CHAPTER 4

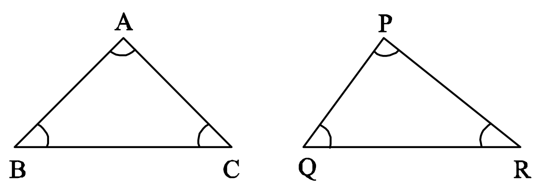
GEOMETRY

Exercise 4.1

KEY POINTS

I. Congruency and Similarity of Triangles

1. Two triangles said to be congruent if they have same shape and same size.



$\Delta ABC \equiv \Delta PQR$, then

* $\angle A = \angle P$ (Corresponding angles are equal).

$\angle B = \angle Q$

$\angle C = \angle R$

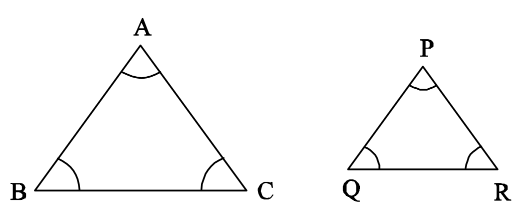
* $AB = PQ$ (Corresponding sides are equal)

$BC = QR$

$CA = RP$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = 1$$

2. Two triangles said to be **similar** if they same shape but not same size.



$\Delta ABC \sim \Delta PQR$, then

* $\angle A = \angle P$ (Corresponding angles are equal).

$\angle B = \angle Q$

$\angle C = \angle R$

* $AB \neq PQ$ (Corresponding sides are not equal).

$BC \neq QR$

$AC \neq PR$

- $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} > 1$ or < 1

Corresponding sides are proportional.

Criteria of Similarity

AA Criterion of similarity

In ΔABC and ΔPQR

if $\angle A = \angle P$ and $\angle B = \angle Q$ [any two corresponding angles are equal]

then $\Delta ABC \sim \Delta PQR$

SAS Criterian of similarity

In ΔABC and ΔPQR

if $\angle A = \angle P$ and

$\frac{AB}{PQ} = \frac{AC}{PR}$ then

(one corresponding angle two corresponding sides are equal)

$\Delta ABC \sim \Delta PQR$

SSS Criterian of Similarity

In ΔABC and ΔPQR

if $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

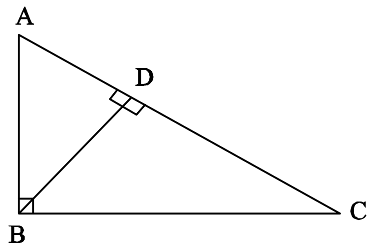
(Corresponding sides are proportional)

then $\Delta ABC \sim \Delta PQR$

Some results on similar triangles

1. ΔABC right angled at B, and $BD \perp AC$, then

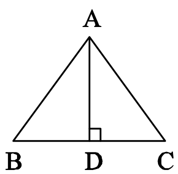
- $\Delta ADB \sim \Delta BDC$
- $\Delta ABC \sim \Delta ADB$
- $\Delta ABC \sim \Delta BDC$

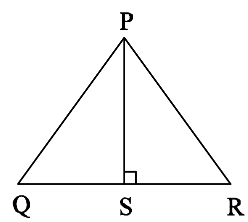


2. If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of their corresponding altitudes and (Medians) also.

$$\Delta ABC \sim \Delta PQR, \text{ then}$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$





3. If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of the corresponding perimeters.

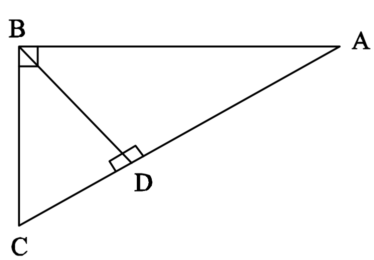
$$\Delta ABC \sim \Delta PQR \text{ then}$$

$$\frac{\text{Perimeter } \Delta ABC}{\text{Perimeter } \Delta PQR} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

4. If two triangle are similar, then the area of two similar triangles are equal to the ratio of the squares of their corresponding sides.

$$\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

5. If two triangles have common vertex and their bases are on the same straight line, the ratio between their areas is equal to the ratio between the length of their bases.



$$\frac{\text{area } (\Delta ABD)}{\text{area } (\Delta BDC)} = \frac{AD}{DC}$$

6. Two triangles are said to be similar if their corresponding sides are proportional.

- Two triangles are equiangular if the corresponding angles are equal.
- If two triangles are similar, then they are equiangular.

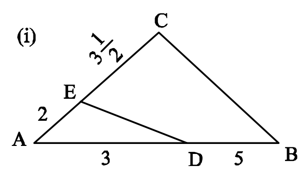
Exercise 4.1

Type I: Problems Based on Similarity

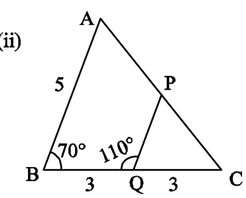
Q.No. 1(i)(ii), Example 4.1, 4.2, 4.3, 4.5, 4.6, 4.4, 2, 3, 5, 6, 8, 4.7, 4.8

1. Check whether the which triangles are similar and find the value of *x*.

(i)



(ii)



✍ **Solution:**

In ΔABC and ΔADE

(i) $\dfrac{AE}{AC} = \dfrac{2}{2+3.5} = \dfrac{2}{5.5} = \dfrac{4}{11}$

$\dfrac{AD}{AB} = \dfrac{3}{3+5} = \dfrac{3}{8}$

$\therefore \dfrac{AE}{AC} \neq \dfrac{AD}{AB}$

$\therefore$ the 2 triangles are not similar.

(ii) Given In ΔABC and ΔPQC

$\angle PQB = 110^\circ \Rightarrow \angle PQC = 70^\circ = \angle QBA$

$\therefore$ Corresponding angles are equal.

$$\therefore \Delta ABC \sim \Delta PQC$$

$\Rightarrow \dfrac{AB}{PQ} = \dfrac{BC}{QC}$

$\Rightarrow \dfrac{5}{x} = \dfrac{6}{3}$

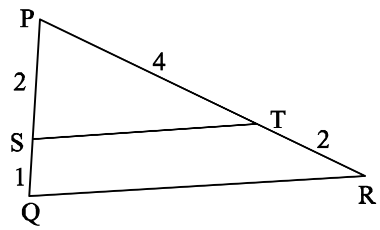
$\therefore 2x = 5$

$x = 2.5$ cm

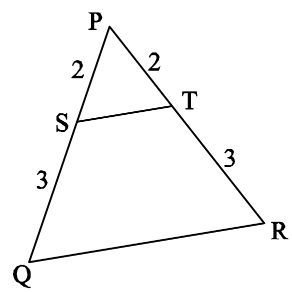
Example 4.1

Show that $\Delta PST \sim \Delta PQR$

(i)



(ii)



Solution:

(i) In ΔPST and ΔPQR,

$$\frac{PS}{PQ} = \frac{2}{2+1} = \frac{2}{3}, \frac{PT}{PR} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common

Therefore, by SAS similarity,

$$\Delta PST \sim \Delta PQR$$

(ii) In ΔPST and ΔPQR,

$$\frac{PS}{PQ} = \frac{2}{2+3} = \frac{2}{5}, \frac{PT}{PR} = \frac{2}{2+3} = \frac{2}{5}$$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common

Therefore, by SAS similarity,

$$\Delta PST \sim \Delta PQR$$

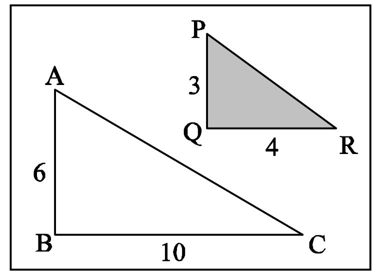
Example 4.2

Is $\Delta ABC \sim \Delta PQR$?

Solution:

In ΔABC and ΔPQR,

$$\frac{PQ}{AB} = \frac{3}{6} = \frac{1}{2}, \frac{QR}{BC} = \frac{4}{10} = \frac{2}{5}$$



Since $\frac{1}{2} \neq \frac{2}{5}, \frac{PQ}{AB} \neq \frac{QR}{BC}$

The corresponding sides are not proportional.

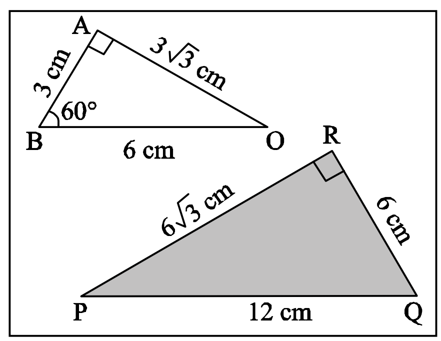
Therefore ΔABC is not similar to ΔPQR

Example 4.3

Observe Fig. 4.18 and find $\angle P$

Solution:

In ΔBAC and ΔPRQ,



$$\frac{AB}{RQ} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{BC}{QP} = \frac{6}{12} = \frac{1}{2}; \frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

Therefore, $\frac{AB}{RQ} = \frac{BC}{QP} = \frac{CA}{PR}$

By SSS similarity, we have $\Delta BAC \sim \Delta QRP$

$\angle P = \angle C$ (since the corresponding parts of similar triangle)

$$\angle P = \angle C = 180° - (\angle A + \angle B)$$

$$= 180° - (90° + 60°)$$

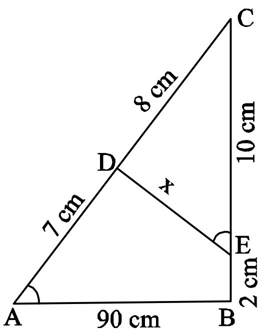
$$\angle P = 180° - 150° = 30°$$

Example 4.5

In Fig. $\angle A = \angle CED$ ***prove that*** $\Delta\, CAB \sim \Delta\, CED$***. Also find the value of x.***

✍ **Solution:**

In $\Delta\, CAB$ and $\Delta\, CED$, $\angle C$ is common, $\angle A = \angle CED$



Therefore, $\Delta\, CAB \sim \Delta\, CED$ (By AA similarity)

Hence

$$\frac{CA}{CE} = \frac{AB}{DE} = \frac{CB}{CD}$$

$$\frac{AB}{DE} = \frac{CE}{CD} \text{ gives } \frac{9}{x} = \frac{10+12}{8}$$

$$\frac{9}{x} = \frac{12}{8}$$

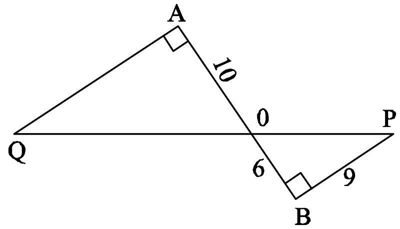
so, $x = \dfrac{8 \times 9}{12} = 6$ cm

Example 4.6

In Fig. QA and PB are perpendicular to AB. If $AO = 10$ **cm,** $BO = 6$ **cm and** $PB = 9$ **cm.** ***Find AQ.***

✍ **Solution:**

In $\Delta\, AOQ$ and $\Delta\, BOP$, $\angle OAQ = \angle OBP = 90°$



$\angle AOQ = \angle BOP$ (Vertically opposite angles)

Therefore, by AA Criterion of similarity,

$\Delta\, AOQ \sim \Delta\, BOP$

$$\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$$

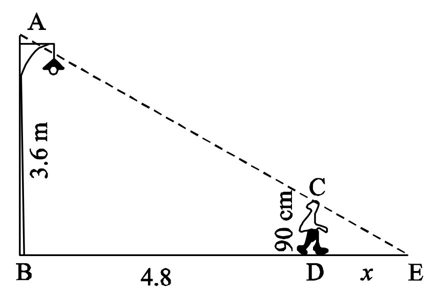
$$\frac{10}{6} = \frac{AQ}{9} \text{ gives } AQ = \frac{10 \times 9}{6} = 15 \text{ cm}$$

Example 4.4

A boy a height 90cm is walking away from the base of a lamp post at a speed of 1.2m/sec. If the lamp post is 3.6m above the ground, find the length of his shadow cast after 4 seconds.

✍ **Solution:**

Given, speed = 1.2 m/s,



time = 4 seconds

distance = speed × time

= 1.2 × 4 = 4.8 m

Let x be the length of the shadow after 4 seconds.

