4. Matrices

Exercise 4.1

1. Question

The rates for the entrance tickets at a water theme park are listed below:

	week days rates (₹)	Week End Rates (₹)
Adult	400	500
Children	200	250
Senior Citizen	300	400

Write down the matrices for the rates of entrance tickets for adults, children and senior citizens. Also find the dimensions of the matrices.

Answer

The given table can be expressed as a matrix where each column denotes the week and weekend rates for Adult, Children and Senior Citizen.

$$\Rightarrow \begin{pmatrix} 400 & 500 \\ 200 & 250 \\ 300 & 400 \end{pmatrix}$$

The dimension of above matrix is 3 × 2.

Also, the same information can also be expressed as a matrix where each row denotes the week and weekend rates for Adult, Children and Senior Citizen.

$$\Rightarrow \begin{pmatrix} 400 & 200 & 300 \\ 500 & 250 & 400 \end{pmatrix}$$

The dimension of above matrix is 2 × 3.

2. Question

There are 6 Higher Secondary Schools, 8 High Schools and 13 Primary Schools in a town. Represent these data in the form of 3 × 1 and 1 × 3 matrices.

Answer

Representing the given information in a 3 × 1 matrix, we get,

$$\Rightarrow \begin{pmatrix} 6 \\ 8 \\ 13 \end{pmatrix}$$

Also, representing the given information in a 1 × 3 matrix, we get,

$\Rightarrow (6\ 8\ 13)$

3. Question

Find the order of the following matrices.

(i) $\begin{pmatrix} 1 & -1 & 5 \\ -2 & 3 & 4 \end{pmatrix}$

(ii) $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & -2 & 6 \\ 6 & -1 & 1 \\ 2 & 4 & 5 \end{pmatrix}$

(iv) (3 4 5)

(v) $\begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 9 & 7 \\ 6 & 4 \end{pmatrix}$

Answer

(i) Order is 2 × 3

(ii) Order is 3 × 1

(iii) Order is 3 × 3

(iv) Order is 1 × 3

(v) Order is 4 × 2

4. Question

A matrix has 8 elements. What are the possible orders it can have?

Answer

Since there are 8 elements, we can make the multiples of 8, which are:- 1, 8, 2, 4

Therefore, the possible orders of a matrix are 1×8; 8×1; 2×4 and 4×2.

5. Question

Matrix consists of 30 elements. What are the possible orders it can have?

Answer

Since there are 30 elements, we can make the multiples of 30, which are:- 1×30; 30×1; 2×15; 15×2; 3×10; 10×3; 5×6 and 6×5

Therefore, the possible orders of a matrix are 1×30; 30×1; 2×15; 15×2; 3×10; 10×3; 5×6 and 6×5

6. Question

Construct a 2x2 matrix A=[a_{ij}] whose elements are given by

(i) a_{ij} = ij

(ii) a_{ij} = 2i – j

(iii) $a_{ij} = \frac{i-j}{(i+j)}$

Answer

(i) Since a_{ij} = i × j, and the general of matrix is:

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

On substituting the values, we get,

$\Rightarrow \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

(ii) Since the general of matrix is:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

On substituting the values, we get,

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

(iii) Since the general of matrix is:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

On substituting the values, we get,

$$\Rightarrow \begin{pmatrix} 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 \end{pmatrix}$$

7. Question

Construct a 3 x 2 matrix A=[a_{ij}] whose elements are given by

(i) $a_{ij} = \frac{i}{j}$

(ii) $a_{ij} = \frac{(i-2j)^2}{2}$

(iii) $a_{ij} = \frac{|2i-3j|}{2}$

Answer

(i) Since the general of matrix is:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

On substituting the values, we get,

$$\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$$

(ii) Since the general of matrix is:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

On substituting the values, we get,

$$\Rightarrow \begin{pmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{pmatrix}$$

(iii) Since the general of matrix is:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

On substituting the values, we get,

$\Rightarrow \begin{pmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{pmatrix}$

8. Question

If A = $\begin{pmatrix} 1 & -1 & 3 & 2 \\ 5 & -4 & 7 & 4 \\ 6 & 0 & 9 & 8 \end{pmatrix}$, (i) find the order of the matrix

(ii) write down the elements A_{24} and a_{32}

(iii) in which row and column does the element 7 occur?

Answer

(i) Order of matrix is 3 × 4.

(ii) Since the general of matrix is:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$

⇒ $a_{24} = 4$ and $a_{32} = 0$

(iii) Element 7 occurs in 2^{nd} row 3^{rd} column

9. Question

If $A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \\ 5 & 0 \end{pmatrix}$, then find the transpose of *A*.

Answer

For the transpose of a matrix, we know that,

$A^T_{ij} = A_{ji}$, therefore the transpose of matrix A is:-

$$A^T = \begin{pmatrix} 2 & 4 & 5 \\ 3 & 1 & 0 \end{pmatrix}$$

10. Question

If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & -5 & 6 \end{pmatrix}$, then verify that $(a^T)^T = A$.

