Fundamentals of Mathematics-1

Contents

Topic				Page No.
Theory		:		01 – 18
Exercise # 1	Part - I	:	Subjective Questions	19 – 29
	Part - II	:	Single option correct type	
	Part - III	:	Match the Column	
Exercise # 2	Part - I	:	Single option correct type	30 – 35
	Part - II	:	Numerical Type	
	Part - III	:	One or More than one option correct type	
	Part - IV	:	Comprehension	
Exercise # 3	Part - I	:	Previous Years JEE Advanced Problems	36 – 37
	Part - II	:	Previous Years AIEEE & JEE Mains Problems	
Answers		:		38 – 40
Self Assessment Paper (SAP)		:		41 – 42
Answers	(SAP)	:		42 – 42
Logarithm table		:		43 – 46

FUNDAMENTALS OF MATHEMATICS-1

1. SETS

1.1 Definition of Set

A set is well defined collection of distinct objects. The objects that make up a set (also known as sets element or member) can be anything: numbers, people, letters, other set and so on. Set are conventionally denoted with capital letters A, B, C, ... etc. and the elements of the set by small letters p, q, r .... etc.

If p is an element of a set A, then we write $p \in A$ and say p belongs to A.

If p does not belong to A then we write $p \notin A$,

e.g. the collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

1.2 Methods to write a set

(a) Roster Method or Tabular Method : In this method a set is described by listing elements, separated by commas and enclose then by curly brackets. Note that while writing the set in roster form, an element is not generally repeated e.g. the set of letters of word SCHOOL may be written as {S, C, H, O, L}.

(b) Set builder form (Property Method) : In this we write down a property or rule which gives us all the element of the set.

$A = \{x : P(x)\}$ where P(x) is the property by which $x \in A$ and colon (:) stands for 'such that'

SOLVED EXAMPLE

Example 1 : Express set $A = \{x : x \in N$ and $x = 3n$ for $n \in N\}$ in roster form

Solution : $A = \{3, 6, 9, ...\}$

Example 2 : Express set $B = \{x^2 : x \leq 4, x \in W\}$ in roster form

Solution : $B = \{0, 1, 4, 9, 16\}$

Example 3 : Express set A = {2, 5, 10, 17, 26} in set builder form

Solution : $A = \{x : x = n^2 + 1, n \in N, 1 \leq n \leq 5\}$

1.3 Types of sets

(a) Null set or empty set : A set having no element in it is called an empty set or a null set or void set, it is denoted by ϕ or { }. A set consisting of at least one element is called a non-empty set or a non-void set.

(b) Singleton set : A set consisting of a single element is called a singleton set.

(c) Finite set : A set which has only finite number of elements is called a finite set.

Note : Order of a finite set : The number of elements in a finite set A is called the order of this set and denoted by O(A) or n(A). It is also called cardinal number of the set.

e.g. $A = \{p, q, r, s\} \Rightarrow n(A) = 4$

(d) Infinite set : A set which has an infinite number of elements is called an infinite set.

Note :

(i) Equal sets : Two sets A and B are said to be equal if every element of A is member of B, and every element of B is a member of A. If sets A and B are equal, we write A = B and if A and B are not equal then $A \neq B$

(ii) Equivalent sets : Two finite sets A and B are equivalent if their number of elements are same

i.e. n(A) = n(B)

e.g. A = {1, 2, 3, 4}, B = {p, q, r, s}

⇒ n(A) = 4 and n(B) = 4

⇒ A and B are equivalent sets

Note : Equal sets are always equivalent but equivalent sets may not be equal

SOLVED EXAMPLE

Example 4 : Identify the type of set :

(i) $A = \{x \in W : 5 < x < 6\}$ (ii) $A = \{a, b, c\}$

(iii) $A = \{1, 2, 3, 4, ...\}$ (iv) $A = \{1, 2, 6, 7\}$ and $B = \{6, 1, 2, 7, 7\}$

(v) $A = \{0\}$

Solution : (i) Null set, finite set (ii) finite set

(iii) infinite set (iv) A & B are finite sets and they are equal sets also

(v) singleton set, finite set

<u>Problems for Self Practice</u> -1:

(1) Write the set of all integers 'x' such that $-8 < x - 3 < 8$

(2) Write the set {1, 2, 3, 6} in set builder form.

(3) If $A = \{x : |x| < 2, x \in Z\}$ and $B = \{-1, 1\}$ then find whether sets A and B are equal or not.

Answers (1) [–4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

(2) {x : x is a natural number and a divisor of 6}

(3) Not equal sets

1.4 Some important number sets

(a) Set of all natural numbers = {1, 2, 3, 4, ...} = N

(b) Set of all whole numbers = {0, 1, 2, 3, ...} = W

(c) set of all integers = {.... –3, –2, –1, 0, 1, 2, 3, ...} = I = Z

(d) Set of all +ve integers = {1, 2, 3, ...} = $Z^+ = N = I^+$

(e) Set of all –ve integers = (–1, –2, –3, ...} = $Z^- = I^-$

(f) The set of all non-zero integers = {±1, ±2, ±3, ...} = Z_0

(g) The set of all rational numbers = $\left\{\frac{p}{q} : p, q \in I, q \neq 0\right\} = Q$

(h) The set of all positive rational numbers = $\left\{\frac{p}{q} : p, q \in I^+\right\} = Q^+$

(i) The set of all negative rational numbers = $\left\{\frac{p}{q} : p \in I^-, q \in I^+\right\} = Q^-$

(j) The set of all real numbers = R

(k) The set of all positive real numbers = R^+

(ℓ) The set of all negative real numbers = R^-

(m) The set of all irrational numbers = R–Q

e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... π, e, $\log_{10}2$ etc. are all irrational numbers.

1.5 Intervals

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains.
If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

Name	Representation	Discription
Open Interval	(a, b)	$\{x : a < x < b\}$ i.e. end points are not included.
Close Interval	[a, b]	$\{x : a \le x \le b\}$ i.e. end points are also included. This is possible only when both a and b are finite.
Open - Closed Interval	(a, b]	$\{x : a < x \le b\}$ i.e. a is excluded and b is included.
Close - Open Interval	[a, b)	$\{x : a \le x < b\}$ i.e. a is included and b is excluded.

Note : (1) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x : x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = R$

(2) $x \in \{1, 2\}$ denotes some particular values of x, i.e. x = 1, 2

(3) If there is no value of x, then we say $x \in \phi$ (null set)

1.6 Subset, Superset, Universal, Set, Power set

1.6.1 SUBSET

Let A and B be two sets. If every element of A is an element B then A is called a subset of B and B is called superset of A. We write it as $A \subseteq B$ or $A \subset B$

e.g. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$ $\Rightarrow A \subseteq B$

If A is not a subset of B then we write $A \nsubseteq B$ or $A \not\subset B$

1.6.2 PROPER SUBSET :

If A is a subset of B but $A \neq B$ then A is a proper subset of B. Set A is not proper subset of A so this is improper subset of A

Note : (i) Every set is a subset of itself

(ii) Empty set ϕ is a subset of every set

(iii) $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$

(iv) The total number of possible subsets of a finite set containing n elements is 2^n.

(v) Number of possible proper subsets of a set having n elements is $2^n - 1$.

(vi) Empty set ϕ is proper subset of every set except itself.

1.6.3 UNIVERSAL SET

A set consisting of atleast all possible elements which occur in the discussion is called a universal set and is denoted by U. Basically universal set is superset of all the sets mentioned in the discussion.

e.g. if $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 3, 5, 7\}$ then $U = \{1, 2, 3, 4, 5, 6, 7\}$ can be taken as the universal set.

1.6.4 POWER SET

Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

SOLVED EXAMPLE

Example 5: Examine whether the following statements are true or false :

(i) $\{a, b\} \nsubseteq \{b, c, a, d\}$ (ii) $\{a, e\} \nsubseteq \{x : x \text{ is a vowel in the English alphabet}\}$

(iii) $\{1, 2, 3\} \subseteq \{1, 3, 5, 7\}$ (iv) $\{a\} \in \{a, b, c\}$

Solution : (i) False as {a, b} is subset of {b, c, a}

(ii) False as a, e are vowels

(iii) False as element 2 is not in the set {1, 3, 5}

(iv) False as $a \in \{a, b, c\}$ and $\{a\} \subseteq \{a, b, c\}$

Example 6: Find power set of set A = {1, 2}

Solution : $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Example 7: If ϕ denotes null set then find $P(P(P(\phi)))$

Solution : Let $P(\phi) = \{\phi\}$

$P(P(\phi)) = \{\phi, \{\phi\}\}$

$P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Problems for Self Practice- 2 :

(1) State true/false : A = {1, 3, 4, 5}, B = {1, 3, 5} then $A \subseteq B$.

(2) State true/false : A = {1, 3, 7, 5}, B = {1, 3, 5, 7} then $A \subset B$.

