CHAPTER 03

Crashing of Network, Resource allocation and CPM Updating

3.1 Cost Model Analysis

- In CPM, time is related to cost and the object is to develop an optimum time-cost relationship.
- The overall project duration can be reduced by reducing the duration of only the critical activities in the project network. The durations of such activities may be reduced in two ways.
  (a) by deploying more resources for the early completion of such activities.
  (b) by relaxing the technical specifications for such activities.
- In whole of CPM Cost Model, we will be assuming that project duration is reduced by deploying more resources on critical activities.
- In CPM, there are two time and cost estimates for each activity: 'normal estimate' and crash estimate'. In the normal estimate, the emphasis is on cost with time being associated with minimum cost. The 'crash' estimate involves the absolute minimum time required for the job and the cost necessary to achieve it. Here the emphasis is on 'time'.

3.2 Project Cost

Total project cost is the sum of two separate costs:

(a) the direct cost for accomplishing the work, and

(b) the indirect cost related to the control or direction of that work, financial overhead, lost production, and the hike etc.

The components of the total cost are shown in figure below.

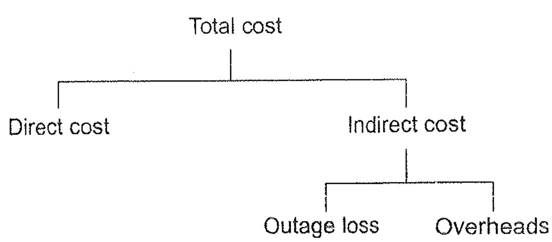


Fig.3.2 Various Types of Costs

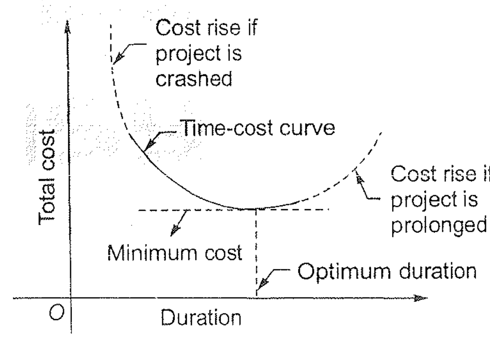


Fig. 3.1 Cost-Time Relationship of a project

3.3 Components of Project Cost

3.3.1 Indirect Project Cost

- Indirect costs on a project are those expenditures which cannot be apportioned or clearly allocated to the individual activities of a project, but are assessed as a whole. The indirect cost includes the expenditure related to administrative and establishment charges, overhead, supervision, expenditure on a central store organization, loss of revenue, lost profit, penalty etc.
- Indirect cost rises with increased duration, considering only overhead and supervision. It is represented by a straight line, with a slope equal to daily overhead.
- But when there is a loss in profits, due to inability to meet demand or due to some penalty due to delay, a corresponding cost increase must be added to the cost of overheads, producing the curve. Such a loss is called the outage loss.
- The total indirect cost curve will thus be curved.

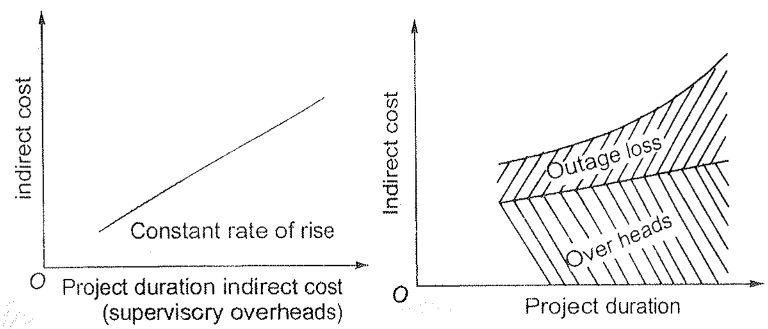
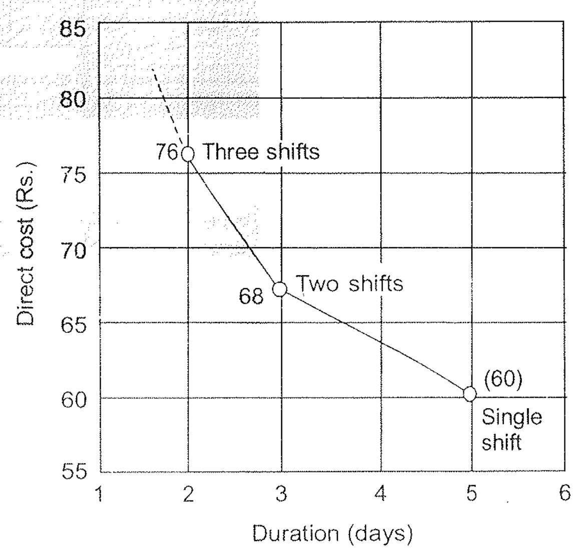