Answer

For the transpose of a matrix, we know that,

$A^T_{ij} = A_{ji}$, therefore the transpose of matrix A is:-

$$A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & 5 & 6 \end{pmatrix}$$

Now, applying transpose on A^T , we get,

$$(A^T)^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & -5 & 6 \end{pmatrix}$$

∴ We see that $(A^T)^T = A$

Exercise 4.2

1. Question

Find the values of x, y and z from the matrix equation $\begin{pmatrix} 5x+2 & y-4 \\ 0 & 4z+6 \end{pmatrix} = \begin{pmatrix} 12 & -8 \\ 0 & 2 \end{pmatrix}$

Answer

Since given is the matrix equation we would equate the right hand side elements with left hand side elements

⇒ 5x + 2 = 12 , y – 4 = – 8 and 4z + 6 = 2

⇒ 5x = 10 , y = – 4 and 4z = – 4

⇒ x = 2 , y = – 4 and z = – 1

2. Question

Solve for x and y if $\begin{pmatrix} 2x+y \\ x-3y \end{pmatrix} = \begin{pmatrix} 5 \\ 13 \end{pmatrix}$

Answer

Since given is the matrix equation we would equate the right hand side elements with left hand side elements

2x + y = 5 …1

and x – 3y = 13 ….2

Multiplying equation 2 by 2

we get (x – 3y = 13) 2

2x – 6y = 26 ….3

Subtracting equation 1 and 3

$$\begin{array}{l} 2x + y = 5 \\ 2x - 6y = 26 \\ -\quad\ +\quad\ - \\ \hline 7y = -21 \end{array}$$

Or

Y = – 3

Substituting value of y in 1

2x – 3 = 5 ⇒ 2x = 8 ⇒ x = 4

Hence the solution is x = 4 and y = – 3

3. Question

If $A = \begin{pmatrix} 2 & 3 \\ -9 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 5 \\ 7 & -1 \end{pmatrix}$, then find the additive inverse of A.

Answer

Let us first solve for the value of matrix A =

$\begin{pmatrix} 2 & 3 \\ -9 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 5 \\ 7 & -1 \end{pmatrix} = \begin{pmatrix} 2-1 = 1 & 3-5 = -2 \\ -9-7 = -16 & 5+1 = 6 \end{pmatrix}$

$\Rightarrow A = \begin{pmatrix} 1 & -2 \\ -16 & 6 \end{pmatrix}$

The additive inverse of A = negative of the matrix

– A = additive inverse of A = $\begin{pmatrix} -1 & 2 \\ 16 & -6 \end{pmatrix}$

4. Question

Let $A = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$. Find the matrix C if C = 2A + B.

Answer

Given A = $\begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 8 & -1 \\ 4 & 3 \end{bmatrix}$

We have to find matrix C where C = 2A + B

Now 2A = 2 $\begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$

We would multiply each term of A with 2

2A = $\begin{bmatrix} 6 & 4 \\ 10 & 2 \end{bmatrix}$

2A + B = $\begin{bmatrix} 6 & 4 \\ 10 & 2 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 6+8 = 14 & 4-1 = 3 \\ 10+4 = 14 & 2+3 = 5 \end{bmatrix}$

C = $\begin{bmatrix} 14 & 3 \\ 14 & 5 \end{bmatrix}$

5. Question

If $A = \begin{pmatrix} 4 & -2 \\ 5 & -9 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 2 \\ -1 & -3 \end{pmatrix}$ find 6A – 3B.

Answer

Given A = $\begin{bmatrix} 4 & -2 \\ 5 & -9 \end{bmatrix}$

B = $\begin{bmatrix} 8 & 2 \\ -1 & -3 \end{bmatrix}$

6A = 6 $\begin{bmatrix} 4 & -2 \\ 5 & -9 \end{bmatrix} = \begin{bmatrix} 24 & -12 \\ 30 & -54 \end{bmatrix}$

3B = 3 $\begin{bmatrix} 8 & 2 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 24 & 6 \\ -3 & -9 \end{bmatrix}$

6A – 3B = $\begin{bmatrix} 24 & -12 \\ 30 & -54 \end{bmatrix} - \begin{bmatrix} 24 & 6 \\ -3 & -9 \end{bmatrix} = \begin{bmatrix} 24-24 = 0 & -12-6 = -18 \\ 30+3 = 33 & -54+9 = -45 \end{bmatrix}$

6A – 3B = $\begin{bmatrix} 0 & -18 \\ 33 & -45 \end{bmatrix}$

6. Question

Find a and b if $a\begin{pmatrix} 2 \\ 3 \end{pmatrix} + b\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$.

Answer

$a\begin{pmatrix}2\\3\end{pmatrix} + b\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix}$

$\Rightarrow\begin{pmatrix}2a\\3a\end{pmatrix} + \begin{pmatrix}-b\\b\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix}$

$\Rightarrow \begin{pmatrix}2a-b\\3a+b\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix}$

Equating both the sides of the matrix equation

We get

2a –b = 10 1

And 3a + b = 5 2

Adding equation 1 and 2

$$\begin{array}{l} 2a - b = 10 \\ + \; 3a + b = 5 \\ \hline \quad 5a = 15 \end{array}$$

⇒ a = 3

Putting value of a in 1

2a –b = 10

⇒ 6 –b = 10 ⇒ b = – 4

Thus the required solutions are a = 3 and b = – 4

7. Question

Find X and Y if 2X + 3Y = $\begin{pmatrix}2 & 3\\4 & 0\end{pmatrix}$ and 3X + 2Y = $\begin{pmatrix}2-2\\-1 \quad 5\end{pmatrix}$.

