PTA-4_Q'P

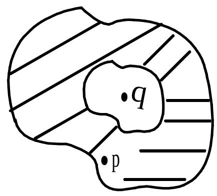
PHYSICS **Max Marks:60**

SECTION-I

(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A) (B)(C) and (D) out of which ONE or MORE are correct.

1. Inside an electrical conductor, there is a cavity and a charge q is suspended fixed inside the cavity without any electrical contact with conductor. P is a point inside the conductor. Let $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ be the electrostatic field at P only due to q, only due to charge on inner surface of the conductor and only due to outer surface of conductor respectively. Then,



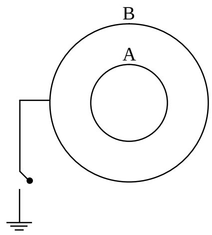
A) $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{0}$

B) $\vec{E}_1 + \vec{E}_2 = \vec{0}$

C) $\vec{E}_2 + \vec{E}_3 = \vec{0}$

D) $\vec{E}_3 = \vec{0}$

2. Two concentric shells A and B have radii R and 2R, charges q_A and q_B and potential 2V and $\left(\frac{3}{2}\right)V$ respectively. Now shell B is earthed and let charges on them become q'_A & q'_B. Then



A) $\frac{q_A}{q_B} = \frac{1}{2}$

B) $\frac{q'_A}{q'_B} = 1$

C) Potential of A after earthing becomes $\left(\frac{3}{2}\right)V$

D) Potential difference between A and B$\left(V_A - V_B\right)$ after earthing becomes $\frac{V}{2}$

3. An uncharged conducting negligibly thin square plate of side 1 m is placed in an uniform electric field of magnitude 200 N/C. The field direction is perpendicular to the plane of plate. For this situation, mark out the correct statement(s). [Ignore edge effects]

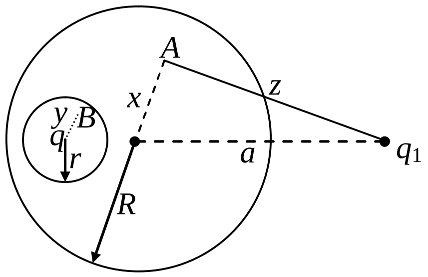
A) The net charge on each face of the plate is zero

B) The net charge on each face of the plate is non-zero and having the magnitude equal to 1.77×10^{-9}C

C) The net electric field intensity inside the plate is zero

D) The electric field intensity across the surface of plate is discontinuous while potential is continuous.

4. A cavity of radius 'r' is made inside a solid neutral conducting sphere of radius 'R' and a positive charge q is placed(at a distance 'y' from the center of cavity) inside the cavity at the center. *A*nother positive charge q_1 is placed (at a distance 'a' from the center of sphere) outside the sphere as shown in the figure. Point *A* is inside the sphere and point *B* is inside the cavity.



Then, which of the following is correct.

A) Electric field at point *A* is zero.

B) Electric field at point *B* is $\frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}$.

C) Potential at point *A* is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a}+\frac{q}{R}\right]$

D) Potential at point *B* is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a}+\frac{q}{R}+\frac{q}{y}-\frac{q}{r}\right]$

5. At a distance of 5cm and 10cm from surface of a uniformly charged solid sphere, the potentials are 100V and 75V respectively. Then,

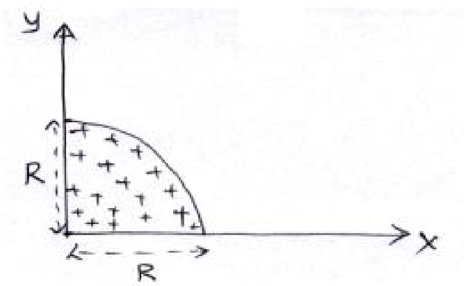
A) potential at its surface is 150 V

B) The charge on the sphere is $\frac{50}{3}\times 10^{-10} C$

C) The electric field on the surface is 1500 V/m

D) The electric potential at its centre is 25V

6. The quarter disc of radius ‘R’ (shown in figure) has a uniform surface charge density ‘σ’.



A) Electric potential at (0, 0, Z) is $\frac{\pm}{8£_0}\left(\sqrt{R^2+Z^2}-Z\right)$

B) Z- component of electric field at (0, 0, Z) is $\frac{\pm}{8£_0}\left(1-\frac{Z}{\sqrt{R^2+Z^2}}\right)$

C) Electric potential (0, 0, Z) is $\frac{\pm}{4£_0}\left(\sqrt{R^2+Z^2}-Z\right)$

D) x and y components of electric field at (0, 0, Z) are zero

7. A conducting bubble of radius a, thickness t (<<a) has a potential ‘V’. Now the bubble collapses into a drop. Assuming that there is no leakage of charge

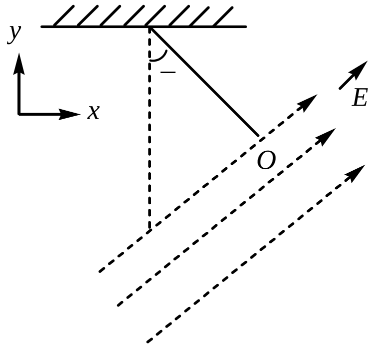
A) Radius of drop $=\sqrt[3]{3a^2t}$

B) Potential of drop $=V\left(\frac{a}{3t}\right)^{1/3}$

C) Electric field on surface of drop $=\frac{V}{\left(9at^2\right)^{1/3}}$

D) The charge on drop is Va/R

8. A charged cork ball of mass 'm' is suspended by means of a light string in the presence of an electric field as shown in the figure. When the electric field given by $\vec{E} = A\hat{i} + B\hat{j}$ is switched on $(A > 0, B > 0)$ the ball attains equilibrium as shown.



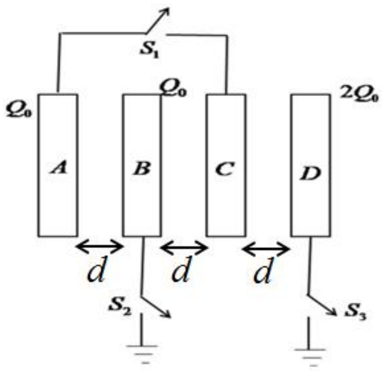
A) Charge on ball is $\frac{mg}{A\cos\theta + B}$

B) Charge on ball is $\frac{mg}{A\cot\theta + B}$

C) Tension in string is $\frac{mgA}{A\cos\theta + B\sin\theta}$

D) $\frac{Tension}{mg} = \cos\theta$

9. In the given arrangment the charges given to the metallic plates A,B and D are Q_0, Q_0 and $2Q_0$ respectively



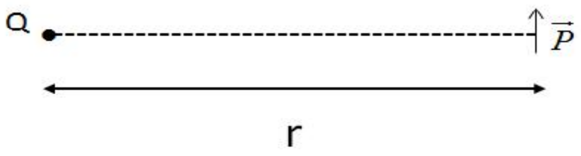
A) If all switch are closed then charge passing through switche S_1 is $\frac{2}{3}Q_0$

B) If only S_3 is closed then '$4Q_0$' charge flown into the earth

C) If only 'S_1' is closed 'Q_0'charge will flow from 'C' to 'A'

D) If only 'S_1' is closed 'Q_0'charge will flow from A to C.

10. For the situation shown in the figure (assume r >> length of the dipole), select the correct statement (s). [$P=$ dipole moment, Q= charge on the particle which is on equatorial line of dipole)



A) Force acting on the dipole is zero

B) Force acting on the dipole is approximately $\frac{PQ}{4®£_0 r^3}$ and is acting upward

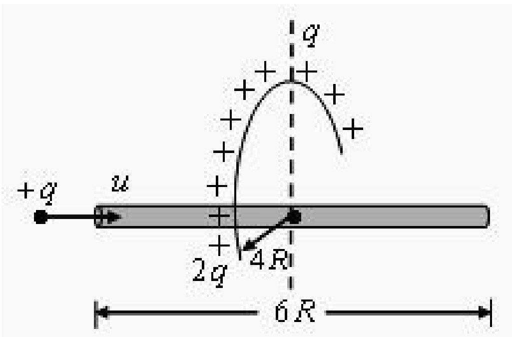
C) Torque acting on the dipole is $\frac{PQ}{4®£_0 r^2}$ is clockwise direction

D) Torque acting on the dipole is $\frac{PQ}{4®£_0 r^2}$ is anti –clockwise direction

SECTION-II
(Integer Value Correct Type)

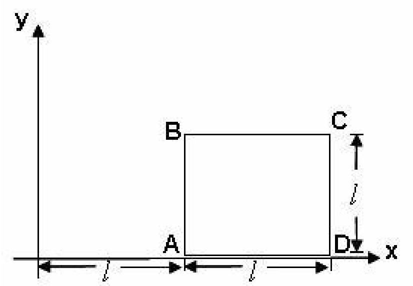
This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

11. On a semicircular ring of radius $= 4R$, charge $+3q$ is distributed in such a way that on one quarter $+q$ is uniformly distributed and on another quarter $+2q$ is uniformly distributed. Along its axis a smooth non-conducting and uncharged pipe of length 6 R is fixed in such a way that the centres of both coincide as shown. A small ball of mass m and charge $+q$ is thrown from one end of pipe with speed u. The minimum speed u so that ball will come out from the other end of pipe should be $\sqrt{\frac{xq^2}{40\pi\varepsilon_0 Rm}}$. Find the value of *x*, taking all in SI units.



