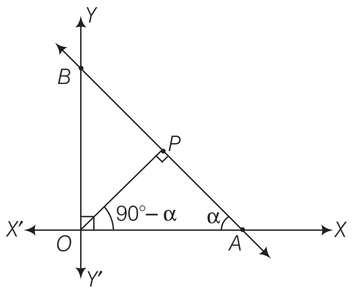
Hence, the required four points have the coordinates

$$\left(\frac{6}{5}, -\frac{1}{10}\right), \left(-\frac{2}{5}, -\frac{13}{10}\right), \left(0, \frac{3}{2}\right), \left(-\frac{8}{5}, \frac{3}{10}\right).$$

86. Let $\angle OAB = \alpha$

$\therefore \quad OA = AB \cos\alpha$ and $OB = AB \sin\alpha$

$\because \quad (OA)^2 + (OB)^2 = k^2$



i.e. $\quad (AB)^2 (\cos^2\alpha + \sin^2\alpha) = k^2$

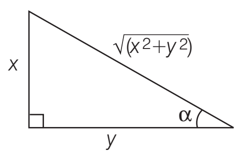
or $\quad AB = k$

then $\quad OA = k\cos\alpha$ and $OB = k\sin\alpha$

$\therefore$ Equation of AB is $\dfrac{x}{k\cos\alpha} + \dfrac{y}{k\sin\alpha} = 1$

or $\quad \dfrac{x}{\cos\alpha} + \dfrac{y}{\sin\alpha} = k$ ...(i)

Let P be the foot of perpendicular from O on AB.



$\therefore$ Equation of OP is $y = x\tan(90° - \alpha)$

or $\quad \cot\alpha = \dfrac{y}{x}$

$\therefore \quad \sin\alpha = \dfrac{x}{\sqrt{(x^2 + y^2)}}$

and $\quad \cos\alpha = \dfrac{y}{\sqrt{(x^2 + y^2)}}$ ...(ii)

Substituting the values of $\sin\alpha$ and $\cos\alpha$ from Eq. (i) in (i) then we get the required locus of P

$$\therefore \quad \frac{x}{y/\sqrt{(x^2+y^2)}} + \frac{y}{x/\sqrt{(x^2+y^2)}} = k$$

$\Rightarrow \quad (x^2 + y^2)\sqrt{(x^2 + y^2)} = kxy$

Squaring both sides, we get

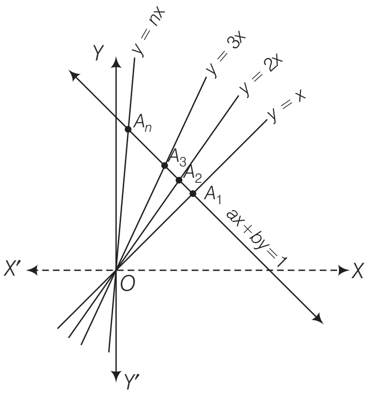
$$(x^2 + y^2)^2 (x^2 + y^2) = k^2x^2y^2$$

or $\quad (x^2 + y^2)^2 \left(\dfrac{x^2}{x^2y^2} + \dfrac{y^2}{x^2y^2}\right) = k^2$

or $\quad (x^2 + y^2)^2 (x^{-2} + y^{-2}) = k^2$

87. Let the equation of variable line be $ax + by = 1$. Then the coordinates of A_p will be

$$A_p \equiv \left(\frac{1}{a + bp}, \frac{p}{a + bp}\right)$$



$\therefore \quad OA_p = \dfrac{\sqrt{(1 + p^2)}}{|a + bp|}$ ...(i)

Given, $\quad \displaystyle\sum_{p=1}^{n} \frac{1}{OA_p} = c$

$\Rightarrow \quad \displaystyle\sum_{p=1}^{n} \frac{|a + bp|}{\sqrt{(1 + p^2)}} = c$ [from Eq. (1)]

$$\Rightarrow a\left(\pm \sum_{p=1}^{n} \frac{1}{\sqrt{(1+p^2)}}\right) + b\left(\pm \sum_{p=1}^{n} \frac{p}{\sqrt{(1+p^2)}}\right) = c$$

$$\text{or} \quad a\left(\pm \frac{\sum_{p=1}^{n} \frac{1}{\sqrt{(1+p^2)}}}{c}\right) + b\left(\pm \frac{\sum_{p=1}^{n} \frac{p}{\sqrt{(1+p^2)}}}{c}\right) = 1$$

So, line always passes through a fixed point whose coordinates are

$$\left(\frac{\pm \sum_{p=1}^{n} \frac{1}{\sqrt{(1+p^2)}}}{c}, \frac{\pm \sum_{p=1}^{n} \frac{p}{\sqrt{(1+p^2)}}}{c} \right)$$

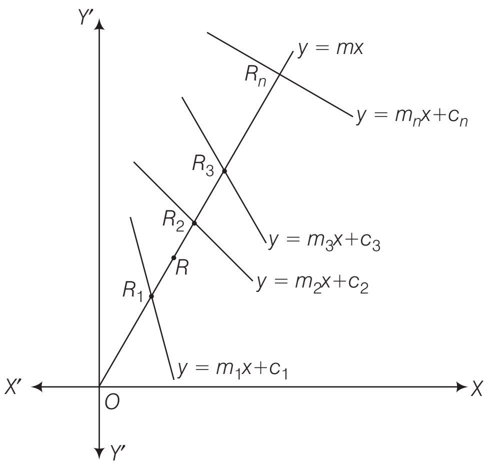
88. Let the equation of given n lines be

$$y = m_r\, x + c_r,$$

where $r = 1, 2, 3, \ldots, n$ ...(i)

