CHAPTER 10

STRAIGHT LINES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Slope of non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by :

(a) $m = \frac{y_2 - y_1}{x_2 - x_1}$ (b) $m = \frac{x_2 - x_1}{y_2 - y_1}$

(c) $m = \frac{x_2 + x_1}{y_2 + y_1}$ (d) $m = \frac{y_2 + y_1}{x_2 + x_1}$

2. If a line makes an angle α in anti-clockwise direction with the positive direction of x-axis, then the slope of the line is given by :

(a) $m = \sin\alpha$ (b) $m = \cos\alpha$

(c) $m = \tan\alpha$ (d) $m = \sec\alpha$

3. The point (x, y) lies on the line with slope m and through the fixed point (x_0, y_0) if and only if its coordinates satisfy the equation $y - y_0$ is equal to ....

(a) $m(x - x_0)$ (b) $m(y - x_0)$

(c) $m(y - x)$ (d) $m(x - y_0)$

4. If a line with slope m makes x-intercept d. Then equation of the line is :

(a) $y = m(d - x)$ (b) $y = m(x - d)$

(c) $y = m(x + d)$ (d) $y = mx + d$

5. The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by :

(a) $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ (b) $d = \frac{|Ax_1 - By_1 + C|}{\sqrt{A^2 + B^2}}$

(c) $d = \frac{\sqrt{A^2 + B^2}}{|Ax_1 + By_1 + C|}$ (d) $d = \frac{\sqrt{A^2 + B^2}}{|Ax_1 - By_1 + C|}$

6. Distance between the parallel lines
$Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by:

(a) $d = \frac{\sqrt{A^2 + B^2}}{|C_1 - C_2|}$ (b) $d = \frac{\sqrt{A^2 - B^2}}{|C_1 - C_2|}$

(c) $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$ (d) $d = \frac{|C_1 + C_2|}{\sqrt{A^2 + B^2}}$

7. The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is

(a) 45° (b) 135° (c) –45° (d) –135°

8. Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) –1 (b) 0 (c) 2 (d) $\sqrt{3}$

9. Which of the following lines is farthest from the origin?

(a) $x - y + 1 = 0$ (b) $2x - y + 3 = 0$

(c) $x + 2y - 2 = 0$ (d) $x + y - 2 = 0$

10. Equation of the straight line making equal intercepts on the axes and passing through the point (2, 4) is :

(a) $4x - y - 4 = 0$ (b) $2x + y - 8 = 0$

(c) $x + y - 6 = 0$ (d) $x + 2y - 10 = 0$

11. A line passes through $P(1, 2)$ such that its intercept between the axes is bisected at P. The equation of the line is

(a) $x + 2y = 5$ (b) $x - y + 1 = 0$

(c) $x + y - 3 = 0$ (d) $2x + y - 4 = 0$

12. The tangent of angle between the lines whose intercepts on the axes are $a, -b$ and $b, -a$ respectively, is

(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{b^2 - a^2}{2}$

(c) $\frac{b^2 - a^2}{2ab}$ (d) None of these

13. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2) then the equation of the line will be

(a) $2x + 3y = 12$ (b) $3x + 2y = 12$

(c) $4x - 3y = 6$ (d) $5x - 2y = 10$

14. The intercept cut off by a line from y-axis twice than that from x-axis, and the line passes through the point (1, 2). The equation of the line is

(a) $2x + y = 4$ (b) $2x + y + 4 = 0$

(c) $2x - y = 4$ (d) $2x - y + 4 = 0$

15. Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and $(x, 24)$. Find the value of x.

(a) 2 (b) 3 (c) 4 (d) 5

16. Let the perpendiculars from any point on the line $7x + 56y = 0$ upon $3x + 4y = 0$ and $5x - 12y = 0$ be p and p', then

(a) $2p = p'$ (b) $p = 2p'$

(c) $p = p'$ (d) None of these

17. The lines $x + 2y - 5 = 0$, $2x - 3y + 4 = 0$, $6x + 4y - 13 = 0$

(a) are concurrent.

(b) form a right angled triangle.

(c) form an isosceles triangle.

(d) form an equilateral triangle.

18. A triangle ABC is right angled at A has points A and B as (2, 3) and (0, –1) respectively. If $BC = 5$, then point C may be
(a) (–4, 2) (b) (4, – 2) (c) (0, 4) (d) (0, – 4)

19. The relation between a, b, a' and b' such that the two lines $ax + by = c$ and $a'x + b'y = c'$ are perpendicular is
(a) $aa' - bb' = 0$ (b) $aa' + bb' = 0$
(c) $ab + a'b' = 0$ (d) $ab - a'b' = 0$

20. The equation of a straight line which cuts off an intercept of 5 units on negative direction of y-axis and makes an angle of 120° with the positive direction of x-axis is
(a) $\sqrt{3}x + y + 5 = 0$ (b) $\sqrt{3}x + y - 5 = 0$
(c) $\sqrt{3}x - y - 5 = 0$ (d) $\sqrt{3}x - y + 5 = 0$

21. The equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$ is
(a) $2x + 3y + 6 = 0$ (b) $2x - 3y - 6 = 0$
(c) $2x - 3y + 6 = 0$ (d) $2x + 3y - 6 = 0$

22. Which one of the following is the nearest point on the line $3x - 4y = 25$ from the origin?
(a) (–1, –7) (b) (3, –4)
(c) (–5, –8) (d) (3, 4)

23. If the mid-point of the section of a straight line intercepted between the axes is (1, 1), then what is the equation of this line?
(a) $2x + y = 3$ (b) $2x - y = 1$
(c) $x - y = 0$ (d) $x + y = 2$

24. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?
(a) 60° (b) 45° (c) 30° (d) 15°

25. If the points (x, y), (1, 2) and (–3, 4) are collinear, then
(a) $x + 2y - 5 = 0$ (b) $x + y - 1 = 0$
(c) $2x + y - 4 = 0$ (d) $2x - y + 10 = 0$

26. If p be the length of the perpendicular from the origin on the straight line $x + 2by = 2p$, then what is the value of b?
(a) $\frac{1}{p}$ (b) p (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

27. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is
(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

28. The coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are
(a) (–6, 5) (b) (5, 6) (c) (–5, 6) (d) (6, 5)

29. The length of the perpendicular from the origin to a line is 7 and line makes an angle of 150° with the positive direction of y-axis then the equation of the line is
(a) $4x + 5y = 7$ (b) $-x + 3y = 2$
(c) $\sqrt{3}x - y = 10\sqrt{2}$ (d) $\sqrt{3}x + y = 14$

30. A straight line makes an angle of 135° with x-axis and cuts y-axis at a distance of – 5 from the origin. The equation of the line is
(a) $2x + y + 5 = 0$ (b) $x + 2y + 3 = 0$
(c) $x + y + 5 = 0$ (d) $x + y + 3 = 0$

31. The equation of a line through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$ is
(a) $2x + y - 5 = 0$ (b) $x - 3y + 6 = 0$
(c) $x + 2y - 7 = 0$ (d) $x + 3y + 8 = 0$

32. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other
(a) $a_1b_1 - b_1a_2 = 0$ (b) $a_1^2b_2 + b_1^2a_2 = 0$
(c) $a_1b_1 + a_2b_2 = 0$ (d) $a_1a_2 + b_1b_2 = 0$

33. If the coordinates of the points A and B be (3, 3) and (7, 6), then the length of the portion of the line AB intercepted between the axes is
(a) $\frac{5}{4}$ (b) $\frac{\sqrt{10}}{4}$ (c) $\frac{\sqrt{13}}{3}$ (d) None of these

34. The line $(3x - y + 5) + \lambda(2x - 3y - 4) = 0$ will be parallel to y-axis, if $\lambda =$
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

