Sample Paper 9

Class IX 2022-23

Mathematics

Time: 3 Hours **Max. Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 3 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

1. The value of $\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$ is equal to

(a) $x^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (b) 0

(c) $x^{pq+qr+rp}$ (d) 1

2. For the polynomial $p(x) = x^5 + 4x^3 - 5x^2 + x - 1$, one of the factors is

(a) $(x+1)$ (b) $(x-1)$

(c) x (d) $(x+2)$

3. The point for which the abscissa and ordinate have same signs will lie in

(a) I and II quadrants (b) I and III quadrants

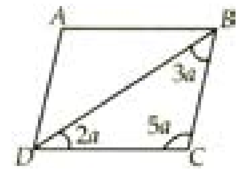
(c) I and IV quadrants (d) III and IV quadrants

4. Which of the following equation has graph parallel to y-axis?

(a) $y = -2$ (b) $x = 1$

(c) $x - y = 2$ (d) $x + y = 2$

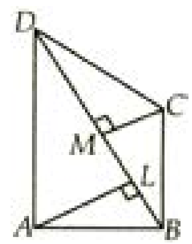
5. In the given figure, the measure of $\angle C$ is equal to



(a) 90° (b) 80°

(c) 75° (d) 95°

6. In the adjoining figure, $ABCD$ is a quadrilateral in which diagonal $BD = 14\text{ cm}$. If $AL \perp BD$ and $CM \perp BD$ such that $AL = 8\text{ cm}$ and $CM = 6\text{ cm}$, then area of quadrilateral $ABCD$ is



(a) 60 cm^2 (b) 72 cm^2

(c) 84 cm^2 (d) 98 cm^2

7. The length of the sides of a triangle are 4 cm, 6 cm and 8 cm. The length of perpendicular from the opposite vertex to the side whose length is 8 cm, is equal to .... cm.

(a) $\frac{3}{5}\sqrt{15}$ cm (b) $\frac{3}{2}\sqrt{15}$ cm

(c) $\frac{3}{6}\sqrt{15}$ cm (d) $\frac{3}{4}\sqrt{15}$ cm

8. The hollow sphere, in which the circus motorcyclist performs his stunt, has a diameter of 7 m. Find the area available to the motorcyclist for riding?

(a) 154 m^2 (b) 152 m^2

(c) 153 m^2 (d) 151 m^2

9. Find k, if $x^{51} + 2x^{60} + 3x + k$ is divisible by $x + 1$.

(a) 1 (b) 2

(c) 3 (d) 4

10. The radius of a cone is 3 cm and vertical heights is 4 cm. Find the area of the curved surface.

(a) 62.85 cm^2 (b) 61.85 cm^2

(c) 63.85 cm^2 (d) 64.85 cm^2

11. Let U be the upper class boundary of a class in a frequency distribution and M be the midpoint of the class. Which one of the following is the lower class boundary of the class?

(a) $M+\frac{(M+L)}{2}$

(b) $L+\frac{M+L}{2}$

(c) $2M-U$

(d) $M-2L$

12. At Middle School, 3 out of 5 students make honour roll. What is the probability (in%) that a student does not make honour roll?

(a) 65%

(b) 40%

(c) 60%

(d) None of these

13. A rational number equivalent to a rational number $\frac{7}{19}$ is

(a) $\frac{17}{119}$

(b) $\frac{14}{57}$

(c) $\frac{21}{38}$

(d) $\frac{21}{57}$

14. Factors of $(a+b)^3-(a-b)^3$ are

(a) $2ab, (3a^2+b^2)$

(b) $ab, (3a^2+b^2)$

(c) $2b, (3a^2+b^2)$

(d) $(3a^2+b^2), 2a$

15. The three vertices of a square $ABCD$ are $A(3,2)$, $B(-2,2)$ and $D(3,-3)$. Find the coordinates of C and the area of square $ABCD$.

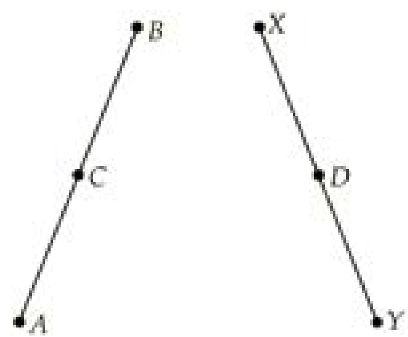
(a) $C(-2,-3)$, 5 sq. units

(b) $C(3,-3)$, 5 sq. units

(c) $C(3,2)$, 25 sq. units

(d) $C(-2,-3)$, 25 sq. units

16. In the given figure, $AC=XD, C$ is mid-point of AB And D is mid-point of XY. Using an Euclid's axiom, we have



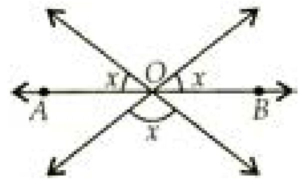
(a) $AB=XY$

(b) $AX=BC$

(c) $BY=AC$

(d) none of these

17. The value of x if AOB is a straight line, is



(a) 36° (b) 60°

(c) 30° (d) 35°

18. If $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔRPQ, then which of the following is not true?

(a) $BC = PQ$ (b) $AC = PR$

(c) $AB = PQ$ (d) $QR = BC$

19. **Assertion :** The point $(1, 1)$ is the solution of $x + y = 2$.
Reason : Every point which satisfy the linear equation is a solution of the equation.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

20. **Assertion :** A fair die is rolled. Then the probability of getting an even number is $\frac{1}{2}$ and probability of getting an odd number is $\frac{1}{2}$.

Reason : Possible outcomes when a fair die is rolled is (1, 2, 3, 4, 5, 6).

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Section B

Section B consists of 5 questions of 2 marks each.

21. Simplify : $\dfrac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

OR

If $\dfrac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = a + b\sqrt{3}$, find the values of a and b.

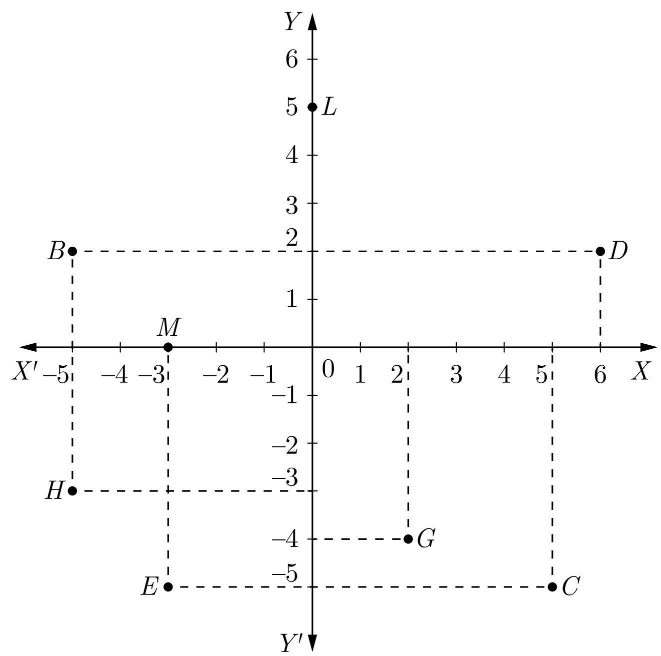
22. Find the area of regular hexagon of side a cm.

OR

The sides of a triangle are 4 cm, 8 cm and 6 cm. Find the length of the perpendicular from the opposite vertex to the longest side.

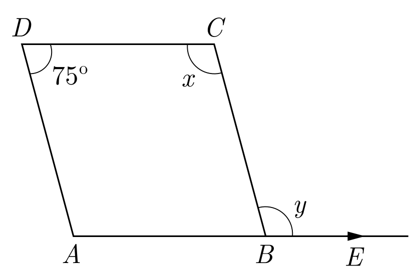
23. If $\left(x+\frac{1}{x}\right)=9$, then find the value of $x^3+\frac{1}{x^3}$.

24. See Figure and write the following :



(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (–3, –5).
(iv) The point identified by the coordinates (2, –4).

25. $ABCD$ is a parallelogram in which $\angle ADC = 75^\circ$ and side AB is produced to point E as shown in the figure. Find $(x+y)$.



Section C

Section C consists of 6 questions of 3 marks each.

26. Find the remainder, when $3x^3 - 6x^2 + 3x - \frac{7}{9}$ is divided by $3x - 4$.

OR

Write the equation of the lines drawn in following graph. Also, find the area enclosed between them.

