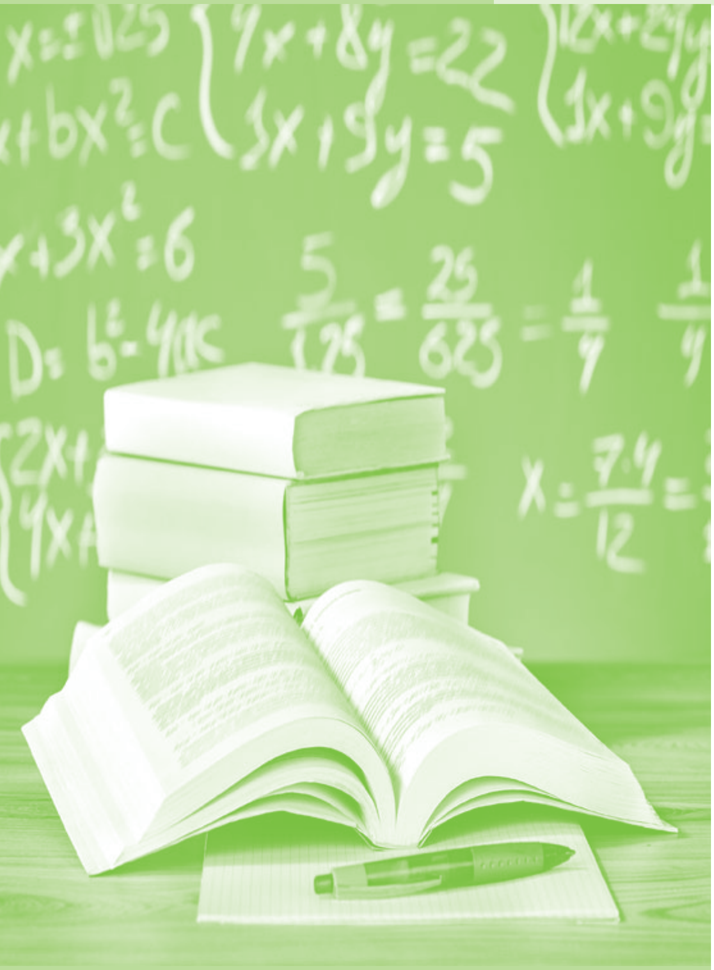
Chapter 15

Locus



REMEMBER

Before beginning this chapter, you should be able to:

- Understand the basic concepts of geometrical shapes and patterns
- Know different types of angles and their values
- Remember theorems related to triangles and angles

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the concept of locus and point of locus
- Calculate the equation of a locus
- Know the concept of congruency and learn the terms related to geometric centres of a triangle

INTRODUCTION

Mark a fixed point O on a sheet of paper. Now, start marking points P_1, P_2, P_3, P_4, … on the sheet of paper such that $OP_1 = OP_2 = OP_3 = \ldots = 4$ cm. What do we observe on joining these points by a smooth curve? We observe a pattern, which is circular in shape and every point is at a distance of 4 cm from point O.

It can be said that whenever points satisfying a certain condition are plotted, a pattern is formed. This pattern formed by all possible points satisfying the given condition is called the locus of points satisfying the given conditions. In the above example, we have the locus of points which are equidistant (4 cm) from the given point O.

The collection (set) of all points which satisfy certain given geometrical conditions is called the **locus of a point** satisfying the given conditions.

Alternatively, a locus can be defined as the path or curve traced by a point in a plane when subjected to some geometrical conditions.

Consider the following examples:

1. The locus of the point in a plane which is at a constant distance r from a fixed point O is a circle with centre O and radius r units (see Fig. 15.1).
2. The locus of the point in a plane which is at a constant distance from a fixed straight line is a pair of lines, parallel to the fixed line. Let the fixed line be l. The lines, m and n, form the set of all points which are at a constant distance from l (see Fig. 15.2).

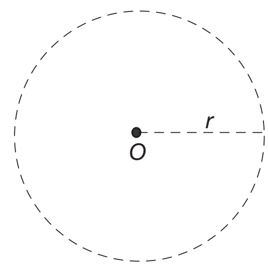


Figure 15.1

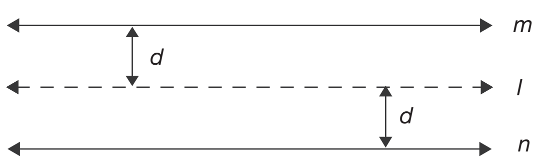


Figure 15.2

3. The locus of a point in a plane, which is equidistant from a given pair of parallel lines is a straight line, parallel to the two given lines and lying midway between them.

   In Fig. 15.3, m and n are the given lines and line l is the locus.

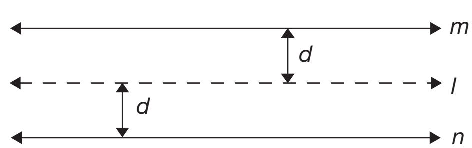


Figure 15.3

Before proving that a given path or curve is the desired locus, it is necessary to prove the following:

1. Every point lying on the path satisfies the given geometrical conditions.
2. Every point that satisfies the given conditions lies on the path.

EXAMPLE 15.1

Show that the locus of a point, equidistant from the endpoints of a line segment, is the perpendicular bisector of the segment.

SOLUTION

The proof will be taken up in two steps.

Step 1: We initially prove that any point equidistant from the endpoints of a line segment lies on the perpendicular bisector of the line segment.