Answer

Given 2x + 3y = $\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$ 1

and 3x + 2y = $\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}$ ...2

Adding 1 and 3 equations we get,

$$\begin{array}{l} \quad 2x + 3y = \begin{bmatrix}2 & 3\\4 & 0\end{bmatrix} \\ + \; 3x + 2y = \begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix} \\ \hline \end{array}$$

5x + 5y = $\begin{bmatrix}2+2 = 4 & 3-2 = 1\\4-1 = 3 & 0+5 = 5\end{bmatrix}$

$= \begin{bmatrix}4 & 1\\3 & 5\end{bmatrix}$

Dividing both the sides by 5

x + y = $\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\\frac{3}{5} & 1\end{bmatrix}$ ...3

Now subtracting 1 from 2

$$3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$
$$- 2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$X - y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix} \text{......4}$$

Adding 3 and 4

$$x + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix}$$

$+$

$$X - y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$2x = \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

Subtracting 4 from 3

$$x + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix}$$
$$X - y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$
$$- \quad + \quad -$$

$$2y = \begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix}$$

$$Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Hence x = = $\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$ and y = $\begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

8. Question

Solve for x and y if $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 3\begin{pmatrix} 2x \\ -y \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$

Answer

given

$$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 3\begin{pmatrix} 2x \\ -y \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$$

$\Rightarrow \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + \begin{pmatrix} 6x \\ -3y \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x^2 + 6x \\ y^2 - 3y \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$

Equating each element of the matrix equation with corresponding element

$x^2 + 6x = -9$

$\Rightarrow x^2 + 6x + 9 = 0$

$\Rightarrow x(x + 3) + 3(x + 3) = 0$

$\Rightarrow (x + 3)(x + 3) = 0$

$x = -3, -3$

$y^2 - 3y = 4$

$\Rightarrow y^2 - 3y - 4 = 0$

$\Rightarrow y(y - 4) + 1(y - 4) = 0$

$\Rightarrow (y + 1)(y - 4) = 0$

$\Rightarrow y = -1$ or 4

Hence the values of $x = -3, -3$ and $y = -1, 4$

9. Question

if $A = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. then

Verify: (i) $A + B = B + A$ (ii) $A + (-A) = O = (-A) + A$.

Answer

9) given

$A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(i) To verify $A + B = B + A$

LHS:

$A + B = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 + 1 = 4 & 2 - 2 = 0 \\ 5 + 2 = 7 & 1 + 3 = 4 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 7 & 4 \end{bmatrix}$

RHS = B + A

$= \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 7 & 4 \end{bmatrix}$

Here, LHS = RHS hence proved

(ii) $A + (-A) = 0 = (-A) + A$

$-A = -\begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -5 & -1 \end{bmatrix}$

LHS:

= A + (– A)

$$= \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 = 0 & 2-2 = 0 \\ 5-5 = 0 & 1-1 = 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

RHS (– A) + A =

$$= \begin{bmatrix} -3 & -2 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+3 = 0 & -2+2 = 0 \\ -5+5 = 0 & -1+1 = 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence LHS = RHS = 0 proved

10. Question

If $A = \begin{pmatrix} 4 & 1 & 2 \\ 1 & -2 & 3 \\ 0 & 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, then

verify that A + (B + C) = (A + B) + C.

Answer

10) given

A = $\begin{pmatrix} 4 & 1 & 2 \\ 1 & -2 & 3 \\ 0 & 3 & 2 \end{pmatrix}$, B = $\begin{pmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{pmatrix}$ and C = $\begin{pmatrix} 1 & 0 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$

LHS = A + (B + C)

$$\Rightarrow \begin{pmatrix} 4 & 1 & 2 \\ 1 & -2 & 3 \\ 0 & 3 & 2 \end{pmatrix} + \left(\begin{pmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 0 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} \right)$$

$$\Rightarrow \begin{pmatrix} 4 & 1 & 2 \\ 1 & -2 & 3 \\ 0 & 3 & 2 \end{pmatrix} + \begin{pmatrix} 2+1 = 3 & 0+0 = 0 & 4-3 = 1 \\ 6+5 = 11 & 2+0 = 2 & 8+2 = 10 \\ 2+1 = 3 & 4-1 = 3 & 6+1 = 7 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4+3 = 7 & 1+0 = 1 & 2+1 = 3 \\ 1+11 = 12 & -2+2 = 0 & 3+10 = 13 \\ 0+3 = 3 & 3+3 = 6 & 2+7 = 9 \end{pmatrix} = \begin{pmatrix} 7 & 1 & 3 \\ 12 & 0 & 13 \\ 3 & 6 & 9 \end{pmatrix}$$

RHS = (A + B) + C

$$\Rightarrow \left(\begin{pmatrix} 4 & 1 & 2 \\ 1 & -2 & 3 \\ 0 & 3 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{pmatrix} \right) + \begin{pmatrix} 1 & 0 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 6 & 1 & 6 \\ 7 & 0 & 11 \\ 2 & 7 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 0 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 7 & 1 & 3 \\ 12 & 0 & 13 \\ 3 & 6 & 9 \end{pmatrix}$$

LHS = RHS

Hence proved

11. Question

An electronic company records each type of entertainment device sold at three of their branch stores so that they can monitor their purchases of supplies. The sales in two weeks are shown in the following spreadsheets.

		T.V	DVD	Videogames	CD Players
Week I	Store I	30	15	12	10
	Store II	45	20	15	15
	Store III	25	18	10	12
Week II	Store I	25	12	8	6
	Store II	32	10	10	12
	Store III	22	15	8	10

Find the sum of the items sold out in two weeks using matrix addition.

Answer

Here we consider the types of items sold along the column and the store in which they are stored along the rows

Thus week 1 and week 2 matrices can be written as

$$W_1 = \begin{bmatrix} 30 & 15 & 12\ 10 \\ 45 & 20 & 15\ 15 \\ 25 & 18 & 10\ 12 \end{bmatrix} \text{ and } W_2 = \begin{bmatrix} 25 & 12 & 8\ 6 \\ 32 & 10 & 10\ 12 \\ 22 & 15 & 8\ 10 \end{bmatrix}$$

The sum of the items sold in two weeks is the sum of the above two matrices, which is sum of each corresponding elements of the two matrices

$$W_1 + W_2 = \begin{bmatrix} 30 & 15 & 12\ 10 \\ 45 & 20 & 15\ 15 \\ 25 & 18 & 10\ 12 \end{bmatrix} + \begin{bmatrix} 25 & 12 & 8\ 6 \\ 32 & 10 & 10\ 12 \\ 22 & 15 & 8\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 55 & 27 & 20\ 16 \\ 77 & 30 & 25\ 27 \\ 47 & 33 & 18\ 22 \end{bmatrix}$$