27. A family with monthly income of ₹ $30,000$ had planned the following expenditures per month under various heads :

Heads	**Expenditure (in ₹ 1000)**
Rent	5
Grocery	4
Clothings	3
Education of children	5
Medicine	2
Entertainment	3
Miscellaneous	6
Savings	2

Draw a bar graph for the above data.

OR

If the mean of five observations x, $x+2$, $x+4$, $x+6$ and $x+8$ is 11. Find the value of x.

28. Find the curved surface area and total surface area of a hemisphere of radius $35\,\text{cm}$.

OR

The volume of a cylindrical rod is $628\,\text{cm}^3$. If its height is 20 cm, find the radius of its cross section. (Use $\pi = 3.14$).

29. Write true or false and justify your answer. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is $96\,\text{cm}^2$.

30. $ABCD$ is a parallelogram. A circle through A and B is drawn, so that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

31. Draw a right angled triangle whose hypotenuse measure 6 cm and the length of one of whose sides containing the right angle is 4 cm.

Section D

Section D consists of 4 questions of 5 marks each.

32. A recent survey found that the age of workers in a factory as follows :

Age (in yrs)	**Number of workers**
20-29	38
30-39	27
40-49	86
50-59	46
60 and above	3

If a person is selected at random, then find the probability that the person is

OR

The mean of the following frequency distribution is 16.6.

x_i	8	12	15	18	20	25	30	Total
f_i	12	16	p	24	16	q	4	100

Find the missing frequencies p and q.

33. Water flows in a tank 150 m $\times$ 100 m at the base through a pipe whose cross-section is 2 dm $\times$ 1.5 dm at the speed of 15 km/h. In what time, will the water be 3 m deep ?

OR

An open rectangular cistern is made of iron 2.5 cm thick. When measured from outside, it is 1 m 25 cm long, 1 m 5 cm broad and 90 cm deep.

Find :

(i) the capacity of the cistern in litres

(ii) the volume of iron used

(iii) the total surface area of the cistern

34. If $x = \dfrac{1}{2-\sqrt{3}}$, then find the value of $x^3 - 2x^2 - 7x + 5$.

OR

Find the zeroes of the given polynomial $f(x) = 2x^3 + 3x^2 - 11x - 6$.

35. AB and AC are two chords of a circle of radius r such that $AB = 2AC$. If p and q are the distances of AB and AC from the centre then prove that $4q^2 = p^2 + 3r^2$.

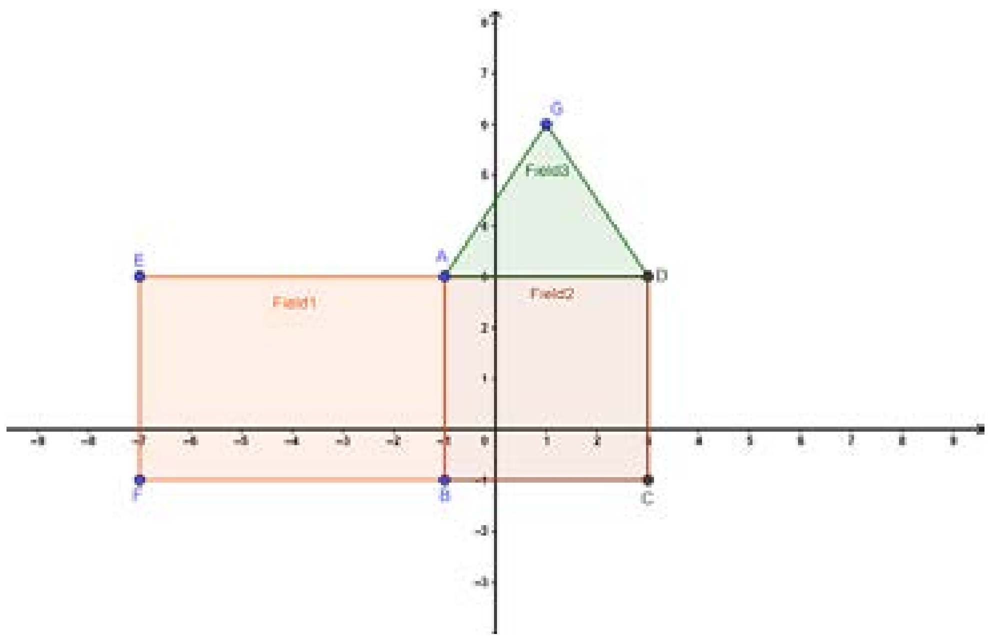
Section E

Case study based questions are compulsory.

36. FARMER : A farmer Naresh has the field AGDCFE as shown in the map. He decided to grow three crops in his field. Naresh divided his field into three fields for each crop. There is shortage of onions in the country and he decided to grow onions on the field which has more area. In the field having smallest area he decided to grow wheat and in remaining field he decided to grow tomatoes.

Map showing field owned by a Naresh FARMER

1unit = 25m



(i) What are the coordinates of field in which onion is grown?

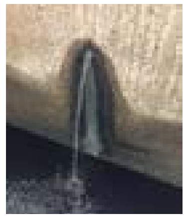
(ii) Naresh needs to water the fields of onion and tomatoes with a sprinkler. He decides to place sprinkler in the middle of each field. Write the coordinates where Naresh will put sprinkler in the field of tomato.

(iii) Annual yield of wheat in India in FY 2018 is 3,371 kilograms per hectare. What is expected yield of wheat in Naresh field?

OR

(iv) The revised purchase price of wheat is 1,840 rupees ($25.09) per 100 kg for 2019 compared with 1,735 rupees a year ago, Farm Minister Radha Mohan Singh said. India, the world's second-biggest rice and wheat producer, buys the grain from local farmers at state-set prices to build stocks to run a major food welfare programme.
How much earning is expected by Naresh?

37. Water is leaking from water tanks at a linear rate. The amount of water, in liters, is measured at the start of each day. At the end of a day and its alternate day the capacity of water in the tanks are given as shown below.



Tank A

Tank B

Tank C

Tank/day	**1**	**2**	**3**	**4**	**5**
A	2950 lit		2850 lit		
B		3800 lit		3600 lit	
C			4400 lit		4400 lit

(i) Determine the amount of water that was initially in the tanks.

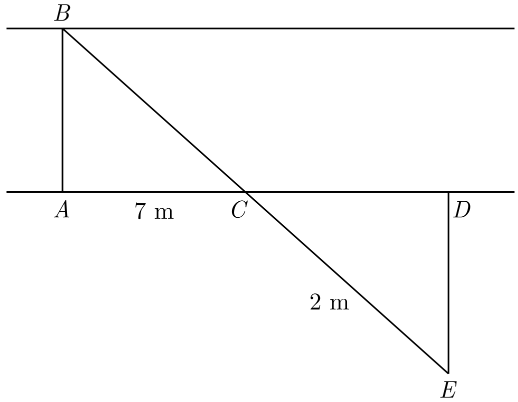
(ii) Frame equations that find amount of water (w) in liters, at the end of any day (d). Use variables w for water and d for days.

(iii) Assume that once water is filled in the tanks, then later water is not used and not filled but the water is being leaked. Which tank will be emptied in 25 days ?

OR

(iv) After how many days the capacity of water is equal in tank B and C?

38. Breadth of a river : There is a river is in a village. The villagers want to measure its breadth without crossing the river as force of water's current is very high. Aniket a student of class IX of their village came and told "I can measure the breadth of the river without crossing it." He came on the bank of river at a point A and imagines a point B just opposite on the other bank. He moved to C and then D such that C is the equidistant from A and D. Then he moves to E such that B and E are the on the same line.



(i) How can it be possible? Explain.

(ii) What congruence criteria she uses to find the breadth of the river?

(iii) What is the length of BC?

OR

(iv) If AC = 7 m and CE = 25 m, find the breadth of the river.