Given: M and N are two points on a plane. A is a point in the same plane such that $AM = AN$ (see Fig. 15.4).

To prove: A lies on the perpendicular bisector of MN.

Proof: Let L be the mid-point of $\overline{MN}$.

If A coincides with L, then A lies on the bisector of MN.

Suppose A is different from L.

Then, in ΔMLA and ΔNLA,

$ML = NL$, $AM = AN$ and AL is a common side.

$\therefore$ By SSS congruence property, $\Delta MLA \cong \Delta NLA$.

$\Rightarrow \angle MLA = \angle NLA$ ($\because$ The corresponding elements of congruent triangles are equal.) (1)

But, $\angle MLA + \angle NLA = 180°$ ($\because$ Linear pair)

$\Rightarrow 2\,\angle MLA = 180°$ (From Eq. (1))

$\therefore \angle MLA = \angle NLA = 90°$.

So, $\overline{AL} \perp \overline{MN}$. Hence, $\overline{AL}$ is the perpendicular bisector of $\overline{MN}$.

$\therefore$ A lies on the perpendicular bisector of $\overline{MN}$.

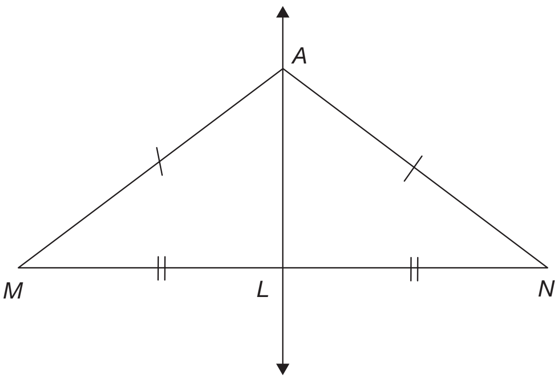


Figure 15.4

Step 2: Now, we prove that any point on the perpendicular bisector of the line segment is equidistant from the end points of the line segment.

Given: MN is a line segment and P is a point on the perpendicular bisector. L is the mid-point of MN (see Fig. 15.5).

To prove: $MP = NP$.

Proof: If P coincides with L, then $MP = NP$.

Suppose P is different from L. Then, in ΔMLP and ΔNLP, $ML = LN$.

LP is the common side and $\Delta MLP = \Delta NLP = 90°$.

$\therefore$ By the SAS congruence property, $\Delta MLP \cong \Delta NLP$.

So, $MP = PN$ ($\because$ The corresponding elements of congruent triangles are equal.)

$\therefore$ Any point on the perpendicular bisector of $\overline{MN}$ is equidistant from points M and N.

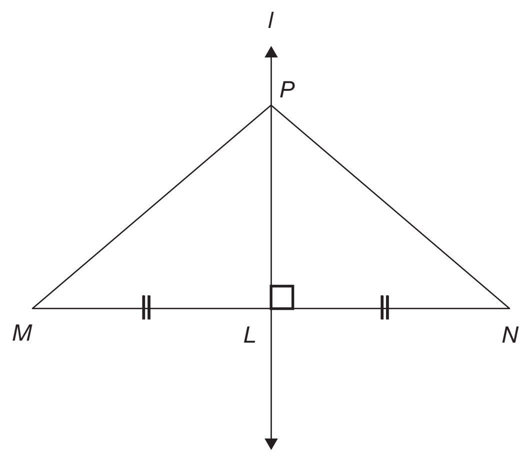


Figure 15.5

Hence, from Step I and Step II of the proof, it can be said that the locus of point equidistant from two fixed points is the perpendicular bisector of the line segment joining the two points.

EXAMPLE 15.2

Show that the locus of a point, equidistant from two intersecting lines in the plane, is a pair of lines bisecting the angles formed by the given lines.

SOLUTION

Step 1: We initially prove that any point, equidistant from two given intersecting lines, lies on one of the lines bisecting the angles formed by the given lines.

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two lines intersecting at O. P is the point on the plane such that $PM = PN$. Line l is the bisector of $\angle BOD$ and $\angle AOC$.

Line m is the bisector of $\angle BOC$ and $\angle AOD$ (see Fig. 15.6).

To prove: P lies either on the line l or on the line m.

Proof: In ΔPOM and ΔPON, $PM = PN$,

OP is a common side and $\angle PMO = \angle PNO = 90°$.

$\therefore$ By RHS congruence property, $\Delta POM \cong \Delta PON$.

So, $\angle POM = \angle PON$, i.e., P lies on the angle bisector of $\angle BOD$.

As l is the bisector of $\angle BOD$ and $\angle AOC$, P lies on the line l.

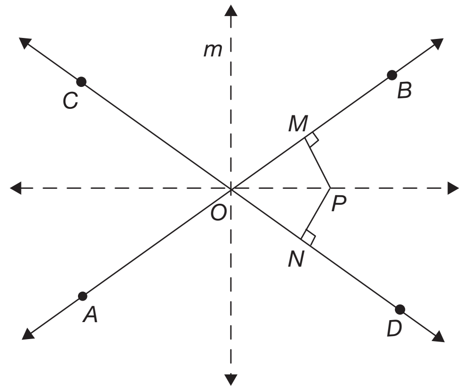


Figure 15.6

Similarly, if P lies in any of the regions of $\angle BOC$, $\angle AOC$ or $\angle AOD$, such that it is equidistant from $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, then we can conclude that P lies on the angle bisector l or on the angle bisector m.