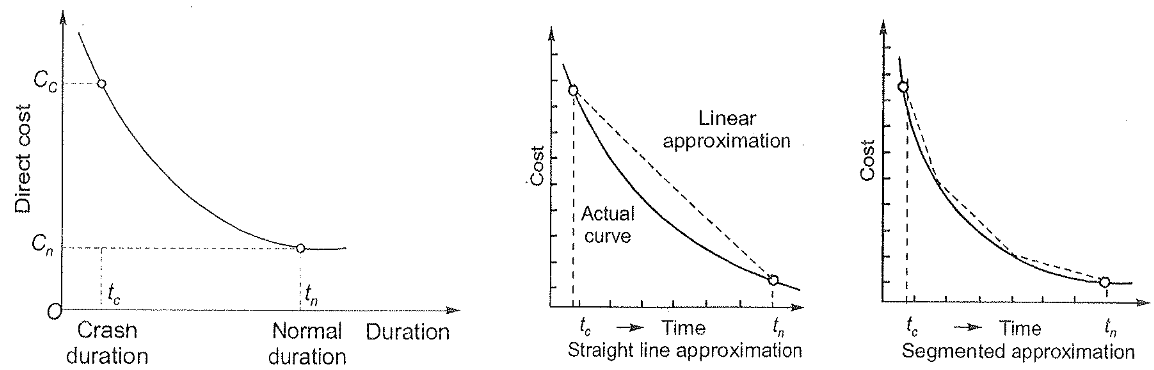
Fig. 3.3 Indirect Project Cost

3.3.2 Direct Project Cost

- It is the cost which is directly dependent on the amount of resources involved for completion of activities.
- It includes labour, materials, plants and machining etc.
- To get the same work done in less time, we have to increase amount of labour, equipment and time saving material that too at extra charges which simply means increase in direct cost.
- The project has the highest cost corresponding to the crash duration, and has normal cost corresponding to the normal duration.
- Normal time (t_n) : Normal time is the standard time that an estimator would usually allow for an activity.



- Crash time (t_c) : Crash time is the minimum possible time in which an activity can be completed, by employing extra resources. Crash time is that time, beyond which the activity cannot be shortened by any amount of increase in the resources.
- Normal cost (C_n) : This is direct cost required to complete the activity in normal time duration.
- Crash cost (C_c) : This is the direct cost corresponding to the completion of the activity within crash time.



Generalized direct cost-time curve *Direct cost curve approximation*

Fig. 3.4 Direct Project Cost

- The straight line or segmented approximation of the direct cost curve is helpful in carrying out the project cost analysis. In such analysis, the cost slope is used.

Cost Slope:

- The cost slope is the slope of the direct cost curve, approximated as straight line. It is defined as follows :

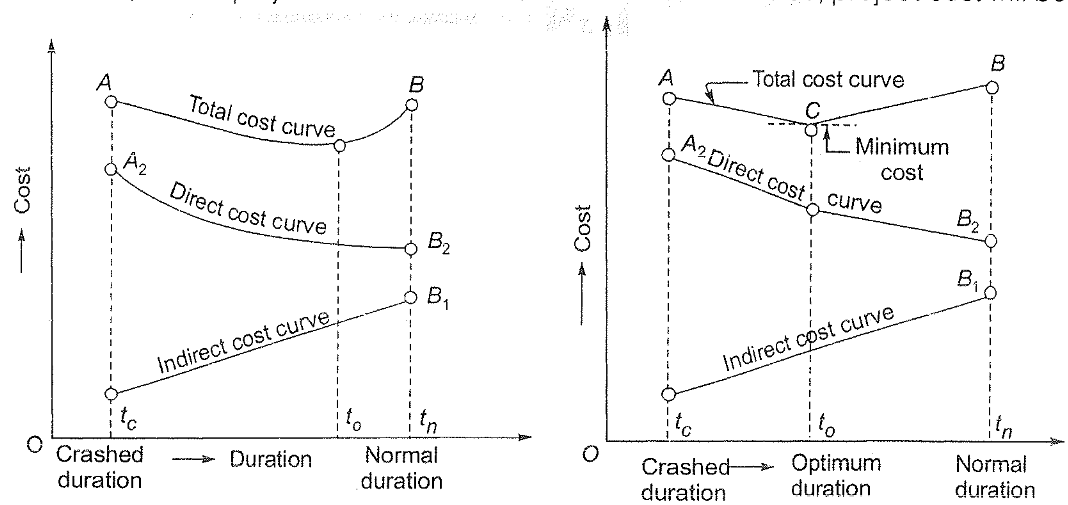
$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}} \quad \text{or} \quad C_S = \frac{C_c - C_n}{t_n - t_c} = \frac{\Delta c}{\Delta t}$$

Do you know? Crashing of critical activity is started in systematic manner i.e., starting with that activity which has the least cost slope.