Sample Paper 9 Solutions

Class- IX Exam - 2022-23

Mathematics

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
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7. All Questions are compulsory. However, an internal choice in 3 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The value of $\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$ is equal to

(a) $x^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (b) 0

(c) $x^{pq+qr+rp}$ (d) 1

Sol : (d) 1

$$\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}} = \frac{x^{\frac{1}{r}}}{x^{\frac{1}{q}}} \times \frac{x^{\frac{1}{p}}}{x^{\frac{1}{r}}} \times \frac{x^{\frac{1}{q}}}{x^{\frac{1}{p}}} = 1$$

2. For the polynomial $p(x) = x^5 + 4x^3 - 5x^2 + x - 1$, one of the factors is

(a) $(x+1)$ (b) $(x-1)$

(c) x (d) $(x+2)$

Sol : (b) $(x-1)$

$$p(x) = x^5 + 4x^3 - 5x^2 + x - 1$$

$$p(1) = 1 + 4 - 5 + 1 - 1 = 0$$

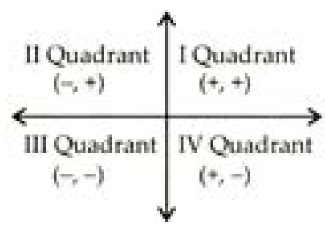
Hence, $x = 1$ is the solution of $p(x)$.

3. The point for which the abscissa and ordinate have same signs will lie in

(a) I and II quadrants (b) I and III quadrants

(c) I and IV quadrants (d) III and IV quadrants

Sol : (b) I and III quadrants



Abscissa and ordinate have same sign in I and III quadrants.

4. Which of the following equation has graph parallel to y-axis?

(a) $y = -2$ (b) $x = 1$

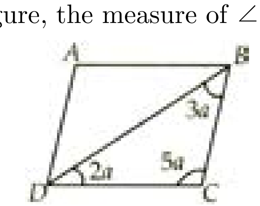
(c) $x - y = 2$ (d) $x + y = 2$

Sol : (b) $x = 1$

$x = a$ has the graph which is parallel to y-axis.

$x = 1$ is the required equation that has graph parallel to y-axis.

5. In the given figure, the measure of $\angle C$ is equal to



(a) 90° (b) 80°

(c) 75° (d) 95°

Sol : (a) 90°

In, ΔBCD $\quad \angle BCD + \angle CDB + \angle DBC = 180^\circ$

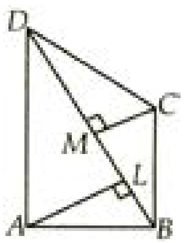
$$2a + 5a + 3a = 180^\circ$$

$$a = 18^\circ$$

$$\angle C = 5a$$

$$= 5 \times 18^\circ = 90^\circ$$

6. In the adjoining figure, $ABCD$ is a quadrilateral in which diagonal $BD = 14\,\text{cm}$. If $AL \perp BD$ and $CM \perp BD$ such that $AL = 8\,\text{cm}$ and $CM = 6\,\text{cm}$, then area of quadrilateral $ABCD$ is



(a) 60 cm^2 (b) 72 cm^2

(c) 84 cm^2 (d) 98 cm^2

Sol : (d) 98 cm^2

Area of quadrilateral $ABCD$

$$= \text{area}(\Delta ABD) + (\Delta BCD)$$

$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$= \left[\frac{1}{2} \times 14 \times 8 + \frac{1}{2} \times 14 \times 6 \text{ cm}^2\right.$$

$$= 98\,\text{cm}^2$$

7. The length of the sides of a triangle are 4 cm, 6 cm and 8 cm. The length of perpendicular from the opposite vertex to the side whose length is 8 cm, is equal to .... cm.

(a) $\frac{3}{5}\sqrt{15}$ cm (b) $\frac{3}{2}\sqrt{15}$ cm

(c) $\frac{3}{6}\sqrt{15}$ cm (d) $\frac{3}{4}\sqrt{15}$ cm

Sol : (d) $\frac{3}{4}\sqrt{15}\,\text{cm}$

$$s = \frac{1}{2}(4 + 6 + 8)\,\text{cm} = 9 \text{ cm}$$

$$\text{Area} = \sqrt{9(9-4)(9-6)(9-8)}$$

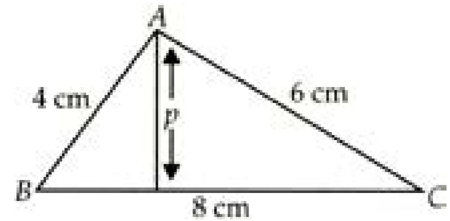
$$= \sqrt{9 \times 5 \times 3 \times 1}$$

$$= 3\sqrt{15} \text{ cm}^2$$

Also, $\quad \text{area} = \frac{1}{2} \times 8 \times p$

$$4p = 3\sqrt{15}$$

$$p = \frac{3\sqrt{15}}{4} \text{ cm}$$



8. The hollow sphere, in which the circus motorcyclist performs his stunt, has a diameter of 7 m. Find the area available to the motorcyclist for riding?

(a) 154 m^2 (b) 152 m^2

(c) 153 m^2 (d) 151 m^2

Sol : (a) 154 m^2

Given,

Diameter of the sphere = 7 m.

Therefore, radius is 3.5 m, So, the riding space available for the motorcyclist is the surface area ot the sphere.

$$4\pi r^2 = 4 \times \frac{22}{7} \times 3.5 \times 3.5\,\text{m}^2$$

$$= 154\,\text{m}^2$$

9. Find k, if $x^{51} + 2x^{60} + 3x + k$ is divisible by $x + 1$.

(a) 1 (b) 2

(c) 3 (d) 4

Sol : (b) 2

Let, $\quad p(x), = x^{51} + 2x^{60} + 3x + k$

Given that, $p(x)$, is divisible by $x + 1$.

$$p(-1) = 0$$

$$(-1)^{51} + 2(-1)^{60} + 3(-1) + k = 0$$

$$-1 + 2 - 3 + k = 0$$

$$k - 4 + 2 = 0$$

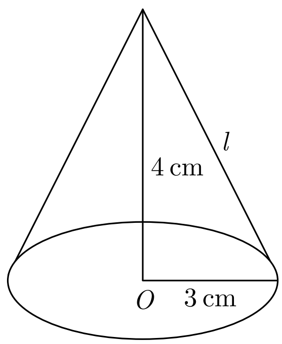
$$k - 2 = 0$$

$$k = 2$$

10. The radius of a cone is $3\,\text{cm}$ and vertical heights is $4\,\text{cm}$. Find the area of the curved surface.

(a) $62.85\,\text{cm}^2$ (b) $61.85\,\text{cm}^2$
(c) $63.85\,\text{cm}^2$ (d) $64.85\,\text{cm}^2$

Sol : (a) $62.85\,\text{cm}^2$



We have, $r = 3\,\text{cm}$ and $h = 4\,\text{cm}$.

Let $l\,\text{cm}$ be the slant height of the cone.

Then, $l^2 \ - r^2 + h^2$

$l^2 = \sqrt{25}\ \text{cm} = 5\,\text{cm}$

Area of the curved surface $= \pi rl$

Area of the curved surface $= \left(\frac{22}{7} \times 4 \times 5\right)\text{cm}^2$

$= 62.85\,\text{cm}^2$

11. Let U be the upper class boundary of a class in a frequency distribution and M be the midpoint of the class. Which one of the following is the lower class boundary of the class?

(a) $M + \frac{(M+L)}{2}$ (b) $L + \frac{M+L}{2}$
(c) $2M - U$ (d) $M - 2L$

Sol : (c) $2M - U$

Class mark

$$= \frac{\text{Upper class boundary} + \text{lower class boundary}}{2}$$

$$M = \frac{U+L}{2}$$

$$2M - U = L$$

12. At Middle School, 3 out of 5 students make honour roll. What is the probability (in%) that a student does not make honour roll?