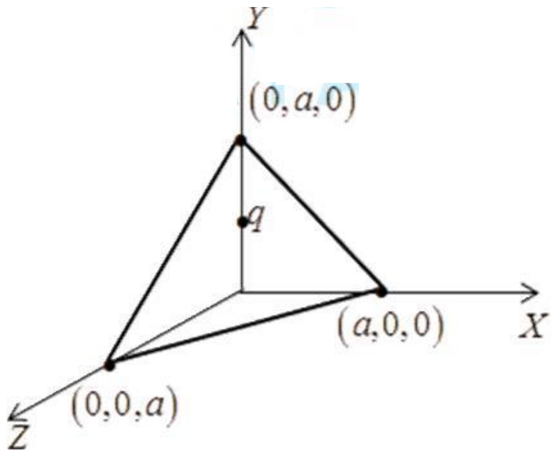
12. A particle that carries a negative charge $'-q'$ C is placed at rest in a uniform electric field 10 N/C . It experiences a force and moves. In a certain time t, it is observed to acquire a velocity $10\hat{i}-10\hat{j}$ m/s . The given electric field intersects a surface of area A m^2 in the $x-z$ plane. Electric flux through the surface is $5\sqrt{x}$ A Nm^2/C . Find the value of x.

13. A square loop of side '*l*' having uniform linear charge density 'ª' is placed in 'xy' plane as shown in the figure. There is a non-uniform electric field $\vec{E}=\frac{a}{l}(x+l)\hat{i}$ where a is a constant. Find the resultant electric force in «N on the loop if $l=10$ cm, a = 2 N/C and charge density ª $=2$«c/m .



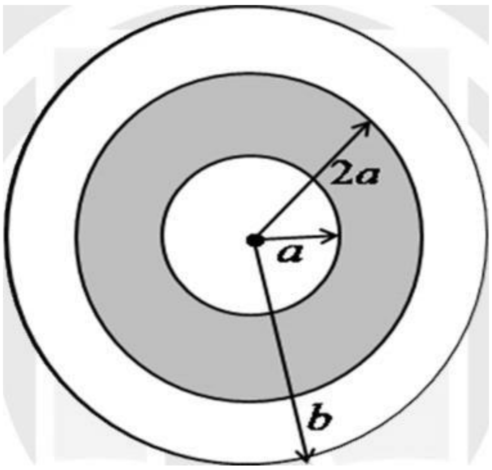
14. Potential function $V(x,y)$ of an electrastatic field $\vec{E}=2axy\hat{i}+a\left(x^2-y^2\right)\hat{j}$ (a is a constant) is given by $V_0+(-1)^n ax^m y+\frac{ay^3}{p}$ (n is either '0' or '1') then value of $(n+m+p)$ is

15. In the given figure a charge $'q'$ is placed at $\left(0,\frac{a}{\sqrt{2}},0\right)$. Then electric flux through the given triangular face is $\frac{nq}{6m£_0}$ then $(n+m)$ is

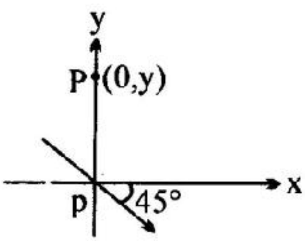


16. There exist a non-conducting disc of radius $'R'$ and having a charge density of ± . What will be the work done by electrostatic repulsion when radius of the disc becomes $2R$. If work done by electric force is of the form $\frac{2\times m}{3}\frac{\pm^2 R^3}{£_0}$ in simplest ratio. Then the value of m is (assume charge is constant on the disc)

17. In the given figure a shell of inner radius $'a'$ & outer radius $'2a'$ is sorrounded by a thin shell of radius $'b'$. Both shells are conducting and concentric. If a charge $'Q'$ is placed at the centre of shell. Then total potential energy of the system is given by $\frac{-kQ^2}{na}$ where n is



18. A dipole is placed at origin of coordinate system as shown in figure, electric field at point $p(o,y)$ is given as $\frac{kp}{\sqrt{c}y^3}\left(a\hat{i}-b\hat{j}\right)$ then value of $a+b+c$ is



19. In a certain region, there are non-uniform electrical potential (V_e) as well as gravitational potential (V_g). The electrical potential varies only with x as shown in figure (i) and the gravitational potential varies only with y as shown in figure (ii). Consider a particle of mass 200 kg and charge $20«C$ in this field.

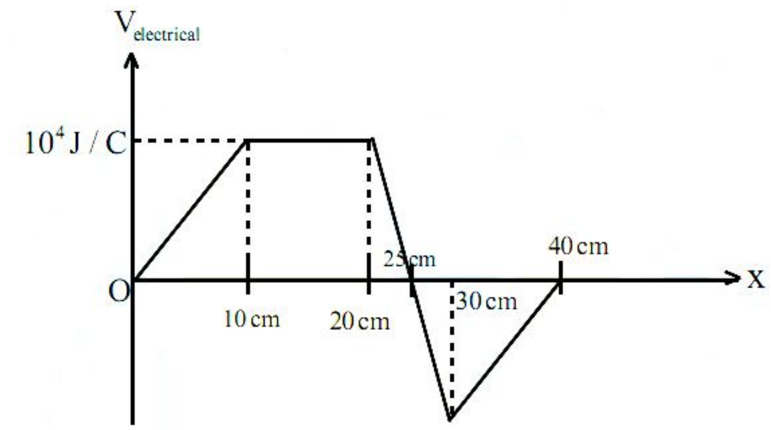


Figure-i

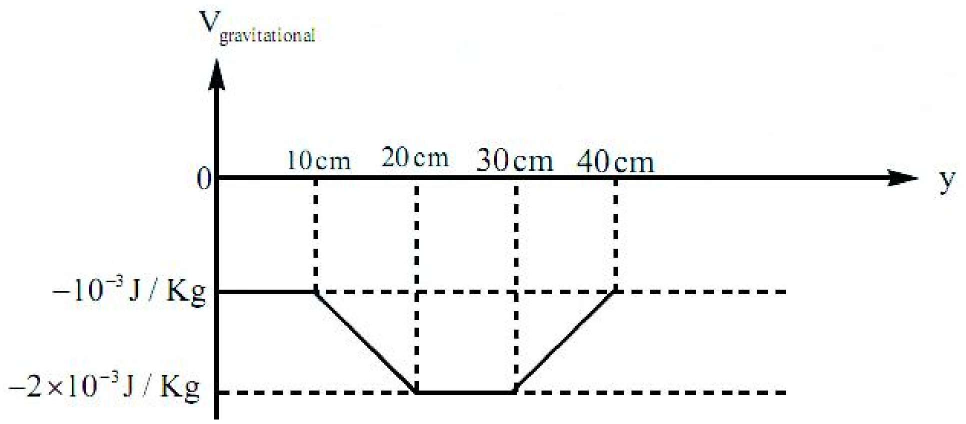


Figure-ii

Then acceleration of the particle at point $(25,35)$ cm is $\left(a\hat{i}+b\hat{j}\right)\times 10^{-2}\ m/\sec^2$ then value of $(a+b)$ is

20. A charged particle of charge 'Q' is held fixed and another charged particle of mass 'm' and charge 'q' (of the same sign) is released from a distance 'r'. The impulse of the force exerted by the external agent on the fixed charge by the time distance between 'Q' and 'q' becomes 2r is $\sqrt{\dfrac{Qqm}{x® \in_0 r}}$ then value of x is –

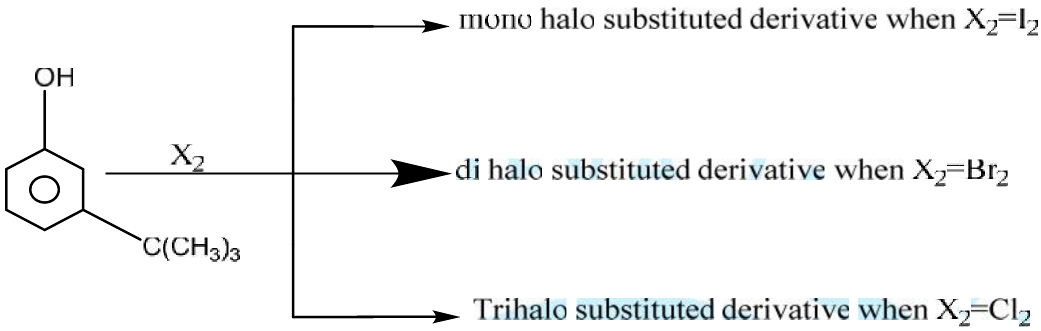
CHEMISTRY **Max Marks:60**

SECTION-I

(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A) (B)(C) and (D) out of which ONE or MORE are correct.

21. The reactivity of compound Z with different halogens under appropriate condition is given below



The observed pattern of electrophilic substitution can be explained by

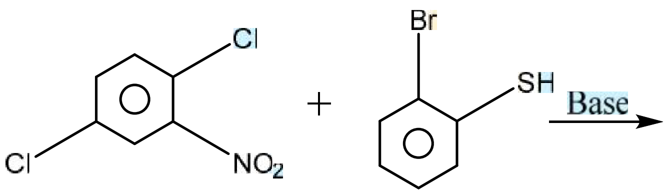
A) The steric effect of the halogen

B) The steric effect of the tert-butyl group

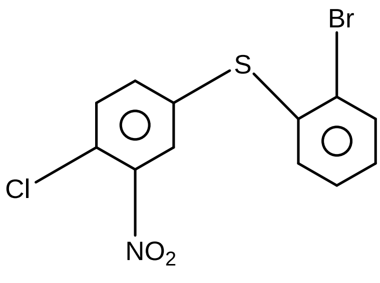
C) The electronic effect of the phenolic group

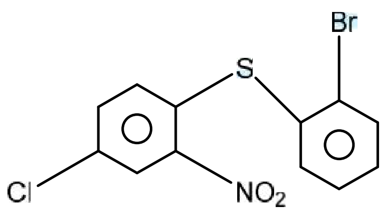
D) The electronic effect of the tert-butyl group

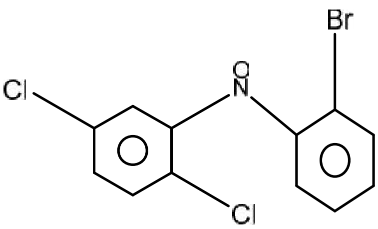
22.