(3) State true/false : $[3, 7] \subseteq (2, 10)$

Answers (1) False (2) False (3) True

1.7 Some Operations on sets

(a) Union of two sets : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.g. A = {1, 2, 3}, B = {2, 3, 4} then $A \cup B = \{1, 2, 3, 4\}$

(b) Intersection of two sets : $A \cap B = \{x : x \in A \text{ and } x \in B\}$

e.g. A = {1, 2, 3}, B = {2, 3, 4} then $A \cap B = \{2, 3\}$

(c) Difference of two sets : $A - B = \{x : x \in A \text{ and } x \notin B\}$. It is also written as $A \cap B'$.

Similarly $B - A = B \cap A'$

e.g. A = {1, 2, 3}, B = {2, 3, 4} ; A – B = {1}

(d) Symmetric difference of sets : It is denoted by $A \Delta B$ and $A \Delta B = (A - B) \cup (B - A)$

(e) Complement of a set : $A' = A^c = \{x : x \notin A \text{ but } x \in U\} = U - A$

e.g. U = {1, 2,........, 10}, A = {1, 2, 3, 4, 5} then A' = {6, 7, 8, 9, 10}

(f) Disjoint sets : If $A \cap B = \phi$, then A, B are disjoint

e.g. If A = {1, 2, 3}, B = {7, 8, 9} then $A \cap B = \phi$

1.8 Laws of Algebra of sets (Properties of sets)

(a) Commutative law : $(A \cup B) = B \cup A$; $A \cap B = B \cap A$

(b) Associative law : $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(c) Distributive law : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(d) De-morgan law : $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

(e) Identity law : $A \cap U = A$; $A \cup \phi = A$

(f) Complement law : $A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$

(g) Idempotent law : $A \cap A = A$, $A \cup A = A$

NOTE :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cap \phi = \phi$, $A \cup U = U$

SOLVED EXAMPLE

Example 8: Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12} then find $A \cup B$

Solution : $A \cup B$ = {2, 4, 6, 8, 10, 12}

Example 9: Let A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8}. Find A – B and B – A.

Solution: A – B = {x : x $\in$ A and x $\notin$ B} = {1, 3, 5}

similarly B – A = {8}

Example 10: State true or false :

(i) $A \cup A' = \phi$ (ii) $\phi' \cap A = A$

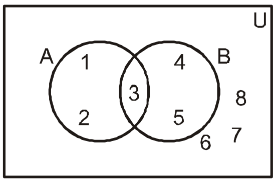
Solution : (i) false because $A \cup A' = U$

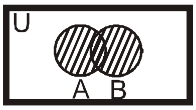
(ii) true as $\phi' \cap A = U \cap A = A$

1.9 Venn diagram

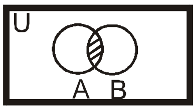
Most of the relationships between sets can be represented by means of diagrams which are known as venn diagrams.These diagrams consist of a rectangle for universal set and circles in the rectangle for subsets of universal set. The elements of the sets are written in respective circles.

For example If A = {1, 2, 3}, B = {3, 4, 5}, U = {1, 2, 3, 4, 5, 6, 7, 8} then their venn diagram is

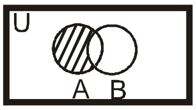




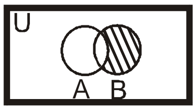
$A \cup B$



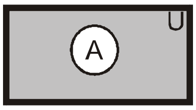
$A \cap B$



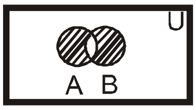
A – B



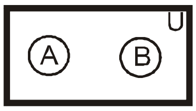
B – A



A'



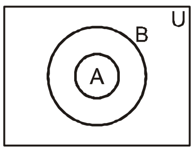
$(A \Delta B) = (A - B) \cup (B - A)$



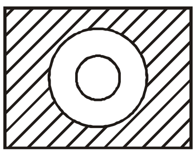
Disjoint

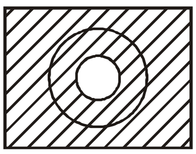
SOLVED EXAMPLE

Example 11: Use Venn diagram to prove that $A \subseteq B \Rightarrow B' \subseteq A'$.



Solution :


B′


A′

From venn diagram we can conclude that $B' \subseteq A'$.

Example 12: Prove that if $A \cup B = C$ and $A \cap B = \phi$ then A = C – B.

Solution: Let $x \in A$ $\Rightarrow x \in A \cup B$ $\Rightarrow x \in C$ $(\because A \cup B = C)$

	Now	$A \cap B = \phi$	$\Rightarrow x \notin B$	$(\because x \in A)$
	$\Rightarrow$	$x \in C - B$		$(\because x \in C$ and $x \notin B)$
	$\Rightarrow$	$A \subseteq C - B$		
Let	$x \in C - B$	$\Rightarrow x \in C$ and $x \notin B$		
$\Rightarrow$	$x \in A \cup B$	and $x \notin B$	$\Rightarrow x \in A$	$\Rightarrow$ $C - B \subseteq A$
$\therefore$	$A = C - B$			

Problems for Self Practice - 3 :

(1) Find $A \cup B$ if $A = \{x : x = 2n + 1, n \le 5, n \in N\}$ and $B = \{x : x = 3n - 2, n \le 4, n \in N\}$.

(2) Find $A - (A - B)$ if $A = \{5, 9, 13, 17, 21\}$ and $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$

Answers (1) {1, 3, 4, 5, 7, 9, 10, 11} (2) {9, 21}

1.10 Some important results on number of elements in sets

If A, B, C are finite sets and U be the finite universal set then

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(b) $n(A - B) = n(A) - n(A \cap B)$

(c) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(d) Number of elements in exactly two of the sets A, B, C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(e) Number of elements in exactly one of the sets A, B, C
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

SOLVED EXAMPLE

Example 13: In a group of 40 students, 26 take tea, 18 take coffee and 8 take neither of the two. How many take both tea and coffee ?

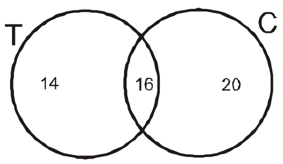
Solution: $n(U) = 40, n(T) = 26,$ $n(C) = 18$ $n(T' \cap C') = 8 \Rightarrow n(T \cup C)' = 8$

$\Rightarrow n(U) - n(T \cup C) = 8$ $\Rightarrow n(T \cup C) = 32$

$\Rightarrow n(T) + n(C) - n(T \cap C) = 32$ $\Rightarrow n(T \cap C) = 12$

Example 14: In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find
(i) How many drink tea and coffee both ? (ii) How many drink coffee but not tea ?

Solution : T : people drinking tea
C : people drinking coffee
(i) $n(T) = n(T - C) + n(T \cap C) \Rightarrow 30 = 14 + n(T \cap C)$ $\Rightarrow n(T \cap C) = 16$



(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

Problems for Self Practice-4 :

(1) Let A and B be two finite sets such that $n(A - B) = 15$, $n(A \cup B) = 90$, $n(A \cap B) = 30$. Find n(B)

(2) A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products ?

Answers (1) 75 (2) 170

2. SOLUTION OF RATIONAL INEQUALITIES

Let $y=\dfrac{f(x)}{g(x)}$ be an expression in x where f(x) & g(x) are polynomials in x. If it is given that y > 0 (or y < 0), then we have to write set of all the values of x for which y is positive (or y is negative). This solution set can be found by following steps :

Step I : Let after factorization of f(x) & g(x), $y=\dfrac{f(x)}{g(x)}$ becomes

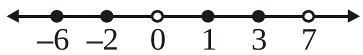
$$y=\frac{f(x)}{g(x)}=\frac{(x-1)^3(x+2)^4(x-3)^5(x+6)}{x^2(x-7)^3}$$

Clearly, here 1, –2, 3, –6 are roots of f(x) = 0 and 0, 7 are roots of g(x) = 0.

Step II : Here y vanishes (becomes zero) for 1, –2, 3, –6. These points are marked on the number line with a black dot. They are solution of y = 0.

If g(x) = 0, $y=\dfrac{f(x)}{g(x)}$ attains an undefined form, hence 0, 7 are excluded from the solution. These points are marked with white dots.

e.g. $f(x)=\dfrac{(x-1)^3(x+2)^4(x-3)^5(x+6)}{x^2(x-7)^3}$



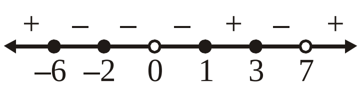
Step-III : Check the value of y for any real number greater than the right most marked number on the number line. If it is positive, then y is positive for all the real numbers greater than the right most marked number and vice versa.

Step-IV : If the exponent of a factor is odd, then the point is called simple point and if the exponent of a factor is even, then the point is called double point

$$\frac{(x-1)^3(x+2)^4(x-3)^5(x+6)}{x^2(x-7)^3}$$

Here 1,3,–6 and 7 are simple points and –2 & 0 are double points.

Now sign in neigbouring interval of simple point is different and sign in the neigbouring interval of double point is same. Hence in above example sign scheme of $y=\dfrac{f(x)}{g(x)}$ is



Step-V : y will be positive for the values of x which lies in the intervals where + mark is present & y will be negative for the values of x which lies in the intervals where – mark is present. The appropriate intervals are chosen in accordance with the sign of inequality & their union represents the solution of inequality.