(a) 65% (b) 40%
(c) 60% (d) None of these

Sol : (b) 40%

Number of students make honour roll $= 3$

P(Student does not make honour roll) $= 2/5$

Probability (in%) that a student does not make

honour roll $= \left(\frac{2}{5} \times 100\right)\%$

$= 40\%$

13. A rational number equivalent to a rational number $\frac{7}{19}$ is

(a) $\frac{17}{119}$ (b) $\frac{14}{57}$
(c) $\frac{21}{38}$ (d) $\frac{21}{57}$

Sol : (d) $\frac{21}{57}$

Simplest form of $\frac{17}{119} = \frac{17}{119}$

Simplest form of $\frac{14}{57} = \frac{14}{57}$

Simplest form of $\frac{21}{38} = \frac{21}{38}$

Simplest form of $\frac{21}{57} = \frac{7}{19}$

14. Factors of $(a+b)^3 - (a-b)^3$ are

(a) $2ab, (3a^2 + b^2)$ (b) $ab, (3a^2 + b^2)$
(c) $2b, (3a^2 + b^2)$ (d) $(3a^2 + b^2), 2a$

Sol : (c) $2b, (3a^2 + b^2)$

$(a+b)^3 - (a-b)^3$

$$= [(a+b) - (a-b)][(a+b)^2 + (a+b)(a-b) + (a-b)^2]$$

$$= 2b[a^2 + 2ab + b^2 + a^2 - b^2 + a^2 - 2ab + b^2]$$

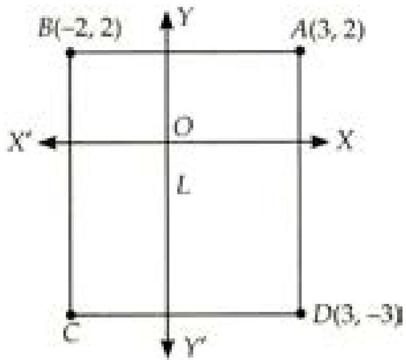
$$= 2b[3a^2 + b^2]$$

15. The three vertices of a square $ABCD$ are $A(3,2)$, $B(-2,2)$ and $D(3,-3)$. Find the coordinates of C and the area of square $ABCD$.

(a) $C(-2,-3)$, 5 sq. units
(b) $C(3,-3)$, 5 sq. units
(c) $C(3,2)$, 25 sq. units
(d) $C(-2,-3)$, 25 sq. units

Sol : (d) $C(-2,-3)$, 25 sq. units

Here, $A(3,2)$, $B(-2,2)$ and $D(3,-3)$ are the three vertices of square $ABCD$.



$$\text{Clearly, abscissa of } C = \text{abscissa of } B = -2,$$

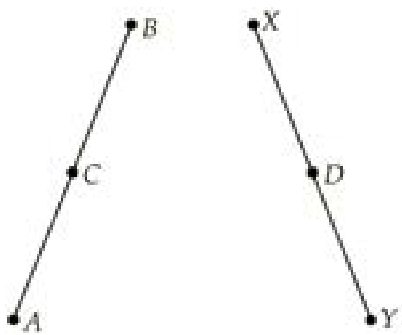
$$\text{Ordinate of } C = \text{ordinate of } D = -3.$$

Coordinates of C are $(-2,-3)$.

$$CD = (2+3) = 5 \text{ units.}$$

$$\text{Area of square } ABCD = 5 \times 5 = 25 \text{ sq. units.}$$

16. In the given figure, $AC = XD, C$ is mid-point of AB And D is mid-point of XY. Using an Euclid's axiom, we have



(a) $AB = XY$ (b) $AX = BC$

(c) $BY = AC$ (d) none of these

Sol : (a) $AB = XY$

C is the mid-point of AB

$$AB = 2AC$$

Also, D is the mid-point of XY

$$XY = 2XD$$

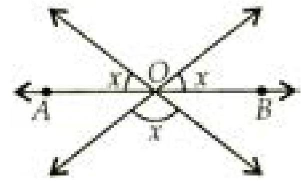
By Euclid's sixth axiom "Things which are double of same things are equal to one another."

$$AC = XD$$

$$2AC = 2XD$$

$$AB = XY$$

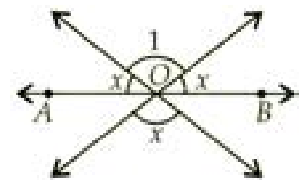
17. The value of x if AOB is a straight line, is



(a) 36° (b) 60°

(c) 30° (d) 35°

Sol : (b) 60°



$$\angle 1 = x \quad \text{[Vertically opposite angles]}$$

Since, AOB is a straight line

$$x + x + x = 180^\circ$$

$$3x = 180^\circ$$

$$x = 60^\circ$$

18. If $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔRPQ, then which of the following is not true?

(a) $BC = PQ$ (b) $AC = PR$

(c) $AB = PQ$ (d) $QR = BC$

Sol : (a) $BC = PQ$

If $\Delta ABC \cong \Delta PQR$, then their respective congruent sides and angles will be as follows

$$AB = PQ, \angle A = \angle P$$

$$BC = QR, \angle B = \angle Q$$

$$AC = PR, \angle C = \angle R$$

Thus, only (a) is not true.

19. **Assertion :** The point $(1,1)$ is the solution of $x + y = 2$.

Reason : Every point which satisfy the linear equation is a solution of the equation.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Sol : (a) Both assertion and reason are true and reason is the correct explanation of assertion.

Putting $(1,1)$ in the given equation, we have

$$\text{L.H.S} = 1+1 = 2 = \text{R.H.S}$$

$$\text{L.H.S} = \text{R.H.S}$$

Hence $(1,1)$ satisfy the $x+y=2$. So it is the solution of $x+y=2$.

20. **Assertion :** A fair die is rolled. Then the probability of getting an even number is $\frac{1}{2}$ and probability of getting an odd number is $\frac{1}{2}$.

Reason : Possible outcomes when a fair die is rolled is (1, 2, 3, 4, 5, 6).

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Sol : (a) Both assertion and reason are true and reason is the correct explanation of assertion.

Possible outcomes when a die is thrown

$$= \{1,2,3,4,5,6\}$$

$$P \text{ (getting an odd number)} = \frac{3}{6} = \frac{1}{2}$$

$$P \text{ (getting an even number)} = \frac{3}{6} = \frac{1}{2}$$

Section B

21. Simplify : $\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

or

If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$, find the values of a and b.

Sol :

We have

$$\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \frac{\sqrt[3]{6^2} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$$

$$= \frac{\sqrt[3]{6^2 \times 6^7}}{\sqrt[3]{6^6}} \quad [\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}]$$

$$= \frac{\sqrt[3]{6^9}}{\sqrt[3]{6^6}} \quad [\because a^m \times a^n = (a)^{m+n}]$$

$$= \sqrt[3]{\frac{6^9}{6^6}} = \sqrt[3]{6^{9-6}}$$

$$\left[\because \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \text{ and } a^m + a^n = (a)^{m-n}\right]$$

$$= \sqrt[3]{6^3} = 6 \quad [\because \sqrt[m]{a^m} = a]$$

or

We have,

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$$

$$\Rightarrow \quad \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a+b\sqrt{3}$$

$$\frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48} = a+b\sqrt{3}$$

$$11-6\sqrt{3} = a+b\sqrt{3}$$

$$\Rightarrow \quad a = 11 \text{ and } b = -6$$

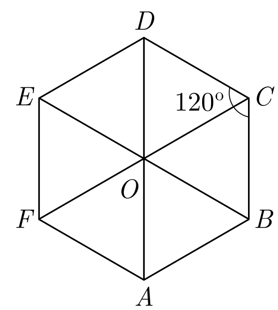
22. Find the area of regular hexagon of side a cm.

or

The sides of a triangle are 4 cm, 8 cm and 6 cm. Find the length of the perpendicular from the opposite vertex to the longest side.

Sol :

We know that, regular hexagon is divided into six equilateral triangles.



$\therefore$ Area of regular hexagon of side a

$$= \text{Sum of the area of six equilateral triangles}$$

$$= 6 \times \frac{\sqrt{3}}{4} \times a^2 = \frac{3\sqrt{3}}{2} a^2 \text{ cm}^2$$

$$[\because \text{ Area of equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2]$$

or

$$s = \frac{4+8+6}{2} \text{ cm} = 9 \text{ cm}$$

$\therefore$Area of the triangle$= \sqrt{9(9-4)(9-8)(9-6)}$ cm^2

$$= \sqrt{9 \times 5 \times 1 \times 3} \text{ cm}^2$$

$$= 3\sqrt{15} \text{ cm}^2$$

Also, $\frac{1}{2} \times 8 \times \text{Altitude} = 3\sqrt{15}$

$$\text{Altitude} = \frac{3\sqrt{15}}{4} \text{ cm}$$

23. If $\left(x+\frac{1}{x}\right)=9$, then find the value of $x^3+\frac{1}{x^3}$.

Sol :