TV DVD video CD

$$\Rightarrow \text{The sum of items sold in two weeks} = \begin{pmatrix} \text{store 1 } 55 & 27 & 20\ 16 \\ \text{store 2 } 77 & 30 & 25\ 27 \\ \text{store 3 } 47 & 33 & 18\ 22 \end{pmatrix}$$

12. Question

The fees structure for one – day admission to a swimming pool is as follows:

Daily Admission Fees in ₹		
Member	Children	Adult
Before 2.00 p.m.	20	30
After 2.00 p.m.	30	40
Non - Member		
Before 2.00 p.m.	25	35
After 2.00 p.m.	40	50

Write the matrix that represents the additional cost for non – membership.

Answer

Here we consider the type of member along the columns and the timings in the rows. Thus members and non – members matrices can be written as:

M = $\begin{bmatrix} 20 & 30 \\ 30 & 40 \end{bmatrix}$ and N = $\begin{bmatrix} 25 & 35 \\ 40 & 50 \end{bmatrix}$ respectively

The additional cost for non – members as compared to the members is the difference of the above two matrices, which is the difference of each element of the matrices to its corresponding element in the other matrix

N –M = $\begin{bmatrix} 25 & 35 \\ 40 & 50 \end{bmatrix} - \begin{bmatrix} 20 & 30 \\ 30 & 40 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 25-20 = 5 & 35-30 = 5 \\ 40-30 = 10 & 50-40 = 10 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 10 & 10 \end{bmatrix}$

The additional cost of non – members as compared to the members is

$$\begin{array}{ll} & \text{Children} \quad \text{Adult} \\ & \begin{bmatrix} \text{before 2 pm } 5 & 5 \\ \text{after 2.00pm } 10 & 10 \end{bmatrix} \end{array}$$

Exercise 4.3

1. Question

Determine whether the product of the matrices is defined in each case. If so, state the

order of the product.

(i) AB, where A = $[a_{ij}]_{4x3}$, B = $[b_{ij}]_{3x2}$

(ii)PQ, where P = $[p_{ij}]_{4x3}$, Q = $[q_{ij}]_{4x3}$

(iii)MN, where M = $[m_{ij}]_{3x1}$, N = $[n_{ij}]_{1x5}$

(iv) RS, where R = $[r_{ij}]_{2x2}$, S = $[s_{ij}]_{2x2}$

Answer

(i) The multiplication of 2 matrices is possible if number of columns in first matrix is equal to number of rows in second.

⇒ Here A$[a_{ij}]_{4 \times 3}$ and B = $[b_{ij}]_{3x2}$

⇒ Number of columns in A = 3

⇒ Number of rows in B = 3

Thus the product is defined and the order if product is

Number of rows in A × Number of columns in B

∴ AB = 4 × 3

(ii) The multiplication of 2 matrices is possible if number of columns in first matrix is equal to number of rows in second.

⇒ Here $P[p_{ij}]_{4 \times 3}$ and $Q = [q_{ij}]_{4 \times 3}$

⇒ Number of columns in P = 3

⇒ Number of rows in Q = 4

Thus the product is not defined.

(iii) The multiplication of 2 matrices is possible if number of columns in first matrix is equal to number of rows in second.

⇒ Here $M[m_{ij}]_{3 \times 1}$ and $N = [n_{ij}]_{1 \times 5}$

⇒ Number of columns in M = 1

⇒ Number of rows in N = 1

Thus the product is defined and the order if product is

Number of rows in M × Number of columns in N

∴ MN = 3 × 5

(iv) The multiplication of 2 matrices is possible if number of columns in first matrix is equal to number of rows in second.

⇒ Here $R[r_{ij}]_{2 \times 2}$ and $S = [s_{ij}]_{2 \times 2}$

⇒ Number of columns in R = 2

⇒ Number of rows in S = 2

Thus the product is defined and the order if product is

Number of rows in R × Number of columns in S

∴ RS = 2 × 2

2. Question

Find the product of the matrices, if exists,

(i) $\begin{pmatrix} 2 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 3 & -2 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 2 & 7 \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 6 \\ -3 \end{pmatrix}\begin{pmatrix} 2 & -7 \end{pmatrix}$

Answer

(i) ⇒ let A : [2 -1] ∴ $A[a_{ij}]_{1 \times 2}$

⇒ let B : $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ ∴ $B[b_{ij}]_{2 \times 1}$

Number of columns in A = 2

Number of rows in B = 2

Thus the product is defined and the order if product is

Number of rows in A × Number of columns in B

∴ AB = 1 × 1

$\Rightarrow [2 \ -1] \times \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

⇒ [2 × 5 + (-1) × 4]

⇒ [10-4]

⇒ [6]

(ii) ⇒ let A : $\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$ ∴ $A[a_{ij}]_{2 \times 2}$

⇒ let B : $\begin{bmatrix} 4 & 1 \\ 2 & 7 \end{bmatrix}$ ∴ $B[b_{ij}]_{2 \times 2}$

Number of columns in A = 2

Number of rows in B = 2

Thus the product is defined and the order if product is

Number of rows in A × Number of columns in B

∴ AB = 2 × 2

$\Rightarrow \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 1 \\ 2 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 \times 4 + (-2) \times 2 & 3 \times 1 + (-2) \times 7 \\ 5 \times 4 + 1 \times 2 & 5 \times 1 + 1 \times 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & -11 \\ 22 & 12 \end{bmatrix}$