In above example solution of y > 0 is $x \in (-\infty, -6) \cup (1, 3) \cup (7, \infty)$, Solution of $y \geq 0$ is $x \in (-\infty, -6] \cup \{-2\} \cup [1, 3] \cup (7, \infty)$,

Solution of $y < 0$ is $x \in (-6, -2) \cup (-2, 0) \cup (0, 1) \cup (3, 7)$,

Solution of $y \leq 0$ is $x \in [-6, 0) \cup (0, 1] \cup [3, 7)$

Note :

(i) Points where denominator is zero will never be included in the answer.

(ii) If you are asked to find the intervals where f(x) is non-negative or non-positive then make the intervals closed corresponding to the roots of the numerator and let it remain open corresponding to the roots of denominator.

(iii) Normally we cannot cross-multiply in inequalities. But we cross multiply if we are sure that quantity in denominator is always positive.

(iv) Normally we cannot square in inequalities. But we can square if we are sure that both sides are non negative.

(v) We can multiply both sides with a negative number by changing the sign of inequality.

(vi) We can add or subtract equal quantity to both sides of inequalities without changing the sign of inequality.

SOLVED EXAMPLE

Example 15: Find x such that $3x^2 - 7x + 6 < 0$

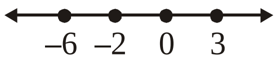
Solution : $D = 49 - 72 < 0$

As $D < 0$, $3x^2 - 7x + 6$ will always be positive. Hence $x \in \phi$.

Example 16 : $(x^2 - x - 6)(x^2 + 6x) \geq 0$

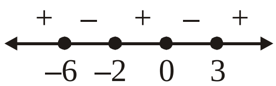
Solution : $(x-3)(x+2)(x)(x+6) \geq 0$

Consider $E = x(x-3)(x+2)(x+6)$, $E = 0 \Rightarrow x = 0, 3, -2, -6$ (all are simple points)



For $x \geq 3$ $E = \underbrace{x}_{+ve}\ \underbrace{(x-3)}_{+ve}\ \underbrace{(x+2)}_{+ve}\ \underbrace{(x+6)}_{+ve}$

= positive



Hence for $x(x-3)(x+2)(x+6) \geq 0$

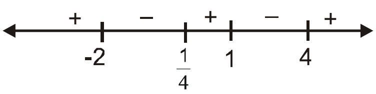
$x \in (-\infty, -6] \cup [-2, 0] \cup [3, \infty)$

Example 17 : Solve the inequality $\dfrac{x-2}{x+2} > \dfrac{2x-3}{4x-1}$

Solution : $\dfrac{x-2}{x+2} - \dfrac{2x-3}{4x-1} > 0 \Rightarrow \dfrac{(x-2)(4x-1)-(2x-3)(x+2)}{(x+2)(4x-1)} > 0$

$\Rightarrow \dfrac{4x^2 - x - 8x + 2 - 2x^2 - 4x + 3x + 6}{(x+2)(4x-1)} > 0 \Rightarrow \dfrac{2x^2 - 10x + 8}{(x+2)(4x-1)} > 0 \Rightarrow \dfrac{x^2 - 5x + 4}{(x+2)(4x-1)} > 0$

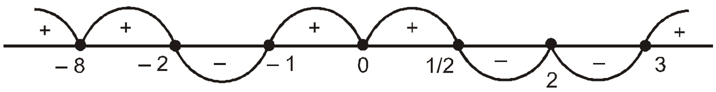
$\Rightarrow \dfrac{(x-1)(x-4)}{(x+2)(4x-1)} > 0$



Ans. : $x \in (-\infty, -2) \cup \left(\dfrac{1}{4}, 1\right) \cup (4, \infty)$

Example 18: Solve the inequality if $f(x) = \dfrac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$ is > 0 or < 0.

Solution. Let $f(x) = \dfrac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$ the poles and zeros are 0, 3, – 2, – 1, $\frac{1}{2}$, – 8, 2



If f(x) > 0, then $x \in (-\infty, -8) \cup (-8, -2) \cup (-1, 0) \cup \left(0, \frac{1}{2}\right) \cup (3, \infty)$

and if f(x) < 0, then $x \in (-2, -1) \cup \left(\frac{1}{2}, 2\right) \cup (2, 3)$ **Ans.**

Problems for Self Practice - 5 :

(1) Find complete set of value of x in following inequation

(a) $(x-2)(x+3) \geq 0$ (b) $\dfrac{x}{x+1} > 2$ (c) $\dfrac{3x-1}{4x+1} \leq 0$

(d) $\dfrac{(2x-1)(x+3)(2-x)(1-x)^2}{x^4(x+6)(x-9)(2x^2+4x+9)} < 0$ (e) $\dfrac{7x-17}{x^2-3x+4} \geq 1$

(f) $x^2 + 2 \leq 3x \leq 2x^2 - 5$ (g) $\dfrac{x^2+6x-7}{x^2+1} \leq 2$ (h) $\dfrac{1}{x+2} < \dfrac{3}{x-3}$

(i) $\dfrac{x^2+2}{x^2-1} < -2$ (j) $\dfrac{5-4x}{3x^2-x-4} < 4$ (k) $\dfrac{2x}{x^2-9} \leq \dfrac{1}{x+2}$

(ℓ) $\dfrac{1}{x-2} + \dfrac{1}{x-1} > \dfrac{1}{x}$ (m) $\dfrac{20}{(x-3)(x-4)} + \dfrac{10}{x-4} + 1 > 0$

(n) $\dfrac{1}{x-2} - \dfrac{1}{x} \leq \dfrac{2}{x+2}$

Answers

(1) **(a)** $(-\infty, -3] \cup [2, \infty)$ **(b)** $(-2, -1)$ **(c)** $\left(-\frac{1}{4}, \frac{1}{3}\right]$

(d) $(-6, -3) \cup \left(\frac{1}{2}, 2\right) - \{1\} \cup (9, \infty)$ **(e)** $[3, 7]$ **(f)** ϕ

(g) $(-\infty, +\infty)$ **(h)** $(-9/2, -2) \cup (3, +\infty)$

(i) $(-1, 0) \cup (0, 1)$ **(j)** $\left(-\infty, -\sqrt{7}/2\right) \cup \left(-1, \sqrt{7}/2\right) \cup (4/3, +\infty)$

(k) $(-\infty, -3) \cup (-2, 3)$ **(l)** $\left(-\sqrt{2}, 0\right) \cup \left(1, \sqrt{2}\right) \cup (2, +\infty)$

(m) $(-\infty, -2) \cup (-1, 3) \cup (4, +\infty)$ **(n)** $\left(-2, \frac{(3-\sqrt{17})}{2}\right] \cup (0, 2) \cup \left[\frac{(3+\sqrt{17})}{2}, +\infty\right)$

3. LOGARITHM

3.1 Definition

Every positive real number N can be expressed in exponential form as $a^x = N$ where 'a' is also a positive real number different than unity and is called the base and 'x' is called an exponent.

We can write the relation $a^x = N$ in logarithmic form as $\log_a N = x$. Hence $a^x = N \Leftrightarrow \log_a N = x$

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number.

Limitations of logarithm: $\log_a N$ is defined only when

(i) $N > 0$ (ii) $a > 0$ (iii) $a \neq 1$

Note :

(i) For a given value of N, $\log_a N$ will give us a unique value.

(ii) Logarithm of zero does not exist.

(iii) Logarithm of negative reals are not defined in the system of real numbers.

SOLVED EXAMPLE

Example 19 : If $\log_4 m = 1.5$, then find the value of m.

Solution : $\log_4 m = 1.5 \Rightarrow m = 4^{3/2} \Rightarrow m = 8$

Example 20 : If $\log_5 p = a$ and $\log_2 q = a$, then prove that $\frac{p^4q^4}{100} = 100^{2a-1}$

Solution : $\log_5 p = a \Rightarrow p = 5^a$ $\quad\log_2 q = a \Rightarrow q = 2^a$

$$\Rightarrow \frac{p^4q^4}{100} = \frac{5^{4a}.2^{4a}}{100} = \frac{(10)^{4a}}{100} = \frac{(100)^{2a}}{100} = 100^{2a-1}$$

Example 21 : Show that $\log_4 18$ is an irrational number.

Solution : $\log_4 18 = \log_4(3^2 \times 2) = 2\log_4 3 + \log_4 2 = 2\frac{\log_2 3}{\log_2 4} + \frac{1}{\log_2 4} = \log_2 3 + \frac{1}{2}$

assume the contrary, that this number $\log_2 3$ is rational number.

$\Rightarrow \log_2 3 = \frac{p}{q}$. Since $\log_2 3 > 0$ both numbers p and q may be regarded as natural number

$\Rightarrow 3 = 2^{p/q} \Rightarrow 2^p = 3^q$

But this is not possible for any natural number p and q. The resulting contradiction completes the proof.