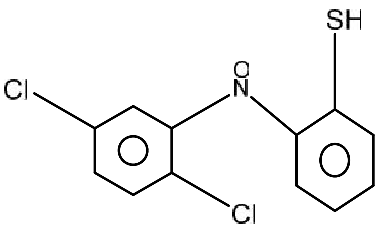


Major product of the reaction is

A) 

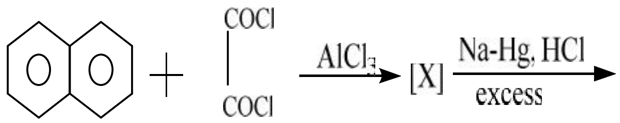
B) 

C) 

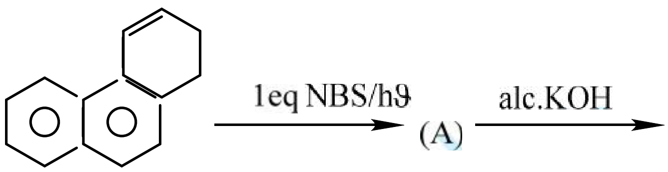
D) 

23. In which of the following reaction final product is phenanthrene

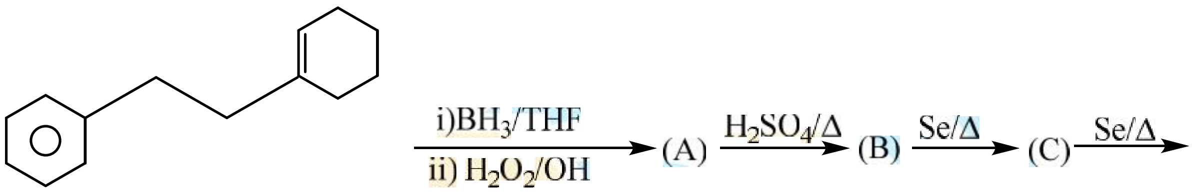
A)



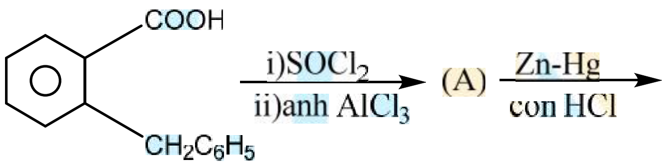
B)



C)



D)



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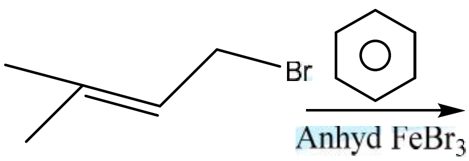
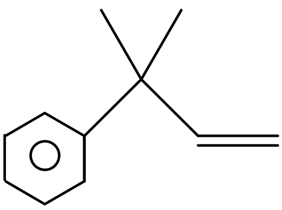
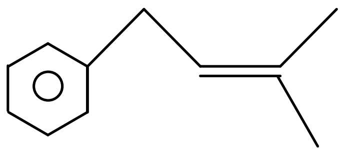
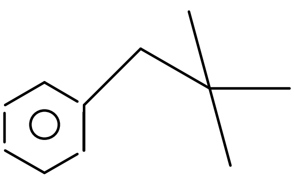
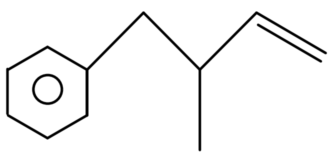


The products formed from the above reaction are

A) 

B) 

C) 

D) 

25. Which of the following statement(s) regarding friedal craft Alkylation is correct?

A) The reaction of $Me_3CCH_2Cl / Anh\ AlCl_3$ on Benzene, yields almost wholly the rearranged product $PhCMe_2CH_2Me$

B) Initial electrophile complex being polarized enough to allow the rearrangement of $Me_3CC\overset{+¢}{H}_2----Cl----\overset{-¢}{Al}Cl_3$ to the more stable $[Me_2CCH_2Me]^{+¢}---Cl---\overset{-¢}{Al}Cl_3$

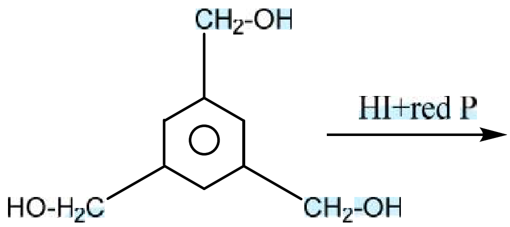
C) By contrast $Me_2CCH_2Cl / Anh\ FeCl_3$ on benzene is found to yield almost wholly unrearranged product

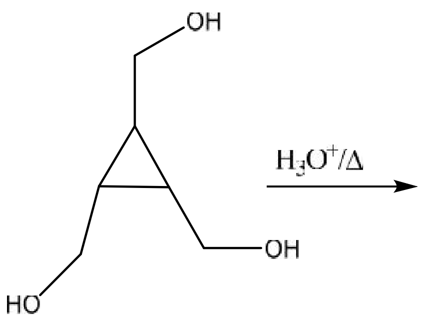
D) The % of rearranged product from a given halid and lewis acid being less at lower temperature as compared to that at high temperature

26. Which of the following will produce aromatic compound

A) $CH_3-C\equiv CH \xrightarrow[\Delta]{Red\ hot\ Fe\ tube}$

B) $nC_7H_{16} \xrightarrow[Cr_2O_3/600^0C]{Al_2O_3}$

C) 

D) 

27.

Which of the following statement(s) are correct regarding the given reaction

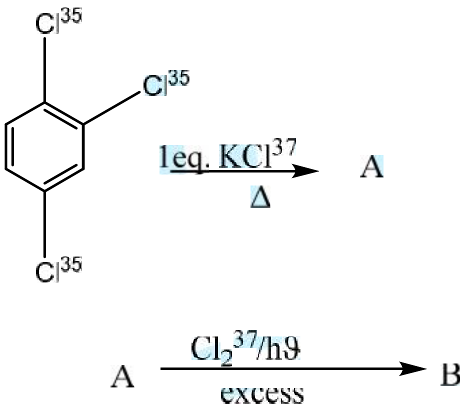
A) If ‘A’ is activating group incoming electrophile goes preferably to para position

B) If ‘A’ is strong deactivating group the reactions is almost hindered

C) If ‘A’ is activating group, product formed is less reactive for further reaction with Cl_2 / Fe as compared to starting reactant

D) If ‘A’ is deactivating group, the reaction if at all happens may follow nucleophilic substitution path

28.



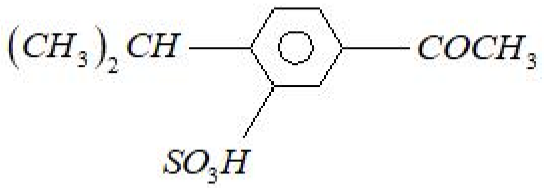
A) A is aromatic while B is not

B) In product B the ratio of Cl^{35}:Cl^{37} is 2:7

C) The sequence involves isotopic electrophilic exchange and free radical substitution

D) The sequence involves electrophilic substitution and free additions

29. The best sequence of reactions for preparation of the following compound from benzene is



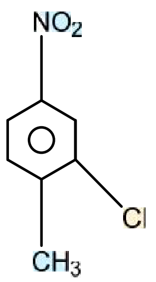
A) (i) $CH_3COCl / Anh\ AlCl_3$ (ii) Oleum (iii) $(CH_3)_2CH-Cl(1mole)/AnhAlCl_3$

B) (i) $(CH_3)_2CH-Cl(1mole)/AnhAlCl_3$ (ii) $CH_3COCl / Anh\ AlCl_3$ (iii) Oleum

C) (i)Oleum(ii) $CH_3COCl / Anh\ AlCl_3$ (iii) $(CH_3)_2CH-Cl(1mole)/AnhAlCl_3$

D) (i) $(CH_3)_2CH-Cl(1mole)/AnhAlCl_3$ (ii) Oleum(iii) $CH_3COCl / Anh\ AlCl_3$

30. Which of the following is/are correct IUPAC name of given compound



A) 2-chloro-1-methyl-4-nitrobenzene

B) 4-methyl-5-chloronitrobenzene

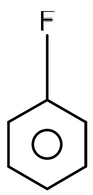
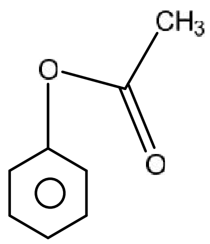
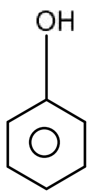
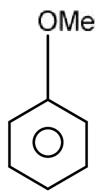
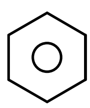
C) 2-chloro-4-nitrotoluene

D) 2-methyl-5-nitrochlorobenzene

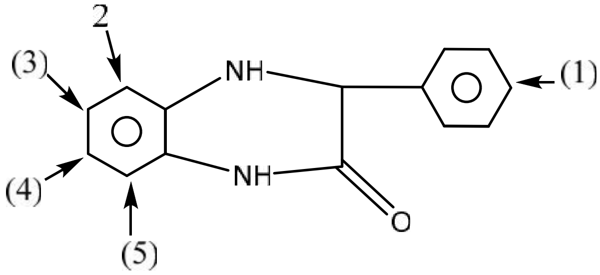
SECTION-II
(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

31. How many of the following compounds are more reactive than chlorobenzene towards nitration ($HNO_3 + H_2SO_4$)

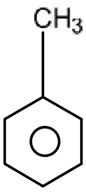
i)  ii)  iii)  iv)  v) 

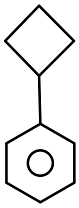
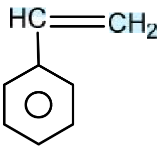
32.