Step 2: Now, we prove that any point on the bisector of one of the angles formed by two intersecting lines is equidistant from the lines.

Given: Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, intersect at O. Lines l and m are the angle bisectors.

Proof: Let l be the angle bisector of $\angle BOD$ and $\angle AOC$, and m be the angle bisector of $\angle BOC$ and $\angle AOD$.

Let P be a point on the angle bisector l, as shown in Fig. 15.7.

If P coincides with O, then P is equidistant from line $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

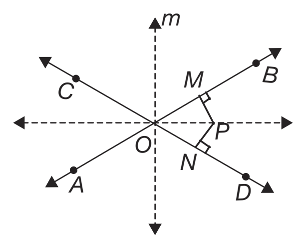


Figure 15.7

Suppose P is different from O.

Draw the perpendiculars $\overline{PM}$ and $\overline{PN}$ from the point P onto the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ respectively.

Then in ΔPOM and ΔPON,

$\angle POM = \angle PON$, $\angle PNO = \angle PMO = 90°$ and OP is a common side.

$\therefore$ By the AAS congruence property,

$\Delta POM \cong \Delta PON$.

So, $PN = PM$ ($\because$ The corresponding sides in congruent triangles.)

That is, P is equidistant from lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Hence, from Step I and Step II of the proof, it can be said that the locus of the point, which is equidistant from the two intersecting lines is the pair of the angle bisectors of the two pairs of vertically opposite angles formed by the lines.

EQUATION OF A LOCUS

We know that the locus is the set of points that satisfy a given geometrical condition. When we express the geometrical condition in the form of an algebraic equation, the equation is called the equation of the locus.

Steps to Find the Equation of a Locus

1. Consider any point (x_1, y_1) on the locus
2. Express the given geometrical condition in the form of an equation using x_1 and y_1.
3. Simplify the equation obtained in Step 2.
4. Replace (x_1, y_1) by (x, y) in the simplified equation, which gives the required equation of the locus.

The following formulae will be helpful in finding the equation of a locus:

1. Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. Area of the triangle formed by joining points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

   $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$, where the value of $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.
3. Equation of the circle with centre (a, b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$.
4. The perpendicular distance from a point $P(x_1, y_1)$ to a given line $ax + by + c = 0$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|.$$

EXAMPLE 15.3

Find the equation of the locus of a point which forms a triangle of area 5 square units with the points $A(2, 3)$ and $B(-1, 4)$.

SOLUTION

Let $P(x_1, y_1)$ be point on the locus, $(x_2, y_2) = (2, 3)$ and $(x_3, y_3) = (-1, 4)$.

Given area of $\Delta PAB = 5$ sq. units.

$$\therefore \frac{1}{2}\begin{vmatrix} x_1 - 2 & 2 - (-1) \\ y_1 - 3 & 3 - 4 \end{vmatrix} = 5$$

$$\Rightarrow \begin{vmatrix} x_1 - 2 & 3 \\ y_1 - 3 & -1 \end{vmatrix} = 10 \Rightarrow -(x_1 - 2) - 3(y_1 - 3) = \pm 10$$

$$\Rightarrow x_1 + 3y_1 - 11 = \pm 10.$$

The required equation is,

$$x + 3y = 10 + 11 \quad \text{or} \quad x + 3y = -10 + 11$$
$$x + 3y - 21 = 0 \quad \text{or} \quad x + 3y - 1 = 0$$

CONCURRENCY—GEOMETRIC CENTRES OF A TRIANGLE

Let us recall that if three or more lines pass through a fixed point, then the lines are said to be **concurrent** and the fixed point is called the point of **concurrence**. In this context, we recall different concurrent lines and their points of concurrence associated with a triangle.

Geometric Centres of a Triangle

Circum-centre

The locus of the point equidistant from the endpoints of the line segment is the perpendicular bisector of the line segment. The three perpendicular bisectors of the three sides of a triangle are concurrent. The point of their concurrence is called the **circum-centre** of the triangle. It is usually denoted by S. The circum-centre is equidistant from all the vertices of the triangle. The circum-centre of the triangle is the locus of the point in the plane of the triangle equidistant from the vertices of the triangle (see Fig. 15.8).

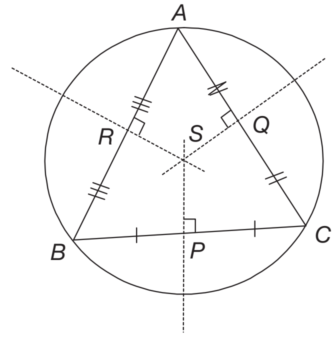


Figure 15.8

In-centre

The angle bisectors of the triangle are concurrent. The point of concurrence is called in-centre. It is usually denoted by I. The point I is equidistant from the sides of the triangle. The **in-centre** of the triangle is the locus of the point, in the plane of the triangle, equidistant from the sides of the triangle (see Fig. 15.9).

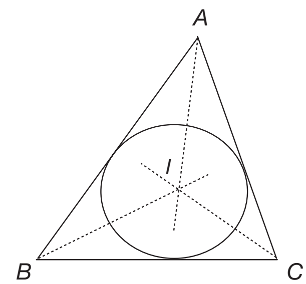


Figure 15.9

Ex-centre

The point of concurrence of the external bisector of two angles of a triangle and the internal bisector of the third angle is called **ex-centre** of the triangle (see Fig. 15.10).