Problems for Self Practice - 6 :

(1) Express the following in logarithmic form :

(a) $81 = 3^4$ (b) $0.001 = 10^{-3}$ (c) $2 = 128^{1/7}$

(2) Express the following in exponential form :

(a) $\log_2 32 = 5$ (b) $\log_{\sqrt{2}} 4 = 4$ (c) $\log_{10} 0.01 = -2$

(3) If $\log_{2\sqrt{3}} 1728 = x$, then find x.

Answers: **(1) (a)** $\log_3 81 = 4$ **(b)** $\log_{10}(0.001) = -3$ **(c)** $\log_{128} 2 = 1/7$

(2) (a) $32 = 2^5$ **(b)** $4 = (\sqrt{2})^4$ **(c)** $0.01 = 10^{-2}$

(3) 6

3.2 Fundamental identities

Using the basic definition of logarithm we have 3 important deductions :

(a) $\log_a 1 = 0$ i.e. logarithm of unity to any base is zero.

(b) $\log_N N = 1$ i.e. logarithm of a number to the same base is 1.

(c) $\log_{\frac{1}{N}} N = -1 = \log_N \frac{1}{N}$ i.e. logarithm of a number to the base as its reciprocal is –1.

SOLVED EXAMPLE

Example 22 : The value of N, satisfying $\log_a[1 + \log_b\{1 + \log_c(1 + \log_p N)\}] = 0$ is -

(A) 4 (B) 3 (C) 2 (D) 1

Solution : $1 + \log_b\{1 + \log_c(1 + \log_p N)\} = a^0 = 1$

$\Rightarrow \log_b\{1 + \log_c(1 + \log_p N)\} = 0 \Rightarrow 1 + \log_c(1 + \log_p N) = 1$

$\Rightarrow \log_c(1 + \log_p N) = 0 \Rightarrow 1 + \log_p N = 1$

$\Rightarrow \log_p N = 0 \Rightarrow N = 1$

Problems for Self Practice - 7 :

(1) (a) Find the value of the following :

(i) $\log_{1.4\bar{3}} \frac{43}{30}$ **(ii)** $\left(\frac{1}{2}\right)^{\log_2 5}$

(b) If $4^{\log_2 2x} = 36$, then find x.

Answers : (1) (a) (i) 1 (ii) $\frac{1}{5}$ **(b)** 3

3.3 The principal properties of logarithms

If m,n are arbitrary positive numbers where a > 0, a ≠ 1 and x is any real number, then-

(a) $\log_a mn = \log_a m + \log_a n$ (b) $\log_a \frac{m}{n} = \log_a m - \log_a n$ (c) $\log_a m^x = x \log_a m$

SOLVED EXAMPLE

Example 23 : Find the value of $2\log\frac{2}{5} + 3\log\frac{25}{8} - \log\frac{625}{128}$

Solution : $2\log\frac{2}{5} + 3\log\frac{25}{8} + \log\frac{128}{625} = \log\frac{2^2}{5^2} + \log\left(\frac{5^2}{2^3}\right)^3 + \log\frac{2^7}{5^4}$

$= \log\frac{2^2}{5^2}.\frac{5^6}{2^9}.\frac{2^7}{5^4} = \log 1 = 0$

Example 24 : If $\log_e x - \log_e y = a$, $\log_e y - \log_e z = b$ & $\log_e z - \log_e x = c$, then find the value of

$\left(\frac{x}{y}\right)^{b-c} \times \left(\frac{y}{z}\right)^{c-a} \times \left(\frac{z}{x}\right)^{a-b}$

Solution : $\log_e x - \log_e y = a \Rightarrow \log_e \frac{x}{y} = a \Rightarrow \frac{x}{y} = e^a$

$\log_e y - \log_e z = b \Rightarrow \log_e \frac{y}{z} = b \Rightarrow \frac{y}{z} = e^b$

$\log_e z - \log_e x = c \Rightarrow \log_e \frac{z}{x} = c \Rightarrow \frac{z}{x} = e^c$

$\therefore \left(e^a\right)^{b-c} \times \left(e^b\right)^{c-a} \times \left(e^c\right)^{a-b} = e^{a(b-c)+b(c-a)+c(a-b)} = e^0 = 1$

Example 25 : If $a^2 + b^2 = 23ab$, then prove that $\log\frac{(a+b)}{5} = \frac{1}{2}(\log a + \log b)$.

Solution : $a^2 + b^2 = (a + b)^2 - 2ab = 23ab$

$\Rightarrow (a + b)^2 = 25ab \Rightarrow a+b = 5\sqrt{ab}$ ....(i)

Using (i)

L.H.S. = $\log\frac{(a+b)}{5} = \log\frac{5\sqrt{ab}}{5} = \frac{1}{2}\log ab = \frac{1}{2}(\log a + \log b)$ = R.H.S.

Example 26 : If $\log_a x = p$ and $\log_b x^2 = q$, then $\log_x \sqrt{ab}$ is equal to (where a, b, x $\in R^+ - \{1\}$)-

(A) $\frac{1}{p} + \frac{1}{q}$ (B) $\frac{1}{2p} + \frac{1}{q}$ (C) $\frac{1}{p} + \frac{1}{2q}$ (D) $\frac{1}{2p} + \frac{1}{2q}$

Solution : $\log_a x = p \Rightarrow a^p = x \Rightarrow a = x^{1/p}$.

similarly $b^q = x^2 \Rightarrow b = x^{2/q}$

Now, $\log_x \sqrt{ab} = \log_x \sqrt{x^{1/p}x^{2/q}} = \log_x x^{\left(\frac{1}{p}+\frac{2}{q}\right)\cdot\frac{1}{2}} = \frac{1}{2p} + \frac{1}{q}$

Problems for Self Practice - 8 :

(1) Show that $\frac{1}{2}\log 9 + 2\log 6 + \frac{1}{4}\log 81 - \log 12 = 3\log 3$

3.4 Base changing theorem

Can be stated as "quotient of the logarithm of two numbers is independent of their common base."

Symbolically, $\log_b m = \frac{\log_a m}{\log_a b}$, where a > 0, a ≠ 1, b > 0, b ≠ 1, m > 0

(a) $\log_b a . \log_a b = \frac{\log a}{\log b} . \frac{\log b}{\log a} = 1$; hence $\boxed{\log_b a = \frac{1}{\log_a b}}$.

(b) **Base power formula :** $\log_{a^k} m = \frac{1}{k}\log_a m$

(c) $a^{\log_b c} = c^{\log_b a}$ or $a^{d\log_b c} = c^{d\log_b a}$ (c > 0) or $m = (a)^{\log_a m}$

Note :

(i) The base of the logarithm can be any positive number other than 1, but in normal practice, only two bases are popular, these are 10 and e(=2.718 approx). Logarithms of numbers to the base 10 are named as 'common logarithm' and the logarithms of numbers to the base e are called Natural or Napierian logarithm. **We will consider logx as $\log_e x$ or ℓnx.**

(ii) Conversion of base e to base 10 & viceversa :

$$\log_e a = \frac{\log_{10} a}{\log_{10} e} = 2.303 \times \log_{10} a \;; \quad \log_{10} a = \frac{\log_e a}{\log_e 10} = \log_{10} e \times \log_e a = 0.434 \log_e a$$

(iii) It must be noted that whenever the number and the base are on the same side of unity then logarithm of that number to that base is positive, however if the number and the base are located on different side of unity then logarithm of that number to that base is negative.

e.g. $\log_{10}\sqrt[3]{10} = \frac{1}{3}$; $\log_{\sqrt{7}} 49 = 4$; $\log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$; $\log_2\left(\frac{1}{32}\right) = -5$; $\log_{10}(0.001) = -3$

SOLVED EXAMPLE

Example 27 : If a, b, c are distinct positive real numbers different from 1 such that $(\log_b a \,.\, \log_c a - \log_a a) + (\log_a b \,.\, \log_c b - \log_b b) + (\log_a c \,.\, \log_b c - \log_c c) = 0$, then abc is equal to -

(A) 0 (B) e (C) 1 (D) none of these

Solution : $(\log_b a \log_c a - 1) + (\log_a b \,.\, \log_c b - 1) + (\log_a c \log_b c - 1) = 0$

$$\Rightarrow \frac{\log a}{\log b}.\frac{\log a}{\log c} + \frac{\log b}{\log a}.\frac{\log b}{\log c} + \frac{\log c}{\log a}.\frac{\log c}{\log b} = 3$$

$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3\log a \log b \log c$

$\Rightarrow (\log a + \log b + \log c) = 0$ [$\because$ If $a^3 + b^3 + c^3 - 3abc = 0$, then $a + b + c = 0$ if $a \neq b \neq c$]

$\Rightarrow \log abc = \log 1 \Rightarrow abc = 1$

Example 28 : Evaluate : $81^{1/\log_5 3} + 27^{\log_9 36} + 3^{4/\log_7 9}$

Solution : $81^{\log_3 5} + 3^{3\log_9 36} + 3^{4\log_9 7} = 3^{4\log_3 5} + 3^{\log_3 (36)^{3/2}} + 3^{\log_3 7^2}$ = 625 + 216 + 49 = 890.