Let equation of line through origin O is

$$y = mx \quad \text{...(ii)}$$



Solving Eqs. (i) and (ii), we get

$$R_r \equiv \left(\frac{c_r}{m - m_r}, \frac{mc_r}{m - m_r} \right)$$

$$\therefore \quad OR_r = \sqrt{\left(\frac{c_r}{m - m_r} \right)^2 + \left(\frac{m\,c_r}{m - m_r} \right)^2}$$

$$= \left| \frac{c_r}{m - m_r} \right| \sqrt{(1+m^2)} \quad \text{...(iii)}$$

Let $R \equiv (x_1, y_1)$

$$\therefore \quad y_1 = mx_1 \Rightarrow m = \frac{y_1}{x_1} \quad \text{...(iv)}$$

Given, $\frac{n}{OR} = \sum_{r=1}^{n} \frac{1}{OR_r}$

$$\Rightarrow \frac{n}{\sqrt{(x_1^2 + y_1^2)}} = \sum_{r=1}^{n} \left| \frac{m - m_r}{c_r} \right| \frac{1}{\sqrt{(1+m^2)}} \quad \text{[from Eq . (iii)]}$$

$$= \frac{1}{\sqrt{(1+m^2)}} \left\{ m \left(\sum_{r=1}^{n} \left(\pm \frac{1}{c_r} \right) \right) + \sum_{r=1}^{n} \left(\mp \frac{m_r}{c_r} \right) \right\}$$

$$= \frac{1}{\sqrt{(1+m^2)}} (ma + b)$$

$$\left\{ \text{where } a = \sum_{r=1}^{n} \left(\pm \frac{1}{c_r} \right) \text{ and } b = \sum_{r=1}^{n} \left(\mp \frac{m_r}{c_r} \right) \right.$$

then $$\frac{n}{\sqrt{(x_1^2 + y_1^2)}} = \frac{\frac{y_1}{x_1} a + b}{\sqrt{1 + \left(\frac{y_1}{x_1} \right)^2}} \quad \text{[from Eq. (iv)]}$$

$$\Rightarrow \quad n = ay_1 + bx_1$$

Hence, locus of point R is $bx + ay = n$.

89. First equation can be expressed as

$$(2x + 3y - 5) \cos\theta + (3x - 5y + 2) \sin\theta = 0$$

$$\Rightarrow \quad (2x + 3y - 5) + (3x - 5y + 2) \tan\theta = 0$$

It is clear that these lines will pass through the point of intersection of the lines

$$\left. \begin{aligned} 2x + 3y - 5 &= 0 \\ 3x - 5y + 2 &= 0 \end{aligned} \right\} \quad \text{...(i)}$$

for all values of θ.

Solving the system of Eq. (i), we get (1, 1).

Hence, the fixed point is P (1, 1). Let Q (α, β) be the reflection of P (1, 1) in the line $x + y = \sqrt{2}$.

Then $$\frac{\alpha - 1}{1} = \frac{\beta - 1}{1} = \frac{-2(1 + 1 - \sqrt{2})}{1^2 + 1^2} = \sqrt{2} - 2$$

$$\therefore \quad \alpha = \sqrt{2} - 1, \beta = \sqrt{2} - 1$$

i.e. $$Q \equiv (\sqrt{2} - 1, \sqrt{2} - 1)$$

If the required family of lines is

$$(2 \cos\theta + 3 \sin\theta)\, x + (3 \cos\theta - 5 \sin\theta)\, y = \lambda$$

in order that each member of the family pass through Q, we have

$$\lambda = (\sqrt{2} - 1)(2 \cos\theta + 3 \sin\theta + 3 \cos\theta - 5 \sin\theta)$$

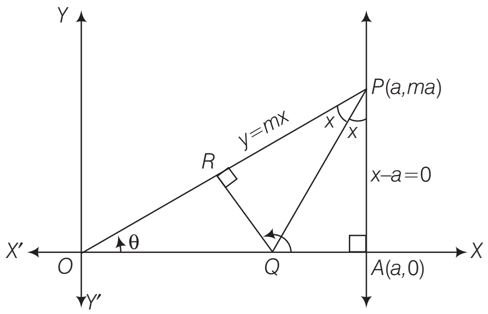
$$\lambda = (\sqrt{2} - 1)(5 \cos\theta - 2 \sin\theta)$$

Hence, equation of required family is

$$(2 \cos\theta + 3 \sin\theta)\, x + (3 \cos\theta - 5 \sin\theta)\, y = (\sqrt{2} - 1)(5 \cos\theta - 2 \sin\theta).$$

90. Let R (h, k) be the foot of perpendicular from Q on OP.

Let equation of OP be



$$y = mx$$

then $$k = mh$$

or $$m = \frac{k}{h} \quad \text{...(i)}$$

and coordinates of $P \equiv (a, ma)$

$\because$ PQ is the bisector of OPA

$\therefore$ $\angle APQ = \angle RPQ$

and $\angle PAQ = \angle QRP = 90°$

$\therefore$ $PA = PR$

then $|ma| = \sqrt{(h-a)^2 + (k-ma)^2}$

From Eq. (i), $\left|\frac{ak}{h}\right| = \sqrt{(h-a)^2 + \left(k - \frac{ak}{h}\right)^2}$

$\Rightarrow$ $a|k| = |(h-a)| \sqrt{(h^2+k^2)}$

Hence, required locus is

$$(x-a)^2 (x^2+y^2) = a^2 y^2$$

91. Let the coordinates of the vertex be (h, k) and equations of the bases be

$$x \cos \alpha_r + y \sin \alpha_r - p_r = 0 \quad \text{where } r = 1, 2, 3, \ldots, n$$

and their lengths be respectively $l_1, l_2, l_3, \ldots, l_n$.