35. The equation of a straight line passing through (–3, 2) and cutting an intercept equal in magnitude but opposite in sign from the axes is given by
(a) $x - y + 5 = 0$ (b) $x + y - 5 = 0$
(c) $x - y - 5 = 0$ (d) $x + y + 5 = 0$

36. The points A(1, 3) and C(5, 1) are the opposite vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is
(a) $2x + y - 8 = 0$ (b) $2x - y - 4 = 0$
(c) $2x - y + 4 = 0$ (d) $2x + y + 7 = 0$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

37. Consider the following statements about straight lines :
I. Slope of horizontal line is zero and slope of vertical line is undefined.
II. Two lines are parallel if and only if their slopes are equal.
III. Two lines are perpendicular if and only if product of their slope is – 1.
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All the above

38. The distances of the point (1, 2, 3) from the coordinate axes are A, B and C respectively. Now consider the following equations:
I. $A^2 = B^2 + C^2$ II. $B^2 = 2C^2$
III. $2A^2C^2 = 13B^2$
Which of these hold(s) true?
(a) Only I (b) I and III (c) I and II (d) II and III

39. Consider the following statements.
I. Equation of the line passing through (0, 0) with slope m is $y = mx$
II. Equation of the x-axis is $x = 0$.

Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

40. Consider the following statements.
I. The distance between two points (x_1, y_1) and (x_2, y_2) is given by
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
II. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2)
$=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

41. Consider the following statements.
The three given points *A, B, C* are collinear i.e., lie on the same straight line, if
I. area of ΔABC is zero.
II. slope of AB = Slope of BC.
III. any one of the three points lie on the straight line joining the other two points.
Choose the correct option
(a) Only I is true (b) Only II is true
(c) Only III is true (d) All are true

42. Consider the following statements.
I. Slope of horizontal line is zero and slope of vertical line is undefined.
II. Two lines whose slopes are m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$
Choose the correct option.
(a) Both are true (b) Both are false
(c) Only I is true (d) Only II is true

43. Consider the following statements.
I. The length of perpendicular from a given point (x_1, y_1) to a line $ax+by+c=0$ is
$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$
II. Three or more straight lines are said to be concurrent lines, if they meet at a point.
Choose the correct option
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

44. Consider the following statements.
I. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle then centroid is
$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
II. If the point $P(x, y)$ divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ (internally), then
$$x=\frac{mx_2+nx_1}{m+n},\ y=\frac{my_2+ny_1}{m+n}$$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

45. Consider the following statements.
I. The equation of a straight line passing through the point (x_1, y_1) and having slope m is given by $y-y_1=m(x-x_1)$
II. Equation of the y-axis is $x=0$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false.

46. Consider the following statements.
I. The equation of a straight line making intercepts *a* and *b* on *x* and *y*-axis respectively is given by
$\frac{x}{a}+\frac{y}{b}=1$
II. If $ax+by+c_1=0$ and $ax+by+c_2=0$ be two parallel lines, then distance
between two parallel lines, $d=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

47. Consider the following statements.
I. If (a, b), (c, d) and (a – c, b – d) are collinear, then bc – ad = 0
II. If the points A (1, 2), B (2, 4) and C (3, a) are collinear, then the length BC = 5 unit.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

48. Consider the following statements.
I. Centroid of a triangle is a point where angle bisectors meet.
II. If value of area after calculations is negative then we take its negative value.
Choose the correct option
(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

49. Consider the following statements.
I. Two lines are parallel if and only if their slopes are equal.
II. Two lines are perpendicular if and only if product of their slopes is 1.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false.

50. Equation of a line is $3x-4y+10=0$
I. Slope of the given line is $\frac{3}{4}$.
II. x-intercept of the given line is $-\frac{10}{3}$.
III. y-intercept of the given line is $\frac{5}{2}$.
Choose the correct option.
(a) Only I and II are true
(b) Only II and III are true
(c) Only I and III are true
(d) All I, II and III are true

51. Consider the equation $\sqrt{3}x + y - 8 = 0$

I. Normal form of the given equation is $\cos 30° x + \sin 30° y = 4$

II. Values of p and w are 4 and 30° respectively.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

52. Slope of the lines passing through the points

I. (3, – 2) and (– 1, 4) is $-\frac{3}{2}$

II. (3, – 2) and (7, – 2) is 0.

III. (3, – 2) and (3, 4) is 1.

Choose the correct option.

(a) Only I and III are true
(b) Only I and II are true
(c) Only II and III are true
(d) None of these

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

53. The value of x for which the points $(x, -1)$, (2, 1) and (4, 5) are collinear, is

(a) 1 (b) 2 (c) 3 (d) 4

54. The distance of the point (–1, 1) from the line $12(x + 6) = 5(y - 2)$ is

(a) 2 (b) 3 (c) 4 (d) 5

55. The perpendicular from the origin to the line $y = mx + c$ meets it at the point (–1, 2). Find the value of m + c.

(a) 2 (b) 3 (c) 4 (d) 5

56. The values of k for which the line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ is parallel to the x-axis, is

(a) 3 (b) 2 (c) 1 (d) 4

57. The line joining (–1, 1) and (5, 7) is divided by the line x + y = 4 in the ratio 1 : k. The value of 'k' is

(a) 2 (b) 4 (c) 3 (d) 1

58. If three points (h, 0), (a, b) and (0, k) lies on a line, then the value of $\frac{a}{h} + \frac{b}{k}$ is

(a) 0 (b) 1 (c) 2 (d) 3

59. Value of x so that 2 is the slope of the line through (2, 5) and (x, 3) is

(a) 0 (b) 1 (c) 2 (d) 3

60. What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (– 1, 4) and (0, 6)?

(a) 6 (b) 7 (c) 5 (d) 9

61. Reduce the equation $\sqrt{3}x + y - 8 = 0$ into normal form. The value of p is

(a) 2 (b) 3 (c) 4 (d) 5

62. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is $\frac{a}{b}$. Value of a + b is

(a) 2 (b) 5 (c) 7 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

63. **Assertion:** If θ is the inclination of a line l, then the slope or gradient of the line l is tan θ.

Reason: The slope of a line whose inclination is 90°, is not defined.

64. **Assertion:** The inclination of the line l may be acute or obtuse.

Reason: Slope of x-axis is zero and slope of y-axis is not defined.

65. **Assertion:** Slope of the line passing through the points (3, –2) and (3, 4) is 0.

Reason: If two lines having the same slope pass through a common point, then these lines will coincide.

66. **Assertion:** If A (– 2, –1), B (4, 0), C (3, 3) and D (–3, 2) are the vertices of a parallelogram, then mid-point of AC = Mid-point of BD

Reason: The points A, B and C are collinear ⇔ Area of ΔABC = 0.

67. **Assertion:** Pair of lines $x + 2y - 3 = 0$ and $-3x - 6y + 9 = 0$ are coincident.

Reason: Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

68. **Assertion:** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, are parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

Reason: If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular, then $a_1a_2 - b_1b_2 = 0$.

69. **Assertion:** The equation of the line making intercepts a and b on x and y-axis respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Reason: The slope of the line ax + by + c = 0 is $\frac{b}{a}$.

70. **Assertion:** The equation of a line parallel to the line ax + by + c = 0 is ax – by – λ = 0, where λ is a constant.

Reason: The equation of a line perpendicular to the line ax + by + c = 0 is bx – ay + λ = 0, where λ is a constant.

71. **Assertion:** The distance between the parallel lines $3x - 4y + 9 = 0$ and $6x - 8y - 15 = 0$ is $\frac{33}{10}$.