Example 29 : Find the value of the followings :

(i) $\log_a 2 + \log_a\left(1 - \frac{1}{2}\right) + \log_a\left(1 - \frac{1}{3}\right) + ... + \log_a\left(1 + \frac{1}{n}\right)$

(ii) $\log_2 72 + \log_2\left(\frac{32}{81}\right) + \log_2\left(\frac{9}{64}\right)$

(iii) $7^{\frac{1}{\log_{25} 49}}$

Solution: (i) $\log_a 2 + \left(1 + \frac{1}{2}\right) + \log_a\left(1 + \frac{1}{3}\right) + ... + \log_a\left(1 + \frac{1}{n}\right)$

$= \log_a\left(\frac{2}{2}\right) + \log_a\left(\frac{3}{2}\right) + ... + \log_a\left(\frac{n+1}{n}\right) = \log_a\left(\frac{2}{1}.\frac{3}{2}.\frac{4}{3}.......\frac{n+1}{n}\right) = \log_a(n+1)$

(ii) $\log_2 72 + \text{..........} = \log_2\left\{2^3.3^2.\frac{2^5}{3^4}.\frac{3^2}{2^6}\right\} = \log_2 4 = 2$

(iii) $7^{\frac{1}{\log_{25}49}} = 7^{\log_{49}25} = 7^{\frac{2}{2}\log_7 5} = 5^{\log_7 7} = 5$

Example 30 : If in a right angled triangle, a and b are the lengths of sides and c is the length of hypotenuse and $c - b \neq 1$, $c + b \neq 1$, then show that $\log_{c+b}a + \log_{c-b}a = 2\log_{c+b}a \,.\, \log_{c-b}a$.

Solution : We know that in a right angled triangle

$c^2 = a^2 + b^2$

$c^2 - b^2 = a^2$ .... (i)

LHS = $\frac{1}{\log_a(c+b)} + \frac{1}{\log_a(c-b)} = \frac{\log_a(c-b)+\log_a(c+b)}{\log_a(c+b).\log_a(c-b)}$

$= \frac{\log_a(c^2-b^2)}{\log_a(c+b).\log_a(c-b)} = \frac{\log_a a^2}{\log_a(c+b).\log_a(c-b)}$ (using (i))

$= \frac{2}{\log_a(c+b).\log_a(c-b)}$

$= 2\log_{(c+b)}a \,.\, \log_{(c-b)}a$ = RHS

Problem for Self practice - 9 :

(1) Find the value of the followings :

(i) $\log_{49}343$ (ii) $4\log_{27}243$ (iii) $\log_{(1/100)}1000$

(iv) $\log_{(7-4\sqrt{3})}(7+4\sqrt{3})$ (v) $\log_{125}625$ (vi) $\log_8 9.\log_9 10$ .... $\log_{63}64$

(vii) log cot1° + log cot2° + log cot3° + ... + log cot89°

(2) Evaluate : **(i)** $\frac{\log_3 135}{\log_{15}3} - \frac{\log_3 5}{\log_{405}3}$

(ii) $\log_9 27 - \log_{27}9$

(iii) $2^{\log_3 5} - 5^{\log_3 2}$

(iv) $\log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8 \,.\, \log_8 9$

(3) If $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > x$, then x can be ....

(4) If $\log_a 3 = 2$ and $\log_b 8 = 3$, then $\log_a b$ is..........

Answers : (1) (i) 3/2 (ii) 20/3 (iii) – 3/2 (iv) –1 (v) 4/3 (vi) 2 (vi) 0

(2) (i) 3 (ii) 5/6 (iii) 0 (iv) 2

(3) 2 **(4)** $\log_3 4$

3.5. Logarithmic equalities

If $x > 0$, $y > 0$, $a > 0$, $a \neq 1$, then the equality $\log_a x = \log_a y$ is possible if and only if $x = y$

i.e. $\mathbf{\log_a x = \log_a y \Leftrightarrow x = y}$.

Always check validity of given equation, ($x > 0$, $y > 0$, $a > 0$, $a \neq 1$)

SOLVED EXAMPLE

Example 31: $\log_x(4x - 3) = 2$

Solution : Domain : $x > 0$, $4x - 3 > 0$, $x \neq 1$

Hence $4x - 3 = x^2 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow x = 3$ or $x = 1$ (rejected as not in domain)

Exmaple 32: $\log_2(\log_3\{\log_5(x^2 + 4)\} = 0$

Solution : $\log_3(\log_5(x^2 + 4)\} = 2° = 1 \Rightarrow \log_5(x^2 + 4) = 3^1 = 3 \Rightarrow (x^2 + 4) = 5^3 = 125 \Rightarrow x^2 = 121 \Rightarrow x = \pm 11$

Example 33: $\log_2(x^2) + \log_2(x + 2) = 4$

Solution: $\log_2(x^2(x + 2) = 4 \Rightarrow x^3 + 2x^2 - 16 = 0 \Rightarrow (x - 2)\underbrace{(x^2 + 4x + 8)}_{D<0} = 0 \Rightarrow x = 2$

Problem for Self practice –10

(1) $3^{3\log_3 x} = 27$ (2) $(\log_{10}x)^2 - (\log_{10}x) - 6 = 0$

(3) $3(\log_7 x + \log_x 7) = 10$ (4) $(x+2)^{\log_2(x+2)} = 8(x+2)^2$

Ans. (1) $x = 3$ (2) $x = 10^3, \dfrac{1}{10^2}$

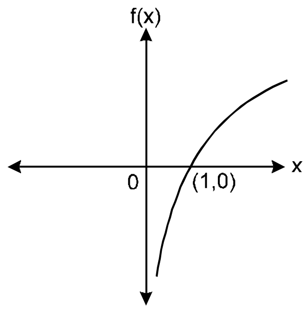
(3) $x = 343, \sqrt[3]{7}$ (4) $x = 6$ or $-3/2$

3.6 Logarithmic and Exponential inequalities

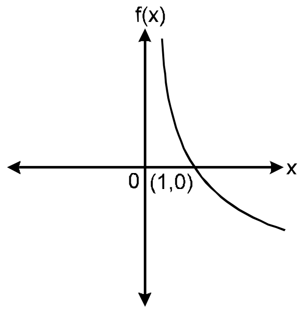
3.6.1 LOGARITHMIC INEQUALITIES

$f(x) = \log_a x$ is called logarithmic function where $a > 0$ and $a \neq 1$ and $x > 0$. Its graph can be as follows:

Case- I
For a > 1



Case- II
For 0 < a < 1



If $x, y \in R^+$ then **(i)** $\log_a x < \log_a y \Leftrightarrow \begin{cases} x < y & \text{if} \quad a > 1 \\ x > y & \text{if} \quad 0 < a < 1 \end{cases}$

(ii) If $a > 1$, then (a) $\log_a x < p \Rightarrow 0 < x < a^p$ (b) $\log_a x > p \Rightarrow x > a^p$

(iii) If $0 < a < 1$, then (a) $\log_a x < p \Rightarrow x > a^p$ (b) $\log_a x > p \Rightarrow 0 < x < a^p$

SOLVED EXAMPLE

Example 34 : Solve for x : (a) $\log_{0.5}(x^2 - 5x + 6) \geq -1$ (b) $\log_{1/3}(\log_4(x^2 - 5)) > 0$

Solution : (a) $\log_{0.5}(x^2 - 5x + 6) \geq -1 \Rightarrow 0 < x^2 - 5x + 6 \leq (0.5)^{-1}$

$\Rightarrow 0 < x^2 - 5x + 6 \leq 2$ $\begin{cases} x^2 - 5x + 6 > 0 \\ x^2 - 5x + 6 \leq 2 \end{cases} \Rightarrow x \in [1, 2) \cup (3, 4]$

Hence, solution set of original inequation : $x \in [1,2) \cup (3,4]$

(b) $\log_{1/3}(\log_4(x^2 - 5)) > 0 \Rightarrow 0 < \log_4 (x^2 - 5) < 1$

$$\begin{cases} 0 < \log_4(x^2 - 5) \Rightarrow x^2 - 5 > 1 \\ \log_4(x^2 - 5) < 1 \Rightarrow 0 < x^2 - 5 < 4 \end{cases}$$

$\Rightarrow 1 < (x^2 - 5) < 4 \Rightarrow 6 < x^2 < 9 \Rightarrow x \in \left(-3,-\sqrt{6}\right)\cup\left(\sqrt{6},3\right)$

Hence, solution set of original inequation : $x \in \left(-3,-\sqrt{6}\right)\cup\left(\sqrt{6},3\right)$

Example 35 : Solve for x : $\log_2 x \le \dfrac{2}{\log_2 x - 1}$.