Since, $\Delta\, ABE \sim CDE$, $\dfrac{BE}{DE} = \dfrac{AB}{CD}$

$$\frac{4.8 + x}{x} = \frac{3.6}{0.9}$$

$$\frac{4.8 + x}{x} = 4$$

$$4x = 4.8 + x$$

$$4x - x = 4.8$$

$$3x = 4.8$$

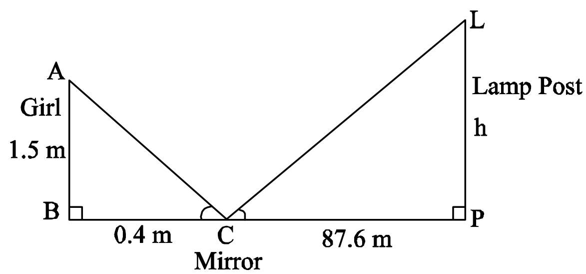
$$x = \frac{4.8}{3}$$

$$x = 1.6$$

$DE = 1.6$ m

2. A girl looks the reflection of the top of the lamp post on the mirror which is 66 m away from the foot of the lamppost. The girl whose height is 12.5 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamppost are in a same line, find the height of the lamp post.

✍ **Solution:**



Given $AB =$ height of girl = 1.5 m

$BC =$ Dist. between girl and Mirror = 0.4 m

$LP =$ height of lamp post $= h$

$CP =$ dist. between Mirror and Post = 87.6 m

In $\Delta ABC, \Delta LPC$, $\angle B = LP = 90°$,

$\angle ACB = \angle LCP$ (angle of incidence and angle of reflection)

$\therefore$ ΔABC and ΔLPC are similar.

$\therefore \dfrac{AB}{LP} = \dfrac{BC}{CP}$ (By AA similarity)

$\Rightarrow \dfrac{1.5}{h} = \dfrac{0.4}{87.6}$

$\Rightarrow h = \dfrac{87.6 \times 1.5}{0.4}$

$= \dfrac{131.4}{0.4}$

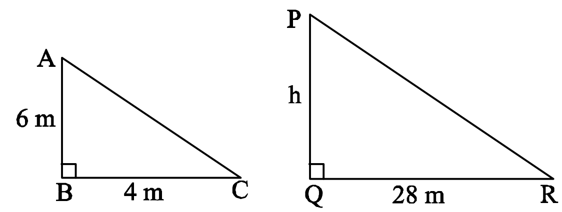
$= \dfrac{1314}{4}$

$= 328.5$

$\therefore$ Height of the lamp post = 328.5 m

3. A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.

✍ **Solution:**



In ΔABC and ΔPQR

$\angle B = \angle Q = 90°$

$\angle C = \angle R$ $(AC \,||\, PR)$

$\therefore$ By AA similarity, $\Delta ABC \sim \Delta PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\Rightarrow \dfrac{6}{h} = \dfrac{4}{28}$

$\Rightarrow \dfrac{6}{h} = \dfrac{1}{7}$

$\Rightarrow h = 42$ m

$\therefore$ Height of the tower = 42 m.

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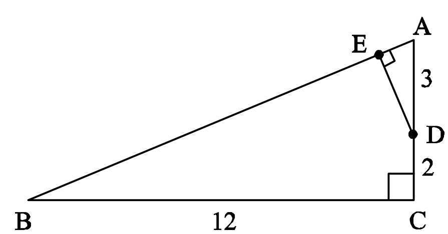
5. In the adjacent figure, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.



✍ **Solution:**

In ΔABC and ΔADE,

(i) $\angle AED = \angle ACB = 90°$

(ii) $\angle A$ is common

$\therefore$ By AA similarly,

$$\Delta ABC \sim \Delta ADE$$

Also, $AB^2 = AC^2 + BC^2$

$$= 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169$$

$\therefore$ $AB = 13$

$\therefore$ By similarily,

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

$$\frac{13}{3} = \frac{12}{DE} \qquad \frac{13}{3} = \frac{5}{AE}$$

$$13DE = 36 \qquad AE = \frac{15}{13}$$

$$DE = \frac{36}{13}$$

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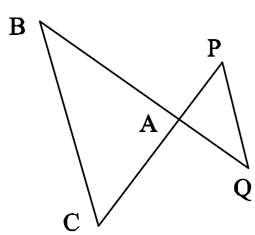
6. In the adjacent figure, $\Delta ACB \sim \Delta APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, find CA and AQ.



✍ **Solution:**

Given $\Delta ACB \sim \Delta APQ$

$$\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$$

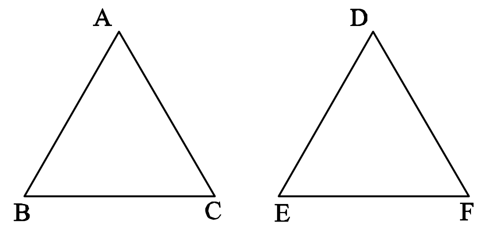
$$\frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$$

$$\frac{AC}{2.9} = \frac{8}{4}$$
$$\frac{AC}{2.8} = \frac{2}{1}$$
$$AC = 5.6 \text{ cm}$$

$$\frac{8}{4} = \frac{6.5}{AQ}$$
$$\frac{2}{1} = \frac{6.5}{AQ}$$
$$AQ = \frac{6.5}{2}$$
$$AQ = 3.25 \text{ cm}$$

8. If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is 9 cm^2 and the area of ΔDEF is 16 cm^2 and $BC = 2.1$ cm. Find the length of EF.

✍ **Solution:**



Given $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\left(\frac{3}{4}\right)^2 = \left(\frac{2.1}{EF}\right)^2$$

$$\frac{3}{4} = \frac{2.1}{EF}$$

$$EF = \frac{201 \times 4}{3}$$

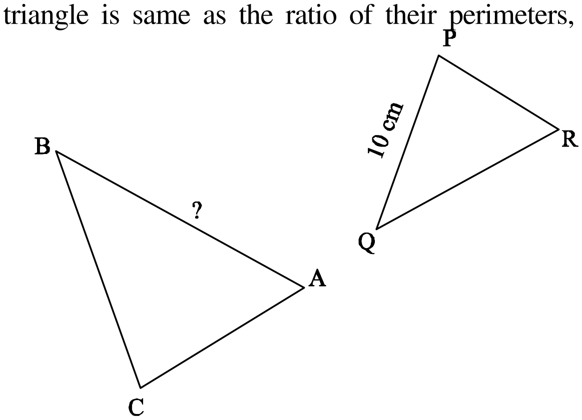
$$= 2.8 \text{ cm}$$

Example 4.7

The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If $PQ = 10$ cm , find AB.

✍ **Solution:**

The ratio of the corresponding sides of similar triangle is same as the ratio of their perimeters,



Since $\Delta ABC \sim \Delta PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24}$$

$$\frac{AB}{PQ} = \frac{36}{24} \text{ gives } \frac{AB}{10} = \frac{36}{24}$$

$$AB = \frac{36 \times 10}{24} = 15 \text{ cm}$$

Example 4.8

If ΔABC is similar to ΔDEF such that $BC = 3$ cm, $EF = 4$ cm and area of $\Delta ABC = 54$ cm^2. Find the area of ΔDEF.

✍ **Solution:**

Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC^2}{EF^2} \text{ gives } \frac{54}{\text{Area}(\Delta DEF)} = \frac{3^2}{4^2}$$

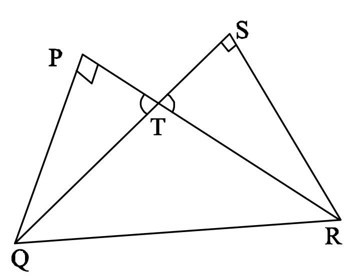
$$\text{Area}(\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

Type II: Prove the following based on similarity

Q.No. 4, 7, 9, Example 4.9

4. Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that $PT \times TR = ST \times TQ$.

✍ **Solution:**



Consider ΔPQT and ΔSRT

(i) $\angle P = \angle S = 90°$

(ii) $\angle PTQ = \angle STR$ (Vertically Opp.angle)

$\therefore$ By AA similarity,

$$\Delta PQT \sim \Delta SRT$$

$$\therefore \frac{QT}{TR} = \frac{PT}{ST}$$

$\Rightarrow PT \times TR = ST \times TQ$

Hence proved.

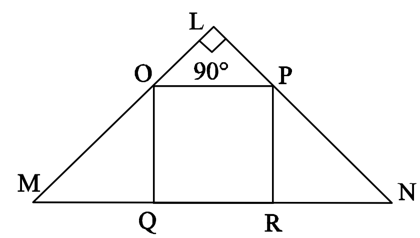
7. If figure $OPRQ$ is a square and $\angle MLN = 90°$. Prove that

(i) $\Delta LOP \sim \Delta QMO$

(ii) $\Delta LOP \sim \Delta RPN$

(iii) $\Delta QMO \sim \Delta RPN$

(iv) $QR^2 = MQ \times RN$



✍ **Solution:**

(i) In $\Delta LOP, \Delta QMO$

$\angle OLP = \angle OQM = 90°$

$\angle LOP = \angle OMQ$ (Corresponding angles)

$\therefore$ By AA similarity,

$$\Delta LOP \sim \Delta QMO$$

(ii) In $\Delta LOP, \Delta RPN$

$\angle OLP = \angle PRN = 90°$

$\angle LPO = \angle PNR$ (Corresponding angles)

$\therefore$ By AA similarity,

$$\Delta LOP \sim \Delta RPN$$

$\therefore$ From (i) and (ii)

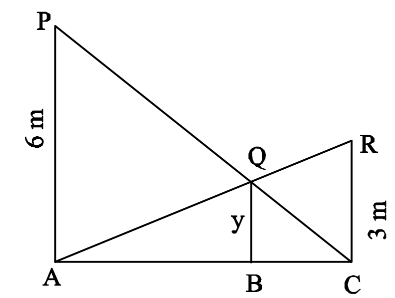
$$\Delta QMO \sim \Delta RPN$$

$$\therefore \frac{QM}{RP} = \frac{QO}{RN} \Rightarrow \frac{QM}{QR} = \frac{QR}{RN}$$

($RP = QR$) Square sides are equal.

$$\Rightarrow QR^2 = MQ \times RN$$

9. Two vertical poles of heights 6m and 3m are erected above a horizontal ground *AC*. Find the value of *y*.



✍ **Solution:**

From the fig.

$\Delta\, PAC$ and $\Delta\, QBC$)

$\angle PAC = \angle QBC \;= 90°$

$\angle C$ is common

$\therefore$ by AA similarity

$\Delta\, PAC \sim QBC$

$$\therefore \; \frac{CB}{CA} = \frac{QB}{PA}$$

$$\Rightarrow \; \frac{CB}{CA} = \frac{y}{6} \qquad ...(1)$$

$\Delta\, RCA$ and $\Delta\, QBA$

$\angle RCA = \angle QBA = 90°$

$\angle A$ is common

$\therefore$ by AA similarity

$\Delta\, RCA \sim \Delta\, QBA$

$$\therefore \; \frac{AB}{AC} = \frac{BQ}{RC} \Rightarrow \frac{AB}{BC} = \frac{y}{3} \qquad ...(2)$$

Adding (1) and (2),

$$\frac{AB + BC}{AC} = \frac{y}{6} + \frac{y}{3}$$

$$\Rightarrow \; \frac{AC}{AC} = y\left(\frac{1}{6} + \frac{1}{3}\right)$$

$$\Rightarrow \; y\left(\frac{1+2}{6}\right) = 1$$

$$\Rightarrow \; y\left(\frac{1}{2}\right) = 1$$

$$\therefore \; y = 2m$$

(or) Using formula $y = \dfrac{ab}{a+b}$

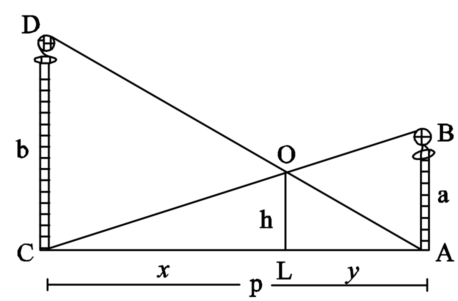
$$= \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\text{m}$$

Example 4.9

Two poles of height 'a' meters and 'b' meters are 'p' meters apart. Prove that the height of the point of intersection of the lines joining the top of each pole of the foot of the opposite pole is given by $\dfrac{ab}{a+b}$ meters.