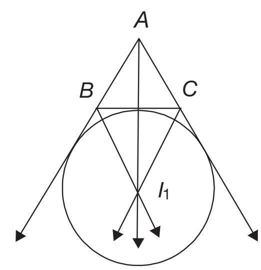


Figure 15.10

Ortho-centre

The altitudes of the triangle are concurrent and the point of concurrence of the altitudes of a triangle is called **ortho-centre**. It is usually denoted by O (see Fig. 15.11).

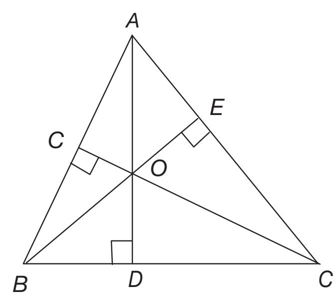


Figure 15.11

Centroid

The medians of a triangle are concurrent and the point of concurrence of the medians of a triangle is called **centroid**. It is usually denoted by G. The centroid divides each of the medians in the ratio 2 : 1, beginning from vertex, i.e., $AG : GD = 2 : 1$, $BG : GE = 2 : 1$ and $CG : GF = 2 : 1$, as shown in Fig. 15.12.

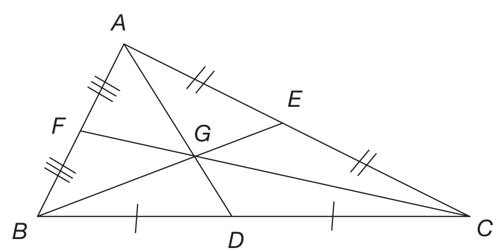


Figure 15.12

Some Important Points

1. In an equilateral triangle, the centroid, the ortho-centre, the circum-centre and the in-centre all coincide.
2. In an isosceles triangle, the centroid, the ortho-centre, the circum-centre and the in-centre all lie on the median to the base.
3. In a right triangle, the length of the median drawn to the hypotenuse is equal to half of the hypotenuse. The median is also equal to the circum-radius. The mid-point of the hypotenuse is the circum-centre.
4. In an obtuse-angled triangle, the circum-centre and ortho-centre lie outside the triangle. For an acute-angled triangle, the circum-centre and the ortho-centre lie inside the triangle.
5. For all triangles, the centroid and the in-centre lie inside the triangle.
6. For all triangles, the ex-centre lies outside the triangle.

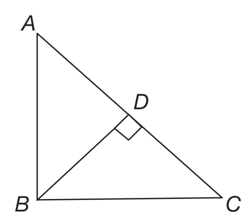
TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Centroid divides the median from the vertex in the ratio ______.
2. Any point on the perpendicular bisector of a line segment joining two points is ______ from the two points.
3. What is the locus of a point in a plane which is at a distance of p units from the circle of radius q units $(p \neq q)$?
4. If A and B are two fixed points, then the locus of a point P, such that $\angle APB = 90°$ is ______.
5. The locus of the point equidistant (in a plane) from the three vertices of a triangle is ______.
6. The locus of the point in a plane which is equidistant from two intersecting lines is ______.
7. The line segment from the vertex of a triangle perpendicular to its opposite side is called ______.
8. The path traced out by a moving point which moves according to some given geometrical conditions is ______.
9. Is the statement 'In ΔABC, a point equidistant from AB and AC lies on the median', true?
10. The path of a freely falling stone is ______.
11. The orthocentre of a right triangle is the ______.
12. The line segment joining the vertex of a triangle and the mid-point of its opposite side is called ______.
13. The locus of the centre of the circles (in a plane) passing through two given points is the ______ of the line segment joining the two points.
14. The circum-centre of a right triangle always lies ______.
15. The locus of the tip of the hour hand is ______.

Short Answer Type Questions

16. Find the locus of the vertex of a triangle with fixed base and having constant area.
17. Find the locus of the centre of a circle passing through two fixed points A and B.
18. Let A and B be two fixed points in a plane. Find the locus of a point P, such that $PA^2 + PB^2 = AB^2$.
19. Find the locus of the point P, such that $TP : MP = 3 : 2$, where T is $(-2, 3)$ and M is $(4, -5)$.
20. Find the locus of the point which is equidistant from sides AB and AD of a rhombus $ABCD$.
21. In the following figure (not to scale), ABC is a right isosceles triangle, right-angled at B and $\overline{BD} \perp \overline{AC}$. If the triangle ABC is rotated about the hypotenuse, then find the locus of the triangle ABC.



22. Show that the locus of a point, equidistant from three distinct given collinear points in a plane does not exist.
23. If ZQ and RU be two lines intersecting at point E, then find the locus of a point moving in the interior of $\angle UEZ$, such that the sum of its distances from the lines ZQ and RU is b units.
24. Find the locus of the point which is equidistant from the sides AB and AC of triangle ABC.
25. ΔAPQ, ΔBPQ and ΔCPQ are three isosceles triangles with the same base PQ. Show that the points A, B and C are collinear.

Essay Type Questions

26. In a square $ABCD$, if A and B are $(5, 1)$ and $(7, 1)$ respectively, then what is the locus of the mid-point of diagonal AC?
27. If $P(x, y)$ and $Q(1, 4)$ are the points on the circle whose centre is $C(5, 7)$, then find the locus of P.

PRACTICE QUESTIONS

28. If two lines intersect at P at right angles and pass through $A(1, 1)$ and $B(1, 0)$ respectively then what is the locus of P?