We have, $x+\frac{1}{x} = 9$

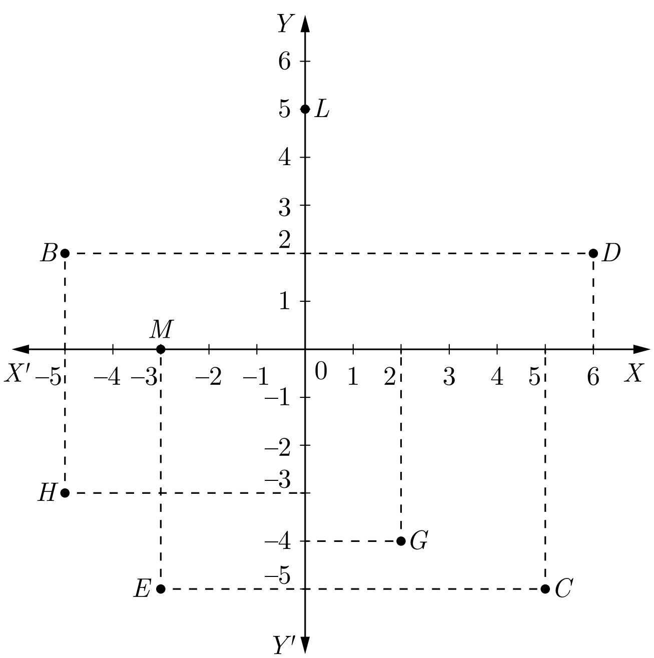
$$\left(x+\frac{1}{x}\right)^3 = 9^3$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = 729$$

$$x^3+\frac{1}{x^3}+3\times 9 = 729$$

$\Rightarrow$ $x^3+\frac{1}{x^3} = 729-27=702$

24. See Figure and write the following :

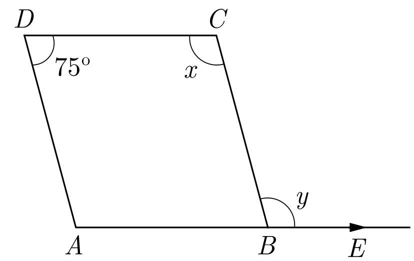


(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (–3, –5).
(iv) The point identified by the coordinates (2, –4).

Sol :

(i) (–5, 2)
(ii) (5, –5)
(iii) E
(iv) G

25. $ABCD$ is a parallelogram in which $\angle ADC = 75^\circ$ and side AB is produced to point E as shown in the figure. Find $(x+y)$.



Sol :

Given, $ABCD$ is a parallelogram, in which $\angle ADC = 75^\circ$

$\therefore$ $\angle ABC = 75^\circ$

[In a parallelogram, opposite sides are equal]

$$\angle CBE = y = 180^\circ - \angle ABC$$

[Linear pair axiom]

$$= 180^\circ - 75^\circ = 105^\circ$$

Also, $\angle x = 180^\circ - 75^\circ = 105^\circ$

[$\because$ $\angle D + \angle x = 180^\circ$ as $DA || CB$ and DC is a transversal]

$\therefore$ $x+y = 105^\circ + 105^\circ = 210^\circ$

Section C

26. Find the remainder, when $3x^3-6x^2+3x-\frac{7}{9}$ is divided by $3x-4$.

or

Write the equation of the lines drawn in following graph. Also, find the area enclosed between them.

Sol :

Let $p(x) = 3x^3-6x^2+3x-\frac{7}{9}$ and it is divided by $3x-4$.

Put $3x-4 = 0$

$\Rightarrow$ $3x = 4$

$$x = \frac{4}{3}$$

On putting $x=\frac{4}{3}$ in $p(x)$, we get

$$p\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^3 - 6\left(\frac{4}{3}\right)^2 + 3\left(\frac{4}{3}\right) - \frac{7}{9}$$

$$= 3 \times \left(\frac{64}{27}\right) - 6 \times \left(\frac{16}{9}\right) + 4 - \frac{7}{9}$$

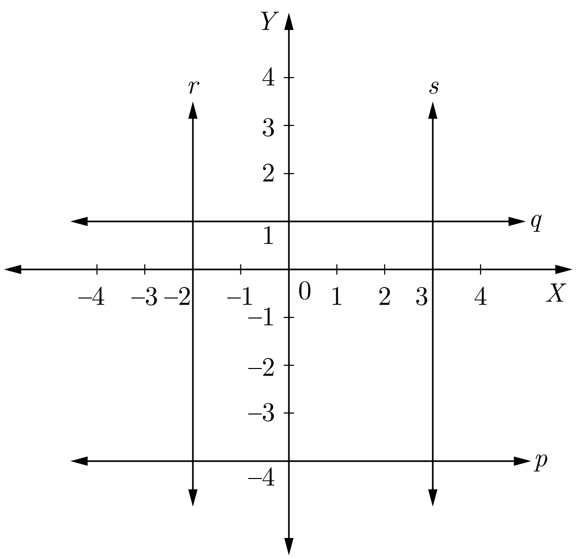
$$= \frac{64}{9} - \frac{32}{3} + 4 - \frac{7}{9}$$

$$= \frac{64 - 96 + 36 - 7}{9}$$

$$= -\frac{3}{9} = -\frac{1}{3}$$

Hence, the remainder is $-\frac{1}{3}$.

or



The line q is parallel to X-axis and at 1 unit distance from X-axis in the positive direction of Y-axis.

So, equation of line q is $y = 1$.

The line p is parallel to X-axis and at 4 units distance from X-axis in the negative direction of Y-axis.

So, the equation of line p is $y = -4$.

Again, the line r is parallel to X-axis and at a distance of 2 units from Y-axis in the negative direction of X -axis.

So, the equation of line r is $x = -2$.

Similarly, the equation of line s is $x = 3$.

Thus, we get the equation of lines as $y = 1$, $y = -4$, $x = -2$, $x = 3$.

Thus, formed figure by these lines is of a square of length 5 units.

$\therefore$ Area of formed figure $= 5 \times 5 = 25$ sq units.

27. A family with monthly income of ₹ 30,000 had planned the following expenditures per month under various heads :

Heads	**Expenditure (in ₹ 1000)**
Rent	5
Grocery	4
Clothings	3
Education of children	5
Medicine	2
Entertainment	3
Miscellaneous	6
Savings	2

Draw a bar graph for the above data.

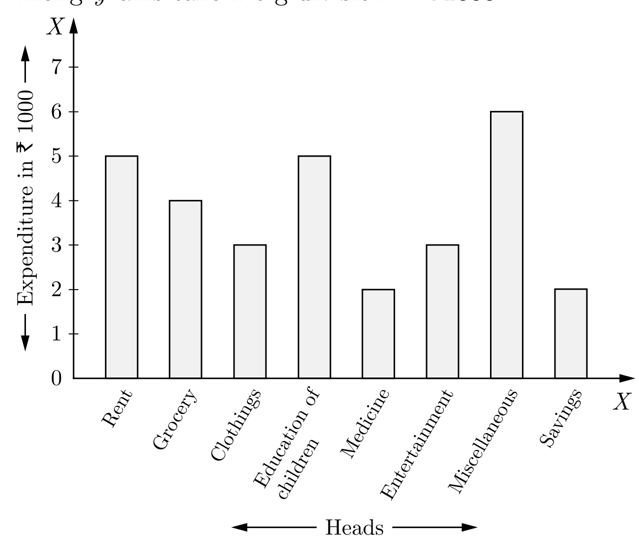
or

If the mean of five observations x, $x+2$, $x+4$, $x+6$ and $x+8$ is 11. Find the value of x.

Sol :

Let us take heads along x-axis and expenditure (in ₹ 1000) among y-axis.

Along y-axis take 1 big division = ₹ 1000



or

Mean of the given observations

$$= \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5}$$

$$= \frac{5x + 20}{5}$$

But mean = 11 [Given]

$$\therefore \quad \frac{5x + 20}{5} = 11$$

$\Rightarrow \qquad 5x+20 = 55$

$x = 7$

Hence, $x = 7$

28. Find the curved surface area and total surface area of a hemisphere of radius $35\,\text{cm}$.

or

The volume of a cylindrical rod is $628\,\text{cm}^3$. If its height is 20 cm, find the radius of its cross section. (Use $\pi = 3.14$).

