Chapter 12

Alternating Current

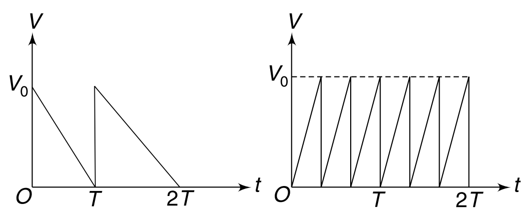
LEVEL 1

Q. 1: An electric appliance draws 3*A* current from a 200 V, 50 Hz power supply.

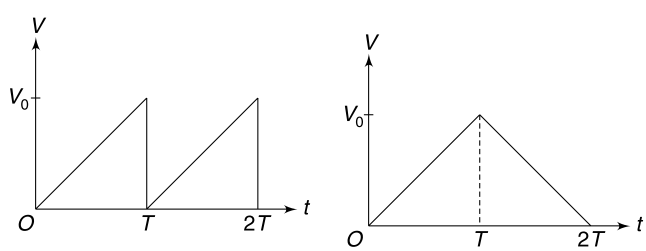
(a) Find the average of square of the current.

(b) Find the amplitude of the supply voltage.

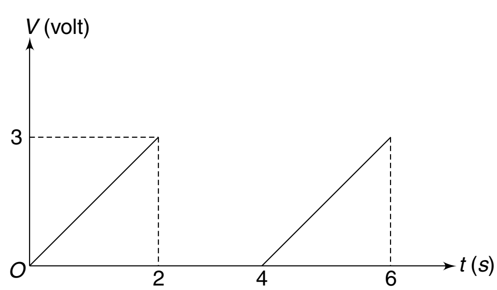
Q. 2: Which of the two waveforms shown in Figure has a higher average value?



Q. 3: Which of the two waveforms shown in Figure has a higher rms value?



Q. 4: A voltage waveform is as shown in the Figure calculate the ratio of rms value and average value of the voltage.

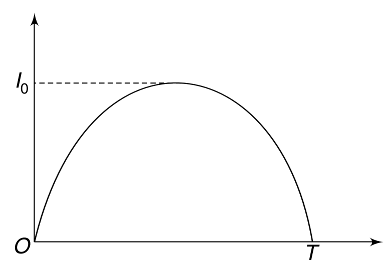


Q. 5: The graph given here represents an idealized sinusoidal current flow between a cloud at a height of 1 km and the earth, during a lightning discharge. Value of I_0 is 157 kA and $T = 0.2$ ms. Assume that discharge happens when the electric field in the air between the cloud and the earth becomes equal to the breakdown field of air i.e., $E_0 = 3 \times 10^6$ V/m

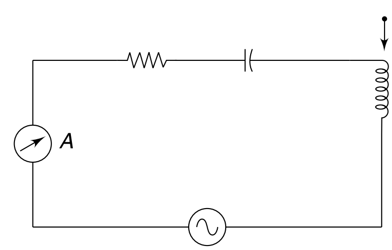
(a) Calculate the total charge flow due to lightning.

(b) Calculate the average current between the cloud and the earth during the lightning.

(c) Assume that the entire charge on the cloud is released during the lightning and estimate the capacitance of the cloud – Earth system.



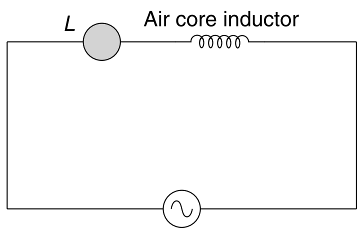
Q. 6: In the circuit shown, the frequency of the source is adjusted so that the reading of the *ac* ammeter is maximum. The inductor shown is a short coil in vertical orientation. A steel ball is dropped through the coil. How is the reading of ammeter affected when the ball enters the coil?



Q. 7: A lamp *L* is connected to an ac source along with an air core inductor as shown in the Figure. How is the brilliance of the lamp affected if core made of following material is inserted inside the inductor?

(a) Iron (b) Copper (c) Iron sheets pasted together with insulation in between [laminated Iron core]

In which case the lamp will be least bright?

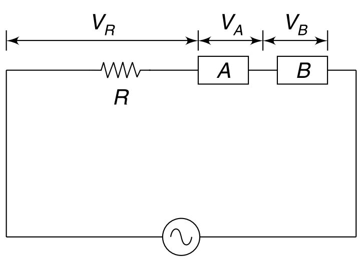


Q. 8: In the circuit shown in the Figure, the voltage across resistance R, box A and box B are represented as

$$v_R = V\sin(\omega t),\ v_A = \sqrt{2}\,V\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and}$$

$$v_B = V\sin\left(\omega t + \frac{\pi}{2}\right)$$

Find the phase difference between current and the applied voltage.



Level 2

Q. 9: A series LCR circuit has 120 Ω resistance. When the angular frequency of the source is 4×10^5 rad s^{-1} the voltage across resistance, inductance and capacitance are 60 V, 40 V and 40 V respectively. At what angular frequency of the source the current in the circuit will lag behind the source voltage by $\frac{\pi}{4}$.