$\because$ Length of perpendicular from (h, k) on $x \cos \alpha_r + y \sin \alpha_r - p_r = 0$ is

$$\frac{|h \cos \alpha_r + k \sin \alpha_r - p_r|}{\sqrt{(\cos^2 \alpha + \sin^2 \alpha)}},$$

i.e. $|h \cos \alpha_r + k \sin \alpha_r - p_r|$

Given, sum of areas of all triangles = constant

then

$$\sum_{r=1}^{n} \frac{1}{2} l_r \cdot |h \cos \alpha_r + k \sin \alpha_r - p_r| = C'$$

$$\Rightarrow \sum_{r=1}^{n} \frac{1}{2} . l_r . (\pm (h \cos \alpha_r + k \sin \alpha_r - p_r)) = C'$$

$$\Rightarrow h\left(\sum_{r=1}^{n} \pm \frac{1}{2} l_r \cos \alpha_r\right) + k\left(\sum_{r=1}^{n} \pm \frac{1}{2} l_r . \sin \alpha_r\right)$$

$$= \sum_{r=1}^{n} \pm \frac{1}{2} l_r . p_r + C'$$

$\Rightarrow Ah + Bk = -C$

$\therefore$ Required locus is

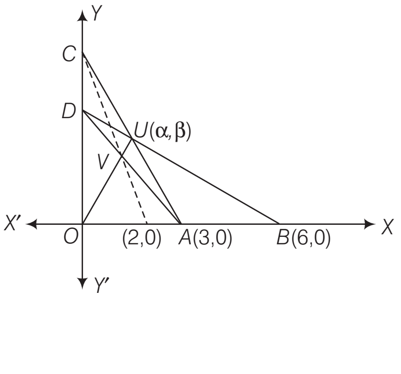
$$Ax + By + C = 0$$

where A, B, C are constants.

92. The equation of BU is

$$y - \beta = \frac{0-\beta}{6-\alpha}(x-\alpha)$$

So that the coordinates of D are $\left(0, \frac{6\beta}{6-\alpha}\right)$



Similarly, the coordinates of C are $\left(0, \frac{3\beta}{3-\alpha}\right)$

Now, the equation of AD is

$$\frac{x}{3} + \frac{(6-\alpha)}{6\beta} y = 1 \quad \text{...(i)}$$

and the equation of OU is

$$\beta x = \alpha y \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$x = \frac{6\alpha}{6+\alpha}, y = \frac{6\beta}{6+\alpha}$$

Hence, coordinates of V are $\left(\frac{6\alpha}{6+\alpha}, \frac{6\beta}{6+\alpha}\right)$

Then, the equation of CV is

$$y - \frac{3\beta}{3-\alpha} = \frac{\frac{6\beta}{6+\alpha} - \frac{3\beta}{3-\alpha}}{\frac{6\alpha}{6+\alpha} - 0}(x-0)$$

$$\Rightarrow y - \frac{3\beta}{3-\alpha} = \frac{-9\alpha\beta}{6\alpha(3-\alpha)} x$$

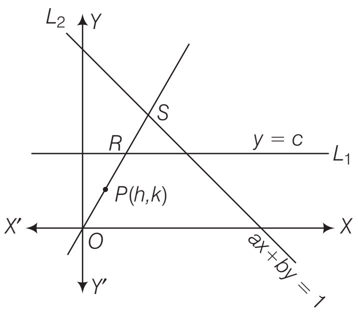
$$\Rightarrow y = \frac{3\beta}{(3-\alpha)}\left(1 - \frac{x}{2}\right)$$

which pass through the point (2, 0) for all values of (α, β).

93. Let the equation of the variable line through `O' be

$$\frac{x}{\cos \theta} = \frac{y}{\sin \theta}$$

and let $OR = r_1$, $OS = r_2$ and $OP = r_3$



Then coordinates of R, S and P are :

$R\,(r_1 \cos \theta, r_1 \sin \theta)$, $S\,(r_2 \cos \theta, r_2 \sin \theta)$, $P\,(r_3 \cos \theta, r_3 \sin \theta)$

R lies on L_1 and S lies on L_2.

Let $L_1 \equiv y - c = 0$

and $L_2 \equiv ax + by - 1 = 0$

$\therefore$ $r_1 \sin \theta = c$ and $ar_2 \cos \theta + br_2 \sin \theta = 1$

$\therefore$ $r_1 = \frac{c}{\sin \theta}$ and $r_2 = \frac{1}{a \cos \theta + b \sin \theta}$

From the given condition

$$\frac{m+n}{r_3} = \frac{m}{r_1} + \frac{n}{r_2}$$

$$\Rightarrow \frac{m+n}{r_3} = \frac{m \sin \theta}{c} + n(a \cos \theta + b \sin \theta) \quad \text{...(i)}$$

Let the coordinates of P be (h, k), then

$$h = r_3 \cos\theta, k = r_3 \sin\theta$$

From Eq. (i), $\quad m + n = \dfrac{mr_3 \sin\theta}{c} + n(ar_3 \cos\theta + br_3 \sin\theta)$

$$\Rightarrow \quad m + n = \frac{mk}{c} + n(ah + bk)$$

Locus of P is $n(ax + by) + \dfrac{my}{c} = (m + n)$

$$\Rightarrow \quad n(ax + by - 1) + \frac{m}{c}(y - c) = 0$$

$$\Rightarrow \quad (ax + by - 1) + \frac{m}{nc}(y - c) = 0$$

$$\Rightarrow \quad L_2 + \lambda L_1 = 0 \quad \left(\text{where, } \lambda = \frac{m}{nc}\right)$$

Hence, locus of P is a point of intersection of L_1 and L_2.