Sol :

Here, radius of the hemisphere $(r) = 35\,\text{cm}$

$\therefore$ Curved surface area of hemisphere $= 2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times 35 \times 35\right)\text{cm}^2$$

$$= 44 \times 5 \times 35 = 7700\,\text{cm}^2$$

Total surface area of hemisphere $= 3\pi r^2$

$$= \left(3 \times \frac{22}{7} \times 35 \times 35\right)\text{cm}^2$$

$$= 66 \times 5 \times 35 = 11550\,\text{cm}^2$$

or

Let radius of cross section of rod $= r$ cm

Height of cylindrical rod $= 20\,\text{cm}$

Volume of cylindrical rod $= 628\,\text{cm}^3$

$\Rightarrow \qquad \pi r^2 h = 628$

$$3.14 \times r^2 \times 20 = 628$$

$$r^2 = \frac{628 \times 100}{314 \times 20} = 10$$

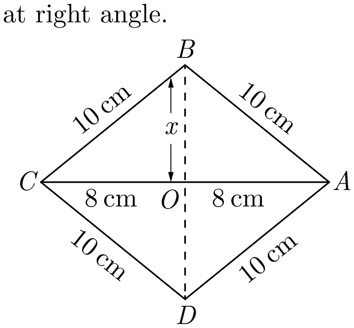
$$r = \sqrt{10}\text{ cm} = 3.16\,\text{cm}$$

$\therefore$ Radius of its cross section $= 3.16\,\text{cm}$

29. Write true or false and justify your answer. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is $96\,\text{cm}^2$.

Sol :

True. We know that diagonals of a rhombus bisect each other at right angle.



$\therefore \qquad OA = OC = 8\,\text{cm}$

In ΔOAB,

$$AB^2 = OA^2 + OB^2$$

$$(10)^2 = (8)^2 + (x)^2$$

$$x = \sqrt{36} = 6\,\text{cm}$$

$$DB = 2(OB)$$

$$= 2 \times 6 = 12\,\text{cm}$$

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 16 \times 12 = 96\,\text{cm}^2$$

30. $ABCD$ is a parallelogram. A circle through A and B is drawn, so that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

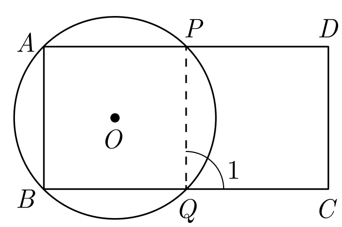
Sol :

Here, join PQ.

Now, $\qquad \angle 1 = 180^\circ - \angle BQP$

$\Rightarrow \qquad \angle 1 = \angle A$

[By property of cyclic quadrilateral]



But $\qquad \angle A = \angle C$

[Opposite angles of a parallelogram]

$\therefore \qquad \angle 1 = \angle C \qquad$ ...(1)

But $\qquad \angle C + \angle D = 180^\circ$

[Sum of co-interior angles on same side is 180°]

[From eq.(1)]

$\Rightarrow \qquad \angle 1 + \angle D = 180^\circ$

Thus, the quadrilateral $QCDP$ is cyclic.

So, the points P, Q, C and D are concyclic.

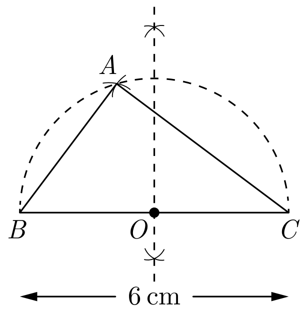
31. Draw a right angled triangle whose hypotenuse measure 6 cm and the length of one of whose sides containing the right angle is 4 cm.

Sol :

Steps of construction :

(i) Draw a line segment $BC = 6\,\text{cm}$.

(ii) Draw perpendicular bisector of BC which intersects BC at O.



(iii) Taking O as centre and radius OB, draw a semi-circle on BC.

(iv) Taking B as centre and radius equal to 4 cm. draw an arc, cutting the semi-circle at A.

(v) Join AB and AC.

Thus, ABC is the required right and angled triangle.

Section D

32. A recent survey found that the age of workers in a factory as follows :

Age (in yrs)	**Number of workers**
20-29	38
30-39	27
40-49	86
50-59	46
60 and above	3

If a person is selected at random, then find the probability that the person is

or

The mean of the following frequency distribution is 16.6.

x_i	8	12	15	18	20	25	30	Total
f_i	12	16	p	24	16	q	4	100

Find the missing frequencies p and q.

Sol :

Total number of workers in a factory,

$$n(S) = 38 + 27 + 86 + 46 + 3$$
$$= 200$$

(i) Number of persons having age of 40 yrs or more,

$$n(E_1) = 86 + 46 + 3 = 135$$

$\therefore$ Probability that the person selected at the age of 40 yrs or more.

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{135}{200} = 0.675$$

Hence, the probability that the person selected at the age of 40 yrs or more is 0.675.

(ii) Number of persons under the age of 40 yrs.

$$n(E_2) = 38 + 27 = 65$$

$\therefore$ Probability that the selected person under the age of 40 yrs,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{65}{200} = 0.325$$

Hence, the probability that the selected person under 40 yrs is 0.325.

(iii) Number of persons having age from 30 to 39 yrs,

$$n(E_3) = 27$$

$\therefore$ Probability that the selected person have age from 30 to 39 yrs.

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{27}{200} = 0.135$$

Hence, the probability that the selected person have age from 30 to 39 yrs is 0.135.

or

We prepare the table given below :

x_i	f_i	$f_i x_i$
8	12	96
12	16	192
15	p	$15p$
18	24	432
20	16	320
25	q	$25q$
30	4	120
	$\Sigma f_i = 72 + p + q$	$\Sigma f_i x_i = 1160 + 15p + 25q$

Here, $\Sigma f_i = 72 + p + q$

But, $\Sigma f_i = 100$ (Given)

$\therefore \quad 72 + p + q = 100$

$p + q = 28$

Also, $\text{Mean} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{1160 + 15p + 25q}{72 + p + q}$

$= \dfrac{1160 + 15(p+q) + 10q}{72 + (p+q)}$

$= \dfrac{1160 + 15 \times 28 + 10q}{72 + 28}$

$= \dfrac{1580 + 10q}{100}$

But mean $= 16.6$ (Given)

$\therefore \quad \dfrac{1580 + 10q}{100} = 16.6$

$1580 + 10q = 1660$

$10q = 80$

$q = 8$

$\Rightarrow \quad p + q = 28$

$p = 28 - q$

$= 28 - 8 = 20$

Hence, $p = 20$ and $q = 8$.

33. Water flows in a tank 150 m × 100 m at the base through a pipe whose cross-section is 2 dm × 1.5 dm at the speed of 15 km/h. In what time, will the water be 3 m deep ?

or

An open rectangular cistern is made of iron 2.5 cm thick. When measured from outside, it is 1 m 25 cm long, 1 m 5 cm broad and 90 cm deep.

Find :

(i) the capacity of the cistern in litres

(ii) the volume of iron used

(iii) the total surface area of the cistern

Sol :

Suppose in x hours water will be 3 m deep in tank.

Volume of water in the tank

$= 150 \times 100 \times 3 = 45000 \text{ m}^3$

Area of cross-section of the pipe

$= \dfrac{2}{10} \times \dfrac{1.5}{10} = \dfrac{1}{5} \times \dfrac{15}{100}$

$= \dfrac{3}{100} \text{ m}^2 \qquad \left[\therefore 1 \text{ dm} = \dfrac{1}{10} \text{m}\right]$

Volume of water that flows in the tank in x hours

$=$ Area of cross-section of the pipe

$\times$ Speed of water $\times$ Time

$= \dfrac{3}{100} \times 15000 \times x$

$[\because \text{speed} = 15 \text{ km/h} = 15000 \text{ m/h}]$

$= 450x \text{ m}^3$

Since, the volume of water in the tank is equal to the volume that flows in the tank in x hours.

$\therefore$ Volume of water in the tank

$=$ Volume of water that flows in x hours

$\therefore \quad 450x = 45000 \text{ h}$

$x = 100$ hours

or

External dimensions of the cistern are :

Length $= 125 \text{ cm}$

Breadth $= 105 \text{ cm}$

and Depth $= 90 \text{ cm}$

Internal dimensions of the cistern are :

Length $= 120 \text{ cm}$

Breadth $= 100 \text{ cm}$

and Depth $= 87.5 \text{ cm}$

(i) Capacity $=$ Internal volume

$= (120 \times 100 \times 87.5) \text{cm}^3$

$= \left(\dfrac{120 \times 100 \times 87.5}{1000}\right)$

$= 1050$ litres

(ii) Volume of iron $=$ (External volume)

$-$ (Internal volume)

$= \{(125 \times 105 \times 90)$

$$-(120 \times 100 \times 87.5)\}$$

$$= (1181250 - 1050000)$$

$$= 131250 \text{ cm}^3$$

(iii) External area = (Area of 4 faces) + (Area of the base)

$$= \{[2(125 + 105) \times 90] + (125 \times 105)\}$$

$$= (41400 + 13125)$$

$$= 54525 \text{ cm}^2$$

Internal area $= \{[2(120 + 100) \times 87.5] + (120 \times 100)\}$

$$= (38500 + 12000)$$

$$= 50500 \text{ cm}^2$$

Area at the top = Area between outer and inner rectangles

$$= \{(125 \times 105) - (120 \times 100)\}$$

$$= (13125 - 12000)$$

$$= 1125 \text{ cm}^2$$

$\therefore$ Total surface area $= (54525 + 50500 + 1125)$

$$= 106150 \text{ cm}^2$$

34. If $x = \frac{1}{2-\sqrt{3}}$, then find the value of $x^3 - 2x^2 - 7x + 5$.

or

Find the zeroes of the given polynomial $f(x) = 2x^3 + 3x^2 - 11x - 6$.