(iii) ⇒ let A : $\begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix}$ ∴ $A[a_{ij}]_{2 \times 3}$

⇒ let B : $\begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$ ∴ $B[b_{ij}]_{3 \times 2}$

Number of columns in A = 3

Number of rows in B = 3

Thus the product is defined and the order if product is

Number of rows in A × Number of columns in B

∴ AB = 2 × 2

$\Rightarrow \begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 \times 4 + 9 \times (-6) + (-3) \times (-2) & 2 \times 2 + 9 \times 7 + (-3) \times 1 \\ 4 \times 4 + (-1) \times (-6) + 0 \times (-2) & 4 \times 2 + (-1) \times 7 + 0 \times 1 \end{bmatrix}$

$\Rightarrow \Rightarrow \begin{bmatrix} -40 & 64 \\ 22 & 1 \end{bmatrix}$

(iv) let A : $\begin{bmatrix} 6 \\ -3 \end{bmatrix}$ ∴ $A[a_{ij}]_{2 \times 1}$

let B : [2 7] ∴ $B[b_{ij}]_{1 \times 2}$

Number of columns in A = 1

Number of rows in B = 1

Thus the product is defined and the order if product is

Number of rows in A × Number of columns in B

∴ AB = 2 × 2

$\Rightarrow \begin{bmatrix} 6 \\ -3 \end{bmatrix} \times [2\ -7]$

$\Rightarrow \begin{bmatrix} 6 \times 2 & 6 \times -7 \\ -3 \times 2 & -3 \times -7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}$

3. Question

A fruit vendor sells fruits from his shop. Selling prices of Apple, Mango and Orange are ₹20, ₹10 and ₹5 each respectively. The sales in three days are given below

Day	Apples	Mangoes	Oranges
1	50	60	30
2	40	70	20
3	60	40	10

Write the matrix indicating the total amount collected on each day and hence find the total amount collected from selling of all three fruits combined.

Answer

⇒ Let the Sales matrix be A = $\begin{bmatrix} 50 & 60 & 30 \\ 40 & 70 & 20 \\ 60 & 40 & 10 \end{bmatrix}$

⇒ Selling price matrix B = $\begin{bmatrix} 20 \\ 10 \\ 5 \end{bmatrix}$

$AB = \begin{bmatrix} 50 & 60 & 30 \\ 40 & 70 & 20 \\ 60 & 40 & 10 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 50 \times 20 + 60 \times 10 + 30 \times 5 \\ 40 \times 20 + 70 \times 10 + 20 \times 5 \\ 60 \times 20 + 40 \times 10 + 10 \times 5 \end{bmatrix}$

$= \begin{bmatrix} 1750 \\ 1600 \\ 1650 \end{bmatrix}$

These are the amounts earned on each day.

Total amount earned = 1750 + 1600 + 1650 = 5000 Rs

4. Question

Find the values of x and y if $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 9 & 0 \end{pmatrix}$.

Answer

$= \begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$

$= \begin{bmatrix} 1 \times x + (2) \times (0) & 1 \times (0) + (2) \times y \\ (3) \times x + 3 \times (0) & (3) \times (0) + 3 \times y \end{bmatrix}$

$= \begin{bmatrix} x & 2y \\ 3x & 3y \end{bmatrix}$

Comparing with $\begin{bmatrix} x & 0 \\ 9 & y \end{bmatrix}$

3x = 9

∴ x = 3

And Y = 0

5. Question

If $A = \begin{pmatrix} 5 & 3 \\ 7 & 5 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $C = \begin{pmatrix} -5 \\ -11 \end{pmatrix}$ and if AX = C, then find the values of x and y.

Answer

x = 2, y = - 5

⇒ AX = $\begin{bmatrix} 5 & 3 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

= $\begin{bmatrix} 5x + 3y \\ 7x + 5y \end{bmatrix}$

Comparing with $\begin{bmatrix} -5 \\ -11 \end{bmatrix}$

5x + 3y = -5 ---1

7x + 5y = -11 ---2

Multiply 1 by 5 and multiply 2 by 3 and subtract,

25x + 15y = -25

- 21x + 15y = -33

4x = 8

x = 2 and y = -5

6. Question

If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then show that $A^2 = 4A + 5I_2 = O$.

Answer

⇒ A^2 = AA = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

= $\begin{bmatrix} 1 \times 1 + (-1) \times (2) & 1 \times (-1) + (-1) \times 3 \\ 2 \times 1 + 3 \times 2 & 2 \times (-1) + 3 \times 3 \end{bmatrix}$

= $\begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$

⇒ 4A = $4 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

= $\begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix}$

⇒ 5I = $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

⇒ $A^2 - 4A + 5I_2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

7. Question

If $A = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$ and , $B = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$ then find AB and BA. Are they equal?

Answer

$$\Rightarrow AB = \begin{bmatrix} 3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 3 + 2 \times 3 & 3 \times (0) + 2 \times 2 \\ 4 \times 3 + 0 \times 3 & 4 \times (0) + 0 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 4 \\ 12 & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 3 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 3 + 0 \times 4 & 3 \times (2) + 0 \times 0 \\ 3 \times 3 + 2 \times 4 & 3 \times (2) + 2 \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 \\ 17 & 6 \end{bmatrix}$$

AB not equal to BA

8. Question

If $A = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \end{pmatrix}$ verify (AB) C = A (BC).