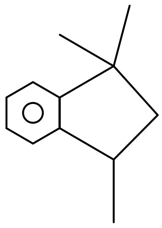


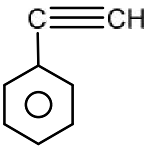
The mono nitration ($HNO_3 + H_2SO_4$) will mainly take place at which position

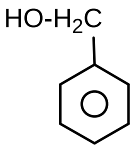
33. Examine the structural formulas shown below and find out how many following compounds give aromatic carboxylic acid with acidic $KMnO_4$

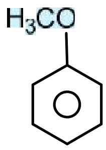
I) 

, II)  , III) 

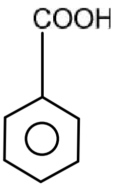
, IV)  ,

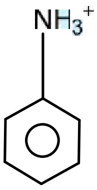
V) 

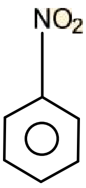
, VI) 

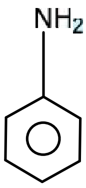
, VII) 

34. How many of the following aromatic compounds cannot give friedal cfraft's reaction (in presence of Anhydrous $AlCl_3$)

I) 

, II) 

, III) 

, IV) 

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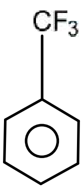
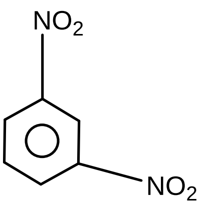
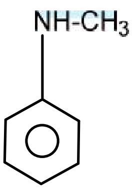
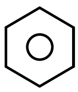
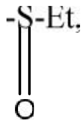
V) 

, VI) 

, VII) 

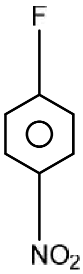
35.  + Cl-CH_2-CH_2-CH_2-CH_2-Cl $\xrightarrow{\text{Anh } AlCl_3}$

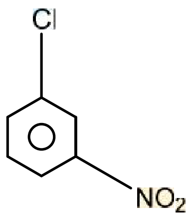
In the above reaction, how many number of inter molecular friedal craft's alkylation products are possible? (excluding stereoisomers)

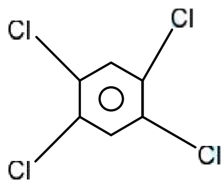
36. Identify the number of O.P directing groups among the following (towards electrophilic aromatic substitution -F, $-CH=CH_2$-COOH,-Br,-NO 

37. A Hydro carbon (x) contains 91.3% carbon and 8.7% hydrogen. The compound on chlorination using $Cl_2 / h\nu$ and $Cl_2 / Anh\ AlCl_3$ separately gives four isomeric monochloro substituted product. Calculate $\frac{total\ number\ of\ atoms\ present\ in\ (x)}{5} = ?$

38. How many of the following molecules can give activated aromatic nucleophilic addition elimination reactions

I) 

, II) 

, III) 

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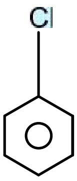
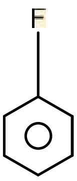
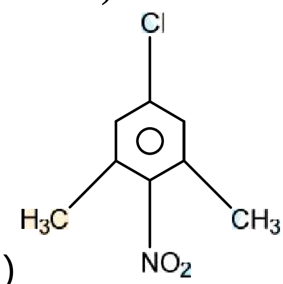
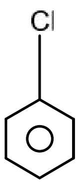
IV) 

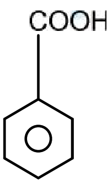
, V) 

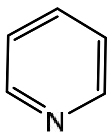
, VI) 

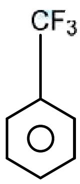
39. The number of isomeric benzene ring substituted compounds of C_9H_{12} is

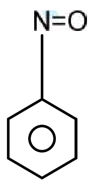
40. How many compounds are more reactive than benzene towards electrophilic substitution



, 

, 

, 

, 

MATHEMATICS **Max Marks:60**

SECTION-I

(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A) (B)(C) and (D) out of which ONE or MORE are correct.

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41. If $\int \frac{dx}{\sin x + \sec x} = \frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3}+s}{\sqrt{3}-s}\right| + Tan^{-1}(t) + c$ then

A) $s = \sin x + \cos x$

B) $s = \sin x - \cos x$

C) $t = \sin x + \cos x$

D) $t = \sin x - \cos x$

42. If $\int \frac{3\cot 3x - \cot x}{\tan x - 3\tan 3x} dx = pf(x) + qg(x) + c$ where ‘c’ is a constant of integration, then

A) $p = 1;\ q = \frac{1}{\sqrt{3}};\ f(x) = x,\ g(x) = \ln\left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$

B) $p = 1;\ q = \frac{2}{\sqrt{3}};\ f(x) = x,\ g(x) = \ln\left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$

C) $p = 1;\ q = -\frac{2}{\sqrt{3}};\ f(x) = x,\ g(x) = \ln\left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

D) $p = 1;\ q = -\frac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ln\left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

43. Let $f(x) = \int x^2 \cos^2 x \left(2x + 6\tan x - 2x\tan^2 x\right) dx$ and $f(x)$ passes through the point $(\text{®}, 0)$

A) If $f : R - (2n+1)\frac{\text{®}}{2} \to R$ then $f(x)$ be an even function:

B) If $f : R - (2n+1)\frac{\text{®}}{2} \to R$ then $f(x)$ be an odd function

C) The number of solution(s) of the equation $f(x) = x^3$ in $[0, 2\text{®}]$ is 3

D) The number of solution(s) of the equation $f(x) = x^3$ in $[0, 2\text{®}]$ is 2

44. If $\int \cot\left(2\tan^{-1}\sqrt{\frac{\sqrt{1+\sqrt{x}}-x^{1/4}}{\sqrt{1+\sqrt{x}}+x^{1/4}}}\right)dx = \frac{qx^{p/q}}{p}+C;$ (where p and q are relatively prime and C is constant of integration), then which of the following option(s) is/are correct?

A) $p=5$ B) $p+2q=13$ C) $p-q=1$ D) $q=5$

45. $\int \frac{(x^2+20)}{(x\sin x+5\cos x)^2}dx = \frac{-xf(\text{x})}{x\sin x+5\cos x}+g(x)+c$

A) f(x)=sec(x) B) g(x)=tan(x) C) f(x)=tan(x) D) g(x)=cos(x)

46. $\int \frac{\sqrt{4+x^2}}{x^6}dx = \frac{A(4+x^2)^{3/2}(Bx^2-6)}{x^5}+C$

A) $A=\frac{1}{120}$ B) $B=1$ C) $A=\frac{-1}{120}$ D) $B=-1$

47. If $I=\int \frac{(1-y)\,dx}{lnx^x+xy^{-1}},\ J=\int \frac{\ln x^x+xy^{-1}}{1-y}dy,$ where $xy^{-1}=x^y$

A) $I=x+c_1$ B) $J=y+c_2$ C) $I=y+c_2$ D) $J=x+c_2$

48. $\int \frac{x^4+1}{x^6+1}dx = Tan^{-1}(g(\text{x}))-\frac{2}{3}\tan^{-1}(f(\text{x}))+C$

A) $g:R-\{0\}\to R$ is a one-one function& onto

B) $g:R-\{0\}\to R$ is a many one & onto

C) $f:R\to R$ is increasing function

D) $f:R\to R$ is one-one & onto function

49. $\int \frac{xdx}{\sqrt{x^2+1}\left(x+\sqrt{x^2+1}\right)^2} = \frac{-1}{a\left(x+\sqrt{x^2+1}\right)}+\frac{1}{b\left(x+\sqrt{x^2+1}\right)^d}+c$

A) a+d=5 B) b+d=9 C) a=3 D) d =2

50. Let $f(x)$ be a differentiable function so that $f^1(x)$ is continuous function and

$\int \frac{\left(f(x)-f^1(x)\right)e^x}{\left(e^x+f(x)\right)^2}dx = g(x)+c$, where c is integration constant, then which of the

Following is/are true?

A) $g(x)=\frac{1}{1+e^x}, if\ f(x)=e^{2x}$

B) $g(®)=g(2®), if\ f(x)=\sin x$

C) $g^1(x)=0$ has at least one solution in $\left(\frac{®}{2},\frac{3®}{2}\right), if\ f(x)=\cos x$

D) $g(x)$ is a bounded function when $f(x)=e^x$

SECTION-II

(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

---

51. If $\int \frac{3x^2+2x}{x^6+2x^5+x^4+2x^3+2x^2+5}dx = F(x),$ then the value of [F (1) – F (0)],

where [.] represents greatest integer function.