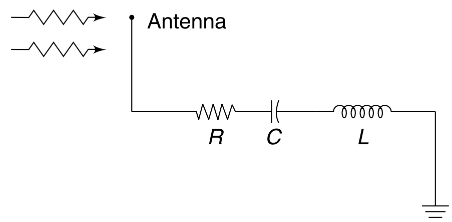
Q. 10: A resistance (R), inductance (L) and capacitance (C) are connected in series to an *ac* source of voltage V having variable frequency. Calculate the energy delivered by the source to the circuit during one period if the operating frequency is twice the resonance frequency.

Q. 11: A FM radio receiver has a series LCR circuit with $L = 1\ \mu$H, and $R = 100$ Ω. The antenna receives radio waves and induces a sinusoidally alternating emf of amplitude 10 μV. The induced voltage is fed to the series LCR circuit. The capacitance in the circuit is adjusted to a value of $C = 2$ pF.

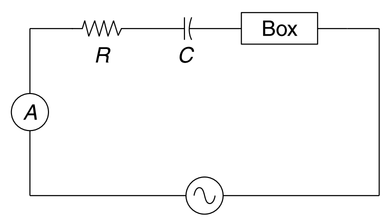
(a) Find the frequency of radio wave to which the radio will tune to.

(b) Find the rms current in the circuit.

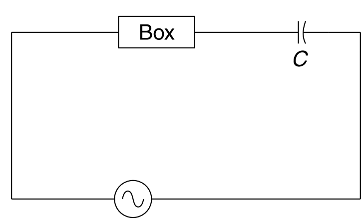
(c) Find quality factor of the resonance.



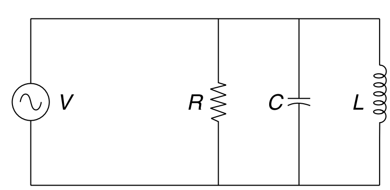
Q. 12: In the circuit shown in Figure, the source has a rating of 15 V, 100 Hz. The resistance R is 3 Ω and the reactance of the capacitor is 4 Ω. It is known that the box certainly contains one or more element (resistance, capacitance or inductance). Which element/s are present inside the box?



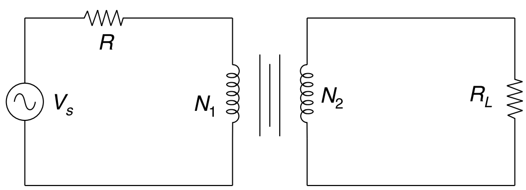
Q. 13: A box has a large electric circuit inside it. When it was connected to an ac generator it was found that it was putting a lot of load on the generator and the power factor of the box was $\frac{1}{\sqrt{2}}$. A capacitor of capacitance C was connected in series with the box and the power factor of the circuit became equal to the ideal value. Find the impedance of the box. The generator has an angular frequency of ω.



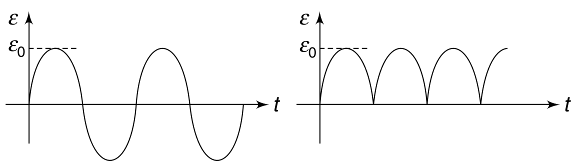
Q. 14: In the circuit shown the source voltage is given as $v = V_0 \sin\omega t$. Find the current through the source as a function of time.



Q. 15: In the circuit shown the transformer is ideal with turn ratio $\frac{N_1}{N_2} = \frac{5}{1}$. The voltage of the source is V_s = 300 Volt. The voltage measured across the load resistance R_L = 100 Ω is 50 Volt. Find the value of resistance R in the primary circuit.



Q. 16: A transformer with 20 turns in its secondary coil is used to step down the input 220 V ac emf. The output of the transformer is fed to a rectifier circuit which convents the ac input into dc output. The input and output of rectifier are as shown in Figure (there is no change in peak voltage). The rectifier output has an average emf of 8.98 volt. Calculate the number of turns in the primary coil of the transformer.



Rectifier input   Rectifier output

Q. 17: Two boys are holding a wire, standing 4 m apart. The wire sags in the shape of a circular arc with a sag of 1.0 m. The students rotate the wire about the horizontal line connecting their hands, as if they were playing jump rope. The boys rotate the wire at a speed of 4 revolutions per second. The earth's magnetic field at the location is 4×10^{-5} T. Calculate the rms value of emf developed between the ends of the wire. Assume that the shape of the wire is maintained as it is rotated.

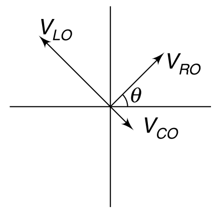
Q. 18: In a series LCR circuit the phasors corresponding to voltage across resistance, capacitor and inductor at an instant are as shown in the Figure and have amplitudes of V_{RO} = 4 volt, V_{CO} = 3 volt, and V_{LO} = 6 volt

(a) Is the source frequency larger than or lesser than the resonance frequency? Does the current (I) lead or lag the source voltage?

(b) Find the voltage amplitude of the source.