Reason: Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

72. **Assertion:** Equation of the horizontal line having distance 'a' from the x-axis is either y = a or y = –a.
Reason: Equation of the vertical line having distance b from the y-axis is either x = b or x = –b.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

73. In what ratio does the line $y - x + 2 = 0$ cut the line joining (3, –1) and (8, 9)?
(a) 2 : 3 (b) 3 : 2 (c) 3 : –2 (d) 1 : 2

74. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is:
(a) $\frac{3}{2}$ (b) $\frac{3}{10}$ (c) 6 (d) $\frac{9}{4}$

75. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y – intercept is:
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

76. If the area of the triangle with vertices (x, 0), (1, 1) and (0, 2) is 4 square unit, then the value of x is :
(a) – 2 (b) – 4 (c) – 6 (d) 8

77. The distance of the line $2x + y = 3$ from the point (–1, 3) in the direction whose slope is 1, is
(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$
(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{2\sqrt{5}}{3}$

78. The straight lines $x + 2y - 9 = 0$, $3x + 5y - 5 = 0$ and $ax + by = 1$ are concurrent if the straight line $35x - 22y + 1 = 0$ passes through :
(a) (a, b) (b) (b, a)
(c) (a, – b) (d) (– a, b)

79. The reflection of the point (4, – 13) in the line $5x + y + 6 = 0$ is
(a) (–1, –14) (b) (3, 4)
(c) (0, 0) (d) (1, 2)

80. If *a, b, c* are in A.P., then the straight lines $ax + by + c = 0$ will always pass through
(a) (1, – 2) (b) (1, 2)
(c) (– 1, 2) (d) (– 1, – 2)

81. What is the image of the point (2, 3) in the line $y = -x$?
(a) (–3, –2) (b) (– 3, 2)
(c) (–2, –3) (d) (3, 2)

82. If p be the length of the perpendicular from the origin on the straight line $ax + by = p$ and $b = \frac{\sqrt{3}}{2}$, then what is the angle between the perpendicular and the positive direction of x-axis?
(a) 30° (b) 45° (c) 60° (d) 90°

83. If (– 4, 5) is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is
(a) $x + 3y = 21$ (b) $2x - 3y = 7$
(c) $x + 7y = 31$ (d) $2x + 3y = 21$

84. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The co-ordinates of the point A is
(a) $\left(\frac{13}{5}, 0\right)$ (b) $\left(\frac{5}{13}, 0\right)$
(c) (–7, 0) (d) None of these

85. The vertices of a triangle ABC are (1, 1), (4, – 2) and (5, 5) respectively. Then equation of perpendicular dropped from C to the internal bisector of angle A is
(a) $y - 5 = 0$ (b) $x - 5 = 0$
(c) $2x + 3y - 7 = 0$ (d) None of these

86. The line L has intercepts a and b on the coordinate axes. When keeping the origin fixed, the coordinate axes are rotated through a fixed angle, then same line has intercepts p and q on the rotated axes, then
(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$
(c) $a^2 + \beta^2 = b^2 + q^2$ (d) $b^2 + q^2 = \frac{1}{b^2} + \frac{1}{q^2}$

87. The equation of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, – 10), then the equation of the third side is (are)
(a) $3x + y + 7 = 0$, $x - 3y - 31 = 0$
(b) $2x + y + 5 = 0$, $x - 2y + 3 = 0$
(c) $3x + y + 7 = 0$, $x + y = 0$
(d) $3x - y = 7$, $x + 3y = 15$

88. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for
(a) exactly one value of p
(b) exactly two values of p
(c) more than two values of p
(d) no value of p

89. The bisector of the acute angle formed between the lines $4x - 3y + 7 = 0$ and $3x - 4y + 14 = 0$ has the equation
(a) $x + y + 3 = 0$ (b) $x - y - 3 = 0$
(c) $x - y + 3 = 0$ (d) $3x + y - 7 = 0$

90. The equations of the lines which cuts off an intercept –1 from y-axis and equally inclined to the axes are
(a) $x - y + 1 = 0$, $x + y + 1 = 0$
(b) $x - y - 1 = 0$, $x + y - 1 = 0$
(c) $x - y - 1 = 0$, $x + y + 1 = 0$
(d) None of these

91. If the coordinates of the points A, B, C be (–1, 5), (0, 0) and (2, 2) respectively and D be the middle point of BC, then the equation of the perpendicular drawn from B to the line AD is
(a) $x + 2y = 0$ (b) $2x + y = 0$
(c) $x - 2y = 0$ (d) $2x - y = 0$

92. The line parallel to the x-axis and passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0, where (a, b) ≠ (0, 0) is

(a) Above the x-axis at a distance of $\frac{3}{2}$ from it

(b) Above the x-axis at a distance of $\frac{2}{3}$ from it

(c) Below the x-axis at a distance of $\frac{3}{2}$ from it

(d) Below the x-axis at a distance of $\frac{2}{3}$ from it

93. Equation of angle bisector between the lines 3x + 4y – 7 = 0 and 12x + 5y + 17 = 0 are

(a) $\frac{3x+4y-7}{\sqrt{25}} = \pm\frac{12x+5y+17}{\sqrt{169}}$

(b) $\frac{3x+4y+7}{\sqrt{25}} = \frac{12x+5y+17}{\sqrt{169}}$

(c) $\frac{3x+4y+7}{\sqrt{25}} = \pm\frac{12x+5y+17}{\sqrt{169}}$

(d) None of these

94. The equation of the line which bisects the obtuse angle between the lines x – 2y + 4 = 0 and 4x – 3y + 2 = 0, is

(a) $\left(4-\sqrt{5}\right)x-\left(3-2\sqrt{5}\right)y+\left(2-4\sqrt{5}\right)=0$

(b) $\left(4+\sqrt{5}\right)x-\left(3+2\sqrt{5}\right)y+\left(2+4\sqrt{5}\right)=0$

(c) $\left(4+\sqrt{5}\right)x+\left(3+2\sqrt{5}\right)y+\left(2+4\sqrt{5}\right)=0$

(d) None of these

95. Choose the correct statement which describe the position of the point (–6, 2) relative to straight lines 2x + 3y – 4 = 0 and 6x + 9y + 8 = 0.

(a) Below both the lines (b) Above both the lines

(c) In between the lines (d) None of these

96. If A and B are two points on the line 3x + 4y + 15 = 0 such that OA = OB = 9 units, then the area of the triangle OAB is

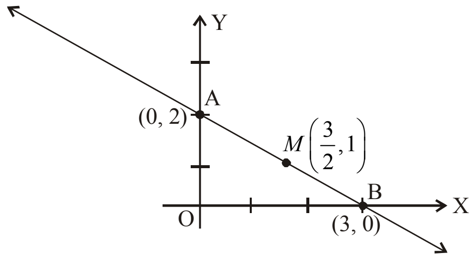
(a) 18 sq. units (b) $18\sqrt{2}$ sq. units