Answer

$$\Rightarrow AB = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \times 0 + 2 \times 1 + 1 \times 2 \\ 1 \times 0 + 2 \times 1 + 3 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\Rightarrow (AB)C = \begin{bmatrix} 4 \\ 8 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \times 2 & 4 \times 1 \\ 8 \times 2 & 8 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 16 & 8 \end{bmatrix}$$

$$\Rightarrow BC = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \times 2 & 0 \times 1 \\ 1 \times 2 & 1 \times 1 \\ 2 \times 2 & 2 \times 1 \end{bmatrix}$$

$= \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 4 & 2 \end{bmatrix}$

$\Rightarrow A(BC) = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1 \times 0 + 2 \times 2 + 1 \times 4 & -1 \times 0 + 2 \times 1 + 1 \times 2 \\ 1 \times 0 + 2 \times 2 + 3 \times 4 & 1 \times 0 + 2 \times 1 + 3 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 8 & 4 \\ 16 & 8 \end{bmatrix}$

⇒ (AB) C = A (BC)

Thus verified.

9. Question

If $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$.

Answer

$\Rightarrow AB = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 \times 3 + 2 \times (-1) & 5 \times (-1) + 2 \times 1 \\ 7 \times 3 + 3 \times (-1) & 7 \times (-1) + 3 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 12 & -3 \\ 18 & -4 \end{bmatrix}$

$\Rightarrow (AB)^T = \begin{bmatrix} 12 & 18 \\ -3 & -4 \end{bmatrix}$

$\Rightarrow B^T = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$

$\Rightarrow B^T A^T = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 5 + (-1) \times (2) & 3 \times (7) + (-1) \times 3 \\ (-1) \times 5 + 1 \times (2) & (-1) \times (7) + 1 \times 3 \end{bmatrix}$

$= \begin{bmatrix} 12 & 18 \\ -3 & -4 \end{bmatrix}$

$\Rightarrow (AB)^T = B^T A^T$

This proved.

10. Question

Prove that $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$ are inverses to each other under matrix multiplication.

Answer

$\Rightarrow AB = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$= \begin{bmatrix} 5 \times 3 + 2 \times (-7) & 5 \times (-2) + 2 \times 5 \\ 7 \times 3 + 3 \times (-7) & 7 \times (-2) + 3 \times 5 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ BA = $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 5 + (-2) \times (7) & 3 \times (2) + (-2) \times 3 \\ (-7) \times 5 + 5 \times (7) & -7 \times (2) + 5 \times 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Thus A and B are inverse to each other

11. Question

Solve $\begin{pmatrix} x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} x \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix}$.

Answer

Let A = [x 1] B = $\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}$ C = $\begin{bmatrix} x \\ 5 \end{bmatrix}$

$\Rightarrow$ BC = $\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix}$

$= \begin{bmatrix} x + 5 \times 0 \\ -2x - 15 \end{bmatrix}$

$= \begin{bmatrix} x \\ -2x - 15 \end{bmatrix}$

$\Rightarrow$ A(BC) = [x 1] $\begin{bmatrix} x \\ -2x - 15 \end{bmatrix}$

$= [x \quad 1] \begin{bmatrix} x \\ -2x - 15 \end{bmatrix}$

$= [x^2 - 2x - 15]$ = 0

$\therefore$ x = -3, x = 5

12. Question

If $A = \begin{pmatrix} 3 & 3 \\ 7 & 6 \end{pmatrix}, B = \begin{pmatrix} 8 & 7 \\ 0 & 9 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & -3 \\ 4 & 6 \end{pmatrix}$, find (A + B)C and AC + BCIs (A + B)C = AC + BC ?

Answer

$\Rightarrow$ A = $\begin{bmatrix} 3 & 3 \\ 7 & 6 \end{bmatrix}$, B = $\begin{bmatrix} 8 & 7 \\ 0 & 9 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

$\Rightarrow$ A + B = $\begin{bmatrix} 3 & 3 \\ 7 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 7 \\ 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} 3 + 8 & 3 + 7 \\ 7 + 0 & 6 + 9 \end{bmatrix}$

$\Rightarrow$ A + B = $\begin{bmatrix} 11 & 10 \\ 7 & 15 \end{bmatrix}$

$\Rightarrow$ (A + B)C = $\begin{bmatrix} 11 & 10 \\ 7 & 15 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 11 \times 2 + 10 \times 4 & 11 \times (-3) + 10 \times 6 \\ 7 \times 2 + 15 \times 4 & 7 \times (-3) + 15 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 62 & 27 \\ 74 & 69 \end{bmatrix}$

$\Rightarrow$ AC = $\begin{bmatrix} 3 & 3 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 2 + 3 \times 4 & 3 \times (-3) + 3 \times 6 \\ 7 \times 2 + 6 \times 4 & 7 \times (-3) + 6 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 18 & 9 \\ 38 & 15 \end{bmatrix}$

$\Rightarrow$ BC = $\begin{bmatrix} 8 & 7 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 8 \times 2 + 7 \times 4 & 8 \times (-3) + 7 \times 6 \\ 0 \times 2 + 9 \times 4 & 0 \times (-3) + 9 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 44 & 18 \\ 36 & 54 \end{bmatrix}$

$\Rightarrow$ AC + BC = $\begin{bmatrix} 18 & 9 \\ 38 & 15 \end{bmatrix} + \begin{bmatrix} 44 & 18 \\ 36 & 54 \end{bmatrix}$

$= \begin{bmatrix} 62 & 27 \\ 74 & 69 \end{bmatrix}$

$\Rightarrow$ Thus (A + B)C = AC + BC is true

Exercise 4.4

1. Question

Which one of the following statements is not true?

A. A scalar matrix is a square matrix

B. A diagonal matrix is a square matrix

C. A scalar matrix is a diagonal matrix

D. A diagonal matrix is a scalar matrix.

Answer

In the above question we see the following terms,

Scalar Matrix, Square Matrix, Diagonal Matrix

To answer this question, we have to know the definition of the above terms.

<u>Square Matrix:</u>

If the rows and columns of the matrices are equal then it will constitute square like structure. So, it is called as square matrix.