- The segmented approximation of cost curve, having multiple cost slopes, is more accurate but calculations involved are more. Generally, single cost slope is assumed.

Total Project Cost and Optimum Duration

- The total project cost is the sum of the direct cost and the indirect costs.
- We find that the minimum total cost is obtained at duration known as the optimum duration. The corresponding cost is known as the minimum cost. If the project duration is increased, total cost will increase, while if project duration is decreased to the crash value, project cost will be the highest.



Total cost curve (Linear approximation)

Fig. 3.5 Total Project Cost and Optimum Duration

Do you know? Crashing of non-critical activities does not serve any purpose as they do not control the project duration and completing them earlier does no benefit rather it increases the project cost.

Steps in Crashing

1. Estimate project duration and find the critical path.
2. Find cost slope of all activities.
3. The critical activity having minimum cost slope is crashed in first stage. The next stage crashing will involve activity having second lower cost slope in critical path.
4. Total cost of project at this stage is calculated.
5. Steps 3 and 4 are repeated till all activities of project are crashed along critical paths and corresponding time is crash time of project.
6. It is to be noted that only critical activities should be crashed.

Do you know? That certain non-critical activities may also become critical in the process of crashing critical activities.

NOTE: In case of formation of new critical path at any stage of crashing, we have to do parallel crushing of activities for which cost slope is minimum.

3.4 Resource Allocation

- 'Resource' is a physical variable required for completion of activities. It can be material, manpower, machinery, space, money or time resources.
- Due to limited resources or limited project duration, it is required to allocate resources in such a way that more or less a uniform demand through out the project duration can be achieved.
- "Resource allocation" is deciding the resources to each activity.
- The diagram which shows variation in the requirement of resources with time is called *"Resource usage profile" or "Histogram"* as shown below.

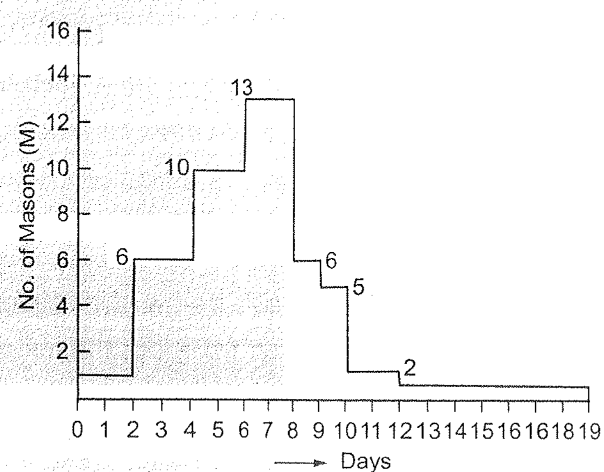


Fig. 3.6 *A Typical Histogram for Resource Allocation*

Histograms are helpful to know the requirement of resources at different times in different activities.

- Resource allocation can be achieved by following two processes:

3.4.1 Resource Levelling

Here resources are considered unlimited. Project duration is maintained and critical activities remain unchanged. Start time of some of non-critical activities are shifted within their available floats to create uniform demand throughout.

3.4.2 Resource Smoothening

Here resources are considered limited. Project duration may be changed. Activities are rescheduled to cut down the peak requirement of resources so that it does not cross the limit of resources. Available resources should never be less than the maximum quantity required for any activity of project. Firstly, available floats are used then if needed duration of some activities is increased or decreased as per the resource requirement.

Do you know? Resource smoothening and resource levelling both are trial and error methods.

NOTE: Conflict in Definitions
Resource levelling and resource smoothening defined in this book are as per definitions/concepts explained in Foreign author books such or Roury Bourke, Harald Kerzer and some Indian authors also have followed the same concept like M.R. Gopalan, K. Nagranjan, Prasanna Chandra, NPTEL Lectures etc. but some of the Indian authors have given just opposite explanation of these terms which include B.C. Puramia, R. Seetharaman and U.K. Shrivastava.

3.5 CPM Updating

- The process of reviewing the progress of project execution and redrafting the network according to latest requirements is called "updating".
- During redrafting, scheduled dates are revised. New critical path may emerge and hence project priorities may change.
- Crashing of new critical activities may be required to make project on schedule.
- Updating is necessary to compensate for deviations in actual execution of works and original plans.
- During the process of updating, neither activities are deleted nor new activities added.

When to Update?