(c) $\frac{18}{\sqrt{2}}$ sq. units (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}, \; x_1 \neq x_2$
2. (c) $m = \tan\alpha, \alpha \neq 90°$
3. (a) $y - y_0 = m(x - x_0)$
4. (b) $y = m(x - d)$
5. (a) $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$
6. (c) $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$
7. (a) The equation of the line $x - y + 3 = 0$ can be rewritten as $y = x + 3$
   $\Rightarrow m = \tan\theta = 1$ and hence $\theta = 45°$.
8. (a) Equation of line in intercept form is $\frac{x}{a} + \frac{y}{a} = 1$
   ($\because$ Intercept has equal length)
   $\Rightarrow x + y = a$
   $\Rightarrow y = -x + a$
   $\Rightarrow$ slope $= -1$
9. (d) Let d_1, d_2, d_3, d_4 are distances of equations $x - y + 1 = 0$, $2x - y + 3 = 0$, $x + 2y - 2 = 0$ and $x + y - 2 = 0$ respectively from the origin.
   $$d_1 = \left|\frac{-0 + 1}{\sqrt{1^2 + (-1)^2}}\right| = \frac{1}{\sqrt{2}}$$
   $$d_2 = \left|\frac{2(0) - 0 + 3}{\sqrt{2^2 + (-1)^2}}\right| = \frac{3}{\sqrt{5}}$$
   $$d_3 = \left|\frac{1(0) + 2(0) - 2}{\sqrt{1^2 + 2^2}}\right| = \frac{2}{\sqrt{5}}$$
   $$d_4 = \left|\frac{0 + 0 - 2}{\sqrt{1^2 + 1^2}}\right| = \frac{2}{\sqrt{2}} = \sqrt{2}$$
   Hence, line corresponding to d_4 (1.414) is farthest from origin.
10. (c) Let intercept on x-axis and y-axis be a and b respectively so that the equation of line is
   $\frac{x}{a} + \frac{y}{b} = 1$
   But a = b [given]
   so; x + y = a
   Also it passes through (2, 4) (given)
   Thus 2 + 4 = a
   $\Rightarrow$ a = 6
   Now, the reqd. equation of the straight line
   x + y = 6
   or, x + y – 6 = 0.
11. (d) We know that the equation of a line making intercepts a and b with x-axis and y-axis, respectively, is given by
   $\frac{x}{a} + \frac{y}{b} = 1.$
   Here we have $1 = \frac{a + 0}{2}$ and $2 = \frac{0 + b}{2}$, which give $a = 2$ and $b = 4$. Therefore, the required equation of the line is given by
   $\frac{x}{2} + \frac{y}{4} = 1$ or $2x + y - 4 = 0$
12. (c) Equations of lines are
   $\frac{x}{a} + \frac{y}{-b} = 1$ and $\frac{x}{b} + \frac{y}{-a} = 1$
   $\Rightarrow$ $bx - ay = ab$ and $ax - by = ab$
   $\Rightarrow$ $m_1 = \frac{b}{a}$ and $m_2 = \frac{a}{b}$ (slopes)
   $$\therefore \tan\theta = \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{b}{a} \times \frac{a}{b}} = \frac{b^2 - a^2}{2ab}$$
13. (a) Equation of line AB is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$



   $\Rightarrow$ $-\frac{2}{3} = \frac{y - 2}{x - 3}$
   $\Rightarrow$ $-2(x - 3) = 3(y - 2)$
   $\Rightarrow$ $2x + 3y = 12$
14. (a) Let the line make intercept 'a' on x-axis. Then, it makes intercept '$2a$' on y-axis. Therefore, the equation of the line is given by
   $\frac{x}{a} + \frac{y}{2a} = 1$
   It passes through (1, 2), so, we have
   $\frac{1}{a} + \frac{2}{2a} = 1$ or $a = 2$
   Therefore, the required equation of the line is given by
   $\frac{x}{2} + \frac{y}{4} = 1$ or $2x + y = 4$
15. (c) Slope of the line through the points (–2, 6) and (4, 8) is,
   $m_1 = \frac{8 - 6}{4 - (-2)} = \frac{2}{6} = \frac{1}{3}$
   Slope of the line through the points (8, 12) and $(x, 24)$ is :
   $m_2 = \frac{24 - 12}{x - 8} = \frac{12}{x - 8}$

Since, two lines are perpendicular,
$m_1 m_2 = -1$, which gives

$$\frac{1}{3} \times \frac{12}{x-8} = -1$$

$\Rightarrow -x + 8 = 4 \Rightarrow 8 - 4 = x \Rightarrow x = 4$

16. (c) Any point on the line $7x + 56y = 0$ is

$$\left(x_1, -\frac{7x_1}{56}\right), \text{ i.e., } \left(x_1, -\frac{x_1}{8}\right)$$

$\therefore$ The perpendicular distance p and p' are

$$p = \frac{3x_1 - \frac{4x_1}{8}}{5} = \frac{x_1}{2}$$

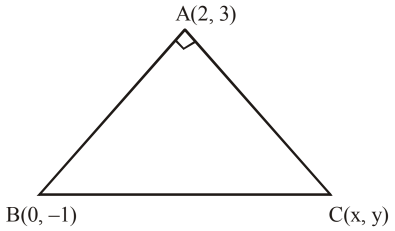
and $p' = \dfrac{5x_1 + \frac{12x_1}{8}}{13} = \dfrac{x_1}{2} \Rightarrow p = p'$

17. (b) Lines II and III are at right angles

$$\left[\because \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1\right]$$

Lines I and II intersect at the point (1, 2) and (1, 2) does not belong to III. Hence, the lines are not concurrent, i.e., they form a right angled triangle.

18. (c) Slope of $AB = 2 \Rightarrow$ slope of $AC = -\dfrac{1}{2}$



$\Rightarrow \dfrac{y-3}{x-2} = -\dfrac{1}{2} \Rightarrow x + 2y - 8 = 0$ ....(i)

Also $x^2 + (y+1)^2 = 25$

$\Rightarrow (8-2y)^2 + (y+1)^2 = 25$ [from (i)]

$\Rightarrow y = 2$ or 4 and correspondingly $x = 4$ and $x = 0$.
Hence, C is (0, 4) or (4, 2).

19. (b) Slope of the line $ax + by = c$ is $\dfrac{-a}{b}$, and the slope of the line $a'x + b'y = c'$ is $\dfrac{-a'}{b'}$. The lines are perpendicular if $\left(\dfrac{-a}{b}\right)\left(\dfrac{-a'}{b'}\right) = -1$ or $aa' + bb' = 0$

20. (a) Here, $m = \tan 120° = \tan(90 + 30°) = -\cot 30° = -\sqrt{3}$
and $c = -5$
So, the equation of the line is
$y = -\sqrt{3}\,x - 5$ [Using : $y = mx + c$]
$\Rightarrow \sqrt{3}\,x + y + 5 = 0$

21. (c) The equation of a line perpendicular to $3x + 2y + 5 = 0$ is
$2x - 3y + \lambda = 0$ ...(i)
This passes through the point (3, 4).
$\therefore 3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$
Putting $\lambda = 6$ in (i), we get $2x - 3y + 6 = 0$, which is the required equation.

22. (b) Only two point A(–1, – 7) and B (3, 4) satisfy the given equation of the line $3x - 4y = 25$
Distance of A (– 1, – 7) from the origin O.

$$= \sqrt{(0+1)^2 + (0+7)^2} = \sqrt{50} = 5\sqrt{2}$$

Distance of B (3, – 4) from the origin O.

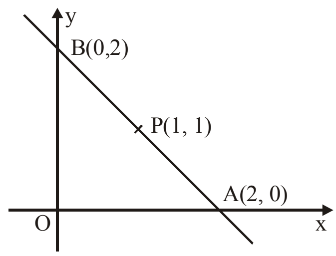
$$= \sqrt{(0-3)^2 + (0+4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

The nearest point is (3, – 4)

23. (d) Let intercept on x-axis be a and that on y axis be b, the coordinate of these end points are (a, 0) and (b, 0).

Since, P(1, 1) is the mid point therefore $1 = \dfrac{a+0}{2}$ and

$1 = \dfrac{0+b}{2} \Rightarrow a = 2, b = 2.$



So, equation of straight line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$\Rightarrow \dfrac{x}{2} + \dfrac{y}{2} = 1 \Rightarrow x + y = 2$

24. (a) The given lines are

$y = (2-\sqrt{3})\,x + 5$

and $y = (2+\sqrt{3})\,x - 7$

Therefore, slope of first line $= m_1 = 2-\sqrt{3}$ and slope of second line $= m_2 = 2+\sqrt{3}$

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$$

$$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \quad \Rightarrow \quad \theta = \frac{\pi}{3} = 60°$$

25. (a) If (x, y), (1, 2) and (–3, 4) are collinear then slope of line joining (x, 4) and (1, 2) is same as line joining points (1, 2) and (–3, 4) or line joining (x, 4) to (– 3, 4).