52. $\int \frac{\cos 9x+\cos 6x}{2\cos 5x-1}dx = A\sin 4x + B\sin x + c$ then the value of 4(A+B) is

53. If $\int\left(x^{2010}+x^{804}+x^{402}\right)\left(2x^{1608}+5x^{402}+10\right)^{\frac{1}{402}}dx = \frac{1}{10a}\left(2x^{2010}+5x^{804}+10x^{402}\right)^{\frac{a}{402}}$, Then $(a-400)$

is equal to ….

54. $\int \frac{\sec x\left(2+\sec x\right)}{\left(1+2\sec x\right)^2}dx = \frac{\sin x}{p+\cos x}+C,$ then $P=$

55. For $x\in\left(o,{}^{®}\!/_{2}\right)$, If $\int \frac{\tan x\sqrt{\sec x}\left(1+\cos^2 x\right)}{\sqrt{\cos x+\sin^2 x}}dx = 2\sqrt{f}+c$ Then $\left[f\left({}^{®}\!/_{4}\right)\right]=$

(where [.] denotes G.I.F)

56. If $\int \frac{(x-1)\,dx}{\left(x+x\sqrt{x}+\sqrt{x}\right)\left(\sqrt{\sqrt{x}(x+1)}\right)} = 4\tan^{-1}\left[g(x)\right]+c$, where C is an arbitrary constant of Integration. Find $g^2(1)$

57. If $\int \frac{\cos ec^2 x - 2020}{\cos^{2020} x}\,dx = \frac{Af(x)^B}{\left(g(x)\right)^{2020}} + c$; where $f(®) = \sqrt{3}$, then value of $A^2 + B^2 + f\left(\frac{®}{4}\right)$ is equal to

58. $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}\,dx = \frac{Ax^B}{\left(1+x^5+x^3\right)^C} + D$ then $A \cdot B + C =$

59. If $\int \left(x^6+7x^5+6x^4+5x^3+4x^2+3x+1\right)e^x dx$ is equal to $\sum_{k=1}^{\sim} x^k.e^x + C$ (where C is constant of integration) then $(\sim +\ \ )$ is

60. $\int \frac{\sin 7x}{\sin x}\,dx = Ax + B\sin 2x + C\sin 4x + D\sin 6x + E$ then $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D} =$

PTA-4

KEY SHEET

PHYSICS

1	**ABD**	2	**AD**	3	**BCD**	4	**ABCD**	5	**ABC**
6	**AB**	7	**ABCD**	8	**BC**	9	**AB**	10	**BC**
11	**3**	12	**2**	13	**4**	14	**6**	15	**9**
16	**7**	17	**4**	18	**3**	19	**1**	20	**4**

CHEMISTRY

21	**ABC**	22	**B**	23	**BC**	24	**AB**	25	**ABCD**
26	**ABCD**	27	**ABC**	28	**AB**	29	**B**	30	**AC**
31	**5**	32	**3**	33	**6**	34	**7**	35	**3**
36	**5**	37	**3**	38	**2**	39	**8**	40	**0**

MATHS

41	**BC**	42	**AD**	43	**AC**	44	**ABC**	45	**AB**
46	**AB**	47	**CD**	48	**BCD**	49	**AB**	50	**ABCD**
51	**0**	52	**5**	53	**3**	54	**2**	55	**1**
56	**2**	57	**1**	58	**7**	59	**7**	60	**7**

SOLUTIONS
PHYSICS

1. Inner world is electrically shielded from outer world

2. $$\frac{Kq_A}{R}+\frac{Kq_B}{2R}=2V \quad \ldots(i)$$

and $$\frac{Kq_A}{2R}+\frac{Kq_B}{2R}=\frac{3}{2}V \quad \ldots(ii)$$

Solving eqn. (i) and (ii):

$$\frac{q_A}{q_B}=\frac{1}{2}$$

After B is earthed.

$V_B=0$

$\therefore \quad q'_B=-q'_A=-q_A$

(charge on A remains same)

Also after earthing

$$V_A-V_B=Kq_A\left(\frac{1}{R}-\frac{1}{2R}\right)=\frac{Kq_A}{2R}$$

Substituting $q_B=2q_A$ in equation (i)

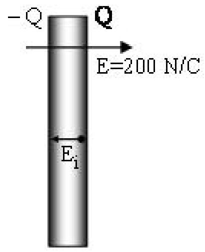
$$\frac{Kq_A}{2R}=\frac{V}{2}$$

$\Rightarrow \quad V_A-V_B=\frac{V}{2}$

$\because \quad V_B=0$

So, $V_A=\frac{V}{2}$

3. As in electrostatics the electric field inside the bulk of the material of the conductor is zero. So some charge gets induced on two faces of the plate to make electric field zero inside the conductor. Let Q and $-Q$ appears on right and left face as shown, then induced electric field,



$$E_i=\frac{Q}{2\varepsilon_0 A}+\frac{Q}{2\varepsilon_0 A}$$

$$=\frac{Q}{\varepsilon_0 A} \text{ (towards left)}$$

For $\vec{E}$ inside plate to be zero $E_i=E$

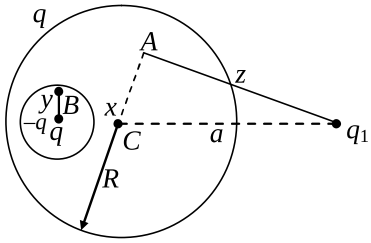
$\Rightarrow Q=\varepsilon_0 A\times 200=1.77\times 10^{-9}C$

The electric field is discontinuous at surface but potential is continuous

4. The charge distribution is shown in figure.

Due to q_1, the charge induction is non – uniform but net induced charge due to q_1 is zero.

At point A field is zero and potential is equal to the potential at center.



$$V_A = V_C = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a}$$

At point B, field is due to charge q only

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}$$

At point B, potential is due to charge q at center of cavity, $-q$ induced on inner surface of cavity in addition to the potential at the center.

$$\therefore \qquad V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a} + \frac{q}{R} + \frac{q}{y} - \frac{q}{r}\right].$$

5. $$100 = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(R+0.05)}$$

$$75 = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(R+0.1)}$$

Solving these equations, we get

$$q = \frac{5}{3}\times 10^{-9}C \text{ and } R = 0.1m$$

$$(a)\ V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{R}$$

$$= \frac{(9\times 10^9)\left(\frac{5}{3}\times 10^{-9}\right)}{0.1} = 150V$$

$$(c)\ E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{R^2} = \frac{V}{R} = \frac{150}{0.1} = 1500V/m$$

$$(d)\ V_{centre} = 1.5V_{surface}$$

6. We know that electric potential due to entire disc is

$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+Z^2} - Z\right)$$

$\Rightarrow V$ due to quarter disc is

$$V = \frac{\sigma}{8\epsilon_0}\left(\sqrt{R^2+Z^2} - Z\right)$$

$\Rightarrow$ Electric field (Z component is)

$$E = -\frac{\partial V}{\partial Z}$$

$$E = -\frac{\sigma}{8\varepsilon_0}\left(\frac{1}{2\sqrt{R^2+Z^2}}(2Z)-1\right)$$

$$E = -\frac{\sigma}{8\varepsilon_0}\left(1-\frac{Z}{\sqrt{R^2+Z^2}}\right)$$

7. $$\frac{4}{3}\pi R^3 = 4\pi a^2 t$$

$$\Rightarrow R^3 = 3a^2 t$$

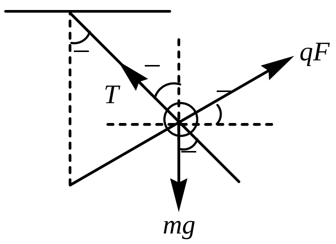
Charge on drop is same as that on bubble.

Potential on bubble $V = \dfrac{Q}{4\pi\varepsilon_0 a}$

Potential on drop $= \dfrac{Q}{4\pi\varepsilon_0 R} = \dfrac{Va}{R} = \dfrac{Va}{\left(3a^2t\right)^{1/3}} = V\left(\dfrac{a}{3t}\right)^{1/3}$

E on drop $= \dfrac{Q}{4\pi\varepsilon_0 R^2} = \dfrac{Va}{R^2} = \dfrac{Va}{\left(3a^2t\right)^{2/3}} = \dfrac{V}{\left(9at^2\right)^{1/3}}$

8.



$$T\cos\theta + qE\sin\theta = mg$$

$$T\sin\theta = qE\cos\theta$$

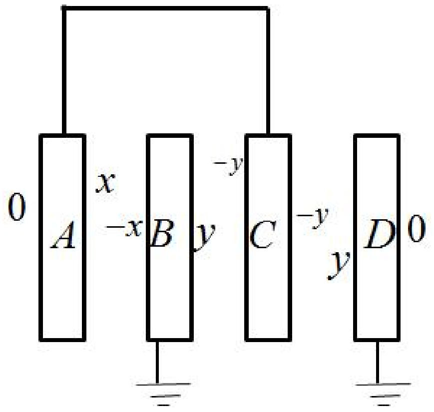
$$E\cos\theta = A, E\sin\theta = B$$

$$\frac{qA\cos\theta}{\sin\theta} + qB = mg$$

$$q = \frac{mg}{A\cot\theta + B}$$

$$T = \frac{mgA}{A\cos\theta + B\sin\theta}$$

9. When a all switches are closed



Since A & c are at same potential

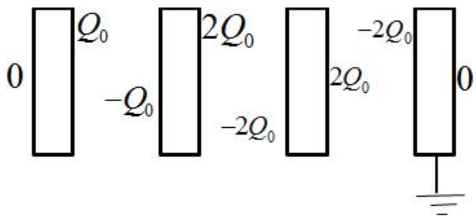
$y = -x$

$x - y - y = Q_0$ [charge concentration between A&C]