94. The given lines are

$$x + 3y + 2 = 0 \quad \text{...(i)}$$
$$2x + y + 4 = 0 \quad \text{...(ii)}$$
$$x - y - 5 = 0 \quad \text{...(iii)}$$

Equation of the line passing through $A(-5, -4)$ and making an angle θ with the positive direction of X-axis is

$$\frac{x + 5}{\cos\theta} = \frac{y + 4}{\sin\theta} = r(AB, AC, AD) \quad \text{...(iv)}$$

$\therefore$ Points $(-5 + AB\cos\theta, -4 + AB\sin\theta)$,
$(-5 + AC\cos\theta, -4 + AC\sin\theta)$ and
$(-5 + AD\cos\theta, -4 + AD\sin\theta)$ lie on Eqs. (i), (ii) and (iii) respectively.

$$(-5 + AB\cos\theta) + 3(-4 + AB\sin\theta) + 2 = 0$$

$$\Rightarrow \quad AB(\cos\theta + 3\sin\theta) = 15$$

$$\Rightarrow \quad \frac{15}{AB} = \cos\theta + 3\sin\theta$$

Similarly, $\quad \dfrac{10}{AC} = 2\cos\theta + \sin\theta$

and $\quad \dfrac{6}{AD} = \cos\theta - \sin\theta$

From given condition

$$\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$$

we get $(\cos\theta + 3\sin\theta)^2 + (2\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$

$$\Rightarrow \quad 4\cos^2\theta + 9\sin^2\theta + 12\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad (2\cos\theta + 3\sin\theta)^2 = 0$$

$$\therefore \quad \tan\theta = -\frac{2}{3}$$

Hence the equation of the line from Eq. (iv) is

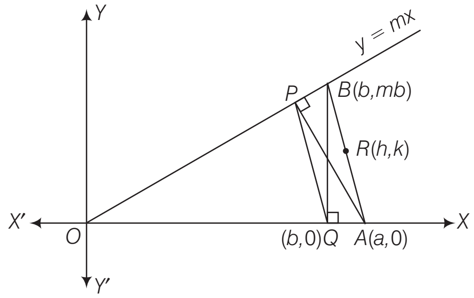
$$y + 4 = -\frac{2}{3}(x + 5) \Rightarrow 2x + 3y + 22 = 0$$

95. $\because$ A, R and B are collinear

then, $\quad \dfrac{k - 0}{h - a} = \dfrac{mb - 0}{b - a}$

$$\therefore \quad \frac{a}{b}k - am + mh - k = 0 \quad \text{...(i)}$$

Let $P \equiv (\alpha, \beta)$



$\because$ P be the foot of perpendicular from A on $y = mx$, then

$$\frac{\alpha - a}{-m} = \frac{\beta - 0}{1} = \frac{-(0 - ma)}{(1 + m^2)}$$

$$\therefore \quad \alpha = \frac{a}{1 + m^2}, \ \beta = \frac{am}{1 + m^2}$$

i.e. $\quad P \equiv \left(\dfrac{a}{1 + m^2}, \dfrac{am}{1 + m^2}\right)$

$\therefore$ Equation of PQ is

$$y - 0 = \frac{\dfrac{am}{1 + m^2} - 0}{\dfrac{a}{1 + m^2} - b}(x - b)$$

$$\Rightarrow \quad \frac{a}{b}(y - mx) + am - (1 + m^2)y = 0 \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), then

$$\frac{a}{b}(y - mx + k) + (mh - k - (1 + m^2)y) = 0$$

$$\Rightarrow \quad (mh - k - (1 + m^2)y) + \lambda(y - mx + k) = 0$$

$$\left(\text{where, } \lambda = \frac{a}{b}\right)$$

Hence PQ pass through a fixed point.

For fixed point

$$mh - k - (1 + m^2)y = 0, y - mx + k = 0$$

$$y = \frac{mh - k}{(1 + m^2)}, x = \frac{h + mk}{(1 + m^2)}$$

Hence, fixed point is $\left(\dfrac{h + mk}{1 + m^2}, \dfrac{mh - k}{1 + m^2}\right)$.

96. Given lines are parallel and distance between them < 2

Given lines are

$$2x + y = 3 \quad \text{...(i)}$$

and $\quad 2x + y = 5 \quad$ ...(ii)

Equation of any line through Eqs. (ii) and (iii) is

$$y - 3 = m(x - 2)$$

or $\quad y = mx - 2m + 3 \quad$ ...(iii)

Let line (iii) cut lines (i) and (ii) at A and B respectively.

Solving Eqs. (i) and (iii), we get

$$A \equiv \left(\frac{2m}{m + 2}, \frac{6 - m}{m + 2}\right)$$

and solving Eqs. (ii) and (iii), we get

$$B \equiv \left(\frac{2m+2}{m+2}, \frac{m+6}{m+2}\right)$$

According to question $AB = 2$

$\Rightarrow \quad (AB)^2 = 4$

$\Rightarrow \quad \left(\frac{2}{m+2}\right)^2 + \left(\frac{2m}{m+2}\right)^2 = 4$

$\Rightarrow \quad 1 + m^2 = m^2 + 4m + 4 \quad$ ...(iv)

Case I : When m is finite (line is not perpendicular to X-axis) then from Eq. (iv).

$$1 = 4m + 4$$

$\therefore \quad m = -\frac{3}{4}$

Case II : When m is infinite (line is perpendicular to X-axis) then from Eq. (iv),

$$\frac{1}{m^2} + 1 = 1 + \frac{4}{m} + \frac{4}{m^2}$$

$$0 + 1 = 1 + 0 + 0$$

$$1 = 1 \text{ which is true}$$

Hence $m \to \infty$ acceptable.