29. In ΔPAB, D, E and F are the mid-points of PA, AB and BP, respectively. The area of DEF is 8 sq. units. If A is (2, 5) and B is (3, 4), then what is the locus of P?

30. If P is a point on the circle with AB as a diameter, where A and B are (0, 2) and (2, 4) respectively, then the locus of P is ______.

CONCEPT APPLICATION

Level 1

1. The locus of a point equidistant from three fixed points is a single point. The three points are ______.
   (a) collinear (b) non-collinear
   (c) coincidental (d) None of these

2. The locus of a point moving in a space which is at a constant distance from a fixed point in space is called a ______.
   (a) square (b) sphere
   (c) circle (d) triangle

3. The locus of the centre of the circle that touch a given circle internally is a ______.
   (a) straight line (b) hellix
   (c) circle (d) None of these

4. In ΔABC, $\angle A = \angle B + \angle C$, then the circum-centre is at ______.
   (a) A (b) B
   (c) C (d) the mid-point of BC

5. The locus of a point equidistant from three points does not exist. This implies that the three points are ______.
   (a) collinear (b) non-collinear
   (c) coincidental (d) None of these

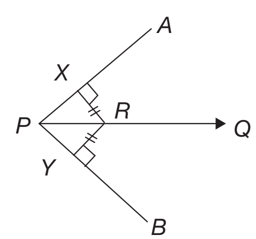
6. The locus of a point which is equidistant from two non-intersecting lines l and m is a ______.
   (a) straight line parallel to the line l
   (b) straight line parallel to the line m
   (c) straight line parallel to the lines l and m and midway between them
   (d) straight line that intersects both the lines l and m

7. The locus of a point which is at a constant distance k from Y-axis is ______.
   (a) $x = \pm k$ (b) $y = \pm k$
   (c) $x = 0$ (d) $y = 0$

8. The locus of the centre of a wheel rolling on a straight road is a ______.
   (a) concentric circle (b) straight line
   (c) curve path (d) parabola

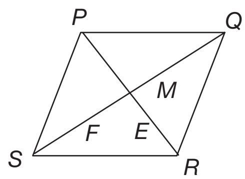
9. If A and B are two fixed points, then the locus of a point P, such that $PA^2 + PB^2 = AB^2$ is a/an ______.
   (a) circle with AB as the diameter
   (b) right triangle with $\angle P = 90°$
   (c) semi-circle with AB as the diameter
   (d) circle with AB as the diameter, excluding points A and B

10. In the figure, $\overline{RX} \perp \overline{PA}$, $\overline{RY} \perp \overline{PB}$, $RX = RY$ and $\angle APB = 70°$. Find $\angle APQ$.
   (a) 70° (b) 140°
   (c) 35° (d) 50°



11. Consider a point M inside a quadrilateral $PQRS$. If M be the point of intersection of angle bisectors PE and QF, then ______.
   (a) M is nearer to $\overline{PS}$ than to $\overline{QR}$
   (b) M is equidistant from opposite sides $\overline{PS}$ and $\overline{QR}$
   (c) M is nearer to $\overline{QR}$ than to $\overline{PS}$
   (d) M is equidistant from opposite sides $\overline{PQ}$ and $\overline{SR}$

PRACTICE QUESTIONS



12. The locus of the point which is equidistant from the three lines determined by the sides of a triangle is ______.

(a) the in-centre

(b) the ex-centre

(c) the ortho-centre

(d) either (a) or (b)

13. Find the locus of any fixed point on the circumference of a coin when the coin is rolling on a straight path.

(a) Circle (b) Straight line

(c) Sphere (d) Helix

14. The locus of a point which is equidistant from the coordinate axes can be a ______.

(a) line making a non-zero intercept on X-axis

(b) line making a non-zero intercept on Y-axis

(c) line passing through the origin making an angle of 45° with X-axis

(d) None of the above

15. The locus of a point equidistant from two intersecting lines PQ and RS, and at a distance of 10 cm from their point of intersection O is ______.

(a) four points lying on the angle bisectors at a distance of 5 cm from O

(b) two points lying on the angle bisectors at a distance of 10 cm from O

(c) four points lying on the angle bisectors at a distance of 10 cm from O

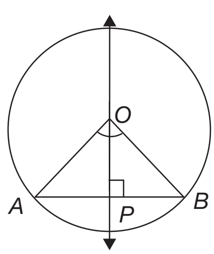
(d) two points lying on the angle bisectors at a distance of 5 cm from O

Level 2

16. In the figure following figure, O is the centre of the circle and $\overline{AL} \perp \overline{MN}$. If $\angle AOB = 90°$, then find $\angle AOP$.

(a) 60° (b) 20°

(c) 30° (d) 45°



17. The solid formed when a right triangle is rotated about one of the sides containing the right angle is a ______.

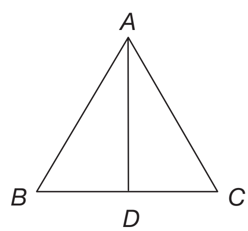
(a) prism (b) cylinder

(c) cone (d) sphere

18. In the figure (not to scale), $AB = AC$ and $BD = CD$. Find $\angle ADB$.

(a) 60° (b) 90°

(c) 120° (d) Cannot be determined



19. A part of the locus of a point P, which is equidistant from two intersecting lines $ax + by + c = 0$ and $px + qy + r = 0$, is ______.