So, $\dfrac{2-y}{1-x} = \dfrac{4-2}{-3-1} = \dfrac{4-y}{-3-x}$

$\Rightarrow \dfrac{2-y}{1-x} = -\dfrac{1}{2} \Rightarrow \dfrac{y-2}{1-x} = \dfrac{1}{2}$

$\Rightarrow -4 + 2y = 1 - x \Rightarrow x + 2y - 5 = 0$

26 . (d) Length of perpendicular from the origin on the straight line $x + 2by - 2p = 0$ is

$$\left|\frac{0 + 2b \times 0 - 2p}{\sqrt{1^2 + (2b)^2}}\right| = p$$

or $p = \left| \dfrac{-2p}{\sqrt{1^2 + 4b^2}} \right|$ or $p^2 = \dfrac{4p^2}{1+4b^2}$

$\Rightarrow \dfrac{4}{1+4b^2} = 1$

$\Rightarrow 1 + 4b^2 = 4$ or $4b^2 = 3 \Rightarrow b^2 = \dfrac{3}{4}$

$\Rightarrow b = \pm \dfrac{\sqrt{3}}{2}$

$\Rightarrow b = \dfrac{\sqrt{3}}{2}$ matches with the given option.

27. (a) Let the required line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ....(i)

then $a + b = -1$ ....(ii)

(i) passes through (4, 3), $\Rightarrow \dfrac{4}{a} + \dfrac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(iii)

Eliminating b from (ii) and (iii), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\dfrac{x}{2} + \dfrac{y}{-3} = 1$ or $\dfrac{x}{-2} + \dfrac{y}{1} = 1$

28. (b) Let (h, k) be the coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$. Then, the slope of the perpendicular line is $\dfrac{k-3}{h-2}$. Again the slope of the given line $x + y - 11 = 0$ is -1

Using the condition of perpendicularity of lines, we have

$\left(\dfrac{k-3}{h-2}\right)(-1) = -1$

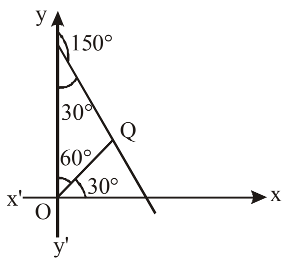
or $k - h = 1$ ...(i)

Since (h, k) lies on the given line, we have,

$h + k - 11 = 0$ or $h + k = 11$ ...(ii)

Solving (i) and (ii), we get $h = 5$ and $k = 6$. Thus (5, 6) are the required coordinates of the foot of the perpendicular.

29. (d) Here $p = 7$ and $\alpha = 30°$



$\therefore$ Equation of the required line is

$x \cos 30° + y \sin 30° = 7$

or $x\dfrac{\sqrt{3}}{2} + y \times \dfrac{1}{2} = 7$

or $\sqrt{3}x + y = 14$

30. (c) The equation of a line making an angle θ with positive x-axis and cutting intercept c on y-axis is given by $y = \tan\theta\ x + c$

Here, $\theta = 135° \Rightarrow \tan\theta = -1$ and $c = -5$

$\therefore y = -x - 5 \Rightarrow x + y + 5 = 0$

31. (a) Let the required line by method $P + \lambda Q = 0$ be $(x - 3y + 1) + \lambda(2x + 5y - 9) = 0$

$\therefore$ Perpendicular from (0, 0) = $\sqrt{5}$ gives

$\dfrac{1 - 9\lambda}{\sqrt{(1+2\lambda)^2 + (5-3\lambda)^2}} = \sqrt{5}$

Squaring and simplifying, $(8\lambda - 7)^2 = 0 \Rightarrow \lambda = 7/8$

Hence the line required is

$(x - 3y + 1) + 7/8(2x + 5y - 9) = 0$

or $22x + 11y - 55 = 0 \Rightarrow 2x + y - 5 = 0$

32. (d) The two lines having the slopes m_1 and m_2 are perpendicular iff $m_1.m_2 = -1$

Now $a_1x + b_1y + c_1 = 0$

$\Rightarrow y = \dfrac{-a_1}{b_1}x - \dfrac{-c_1}{b_1} \Rightarrow$ slope $(m_1) = \dfrac{-a_1}{b_1}$

Similarly, $a_2x + b_2y + c_2 = 0$

Gives the slope, $m_2 = \dfrac{-a_2}{b_2}$

Now, we know the lines $\perp$ when $m_1 . m_2 = -1$

$\Rightarrow \dfrac{-a_1}{b_1} \cdot \dfrac{-a_2}{b_2} = -1$

$\Rightarrow a_1a_2 = -b_1b_2 \Rightarrow a_1a_2 + b_1b_2 = 0.$

33. (a) Equation of line AB is $y - 3 = \dfrac{6-3}{7-3}(x - 3)$

$\Rightarrow 3x - 4y + 3 = 0 \Rightarrow \dfrac{x}{-1} + \dfrac{y}{\frac{3}{4}} = 1$

Hence, required length is $\sqrt{(-1)^2 + \left(\dfrac{3}{4}\right)^2} = \dfrac{5}{4}.$

34. (b) The given line can be written in this form $(3 + 2\lambda)x + (-1 - 3\lambda)y + (5 - 4\lambda) = 0$

It will be parallel to y-axis, if

$-1 - 3\lambda = 0 \Rightarrow \lambda = -\dfrac{1}{3}.$

35. (a) Let the equation be $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$\Rightarrow x - y = a$

But it passes through (–3, 2), hence a = –3 – 2 = –5.

Hence, the equation is $x - y + 5 = 0$.

36. (b) Mid point $\equiv$ (3, 2). Equation is $2x - y - 4 = 0$.

STATEMENT TYPE QUESTIONS

37. (d)

38. (d) Given: A = distance of point from x-axis

$A^2 = 2^2 + 3^2 = 4 + 9 = 13$

$B^2 = 3^2 + 1^2 = 9 + 1 = 10$

$C^2 = 1^2 + 2^2 = 1 + 4 = 5$

From above, we get
$B^2 = 10 = 2 \times 5 = 2C^2$
$\Rightarrow \quad B^2 = 2C^2 \qquad [\because C^2 = 5]$
and $2A^2C^2 = 2.13.5 = 13.10 = 13B^2 \qquad [\because B^2 = 10]$
$\Rightarrow \quad 2A^2C^2 = 13B^2$

39. **(a)** I. Equation of line is
$y - 0 = m(x - 0)$
$\Rightarrow y = mx$
II. Equation of the x-axis is y = 0.

40. **(c)** Both are true.

41. **(d)** All are true statements.

42. **(a)** Both the given statements are true.

43. **(c)**1

44. **(c)** Both the given statements are true.

45. **(c)** Both the given statements are true.

46. **(c)** Both the given statements are true.

47. **(a)** I. Let A, B and C having coordinates (a, b), (c, d) and {(a – c), (b – d)} respectively be the points
If these points are collinear then

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

On solving this expression we get
1. {a (d – b) – b (c – a)} = 0
$\Rightarrow$ ad – ab – bc + ab = 0
$\Rightarrow$ bc – ad = 0

II. Since the points are collinear.

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & a & 1 \end{vmatrix} = 0$$

Expanding the above expression

$$\Rightarrow 1\begin{vmatrix} 4 & 1 \\ a & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 3 & a \end{vmatrix} = 0$$

$\Rightarrow (4 - a) - 2(2 - 3) + 1(2a - 12) = 0$
$\Rightarrow 4 - a + 2 + 2a - 12 = 0$
$\Rightarrow a - 6 = 0$
$\Rightarrow a = 6$

Thus, coordinates of C are (3, 6).