Solution : Let $\log_2 x = t$ $\Rightarrow t \le \dfrac{2}{t-1}$

$\Rightarrow t - \dfrac{2}{t-1} \le 0$ $\Rightarrow \dfrac{t^2 - t - 2}{t-1} \le 0$

$\Rightarrow \dfrac{(t-2)(t+1)}{(t-1)} \le 0$

$\Rightarrow t \in (-\infty, -1] \cup (1,2]$ or $\log_2 x \in (-\infty,-1] \cup (1,2]$ or $x \in \left(0, \dfrac{1}{2}\right] \cup (2,4]$

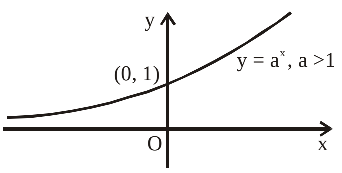
Example 36 : Solve the inequation : $\log_{2x+3} x^2 < \log_{2x+3}(2x+3)$

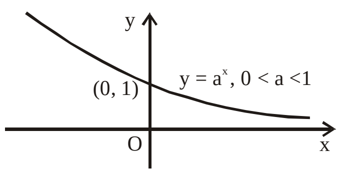
Solution : This inequation is equivalent to the collection of the systems

$$\left[\begin{array}{l} \begin{cases} 2x+3>1 \\ 0<x^2<2x+3 \end{cases} \\ \text{or} \\ \begin{cases} 0<2x+3<1 \\ x^2>2x+3>0 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} x>-1 \\ (x-3)(x+1)<0 \ \& \ x\ne 0 \end{cases} \\ \text{or} \\ \begin{cases} -\dfrac{3}{2}<x<-1 \\ (x-3)(x+1)>0 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} x>-1 \\ -1<x<3 \end{cases} \Rightarrow -1<x<3 \ \& \ x\ne 0 \\ \text{or} \qquad\qquad\qquad \text{or} \\ \begin{cases} -\dfrac{3}{2}<x<-1 \\ x<-1 \text{ or } x>3 \end{cases} \Rightarrow -\dfrac{3}{2}<x<-1 \end{array}\right.$$

Hence, solution of the original inequation is $x \in \left(-\dfrac{3}{2}, -1\right) \cup (-1, 0) \cup (0, 3)$

3.6.2 Exponential inequalities





If $a^{f(x)} > b \Rightarrow \left[\begin{array}{l} f(x) > \log_a b \text{ when } a > 1 \\ f(x) < \log_a b \text{ when } 0 < a < 1 \end{array}\right.$

SOLVED EXAMPLE

Example 37 : Solve for x : (a) $2^{x+2} > \left(\frac{1}{4}\right)^{\frac{1}{x}}$ (b) $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

Solution : (a) We have $2^{x+2} > 2^{-2/x}$. Since the base 2 > 1, we have $x + 2 > -\frac{2}{x}$ (the sign of the inequality is retained).

Now $x+2+\frac{2}{x} > 0 \Rightarrow \frac{x^2+2x+2}{x} > 0 \Rightarrow \frac{(x+1)^2+1}{x} > 0 \Rightarrow x \in (0, \infty)$

(b) We have $\left(\frac{5}{4}\right)^{1-x} < \left(\frac{16}{25}\right)^{2(1+\sqrt{x})}$ or $\left(\frac{4}{5}\right)^{x-1} < \left(\frac{4}{5}\right)^{4(1+\sqrt{x})}$

Since the base $0 < \frac{4}{5} < 1$, the inequality is equivalent to the inequality $x - 1 > 4(1+\sqrt{x})$

$\Rightarrow \frac{x-5}{4} > \sqrt{x}$

Now, R.H.S. is positive

$\Rightarrow \frac{x-5}{4} > 0 \Rightarrow x > 5$ ....(i)

we have $\frac{x-5}{4} > \sqrt{x}$

both sides are positive, so squaring both sides

$\Rightarrow \frac{(x-5)^2}{16} > x$ or $\frac{(x-5)^2}{16} - x > 0$

or $x^2 - 26x + 25 > 0$ or $(x - 25)(x - 1) > 0$

$\Rightarrow x \in (-\infty, 1) \cup (25, \infty)$ ....(ii)

intersection (i) & (ii) gives $x \in (25, \infty)$

Problems for Self Practice-11 :

(1) Solve for x : (a) $\log_{0.3}(x^2 + 8) > \log_{0.3}(9x)$ (b) $\log_7\left(\frac{2x-6}{2x-1}\right) > 0$

Answers : (1) (a) $x \in (1,8)$ (b) $x \in (-\infty, 1/2)$

3.7 Log table and antilog table

3.7.1 LOG

For any given number N, logarithm can be expressed as

$\log_a N$ = Integer(characteristic) + Fraction(mantissa)

The integer part is called characteristic and the fractional part is called mantissa.

Note :

(i) The mantissa part of logarithm of a number is always non-negative $(0 \le m < 1)$

(ii) If the characteristic of $\log_{10}N$ be n, then the number of digits in N is (n + 1)

(iii) If the characteristic of $\log_{10}N$ be (–n), then there exist (n – 1) zeros after decimal in N.

3.7.2 ANTILOGARITHM :

The positive real number 'n' is called the antilogarithm of a number 'm' if log n = m

Thus, **log n = m $\Leftrightarrow$ n = antilog m**

SOLVED EXAMPLE

Example 38 : Find the total number of digits in the number 12^{50}.

(Given $\log_{10}2 = 0.3010$; $\log_{10}3 = 0.4771$)

Solution : $N = 12^{50}$

$\log_{10}N = 50 \log_{10}12 = 50\,(0.6020 + 0.4771)$

$= 50(1.0791) = 53.9550$

Characterstic = $[\log_{10}N] = 53$

No. of digits = 53 + 1 = 54

Problems for Self Practice-12 :

(1) Evaluate : $\log_{10}(0.06)^6$

(2) Find number of digits in 18^{20}

(3) Determine number of cyphers (zeros) between decimal & first significant digit in $\left(\frac{1}{6}\right)^{200}$

(4) Find antilog of $\frac{5}{6}$ to the base 64.

Answers: (1) $\bar{8}.6686$ (2) 26

(3) 155 (4) 32

Exercise # 1

PART-I : SUBJECTIVE QUESTIONS

SECTION-(A) : Representation of Sets, Types of Sets, Subset, Power Set:-

A-1 Which of the following is not a set ?

(i) The collection of naturual numbers from 1 to 100

(ii) The collection of good Hockey players in Haryana

(iii) The collection of integers between 1/2 and 3/4

(iv) The collection of all intelligent students in Kota

(v) The collection of numbers which satisfies the equation $x^2 - 4x + 3 = 0$

A-2 Write the following set in tabular form

(i) A = {x : x is a positive prime < 9}

(ii) $B = \{x : x = 3\lambda, x \in \mathbf{I}, 1 \le \lambda \le 3\}$

(iii) C = {2x : x+1 is a prime number less than 10}

A-3 Write the following set in builder form

(i) set of all rational number

(ii) {2, 5, 10, 17, 26, 37, ... }

A-4 Which of the following is the empty set :-

(i) {x : x is a real number and $x^2 - 1 = 0$}

(ii) {x : x is a real number and $x^2 + 1 = 0$}

(iii) {x : x is a rational number and $7\pi x^2 - (7\pi\sqrt{2} + 22)x + 22\sqrt{2} = 0$}

(iv) {x : x is a real number and $x^2 = x + 2$}

A-5 Which of the following sets is a finite/infinite set ?

(i) Set of divisors of 12

(ii) Set of rational numbers between 5 & 7

(iii) Set of human beings living in India

A-6 If $A = \{x : -3 < x < 3, x \in Z\}$ then find the number of proper subsets of A -

A-7 Write the power set of set $A = \{\phi, 0, \{\phi\}\}$.

Section-(B) : Operations and Algebra of Sets

B-1 If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12}, C = {4, 5, 6, 12, 14} then find $(A \cap B) \cup (A \cap C)$

B-2 If $A = \{x : x = 4n + 1, n \le 5, n \in N\}$ and $B \{3n : n \le 8, n \in N\}$, then find A – (A – B) :-

B-3 Let A = {x : x $\in$ R, –1 < x < 1} , B = {x : x $\in$ R, x $\le$ 0 or x $\ge$ 2} and A $\cup$ B = R – D, then find set D.

B-4 Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5}, B = {6, 7}, then find $A \cap B'$

SECTION-(C) : Cardinal Number Problems

C-1 Let n(U) = 700, n(A) = 200, n(B) = 300 and n(A $\cap$ B) = 100, then find n(A' $\cap$ B')

C-2 If A and B are two sets such that n(A) = 8, n(B) = 9 and n(A $\cup$ B) = 12 then find the minimum and maximum value of n(A $\cap$ B)

C-3 In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then number of families which buy newspaper A only is

C-4 Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. The total number of members in the three athletic teams is:

C-5 In a class of 42 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Find the number of students who have taken exactly one subject.

C-6 A class has 175 students. The following data shows the number of students opting one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have opted Mathematics alone ?

