Quadratic Equations

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1. Solve the following problems given:
(i) $x^2-45x+324=0$
(ii) $x^2-55x+750=0$

Ans. (i) $x^2-45x+324=0$
$\Rightarrow x^2-36x-9x+324=0$
$\Rightarrow x(x-36)-9(x-36)=0$
$\Rightarrow (x-9)(x-36)=0$
$\Rightarrow x=9,36$

(ii) $x^2-55x+750=0$
$\Rightarrow x^2-25x-30x+750=0$
$\Rightarrow x(x-25)-30(x-25)=0$
$\Rightarrow (x-30)(x-25)=0$
$\Rightarrow x=30,25$

2. Find two numbers whose sum is 27 and product is 182.
Ans. Let first number be x and let second number be $(27-x)$
According to given condition, the product of two numbers is 182.
Therefore,
$x(27-x)=182$
$\Rightarrow 27x-x^2=182$
$\Rightarrow x^2-27x+182=0$
$\Rightarrow x^2-14x-13x+182=0$
$\Rightarrow x(x-14)-13(x-14)=0$
$\Rightarrow (x-14)(x-13)=0$
$\Rightarrow x=14,13$
Therefore, the first number is equal to 14 or 13
And, second number is = $27-x=27-14=13$ or Second number = $27-13=14$
Therefore, two numbers are 13 and 14.

3. Find two consecutive positive integers, sum of whose squares is 365.
Ans. Let first number be x and let second number be $(x+1)$
According to given condition,
$x^2+(x+1)^2=365$ $\{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow x^2+x^2+1+2x=365$
$\Rightarrow 2x^2+2x-364=0$
Dividing equation by 2
$\Rightarrow x^2+x-182=0$
$\Rightarrow x^2+14x-13x-182=0$
$\Rightarrow x(x+14)-13(x+14)=0$
$\Rightarrow (x+14)(x-13)=0$
$\Rightarrow x=13,-14$
Therefore, first number = 13{We discard -14 because it is negative number)
Second number = $x+1=13+1=14$
Therefore, two consecutive positive integers are 13 and 14 whose sum of squares is equal to 365.

4. The altitude of right triangle is 7 cm less than its base. If, hypotenuse is 13 cm. Find the other two sides.
Ans. Let base of triangle be x cm and let altitude of triangle be $(x-7)$ cm
It is given that hypotenuse of triangle is 13 cm
According to Pythagoras Theorem,
$13^2=x^2+(x-7)^2$ $(a+b)^2=a^2+b^2+2ab$
$\Rightarrow 169=x^2+x^2+49-14x$
$\Rightarrow 169=2x^2-14x+49$
$\Rightarrow 2x^2-14x-120=0$
Dividing equation by 2
$\Rightarrow x^2-7x-60=0$
$\Rightarrow x^2-12x+5x-60=0$
$\Rightarrow x(x-12)+5(x-12)=0$
$\Rightarrow (x-12)(x+5)$
$\Rightarrow x=-5,12$
We discard $x=-5$ because length of side of triangle cannot be negative.
Therefore, base of triangle = 12 cm
Altitude of triangle = $(x-7)=12-7=5$ cm

5. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.
Ans. Let cost of production of each article be Rs x
We are given total cost of production on that particular day = Rs 90
Therefore, total number of articles produced that day = 90/x
According to the given conditions,

$x = 2\left(\frac{90}{x}\right) + 3$

$\Rightarrow x = \frac{180}{x} + 3$

$\Rightarrow x = \frac{180 + 3x}{x}$

$\Rightarrow x^2 = 180 + 3x$

$\Rightarrow x^2 - 3x - 180 = 0$

$\Rightarrow x^2 - 15x + 12x - 180 = 0$

$\Rightarrow x(x - 15) + 12(x - 15) = 0$

$\Rightarrow (x - 15)(x + 12) = 0$

$\Rightarrow x = 15, -12$

Cost cannot be in negative, therefore, we discard $x = -12$

Therefore, $x = Rs 15$ which is the cost of production of each article.