1. Updating should be more frequent for shorter duration projects.
2. For larger duration projects, frequency should be increased as project is nearing completion.
3. Whenever major change in the duration of any of activity occurs then updating should be done.
4. If change in estimated duration occurs, updating is essential.

Example 3.1 Table below shown information about a two activity network. The project overhead costs are Rs. 400 per day. Work out the least cost plan for the network.

Activity	Normal duration (days)	Normal Cost (Rs.)	Crashing duration (days)	Crash Cost (Rs.)
1-2	6	7000	4	9000
2-3	10	9000	5	13000

Solution:

$$1 \xrightarrow[6(4)]{} 2 \xrightarrow[10(5)]{} 3$$

In the network shown, normal duration of activities are marked adjacent to arrow and corresponding crash duration in brackets.

The cost slope of each activity in computed below:

Activity	ΔC (Rs.)	Δt (days)	Cost slope = $\frac{\Delta C}{\Delta t}$ (Rs./day)
1-2	9000 – 7000 = 2000	6 – 4 = 2	1000
2-3	13000 – 9000 = 4000	10 – 5 = 5	800

Total normal duration of the project = 6 + 10 = 16 days.

Total normal cost incurred on the project = 7000 + 9000 = Rs. 16000/-

Now activity 2-3 has the minimum cost slope. Thus crash activity 2-3 first. The duration of activity 2-3 be crashed by 10- – 5 = 5 days.

∴ Extra cost incurred in crashing the activity 2-3 = 800 × 5 = Rs. 4000/-

After crashing activity 2-3, revised duration of project = 6 + 5 = 11 days.

∴ Direct cost of project for 11 days duration

= Normal cost of project + Extra cost incurred in crashing the activity 2-3

= 16000 + 4000 = Rs. 20000/-

After crashing activity 2-3 fully, now crash activity 1-2 from its normal duration of 6 days to crash duration of 4 days i.e., crashing activity 1-2 by 2 days.

Extra cost incurred in crashing the activity 1-2 = 1000 × 2 = Rs. 2000/-

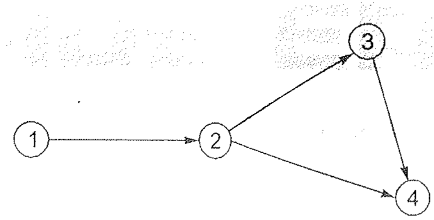
∴ Revised project duration = 4 + 5 = 9 days.

∴ Direct cost of project = 20000 + 2000 = Rs. 22000/-

Duration (days)	16	11	9
Direct cost (Rs.)	16000	20000	22000
Indirect cost (Rs) Rs. 400 per day	16 × 400 = 6400	11 × 400 = 4400	9 × 400 = 3600
Total cost (Rs.)	22400	24400	25600

Thus total cost in minimum for project duration of 16 days and corresponding minimum cost is Rs. 22400/-

Example 3.2 For the network shown below, determine the optimum duration and optimum cost.



Activity	Normal time (days)	Cost time (days)	Normal cost (Rs.)	Crash Cost (Rs.)
1-2	6	3	500	650
2-3	7	4	250	350
2-4	3	1	350	525
3-4	5	3	700	950

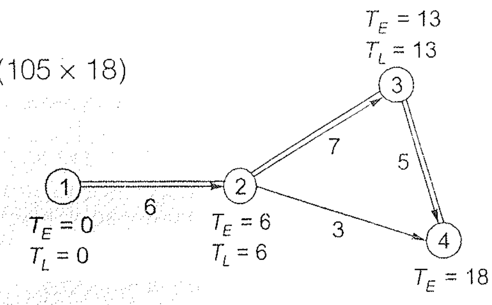
Indirect cost on the project occurs @ Rs. 105 per day.

Solution:

The cost slope of each activity is computed and shown below:

Activity	Normal time (days)	Cost time (days)	Normal cost (Rs.)	Crash Cost (Rs.)	Cost slope (Rs./day)
1-2	6	3	500	650	50
2-3	7	4	250	350	33.33
2-4	3	1	350	525	87.5
3-4	5	3	700	950	125

Network with normal duration activity times



Time duration = 18 days

Total cost = (500 + 250 + 350 + 700) + (105 × 18)

= Rs. 3690/-

Now, activity 2–3 has minimum cost slope

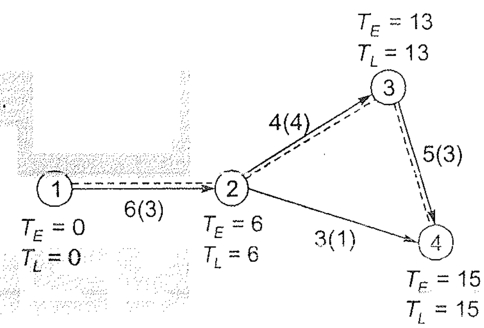
So crashing activity 2–3 by 3 days.