✍ **Solution:**

Let AB and CD be two poles of height 'a' meters and 'b' meters respectively such that the poles are 'p' meters apart. That is $AC = p$ meters. Suppose the lines AD and BC meet at O, such that $OL = h$ meters



Let $CL = x$ and $LA = y$.

Then, $x + y = p$

In $\Delta\, ABC$ and $\Delta\, LOC$, we have $\angle CAB = \angle CLO$ [each equals to 90°]

$\angle C = \angle C$ [C is common]

$\Delta\, CAB \sim \Delta\, CLO$ [By AA similarity]

$$\frac{CA}{CL} = \frac{AB}{LO} \text{ gives } \frac{p}{x} = \frac{a}{h}$$

so, $$x = \frac{ph}{a} \qquad ...(1)$$

In $\Delta\, ALO$ and $\Delta\, ACD$, we have

$\angle ALO = \angle ACD$ [each equal to 90°]

$\angle A = \angle A$ [A is common]

Construction: Draw $CE \parallel DA$. Extend BA to meet at E.

No.	Statement	Reason
1.	$\angle BAD = \angle 1$ $\angle DAC = \angle 2$	Assumption
2.	$\angle BAD = \angle AEC$ $= \angle 1$	Since $DA \parallel CE$ and AC is transversal, corresponding angles are equal.
3.	$\angle DAC = \angle ACE$ $= \angle 2$	Since $DA \parallel CE$ and AC is transversal. Alternate angles are equals.
4.	$\frac{BA}{AE} = \frac{BD}{DC} \ldots (2)$	In ΔBCE by Thales theorem
5.	$\frac{AB}{AC} = \frac{BD}{DC}$	From (1)
6.	$\frac{AB}{AC} = \frac{BA}{AE}$	From (1) and (2)
7.	$AC = AE \ldots (3)$	Cancelling AB
8.	$\angle 1 = \angle 2$	ΔACE is isosceles by (3)
9.	AD bisects $\angle A$	Since Hence proved.

Type I: (Problems Based on Thales Theorem or BPT)

Q.No. 1(i)(ii), Example 4.12, 2, 3(i)(ii), Example 4.13, 5

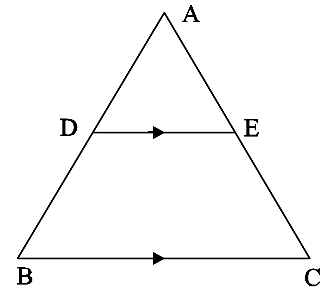
1. In ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm find AE.

(ii) If $AD = 8x - 7$, $DB = 5x - 3$, $AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

✍ **Solution:**

(i) Given $\frac{AD}{DB} = -\frac{3}{4}, AC = 15$



Let $AE = x \Rightarrow EC = 15 - x$

$\therefore$ By $BPT \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{3}{4} = \frac{x}{15 - x}$

$\Rightarrow 4x = 45 - 3x$

$\Rightarrow 7x = 45$

$x = \frac{45}{7} = 6.428$

$= 6.43$

(ii) Given $AD = 8x - 7, DB = 5x - 3$

$AE = 4x - 3, EC = 3x - 1$

By $BPT \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{8x - 7}{5x - 3} = \frac{4x - 3}{3x - 1}$

$\Rightarrow (8x - 7)(3x - 1) = (4x - 3)(5x - 3)$

$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$\Rightarrow 24x^2 - 29x + 7 = 20x^2 - 27x + 9$

$\Rightarrow 4x^2 - 2x - 0 = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\therefore x = 1, \frac{-1}{2}$

(negative not possible)

$\therefore x = 1$ only

-2

$-2 \quad 1$

$\frac{-2}{2}, \quad \frac{1}{2}$

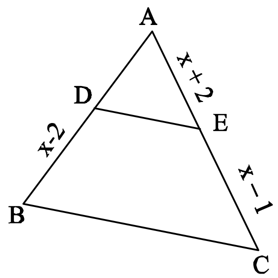
$-1, \quad \frac{1}{2}$

Example 4.12

In ΔABC if $DE \parallel BC, AD = x, DB = x - 2$, and $EC = x - 1$ then find the lengths of the sides AB and AC.

✍ **Solution:**

In ΔABC we have $DE \parallel BC$



By Thales theorem, we have $\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{x}{x-2} = \frac{x+2}{x-1}$ gives $x(x-1) = (x-2)(x+2)$

Hence, $x^2 - x = x^2 - 4$ so, $x = 4$

When $x = 4, AD = 4, DB = x - 2 = 2,$

$AE = x + 2 = 6, EC = x - 1 = 3$

Hence, $AB = AD + DB = 4 + 2 = 6$

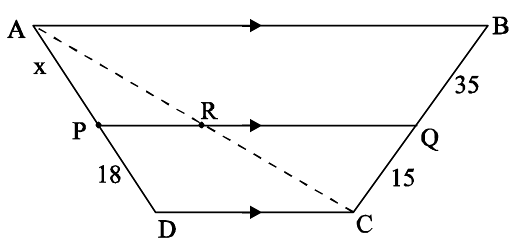
$AC = AE + EC = 6 + 3 = 9$

Therefore, $AB = 6, AC = 9$

---

2. *ABCD* is a trapezium in which $AB \parallel DC$ and P, Q are points on *AD* and *BC* respectively, such that $PQ \parallel DC$ if $PD = 18$ cm, $BQ = 35$ cm and $QC = 15$ cm, find *AD*

✍ **Solution:**



In trapezium $ABCD, AB \parallel DC \parallel PQ$

Join AC, meet PQ at R.

In $\Delta ACD, PR \parallel DC$

$\therefore$ By $BPT \frac{AP}{PD} = \frac{AR}{RC}$

$\Rightarrow \frac{x}{18} = \frac{AR}{RC}$ ...(1)

In $\Delta ABC, RQ \parallel AB$

$\therefore$ By $ABT \frac{BQ}{QC} = \frac{AR}{RC}$

$\Rightarrow \frac{35}{15} = \frac{AR}{RC}$

$\Rightarrow \frac{7}{3} = \frac{AR}{RC}$ ...(2)

$\therefore$ From (1) and (2)

$\frac{x}{18} = \frac{7}{3}$

$\frac{x}{6} = \frac{7}{1}$

$\Rightarrow x = 42$

$\therefore AD = AP + PD$

$= 42 + 18$

$= 60$ m

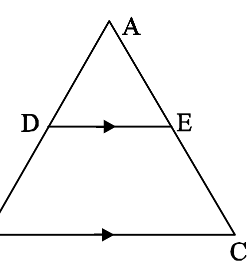
---

3. In $\Delta ABC, D$ and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$

(i) $AB = 12$ cm, $AD = 8$ cm, $AE = 12$ cm and $AC = 18$ cm

(ii) $AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm

✍ **Solution:**



In ΔABC, To Prove: $DE \parallel BC$

(i) $\frac{AD}{AB} = \frac{8}{12} = \frac{2}{3}$

$\frac{AE}{AC} = \frac{12}{18} = \frac{2}{3}$

$\frac{AD}{AB} = \frac{AE}{AC}$

$\therefore$ By converse of $BPT\ DE \parallel BC$

(ii) $\frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4}$

$\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4}$

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$

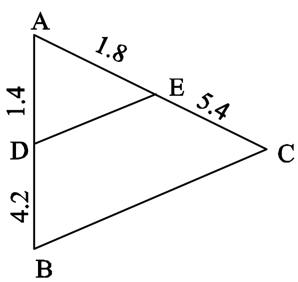
$\therefore$ By Converse of BPT, $DE \parallel BC$

Example 4.13

D and E are respectively the points on the sides AB and AC of a ΔABC such that $AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm AND $AE = 1.8$ cm show that $DE \parallel BC$.

✍ **Solution:**

We have $AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm



$BD = AB - AD = 5.6 - 1.4 = 4.2$ cm

and $EC = AC - AE = 7.2 - 1.8 = 5.4$ cm

$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$

$\frac{AD}{DB} = \frac{AE}{EC}$

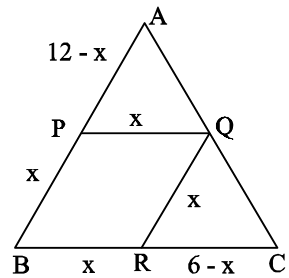
Therefore, by converse of Basic Proportionality Theorem, we have DE is parallel to BC.

Hence proved.

5. Rhombus $PQRB$ is inscribed in ΔABC such that $\angle B$ is one of its angle. P, Q and R lie on AB, AC and BC respectively. If $AB = 12$ cm and $BC = 6$ cm, find the sides PQ, RB of the rhombus.

✍ **Solution:**

Rhombus $PQRS$ is inscribed in ΔABC



Let the side of the rhombus be x

$\therefore AB = 12$ cm $AP = 12 - x$

$BC = 6$ cm $RC = 6 - x$

In ΔABC, $PQ \parallel BC$

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$ ...(1)

In ΔABC, $QR \parallel AB$

$\therefore \frac{BR}{RC} = \frac{AQ}{QC}$ ...(2)

$\therefore$ From (1) and (2)

$\Rightarrow \frac{AP}{PB} = \frac{BR}{RC}$

$\Rightarrow \frac{12 - x}{x} = \frac{x}{6 - x}$

$\Rightarrow x^2 = (6 - x)(12 - x)$

$\Rightarrow x^2 = x^2 - 18x + 72$

$\Rightarrow 18x = 72$

$\Rightarrow x = 4$ cm

$\therefore PQ = RB = 4$ cm

Type II: (Prove the followings based on BPT)

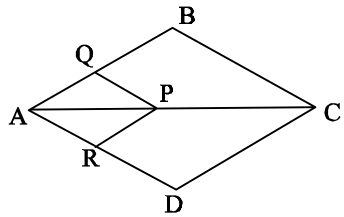
Q.No. 4, 6, 7, Example 4.14

4. In fig. $PQ \parallel BC$ and $PR \parallel CD$ prove that (i) $\frac{AR}{AD} = \frac{AQ}{AB}$ (ii) $\frac{QB}{AQ} = \frac{DR}{AR}$

✍ **Solution:**

(i) In ΔABC, $PQ \parallel BC$

$\therefore$ By $BPT \frac{AQ}{AB} = \frac{AP}{AC}$ ...(1)



In $\Delta\, ADC$, $PR \parallel DC$

$\therefore$ By $BPT\ \dfrac{AR}{AD} = \dfrac{AP}{AC}$ ...(2)

$\therefore$ From (1) and (2),

$$\frac{AQ}{AB} = \frac{AR}{AD}$$

(ii) From (i) $\dfrac{AB}{AQ} = \dfrac{AD}{AR}$ (reciprocal)

$$\Rightarrow \frac{AB}{AQ} - 1 = \frac{AD}{AR} - 1$$

$$\Rightarrow \frac{AB - AQ}{AQ} = \frac{AD - AR}{AR}$$

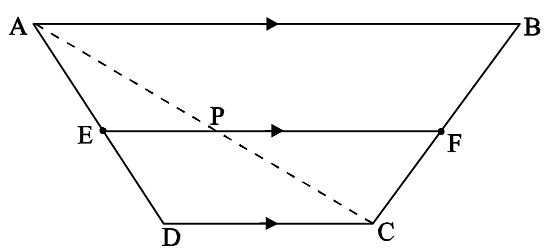
$$\Rightarrow \frac{BQ}{AQ} = \frac{DR}{AR}$$

Hence proved.

---

6. In trapezium $ABCD, AB \parallel DC, E$ and F are points on non-parallel sides AD and BC respectively, such that $EF \parallel AB$. Show that $\dfrac{AE}{ED} = \dfrac{BF}{FC}$

✍ **Solution:**



In trapezium $ABCD, AB \parallel DC \parallel EF$

Join AC to meet EF at P

In $\Delta\, ADC$, $EP \parallel DC$

$\therefore$ By BPT, $\dfrac{AE}{ED} = \dfrac{AP}{PC}$ ...(1)

In $\Delta\, ABC$, $PR \parallel AB$

$\therefore$ By BPT, $\dfrac{BF}{FC} = \dfrac{AP}{PC}$ ...(2)

From (1) and (2)

$$\frac{AE}{ED} = \frac{BF}{FC}$$

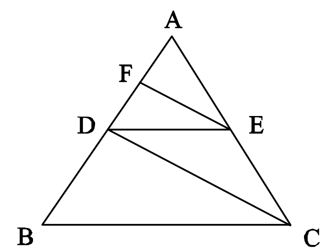
Hence proved.