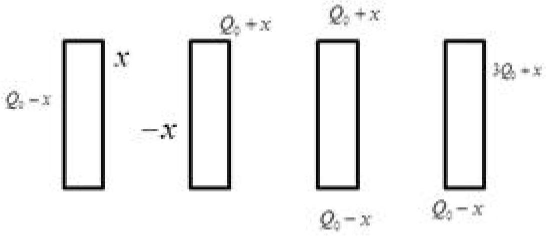
$3x = Q_0$

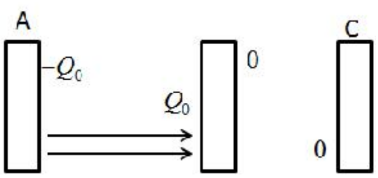
$x = \dfrac{Q_0}{3}$

$2x = \dfrac{2Q_0}{3}$



$$\Delta Q = 4Q_0$$



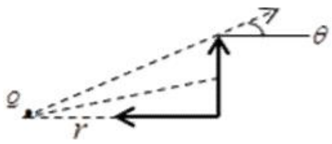


$Q_0 - x = 3Q_0 + x$

$-2x = 2Q_0$

$x = -Q_0$

10.



$$E^{-1} = \frac{-kp^{-1}}{r^3}$$

$$F = \frac{-kpQ^{-1}}{r^3}$$

Force on charge is downward so force on dip upwards

11. To reach the other end, the ball have to cross the max. potential energy barrier formed at the centre of the ring (middle of the rod) applying energy conservation between left end and middle.

$$\frac{1}{2}mu^2 + \left(\frac{K(3q)}{(5R)}\right)q = \left(\frac{K(3q)}{(4R)}\right)q + \frac{1}{2}mv^2$$

To cross the barrier $v > 0$

$$\frac{1}{2}mu^2 + \frac{K3q^2}{5R} - \frac{K3q^2}{4R} > 0 \quad \Rightarrow \quad u > \sqrt{\frac{3q^2}{40\pi\varepsilon_0 Rm}}$$

12. Force on a charge $-q$ in an electric field, $\vec{F} = -q\vec{E}$. This force acts in a direction opposite to $\vec{E}$. Therefore, the particle, initially placed at rest, will move opposite to $\vec{E}$ under the action of force. Obviously, direction of $\vec{v}$ will be opposite to $\vec{E}$.

Now $\vec{v} = 10\hat{i} - 10\hat{j}\ m/s$ (given)

Unit vector in the direction of $\vec{v}$. $\hat{v} = \frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$

$$\vec{E} = 10\left(\frac{-\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$$

$\vec{A} = A\hat{j}$ $\qquad \phi = \vec{E}.\vec{A}$

13. E at $AB = \frac{a}{\ell}(\ell + \ell) = 2a \quad \therefore$ on $AB = 2a\lambda\ell$

$$E \text{ at } CD = \frac{a}{\ell}(2\ell + \ell) = 3a \ \therefore\ F \text{ on } CD = 3a\lambda\ell$$

on BC & AD electric field is nonuniform x is not constant. But on BC & AD electric field will have the same type of variation.

$$\therefore F_{AD} = F_{BC} = \int_{x=\ell}^{2\ell} (\lambda dx).\frac{a}{\ell}(x+\ell)$$

$$= \frac{a\lambda}{\ell}\left[\frac{x^2}{2} + \ell x\right]_{\ell}^{2\ell} = \frac{a\lambda}{\ell}\left[\frac{3\ell^2}{2} + \ell^2\right] = \frac{5}{2}a\lambda\ell$$

$\therefore$ total force on the loop $= 2a\lambda\ell + 3a\lambda\ell + 2\left(\frac{5}{2}a\lambda\ell\right)$

$F = 10a\lambda\ell$

Using values $F = 4x10^6 N$

14. $\vec{E} = 2axy\hat{j} + a\left(x^2 - y^2\right)\hat{j}$

$$Ex = \frac{-dv}{dx}$$

$$\int 2axy\,dx = \int -dv$$

$$-ax^2 y + f(x, y) = V$$

$$\frac{dv}{dy} = -ax^2 + f^1(x, y) = -E_y$$

$$-ax^2 + f^{-1}(x, y) = -a\left(x^2 - y^2\right)$$

$$f^1(x, y)\; ay^2$$

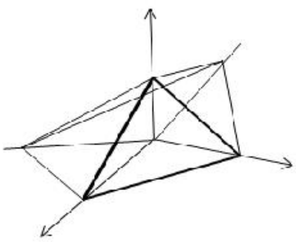
$$f(x, y) = \frac{ay^3}{3} + v_0$$

$$V = -ax^2 y + \frac{ay^3}{3} + V_0$$

$n = 1;\quad m = 2\quad p = 3$

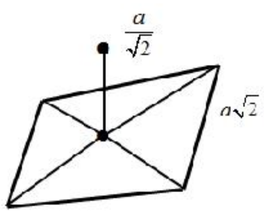
$n + m + p = 6$

15.



Charge centre be enclosed in a pyramid with a square here of $a\sqrt{2}$ side

Flux centre divided in 4 equal parts $a5\ \frac{'q'}{4£_0}$



Through square base $\frac{'q'}{6£_0}$ will base & through $\frac{1^{th}}{4}$ base $\frac{q}{24£_0}$ will bass flux through triangular part will be $\frac{q}{4£_0} - \frac{q}{24£_0}$

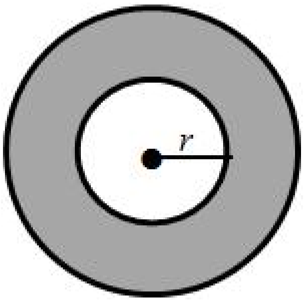
$$Q = \frac{sq}{24£_0}$$

$$n = 5 \quad m = 4$$

$$n + m = 9$$

16. Potential at the edge of disc $= \frac{\pm R}{®£_0}$

At an instant let ‘r’ be radius

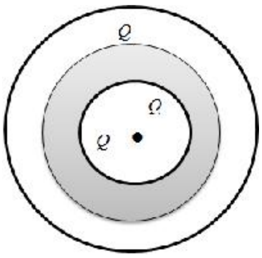


$$dw = \frac{\pm r}{®£_0} 2® r dr \pm$$

$$w = \int_R^{2R} dw \qquad \Rightarrow \frac{2\pm^2}{£_0} \int_R^{2R} r^2 dr$$

$$\Rightarrow \frac{2}{3}\frac{\pm^2}{£_0}\left[8R^3 - R^3\right] = \frac{14}{3}\frac{\pm^2 R^3}{£_0}$$

17.



$$U = \frac{-kQ^2}{a} + \frac{kQ^2}{2a} - \frac{kQ^2}{2a} + \frac{kQ^2 2}{2(2a)} + \frac{kQ^2}{2a}$$

$$U = \frac{-ka^2}{4a}$$

$$n = 4$$

18. Field due to a dipole is

$$\vec{E} = \frac{1}{4\text{®£}_0 r^3}\left[\left(3\vec{p}.\hat{\text{r}}\right)\hat{\text{r}} - \vec{p}\right]$$

19. The electric field at point $x = 25cm$ is

$$E_x = -\frac{dv}{dx} \text{ at } x = 25$$

$$= -\left(\frac{-2\times10^4}{10\times10^{-2}}\right) = 2\times10^5 \ N/C$$

$$a_x = \frac{qE_x}{m} = \frac{20\times10^{-6}\times2\times10^5}{200} = \frac{1}{50} m/s^2 = 2\times10^{-2} m/s^2$$

The gravitational field at point $y = 35 \ cm$ is

$$E_g = -\frac{dv}{dy} = -\left(\frac{+10^{-3}}{10\times10^{-2}}\right) = -10^{-2} \ J/kgm$$

$$a_y = E_g = -10^{-2} \ m/s^2$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(2\hat{i} - \hat{j}\right)\times10^{-2} \ m/s^2$$

20. Applying conservation of energy, we get

$$\frac{kQq}{r} = \frac{kQq}{2r} + \frac{1}{2}\text{mv}^2$$

$$\frac{1}{2}\text{mv}^2 = \frac{kQq}{2r} \ ; \therefore \ \text{v} = \sqrt{\frac{kQq}{mr}}$$

Force required to keep Q fixed = Coulombic repulsion between the charges

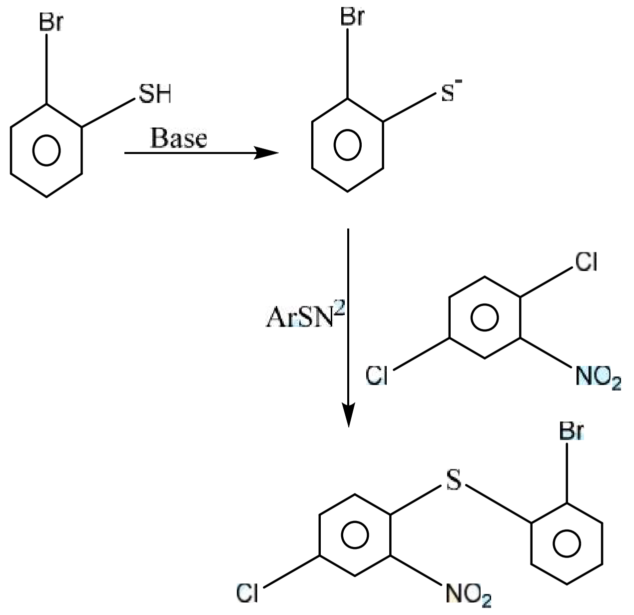
= force on charge q

Impulse = change in momentum of q.= mv = $\sqrt{\frac{mkQq}{r}} = \sqrt{\frac{Qqm}{4\text{®} \in_0 r}}$