Hence revised duration of project = 18 – 3 = 15 days

Indirect cost = 105 × 15 = Rs. 1575

Direct cost = 500 + 250 + 350 + 100 + (33.333 × 3) = Rs. 1900

Crashed network



Still, the critical path is 1–2–3–4

Now, activity 1–2 has minimum cost slope on this critical path.

So, crashing 1–2 by available 3 days

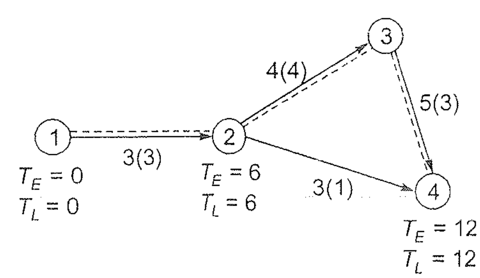
Hence, Revised duration = 12 days

Indirect cost = 105 × 12 = Rs. 1260

Direct cost = 1900 + (50 × 3) = Rs. 2050

Total cost = Rs. 3310

Crashed network



Still critical path is 1–2–3–4

One activity (3–4) along critical path remains to be crushed.

Even if activity 3–4 is crashed by available 2 days.

The critical activities will not change.

So crashing 3–4 activity by 2 days.

Hence, revised duration = 12 – 2 = 10 days

Indirect cost = 105 × 10 = Rs. 1050

Direct cost = 2050 + (125 × 2) = Rs. 2300

Total project cost = Rs. 3350

Now, all the activities along critical path are crushed.

Further crashing not required.

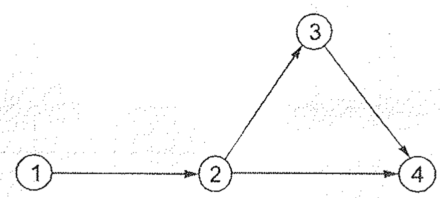
Duration (days)	18	15	12	10
Direct cost (Rs.)	1800	1900	2050	2300
Indirect cost (Rs) Rs. 400 per day	1850	1575	1260	1050
Total cost (Rs.)	3690	3475	3310	3350

Thus is can be seen that for optimum duration

$$t = 12 \text{ days}$$

$$\text{Minimum cost} = \text{Rs. } 3310$$

Example 3.3 Given below is the table showing durations and costs of various activities of the network shown below:



Activity	Normal duration (days)	Normal Cost (Rs.)	Crash duration (days)	Crash Cost (Rs.)
1-2	5	4050	3	9500
2-3	7	3150	4	7520
2-4	9	3750	6	5650
3-4	6	5520	3	10550

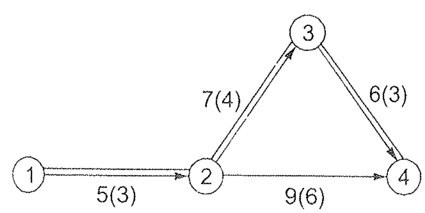
The overhead cost of the project is Rs. 1750 per day. Determine the optimum duration of the project and the corresponding cost.

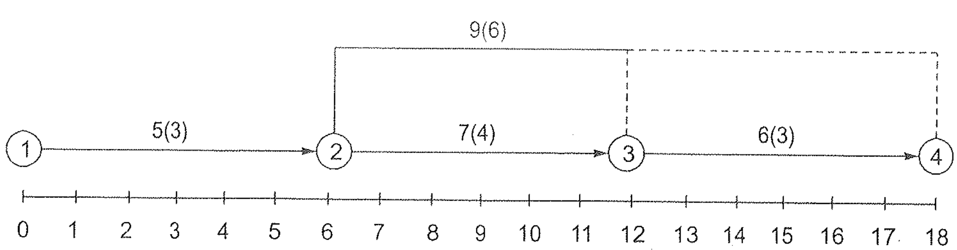
Solution:

The critical path of the network 1-2-3-4

Total normal duration of the project = Sum of normal durations of each activity on critical path = 18 days

A time scaled version of the network is drawn with critical activities on the horizontal line.