---

7. In figure $DE \parallel BC$ and $CD \parallel EF$. Prove that $AD^2 = AB \times AF$

✍ **Solution:**

In figure $DE \parallel BC$ and $CD \parallel EF$



In $\Delta\, ACD$, by BPT, $\dfrac{AF}{AD} = \dfrac{AE}{AC}$ ...(1)

In $\Delta\, ABC$, by BPT, $\dfrac{AD}{AB} = \dfrac{AE}{AC}$ ...(2)

$\therefore$ From (1) and (2)

$$\frac{AF}{AD} = \frac{AD}{AB}$$

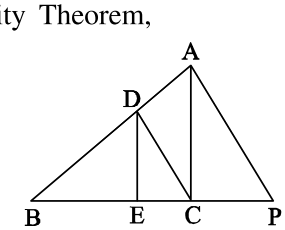
$$\Rightarrow AD^2 = AF \cdot AB$$

Example 4.14

In the Fig. $DE \parallel AC$ and $DC \parallel AP$. Prove that $\dfrac{BE}{EC} = \dfrac{BC}{CP}$.

✍ **Solution:**

In $\Delta\, BPA$, we have $DC \parallel AP$. By Basic Proportionality Theorem,



We have $\frac{BC}{CP}=\frac{BD}{DA}$ ...(1)

In ΔBCA, we have $DE \parallel AC$. By Basic Proportionality Theorem,

We have $\frac{BE}{EC}=\frac{BD}{DA}$ ...(2)

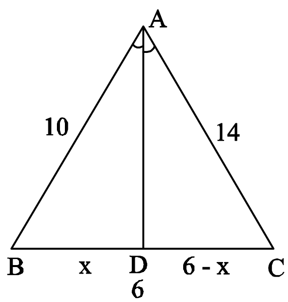
From (1) and (2) we get, $\frac{BE}{EC}=\frac{BC}{CP}$.

Hence proved.

Type III: (Problems based on ABT)
Q.No. 8, Example 4.15, 4.16, 9(i)(ii)

8. In $\Delta ABC, AD$ is the bisector of $\angle A$ meeting side BC at D, if $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm, find BD and DC

✍ **Solution:**



In $\Delta ABC, AD$ is the bisector of $\angle A$

$\therefore$ By ABT, $\frac{AB}{AC}=\frac{BD}{DC}$

$\Rightarrow \frac{10}{14}=\frac{x}{6-x}$

$\Rightarrow \frac{5}{7}=\frac{x}{6-x}$

$\Rightarrow 30-5x=7x$

$\Rightarrow 12x=30$

$x=\frac{5}{2}=2.5$

$\therefore$ $BD = 2.5$ cm and $DC = 6-x = 6-2.5$

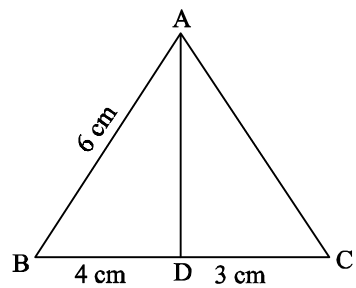
$DC = 3.5$ cm

Example 4.15

In the Fig., AD is the bisector of $\angle A$. If $BD = 4$ cm, $DC = 3$ cm and $AB = 6$ cm, find AC.

✍ **Solution:**

In $\Delta ABC, AD$ is the bisector of $\angle A$



Therefore by Angle Bisector Theorem

$$\frac{BD}{DC}=\frac{AB}{AC}$$

$$\frac{4}{3}=\frac{6}{AC} \text{ gives } 4\,AC=18$$

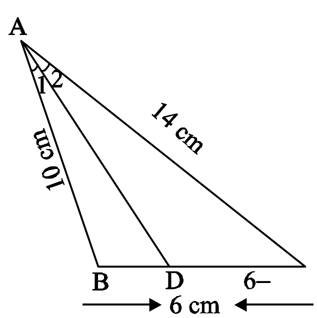
Hence $AC=\frac{9}{2}=4.5$ cm

Example 4.16

In the Fig. AD is the bisector of $\angle BAC$. If $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm, find BD and DC.

✍ **Solution:**

Let $BD = x$ cm, then $DC = (6-x)$ cm



AD is the bisector of $\angle A$

Therefore by Angle Bisector Theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{10}{14} = \frac{x}{6-x} \text{ gives } \frac{5}{7} = \frac{x}{6-x}$$

So, $12x = 30$ we get, $x = \frac{30}{12} = 2.5$ cm

Therefore, $BD = 2.5$ cm, $DC = 6 - x$

$= 6 - 2.5 = 3.5$ cm

---

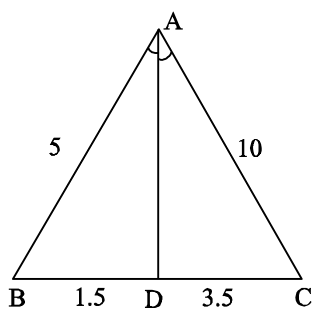
9. Check whether AD is bisector of $\angle A$ of ΔABC in each of the following

(i) $AB = 5$ cm, $AC = 10$ cm, $BD = 1.5$ cm and $CD = 3.5$ cm.

(ii) $AB = 4$ cm, $AC = 6$ cm, $BD = 1.6.$ cm and $CD = 2.4$ cm

✍ **Solution:**

(i)

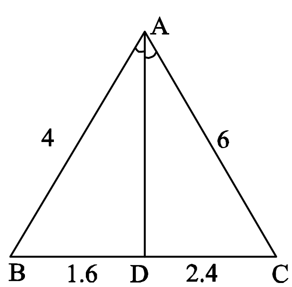


$$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}, \frac{BD}{DC} = \frac{1.5}{3.5} = \frac{3}{7}$$

$$\therefore \frac{AB}{AC} \neq \frac{BD}{DC}$$

$\therefore AD$ is not the bisector of $\angle A$.

(ii)



$$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}, \frac{BD}{DC} = \frac{1.6}{2.4} = \frac{2}{3}$$

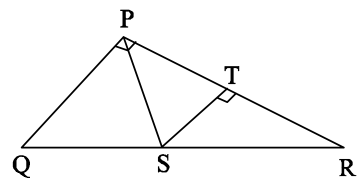
$$\therefore \frac{AB}{AC} = \frac{BD}{DC}$$

$\therefore$ By Converse of ABT,

AD is the bisector of $\angle A$

Type IV: (Prove the following based on ABT)
Q.No. 10, 11

10. In figure $\angle QPR = 90°$, PS is its bisector if $ST \perp PR$. Prove that $ST \times (PQ + PR) = PQ \times PR$



Here $ST = PT$

Area of ΔPQR = Area of ❑ $STPQ$ + Area of ΔSTR

$$\frac{1}{2} PQ \cdot PR = \frac{1}{2} [PT (ST + PQ) + \frac{1}{2} (ST \cdot TR)]$$

$$PQ \cdot PR = PT \cdot ST + PT \cdot PQ + ST \cdot TR$$

$$= ST (PT + TR) + PT \cdot PQ$$

$$= ST \cdot PR + PT \cdot PQ$$

$$= ST \cdot PR + ST \cdot PQ$$

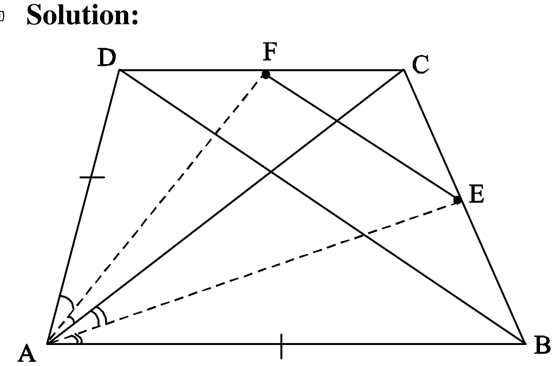
$$= ST (PR + PQ)$$

Hence proved.

---

11. $ABCD$ is a quadrilateral in which $AB = AD$, the bisector of $\angle BAC$ and $\angle CAD$ intersect the sides BC and CD at the points E and F respectively. Prove that $EF \parallel BD$.

✍ **Solution:**



In $\Delta ACD, AF$ is the angle bisector

$\therefore$ By $ABT, \dfrac{AD}{AC} = \dfrac{DF}{FC}$ ...(1)

In $\Delta ABC, AE$ is the angle bisector

$\therefore$ By $ABT, \dfrac{AB}{AC} = \dfrac{BE}{EC}$

$\Rightarrow \dfrac{AD}{AC} = \dfrac{BE}{EC}$ ...(2) (Given $AB = AD$)

$\therefore$ From (1) and (2),

$$\frac{BE}{EC} = \frac{DF}{FC}$$

$\therefore$ By Converse of BPT,

$$EF \parallel BD$$

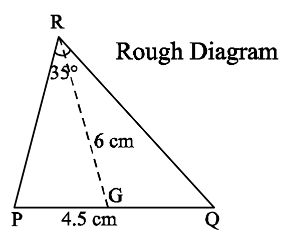
Hence proved.

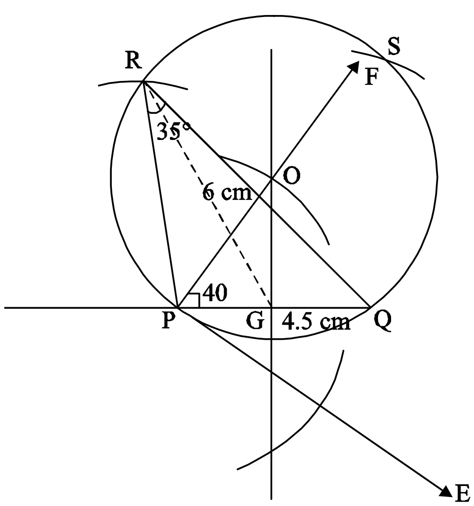
Construction of Triangle

Type I. Given base, vertical angle and median Q.No. 12, 13, Example 4.17, 4.18

12. Construct a ΔPQR which the base $PQ = 4.5$ cm, $\angle R = 35°$ and the median from R to PQ is 6 cm.

✍ **Solution:**





Construction

Step 1: Draw a line segment $PQ = 4.5$ cm

Step 2: At P, draw PE such that $\angle QPE = 35°$.

Step 3: At P, draw PE such that $\angle EPF = 90°$

Step 4: Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.

Step 5: With O as centre and OP as radius draw a circle.

Step 6: From G mark arcs of 6 cm on the circle at RAS.

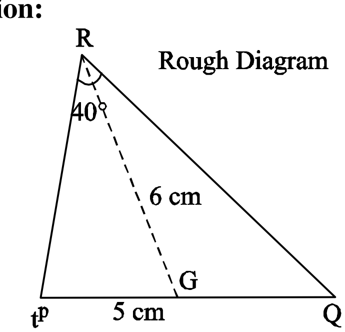
Step 7: Join PR, RQ. Then ΔPQR is the required Δ.

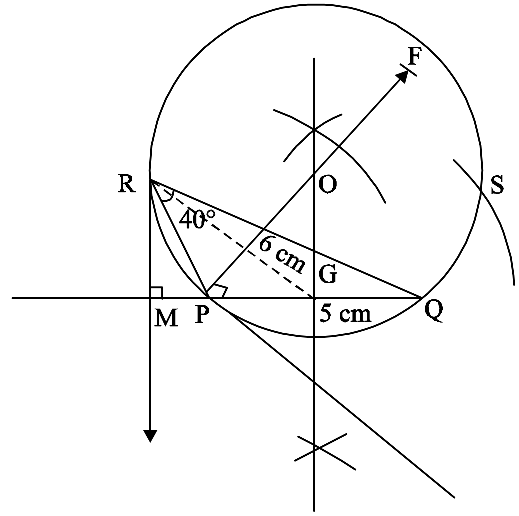
Step 8: Join RG, which is the median.

---

13. Construct a ΔPQR in which $PQ = 5$ cm, $\angle P = 40°$ and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

✍ **Solution:**





Construction

Step 1: Draw a line segment $PQ = 5$ cm

Step 2: At P, draw PE such that $\angle QPE = 40°$

Step 3: At P, draw PF such that $\angle EPF = 90°$

Step 4: Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.

