Vectors

1. Find the unit vector in the direction of sum of vectors $\vec{a}\ \ 2\hat{i}\ \ \hat{j}\ \ \hat{k}$ and $\vec{b}\ \ 2\hat{j}\ \ \hat{k}$.

2. If $\vec{a}\ \ \hat{i}\ \ \hat{j}\ \ 2\hat{k}$ and $\vec{b}\ \ 2\hat{i}\ \ \hat{j}\ \ 2\hat{k}$, find the unit vector in the direction of

   (i) $6\vec{b}$ (ii) $2\vec{a}\ \ \vec{b}$

3. Find a unit vector in the direction of $\overline{PQ}$, where P and Q have co-ordinates (5, 0, 8) and (3, 3, 2), respectively.

4. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that BC = 1.5 BA.

5. Using vectors, find the value of k such that the points (k, – 10, 3), (1, –1, 3) and (3, 5, 3) are collinear.

6. A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, find $\vec{r}$.

7. A vector $\vec{r}$ has magnitude 14 and direction ratios 2, 3, – 6. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.

8. Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i}\ \ \hat{j}\ \ 2\hat{k}$ and $4\hat{i} - \hat{j}\ \ 3\hat{k}$.

9. Find the angle between the vectors $2\hat{i}\ \ \hat{j}\ \ \hat{k}$ and $3\hat{i}\ \ 4\hat{j}\ \ \hat{k}$.

10. If $\vec{a}\ \ \vec{b}\ \ \vec{c}\ \ 0$, show that $\vec{a}\ \ \vec{b}\ \ \vec{b}\ \ \vec{c}\ \ \vec{c}\ \ \vec{a}$. Interpret the result geometrically?

11. Find the sine of the angle between the vectors $\vec{a}\ \ 3\hat{i}\ \ \hat{j}\ \ 2\hat{k}$ and $\vec{b}\ \ 2\hat{i}\ \ 2\hat{j}\ \ 4\hat{k}$.

12. If A, B, C, D are the points with position vectors $\hat{i} \quad \hat{j} \quad \hat{k}$, $2\hat{i} \quad \hat{j} \quad 3\hat{k}$, $2\hat{i} \quad 3\hat{k}$, $3\hat{i} \quad 2\hat{j} \quad \hat{k}$, respectively, find the projection of $\overline{\text{AB}}$ along $\overline{\text{CD}}$.

13. Using vectors, find the area of the triangle ABC with vertices A(1, 2, 3), B(2, – 1, 4) and C(4, 5, – 1).

14. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

15. Prove that in any triangle ABC, $\cos A \quad \dfrac{b^2 \quad c^2 - a^2}{2bc}$, where a, b, c are the magnitudes of the sides opposite to the vertices A, B, C, respectively.

16. If $\vec{a}, \vec{b}, \vec{c}$ determine the vertices of a triangle, show that $\dfrac{1}{2} \vec{b} \quad \vec{c} \quad \vec{c} \quad \vec{a} \quad \vec{a} \quad \vec{b}$ gives the vector area of the triangle. Hence deduce the condition that the three points $\vec{a}, \vec{b}, \vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle.

17. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\dfrac{|\vec{a} \quad \vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i} \quad \hat{j} \quad \hat{k}$ and $\hat{i} \quad 3\hat{j} \quad \hat{k}$.

18. If $\vec{a} = \hat{i} \quad \hat{j} \quad \hat{k}$ and $\vec{b} \quad \hat{j} \quad \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \quad \vec{c} \quad \vec{b}$ and $\vec{a}.\vec{c} \quad 3$.