Sol :

Given, $x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

[By rationalising the denominator]

$$= \frac{2+\sqrt{3}}{(2)^2 - (\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3}$$

$$= 2 + \sqrt{3}$$

$[\because (a-b)(a+b) = a^2 - b^2]$

$\Rightarrow$ $x - 2 = \sqrt{3}$

On squaring both sides, we get

$$(x-2)^2 = (\sqrt{3})^2$$

$\Rightarrow$ $x^2 - 4x + 4 = 3$ $[\because (a-b)^2 = a^2 - b^2 - 2ab]$

$$x^2 - 4x + 1 = 0 \qquad ...(1)$$

Now, divide $(x^3 - 2x^2 - 7x + 5)$ by $(x^2 - 4x + 1)$.

$$\begin{array}{r|l} & x+2 \\ x^2-4x+1 & x^3 - 2x^2 - 7x + 5 \\ & x^3 - 4x^2 + x \\ & -\quad+\quad- \\ \hline & 2x^2 - 8x + 5 \\ & 2x^2 - 8x + 2 \\ & -\quad+\quad- \\ \hline & 3 \end{array}$$

By using long division method, we get

Thus, quotient $= x + 2$ and remainder $= 3$

$\therefore$ $x^3 - 2x^2 - 7x + 5 = (x+2)(x^2 - 4x + 1) + 3$

$= 0 + 3 = 3$ [Using eq.(1)]

Hence, at $x = \frac{1}{2-\sqrt{3}}$, $x^3 - 2x^2 - 7x + 5 = 3$

or

We have $f(x) = 2x^3 + 3x^2 - 11x - 6$

Here, the constant term is 6. Then, the factors of 6 may be ± 1, ± 2, ± 3 and ± 6.

By trial method, put $x = -1$ in $f(x)$, we get

$$f(-1) = 2(-1)^3 + 3(-1)^2 - 11(-1) - 6$$

$$= -2 + 3 + 11 - 6 \neq 0$$

Thus, $x = -1$ is not a zero of $f(x)$.

Now, put $x = 2$ in $f(x)$, we get

$$f(2) = 2(2)^3 + 3(2)^2 - 11(2) - 6$$

$$= 16 + 12 - 22 - 6$$

$$= 28 - 28 = 0$$

$\therefore$ $x = 2$ is zero of $f(x)$.

$\Rightarrow$ $(x-2)$ is a factor of $f(x)$.

Then, $f(x) = (x-2) \cdot q(x)$, where $q(x)$ is a quadratic polynomial of degree 2, which is obtained on dividing, $f(x)$ by $(x-2)$ by using long division method.

$$\begin{array}{r|l} & 2x^2 + 7x + 3 \\ x-2 & 2x^3 + 3x^2 - 11x - 6 \\ & 2x^3 - 4x^2 \\ \hline & 7x^2 - 11x \\ & 7x^2 - 14x \\ \hline & 3x - 6 \\ & 3x - 6 \\ \hline & 0 \end{array}$$

Thus, $q(x) = 2x^2 + 7x + 3$

$\therefore$ $f(x) = (x-2)(2x^2 + 7x + 3)$

$= (x-2)(2x^2 + 6x + x + 3)$

[By spliting the middle term]

$= (x-2)[2x(x+3) + 1(x+3)]$

$= (x-2)(x+3)(2x+1)$

Thus, $f(x) = 0$

if $(x-2) = 0$

or $(x+3) = 0$ or $(2x+1) = 0$

$\Rightarrow$ $x = 2$

$x = -3$

$x = -\frac{1}{2}$

Hence, $2, -3$ and $-\frac{1}{2}$ are zeroes of the given polynomial.

35. AB and AC are two chords of a circle of radius r such that $AB = 2AC$. If p and q are the distances of AB and AC from the centre then prove that $4q^2 = p^2 + 3r^2$.

Sol :

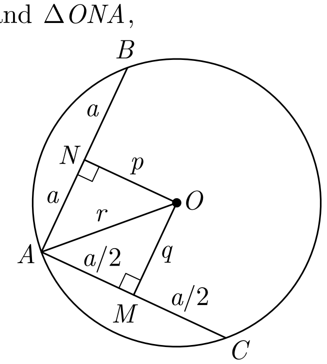
Let $AC = a$, then $AB = 2a$

From centre O, perpendicular is drawn to the chords AC and AB at M and N, respectively.

$\therefore$ $AM = MC = \frac{a}{2}$

and $AN = NB = a$

In ΔOMA and ΔONA,



By Pythagoras theorem,

$AO^2 = AM^2 + MO^2$

$\Rightarrow$ $AO^2 = \left(\frac{a}{2}\right)^2 + q^2$ ...(1)

and $AO^2 = (AN)^2 + (NO)^2$

$\Rightarrow$ $AO^2 = (a)^2 + p^2$ ...(2)

From eqs.(1) and (2), we get

$\left(\frac{a}{2}\right)^2 + q^2 = a^2 + p^2$

$\Rightarrow$ $\frac{a^2}{4} + q^2 = a^2 + p^2$

$a^2 + 4q^2 = 4a^2 + 4p^2$

$4q^2 = 3a^2 + 4p^2$

$4q^2 = p^2 + 3(a^2 + p^2)$

$4q^2 = p^2 + 3r^2$

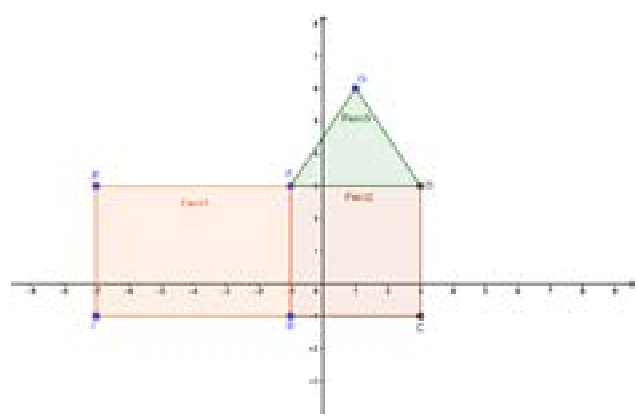
[$\because$ in right angled ΔONA, $r^2 = a^2 + p^2$]

Section E

36. FARMER : A farmer Naresh has the field AGDCFE as shown in the map. He decided to grow three crops in his field. Naresh divided his field into three fields for each crop. There is shortage of onions in the country and he decided to grow onions on the field which has more area. In the field having smallest area he decided to grow wheat and in remaining field he decided to grow tomatoes.

Map showing field owned by a Naresh FARMER

1unit = 25m



(i) What are the coordinates of field in which onion is grown?

(ii) Naresh needs to water the fields of onion and tomatoes with a sprinkler. He decides to place sprinkler in the middle of each field. Write the coordinates where Naresh will put sprinkler in the field of tomato.

(iii) Annual yield of wheat in India in FY 2018 is 3,371 kilograms per hectare. What is expected yield of wheat in Naresh field?

or

(iv) The revised purchase price of wheat is 1,840 rupees ($25.09) per 100 kg for 2019 compared with 1,735 rupees a year ago, Farm Minister Radha Mohan Singh said. India, the world's second-biggest rice and wheat producer, buys the grain from local farmers at state-set prices to build stocks to run a major food welfare programme.
How much earning is expected by Naresh?

Sol :

(i) Given that, Onion is grawn in the field which has maximum area.

In the given cartesian plane, we can see that the field which has maximum are is $ABFE$.