Thus, BC $= \sqrt{(3-2)^2 + (6-4)^2}$
$= \sqrt{1+4} = \sqrt{5}$ unit

48. **(c)** I. Centroid of a triangle is a point where medians meet.
II. If value of area after calculations is negative then we take its absolute value.

49. **(a)** II. Product of slopes = –1

50. **(d)** $y = \frac{3}{4}x + \frac{5}{2} \Rightarrow$ Slope $= \frac{3}{4}$
Also, $3x - 4y = -10$

$$\Rightarrow \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1$$

$\Rightarrow$ x-intercept $= \frac{-10}{3}$ and y-intercept $= \frac{5}{2}$

51. **(c)** $\sqrt{3}x + y - 8 = 0$
$\Rightarrow \frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$ (on dividing by 2)
$\Rightarrow \cos 30°\, x + \sin 30°\, y = 4$

52. **(b)** I. Slope $= \frac{4-(-2)}{-1-3} = \frac{-3}{2}$
II. Slope $= \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0$
III. Slope $= \frac{4-(-2)}{3-3} = \frac{6}{0}$ which is not defined.

INTEGER TYPE QUESTIONS

53. **(a)** We have the points $A(x, -1)$, $B(2, 1)$, C(4, 5).
A, B, C are collinear if the slope of *AB* = Slope of *BC*.

Slope of $AB = \frac{1+1}{2-x} = \frac{2}{2-x}$;

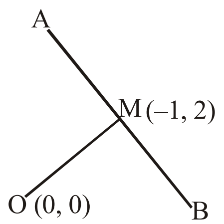
Slope of $BC = \frac{5-1}{4-2} = \frac{4}{2} = 2$

$\therefore \frac{2}{2-x} = 2$ or $2 - x = 1$ or $x = 1$

54. **(d)** The given line is $12(x + 6) = 5(y - 2)$
$\Rightarrow 12x + 72 = 5y - 10$
or $12x - 5y + 72 + 10 = 0$
$\Rightarrow 12x - 5y + 82 = 0$
The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{(ax_1 + by_1 + c)}{\sqrt{a^2 + b^2}}$.
The point (x_1, y_1) is (–1, 1), therefore, perpendicular distance from (–1, 1) to the line $12x - 5y + 82 = 0$ is

$$= \frac{|-12 - 5 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{65}{\sqrt{144+25}} = \frac{65}{\sqrt{169}} = \frac{65}{13} = 5$$

55. **(b)** Let the perpendicular *OM* is drawn from the origin to *AB*.



M is the foot of the perpendicular

Slope of $OM = \frac{2-0}{-1-0} = \frac{2}{-1}$;

Slope of $AB = m$

$OM \perp AB \;\; \therefore m \times (-2) = -1 \;\; \therefore m = \frac{1}{2}$

M(–1, 2) lies on *AB* whose equation is

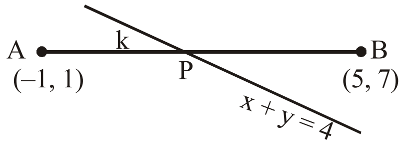
$y = mx + c$ or $y = \frac{1}{2}x + c$

$2 = \frac{1}{2} \times (-1) + c \Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore\ m=\frac{1}{2}$ or $c=\frac{5}{2}\Rightarrow m+c=\frac{6}{2}=3$

56. (a) Any line parallel to x-axis of the form $y=p$
i.e. coefficient of $x=0$
$\therefore$ In equation $(k-3)x-(4-k^2)\,y+k^2-7k+6=0$
Coefficient of $x=k-3=0\ \therefore\ k=3$

57. (a) The line joining the points A (–1, 1) and B(5, 7) is divided by $P(x, y)$ in the ratio k : 1



$\therefore$ Point P is $\left(\frac{5k-1}{k+1},\frac{7k+1}{k+1}\right)$

This point lies on the line $x+y=4$

$\therefore\ \frac{5k-1}{k+1}+\frac{7k+1}{k+1}=4$

$\Rightarrow 5k-1+7k+1=4k+4\Rightarrow 8k=4\Rightarrow k=\frac{1}{2}$

$\therefore$ P divides AB in the ratio 1 : 2

58. (b) The given points are $A(h, 0)$, $B(a, b)$, $C(0, k)$, they lie on the same plane.
$\therefore$ Slope of AB = Slope of BC

$\therefore$ Slope of $AB=\frac{b-0}{a-h}=\frac{b}{a-h}$;

Slope of $BC=\frac{k-b}{0-a}=\frac{k-b}{-a}$

$\therefore\ \frac{b}{a-h}=\frac{k-b}{-a}$ or by cross multiplication

$-ab=(a-h)(k-b)$
or $-ab=ak-ab-hk+hb$
or $0=ak-hk+hb$
or $ak+hb=hk$

Dividing by hk $\Rightarrow\frac{ak}{hk}+\frac{hb}{hk}=1$ or $\frac{a}{h}+\frac{b}{k}=1$

Hence proved.

59. (b) Slope of line through (2, 5) and (x, 3) is $\frac{3-5}{x-2}$

We have, $\frac{3-5}{x-2}=2\Rightarrow x=1$

60. (d) Let A(3, y), B(2, 7), C(–1, 4) and D(0, 6) be the given points.

m_1 = slope of AB = $\frac{7-y}{2-3}=(y-7)$

m_2 = slope of CD = $\frac{6-4}{0-(-1)}=2$

Since AB and CD are parallel
$\therefore\ m_1=m_2\Rightarrow y=9.$

61. (c) Given equation is

$\sqrt{3}x+y-8=0$

Divide this by $\sqrt{\left(\sqrt{3}\right)^2+1^2}=2$,

we get, $\frac{\sqrt{3}}{2}x+\frac{1}{2}y=4$

Which is in the normal form. Hence, p = 4.

62. (c) Given parallel lines are
$3x-4y+7=0$ and $3x-4y+5=0$

Required distance $=\frac{|7-5|}{\sqrt{(3)^2+(-4)^2}}=\frac{2}{5}$

$\Rightarrow a=2, b=5$

ASSERTION - REASON TYPE QUESTIONS

63. (b) Assertion is correct and Reason is also correct

64. (b) Both the Assertion and Reason are true.

65. (c) Assertion is false and Reason is true.

Assertion: Slope $=\frac{4-(-2)}{3-3}=\frac{6}{0}$ which is not defined.

66. (b) Mid-point of AC $=\left(\frac{1}{2},\frac{2}{2}\right)=\left(\frac{1}{2},1\right)$

Mid-point of BD $=\left(\frac{1}{2},1\right)$

67. (a) **Assertion:**
$a_1=1, b_1=2, c_1=-3$
$a_2=-3, b_2=-6, c_2=9$

Clearly, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{-1}{3}$

So, the given lines are coincident.

68. (c) Assertion is correct but Reason is incorrect.
Correct Reason is given lines are perpendicular, if $a_1a_2+b_1b_2=0.$

69. (c) Assertion is correct. Reason is incorrect.

Reason: The slope of the given line is $\frac{-a}{b}$.

70. (d) Assertion is incorrect Reason is correct.
Assertion: The equation of a line parallel to the line $ax+by+c=0$ is $ax+by+\lambda=0$ where λ is a constant.

71. (a) **Assertion:** $A=3, B=-4$

$C_1=9, C_2=-\frac{15}{2}$

$d=\frac{\left|-\frac{15}{2}-9\right|}{\sqrt{9+16}}=\frac{\left|-\frac{33}{2}\right|}{5}=\frac{33}{10}$

72. (b) Both are correct.