(c) If V_R phasor makes $\theta = 53°$ at time $t = 0$, write the source voltage as function of time. Take angular frequency of the source to be ω

$$\left[\sin 53° = \frac{4}{5}\right]$$

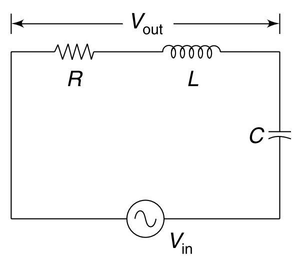


Q. 19: A series LCR circuit having resistance R, capacitance C and inductance L has a voltage source of angular frequency ω and voltage V_{in}. Output voltage (V_{out}) is taken as voltage across the resistor and inductor combined.

(a) Find $\eta = \frac{V_{out}}{V_{in}}$

(b) Find η in the limit of large $\omega\left(\omega >> \frac{1}{RC}, \frac{1}{\sqrt{LC}}, \frac{R}{L}\right)$

(c) Find η in the limit of small $\omega\left(\omega << \frac{1}{RC}, \frac{1}{\sqrt{LC}}, \frac{R}{L}\right)$.



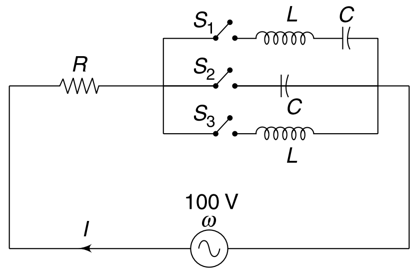
Q. 20: In a series LCR circuit, the frequencies at which the current amplitude is $\frac{1}{\sqrt{2}}$ times the current amplitude at resonance are f_1 and $f_2 (> f_1)$. Find the frequency bandwidth of resonance which is defined as $\Delta f = f_2 - f_1$. Express your answer in terms of R and L. Assume that resonance frequency $f_0 >> \Delta f$.

Q. 21: A series RLC circuit is in resonance with a source of frequency ω_0 = 10 MHz. The current amplitude in the circuit is I_0. It was found that when a different source of frequency $\omega = \omega_0 + \Delta\omega$ [$\Delta\omega$ = 10 KHz] was used the current amplitude in the circuit was only 1% of I_0. Find the inductance in the circuit if it is known that resistance in the circuit is R = 0.314 Ω.

Q. 22: In the circuit shown in the figure, one of the three switches is kept closed and other two are open. The value of resistance is R = 20 Ω. When the angular frequency(ω) of the 100 V source is adjusted to 500 rad/s, 1000 rad/s and 2000 rad/s it was found that the current I was 4A, 5A and 4A respectively.

(a) Which switch is closed? (S_1, S_2 or S_3)

(b) Find the value of L and C.



Q. 23: A village with a demand of 800 kW electric power at 220 V is located 30 km from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.25 Ω/km. The village gets power from the line through a 4000 V – 220 V step down transformer at a sub-station in the village. Assume negligible power loss in the transformers.

(a) Estimate the power loss in form of heat in the transmission line.

(b) How much power must the plant supply?

(c) What is the input and output voltage of step-up transformer at the plant ?

(d) What difference will it make if the village receives power through a 40,000 V – 220 V step down transformer

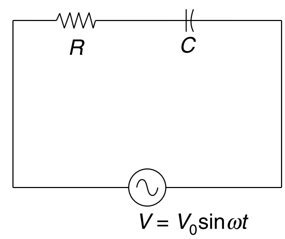
Level 3

Q. 24: A resistance R and a capacitor having capacitance C are connected to an alternating source having emf $v = V_0 \sin(\omega t)$. It is given that $\omega = \dfrac{1}{\sqrt{3}RC}$

(a) Plot the variation of power supplied by the source as a function of time. Mark the maximum and minimum values of power in the graph.

(b) How does the plot change if capacitor is removed and only R remains connected to the source?

(c) Plot the graph when only C remains connected to source and R is removed.



ANSWERS

1. (a) $9A^2$ (b) 282 V

2. Both have same average

3. Both have same value

4. 1.63

5. (a) $Q = 20$ C (b) 100 kA
(c) 6.67 nF

6. Reading of ammeter decreases.

7. (i) Brilliance of lamp decreases
(ii) Brilliance increase
(iii) Brilliance decrease
The lamp is least bright with laminated iron core.

8. 45°

9. $\omega = 8 \times 10^5$ rad s^{-1}

10. $\dfrac{\pi R\sqrt{LC}V^2}{R^2 + 2.25\dfrac{L}{C}}$

11. (a) 112.9 MHz (b) 70 nA
(c) 7.07

12. The box has L and C in series

13. $\dfrac{\sqrt{2}}{\omega C}$

14. $i = V_0\left[\dfrac{1}{R}\sin\omega t + \left(\omega C - \dfrac{1}{\omega L}\right)\cos\omega t\right]$

15. 500 Ω

16. 440

17. 0.97 mV

18. (a) Source frequency is greater than resonant frequency. Current lags.
(b) 5 volt
(c) $V_0 \cos\omega t$

19. (a) $\sqrt{\dfrac{1+\left(\dfrac{\omega L}{R}\right)^2}{1+\left(\dfrac{\omega L}{R} - \dfrac{1}{\omega RC}\right)^2}}$
(b) 1 (c) ωRC