Step 5: With O as centre and OP as radius draw a circle.

Step 6: From G mark arc of 4.4 cm on the circle radius 4.4 m.

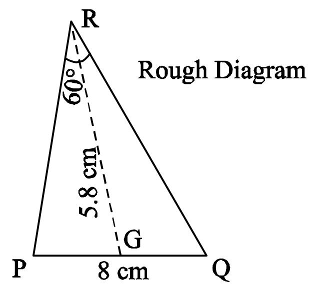
Step 7: Join PR, RQ. Then ΔPQR is the required Δ.

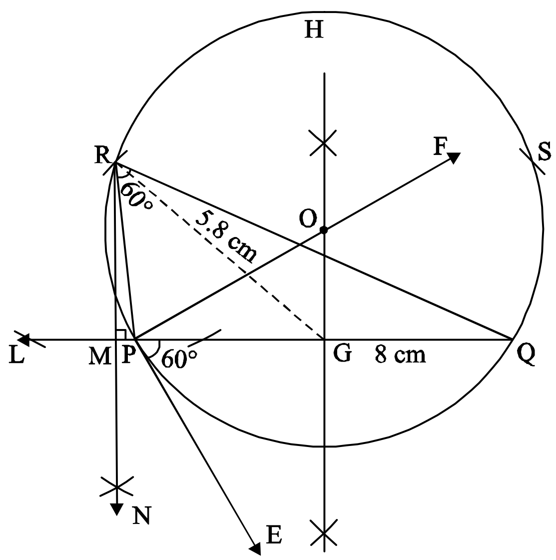
Step 8: Length of altitude is $RM = 3$ cm

Example 4.17

Construct a ΔPQR in which $PQ = 8$ cm, $\angle R = 60°$ and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.

✍ **Solution:**





Construction

Step 1: Draw a line segment $PQ = 8$ cm

Step 2: At P, draw PE such that $\angle QPE = 60°$

Step 3: At P, draw PF such that $\angle EPF = 90°$.

Step 4: Draw the perpendicular bisector to PQ, which intersects PF at O and PQ at G.

Step 5: With O as centre and OP as radius draw a circle.

Step 6: From G mark arcs of radius 5.8 cm on the circle. Mark them as R and S.

Step 7: Join PR and RQ. Then ΔPQR is the required triangle.

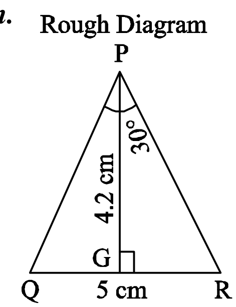
Step 8: From R draw a line RN perpendicular to LQ. LQ meets RN at M

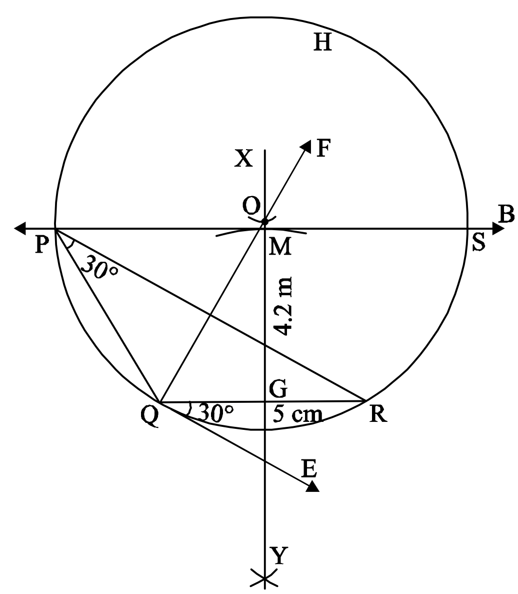
Step 9: The length of the altitude is $RM = 3.5$ cm

Example 4.18

Construct a triangle ΔPQR such that $QR = 5$ cm, $\angle P = 30°$ and the altitude from P to QR is of length 4.2 cm.

✍ **Solution:**





Construction

Step 1: Draw a line segment $QR = 5$ cm

Step 2: At Q, draw QE such that $\angle RQE = 30°$

Step 3: At Q, draw QF such that $\angle EQF = 90°$.

Step 4: Draw the perpendicular bisector XY to QR, which intersects QF at O and QR at G.

Step 5: With O as centre and OQ as radius draw a circle.

Step 6: From G mark an arc in the line XY at M, such that $GM = 4.2$ cm

Step 7: Draw AB through M which is parallel to QR.

Step 8: AB meets the circle at P and S

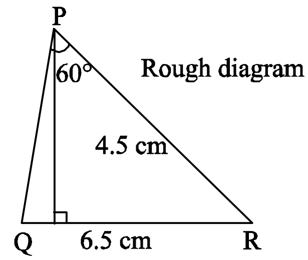
Step 9: Join QP and RP. Then ΔPQR is the required triangle.

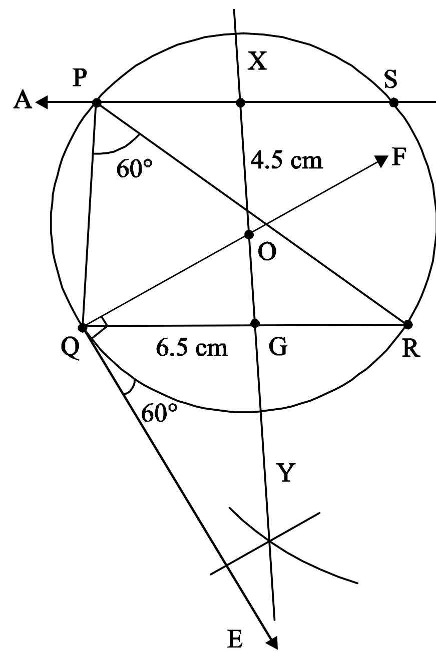
Type II. Given base, vertical angle and altitude

Q.No. 14, 15

14. Construct a ΔPQR such that $QR = 6.5$ cm, $\angle P = 60°$ and the altitude from P to QR is of length 4.5 cm.

✍ **Solution:**





Construction

Step 1: Draw a line segment $QR = 6.5$ cm

Step 2: At Q, draw QE such that $\angle RQE = 60°$

Step 3: At Q, draw QF such that $\angle EQF = 90°$

Step 4: Draw the perpendicular bisector XY to QR intersects QF at O and QR at G.

Step 5: With O as centre and OQ as radius draw a circle.

Step 6: XY intersects QR at G. On XY, from G, mark arc M such that $GM = 4.5$ cm.

Step 7: Draw AB, through M which is parallel to QR.

Step 8: AB meets the circle at P and S.

Step 9: Join QP, RP. Then ΔPQR is the required Δ.

---

15. Construct a ΔABC such that $AB = 5.5$ cm, $\angle C = 25°$ and the altitude from C to AB is 4 cm.

✍ **Solution:**

Construction

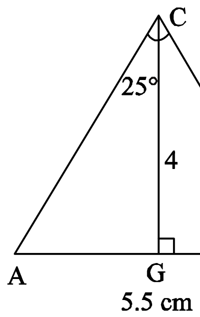
Step 1: Draw a line segment $AB = 5.5$ cm

Step 2: At A, draw AE such that $\angle BAF = 25°$

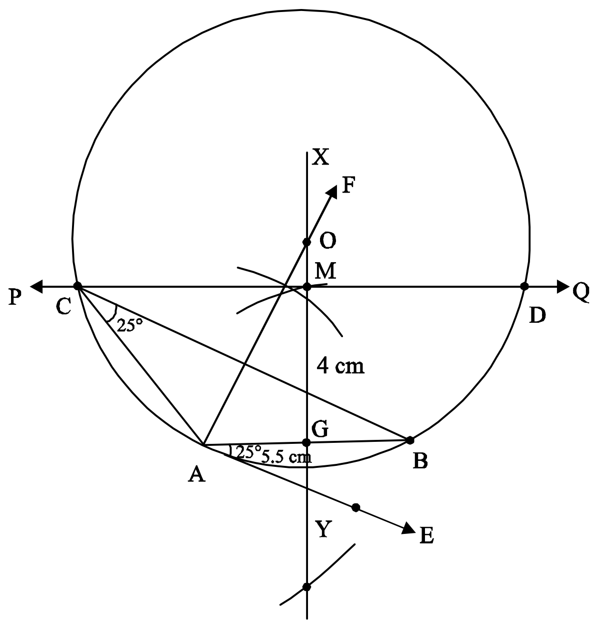
Step 3: At A, draw AF such that $\angle EAF = 90°$

Step 4: Draw the perpendicular bisector XY to AB intersects AF at O and AB at G.

Step 5: With O as centre and OA as radius draw a circle.



Rough diagram



Step 6: *XY* intersects *AB* at *G*. On *XY*, from *G*, mark arc *M* such that $GM = 4$ cm

Step 7: Draw *PQ*, through *M* parallel to *AB* meets the circle at *C* and *D*.

Step 8: Join AC, BC. Then ΔABC is the required Δ.

Type III. Given base, vertical angle and bisector of the vertical angle

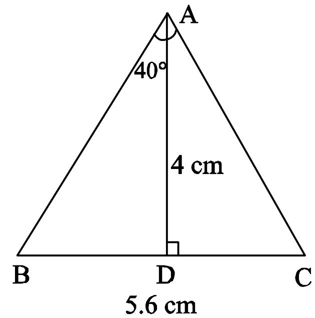
Q.No. 16, 17, Example 4.19

16. Draw a triangle *ABC* of base $BC = 5.6$ cm, $\angle A = 40°$ and the bisector of $\angle A$ meets *BC* at *D* such that $CD = 4$ cm

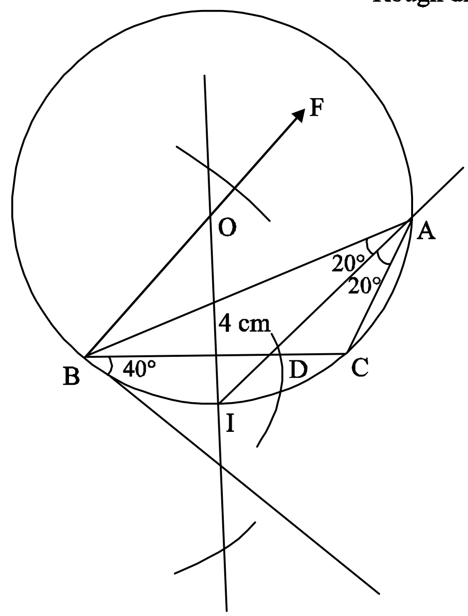
✍ **Solution:**

Construction

Step 1: Draw a line segment $BC = 5.6$ cm



Rough diagram



Step 2: At *B*, draw *BE* such that $\angle CBE = 40°$

Step 3: At *B*, draw *BF* such that $\angle CBF = 90°$

Step 4: Draw the perpendicular bisector to *BC* meets *BF* at *O* and *BC* at *G*

Step 5: With *O* as centre and *OB* as radius draw a circle.

Step 6: From *B*, mark an arc of 4 cm on *BC* at *D*.

Step 7: The $\perp r$ bisector meets the circle at *I* and Join *ID*.

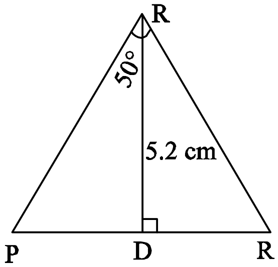
Step 8: *ID* produced meets the circle at *A*. Join *AB* and *AC*.

Step 9: Then ΔABC is the required triangle.

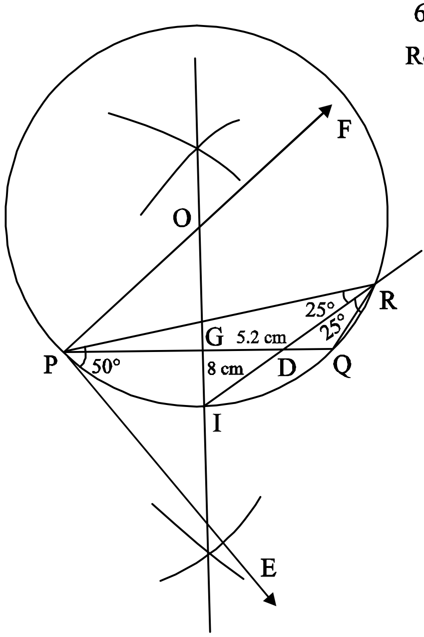
---

17. Draw ΔPQR such that $PQ = 6.8$ cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where $PD = 5.2$ cm

✍ **Solution:**



Rough diagram



Construction

Step 1: Draw a line segment $PQ = 6.8$ cm

Step 2: At P, draw PE such that $\angle QPE = 50°$

Step 3: At P, draw PF such that $\angle QPF = 90°$

Step 4: Draw the perpendicular bisector to PQ meets PF at O and PQ at G.