CRITICALTHINKING TYPE QUESTIONS

73. (a) Let the point of intersection divide the line segment joining points, (3, – 1) and (8, 9) in k : 1 ratio then

The point is $\left(\frac{8k+3}{k+1},\frac{9k-1}{k+1}\right)$

Since this point lies on the line $y-x+2=0$

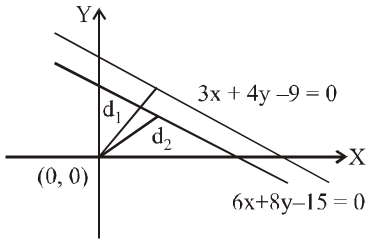
We have, $\frac{9k-1}{k+1}-\frac{8k+3}{k+1}+2=0$

$\Rightarrow\frac{9k-1-8k-3}{k+1}+2=0\Rightarrow\frac{k-4}{k+1}+2=0$

$\Rightarrow k-4+2k+2=0 \Rightarrow 3k-2=0$

$k=\frac{2}{3}:1$ i.e. $2:3$

74. (b) Let d_1 and d_2 be the distances of two lines $3x+4y-9=0$ and $6x+8y-15=0$ respectively from origin.



$\therefore \quad d_1=\frac{|3(0)+4(0)-9|}{\sqrt{3^2+4^2}} \Rightarrow d_1=\frac{9}{5}$

and $d_2=\frac{|6(0)+8(0)-15|}{\sqrt{36+64}}=\frac{15}{10}=\frac{3}{2}$

$\therefore$ distance between these lines is, $d=d_1-d_2$

$\Rightarrow \quad d=\frac{9}{5}-\frac{3}{2}=\frac{18-15}{10}=\frac{3}{10}$

75. (d) Given line is $3x+y=3$

Let the equation of line which is perpendicular to above line is

$x-3y+\lambda=0.$

This line is passing through point (2, 2)

$\therefore \quad 2-3\times 2+\lambda=0$

$\Rightarrow \quad 2-6+\lambda=0 \Rightarrow \lambda=4$

$\therefore$ Equation of line is $x-3y+4=0$

$\Rightarrow \quad 3y=x+4 \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$

Compare the above equation with $y=mx+c$,

We get $c=\frac{4}{3}$

Thus, y –intercept is $\frac{4}{3}$.

76. (c) **Note:** If the vertices of a triangle are $A(a_1, b_1)$, $B(a_2, b_2)$ and $C(a_3, b_3)$, then the area of the triangle ABC

$$=\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

Here in the given question:
we have A(x, 0), B(1, 1), C(0, 2).

and $\frac{1}{2}\begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}=4$

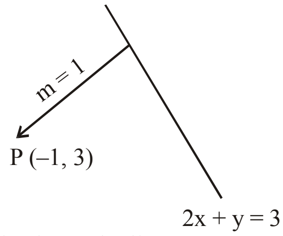
$\Rightarrow \frac{1}{2}[x(1-2)+1(2)]=4$

$\Rightarrow -x+2=8 \Rightarrow x=-6.$

77. (c) The equation of the line through (–1, 3) and having the slope 1 is

$$\frac{x+1}{\cos\theta}=\frac{y-3}{\sin\theta}=r.$$

Any point on this line at a distance r from $P(-1, 3)$ is $(-1+r\cos\theta,\ 3+r\sin\theta)$



This point is on the line $2x+y=3$ if

$2(-1+r\cos\theta)+3+r\sin\theta=3$ ...(i)

But $\tan\theta=1;\ \Rightarrow \theta=45°$

(i) becomes,

$-2+2r.\frac{1}{\sqrt{2}}+3+r.\frac{1}{\sqrt{2}}=3$

$\Rightarrow \frac{3r}{\sqrt{2}}=2;\quad r=\frac{2\sqrt{2}}{3}$

Hence the required distance $=\frac{2\sqrt{2}}{3}$.

78. (a) Given equation of straingh lines are
$x+2y-9=0,\ 3x+5y-5=0$
and $ax+by-1=0$
They are concurrent, if
$-5+5b-2(-3+5a)-9(3b-5a)=0$
$\Rightarrow -5+5b+6-10a-27b+45a=0$
$\Rightarrow 35a-22b+1=0$
Thus, given straight lines are concurrent if the straight line $35x-22y+1=0$ passes through (a, b).

79. (a) Let (h, k) be the point of reflection of the given point $(4, -13)$ about the line $5x+y+6=0$. The mid-point of the line segment joining points (h, k) and $(4, -13)$ is given by

$$\left(\frac{h+4}{2},\frac{k-13}{2}\right)$$

This point lies on the given line, so we have

$$5\left(\frac{h+4}{2}\right)+\frac{k-13}{2}+6=0$$

or $\quad 5h+k+19=0$ …(i)

Again the slope of the line joining points (h, k) and $(4, -13)$ is given by $\frac{k+13}{h-4}$. This line is perpendicular to the given line and hence

$$(-5)\left(\frac{k+3}{h-4}\right)=-1$$

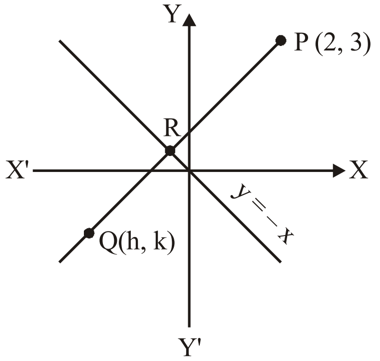
This gives $5k+65=h-4$

or $\quad h-5k-69=0$ …(ii)

On solving (i) and (ii), we get $h=-1$ and $k=-14$. Thus the point $(-1, -14)$ is the reflection of the given point.

80. (a) (1, – 2)

81. (a) Let there be a point P(2,3) on cartesian plane. Image of this point in the line $y=-x$ will lie on a line which is perpendicular to this line and distance of this point from y = –x will be equal to distance of the image from this line.



Let Q be the image of p and let the co-ordinate of Q be (b, k)

Slope of line y = –x is –1

Line joining P, Q will be perpendicular to y = –x so, its slope = 1.

Let the equation of the line be y = x + c since this passes through point (2, 3)

3 = 2 + c ⇒ c = 1

and the equation y = x + 1

The point of intersection R lies in the middle of P & Q.

Point of intersection of line $y = -x$ and $y = x + 1$ is

$2y = 1, \Rightarrow y = \frac{1}{2}$ and $x = -\frac{1}{2}$

Hence, $\frac{h+2}{2} = -\frac{1}{2}$ and $\frac{k+3}{2} = \frac{1}{2}$

⇒ h = –3 and k = –2

So, the image of the point (2, 3) in the line y = – x is (–3, –2).

82. (c) Equation of line is ax + by – p = 0, then length of perpendicular, from the origin is

$$p = \left|\frac{a \times 0 + b \times 0 - p}{\sqrt{a^2 + b^2}}\right| \text{ or } p = \left|\frac{-p}{\sqrt{a^2 + b^2}}\right|$$

$\Rightarrow a^2 + b^2 = 1$

$b = \frac{\sqrt{3}}{2}$ or $b^2 = \frac{3}{4} \Rightarrow a^2 + \frac{3}{4} = 1$

$a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$

[$a = -\frac{1}{2}$ not taken since angle is with + ve direction to x-axis.]

Equation is $\frac{1}{2}x + \frac{\sqrt{3}}{2}y = p$ or $x\cos 60° + y\sin 60° = p$

Angle = 60°

83. (c) One vertex of square is (– 4, 5) and equation of one diagonal is 7x – y + 8 = 0

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be y = mx + c where m is the slope of the line and c is the y-intercept.