Section (D) : Rational Inequalities

D-1 Solve the following Inequalities

(i) $(x-1)^2 (x+1)^3 (x-4) \ge 0$

(ii) $\dfrac{x^2+4x+4}{2x^2-x-1} > 0$ (iii) $\dfrac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)} \ge 0$

(iv) $\dfrac{(x+3)(x^2-2x+1)}{4+3x-x^2}$ (v) $\dfrac{x^6-3x^5+2x^4}{x^2-x-30} > 0$ (vi) $\dfrac{x^4+x^2+2}{x^2-4x-5} > 0$

D-2 Solve the following Inequalities

(i) $\dfrac{7x-5}{8x+3} > 4$ (ii) $\dfrac{2x^2-3x-459}{x^2+1} > 1$ (iii) $\dfrac{x^2-5x+12}{x^2-4x+5} > 3$

(iv) $\dfrac{2x}{2x^2+5x+2} > \dfrac{1}{x+1}$ (v) $\dfrac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1$

D-3 Find the number of integers satisfying the following Inequalities

(i) $x^4 - 5x^2 + 4 \le 0$ (ii) $x^4 - 2x^2 - 63 \le 0$ (iii) $x^2 + 6x - 7 \le 2$

(iv) $\frac{14x}{x+1} < \frac{9x-30}{x-4}$ (v) $\frac{x^2+2}{x^2-1} < -2$

D-4 Find the number of positive integers satisfying the following Inequalities

(i) $\frac{7}{(x-2)(x-3)} + \frac{9}{x-3} + 1 \le 0$ (ii) $\frac{x+7}{x-5} + \frac{3x+1}{2} \ge 0$ & $x < 10$ (iii) $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} > 1$

D-5 Find the number of positive integral values of x satisfying the inequality

$$\frac{(x-4)^{2021}.(x+8)^{2020}(x+1)}{x^{2022}(x-2)^3.(x+3)^5.(x-6)(x+9)^{2022}} \le 0$$

D-6 How many positive integer x are there such that 3x has 3 digits and 4x has four digits ?

Section (E) : Logarithm & its properties

E-1 Which of the following numbers are positive/negative

(i) $\log_{\sqrt{3}}\sqrt{2}$; (ii) $\log_{1/7}(2)$; (iii) $\log_{1/3}(1/5)$; (iv) $\log_3(4)$

(v) $\log_7(2.11)$ (vi) $\log_3(\sqrt{7}-2)$ (vii) $\log_4\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ (viii) $\log_3\left(\frac{2\cdot\sqrt[3]{3}}{3}\right)$

(ix) $\log_{10}(\log_{10} 9)$

E-2 Find the value of

(i) $\log_{10}5.\log_{10}20 + (\log_{10}2)^2$ (ii) $5^{\log_{\sqrt{5}} 2} + 9^{\log_3 7} - 8^{\log_2 5}$

(iii) $\sqrt[3]{5^{\frac{1}{\log_7 5}} + \frac{1}{(-\log_{10} 0.1)}}$ (iv) $\log_{0.75}\log_2\sqrt{\sqrt{\frac{1}{0.125}}}$

(v) $\left(\frac{1}{49}\right)^{1+\log_7 2} + 5^{-\log_{1/5} 7}$ (vi) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(vii) $4^{5\log_{4\sqrt{2}}(3-\sqrt{6}) - 6\log_8(\sqrt{3}-\sqrt{2})}$ (viii) $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}.\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$

(ix) $49^{(1-\log_7 2)} + 5^{-\log_5 4}$. (x) $\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}$

E-3 Let $\log_{10}2 = a$ and $\log_{10}3 = b$ then determinant the following logarithms in terms of a and b.

(i) $\log_{10}\left(\sin^2\frac{\pi}{3}\right)$ (ii) $\log_{100}4 + 2\log_{100}27$ (iii) $\log_2 9 + \log_3 8$ (iv) $\log_{\sqrt{45}} 144$

E-4 (i) Let n = 75600, then find the value of $\frac{4}{\log_2 n}+\frac{3}{\log_3 n}+\frac{2}{\log_5 n}+\frac{1}{\log_7 n}$

(ii) If $\log_2(\log_3(\log_4(x)))=0$ and $\log_3(\log_4(\log_2(y)))=0$ and $\log_4(\log_2(\log_3(z)))=0$ then find the sum of x, y and z is

E-5 Show that the number $\log_2 7$ is an irrational number.

E-6 Suppose n be an integer greater than 1. let $a_n = \frac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then find the value of (b – c)

E-7 If $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}$, show that $a^a. b^b. c^c = 1$.

Section (F) : Logarithmic equations

F-1 Solve the following equations :

(i) $\log_x(4x-3) = 2$

(ii) $4^{\log_2 x}-2x-3=0$

(iii) $\sqrt{5-\log_2 x} = 3-\log_2 x$

(iv) $x^{(\log_{\sqrt{x}} 2x)} = 4$

(v) $\log_{10}(x^2-12x+36) = 2$

(vi) $\log_2(\log_3(x^2-1)) = 0$

(vii) $\log_4\log_3\log_2 x = 0$

(viii) $\log_3\left(\log_9 x+\frac{1}{2}+9^x\right) = 2x$.

(ix) $2\log_4(4-x) = 4-\log_2(-2-x)$.

(xi) $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)}=\frac{1}{2}$

(xii) $\log(\log x) + \log(\log x^3 - 2) = 0$; where base of log is 10.

F-2 Solve the following equations

(i) $\log^2_{10} x + \log_{10}x^2 = \log^2_{10} 2 - 1$

(ii) $\log_4(\log_2 x) + \log_2(\log_4 x) = 2$

(iii) If $9^{1+\log x} - 3^{1+\log x} - 210 = 0$; where base of log is 3.

(iv) $\log_x 2.\ \log_{2x} 2 = \log_{4x} 2$

(v) $x^{0.5\log_{\sqrt{x}}(x^2-x)} = 3^{\log_9 4}$.

(vi) $x^{\log_{10} x+2} = 10^{\log_{10} x+2}$

(vii) $x^{\frac{\log_{10} x+5}{3}} = 10^{5+\log_{10} x}$

F-3 (i) Find the product of roots of the equation $(\log_3 x)^2 - 2(\log_3 x) - 5 = 0$

(ii) Find sum of roots of the equation $4^x - 7.2^x + 6 = 0$

Section (G) : Logarithmic & Exponential inequalities

G-1 Solve the following inequalities

(i) $\log_{\frac{5}{8}}\left(2x^2 - x - \frac{3}{8}\right) \geq 1$

(ii) $\log_{\frac{1}{2}}(x^2 - 5x + 6) > -1$

(iii) $\log_7 \frac{2x-6}{2x-1} > 0$

(iv) $\log_{1/3}(2^{x+2} - 4^x) \geq -2$

(v) $2 - \log_2 (x^2 + 3x) \geq 0$

(vi) $\log_{1/4}(2 - x) > \log_{1/4}\left(\frac{2}{x+1}\right)$

(vii) $\log_{0.5} (x + 5)^2 > \log_{1/2} (3x - 1)^2$.

(viii) $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5}1$

(ix) $\log_{1/2} \log_3 \frac{x+1}{x-1} \geq 0$

(x) $\sqrt{\log_{10}^2 x - 1} > \log_{10}x - 1$

G-2 Solve the following inequalities

(i) $\log_x(4x - 3) \geq 2$ (ii) $\log_{(3x^2+1)} 2 < \frac{1}{2}$ (iii) $\log_{x^2}(2 + x) < 1$

G-3 Find the number of integral solutions of inequality $\left(\frac{1}{10}\right)^{\log_{(x-3)}(x^2-4x+3)} \geq 1$:-

G-4 If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 (\sqrt{x^2+5} + x) > 0$ contains 'n' integral values, then find n

G-5 Find the number of integers satisfying $\log_{1/5}\frac{4x+6}{x} \geq 0$

G-6 Find the number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.

Section (H) : Log & Antilog Table

H-1 Find the log of following with respect to base 10.

(i) 430.1 (ii) 204.01 (iii) .0024

H-2 Find the antilog of following with respect to base 10.

(i) 2.3 (ii) $\overline{1}.3108$ (iii) –1.8123

H-3 Find the antilogarithm of 0.75, if the base of the logarithm is 2401.

H-4 If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then find :

(a) the number of digits in 6^{15}

(b) the number of zeros immediately after the decimal in 3^{-100}

H-5 Evaluate $\sqrt{23.24} \times 38.7$ using log table.

PART-II : OBJECTIVE QUESTIONS

SECTION-(A) : Representation of Sets, Types of Sets, Subset, Power Set:-

A-1 If Q = $\left\{x : x = \frac{1}{y}, \text{ where } y \in N\right\}$, then-

(A) $0 \in Q$ (B) $1 \in Q$ (C) $2 \in Q$ (D) $\frac{2}{3} \in Q$

A-2 Consider the following statements.

S_1 : { } is the subset of all sets
S_2 : $3 \in \{1, 3, 5\}$
S_3 : $3 \subseteq \{1, 3, 5\}$
S_4 : $\{3, 5\} \in \{1, 3, 5\}$
S_5 : $[3, 7] \subseteq (1, 15)$
S_6 : $[3, 7] \in (1, 15)$

Which of the following are **CORRECT** ?