20. $\Delta f = \dfrac{R}{2\pi L}$

21. 0.25 mH

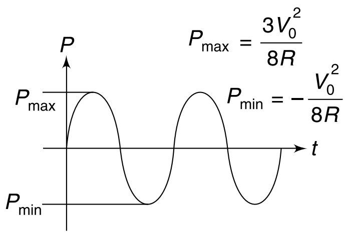
22. (a) S_1 (b) $L = 100$ mH, $C = 100\ \mu$F

23. (a) 600 kW (b) 1400 kW

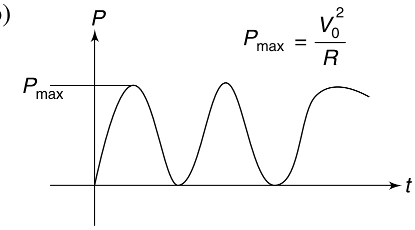
(c) 440 V – 7000 V

(d) Line power loss will drop to 6 kW. Power supply needed from plant = 806 kW

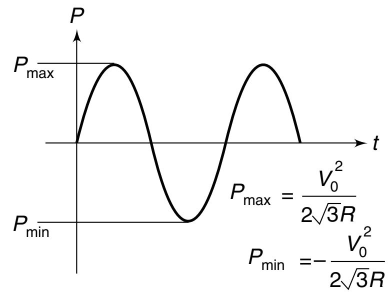
24. (a)



(b)



(c)



SOLUTIONS

1. (a) RMS current = $3A$

$$\Rightarrow \quad \sqrt{\langle i^2 \rangle} = 3A$$

$$\Rightarrow \quad \langle i^2 \rangle = 9A^2$$

(b)
$$V_0 = \sqrt{2}V_{rms} = \sqrt{2} \times 200 = 282 \text{ V}$$

2. Area under the two graphs is same if time interval T is considered. Hence average will be same for both.

4. Time period $T = 4$ s

$$V = 1.5t \quad \text{for} \quad 0 \le t \le 2$$
$$= 0 \quad \text{for} \quad 2 < t \le 4$$

$$\therefore \quad V_{av} = \frac{1}{T}\int_0^T V dt = \frac{1}{4}\int_0^2 1.5t\ dt = \frac{1}{4} \times \left[\frac{1.5t^2}{2}\right]_0^2 = 0.75 \text{ volt}$$

$$V_{rms} = \left[\frac{1}{T}\int_0^T V^2 dt\right]^{1/2} = \left[\frac{1}{4}\int_0^2 (1.5t)^2 dt\right]^{1/2} = 1.22 \text{ V}$$

$$\therefore \quad \frac{V_{rms}}{V_{av}} = \frac{1.22}{0.75} = 1.63$$

5. (a) Charge released

$$Q = \int_0^T I_0 \sin\left(\frac{\pi}{T}t\right) dt$$
$$= \frac{2I_0T}{\pi} = \frac{2 \times 157 \times 10^3 \times 0.2 \times 10^{-3}}{3.14}$$
$$= 20 \text{ C}$$

(b) Average current
$$I_{av} = \frac{Q}{T} = \frac{2I_0}{\pi} = 100 \text{ kA}$$

(c) Electric field at the time of discharge $E = 3 \times 10^6$ V/m

Potential difference between the cloud & the earth

$$V = Ed = 3 \times 10^6 \times 10^3 = 3 \times 10^9 \text{ volt}$$

$\therefore$ Capacitance
$$C = \frac{Q}{V} = \frac{20}{3 \times 10^9} = 6.67 \times 10^{-9} \text{ F}$$

6. Steel is ferromagnetic. Its presence inside the coil increases the inductance. The circuit is initially in resonance as the current is maximum. Reactance of the circuit is zero. The reactance of the circuit becomes non zero as the ball enters the coil. Thus impedance increases and current decrease.

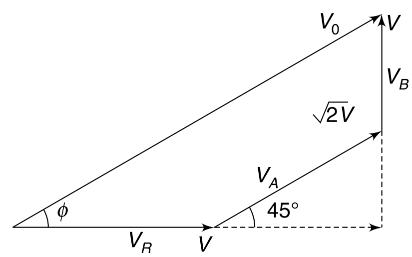
7. With Fe core, inductance of the inductor increases. This is due to strong magnetization of the ferromagnetic material. Due to this inductive reactance (X_L) increases. This increases the impedance (Z) of the circuit. The current drops and the lamp becomes dull.

When Cu core is inserted, periodically changing $\vec{B}$ produces an eddy current in it. The field produced by eddy current will weaken the field inside the coil. This decreases the inductance and hence X_L. Bulb becomes brighter.

Eddy current effect also happens in case of iron core but the property of ferromagnesian proves to be much stronger.

In case of laminated Fe core, the effect of eddy current gets reduced further. This laminated core results in sharp increases in impedance. The brilliance of the bulb decreases a lot.