Number of articles produced on that particular day = $\frac{90}{15}$ = 6

6. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Let Shefali's marks in Mathematics = x

Let Shefali's marks in English = $30 - x$

If, she had got 2 marks more in Mathematics, her marks would be = $x + 2$

If, she had got 3 marks less in English, her marks in English would be = $30 - x - 3 = 27 - x$

According to given condition:

$(x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1, b = -25$ and $c = 156$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{25 \pm \sqrt{(25)^2 - 4(1)(156)}}{2 \times 1}$

$\Rightarrow x = \frac{25 \pm \sqrt{625 - 624}}{2}$

$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$

$\Rightarrow$ $x = \frac{25+1}{2}, \frac{25-1}{2}$
$\Rightarrow x=13,12$
Therefore, Shefali's marks in Mathematics = 13 or 12
Shefali's marks in English = 30 – x=30 – 13=17
Or Shefali's marks in English = 30 – x=30 – 12=18
Therefore, her marks in Mathematics and English are (13,17) or (12,18).

7. The diagonal of a rectangular field is 60 metres more than the shorter side. If, the longer side is 30 metres more than the shorter side, find the sides of the field.
Ans. Let shorter side of rectangle = x metres
Let diagonal of rectangle = $(x+60)$ metres
Let longer side of rectangle = $(x+30)$ metres
According to pythagoras theorem,
$(x+60)^2=(x+30)^2+x^2$
$\Rightarrow x^2+3600+120x=x^2+900+60x+x^2$
$\Rightarrow x^2-60x-2700=0$
Comparing equation $x^2-60x-2700=0$ with standard form $ax^2+bx+c=0$,
We get $a=1, b=-60$ and $c=-2700$
Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{60 \pm \sqrt{(60)^2-4(1)(-2700)}}{2 \times 1}$

$\Rightarrow$
$\Rightarrow$ $x = \frac{60 \pm \sqrt{14400}}{2} = \frac{60 \pm 120}{2}$
$\Rightarrow$ $x = \frac{60+120}{2}, \frac{60-120}{2}$
$\Rightarrow x= 90, -30$
We ignore –30. Since length cannot be in negative.
Therefore, x=90 which means length of shorter side =90 metres
And length of longer side = x+30 = 90+30=120 metres
Therefore, length of sides are 90 and 120 in metres.

8. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
Ans. Let smaller number = x and let larger number = y
According to condition:

$y^2 - x^2 = 180$ ... (1)
Also, we are given that square of smaller number is 8 times the larger number.
⇒ $x^2 = 8y$ ... (2)
Putting equation (2) in (1), we get
$y^2 - 8y = 180$
⇒ $y^2 - 8y - 180 = 0$
Comparing equation $y^2 - 8y - 180 = 0$ with general form $ay^2 + by + c = 0$,
We get $a = 1, b = -8$ and $c = -180$

Using quadratic formula $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-180)}}{2 \times 1}$$

⇒ $y = \frac{8 \pm \sqrt{64 + 720}}{2}$

⇒ $y = \frac{8 \pm \sqrt{784}}{2} = \frac{8 \pm 28}{2}$

⇒ $y = \frac{8 + 28}{2}, \frac{8 - 28}{2}$

⇒ $y = 18, -10$
Using equation (2) to find smaller number:
$x^2 = 8y$
⇒ $x^2 = 8y = 8 \times 18 = 144$
⇒ $x = \pm 12$
And, $x^2 = 8y = 8 \times -10 = -80$ {No real solution for x}
Therefore, two numbers are (12,18) or (−12,18)

9. A train travels 360 km at a uniform speed. If, the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans. Let the speed of the train = x km/hr
If, speed had been 5km/hr more, train would have taken 1 hour less.
So, according to this condition

$$\frac{360}{x} = \frac{360}{x+5} + 1$$

⇒ $360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$

⇒ $360\left(\frac{x+5-x}{x(x+5)}\right) = 1$

⇒ $360 \times 5 = x^2 + 5x$
⇒ $x^2 + 5x - 1800 = 0$

Comparing equation $x^2+5x-1800=0$ with general equation $ax^2+bx+c=0$,
We get $a=1, b=5$ and $c=-1800$

Applying quadratic formula

$$x=\frac{-5\pm\sqrt{(5)^2-4(1)(-1800)}}{2\times1}$$

$\Rightarrow$ $x=\frac{-5\pm\sqrt{25+7200}}{2}$

$\Rightarrow$ $x=\frac{-5\pm\sqrt{7225}}{2}=\frac{-5\pm85}{2}$

$\Rightarrow$ $x=\frac{-5+85}{2},\frac{-5-85}{2}$

$\Rightarrow x=40,-45$

Since speed of train cannot be in negative. Therefore, we discard $x=-45$

Therefore, speed of train = 40 km/hr

10. Find the value of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2+kx+3=0$

(ii) $kx(x-2)+6=0$

Ans. (i) $2x^2+kx+3=0$

We know that quadratic equation has two equal roots only when the value of discriminant isequal to zero.