Hence, equation of the required lines are

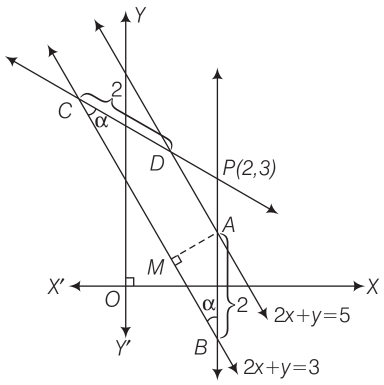
$$y - 3 = -\frac{3}{4}(x-2)$$

and $\quad \frac{y-3}{\infty} = x - 2 \Rightarrow x - 2 = 0$

i.e. $\quad 3x + 4y = 18 \quad$ and $\quad x - 2 = 0$

Aliter I :

$\because \quad 2x + y = 3$ cuts Y-axis at $(0, 3)$ and line $2x + y = 5$ cuts Y-axis at $(0, 5)$



Therefore intercept on Y-axis is 2.

Also, $\quad AM =$ distance between parallel lines

$$= \frac{|-5+3|}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}}$$

$\therefore \quad MB = \sqrt{(AB)^2 - (AM)^2} = \sqrt{4 - \frac{4}{5}} = \frac{4}{\sqrt{5}}$

then $\quad \tan\alpha = \frac{AM}{MB} = \frac{1}{2}$

Also $\quad \tan\theta = -2 \quad$ (slope of $2x + y = 5$)

Now, equation of required lines are

$$y - 3 = \tan(\theta \pm \alpha)(x - 2)$$

$\Rightarrow \quad y - 3 = \left(\frac{\tan\theta \pm \tan\alpha}{1 \mp \tan\theta \tan\alpha}\right)(x-2)$

$\Rightarrow \quad y - 3 = \frac{(-2) \pm \frac{1}{2}}{1 \mp (-2)\left(\frac{1}{2}\right)}(x-2)$

$\Rightarrow \quad y - 3 = \frac{\left(-2 \pm \frac{1}{2}\right)}{1 \mp (-1)}(x-2)$

$\Rightarrow (1 \mp (-1))(y-3) = \left(-2 \pm \frac{1}{2}\right)(x-2)$

$\Rightarrow \quad x - 2 = 0 \quad$ and $\quad 2y - 6 = -\frac{3}{2}(x-2)$

i.e. $\quad x - 2 = 0 \quad$ and $\quad 3x + 4y - 18 = 0$

Aliter II : Any line through $(2, 3)$ is

$$\frac{x-2}{\cos\theta} = \frac{y-3}{\sin\theta} = r$$

Suppose this line cuts $2x + y = 5$ and $2x + y = 3$ at D and C respectively but given $DC = 2$

then $\quad D \equiv (2 + r\cos\theta, 3 + r\sin\theta)$

and $\quad C \equiv (2 + (r+2)\cos\theta, 3 + (r+2)\sin\theta)$

$\because \quad D$ and C lies on

$$2x + y = 5 \quad \text{and} \quad 2x + y = 3$$

then $\quad 2(2 + r\cos\theta) + (3 + r\sin\theta) = 5 \quad$ ... (v)

and $\quad 2(2 + (r+2)\cos\theta) + (3 + (r+2)\sin\theta) = 3 \quad$ ... (vi)

Subtracting Eq. (v) from Eq. (vi), then

$$4\cos\theta + 2\sin\theta = -2$$

or $\quad 2\cos\theta + \sin\theta = -1$

$\Rightarrow \quad 2\left(\frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}\right) + \left(\frac{2\tan\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}\right) = -1$

$\Rightarrow \quad 2 - 2\tan^2\left(\frac{\theta}{2}\right) + 2\tan\left(\frac{\theta}{2}\right) = -1 - \tan^2\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad \tan^2\left(\frac{\theta}{2}\right) - 2\tan\left(\frac{\theta}{2}\right) - 3 = 0$

$\therefore \quad \tan\left(\frac{\theta}{2}\right) = -1 \text{ or } 3$

$\therefore \quad \tan\theta = \infty \quad \text{or} \quad -\frac{3}{4}$

$\therefore$ Required lines are

$$y - 3 = \infty(x-2)$$

and $\quad y - 3 = -\frac{3}{4}(x-2)$

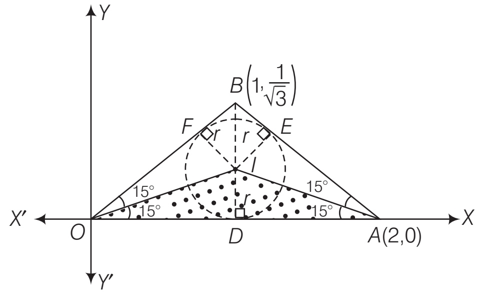
i.e. $\quad x - 2 = 0$

and $\quad 3x + 4y - 18 = 0$

97. If I be the incentre of ΔOAB.

If inradius $= r$

then $\qquad ID = IE = IF = r$



If P at I,then

$$d(P, OA) = d(P, OB) = d(P, AB) = r$$

But $\quad d(P, OA) \le \min\{d(P, OB), d(P, AB)\}$

which is possible only when P lies in the ΔOIA.

$$\therefore \qquad \tan 15^\circ = \frac{ID}{OD} = \frac{r}{1}$$

$$\Rightarrow \qquad r = (2 - \sqrt{3})$$

$\therefore$ Required area $= \frac{1}{2} \cdot 2 \cdot r = r = (2 - \sqrt{3})$ sq units.