Since this line passes through (– 4, 5)

$\therefore \quad 5 = -4m + c \qquad \ldots$(i)

Since this line is at right angle to the line 7x – y + 8 = 0 or y = 7x + 8, having slope = 7,

$\therefore \quad 7 \times m = -1$ or $m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$5 = -4 \times \left(\frac{-1}{7}\right) + c$

or $5 = \frac{4}{7} + c$ or $c = 5 - \frac{4}{7} = \frac{31}{7}$

Hence equation of the other diagonal is

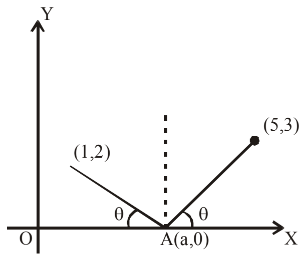
$y = -\frac{1}{7}x + \frac{31}{7}$

or $7y = -x + 31$

or $x + 7y - 31 = 0$

or $x + 7y = 31$.

84. (a) Let the coordinates of A be (a, 0). Then the slope of the reflected ray is



$\frac{3-0}{5-a} = \tan\theta$ (say) $\qquad \ldots$(i)

Then the slope of the incident ray

$= \frac{2-0}{1-a} = \tan(\pi - \theta)$. $\qquad \ldots$(ii)

from (i) and (ii) $\tan\theta + \tan(\pi - \theta) = 0$

$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0 \Rightarrow 3 - 3a + 10 - 2a = 0$

$\Rightarrow a = \frac{13}{5}$

Thus, the co-ordinates of A are $\left(\frac{13}{5}, 0\right)$.

85. (b) $AB = 3\sqrt{2}$, $AC = 4\sqrt{2}$, $BC = 5\sqrt{2}$

$\therefore \frac{AB}{AC} = \frac{3}{4}$. That is the internal bisector of angle A cuts the side BC in ratio 3 : 4 at D. The coordinates of D are

$\left(\frac{4 \times 4 + 3 \times 5}{4+3}, \frac{4 \times -2 + 3 \times 5}{4+3}\right) \equiv \left(\frac{31}{7}, 1\right)$

Slope of AD = 0

∴ Equation of perpendicular from C(5, 5) to AD is x = 5

86. (b) Since the line L has intercepts a and b on the coordinate axes, therefore its equation is

$\frac{x}{a} + \frac{y}{b} = 1 \qquad \ldots$(i)

When the axes are rotated, its equation with respect to the new axes and same origin will become

$\frac{x}{p}+\frac{y}{q}=1$ ...(ii)

In both the cases, the length of the perpendicular from the origin to the line will be same.

$\therefore \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$ or $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

87. (a) Third side passes through (1, – 10), so let its equation be $y+10=m(x-1)$

If it makes equal angle, say θ with given two sides, then

$\tan\theta=\frac{m-7}{1+7m}=\frac{m-(-1)}{1+m(-1)} \Rightarrow m=-3$ or 1/3

Hence possible equations of third side are

$y+10=-3(x-1)$ and $y+10=\frac{1}{3}(x-1)$

or $3x+y+7=0$ and $x-3y-31=0$

88. (a) If the lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore m_1=m_2 \Rightarrow -\frac{p(p^2+1)}{-1}=-\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2+1)(p+1)=0 \Rightarrow p=-1$

$\therefore p$ can have exactly one value.

89. (c) On comparing given equations with $ax+by+c=0$

We get $a_1=4$, $a_2=3$, $b_1=-3$, $b_2=-4$

Now $a_1a_2+b_1b_2=(4\times3+3\times4)=24>0$ (Positive)

Since, $a_1a_2+b_1b_2$ is +ve

$\therefore$ Origin lies in obtuse angle

For acute angle, we find the bisector

Now, equation of bisectors of given lines are

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

The equation of the bisector is

$\left[\frac{4x-3y+7}{5}\right]=-\left[\frac{3x-4y+14}{5}\right] \Rightarrow x-y+3=0$

90. (c) Here, $c=-1$ and $m=\tan\theta=\tan 45°=1$

(Since the line is equally inclined to the axes, so $\theta=45°$)

Also, $m=\tan 135°=-1 \Rightarrow m=\pm 1$

$\therefore \theta=45°$ and $135°$

Hence, equation of straight line is $y=\pm(1\cdot x)-1$

$\Rightarrow x-y-1=0$ and $x+y+1=0$

91. (c) Here, D(1, 1), therefore, equation of line AD is given by $2x+y-3=0$. Thus, the line perpendicular to AD is $x-2y+k=0$ and it passes through B, so $k=0$. Hence, required equation is $x-2y=0$.

92. (c) The lines passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$ is

$ax+2by+3b+\lambda(bx-2ay-3a)=0$

$\Rightarrow (a+b\lambda)x+(2b-2a\lambda)y+3b-3\lambda a=0$ ... (i)

Line (i) is parallel to x-axis,

$\therefore a+b\lambda=0 \Rightarrow \lambda=\frac{-a}{b}=0$

Put the value of λ in (i),

$ax+2by+3b-\frac{a}{b}(bx-2ay-3a)=0$

$y\left(2b+\frac{2a^2}{b}\right)+3b+\frac{3a^2}{b}=0,$

$y\left(\frac{2b^2+2a^2}{b}\right)=-\left(\frac{3b^2+3a^2}{b}\right)$

$y=\frac{-3(a^2+b^2)}{2(b^2+a^2)}=\frac{-3}{2}, \; y=-\frac{3}{2}$

So, it is $\frac{3}{2}$ unit below x-axis.

93. (a) By direct formulae,

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

$\frac{3x+4y-7}{\sqrt{3^2+4^2}}=\pm\frac{12x+5y+17}{\sqrt{(12)^2+(5)^2}}$

$\frac{3x+4y-7}{5}=\pm\frac{12x+5y+17}{13}.$

94. (a) The equations of the bisectors of the angles between the lines are $\frac{x-2y+4}{\sqrt{1+4}}=\pm\frac{4x-3y+2}{\sqrt{16+9}}$

Taking positive sign, then

$\left(4-\sqrt{5}\right)x-\left(3-2\sqrt{5}\right)y-\left(4\sqrt{5}-2\right)=0$ ... (i)

and negative sign gives

$\left(4+\sqrt{5}\right)x-\left(2\sqrt{5}+3\right)y+\left(4\sqrt{5}+2\right)=0$ ... (ii)

Let θ be the angle between the line (i) and one of the given line, then

$\tan\theta=\left|\frac{\frac{1}{2}-\frac{4-\sqrt{5}}{3-2\sqrt{5}}}{1+\frac{1}{2}\cdot\frac{4-\sqrt{5}}{3-2\sqrt{5}}}\right|=\sqrt{5}+2>1$

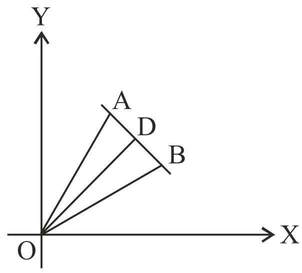
Hence, the line (i) bisects the obtuse angle between the given lines.

95. (a) $L \equiv 2x+3y-4=0$, $L_{(-6,2)}=-12+6-4<0$

$L'=6x+9y+8=0$, $L'_{(-6,2)}=-36+18+8<0$

Hence, the point is below both the lines.

96. (b) $OA=OB=9$, $OD=\frac{15}{\sqrt{25}}=3$



Therefore, $AB=2AD=2\sqrt{81-9}=2\sqrt{72}=12\sqrt{2}$

Hence, $\Delta=\frac{1}{2}\left(3\times12\sqrt{2}\right)=18\sqrt{2}$ sq. units