8. In the Figure shown, V_0 represents the phasor associated with the source voltage.



$$\tan\phi = \frac{2V}{2V} = 1$$

$\Rightarrow$
$$\phi = 45°$$

The current phasor will be parallel to V_R hence the angle between V_0 and I is 45°

9. With $\omega_0 = 4 \times 10^5$ rad s^{-1}, the circuit is in resonance ($\because$ $V_L = V_C$)

$\therefore$
$$Z = R = 120\ \Omega$$

$\therefore$ RMS current
$$I = \frac{V}{Z} = \frac{60}{120} = 0.5 \text{ A}$$

[Note that source voltage = V_R in resonance]

Now
$$IX_L = V_L \Rightarrow 0.5\,(\omega L) = 40$$

$\Rightarrow$
$$L = \frac{40}{0.5 \times 4 \times 10^5} = 0.2 \text{ mH}$$

And
$$IX_C = V_C$$

$\Rightarrow$
$$0.5\,\frac{1}{\omega C} = 40 \Rightarrow C = \frac{0.5}{4 \times 10^5 \times 40} = 31.25 \text{ nF}$$

If current lags behind the voltage by $\pi/4$ we must have

$$\tan\left(\frac{\pi}{4}\right) = \frac{X_L - X_L}{R}$$

$$\omega L - \frac{1}{\omega C} = 1 \times 120$$

$$0.2 \times 10^{-3}\,\omega - \frac{1}{\omega \times 31.25 \times 10^{-9}} = 120$$

$$0.2 \times 31.25 \times 10^{-12}\,\omega^2 - 120 \times 31.25 \times 10^{-9}\,\omega - 1 = 0$$

$\Rightarrow$
$$6.25\omega^2 - 3.75 \times 10^6\omega - 10^{12} = 0$$

Solving this quadratic equation gives

$$\omega = 8 \times 10^5 \text{ rad s}^{-1}$$

10. Resonance angular frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$

When $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$ then–

$$X_L = L\omega = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Impedance at frequency 2ω will be

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z^2 = R^2 + \left(2 - \frac{1}{2}\right)^2 \frac{L}{C} = R^2 + 2.25\,\frac{L}{C}$$

Power supplied is

$$P = VI\cos\phi = \frac{V^2}{Z}\cos\phi$$

$$= \frac{V^2}{Z}\cdot\frac{R}{Z} = \frac{V^2R}{Z^2}$$

$$\therefore \quad P = \frac{V^2R}{R + 2.25\left(\frac{L}{C}\right)}$$

This is actually average power (averaged over one cycle).

$\therefore$ Energy supplied is

$$E = P.T = P\,\frac{2\pi}{\omega} = \pi\sqrt{LC}\,P$$

$$\therefore \quad E = \frac{\pi R\sqrt{LC}\,V^2}{R^2 + 2.25\left(\frac{L}{C}\right)}$$

11. (a) Resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{10^{-6} \times 2 \times 10^{-12}}}$$

$$= \frac{10^9}{2\pi \times 1.41} = 1.129 \times 10^8 \text{ Hz}$$

$$\simeq 112.9 \text{ MHz}$$

(b) Since circuit is in resonance $Z = R = 100\ \Omega$

$$\therefore \quad I = \frac{V}{Z} = \frac{\frac{10}{\sqrt{2}} \times 10^{-6}}{100}$$

$$\therefore \quad I = 70 \text{ nA}$$

(c) $$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{10^{-6}}{2 \times 10^{-12}}}$$

$$= 7.07$$

12. Impedance of the circuit without the box is

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{3^2 + 4^2} = 5\ \Omega$$

$\therefore$ Voltage across R and C combination is $= 5\ \Omega \times 3\ \text{A} = 15$ V

Hence, voltage drop across the box = 0

The box must have a series combination of L and C with $X_L = X_C$

13. The box has inductive load apart from resistance.

$$\tan\phi = \frac{X_L}{R}$$

$$1 = \frac{X_L}{R} \qquad \left[\because \cos\phi = \frac{1}{\sqrt{2}}\right]$$

$$\therefore \quad X_L = R \qquad \text{...(i)}$$

After addition of the capacitor the power factor becomes 1 (the ideal value). This is possible if

$$X_L = X_C = \frac{1}{\omega C} \qquad \text{...(i)}$$

$$\therefore \quad Z_{\text{Box}} = \sqrt{R^2 + X_L^2} = \sqrt{X_L^2 + X_L^2}$$

$$= \sqrt{2}\,X_L = \sqrt{2}\,X_C = \frac{\sqrt{2}}{\omega C}$$

14. The instantaneous current in three branches are–

$$i_R = \frac{V_0}{R}\sin\omega t$$

$$i_C = \frac{V_0}{X_C}\cos\omega t$$

And

$$i_L = -\frac{V_0}{X_L}\cos\omega t$$

$$\therefore \quad i = i_R + i_C + i_L$$

$$= V_0\left[\frac{1}{R}\sin\omega t + \left\{\frac{1}{X_C} - \frac{1}{X_L}\right\}\cos\omega t\right]$$

$$= V_0\left[\frac{1}{R}\sin\omega t + \left(\omega C - \frac{1}{\omega_L}\right)\cos\omega t\right]$$

15.