98. Let $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$ and $C \equiv (x_3, y_3)$ are the vertices of a triangle ABC and $P \equiv (a_1, b_1)$, $Q \equiv (a_2, b_2)$ and $R \equiv (a_3, b_3)$ are the vertices of triangle PQR.

Equation of perpendicular from A to QR is

$$y - y_1 = -\frac{(a_2 - a_3)}{(b_2 - b_3)}(x - x_1)$$

or $\quad (a_2 - a_3)\,x + (b_2 - b_3)\,y - x_1(a_2 - a_3) - y_1(b_2 - b_3) = 0 \quad$ ...(i)

Similarly, equations of perpendiculars from B to RP and C to PQ are respectively,

$(a_3 - a_1)\,x + (b_3 - b_1)\,y - x_2(a_3 - a_1) - y_2(b_3 - b_1) = 0 \quad$ ...(ii)

and $(a_1 - a_2)\,x + (b_1 - b_2)\,y - x_3(a_1 - a_2) - y_3(b_1 - b_2) = 0 \quad$ ...(iii)

Given that lines (i), (ii) and (iii) are concurrent, then adding, we get

$$(x_2 - x_3)\,a_1 + (x_3 - x_1)\,a_2 + (x_1 - x_2)\,a_3 + (y_2 - y_3)\,b_1 + (y_3 - y_1)b_2 + (y_1 - y_2)\,b_3 = 0 \qquad \text{...(iv)}$$

Now, equation of perpendicular from P to BC is

$$y - b_1 = -\frac{(x_2 - x_3)}{(y_2 - y_3)}(x - a_1)$$

or $\qquad (x_2 - x_3)\,x + (y_2 - y_3)\,y - a_1(x_2 - x_3) - b_1(y_2 - y_3) = 0 \quad$ ...(v)

Similarly, equations of perpendiculars from Q to CA and R to AB are respectively,

$(x_3 - x_1)\,x + (y_3 - y_1)\,y - a_2(x_3 - x_1) - b_2(y_3 - y_1) = 0 \qquad$ ...(vi)

and $\qquad (x_1 - x_2)\,x + (y_1 - y_2)\,y - a_3(x_1 - x_2) - b_3(y_1 - y_2) = 0 \qquad$ ...(vii)

Adding Eqs. (v), (vi) and (vii), we get

LHS = 0 (identically) $\qquad$ [from Eq. (iv)]

Hence perpendiculars from P to BC, Q to CA and R to AB are concurrent.

99. The line passing through the intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ is

$$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$$

$$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$$

As this line is parallel to X-axis.

$$\therefore \qquad a + b\lambda = 0 \Rightarrow \lambda = -\frac{a}{b}$$

$$\Rightarrow \quad ax + 2by + 3a - \frac{a}{b}(bx - 2ay - 3a) = 0$$

$$\Rightarrow \quad ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$$

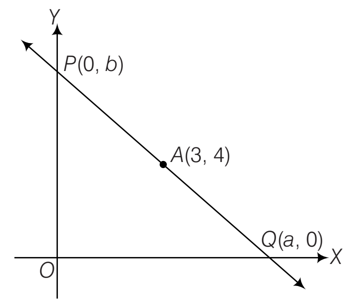
$$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$$

$$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$$

$$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$$

So, it is $\frac{3}{2}$ units below X-axis.

100.



$\because A$ is the mid-point of PQ, therefore

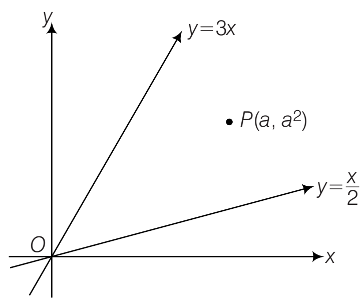
$$\frac{a + 0}{2} = 3, \frac{0 + b}{2} = 4$$

$$\Rightarrow \qquad a = 6, b = 8$$

$\therefore$Equation of line is $\frac{x}{6} + \frac{y}{8} = 1$

or $\qquad 4x + 3y = 24$

101. Clearly for point P,

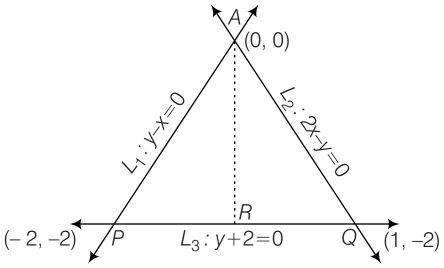


$$a^2 - 3a < 0 \text{ and } a^2 - \frac{a}{2} > 0$$

$$\Rightarrow \qquad \frac{1}{2} < a < 3$$

102. Point of intersection of L_1 and L_2 is $A(0, 0)$.

Also $P(-2, -2), Q(1, -2)$



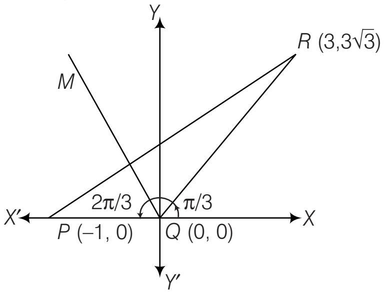
$\because$ AR is the bisector of $\angle PAQ$, therefore R divides PQ in the same ratio as $AP : AQ$.

Thus $PR : RQ = AP : AQ = 2\sqrt{2} : \sqrt{5}$

$\therefore$ Statement I is true.

Statement II is clearly false.