Comparing equation $2x^2+kx+3=0$ with general quadratic equation $ax^2+bx+c=0$, we get$a=2, b=k$ and $c=3$

Discriminant = $b^2-4ac=k^2-4(2)(3)=k^2-24$

Putting discriminant equal to zero

$k^2-24=0$

$\Rightarrow k^2=24$

$\Rightarrow$ $k=\pm\sqrt{24}=\pm2\sqrt{6}$

$\Rightarrow$ $k=2\sqrt{6},-2\sqrt{6}$

(ii) $kx(x-2)+6=0$

$\Rightarrow kx^2-2kx+6=0$

Comparing quadratic equation$kx^2-2kx+6=0$ with general form $ax^2+bx+c=0$, we get $a=k, b=-2k$ and $c=6$

Discriminant = $b^2-4ac=(-2k)^2-4(k)(6)=4k^2-24k$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.

Putting discriminant equal to zero

$4k^2-24k=0$

$\Rightarrow 4k(k-6)=0$
$\Rightarrow k=0,6$
The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2+bx+c=0$, where $a\neq 0$.
Therefore, in equation $kx^2-2kx+6=0$, we cannot have k =0.
Therefore, we discard k=0.
Hence the answer is k=6.

11. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2. If so, find its length and breadth.
Ans. Let breadth of rectangular mango grove = x metres
Let length of rectangular mango grove = $2x$ metres
Area of rectangle = length × breadth = $x \times 2x = 2x^2$ m^2
According to given condition:
$2x^2=800$
$\Rightarrow 2x^2 - 800=0$
$\Rightarrow x^2 - 400=0$
Comparing equation $x^2 - 400=0$ with general form of quadratic equation $ax^2+bx+c=0$, we get $a=1, b=0$ and $c=-400$
Discriminant = $b^2-4ac=(0)^2 - 4(1)(-400)=1600$
Discriminant is greater than 0 means that equation has two disctinct real roots.
Therefore, it is possible to design a rectangular grove.
Applying quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,
$$x=\frac{0\pm\sqrt{1600}}{2\times 1}=\frac{\pm 40}{2}=\pm 20$$
$\Rightarrow x=20,-20$
We discard negative value of x because breadth of rectangle cannot be in negative.
Therefore, x = breadth of rectangle = 20 metres
Length of rectangle = $2x=2\times 20=40$ metres

12. Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.
Ans. Let age of first friend = x years and let age of second friend = $(20-x)$ years
Four years ago, age of first friend = $(x-4)$ years
Four years ago, age of second friend = $(20-x)-4 = (16-x)$ years
According to given condition,
$(x-4)(16-x)=48$
$\Rightarrow 16x-x^2 - 64+4x=48$

$\Rightarrow 20x - x^2 - 112 = 0$
$\Rightarrow x^2 - 20x + 112 = 0$
Comparing equation, $x^2 - 20x + 112 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get$a=1, b=-20$ and $c=112$
Discriminant $= b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$
Discriminant is less than zero which means we have no real roots for this equation.
Therefore, the give situation is not possible.

13. Value of x for $x^2 - 8x + 15 = 0$ is quadratic formula is
(a) 3, 2
(b) 5, 2
(c) 5, 3
(d) 2, 3
Ans. (c) 5, 3

14. Discriminate of $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ is
(a) 30
(b) 31
(c) 32
(d) 35
Ans. (c) 32

15. Solve $12abx^2 - 9a^2x + 8b^2x - 6ab = 0$.
Ans. $12abx^2 - 9a^2x + 8b^2x - 6ab = 0$
$\Rightarrow 3ax(4bx - 3x) + 2b(4bx - 3a) = 0$
$\Rightarrow (4bx - 3a)(3ax + 2b) = 0$
$\Rightarrow 4bx - 3a = 0 \ or \ 3ax + 2b = 0$
$\Rightarrow x = \frac{3a}{4b} \ or \ x = -\frac{2b}{3a}$

16. Solve for x by quadratic formula $p^2x^2 + (p^2 - q^2)x - q^2 = 0$
Ans. $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

17. Find the value of k for which the quadratic equation has real and distinct root.

Ans.