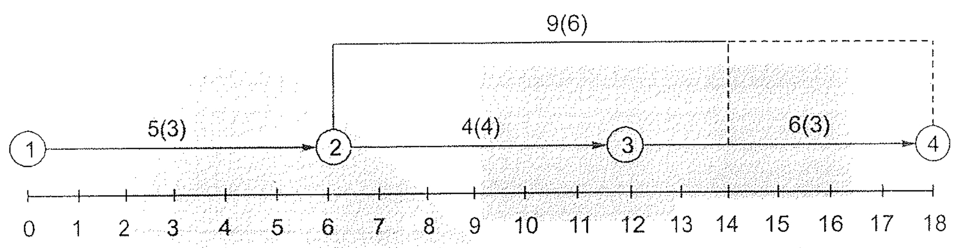


Activity	ΔC(Rs.)	Δt(days)	Cost slope (Rs./day)
1-2	9500 – 4050 = 5450	5 – 3 = 2	2725
2-3	7520 – 3150 = 4370	7 – 4 = 3	1456.67
2-4	5650 – 3750 = 1900	9 – 6 = 3	633.33
3-4	10550 – 5520 = 5030	6 – 3 = 3	1676.67

Total duration of the project = 18 days

Direct cost of the project = Sum of the normal costs of each activities = 4050 + 3150 + 3750 + 5520 = Rs. 16470/-

First Stage Crashing



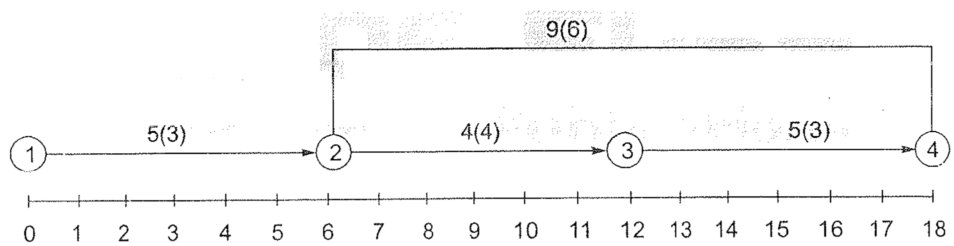
Now for crashing the network, select that critical activity which is the minimum cost slope which is activity 2-3 having the cost slope of Rs. 1456.67 per day. Activity 2-3 has the crash time of 4 days i.e., it can be crashed by 3 days. Crashing activity 2-3 by 3 days will not affect the non-critical activity 2-4.

Thus revised project duration = 18 – 3 = 15 days

Extra cost of crashing activity 2-3 by 3 days = 1456.67 × 3 = Rs. 4370/-

Revised direct cost of the project = 16470 + 4370 = Rs. 20840/-

Second Stage Crashing



Activity 3-4 can be crashed by 6-3 = 3 days but crash it by 1 day only so that activity 2-4 also becomes critical.

Thus revised project duration = 15 – 1 = 14 days

Extra cost of crashing activity 3-4 by 1 day = 1676.67 × 1 = Rs. 1676.67/-

Revised direct cost of the project = 20840 + 1676.67 = 22516.67

But this second stage crashing has also made the activity 2-4 as critical.

Further Stages of Crashing

Further crashing will involve two paths viz. 1-2-4 and 1-2-3-4.

Further crashing can be done with the following three options:

(i) Crashing activities 2-3 and 2-4 simultaneously with the combined cost slope of 1456.67 + 633.33 = 2090.

(ii) Crashing activities 3-4 and 2-4 simultaneously with combined cost slope of 1676.67 + 633.33 = 2310.

(iii) Crashing activity 1-2 alone with the cost slope of 2725.

Of the above three options, option (i) gives the minimum cost slope and thus the extra cost of crashing 2-3 and 2-4 by 1 day = Rs. 2090/- but activity 2-3 cannot be crashed further since its duration now is already equal to crash duration.

Third Stage Crashing

Thus another option is to crash activities 3-4 and 2-4 with a combined cost slope of 2310. Of the activities 2-4 and 3-4, activity 3-4 can be crashed by 5-3 = 2 days after which activity 3-4 cannot be crashed further.

Revised total duration of the project = 14 − 2 = 12 days

So extra cost of crashing by 2 days = 2310 × 2 = 4620

Revised direct cost of the project = 22516.67 + 4620 = Rs. 27136.67

Fourth Stage Crashing

Now the only remaining activity is activity 1-2 which can be crashed by 5 − 3 = 2 day with a cost slope of 2725.

Revised project duration = 12 − 2 = 10 day

Extra cost of crashing by 2 days = 2725 × 2 = 5450

Revised direct cost of the project = 27136.67 + 5450 = Rs. 32586.67

Total cost of the project

Project duration (days)	18	15	14	12	10
Direct cost (Rs.)	16470	20840	22516.67	27136.67	32586.67
Indirect cost (Rs.)	1750 × 18 = 31500	1750 × 15 = 26250	1750 × 14 = 24500	1750 × 12 = 21000	1750 × 10 = 17500
Total cost (Rs.)	47970	47090	47016.67	48136.67	50086.67