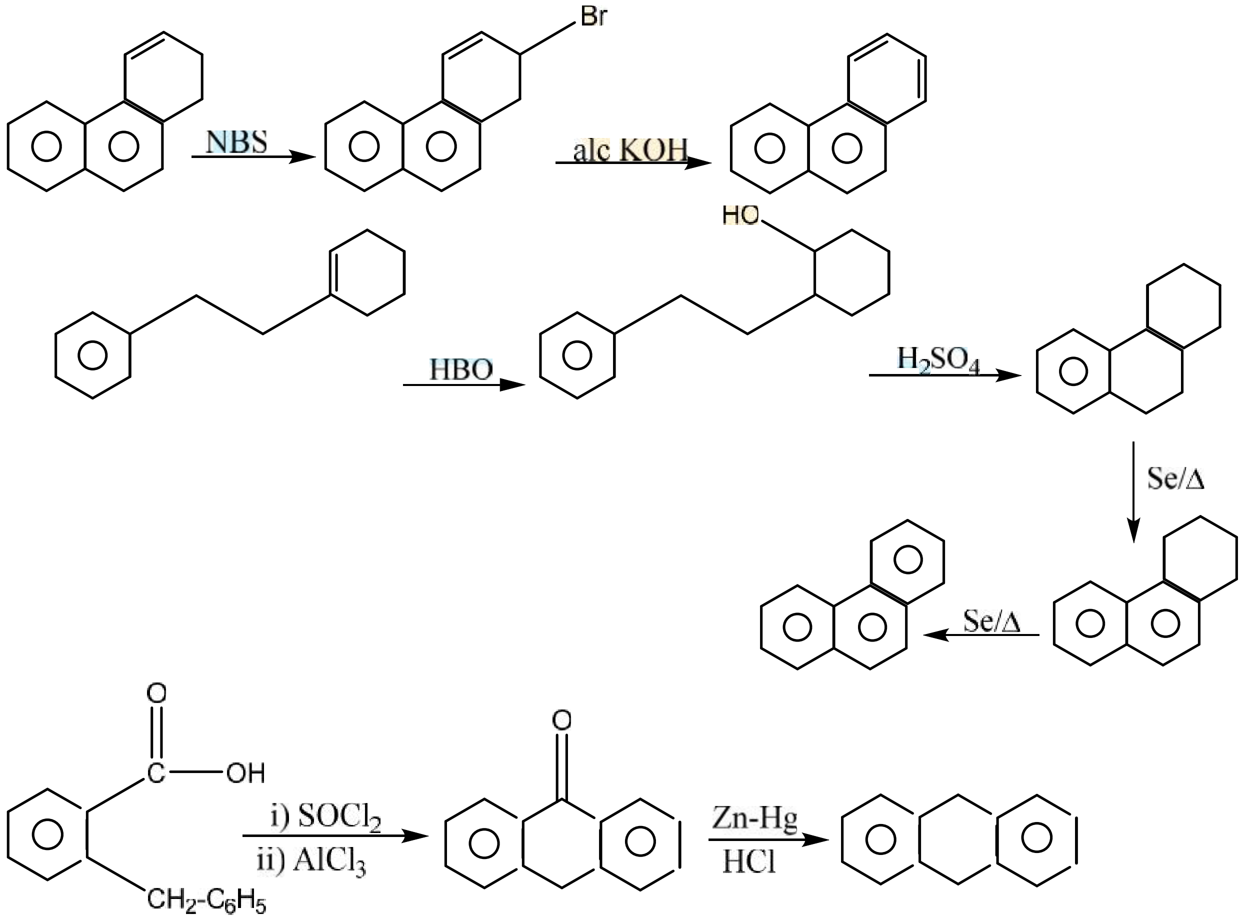
CHEMISTRY

21. Conceptual

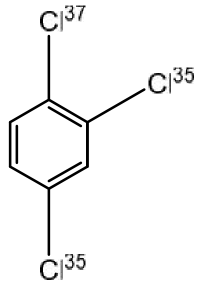
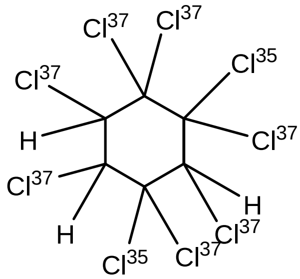
22.



23.



24. Conceptual
25. Conceptual
26. Conceptual
27. Conceptual
28.

A is  , B is 

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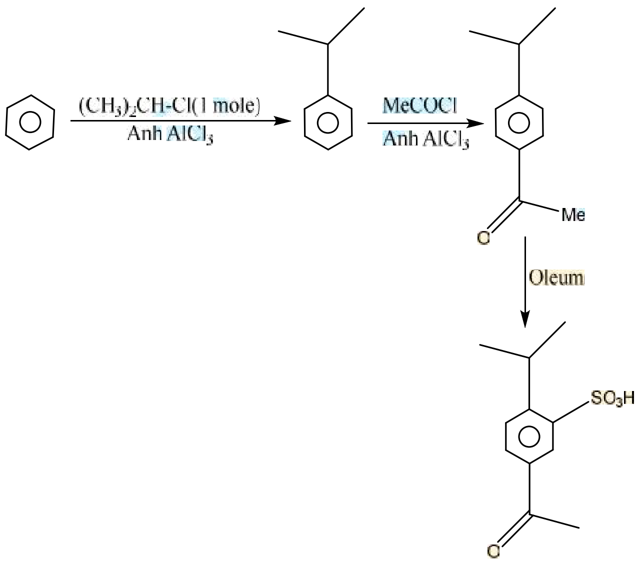
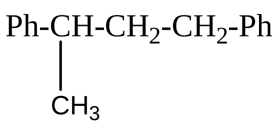
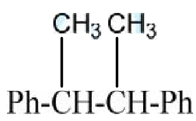


30. Conceptual
31. All are more reactive than chlorobenzene
32. Conceptual
33. I,IV,V,VII,VIII,IX are oxidized by $KMnO_4$
34. Conceptual
35. Due to inter molecular F.C reaction

1) Ph-CH_2-CH_2-CH_2-CH_2-Ph 2)  3) 

36. Conceptual
37. Conceptual
38. I,III are involved in activated $ArSN^2$
39. Conceptual
40. Conceptual

MATHS

41. $I=\int \frac{\cos x}{1+(\sin x-\cos x)}dx$

$=\int \frac{(\cos x+\sin x)}{3-(\sin x-\cos x)}dx+\int \frac{(\cos x-\sin x)}{1+(\sin x-\cos x)}dx$

$=\int \frac{ds}{3-s^2}+\int \frac{dt}{1+t^2}=\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+s}{\sqrt{3}-s}\right|+\tan^{-1}(t)+c$

Where $s=\sin x-\cos x,\ t=\sin x+\cos x$

42. Simplifying gives $I=\int \frac{(1-3\tan^2 x)\sec^2 x}{(3-\tan^2 x)(1+\tan^2 x)}dx$

Put $\tan x=t \Rightarrow I=\int \frac{1-3t^2}{(3-t^2)(1+t^2)}dt$

$=\int\left(\frac{1}{1+t^2}-\frac{2}{3-t^2}\right)dt$

43. $f(x)=\int(2x^3\cos^2 x+6x^2\sin x\cos x-2x^3\sin^2 x)dx$

$f(x)=x^3\sin 2x+c$

$f(®)=0+c=0\Rightarrow c=0$

$f(x)=x^3\sin 2x$

44. $I=\int \cot\left(2Tan^{-1}\sqrt{\frac{\sqrt{1+\sqrt{x}}-x^{\frac{1}{4}}}{\sqrt{1+\sqrt{x}}+x^{\frac{1}{4}}}}\right)dx$, Put $x=Tan^{4}—$

$=\int \cot\left(2Tan^{-1}\sqrt{\frac{\sec— -Tan—}{\sec— +Tan—}}\right)4Tan^{3}— \sec^{2}— d—$

$=\int \cot\left(2\left(\frac{®}{4}-\frac{—}{2}\right)\right)4Tan^{3}— \sec^{2}— d—$

$=\int 4Tan^{4}— \sec^{2}— d— =\frac{4.Tan^{5}—}{5}+c$

$=\left(\frac{4}{5}\right)x^{\frac{5}{4}}+c=\frac{q}{p}x^{\frac{p}{q}+c}$

P=5, q=4

45. $\int \frac{x^2+20}{(x^2+25)\cos^2(x-—)}dx$

Put $y=x-—$ we got

$=\int \sec^2 y.dy \quad =\tan y+c$

46. $$\int \frac{\sqrt{4+x^2}}{x^6}dx = \int \frac{1}{x^5}\sqrt{1+\frac{4}{x^2}}dx$$

$$1+\frac{4}{x^2}=u^2 \quad \to \frac{1}{x^2}=\frac{u^2-1}{4}$$

$$\Rightarrow \frac{-8}{x^3}dx = du.24$$

$$\Rightarrow \frac{dx}{x^3}=\frac{-24du}{8}$$

$$= \int \frac{u^2-1}{4}.\frac{2udx}{8}\times\sqrt{u^2}$$

$$= \int \frac{u^2-u^4}{16}du$$

$$= \frac{1}{16}\left[\frac{u^3}{3}-\frac{u^5}{5}\right]+c$$

$$= \frac{\left(1+\frac{4}{x^2}\right)^{\frac{3}{2}}\left[5-3\left(1+\frac{4}{x^2}\right)\right]}{16\times15}$$

$$= \frac{\frac{\left(4+x^2\right)^{\frac{3}{2}}}{x^3}\times\left(\frac{2x^2-12}{x^2}\right)}{16\times15} = \frac{\left(4+x^2\right)^{\frac{3}{2}}2\left(x^2-6\right)}{16\times15x^5}$$