(a) $(a - p)x + (b - q)y + (c - x) = 0$

(b) $apx + qby + cy = 0$

(c) $\sqrt{a^2 + b^2}(px + qy + r) = \sqrt{p^2 + q^2}(ax + by + c)$

(d) None of these

20. If PAB is a triangle of area 4 sq. units and A is (2, 5) and B is (3, 4), then part of the locus of P is ______.

(a) $x - y + 15 = 0$

(b) $x - y - 15 = 0$

(c) $x + y - 15 = 0$

(d) $x + y + 15 = 0$

21. Given, triangle PBC and parallelogram $ABCD$ lie between the same parallel lines. On the same base, BC and the area of parallelogram is 2 sq. units.

The points B and C are (2, 4) and (4, 4) respectively. Which of the following lines is a part of the locus of P?

(a) $y = 4$ (b) $y = 7$
(c) $y = 5$ (d) $y = 6$

22. If PAB is a triangle in which $\angle B = 90°$ and $A(1, 1)$ and $B(0, 1)$, then the locus of P is ______.

(a) $y = 0$ (b) $xy = 0$
(c) $x = y$ (d) $x = 0$

23. The locus of P whose distance from the X-axis is thrice the distance from the line $x = 5$ is ______.

(a) $x - y - 5 = 0$
(b) $3x - y - 15 = 0$
(c) $3x + y + 15 = 0$
(d) $x + y + 5 = 0$

24. Two of the vertices of a triangle ABC are $A(1, 1)$, $B(-1, -3)$ and the area of ΔABC is 6 sq. units. If P is the centroid of the ΔABC, then find the locus of P.

(a) $2x - y + 1 = 0$ (b) $2x - y - 3 = 0$
(c) $2x + y + 3 = 0$ (d) Both (a) and (b)

25. In a circle with radius 25 cm, what is the area of the region determined by the locus of the mid-points of chords of length 48 cm?

(a) 154 cm^2 (b) 254 cm^2
(c) 72 cm^2 (d) None of these

26. The locus of the centre of a circle that touches the given circle externally is a ______.

(a) curve (b) straight line
(c) circle (d) helix.

27. The locus of a rectangle, when the rectangle is rotated about one of its sides, is a ______.

(a) plane (b) sphere
(c) cone (d) cylinder.

28. If the ortho-centre of a triangle ABC is B, then which of the following is true?

(a) $AC^2 = AB^2 + BC^2$
(b) $AC^2 > AB^2 + BC^2$
(c) $AC^2 < AB^2 + BC^2$
(d) None of these

29. In ΔABC, $\angle A = \angle B + \angle C$. The point which is equidistant from A, B and C is ______.

(a) mid-point of AB
(b) mid-point of AC
(c) mid-point of BC
(d) None of these

30. The locus of a point which is equidistant from (0, 2) and (0, 8) is ______.

(a) $y = 4$ (b) $y = 5$
(c) $x = 4$ (d) $x = 5$

Level 3

31. The area of ΔPQR is 4 sq. units Q and R are (1, 1) and (1, 0), respectively. Which of the following lines is a part of the locus of P?

(a) $x - 6 = 0$ (b) $x - 7 = 0$
(c) $x + 8 = 0$ (d) $x + 7 = 0$

32. What is the locus of the point $P(x, y)$ (where $xy > 0$), which is at a distance of 2 units from the origin?

(a) $x^2 + y^2 = 4$
(b) $x^2 + y^2 = 4, x > 0, y > 0$
(c) $x^2 + y^2 = 4, x < 0, y < 0$
(d) None of these

33. The locus of a point which is twice as far from each vertex of a triangle as it is from the mid-point of the opposite side is a/an/the _______.

(a) median (b) centroid
(c) incentre (d) angle bisector

34. The locus of a point which is collinear with the two given points is ______.

(a) a circle (b) a triangle
(c) a straight line (d) a parabola

35. The locus of a point, which is at a distance of 8 units from (0, –7), is ______.

(a) $x^2 + y^2 + 6x + 14y - 15 = 0$
(b) $x^2 + y^2 + 14y - 15 = 0$
(c) $y^2 + 14y - 8 = 0$
(d) $x^2 + y^2 + 14x + 14y - 15 = 0$

36. The area of ΔABC is 2 sq. units. If $A = (2, 4)$ and $B = (4, 4)$, then find the locus of $C(x, y)$.

(a) $y - 6 = 0$
(b) $y - 2 = 0$
(c) Both (a) and (b)
(d) None of these

37. Find the locus of a point which is a constant distance of 4 units away from the point (2, 4).

(a) $x^2 + y^2 - 4x - 8y + 4 = 0$
(b) $x^2 + 4x + 16 = 0$
(c) $y^2 - 8y + 12 = 0$
(d) None of these

38. P is the point of intersection of the diagonals of a square $READ$. P is equidistant from ______.

(a) the vertices R, E, A and D
(b) $\overline{RE}$ and $\overline{EA}$
(c) $\overline{EA}$ and $\overline{AD}$
(d) All of these

39. A coin of radius 1 cm is moving along the circumference and interior of a square of side 5 cm. Find the locus of centre of the coin.

(a) A square of side 6 cm
(b) A square of side 4 cm
(c) A square of side 3 cm
(d) A square of side 2 cm

40. ABC is a triangle in which $AB = 40$ cm, $BC = 41$ cm and $AC = 9$ cm. Then ortho-centre of ΔABC lies ______.

(a) interior of the triangle
(b) exterior of the triangle
(c) on the triangle
(d) at the mid-point of the triangle

41. O is an interior point of a rhombus, $ABCD$, and O is equidistant from BC and CD. Then O lies on ______.

(a) $\overline{AC}$
(b) $\overline{BD}$
(c) Either (a) or (b)
(d) Neither (a) nor (b)

42. In a triangle ABC, D is a point on BC, such that any point on AD is equidistant from points B and C. Which of the following is necessarily true?