$$V_1 = \left(\frac{N_1}{N_2}\right)V_2 \quad [V_2 = V_L = 50\ \text{V}]$$

And

$$V_s = I_1 R + \left(\frac{N_1}{N_2}\right)V_2$$

Current in secondary circuit is

$$I_2 = \frac{V_2}{R_L}$$

And

$$I_1 = \left(\frac{N_2}{N_1}\right)I_2 = \frac{N_2}{N_1}\frac{V_2}{R_L}$$

$$\therefore \quad V_s\left(\frac{N_2}{N_1}\right)\left(\frac{V_2}{R_L}\right)R + \left(\frac{N_2}{N_1}\right)V_2$$

$$\Rightarrow \qquad R = \frac{N_1 R_L}{N_2 V_2}\left[V_s - V_2\left(\frac{N_1}{N_2}\right)\right]$$

$$= 5 \times \frac{100}{50}[300 - 50 \times 5]$$

$$= 100 \times 50 = 500\ \Omega$$

16. Average of rectifier output is

$$\frac{2\varepsilon_0}{\pi} = 8.98$$

$$\therefore \qquad \varepsilon_0 = \frac{8.98 \times 3.14}{2} = 14.1 \text{ volt}$$

$\therefore$ rms value of transformer output (i.e., same as rectifier input) $= \frac{\varepsilon_0}{\sqrt{2}} = \frac{14.1}{1.41} \simeq 10$ volt

For transformer

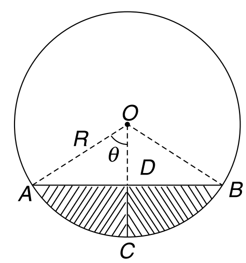
$$\frac{\varepsilon_{\text{input}}}{\varepsilon_{\text{output}}} = \frac{N_P}{N_S}$$

$$\frac{220}{10} \times 20 = N_P$$

$$\therefore \qquad N_P = 440$$

17. Let's Calculate the area occupied by the rotating wire

A and B are boys and ACB is the wire



$$AD = \frac{AB}{2} = \frac{4}{2} = 2.0 \text{ m}$$

$$CD = 1.0 \text{ m}$$

In ΔOAD: $\qquad R^2 = (R-1)^2 + 2^2 \quad \Rightarrow \quad R = \frac{5}{2}$ m

$$\tan\theta = \frac{2}{2.5} = \frac{4}{5} \quad \Rightarrow \quad \theta \simeq 40°$$

Area of sector $OACB = \frac{80}{360} \times \pi R^2 = \frac{2}{9} \times 3.14 \times \frac{25}{4} = 4.36\text{ m}^2$

Area of $\Delta OAB = \frac{1}{2} \times 4 \times \frac{3}{2} = 3\text{ m}^2$

$\therefore$ Area of $ACBDA = 4.36 - 3 = 1.36\text{ m}^2$ (= A say)

Angular speed of rotation $\qquad \omega = 4 \times 2\pi = 8\pi$ rad s^{-1}

Flux through area A at any instant can be written as

$$\phi = BA\sin(\omega t + \delta)$$

$\therefore$ Induced emf $\qquad \varepsilon = \frac{d\phi}{dt} = BA\omega\cos(\omega t + \delta)$

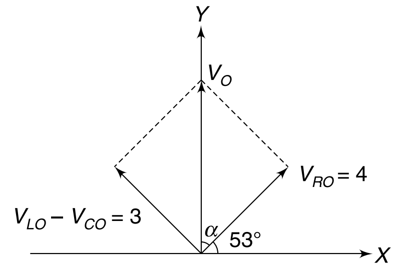
$$\overline{\varepsilon^2} = B^2 A^2 \omega^2 \overline{\cos^2(\omega t + \delta)} = \frac{1}{2} B^2 A^2 \omega^2$$

$$\therefore \qquad \varepsilon_{\text{rms}} = \frac{1}{\sqrt{2}} BA\omega = \frac{1}{\sqrt{2}} \times 4 \times 10^{-5} \times 1.36 \times 8\pi$$

$$= 96.6 \times 10^{-5}\text{ V} = 0.97\text{ mV}$$

18. (a) $V_L > V_C \quad \Rightarrow \quad X_L > X_C \quad \Rightarrow \quad \omega L > \frac{1}{\omega L} \quad \Rightarrow \quad \omega > \frac{1}{\sqrt{LC}}$

The (source) voltage phasor will be represented by vector sum of the three given phasors. It will be somewhere between V_{RO} and V_{LO}. It means the voltage across R (which is in phase with current) is behind the source voltage. It means current lags in the circuit.