For real and distinct roots,

18. If one root of the equations is 1, find the value of a.

(a) = -4

(b) = -5

(c) = -3

(d) = -1

Ans. (b)

19 . Determine the nature of the roots of the quadratic equation $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$

Ans. $D = b^2 - 4ac$

$= (-24abcd)^2 - 4 \times 9a^2b^2 \times 16c^2d^2$

$= 576a^2b^2c^2 - 576a^2b^2c^2d^2 = 0$

20 . Find the discriminant of the equation $(x-1)(2x-1) = 0.$

Ans. $(x-1)(2x-1) = 0$

$\Rightarrow 2x^2 - x - 2x + 1 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

$Here, a = 2, \ \ c = -3, \ \ c = 1$

$D = b^2 - 4ac$

$= (-3)^2 - 4 \times 2 \times 1$

$= 9 - 8 = 1$

21. Find the value of k so that $(x-1)$ is a factor of $k^2x^2 - 2kx - 3.$

Ans. Let $P(x) = k^2x^2 - 2x - 3$

$P(1) = k^2(1)^2 - 2K(1) - 3$

$\Rightarrow 0 = K^2 - 2k - 3$

$\Rightarrow k^2 - 3k + k - 3$

$\Rightarrow k(k-3) + 1(k-3) = 0$

$\Rightarrow (k-3)(k+1) = 0$

$\Rightarrow k = 3 \ or \ k = -1$

22. The product of two consecutive positive integers is 306. Represent these in quadratic equation.

(a) $x^2 + x - 306 = 0$

(b) $x^2 - x + 306 = 0$

(c) $x^2 + 2x - 106 = 0$

(d) $x^2 - x - 306 = 0$

Ans. (a) $x^2 + x - 306 = 0$

23. Which is a quadratic equation?

(a) $x^2 + x + 2 = 0$

(b) $x^3 + x^2 + 2 = 0$

(c) $x^4 + x^2 + 2 = 0$

(d) $x + 2 = 0$

Ans. (a) $x^2 + x + 2 = 0$

24. The sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Ans. Let no. be x

According to question,

$\frac{1}{x} + \frac{1}{16 - x} = \frac{1}{3}$

$\Rightarrow \frac{16}{16x - x^2} = \frac{1}{3}$

$\Rightarrow x^2 - 16x + 48 = 0$

$\Rightarrow x^2 - 12x - 4x + 48 = 0$

$\Rightarrow x = 12$ or $x = 4$

25. Solve for x: $\sqrt{217 - x} = x - 7$

Ans. $\sqrt{217 - x} = (x - 7)$

$\Rightarrow (217 - x) = (x - 7)^2$

$\Rightarrow 217 - x = x2 + 49 - 14x$

$\Rightarrow x^2 - 14x + x + 49 - 217 = 0$

$\Rightarrow x^2 - 13x - 168 = 0$

$\Rightarrow x^2 - 21x + 8x - 168 = 0$

$\Rightarrow x = 21$ or $x = -8$

26. Solve for x by factorization: $x+\frac{1}{x}=11\frac{1}{11}$

Ans. $\frac{x^2+1}{x}=\frac{122}{11}$

$\Rightarrow 11x^2-122x+11=0$

$\Rightarrow 11x^2-121x-1x+11=0$

$\Rightarrow 11x(x-11)-1(x-11)=0$

$\Rightarrow (11x-1)(x-11)=0$

$\Rightarrow x=11 \; or \; x=\frac{1}{11}$

27. Find the ratio of the sum and product of the roots of $7x^2-12x+18=0$.

Ans. $7x^2-12x+18=0$

$\alpha+\beta=\frac{-b}{a}=\frac{12}{7} \; and \; \alpha\beta=\frac{c}{a}=\frac{18}{17}$

$\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{12}{7}}{\frac{18}{17}}=\frac{12}{7}\times\frac{17}{18}=\frac{34}{21}$

28. If α and β are the roots of the equation $x^2+kx+12=0$, such that $\alpha-\beta=1$, then

Ans. $\alpha+\beta=\frac{-k}{1}$,

$\alpha-\beta=1$.

$\alpha\beta=\frac{12}{1}$

$(\alpha+\beta)^2=(\alpha-\beta)^2+4\alpha\beta$

$\Rightarrow (-k)^2=(1)^2+4\times 12$

$\Rightarrow k^2=49$

$\Rightarrow k=\pm 7$