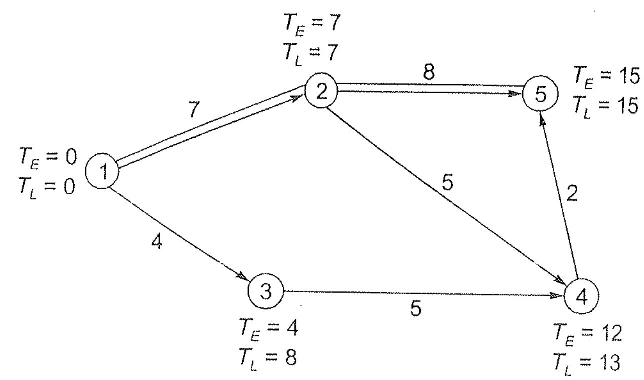
Thus optimum project duration is 14 days which is having the minimum total cost of the project which equals to Rs. 47016.67.

Example 3.4 The table below gives activities for a construction project along with other data.

Activity	Normal		Crash	
	Time (days)	Cost (Rs.)	Time (days)	Cost (Rs.)
1-2	7	60	5	90
1-3	4	80	2	100
2-4	5	50	3	70
2-5	8	100	4	120
3-4	5	150	2	180
4-5	2	70	1	90

The indirect cost of the project is Rs. 30 per day. Crash the activities and find the minimum project duration and associated project cost.

Solution:



The cost slope of each activity is tabulated below:

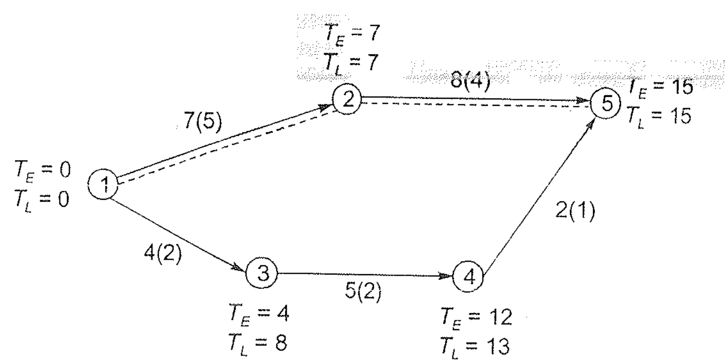
Activity	Normal		Crash		ΔC	Cost slope (Rs./day)
	Time (days)	Cost (Rs.)	Time (days)	Cost (Rs.)		
1-2	7	60	5	90	30	15
1-3	4	80	2	100	20	10
2-4	5	50	3	70	20	10
2-5	8	100	5	110	20	5
3-4	5	150	2	180	30	10
4-5	2	70	1	90	20	20

Total project duration = 15 days

Direct project cost = Rs. 510

Indirect cost = 15×30 = Rs. 450

Total project cost = 510 + 450 = Rs. 960



On the critical path activity2–5 has minimum cost slope.

Activity 2–5 can be crashed by 4 days but doing so will create other critical activities.

So, crashing 2–5 by 1 day

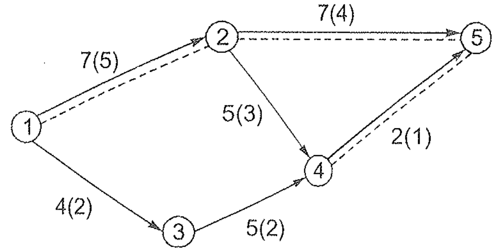
Hence revised duration, t = 14 days

Indirect cost = 30×14 = Rs. 420

Direct cost = 510 + 5 = Rs. 515

Total = Rs. 935

Revised Network



Now, there are 2 critical paths 1–2–5 and 1–2–4–5.

Further crashing will involve option

(i) Crashing 1–2 by 2 days with cost slope of Rs. 15 /day.

(ii) Crashing 2–4 and 2–5 simultaneously with combined cost slope Rs. 15/day

(iii) Crashing 4–5 and 2–5 simultaneously with combined cost slope of Rs. 25 /day

So let say, crashing 1–2 by 2 days first;

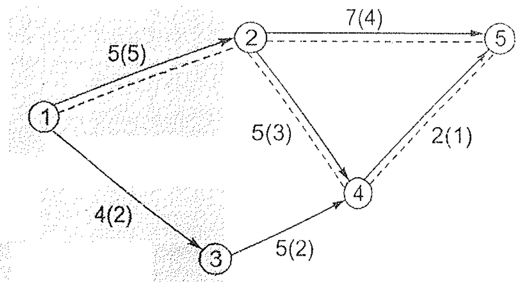
Hence, Revised duration = 14 – 2 = 12 days