So, coordinates of field are $A(-1,3)$, $B(-1,-1)$, $F(-7,-1)$ and $E(-7,3)$.

(ii) Given that, wheat is grawn in the field which has smallest area and this field is in the cartesian plane is AGD.

So, it is clear that the remaining field in which Tomatoes are grawn is $ABCD$.

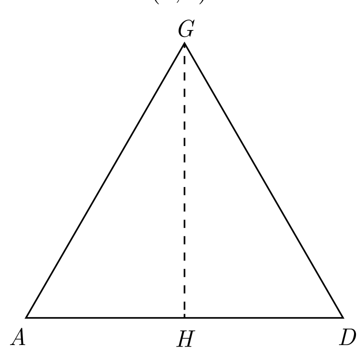
Here, Coordinates of A and C are $(-1,3)$ and $(3,-1)$ respectively.

Therefore, coordinates of that point where Naresh will put sprinkles in the field of Tomato

$$=\left(\frac{-1+3}{2},\frac{3-1}{2}\right)$$

$$=\left(\frac{2}{2},\frac{2}{2}\right)$$

$$=(1,1)$$



(iii) Given that, wheat is grawn in the field of triangle AGD.

To find the area of triangle AGD, draw $GH \perp AD$

So, H is the mid point of AD.

Now, coordinates of H $=\left(\frac{-1+3}{2},\frac{3+3}{2}\right)$

$$=\left(\frac{2}{2},\frac{6}{2}\right)$$

$$=(1,3)$$

Distance between the points G and H

$$GH=\sqrt{(-1+1)^2+(6-3)^2}$$

$$=\sqrt{0+(3)^2}$$

$$=\sqrt{9}$$

$$=3 \text{ units}$$

Distance between points A and D

$$AD=\sqrt{(3+1)^2+(3-3)^2}$$

$$=\sqrt{(4)^2+0}$$

$$=\sqrt{16}$$

$$=4 \text{ units}$$

Since, $1 \text{ unit} = 25\text{ m}$

therefore, $GH = 3\times 25 = 75\text{ m}$

and $AD = 4 \times 25 = 100\text{ m}$

Then, area of field $AGD = \frac{1}{2}\times AD \times GH$

$$=\frac{1}{2}\times 75\times 100$$

$$=75\times 50$$

$$=3750\ sq.\text{ m}$$

Given that, $10000\text{ m}^2 = 3371\text{ kg}$

So, $1\text{ m}^2 = \frac{3371}{10000}$

Now $3750\text{ m}^2 = \frac{3371}{10000}\times 3750$

$$=1264.125\text{ m}^2$$

(iv) Given that,

purchase price of wheat is 1840 rupees per 100 kg.

Since, 100 kg = ₹ 1840

So, $1\text{ kg} = \frac{1840}{100}$

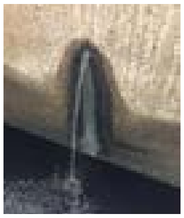
= ₹ 18.40

Now, price of wheat of 1264.125 kg

$$=18.40\times 1264.125$$

= ₹ 23259 (Approx.)

37. Water is leaking from water tanks at a linear rate. The amount of water, in liters, is measured at the start of each day. At the end of a day and its alternate day the capacity of water in the tanks are given as shown below.



Tank A

Tank B

Tank C

Tank/ day	1	2	3	4	5
A	2950 lit		2850 lit		
B		3800 lit		3600 lit	
C			4400 lit		4400 lit

(i) Determine the amount of water that was initially in the tanks.

(ii) Frame equations that find amount of water (w) in liters, at the end of any day (d). Use variables w for water and d for days.

(iii) Assume that once water is filled in the tanks, then later water is not used and not filled but the water is being leaked. Which tank will be emptied in 25 days ?

or

(iv) After how many days the capacity of water is equal in tank B and C?

Sol :

(i) Let the amount of water leaking from tank A = x

amount of water leaking from tank B = y

and amount of water leaking from tank C = z

According to question, for tank A

$$2950 - 2x = 2850$$
$$2x = 2950 - 2850$$
$$2x = 100$$
$$x = \frac{100}{2}$$
$$x = 50$$

then, amount of water initially in the tank A

$$= 50 + 2950$$
$$= 3000 \text{ litre}$$

For tank B,

$$3800 - 2y = 3600$$
$$2y = 200$$
$$y = \frac{200}{2}$$
$$y = 100$$

So, amount of water initially in the tank B,

$$= 3800 + 2y$$
$$= 3800 + 2 \times 100$$
$$= 4000 \text{ litre}$$

For tank C,

$$4400 - 2z = 4000$$
$$2z = 4400 - 4000$$
$$2z = 400$$
$$z = \frac{400}{2}$$
$$z = 200$$

Now, amount of water initially in the tank C,

$$= 4400 + 3z$$
$$= 4400 + 3 \times 200$$
$$= 4400 + 600$$
$$= 5000 \text{ litre}$$

(ii) According to part (i), required equation can be written as,

For tank A,

$$\text{w} = 3000 - 50d$$

For tank B,

$$\text{w} = 4000 - 100d$$

For tank C,

$$\text{w} = 5000 - 200d$$

(iii) For tank A,

$$3000 - 50d = 0$$
$$50d = 3000$$
$$d = \frac{3000}{50}$$
$$d = 60$$

It will be emptied in 60 days

For tank B,

$$4000 - 100d = 0$$

$$100d = 4000$$

$$d = \frac{4000}{100}$$

$$d = 40$$

It will be emptied in 40 days.

For tank C,

$$5000 - 200d = 0$$

$$200d = 5000$$

$$d = \frac{5000}{200}$$

$$d = 25$$

It will be emptied in 25 days.

(iv) According to question,

$$4000 - 100d = 5000 - 200d$$

$$200d - 100d = 5000 - 4000$$

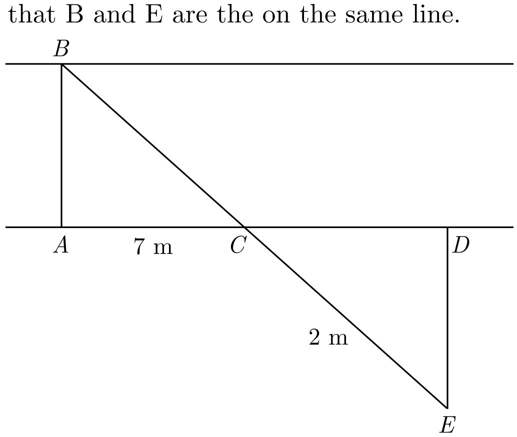
$$100d = 1000$$

$$d = \frac{1000}{100}$$

$$d = 10$$

Hence, after 10 days the capacity of water is equal in tank B and tank C.

38. Breadth of a river : There is a river is in a village. The villagers want to measure its breadth without crossing the river as force of water's current is very high. Aniket a student of class IX of their village came and told "I can measure the breadth of the river without crossing it." He came on the bank of river at a point A and imagines a point B just opposite on the other bank. He moved to C and then D such that C is the equidistant from A and D. Then he moves to E such that B and E are the on the same line.



(i) How can it be possible? Explain.

(ii) What congruence criteria she uses to find the breadth of the river?

(iii) What is the length of BC?

or

(iv) If AC = 7 m and CE = 25 m, find the breadth of the river.

Sol :

(i) Here, it can be possible to measure the river's breadth without crossing the river.

Here, we can see the situation created by Aniket and compare the ΔABC and ΔDCE.

Now, $\angle BAC = \angle EDC$ (each 90^o)

$AC = CD$ (given)

$\angle ACB = \angle ECD$

(vertical opposite angles)

By ASA congruency rule, we get

$$\Delta ABC \cong \Delta DCE$$

So, $BC = CE$ (by CPCT)

After that, we use the Pythagoras theorem to find the river's breadth.

(ii) Here, ASA congruency rule is used to find the required river's breadth.

(iii) Since, $\Delta ABC \cong \Delta DCE$

So, $BC = CE = 25\text{ m}$ (By CPCT)

(iv) Given that, $AC = 7\text{ m}$

and $CE = BC = 25\text{ m}$

Now in ΔABC, by Pythagoras theorem

$$AB^2 + AC^2 = BC^2$$

$$AB^2 + 7^2 = 25^2$$

$$AB^2 = 25^2 - 7^2$$

$$= 625 - 49$$

$$= 576$$

$$AB = \sqrt{576}$$

$$= 24\text{ m}$$

Hence, breadth of river is 24 m.