(b) Voltage amplitude of the source is given by

$$V_0 = \sqrt{V_{RO}^2 + (V_{LO} - V_{CO})^2} = \sqrt{4^2 + (6-3)^2} = 5 \text{ volt}$$

(c) If V_0 makes angle α with V_{RO} then $\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 37°$

This implies that at $t = 0$, phasor V_0 is along Y. The phase angle of this phasor at $t = 0$ is $\pi/2$

$$\therefore \quad V = V_0 \sin\left(\omega t + \frac{\pi}{2}\right) = V_0 \cos\omega t$$

19. (a)
$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I\sqrt{R^2 + X_L^2}}{I\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \sqrt{\frac{R^2 + (\omega L)^2}{R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2}}$$

$$= \sqrt{\frac{1 + \left(\frac{\omega L}{R}\right)^2}{1 + \left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)^2}}$$

(b) when ω is large $\frac{\omega L}{R} >> \frac{1}{\omega RC}$

$$\therefore \quad \frac{\omega L}{R} - \frac{1}{\omega RC} \simeq \frac{\omega L}{R}$$

$$\therefore \quad \frac{V_{\text{out}}}{V_{\text{in}}} \simeq 1$$

(c) When ω is small $\quad \frac{\omega L}{R} - \frac{1}{\omega RC} \simeq -\frac{1}{\omega RC}$

Also $\frac{\omega L}{R} << 1$ and $\frac{1}{\omega RC} >> 1$

$$\therefore \quad \frac{V_{\text{out}}}{V_{\text{in}}} \simeq \sqrt{\frac{1}{1 + \left(\frac{1}{\omega RC}\right)^2}} \simeq \omega RC$$

20. In resonance
$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad I_0 = \frac{V_0}{R}$$

At a different frequency ($\omega = 2\pi f$), the current amplitude is

$$\frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Given
$$\frac{V_0}{\left[\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right]} = \frac{1}{\sqrt{2}}\frac{V_0}{R}$$

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\Rightarrow \qquad \omega L - \frac{1}{\omega L} = \pm R$$

$$2\pi f_1 L - \frac{1}{2\pi f_1 C} = -R \qquad \text{...(i)}$$

And $$2\pi f_2 L - \frac{1}{2\pi f_2 C} = R \qquad \text{...(ii)}$$

Diving (i) by f_2 and (ii) by f_1 we get

$$2\pi L \frac{f_1}{f_2} - \frac{1}{2\pi C f_1 f_2} = -R \qquad \text{...(iii)}$$

$$2\pi L \frac{f_2}{f_1} - \frac{1}{2\pi C f_1 f_2} = R \qquad \text{...(iv)}$$

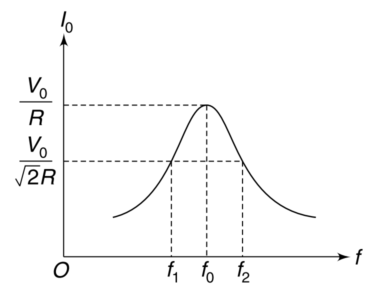
(iv) – (iii) gives

$$2\pi L \left(\frac{f_2^2 - f_1^2}{f_1 f_2}\right) = 2R$$

$$\Rightarrow \qquad \frac{(f_2 - f_1)(f_2 + f_1)}{f_1 f_2} = \frac{R}{\pi L}$$

$$f_1 + f_2 \simeq 2f_0 \quad \text{and} \quad f_1 f_2 \simeq f_0^2$$

$$\therefore \qquad \Delta f = \frac{R}{2\pi L}$$



21. Current amplitude at resonance frequency (ω_0) is $I_0 = \dfrac{V_0}{R}$

Current amplitude at frequency $\omega = \omega_0 + \Delta\omega$ is

$$I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

It is required that $$I = \frac{I_0}{100}$$

$$\Rightarrow \qquad I^2 = \frac{I_0^2}{10^4}$$

$$\Rightarrow \qquad \frac{1}{R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2} = \frac{1}{10^4 R^2}$$

$$\Rightarrow \qquad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 10^4 R^2 \quad \Rightarrow \quad \left(\omega L - \frac{1}{\omega C}\right)^2 \simeq 10^4 R^2$$

$$\Rightarrow \qquad \left(\omega L - \frac{1}{\omega C}\right) = 100\, R$$

We have chosen positive sign because $\omega L > \dfrac{1}{\omega C}$ at frequency higher than resonance frequency

$$\therefore \qquad (\omega_0 + \Delta\omega) L - \frac{1}{(\omega_0 + \Delta\omega) C} = 100 R$$

$$\Rightarrow \qquad \omega_0 L + \Delta\omega L - \frac{1}{\omega_0 C}\left[1 + \frac{\Delta\omega}{\omega_0}\right]^{-1} = 100 R$$

$$\Rightarrow \qquad \omega_0 L + \Delta\omega L - \frac{1}{\omega_0 C}\left[1 - \frac{\Delta\omega}{\omega_0}\right] = 100 R$$

$$\Rightarrow \quad \Delta\omega L + \frac{\Delta\omega}{\omega_0^2 C} = 100R \qquad \left[\because \ \omega_0 L = \frac{1}{\omega_0 C}\right]$$

$$\Rightarrow \quad 2\Delta\omega L = 100R$$

$$\Rightarrow \quad L = \frac{50R}{\Delta\omega} = \frac{50 \times 0.314}{2 \times 3.14 \times 10^4}$$

$$L = 2.5 \times 10^{-4} \text{ H} = 0.25 \text{ mH}$$

22. When current is $5A$, the impedance is

$$Z = \frac{V}{I} = \frac{100}{5} = 20 \ \Omega$$

This is equal to R.
Hence for $\omega_0 = 1000$ rad/s

$$Z = R = 20 \ \Omega$$

This is possible only if S_1 is closed and

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\Rightarrow \quad LC = \frac{1}{10^6} \qquad \text{...(i)}$$