103. Given : The coordinates of points P, Q, R are $(-1, 0), (0, 0), (3, 3\sqrt{3})$ respectively.



Slope of equation $QR = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3\sqrt{3}}{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3} \Rightarrow \theta = \dfrac{\pi}{3}$

$\Rightarrow \quad \angle RQX = \dfrac{\pi}{3}$

$\therefore \quad \angle RQP = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

Let QM bisects the $\angle PQR$,

$\therefore$ Slope of the line $QM = \tan\dfrac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}(x - 0)$

$\Rightarrow \quad y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

104. **(A)** $\because$ L_1, L_2, L_3 are concurrent, then

$$\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 0 \Rightarrow k = 5$$

(B) slope of (L_1) = slope of (L_2)

$\Rightarrow \quad -\dfrac{1}{3} = \dfrac{3}{k} \quad \therefore \; k = -9$

and slope of (L_3) = slope of (L_2)

$\Rightarrow \quad -\dfrac{5}{2} = \dfrac{3}{k} \quad \therefore \; k = -\dfrac{6}{5}$

(C) Lines are not concurrent or not parallel, then

$$k \neq 5, k \neq -9, k \neq -\frac{6}{5}$$

$\therefore \quad k = \dfrac{5}{6}$

(D) The given lines do not form a triangle if they are concurrent or any two of them are parallel.

$\therefore \quad k = 5, k = -9, k = -\dfrac{6}{5}$

105. Slope of $PQ = \dfrac{3-4}{k-1} = \dfrac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of $PQ = (k-1)$

Also mid-point of $PQ \left(\dfrac{k+1}{2}, \dfrac{7}{2}\right)$

$\therefore$ Equation of perpendicular bisector is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow \quad 2y - 7 = 2(k-1)x - (k^2 - 1)$

$\Rightarrow \quad 2(k-1)x - 2y + (8 - k^2) = 0$

$\therefore \quad Y\text{-intercept} = -\dfrac{8-k^2}{-2} = -4$

$\Rightarrow \quad 8 - k^2 = -8 \;$ or $\; k^2 = 16 \Rightarrow k = \pm 4$

106. If the line $p(p^2 + 1)x - y + q = 0$

and $\quad (p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$

are perpendicular to a common line, then these lines must be parallel to each other,

$\therefore \quad m_1 = m_2 \Rightarrow -\dfrac{p(p^2+1)}{-1} = -\dfrac{(p^2+1)^2}{p^2+1}$

$\Rightarrow \quad (p^2 + 1)(p + 1) = 0$

$\Rightarrow \quad p = -1$

$\therefore p$ can have exactly one value.

107. Slope of line $L = -\dfrac{b}{5}$

Slope of line $K = -\dfrac{3}{c}$

Line L is parallel to line K.

$\Rightarrow \quad \dfrac{b}{5} = \dfrac{3}{c} \Rightarrow bc = 15$

(13, 32) is a point on L.

$\therefore \quad \dfrac{13}{5} + \dfrac{32}{b} = 1 \Rightarrow \dfrac{32}{b} = -\dfrac{8}{5}$

$\Rightarrow \quad b = -20 \Rightarrow c = -\dfrac{3}{4}$

Equation of $K : y - 4x = 3$

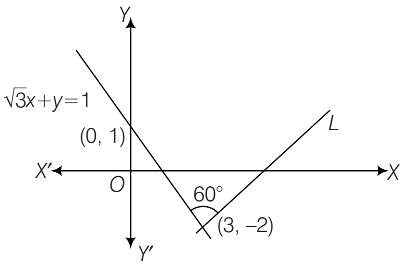
$\Rightarrow \quad 4x - y + 3 = 0$

Distance between L and $K = \dfrac{|52 - 32 + 3|}{\sqrt{17}}$

$$= \frac{23}{\sqrt{17}}$$

108. Let the slope of line L be m.

Then $$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$$



$$\Rightarrow \quad m+\sqrt{3} = \pm(\sqrt{3}-3m)$$

$$\Rightarrow \quad 4m = 0 \text{ or } 2m = 2\sqrt{3}$$

$$\Rightarrow \quad m = 0 \text{ or } m = \sqrt{3}$$

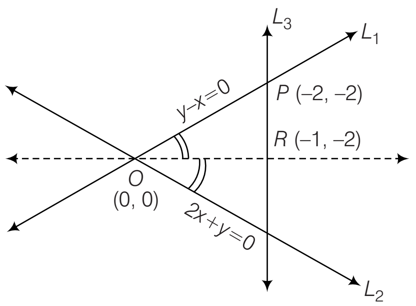
$\because$ L intersects X-axis,

$\therefore \quad m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}(x-3)$

or $\quad \sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$

109.



$$L_1 : y - x = 0, \quad L_2 : 2x + y = 0, \quad L_3 : y + 2 = 0$$

On solving the equation of lines L_1 and L_2, we get their point of intersection (0, 0) i.e. origin O.