Step 5: With O as centre and OP as radius draw a circle.

Step 6: From P mark an arc of 5.2 cm on PQ at D.

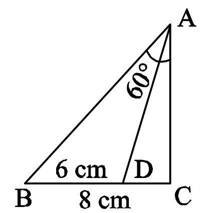
Step 7: The perpendicular bisector meets the circle at R. Join PR and QR.

Step 8: Then ΔPQR is the required triangle.

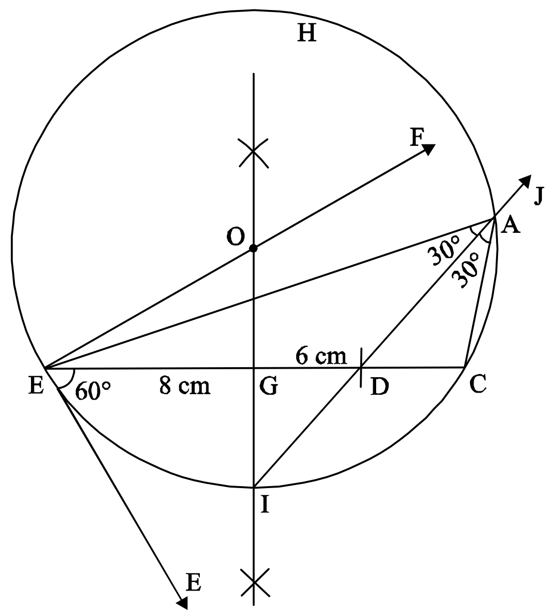
Example 4.19

Draw a triangle ABC of base **$BC = 8$ cm, $\angle A = 60°$** ***and the bisector of $\angle A$ meets BC at D such that*** **$BD = 6$ cm**

✍ **Solution:**



Rough Diagram



Construction

Step 1: Draw a line segment $BC = 8$ cm

Step 2: At B, draw BE such that $\angle CBE = 60°$

Step 3: At B, draw BF such that $\angle EBF = 90°$

Step 4: Draw the perpendicular bisector to BC, which intersects BF at O and BC at G.

Step 5: With O as centre and OB as radius draw a circle.

Step 6: From B mark an arcs of 6 cm on BC at D.

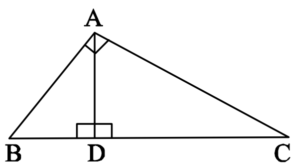
Step 7: The perpendicular bisector intersects the circle at I. Join ID.

Step 8: ID produced meets the circle at A. Now join AB and AC. Then ΔABC is the required triangle.

1. Pythagoras Theorem

Theorem 5: Pythagoras Theorem

Statement



In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof

Given: In ΔABC, $\angle A = 90°$

To prove: $AB^2 + AC^2 = BC^2$

Construction: Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA$ $= 90°$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ...(1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC$ $= 90°$ Therefore $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ...(2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity
3.	$AB^2 + AC^2$ $BC \times BD + BC \times DC$ $= BC(BD + DC)$ $= BC \times BC$ $= BC^2$	Add (1) and (2)

Converse of Pythagoras Theorem

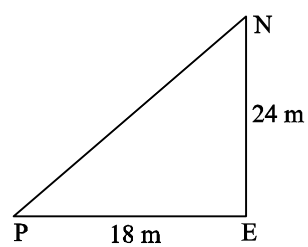
Statement

If the square of the longest side of a triangle is equal to sum of squares of other two sides, then the triangle is a right angle triangle.

Exercise 4.3

Type I: (Problems based on Pythagoras Theorem)

Q.No. 1, 2, 3, 4, 5, 6, Example 4.20, 4.21, 4.22, 4.23



1. A man goes 18m due east and then 24 m due north. Find the distance of his current position from the starting point?

✍ **Solution:**

$P \rightarrow$ Starting Point

By Pythagoras Theorem,

$$PN = \sqrt{18^2 + 24^2}$$

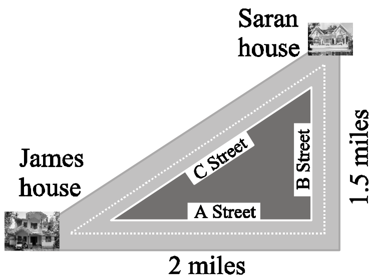
$$= \sqrt{324 + 576}$$

$$= \sqrt{900}$$

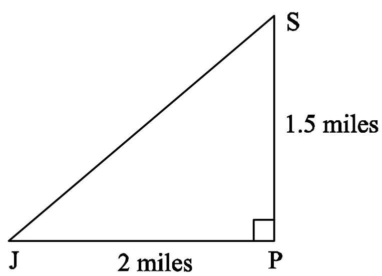
$$= 30 \text{ m}$$

$\therefore$ Distance of his current position from the starting point = 30 m

2. There are two paths that one can choose to go from Sarah's house to James house. One way is to take *C* street, and the other way requires to take *A* street and then *B* street. How much shorter is the direct path along *C* street? (Using figure)



✍ **Solution:**



Path - 1 (Direct *C* Street)

$$SJ = \sqrt{(1.5)^2 + 2^2}$$
$$= \sqrt{2.25 + 4}$$
$$= \sqrt{6.25}$$
$$= 2.5 \text{ miles}$$

Path = 2 (*B* Street and then *A* Street)

$$SP + PJ = 1.5 + 2$$
$$= 3.5 \text{ miles}$$

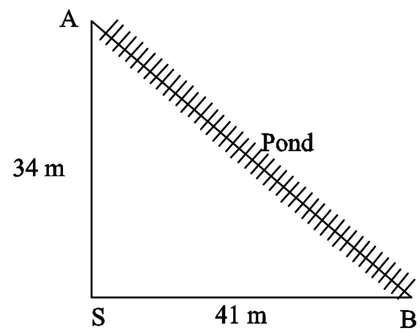
$\therefore$ Required $= 3.5 - 2.5$

$= 1 \text{ mile}$

$\therefore$ 1 mile is shorter along *C* Street.

3. To get from point *A* to point *B* you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?

✍ **Solution:**



Path - 1 (Through pond)

$$AB = \sqrt{34^2 + 41^2}$$
$$= \sqrt{1156 + 1681}$$
$$= \sqrt{2837}$$
$$= 53.26 \text{ m}$$

Path - 2 (South and then East)

Total dist. covered

$$AB = AS + SB$$
$$= 34 + 41$$
$$= 75 \text{ m}$$

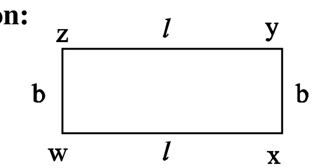
$\therefore$ Reqd. time saving $= 75 - 53.26$

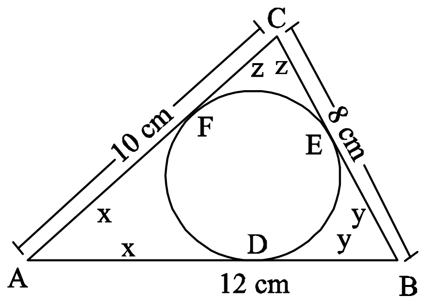
$= 21.74 \text{ m}$

---

4. In the rectangle $WXYZ$, $XY + YZ = 17$ cm, and $XZ + YW = 26$ cm. Calculate the length and breadth of the rectangle.

✍ **Solution:**





$\Rightarrow \quad x + y + z = 15$

$\Rightarrow \quad 12 + z = 15$

$\therefore \quad z = 3$

$\Rightarrow \quad y + 3 = 8$

$\Rightarrow \quad y = 5$

$\therefore \quad x + 5 = 12$

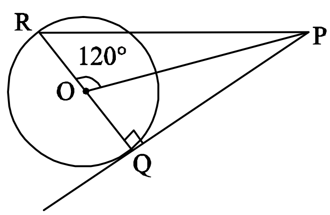
$\Rightarrow \quad x = 7$

$\therefore \quad AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm

---

4. *PQ* is a tangent drawn from a point *P* to a circle with centre *O* and *QOR* is a diameter of the circle such that $\angle PQR = 120°$. Find $\angle OPQ$

✍ **Solution:**



Given $\angle POR = 120°$

$\Rightarrow \quad \angle POQ = 60°$ (linear pair)

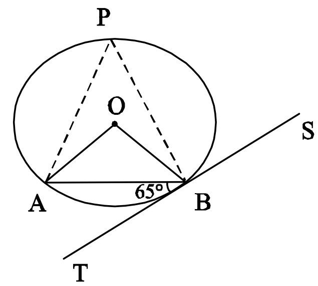
Also $\angle OQP = 90°$ (Radius $\perp$ tangent)

$\therefore \quad \angle OPQ = 90° - 60°$

$= 30°$

5. A tangent *ST* to a circle touches it at *B*. *AB* is a chord such that $\angle ABT = 65°$. Find $\angle AOB$, where "*O*" is the centre of the circle.

✍ **Solution:**



Given $\angle TBA = 65° \Rightarrow \angle APB = 65°$

(angles in ultimate segment).

$\therefore \quad \angle AOB = 2\,\angle APB = 2\,(65°) = 130°$

circumference)

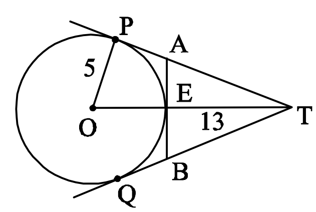
(Angle substantiate at the centre is twice the angle subtended at any point on the remaining.

---

6. In figure, *O* is the centre of the circle with radius 5 cm. *T* is a point such that $OT = 13$ cm and *OT* intersects the circle *E*, if *AB* is the tangent to the circle at *E*, find the length of *AB*.

✍ **Solution:In the figure, given**

$OP = 5$, $OT = 13$



$\therefore \quad PT = \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$

$= TQ$

Also, $OE = 5 \Rightarrow ET = 13 - 5 = 8$

Let $AP = AE = x \Rightarrow TA = 12 - x$

$\therefore$ In ΔAET, $\angle AET = 90°$

$\therefore \quad x^2 + 8^2 = (12 - x)^2$

$\Rightarrow \quad x^2 + 8^2 = 144 + x^2 - 24x$

$$\Rightarrow \quad 64 = 144 - 24x$$

$$\Rightarrow \quad 24x = 80$$

$$x = \frac{80}{24}$$

$$x = \frac{10}{3}$$

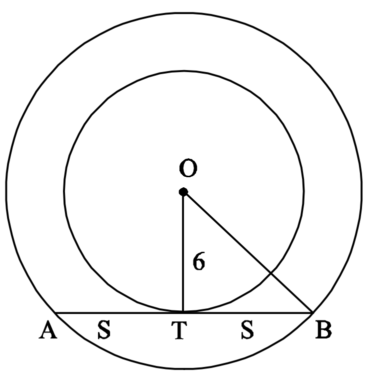
$$\therefore \quad AB = 2x$$

Length of tangent $AB = \frac{20}{3}$ cm

---

7. In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

✍ **Solution:**



Given the chord AB of larger circle is a tangent for the smaller circle and OT is radius.

OT is perpendicular to AB.

$\therefore \quad AT = TB = 8$ cm, $OT = 6$ cm

$\therefore$ In $\Delta\, OBT$,

$$OB = \sqrt{8^2 + 6^2}$$

$$= \sqrt{64 + 36}$$

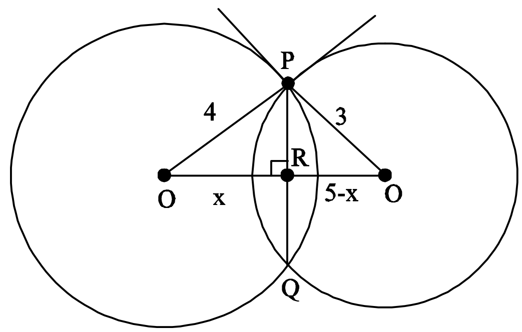
$$= \sqrt{100}$$

$$= 10 \text{ cm}$$

$\therefore$ Radius of the larger circle = 10 cm

8. Two circles with centers O and O' of radii 3 cm and 4 cm, respectively intersect at two point P and Q, such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ.