47. $$\frac{x}{y}=x^y \Rightarrow lnx - lny = ylnx$$

$$\Rightarrow (1-y)dx = \left(lnx^x+\frac{x}{y}\right)dy$$

$$\therefore I = \int dy;\ \ J=\int dx$$

48.

$$I = \int \frac{\left(x^2+1\right)^2-2x^2}{x^6+1}dx$$

$$= \int \frac{\left(x^2+1\right)^2}{x^6+1} - 2\int \frac{x^2}{\left(x^3\right)^2+1}dx$$

$$= \int \frac{\left(x^2+1\right)^2 dx}{\left(x^2+1\right)\left(x^4-x^2+1\right)} - \frac{2}{3}\int \frac{3x^2}{\left(x^3\right)^2+1}dx \qquad \left[x^6+1=\left(x^2\right)^3+1=\left(x^2+1\right)\left(x^4-x^2+1\right)\right]$$

$$= \int \frac{1+\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}dx - \frac{2}{3}\tan^{-1}\left(x^3\right)+C$$

$$= \int \frac{d\left(x+\frac{1}{x}\right)}{\left(x-\frac{1}{x}\right)^2+1} - \frac{2}{3}\tan^{-1}\left(x^3\right)+C$$

$$= Tan^{-1}\left(x-\frac{1}{x}\right) - \frac{2}{3}\tan^{-1}\left(x^3\right)+C$$

49. $$\int \frac{xdx}{\sqrt{x^2+1}\left(x+\sqrt{x^2+1}\right)^2}$$

Put $x+\sqrt{x^2+1}=t$ ……. (1)

$$\Rightarrow \left(1+\frac{1.2x}{2\sqrt{x^2+1}}\right)dx = dt$$

$$\Rightarrow \frac{\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}}dx = dt$$

$$\Rightarrow \frac{dx}{\sqrt{x^2+1}} = \frac{dt}{t}$$

$$\frac{1}{t} = \frac{1}{\sqrt{x^2+1}+x}$$

$$\Rightarrow \frac{1}{t} = \frac{\sqrt{x^2+1}-x}{\left(x^2+1\right)-x^2}$$

$$\Rightarrow \frac{1}{t} = \sqrt{x^2+1}-x$$

Take, $t-\frac{1}{t}=2x$, $\Rightarrow x=\frac{1}{2}\left(t-\frac{1}{t}\right)$

$$\text{G.I} = \int \frac{\frac{1}{2}\left(t-\frac{1}{t}\right)\times\frac{dt}{t}}{t^2}$$

$$= \frac{1}{2}\int \frac{\left(t^2-1\right)}{t^4}dt = \frac{1}{2}\int\frac{dt}{t^2} - \frac{1}{2}\int\frac{dt}{t^4}$$

$$= -\frac{1}{2t} - \frac{1}{2}\left[\frac{-1}{3t^3}\right]+c = -\frac{1}{2t}+\frac{1}{6t^3}+c$$

$$= \frac{-1}{2\left(x+\sqrt{x^2+1}\right)} + \frac{1}{6\left(x+\sqrt{x^2+1}\right)^3}+c$$

50. $I = \int \frac{\left(f(x) - f^{1}(x)\right)e^{x}}{\left(e^{x} + f(x)\right)^{2}} dx = \int \frac{e^{-x}\left(f(x) - f^{1}(x)\right)}{\left(1 + e^{-x} f(x)\right)^{2}} dx$

Put $\frac{1}{1 + e^{-x} f(x)} = t$

Then $I = \frac{1}{1 + e^{-x} f(x)} + c$

$\therefore g(x) = \frac{1}{1 + e^{-x} f(x)}$

(1) $g(x) = \frac{1}{1 + e^{x}}$ $if\ f(x) = e^{2x}$

(2) $g(®) = g(2®)$ $if\ f(x) = \sin x$ as $g(x) = \frac{1}{1 + e^{-x}\sin x}$

(3) $g(x) = \frac{1}{1 + e^{-x}\cos x}$

$\therefore g\left(\frac{®}{2}\right) = g\left(\frac{3®}{2}\right)$ by Rolles theorem $g^{1}(c) = 0$ for some c such that $\frac{®}{1} < c < \frac{3®}{2}$

(4) $g(x)$ is bounded when $f(x) = e^{x}$ since $g(x) = \frac{1}{2}$

51. $\int \frac{3x^{2} + 2x}{\left(x^{3} + x^{2}\right)^{2} + 2\left(x^{3} + x^{2}\right) + 1 + 4} dx$

$= \int \frac{3x^{2} + 2x}{\left(x^{3} + x^{2} + 1\right)^{2} + 4} dx$

52. $\int \frac{2\cos\frac{15x}{2}\cos\frac{3x}{2}}{2\left(2\cos^{2}\frac{5x}{2} - 1\right) - 1} dx$

$= \int \frac{2\cos\frac{15x}{2}\cos\frac{3x}{2}}{4\cos^{2}\frac{5x}{2} - 3} dx$

$= \int \frac{2\cos\frac{15x}{2}\cos\frac{3x}{2}\cos\frac{5x}{2}}{4\cos^{3}\frac{5x}{2} - 3\cos\frac{5x}{2}} dx$

$= \int 2\cos\frac{3x}{2}\cos\frac{5x}{2} dx$

53.

$$= \int \left(x^{2010} + x^{804} + x^{402}\right)\left(2x^{1608} + 5x^{402} + 10\right)^{\frac{1}{402}} dx$$

$$= \int x\left(x^{2009} + x^{508} + x^{401}\right).\left(2x^{1608} + 5x^{402} + 10\right)^{\frac{1}{402}} dx$$

$$= \int \left(x^{2009} + x^{803} + x^{401}\right).\left(2x^{2010} + 5x^{804} + 10^{402}\right)^{\frac{1}{402}} dx$$

Put $2x^{2010} + 5x^{804} + 10^{402} = t$

$$\Rightarrow 4020\left(x^{2009} + x^{803} + x^{401}\right)dx = dt$$

$$\therefore I = \int \frac{1}{4020}.(t)^{\frac{1}{402}} dt = \frac{1}{4020}.\frac{t^{\frac{1}{420}+1}}{\frac{1}{402}+1}$$

$$= \frac{1}{4020}.\frac{(t)^{\frac{403}{402}}}{\frac{403}{402}} = \frac{1}{4030}\left(2x^{2010} + 5x^{804} + 10^{402}\right)^{\frac{403}{402}}$$

54. $\int \frac{2\cos x + 1}{(\cos x + 2)^2} dx$

$$= \int \frac{2\cot x \cos ecx + \cos ec^2 x}{(\cot x + 2\cos ecx)^2} dx$$

55. Put $\cos x = t$

56. Put $x = t^2$

57. A=-1, B=1, $f\left(\frac{®}{4}\right) = -1$

58. $\int \frac{\left(\frac{2x^{12}}{x^{15}} + \frac{5x^9}{x^{15}}\right)dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} = \int \frac{-\left(\frac{2}{x^3} + \frac{5}{x^6}\right)dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$

$$= -\left[\frac{-1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^2}\right] + c$$

$$= -\left[\frac{1.x^{10}}{2\left(x^5 + x^3 + 1\right)^2}\right] + c$$

$$A.B + C = \frac{1}{2}.10 + 2 = 7$$

59 $\Rightarrow \int \{\left(x^6+6x^5\right)+\left(x^5+5x^4\right)+\left(x^3+3x^2\right)+\left(x^2+2x\right)+\left(x+1\right)\}e^x dx$

$\Rightarrow \left(x^6+x^5+x^4+x^3+x^2+x\right)e^x+c$

$\therefore \int e^x\left(f\left(x\right)+f^1\left(x\right)\right)dx=e^x f\left(x\right)+c$

$=\sum_{k=1}^{6} x^k e^x+C$

$\Rightarrow \sim\ =6,\quad =1 \Rightarrow \left(\sim +\quad\right)=7$

60. $I_n=\int \frac{\sin 7x}{\sin x}dx$

$I_n-I_{n-2}=\int \frac{\sin x-\sin\left(n-2\right)x}{\sin x}dx$

$=2\int \cos\left(n-1\right)dx$

$I_n-I_{n-2}=2\frac{\sin\left(n-1\right)x}{n-1}+c$

$I_n=\frac{2\sin\left(n-1\right)x}{n-1}+I_{n-2}$

$I_7=\int \frac{\sin 7x}{\sin x}dx$

$=\frac{2\sin 6x}{6}+I_5$

$=\frac{2\sin 6x}{6}+\frac{2\sin 4x}{4}+I_3$

$=\frac{2\sin 6x}{6}+\frac{2}{4}\sin 4x+\frac{2\sin 2x}{2}+I_1$

$=\frac{\sin 6x}{3}+\frac{2}{4}\sin 4x+\sin 2x+x+c$