On solving the equation of lines L_1 and L_3,

we get $\quad P = (-2, -2)$

Similarly, we get $\quad Q = (-1, -2)$

We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$$\therefore \quad \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

110. Let the joining points be $A(1, 1)$ and $B(2, 4)$.

Let point C divides line AB in the ratio 3 : 2. So, by section formula we have

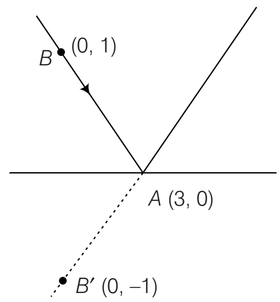
$$C = \left(\frac{3 \times 2 + 2 \times 1}{3+2}, \frac{3 \times 4 + 2 \times 1}{3+2}\right) = \left(\frac{8}{5}, \frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$\therefore C$ satisfies the equation $2x + y = k$.

$$\Rightarrow \quad \frac{2 + 8}{5} + \frac{14}{5} = k \Rightarrow k = 6$$

111. Suppose $B(0, 1)$ be any point on given line and coordinate of A is $(\sqrt{3}, 0)$. So, equation of



Reflected ray is $\dfrac{-1-0}{0-\sqrt{3}} = \dfrac{y-0}{x-\sqrt{3}}$

$$\Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$$

112. The intersection point of two lines is $\left(\dfrac{-c}{a+b}, \dfrac{-c}{a+b}\right)$

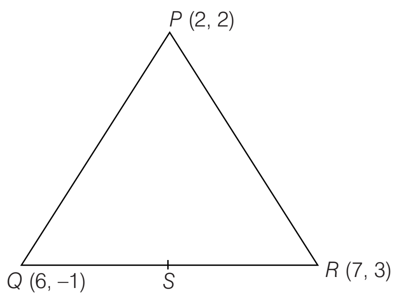
Distance between (1, 1) and $\left(\dfrac{-c}{a+b}, \dfrac{-c}{a+b}\right) < 2\sqrt{2}$

$$\Rightarrow \quad 2\left(1 + \frac{c}{a+b}\right)^2 < 8$$

$$\Rightarrow \quad 1 + \frac{c}{a+b} < 2$$

$$\Rightarrow \quad a + b - c > 0$$

113. Let P, Q, R, be the vertices of ΔPQR



Since, PS is the median, S is mid-point of QR

So, $$S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$$

Now, slope of $PS = \dfrac{2-1}{2-\dfrac{13}{2}} = -\dfrac{2}{9}$

Since, required line is parallel to PS therefore slope of required line = slope of PS Now, eqn of line passing through $(1, -1)$ and having slope $-\dfrac{2}{9}$ is

$$y - (-1) = -\frac{2}{9}(x-1)$$

$$9y + 9 = -2x + 2$$

$$\Rightarrow \quad 2x + 9y + 7 = 0$$

114. Given lines are

$$4ax + 2ay + c = 0$$
$$5bx + 2by + d = 0$$

The point of intersection will be

$$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$$

$\Rightarrow \quad x = \frac{2(ad - bc)}{-2ab} = \frac{bc - ad}{ab}$

$\Rightarrow \quad y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

Also distance from axes is same

So $\quad x = -y \quad$ ($\because$ distance from X-axis is $-y$ as y is negative)

$$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab} \Rightarrow 3bc - 2ad = 0$$

115. Let the point P be (x, y)

Then $\quad d_1(P) = \left|\frac{x - y}{\sqrt{2}}\right|$ and $d_2(P) = \left|\frac{x + y}{\sqrt{2}}\right|$

For P lying in first quadrant $x > 0, y > 0$.

Also $\quad 2 \le d_1(P) + d_2(P) \le 4$

$\Rightarrow \quad 2 \le \left|\frac{x - y}{\sqrt{2}}\right| + \left|\frac{x + y}{\sqrt{2}}\right| \le 4$

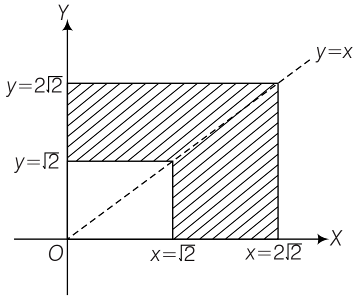
If $x > y$, then

$$2 \le \frac{x - y + x + y}{\sqrt{2}} \le 4 \text{ or } \sqrt{2} \le x \le 2\sqrt{2}$$

If $x < y$, then

$$2 \le \frac{y - x + x + y}{2} \le 4 \text{ or } \sqrt{2} \le y \le 2\sqrt{2}$$

The required region is the shaded region in the figure given below.



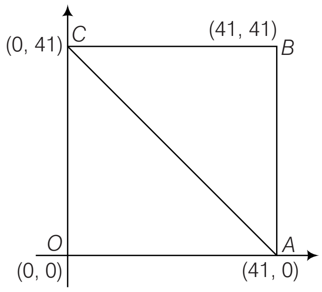
$\therefore$ Required area $= (2\sqrt{2})^2 - (\sqrt{2})^2 = 8 - 2 = 6$ sq units

116. Total number of integral points inside the square $OABC$

$$= 40 \times 40 = 1600$$

Number of integral points on AC

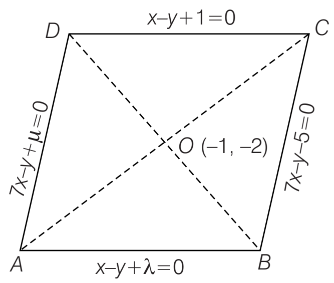
$$= \text{Number of integral points on } OB$$
$$= 40 \text{ [namely (1, 1), (2, 2) ... (40, 40)]}$$



$\therefore \quad$ Number of integral points inside the ΔOAC

$$= \frac{1600 - 40}{2} = 780$$

117.



Let other two sides of rhombus are

$$x - y + \lambda = 0$$

and $\quad 7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore \quad |-1 + 2 + 1| = |-1 + 2 + \lambda| \Rightarrow \lambda = -3$

and $\quad |-7 + 2 - 5| = |-7 + 2 + \mu| \Rightarrow \mu = 15$

$\therefore$Other two sides are

$$x - y - 3 = 0$$

and $\quad 7x - y + 15 = 0$

On solving the equation of sides pairwise, we get the vertices as $\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$