✍ **Solution:**



Given $OP = 4$ cm (radius of 1st circle)

$O'P = 3$ cm (radius of 2nd circle)

Clearly $OP \perp O'P$ (tangent and radius are $\perp$)

$$\therefore \quad OO' = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

Let R be a point of PQ such that

$OR = x$ and $O'R = 5 - x$

Also, $\Delta\, OPO' \sim \Delta\, OQO'$ and

$\Delta\, OPR \sim \Delta\, OQR$ (by similarity)

$$\therefore \quad \angle ORP = 90°$$

$\therefore$ In ΔORP, $PR^2 = 16 - x^2$

In $\Delta\, O'RP$, $PR^2 = 9 - (5 - x)^2$

$$\therefore \quad 16 - x^2 = 9 - (5 - x)^2$$

$$16 - x^2 = 9 - (25 - 10x + x^2)$$

$$16 - x^2 = 9 - 25 + 10x - x^2$$

$$16 - 9 + 25 = 10x$$

$$32 = 10x$$

$$x = \frac{32}{10}$$

$$x = \frac{16}{5}$$

$$\therefore \quad PR = \sqrt{16 - \frac{256}{25}}$$

$$= \sqrt{\frac{144}{25}} = \frac{12}{5}$$

$$= 2.4$$

$$\therefore \quad PQ = 2\,(PR)$$

$$= 2\,(2.4)$$

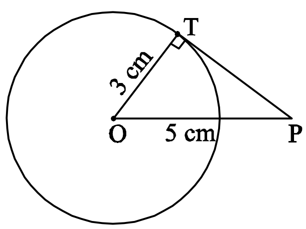
$$= 4.8 \text{ cm}$$

Example 4.24

Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

✍ **Solution:**

Given $OP = 5$ cm, radius $r = 3$ cm



To find the length of tangent PT.

In right angled $\Delta\, OTP$,

$OP^2 = OT^2 + PT^2$ (by Pythagoras theorem)

$5^2 = 3^2 + PT^2$ gives $PT^2 = 25 - 9 = 16$

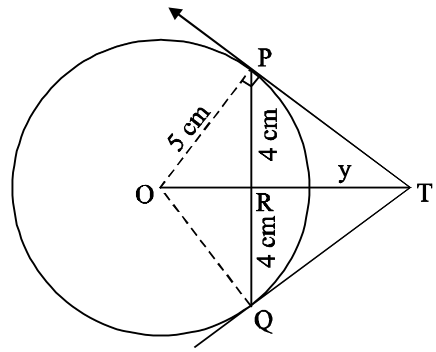
Length of the tangent $PT = 4$ cm

Example 4.25

PQ is a chord of length 8 cm to a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of the tangent TP.

✍ **Solution:**

Let $TR = y$. Since, OT is perpendicular bisector of PQ.



$PR = QR = 4$ cm

In $\Delta\, ORP$, $OP^2 = OR^2 + PR^2$

$OR^2 = OP^2 - PR^2$

$OR^2 = 5^2 - 4^2 = 25 - 16 = 9 \Rightarrow OR = 3$ cm

$OT = OR + RT = 3 + y$ ...(1)

In $\Delta\, PRT$, $TP^2 = TR^2 + PR^2$ ...(2)

and $\Delta\, OPT$ we have, $OT^2 = TP^2 + OP^2$

$OT^2 = (TR^2 + PR^2) + OP^2$ (substitute for TP^2 from (2))

$(3 + y)^2 = y^2 + 4^2 + 5^2$ (substitute for OT from (1))

$9 + 6y + y^2 = y^2 + 16 + 25$

Therefore $y = TR = \frac{16}{3}$

$6y = 41 - 9$ we get $y = \frac{16}{3}$

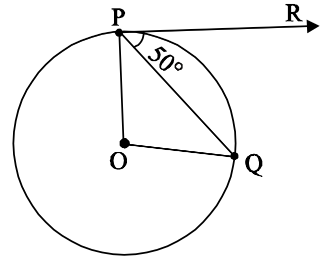
From (2), $TP^2 = TR^2 + PR^2$

$$TP^2 = \left(\frac{16}{3}\right)^2 + 4^2 = \frac{256}{9} + 16 = \frac{400}{9} \text{ so,}$$

$$TP = \frac{20}{3} \text{ cm}$$

Example 4.26

In figure O is the centre of a circle. PQ is a chord and the tangent PR and P makes an angle of 50° with PQ. Find $\angle POQ$



✍ **Solution:**

$\angle OPQ = 90° - 50° = 40°$ (angle between the radius and tangent is 90°)

$OP = OQ$ (Radii of a circle are equal)

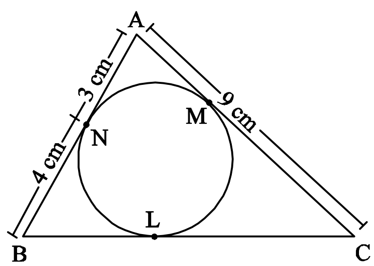
$\angle OPQ = \angle OQP = 40°$ ($\Delta\, OPQ$ is isosceles)

$\angle POQ = 180° - \angle OPQ - \angle OQP$

$\angle POQ = 180° - 40° - 40° = 100°$

Example 4.27

In Fig., ΔABC is circumscribing a circle. Find the length of BC.



✍ **Solution:**

$AN = M = 3$ cm (Tangents drawn from same external point are equal)

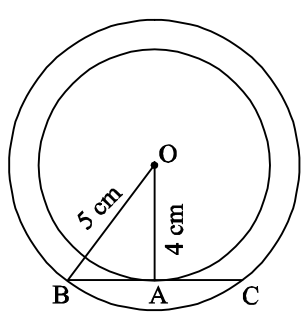
$BN = BL = 4$ cm

$CL = CM = AC - AM = 9 - 3 = 6$ cm

Gives $BC = BL + CL + 4 + 6 = 10$ cm

Example 4.28

If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.



✍ **Solution:**

$OA = 4$ cm, $OB = 5$ cm ; also $OA \perp BC$

$OB^2 = OA^2 + AB^2$

$5^2 = 4^2 + AB^2$ gives $AB^2 = 9$

Therefore $AB = 3$ cm

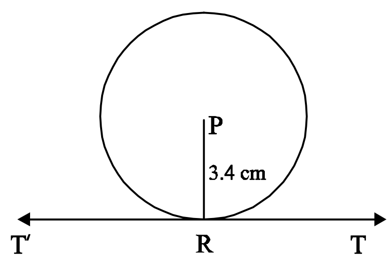
$BC = 2AB$ hence $BC = 2 \times 3 = 6$ cm

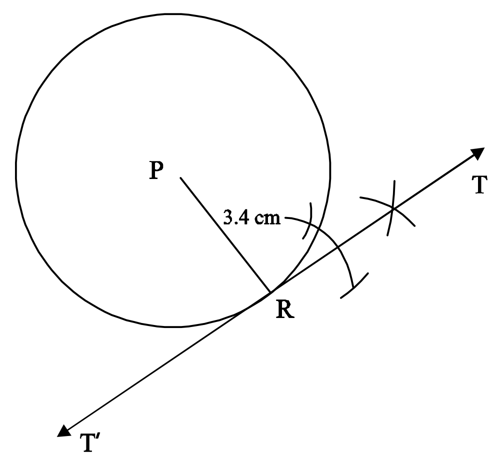
Type II: (Construction of a tangent)

I. Using centre one tangent
Q.No. 12, Example 4.29

12. Draw a tangent at any point *R* on the circle of radius 3.4 cm and centre at *P*?

✍ **Solution:**





Construction

Step 1: Draw a circle with centre at *P* of radius 3.4 cm.

Step 2: Take a point *R* on the circle and Join *PR*.

Step 3: Draw perpendicular line *TT′* to *PR* which passes through *R*.

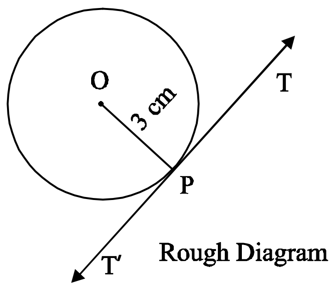
Step 4: *TT′* is the required tangent.

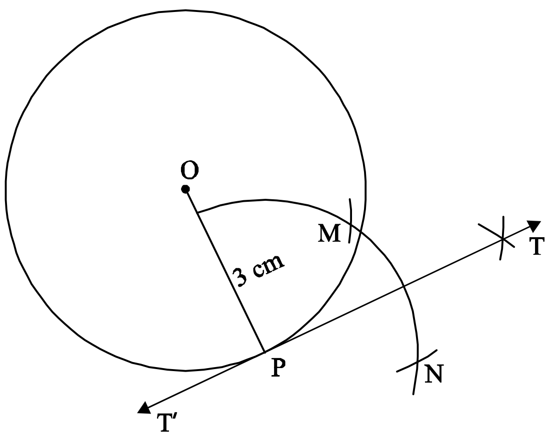
Example 4.29

Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

✍ **Solution:**

Given, radius $r = 3$ cm





Construction

Step 1: Draw a circle with centre at O of radius 3 cm.

Step 2: Take a point P on the circle. Join OP.

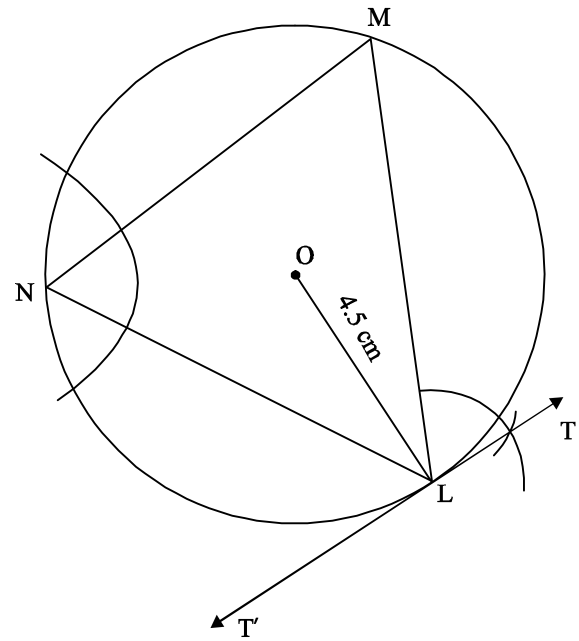
Step 3: Draw perpendicular line TT' to OP which passes through P.

Step 4: TT' is the required tangent.

II. Using alternate segment theorem Q.No. 13, Example 4.30

13. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

✍ **Solution:**



Construction

Step 1: With O as the centre, draw a circle of radius 4.5 cm.

Step 2: Take a point L on the circle. Through L draw any chord LM.

Step 3: Take a point M distinct from L and N on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

Step 4: Through L draw a tangent TT' such that $\angle TLM = \angle MNL$

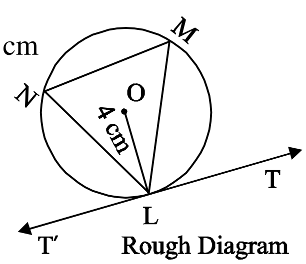
Step 5: TT' is the required tangent.

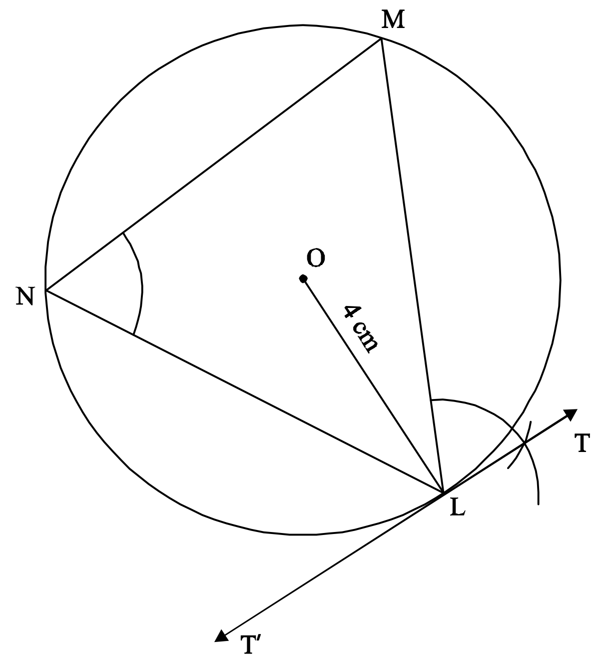
Example 4.30

Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

✍ **Solution:**

Given, radius = 4 cm





Construction

Step 1: With O as the centre, draw a circle of radius 4 cm.