Multiple Choice Questions

Choose the correct answer from the given four options in each of the Exercises from 19 to 33 (M.C.Q)

19. The vector in the direction of the vector $\hat{i} \quad 2\hat{j} \quad 2\hat{k}$ that has magnitude 9 is

(A) $\hat{i} \quad 2\hat{j} \quad 2\hat{k}$

(B) $\dfrac{\hat{i} \quad 2\hat{j} \quad 2\hat{k}}{3}$

(C) $3(\hat{i} \quad 2\hat{j} \quad 2\hat{k})$

(D) $9(\hat{i} \quad 2\hat{j} \quad 2\hat{k})$

20. The position vector of the point which divides the join of points $2\vec{a} \quad 3\vec{b}$ and $\vec{a} \quad \vec{b}$ in the ratio 3 : 1 is

(A) $\dfrac{3\vec{a} \quad 2\vec{b}}{2}$

(B) $\dfrac{7\vec{a} \quad 8\vec{b}}{4}$

(C) $\dfrac{3\vec{a}}{4}$

(D) $\dfrac{5\vec{a}}{4}$

21. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is

(A) $\hat{i} \quad 12\hat{j} \quad 4\hat{k}$

(B) $5\hat{i} \quad 2\hat{j} \quad 4\hat{k}$

(C) $5\hat{i} \quad 2\hat{j} \quad 4\hat{k}$

(D) $\hat{i} \quad \hat{j} \quad \hat{k}$

22. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively,

and $\vec{a}.\vec{b}$ $2\sqrt{3}$ is

(A) $\frac{}{6}$ (B) $\frac{}{3}$ (C) $\frac{}{2}$ (D) $\frac{5}{2}$

23. Find the value of λ such that the vectors $\vec{a}$ $2\hat{i}$ $\hat{j}$ $\hat{k}$ and $\vec{b}$ $\hat{i}$ $2\hat{j}$ $3\hat{k}$ are orthogonal

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) $-\frac{5}{2}$

24. The value of λ for which the vectors $3\hat{i}$ $6\hat{j}$ $\hat{k}$ and $2\hat{i}$ $4\hat{j}$ $\hat{k}$ are parallel is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{2}{5}$

25. The vectors from origin to the points A and B are $\vec{a}$ $2\hat{i}$ $3\hat{j}$ $2\hat{k}$ and $\vec{b}$ $2\hat{i}$ $3\hat{j}$ $\hat{k}$,respectively, then the area of triangle OAB is

(A) 340 (B) $\sqrt{25}$ (C) $\sqrt{229}$ (D) $\frac{1}{2}\sqrt{229}$

26. For any vector $\vec{a}$, the value of $(\vec{a}\ \ \hat{i})^2$ $(\vec{a}\ \ \hat{j})^2$ $(\vec{a}\ \ \hat{k})^2$ is equal to

(A) $\vec{a}^2$ (B) $3\vec{a}^2$ (C) $4\vec{a}^2$ (D) $2\vec{a}^2$

27. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b}$ 12 , then value of $|\vec{a}\ \ \vec{b}|$ is

(A) 5 (B) 10 (C) 14 (D) 16

28. The vectors $\hat{i}\ \ \hat{j}\ \ 2\hat{k}$, $\hat{i}\ \ \hat{j}\ \ \hat{k}$ and $2\hat{i}\ \ \hat{j}\ \ \hat{k}$ are coplanar if

(A) $\lambda = -2$ (B) $\lambda = 0$ (C) $\lambda = 1$ (D) $\lambda = -1$

29. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}\ \ \vec{b}\ \ \vec{c}\ \ \vec{0}$, then the value of $\vec{a}.\vec{b}\ \ \vec{b}.\vec{c}\ \ \vec{c}.\vec{a}$ is

(A) 1 (B) 3 (C) $-\frac{3}{2}$ (D) None of these

30. Projection vector of $\vec{a}$ on $\vec{b}$ is

(A) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\vec{b}$ (B) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (C) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (D) $\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\hat{b}$

31. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}\ \ \vec{b}\ \ \vec{c}\ \ \vec{0}$ and $|\vec{a}|\ \ 2$, $|\vec{b}|\ \ 3$, $|\vec{c}|\ \ 5$, then value of $\vec{a}.\vec{b}\ \ \vec{b}.\vec{c}\ \ \vec{c}.\vec{a}$ is

(A) 0 (B) 1 (C) – 19 (D) 38

32. If $|\vec{a}|\ \ 4$ and $3\ \ \ \ 2$, then the range of $|\ \ \vec{a}|$ is

(A) [0, 8] (B) [– 12, 8] (C) [0, 12] (D) [8, 12]

33. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(A) one (B) two (C) three (D) infinite