Indirect cost = 12 × 30 = Rs. 360

Direct cost = 515 + 30 = Rs. 545

Total cost = Rs. 905

Revised Network



Now, crashing 2–4 and 2–5 simultaneously by 1 day

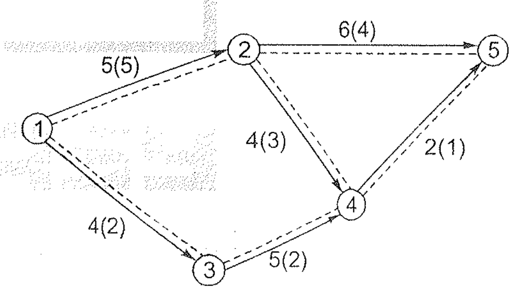
Hence Revised duration = 12 – 1 = 11 day

Indirect cost = 11 × 30 = Rs. 330

Direct cost = 545 + 15 = Rs. 560

Total cost = Rs. 890

Revised Network



Now all three paths, 1–2–5; 1–2–4–5; 1–3–4–5 are critical

So crashing options are

(i) Crash 2–5 and 4–5 by 1 day with combined cost slope of Rs. 25/day

(ii) Crash 2–5, 2–4 and 3–4 by 1 day using combined cost slope of Rs. 25/day

(iii) Crash 2–5, 2–4 and 1–3 by 1 day using combined cost slope of Rs. 25/day

Say, lets crash 2–5 and 4–5 first by 1 day

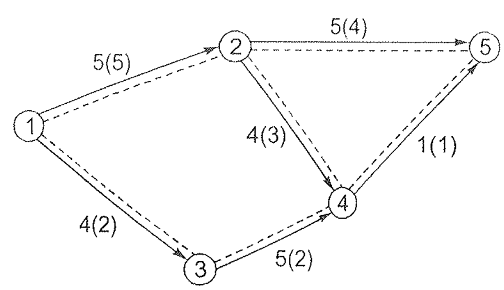
Revised duration = 11 – 1 = 10 day

Indirect cost = 30 × 10 = Rs. 300

Direct cost = 560 + 25 = Rs. 585

Total cost = Rs. 885

Revised Network



Now we can either crash 2–5, 2–4 and 1–3 or 2–5, 2–4 nd 3–4 by 1 day

As combined cost slope is same.

So, let say, crashing 2–5, 2–4 and 1–3 by 1 day

Hence, Revised duration = 10 – 1 = 9 day

Indirect cost = 30 × 9 = Rs. 210

Direct cost = 585 + 25 = Rs. 610

Total cost = Rs. 880

No further crashing is possible.

Hence minimum cost = Rs. 880

For optimum duration = 9 days.

Objective Brain Teasers

Q.1 Total Project Cost versus Time curve is/an
(a) S-shaped curve
(b) Parabola
(c) U-shaped curve
(d) Straight line

Q.2 In the cost optimization procedure the cost slope for each activity can be estimated by the formula

(a) $\dfrac{\text{crash cost} - \text{normal cost}}{\text{crash time} - \text{normal time}}$

(b) $\dfrac{\text{crash time} - \text{normal cost}}{\text{crash cost} - \text{normal cost}}$

(c) $\dfrac{\text{normal time} - \text{normal cost}}{\text{crash cost} - \text{normal cost}}$

(d) $\dfrac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$

Q.3 The normal duration and normal cost of an activity are 10 days and Rs. 350/- respectively. The cost slope is Rs. 75/- per day. If the crash duration is 8 days, then what is the crash cost of the activity?
(a) Rs. 400/-
(b) Rs. 500/-
(c) Rs. 600/-
(d) None of these

Q.4 In the time-cost optimization of a project, the project can be crashed by expending
(a) all activities on the critical path
(b) critical activities having minimum cost slope
(c) activities on subcritical path
(d) critical activities having maximum cost slope

Q.5 Match **List-I** (Cost) with **List-II** (Features) and select the correct answer using the codes given below the lists:

List-I
A. Optimal cost
B. Overhead cost
C. Direct cost
D. Indirect cost

List-II
1. Activity related
2. Developed by crashing process
3. Project-related
4. Contained in or contributing exclusively to the related product

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	4	1	2	3
(d)	2	3	4	1

Q.6 What is the process of incorporating changes and rescheduling or replanning called?
(a) Resource allocation
(b) Resource smoothing
(c) Resource levelling
(d) Updating

Q.7 Scheduling helps in
(a) Planning for the project
(b) Financial control of the project
(c) Preparing the estimate for the project
(d) Carrying out the project in an orderly and effective manner

Q.8 The resources in a construction project are made up of
(a) office staff
(b) construction materials like cement, bricks etc.
(c) office space
(d) skilled and unskilled manpower

Answers

1. (c) 2. (d) 3. (b) 4. (b) 5. (b)
6. (d) 7. (d) 8. (d)