When $\omega = 500$ rad/s

$$I = 4A \quad \Rightarrow \quad \frac{100}{Z} = 4A$$

$$\Rightarrow \quad Z = 25 \ \Omega$$

$$\sqrt{R^2 + X^2} = 25 \quad \Rightarrow \quad X = 15 \ \Omega$$

$$\therefore \quad \frac{1}{\omega C} - \omega L = 15$$

[Note that X is not $\omega L - \frac{1}{\omega C}$, since on increasing the frequency from 500 rad/s to 1000 rad/s the impedance is decreasing]

$$\Rightarrow \quad \frac{1}{500C} - 500L = 15 \qquad \text{...(ii)}$$

Solving (i) and (ii) $L = 10$ mH, $C = 100 \ \mu$F

23. rms current in line $\quad I = \dfrac{800 \times 10^3 \text{ W}}{4000 \text{ V}} = 200$ A

(a) Line loss = $1^2R \qquad [R = 0.25 \times 60 = 15 \ \Omega]$

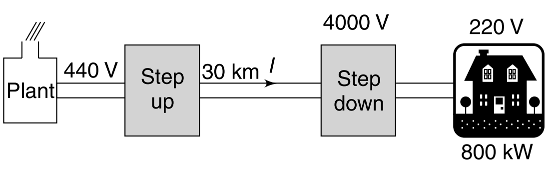
$= (200)^2 \times 15 = 600$ kW

(b) $P_{plant} = 800 + 600 = 1400$ kW

(c) Voltage drop on the line = $200 \times 15 = 3000$ V

$\therefore$ Transformer needed at plant is 440 V – 7000 V

(d) High voltage transmission will cause low power loss



24. (a) Current leads voltage by phase angle

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} = \sqrt{3} \quad \Rightarrow \quad \phi = \pi/3$$

$$\therefore \quad v = V_0 \sin\omega t \quad \text{and} \quad i = i_0 \sin(\omega t + \pi/3) \qquad \left[\text{where} \quad i_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}} = \frac{V_0}{\sqrt{R^2 + 3R^2}} = \frac{V_0}{2R}\right]$$

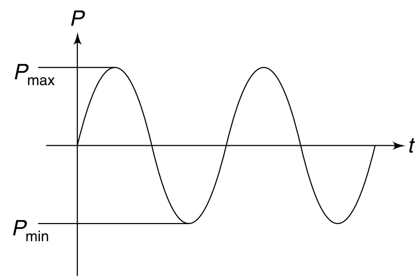
Power supplied by the source

$$P = vi = V_0 i_0 \sin\omega t \sin(\omega t + \pi/3)$$

$$= V_0 i_0 \left[\cos\frac{\pi}{3}\cdot\sin^2\omega t + \sin\frac{\pi}{3}\cdot\sin\omega t\cdot\cos\omega t\right]$$

$$= \frac{V_0 i_0}{2}\left[\cos\frac{\pi}{3}(1 - \cos 2\omega t) + \sin\frac{\pi}{3}\cdot\sin 2\omega t\right]$$

$$= \frac{V_0 i_0}{2}\left[\cos\frac{\pi}{3} - \cos\left(2\omega t + \frac{\pi}{3}\right)\right]$$



$\therefore$ $$P_{max} = \frac{V_0 i_0}{2}\left(\cos\frac{\pi}{3} + 1\right) = \frac{3}{4}V_0 i_0 = \frac{3V_0^2}{8R}$$

And $$P_{min} = \frac{V_0 i_0}{2}\left(\cos\frac{\pi}{3} - 1\right) = -\frac{V_0 i_0}{4} = -\frac{V_0^2}{8R}$$

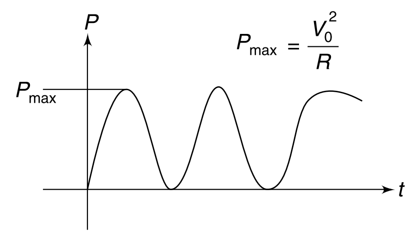
Power oscillates with frequency twice that of current and voltage

(b) with only 'R' the phase difference between current and voltage will be $\phi = 0$

$\therefore$ $$P = v_i = \frac{v^2}{R} = \frac{V_0^2}{R}\sin^2\omega t$$

$$= \frac{V_0^2}{2R}[1 - \cos 2\omega t]$$

$$P_{max} = \frac{V_0^2}{R};\ P_{min} = 0$$



(c) with only 'C'

$$P = \frac{V_0 i_0}{2}\sin(2\omega t)$$

$$P_{max} = \frac{V_0 i_0}{2};\ P_{min} = -\frac{V_0 i_0}{2} = -\frac{V_0^2}{2X_C}$$

$$= -\frac{V_0^2\omega C}{2} = -\frac{V_0^2}{2\sqrt{3}R}$$

$$P_{max} = \frac{V_0^2}{2\sqrt{3}R}$$