(a) $AB = BC$
(b) $BC = AC$
(c) $AC = AB$
(d) $AB = BC = AC$

43. The locus of a point, equidistant from the coordinate axes is ______.

(a) $x = |y|$
(b) $y = |x|$
(c) Both (a) and (b)
(d) None of these

44. P is an interior point of an equilateral triangle ABC. If P is equidistant from AB and BC, BC and AC, then $\angle BPC$ = ______.

(a) 120° (b) 90°
(c) 60° (d) 150°

45. The locus of a point, which is equidistant from (2, 6) and (2, 8), is ______.

(a) $y = 7$ (b) $x = 7$
(c) $x = 2$ (d) $y = 2$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 2 : 1
2. equidistant
3. Another circle of radius $(p + q)$ or $(q - p)$ concentric to it.
4. circle with diameter AB excluding the points A and B.
5. circum-centre
6. angle bisectors of the two pairs of vertically opposite angles of intersecting lines.
7. altitude
8. locus
9. No
10. vertical line
11. vertex containing the right angle
12. median
13. perpendicular bisector
14. on the mid-point of the hypotenuse
15. circle

Short Answer Type Questions

16. The required locus is a line parallel to the given base.
17. The required locus is the perpendicular bisector of $\overline{AB}$, except the mid-point of $\overline{AB}$.
18. The required locus is the circle with diameter AB, excluding points A and B.
19. $5x^2 + 5y^2 - 88x + 114y + 317 = 0$ is the required locus of point P.
20. The required locus is diagonal AC.
21. According to given conditions, two equal cones of base radius BD and slant height AB or BC are formed in such a way that their bases touch one another.
23. The required locus is the angular bisector of the angle RPM.
24. The required locus is the bisector of $\angle BAC$.

Essay Type Questions

26. $y = (2, 0)$
27. $x^2 + y^2 - 10x - 14y + 49 = 0$
28. $x^2 + y^2 - 2x - y + 1 = 0$
29. $x + y - 71 = 0$; $x + y + 57 = 0$
30. $x^2 + y^2 - 2x - 6y + 8 = 0$

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (c)
11. (b)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (c)					

Level 2

16. (d)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (c)	**21.** (c)	**22.** (d)	**23.** (b)	**24.** (d)	**25.** (a)
26. (c)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (b)					

Level 3

31. (d)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (c)	**40.** (c)
41. (a)	**42.** (c)	**43.** (c)	**44.** (a)	**45.** (a)					

CONCEPT APPLICATION

Level 1

1. Three points form a triangle.
2. Recall the definition.
4. Recall the properties of right triangle with respect to geometric centres.
6. Non-intersecting lines are parallel.
7. Equations of the lines which are k units from the Y-axis.
8. When a wheel is rolling on the straight path, the distance from centre to the path remains same.
9. Angle in semicircle is 90°.
10. $\overline{AB}$ is bisector of $\angle APB$.
12. Recall the definitions of geometric centres.
14. Required locus is a line which is in the midway of X-axis and Y-axis.
15. Points on angle bisectors.

Level 2

17. (i) When a right triangle is rotated about one of its perpendicular sides, the other perpendicular side acts as radius of the base and the hypotenuse acts as the slant height of solid.

(ii) The top of a solid is a point (vertex).

18. $\Delta ADB \cong \Delta ADC$.

19. (i) Perpendicular distance of a point $P(x_1, y_1)$ from the line, $px + qy + r = 0$, is

$$\frac{|px_1 + qy_1 + r|}{\sqrt{p^2 + q^2}}.$$

(ii) Perpendicular distance of the point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

20. (i) Given, area of a triangle $ABC = 4$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix} = 4.$$

(ii) Substitute $(x_2, y_2) = (2, 5)$ and $(x_3, y_3) = (3, 4)$ and obtain the relation.

21. (i) The area of the given triangle will be half that of the given parallelogram.

(ii) Area of ΔPBC is half of the area of parallelogram $ABCD$.

(iii) Area of ΔPBC is given by

$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$$

and proceed same as above.

22. (i) Angle in a semicircle is 90°.

(ii) Use, $(PA)^2 = (PB)^2 + (AB)^2$ and obtain the required locus.

23. (i) The perpendicular distance from $P(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

(ii) The distance between P and X-axis is equal to thrice the distance between P and $x = 5$.

24. (i) As P is the centroid of ΔABC, area of ΔPAB is $\frac{1}{3}$(area of ΔABC).

(ii) Area of ΔPAB is given by

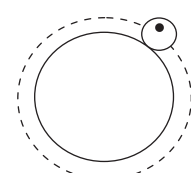
$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}.$$

25. (i) Locus of the mid-points of equal chords in a circle forms a concentric circle with the given circle.

(ii) The required locus is a circle of radius 7 cm.

(iii) Now, find its area.

26. The required locus is a circle.



27. The required locus is a cylinder.

28. In a right triangle, the orthocentre lies at the vertex which contains right angle.

$\therefore$ The given triangle is a right triangle, where $\angle B = 90°$.

$\therefore AC^2 = AB^2 + BC^2$.