E.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

<u>Diagonal Matrix:</u>

In a square matrix all the elements are zero except the diagonal elements of the matrix. Then that matrix is said to be diagonal matrix.

E.g. $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

<u>Scalar Matrix:</u>

In a diagonal matrix, if all the diagonal elements are same then that matrix is said to be a scalar matrix.

E.g. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Option(A):

A scalar matrix should be a square matrix so, it is true

Option(B):

Diagonal matrix forms only with the square matrix so, it is true

Option(C):

A scalar matrix consists of zero except the diagonal elements so it is true.

Option(D):

A scalar matrix should comprise of same diagonal elements but in diagonal matrix it may (or) may not contains same diagonal elements. So it is False.

<u>Option(D) is not True in the given.</u>

2. Question

Matrix A-$[a_{ij}]_{mxn}$ is a square matrix if

A. m < n

B. m > n

C. m = 1

D. m = n

Answer

Given that A is a matrix with m and n as their rows and columns respectively.

We know that for a square matrix both m and n should be equal

So, m = n is the correct answer.

3. Question

If $\begin{pmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{pmatrix} = \begin{pmatrix} 1 & y-2 \\ 8 & 8 \end{pmatrix}$ then the values of x and y respectively are

A. –2, 7

B. $-\frac{1}{3}, 7$

C. $-\frac{1}{3}, -\frac{2}{3}$

D. 2, –7

Answer

Given,

$\begin{pmatrix} 3x + 7 & 5 \\ y + 1 & 2-3x \end{pmatrix} = \begin{pmatrix} 1 & y-2 \\ 8 & 8 \end{pmatrix}$

If the one matrix is equivalent to other matrix, then their elements should be equal.

⇒ 3x + 7 = 1 & 5 = y-2

3x = -6 ; y = 7

x = -2 ; y = 7

∴ Option (A) is the answer.

4. Question

If A = (1 -2 3) and $B = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ then A + B

A. (0 0 0)

B. $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

C. (-14)

D. not defined

Answer

For matrix addition operation their no. of rows and no. of columns should be equal. Otherwise addition is not possible.

In the given matrix A has 1 row and 3 columns

But matrix B has 3 rows and 1 column.

Since rows and columns are not equal it is not possible to add.

So the answer is (d)

5. Question

If a matrix is of order 2 × 3, then the number of elements in the matrix is

A. 5

B. 6

C. 2

D. 3

Answer

Given that a matrix with order 2x3

We know that the no. of elements in the matrix is equal to the product of no. of rows and no. of columns

i.e., No. of elements = No. of rows × No. of columns

∴ No. of elements = 2 x 3

= 6

∴ There will be 6 elements in the given matrix

So, option (B) is the correct answer.

6. Question

If $\begin{pmatrix} 8 & 4 \\ x & 8 \end{pmatrix} = 4\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ then the value of x is

A. 1

B. 2

C. $\frac{1}{4}$

D. 4

Answer

Given,

$\begin{pmatrix} 8 & 4 \\ x & 8 \end{pmatrix} = 4\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$\begin{pmatrix} 8 & 4 \\ x & 8 \end{pmatrix} = \begin{pmatrix} 8 & 4 \\ 4 & 8 \end{pmatrix}$

Since both the matrix is equal to each other, the elements in it also be equal.

By equalizing we will get x = 4

So, option (D) is correct answer.

7. Question

If A is of order 3 × 4 and B is of order 4 × 3, then the order of BA is

A. 3 × 3

B. 4 × 4

C. 4 × 3

D. not defined

Answer

Given matrix orders

[A] with 3 x 4 and [B] with 4 x 3

We have to find the order of [B].[A]

For matrix multiplication the columns of 1st matrix and the rows of 2nd matrix should be equal.

E.g. $[A]_{3x2}.[B]_{2x2}$

The order of new matrix formed after multiplication of matrices will be 1st matrix row will be taken as new matrix rows. The no. of columns in 2nd matrix will be taken as new matrix columns.

E.g. $[A]_{3x2}.[B]_{2x2} = [AB]_{3x2}$

In the same way $[B]_{4x3}.[A]_{3x4} = [AB]_{4x4}$

∴ option (B) is correct answer

8. Question

If $A \times \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} = (1, 2)$, then the order of A is

A. 2 × 1

B. 2 × 2

C. 1 × 2

D. 3 × 2

Answer

Given,

A x $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} = (1\ 2)$

Let us consider $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ as matrix B, it has an order of 2x2

Let us consider the resultant matrix as C, it has an order of 1x2

For matrix multiplication the columns of 1^{st} matrix and the rows of 2^{nd} matrix should be equal.

So, matrix A has got same Columns as Rows of matrix B

The order of new matrix formed after multiplication of matrices will be 1^{st} matrix row will be taken as new matrix rows. The no. of columns in 2^{nd} matrix will be taken as new matrix columns

So, matrix A has got same Rows as Rows of matrix C

So, Matrix A will consists of 1x2 order

∴ option (C) is correct answer

9. Question

If A and B are square matrices such that AB = I and BA = I, then B is

A. Unit matrix

B. Null matrix

C. Multiplicative inverse matrix of A

D. –A

Answer

It is based on a property about multiplication inverse. Only product of a matrix and its own inverse matrix then the resultant matrix will be identity matrix.

AB = I and BA = I

So, B is the multiplicative inverse matrix of A

∴ option (C) is correct answer.