Step 2: Take a point L on the circle. Through L draw any chord LM.

Step 3: Take a point M distinct from L and N on the circle, so that L, M and N are in anticlockwise direction. Join LN and NM.

Step 4: Through L draw a tangement TT' such that $\angle TLM = \angle MNL$.

Step 5: TT' is the required tangent.

III. Using centre two tangent
Q.No. 14, 15, 16, 17, Example 4.31.

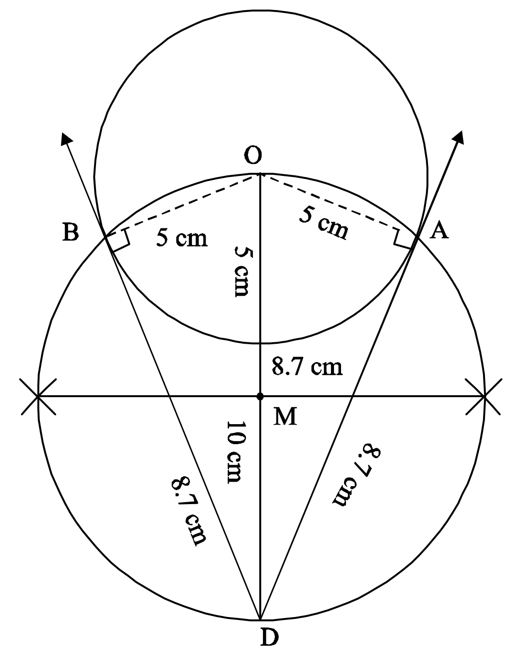
14. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

✍ **Solution:**

Construction

Step 1: With centre at O, draw a circle of radius 5 cm.

Step 2: Draw a line $OP = 10\,\text{cm}$



Step 3: Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4: With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step 5: Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are $PA = PB = 8.7$ cm

Verification: In the right angle triangle $\Delta\, OAP$,

$$PA^2 = \sqrt{OP^2 - OA^2}$$

$$= \sqrt{100 - 25} = \sqrt{75} = 8.7 \text{ cm}$$

15. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

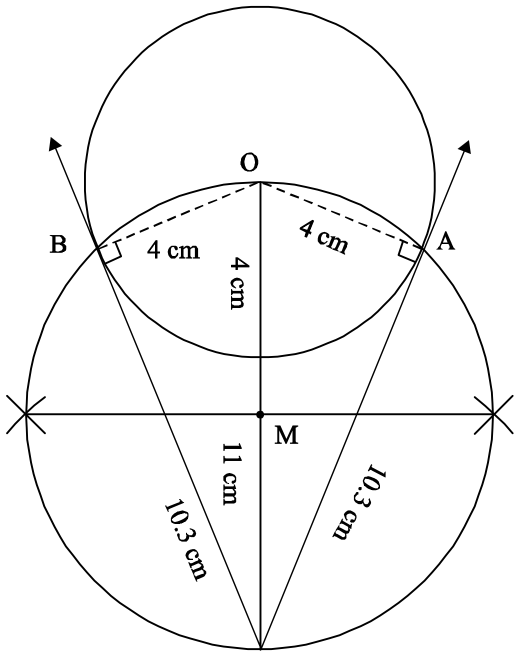
✍ **Solution:**

Construction

Step 1: With centre at O, draw a circle of radius 4 cm.

Step 2: Draw a line $OP = 11\,\text{cm}$

Step 3: Draw a perpendicular bisector of OP, which cuts OP at M.



Step 4: With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step 5: Joint AP and BP. They are the required tangents $AP = BP = 10.3$ cm

Verification: In the right angle triangle $\Delta\, OAP$,

$$AP = \sqrt{OP^2 - OA^2}$$

$$= \sqrt{121 - 16} = \sqrt{105} = 10.3 \text{ cm}$$

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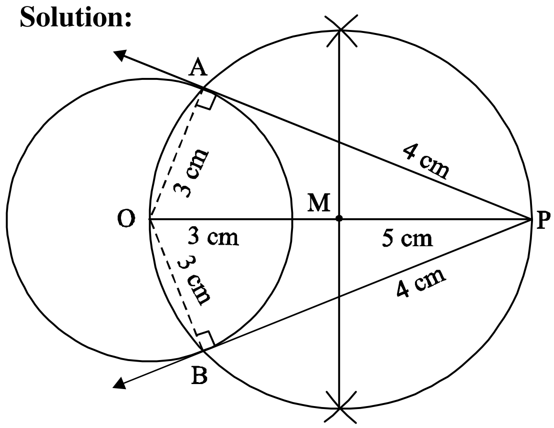
16. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.

✍ **Solution:**



Construction

Step 1: With centre at O, draw a circle of radius 3 cm. with centre at O.

Step 2: Draw a line $OP = 5$ cm

Step 3: Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4: With M as centre and OM as radius, draw a circle which cuts previous circle at A and B.

Step 5: Join AP and BP. They are the required tangents $AP = BP = 4$ cm

Verification:

$$AP = \sqrt{OP^2 - OA^2}$$

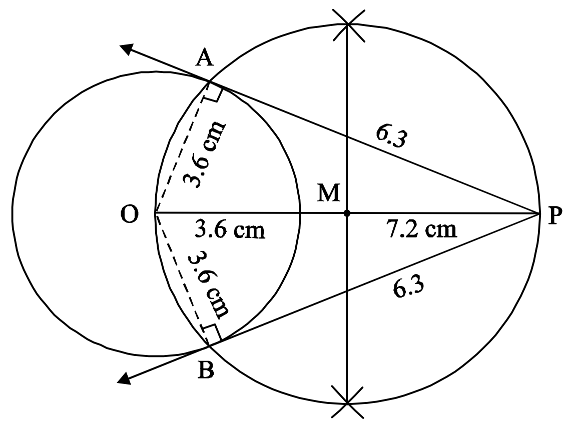
$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16} = 4 \text{ cm}$$

17. Draw a tangent to the circle from the point *P* having radius 3.6 cm, and centre at *O*. Point *P* is at a distance 7.2 cm from the centre.

✍ **Solution:**



Construction

Step 1: Draw a circle of radius 3.6 cm. with centre at O.

Step 2: Draw a line $OP = 7.2$ cm

Step 3: Draw a perpendicular bisector of OP, which cuts it M.

Step 4: With M as centre and OM as radius, draw a circle which cuts previous circle at A and B.

Step 5: Join AP and BP. They are the required tangents $AP = BP = 6.3$ cm

Verification:

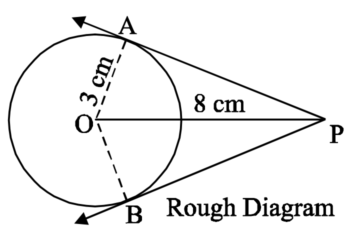
$$AP = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{(7.2)^2 - (3.6)^2}$$
$$= \sqrt{51.84 - 12.96}$$
$$= \sqrt{38.88} = 6.3 \text{ (approx)}$$

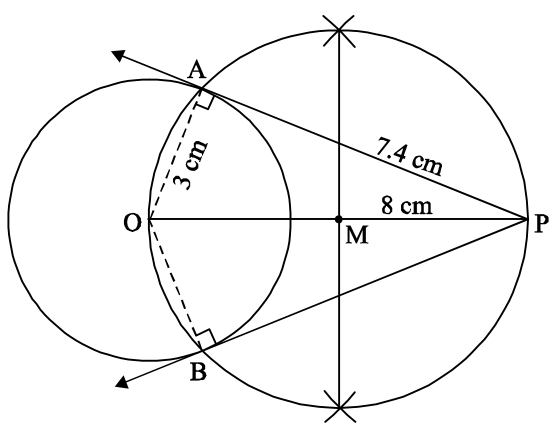
Example 4.31

Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

✍ **Solution:**

Given, diameter $(d) = 6$ cm, we find radius $(r) = \frac{6}{2} = 3$ cm





Construction

Step 1: With centre at O, draw a circle of radius 3 cm.

Step 2: Draw a line OP of length 8 cm.

Step 3: Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4: With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step 5: Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are $PA = PB = 7.4$ cm

Verification: In the right angle triangle OAP,

$$PA^2 = OP^2 - OA^2 = 64 - 9 = 55$$

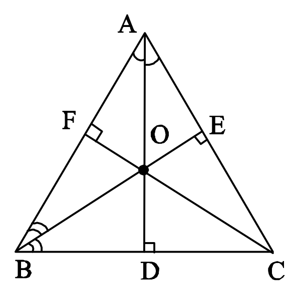
$PA = \sqrt{55} = 7.4$ cm (approximately)

Type III: Concurrency theorems

Q.No. 9. Example 4.32, 10, 11, Example 4.33, 4.34, 4.35

9. Show that the angle bisectors of a triangle are concurrent.

✍ **Solution:**



Consider a ΔABC and let the angular bisectors of A and B meet at 'O'.

From O, draw perpendicular OD, OE, OF to BC, CA, AB respectively.

Now $\Delta BOD = BOF$

$(\because \angle ODB = \angle OFB = 90° \; \angle OBD = \angle OBF)$

$\therefore \quad OD = OF$

IIIy in ΔOAE and ΔOAF, we can prove

$$OE = OF$$

$\therefore \quad OD = OE = OF$

Now, join OC,

Consider $\Delta\, OCD, \Delta\, OCE$

Here (i) $\angle ODC = \angle OEC = 90°$ and OC is common

(ii) $OD = OE$

$\therefore \quad \Delta\, OCD = \Delta\, OCE$

$\therefore \quad \angle OCD = \angle OCE$

CO is angle bisector of $\angle C$

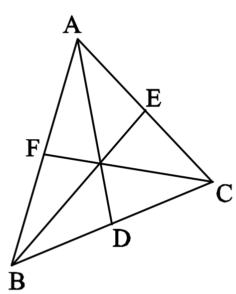
$\therefore$ Angle bisectors of a triangle are concurrent.

Example 4.32

Show that in a triangle, the medians are concurrent.

✍ **Solution:**

Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.



Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

Since D is a mid point of

$$BC, BD = DC \text{ so } \frac{BD}{DC} = 1 \quad ...(1)$$

Since, E is a midpoint of

$$CA, CE = EA \text{ so } \frac{CE}{EA} = 1 \quad ...(2)$$

Since, F is a midpoint of AB,

$$AB, AF = FB \text{ so } \frac{AF}{FB} = 1 \quad ...(3)$$

Thus, multiplying (1), (2) and (3) we get,

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \times 1 \times 1 = 1$$

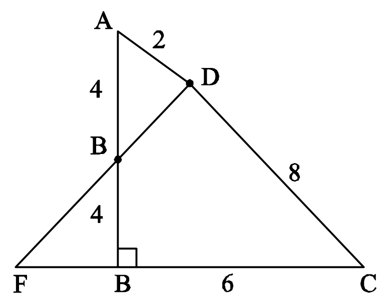
And so, Ceva's theorem is satisfied.

Hence the Medians are concurrent.

---

10. In ΔABC, with $B = 90°, BC = 6$ cm and $AB = 8$ cm, D is a point on AC such that $AD = 2$ cm and E is the midpoint of AB. Join D to E and extend it to meet at F. Find BF.

✍ **Solution:**



Given In $\Delta ABC, AB = 8$ cm, $BC = 6$ cm

$\therefore \quad AC = \sqrt{64+36} = \sqrt{100} = 10$

Also $AD = 2 \Rightarrow CD = 8$ cm

E is the mid point of AB

$$\Rightarrow \quad AE = EB = 4 \text{ cm}$$

By Menelaus Theorem,

$$\frac{AE}{EB} \times \frac{BF}{FC} \times \frac{CD}{DA} = 1$$

$$\Rightarrow \quad \frac{4}{4} \times \frac{BF}{BF+6} \times \frac{8}{2} = 1$$

$$\Rightarrow \quad 4BF = BF + 6$$

$$\Rightarrow \quad 3BF = 6$$

$$\therefore \quad BF = 2 \text{ cm}$$