29. Given, in a ΔABC, $\angle A = \angle B + \angle C$

$\Rightarrow \angle A = 90°$, and BC is the hypotenuse.

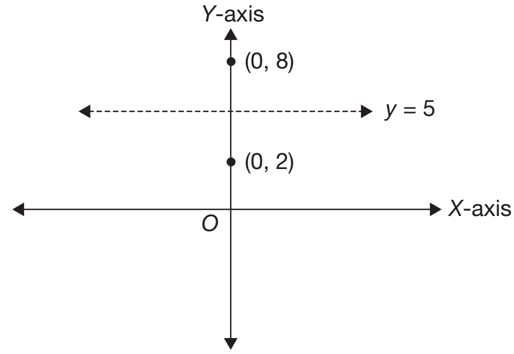
The point equidistant from the vertices of a triangle is circum-centre of the triangle.

For a right triangle, circum-centre lies at the mid-point of the hypotenuse.

$\therefore$ The required point is the mid-point of BC.

30. The locus of a point which is equidistant from two points is the perpendicular bisector of the line segment joining the points.

$\therefore$ The required locus is $y = 5$.



Level 3

31. (i) Area of triangle is given by

$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}.$$

(ii) Substitute $(x_2, y_2) = (1, 1)$, $(x_3, y_3) = (1, 0)$ and obtain the relation in terms of x_1 and y_1.

32. (i) If $xy > 0$, then $x > 0$, $y > 0$ (or) $x < 0$, $y < 0$.

(ii) Given, $OP = 2$, where $O = (0, 0)$ and $P = (x, y)$.

(iii) Substitute the values and obtain the locus.

33. (i) Recall the concept of medians in a triangle.

(ii) The point of concurrence of medians is the centroid.

(iii) Centroid divides each median in the ratio 2 : 1.

36. Area of $A(2, 4)$, $B(4, 4)$ and $C(x, y)$ is 2 sq. units.

$$\frac{1}{2}\begin{vmatrix} 2-4 & 4-x \\ 4-4 & 4-y \end{vmatrix} = 2$$

$$\begin{vmatrix} -2 & 4-x \\ 0 & 4-y \end{vmatrix} = 4 \Rightarrow |2y - 8| = 4$$

$$\Rightarrow |y - 4| = 2 \Rightarrow y - 4 = \pm 2$$

$$\Rightarrow y - 6 = 0 \quad \text{or} \quad y - 2 = 0.$$

37. The required locus is a circle of radius 4 units and passing through point (2, 4).

Let $P(x, y)$ be any point on the locus.

$$\therefore \sqrt{(x-2)^2 (y-4)^2} = 4$$

$$x^2 + y^2 - 4x - 8y + 4 = 0.$$

38. Given, P is the point of intersection of the diagonals of a square $READ$.

In a square, diagonals are equal and bisect to each other.

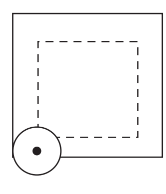
$\therefore$ P is equidistant from the vertices.

As P lies on the diagonals, P is equidistant from any two adjacent sides.

$\therefore$ Hence, the correct answer is option (d).

39. When a coin is moving on the circumference of a square, the path of the center of the coin is a square.

$\therefore$ The required locus is a square of side $(5 - 2)$ cm, i.e., 3 cm.



40. Given, $AB = 40$ cm, $BC = 41$ cm and $AC = 9$ cm.

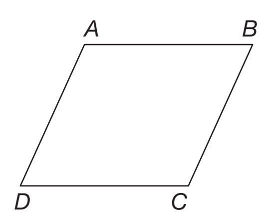
$\therefore$ ABC is right triangle, right angled at A.

$\therefore$ Ortho-centre lies at A.

$\therefore$ Hence, the correct answer is option (c).

41. Given, O is equidistant from BC and CD.

$\therefore$ O lies on the bisector of $\angle BCD$, i.e., $\overline{AC}$.



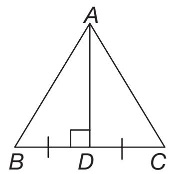
42. Given, any point on AD is equidistant from B and C.

$\therefore$ $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$.

By SAS congruence property,

$\Delta ADB \cong \Delta ADC$.

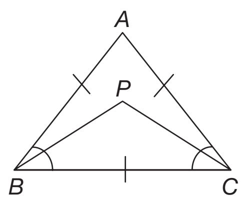
By CPCT, $AB = AC$.



43. The required locus is the union of the locus passing through the origin and making angles of 45° and 135° with the X-axis in positive direction.

That is, $x = |y|$ or $y = |x|$.

44. Here, P is the point of intersection of bisectors of $\angle B$ and $\angle C$.



$\therefore \angle PBC = \angle PCB = 30°$

$\Rightarrow \angle BPC = 180° - (30° + 30°) = 120°$.

45. Let $A = (2, 6)$, $B = (2, 8)$.

The required locus is the perpendicular bisector of $\overline{AB}$.

Let K be the point of intersection of $\overline{AB}$ and its perpendicular bisector.

$$K = \left(\frac{2+2}{2}, \frac{8+6}{2}\right) = (2, 7). \qquad (1)$$

Slope of $\overline{AB} = \dfrac{8-6}{2-2}$ is not defined.

$\therefore$ $\overline{AB}$ is parallel to Y-axis. The required line is parallel to X-axis (2)

$\therefore$ The required line is $y = 7$ (From Eqs. (1) and (2)).