10. Question

If $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, then the values of x and y respectively, are

A. 2, 0

B. 0, 2

C. 0, -2

D. 1, 1

Answer

Given, $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\begin{bmatrix} x + 2y \\ 2x + y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

By equating all the elements in the matrix

We will get 2 equations

x + 2y = 2 ⇒ **1**

2x + y = 4 ⇒ **2**

By solving the both equations we will get

x = 2; y = 0

∴ option (A) is correct answer

11. Question

If A = $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$ and A + B = O, then B is

A. $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$

B. $\begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$

C. $\begin{pmatrix} -1 & -2 \\ -3 & -4 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Answer

Given that

A + B = O ⇒ B = -A

∴ B = $-\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$

B = $\begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$

∴ option (B) is correct answer

12. Question

If A = $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$, then A^2 is

A. $\begin{pmatrix} 16 & 4 \\ 36 & 9 \end{pmatrix}$

B. $\begin{pmatrix} 8 & -4 \\ 12 & -6 \end{pmatrix}$

C. $\begin{pmatrix} -4 & 2 \\ -6 & 3 \end{pmatrix}$

D. $\begin{pmatrix} 4 & -|2 \\ 6 & -3 \end{pmatrix}$

Answer

Given A = $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$

A^2 = A×A = $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$ x $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$

= $\begin{pmatrix} 4(4) + (-2)(6) & 4(-2) + (-2)(-3) \\ 6(4) + (-3)(6) & 6(-2) + (-3)(-3) \end{pmatrix}$

= $\begin{pmatrix} 16 - 12 & -8 + 6 \\ 24 - 18 & -12 + 9 \end{pmatrix}$

= $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$

∴ option (D) is correct answer

13. Question

A is of order m x n and B is of order p x q, addition of A and B is possible only if

A. m = p

B. n = q

C. n = p

D. m = p, n = q

Answer

Given matrix of A is m x n ⇒ $[A]_{mxn}$

Matrix of B is p x q ⇒ $[B]_{pxq}$

We know that addition is possible is possible only the order is same for both the matrices

So, m = p, n = q

So, option (D) is true

14. Question

If $\begin{pmatrix} a & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$, then the value of a is

A. 8

B. 4

C. 2

D. 11

Answer

$\begin{pmatrix} a & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

By multiplying the matrices, we will get the value of ‘a’

$\begin{pmatrix} a(2) + 3(-1) \\ 1(2) + 2(-1) \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 2a - 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

By equating the elements in the matrices

2a-3 = 5

2a = 8

a = 4

So, option (B) is correct answer

15. Question

If A = $\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ is such that $A^2 = I$, then

A. $1 + \alpha^2 + \beta\gamma = 0$

B. $1 - \alpha^2 + \beta\gamma = 0$

C. $1 - \alpha^2 - \beta\gamma = 0$

D. $1 + \alpha^2 - \beta\gamma = 0$

Answer

Given A = $\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$

$A^2 = I$

I is the identity matrix

$$\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} \times \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha^2 + \beta\gamma & \alpha\beta + \beta(-\alpha) \\ \gamma\alpha + (-\alpha)\gamma & \gamma\beta + (-\alpha)(-\alpha) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

By comparing the elements in the matrix

$\alpha^2 + \beta\gamma = 1 \Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

∴ Option (C) is correct answer.

16. Question

If $A = [a_{ij}]_{2x2}$ and $a_{ij} = i + j$, then A =

A. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

B. $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

C. $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

D. $\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$

Answer

Given A = $[a_{ij}]_{2x2}$ and $a_{ij} = i + j$

It means matrix A with order 2x2 and the elements in is given by $a_{ij} = i + j$

$a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{21} = 2 + 1 = 3$

$a_{22} = 2 + 2 = 4$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

∴ Option (B) is correct answer.

17. Question

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, then the values of a, b, c and d

respectively are

A. -1, 0, 0, - 1

B. 1, 0, 0, 1

C. -1,0,1,0

D. 1, 0, 0, 0

Answer

Given

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} . \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

By multiplying the matrices, we will get the values of a, b, c & d

$\begin{pmatrix} -1(a) + 0(c) & -1(b) + 0(d) \\ 0(a) + 1(c) & 0(b) + 1(d) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} -a & -b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

By comparing the matrices on both sides

-a = 1; -b = 0; c = 0; d = -1

∴ Option (A) is correct answer.

18. Question

If $A = \begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}$ and $A + B = \begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix}$, then the matrix B =

A. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 6 & 2 \\ 3 & -1 \end{pmatrix}$

C. $\begin{pmatrix} -8 & -2 \\ 1 & -7 \end{pmatrix}$

D. $\begin{pmatrix} 8 & 2 \\ -1 & 7 \end{pmatrix}$

Answer

Given

A + B = $\begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix}$; A = $\begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}$

B = $\begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix} - A$

B = $\begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}$

= $\begin{pmatrix} -1-7 & 0-2 \\ 2-1 & -4-3 \end{pmatrix}$

= $\begin{pmatrix} -8 & -2 \\ 1 & -7 \end{pmatrix}$

∴ Option (C) is correct answer.

19. Question

If $\begin{pmatrix} 5 & x & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 20 \end{pmatrix}$, then the value of x is

A. 7

B. -7

C. $\frac{1}{7}$

D. 0

Answer

Given $\begin{pmatrix} 5 & x & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = (20)$

$(5(2) + x(-1) + 1(3)) = (20)$

$(10 - x + 3) = (20)$

$(13 - x) = (20)$

By equating the elements

13 – x = 20

x = -7

∴ Option (B) is correct answer.

20. Question

Which one of the following is true for any two square matrices A and B of same order?.

A. $(AB)^T = A^T B^T$

B. $(A^T B)^T = A^T B^T$

C. $(AB)^T = BA$

D. $(AB)^T = B^T A^T$

Answer

This is Reversal law for transpose of matrices

$\therefore (AB)^T = B^T A^T$

$\therefore$ Option (D) is correct answer.