(A) S_2 and S_4 are true. (B) S_5 and S_6 are false.
(C) S_1 and S_2 are true. (D) S_3 and S_4 are true.

A-3 Consider the following statements.

S_1 : A = {x : x > 1 and x < –1] is a null set.
S_2 : C = {ϕ} is a null set.
S_3 : {x : x is a real number and $x^2 + 1 = 0$} is empty set
S_4 : A = {x : $x \in R$, $x^2 = 16$ and 2x = 6} is singleton set

Which of the following are **CORRECT** ?

(A) S_2 and S_4 are true. (B) S_1 and S_3 are false.
(C) S_1 and S_3 are true. (D) S_3 and S_4 are true.

A-4 The number of subsets of the power set of set A = {7, 10, 11} is

(A) 32 (B) 16 (C) 64 (D) 256

A-5 If a set contains m element and another set contains n element. If 56 is the difference between the number of subsets of both sets then find (m, n)

(A) 3, 6 (B) 6, 3 (C) 8, 3 (D) 3, 8

SECTION-(B) : Operations and Algebra of Sets

B-1 Consider the following statements.

S_1 : If A and B are two sets, then $A \cap (A \cup B)'$ is equal to A
S_2 : If A is any set, then $A \cup A' = U$
S_3 : Let A and B be two sets in the universal set. Then A – B equals $A \cap B'$
S_4 : $P(A) = P(B) \Rightarrow A = B$
S_5 : If $A \subseteq B$, then $A \cap B$ is equal to A
S_6 : If A and B are any two sets, then $A \cup (A \cap B)$ is equal to A

Which of the following are **CORRECT** ?

(A) S_2 and S_4 are true. (B) S_1 and S_3 are false.
(C) S_5 and S_6 are false. (D) S_3 and S_4 are false.

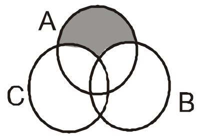
B-2 Given the sets A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, then $A \cup (B \cap C)$ is

(A) {3} (B) {1, 2, 3, 4} (C) {1, 2, 4, 5} (D) {1, 2, 3, 4, 5, 6}

B-3 The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is

(A) $\{2, 3, 5\}$ (B) $\{3, 5, 9\}$ (C) $\{1, 2, 5, 9\}$ (D) None of these

B-4 The shaded region in the given figure is



(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \cap (B - C)$ (D) $A - (B \cup C)$

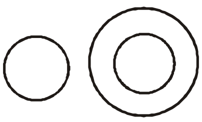
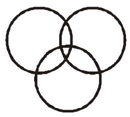
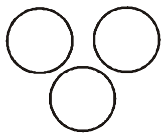
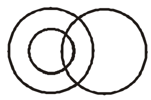
B-5 If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$, $b \geq 2$, $c \geq 2$ are relatively prime, then which one of the following is correct ?

(A) $b = cd$ (B) $c = bd$ (C) $d = bc$ (D) $d^2 = bc$

B-6 $A \cup B = A \cap B$ iff :

(A) $A \subset B$ (B) $A = B$ (C) $A \supset B$ (D) $A \subseteq B$

B-7 Which of the following venn-diagrams best represents the sets of females, mothers and doctors ?

(A)  (B)  (C)  (D) 

SECTION-(C) : Cardinal Number Problems :

C-1 In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

(A) 80 percent (B) 40 percent (C) 60 percent (D) 70 percent

C-2 In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspaper is-

(A) at least 30 (B) at most 20 (C) exactly 25 (D) none of these

C-3 Let A and B be two sets. Then

(A) $n(A \cup B) \leq n(A \cap B)$ (B) $n(A \cap B) \leq n(A \cup B)$ (C) $n(A \cap B) = n(A \cup B)$ (D) None of these

C-4 In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then number of families which buy newspaper A only is

(A) 3100 (B) 3300 (C) 2900 (d) 1400

C-5 A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?

(A) 35 (B) 48 (C) 60 (D) 22

C-6 31 candidates appeared for an examination, 15 candidates passed in Mathematics, 15 candidates passed in physics, 20 candidates passed in Chemistry . 3 candidates passed only in Mathematics. 4. candidates passed only in Physics, 7 candidates passed only in Chemistry. 2 candidates passed in all the three subjects How many candidates passed only in two subjects ?

(A) 17 (B) 15 (C) 22 (D) 14

Section (D) : Rational Inequalities

D-1 Number of integer values of x satisfying $-5 \leq x < 10$ and $0 \leq x \leq 15$ is

(A) 10 (B) 11 (C) 12 (D) 13

D-2 The number of positive integers satisfying the inequality $\frac{x^2-1}{2x+5} < 3$ is

(A) 10 (B) 9 (C) 8 (D) 7

D-3 The complete set of values of 'x' which satisfy the inequations : $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$ is

(A) $(-\infty, 1)$ (B) $(2, 3)$ (C) $(-\infty, 3)$ (D) $(-\infty, 1) \cup (2, 3)$

D-4 The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :

(A) 1 (B) 2 (C) 3 (D) none of these

D-5 The complete solution set of the inequality $\frac{x^4-3x^3+2x^2}{x^2-x-30} \geq 0$ is:

(A) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$ (B) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

(C) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$ (D) none of these

D-6 Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

(A) 0 (B) 1 (C) 2 (D) 3

Section (E) : Logarithm & its properties

E-1 The number $N = 6\log_{10}2 + \log_{10}31$, lies between two successive integers whose sum is equal to

(A) 5 (B) 7 (C) 9 (D) 10

E-2 Which one of the following is the smallest?

(A) $\log_{10}\pi$ (B) $\sqrt{\log_{10}\pi^2}$ (C) $\left(\frac{1}{\log_{10}\pi}\right)^3$ (D) $\left(\frac{1}{\log_{10}\sqrt{\pi}}\right)$

E-3 Let $x = 2^{\log 3}$ and $y = 3^{\log 2}$ where base of the logarithm is 10, then which one of the following holds good?

(A) $2x < y$ (B) $2y < x$ (C) $3x = 2y$ (D) $y = x$

E-4 The value of 'a' for which $\frac{\log_a 7}{\log_6 7} = \log_\pi 36$ holds good, is

(A) $1/\pi$ (B) π^2 (C) $\sqrt{\pi}$ (D) 2

E-5 If $a^4 \cdot b^5 = 1$ then the value of $\log_a(a^5b^4)$ equals

(A*) 9/5 (B) 4 (C) 5 (D) 8/5

E-6 $\frac{1}{1+\log_b a+\log_b c}+\frac{1}{1+\log_c a+\log_c b}+\frac{1}{1+\log_a b+\log_a c}$ has the value equal to

(A) abc (B) $\frac{1}{abc}$ (C) 0 (D) 1

E-7 $\frac{1}{\log_{\sqrt{bc}} abc}+\frac{1}{\log_{\sqrt{ca}} abc}+\frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to :

(A) 1/2 (B) 1 (C) 2 (D) 4

E-8 If $\log_a(ab) = x$, then $\log_b(ab)$ is equal to

(A) $\frac{1}{x}$ (B) $\frac{x}{1+x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x-1}$

E-9 The ratio $\frac{2^{\log_{2^{1/4}} a}-3^{\log_{27}(a^2+1)^3}-2a}{7^{4\log_{49} a}-a-1}$ simplifies to :

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$ (C) $a^2 - a + 1$ (D) $a^2 + a + 1$

E-10 $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$ then 'p' equals

(A) $r^{q/r}$ (B) rq (C) 1 (D) $r^{r/q}$

E-11 $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \log_{10}(\log_4 5) + ... + \log_{10}(\log_{1023} 1024)$ simplifies to

(A) a composite (B) a prime number

(C) rational which is not an integer (D) an integer

E-12 The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{............\sqrt[p]{p}}}}}_{\text{n radical sign}}$, where $p \ge 2$, $p \in N$; $n \in N$ when simplified is

(A) p (B) n (C) – n (D) p^n

Section (F) : Logarithmic equations

F-1 The sum of all the solutions to the equation $2\log_{10} x - \log_{10}(2x - 75) = 2$

(A) 30 (B) 350 (C) 75 (D) 200

F-2 If $\log_x \log_{18}\left(\sqrt{2}+\sqrt{8}\right) = \frac{1}{3}$. Then the value of 1000 x is equal to

(A) 8 (B) 1/8 (C) 1/125 (D) 125

F-3 Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :

(A) zero (B) exactly 1 (C) exactly 2 (D) 4

F-4 If $\log_y x + \log_x y = 7$, then the value of $(\log_y x)^2 + (\log_x y)^2$, is

(A) 43 (B) 45 (C) 47 (D) 49

F-5 If $\log_9 x + \log_4 y = \frac{7}{2}$ and $\log_9 x - \log_8 y = -\frac{3}{2}$, then x + y equals

(A) 35 (B) 41 (C) 67 (D) 73

F-6 Number of real solution(s) of the equation $|x-3|^{3x^2-10x+3} = 1$ is -

(A) exactly four (B) exactly three (C) exactly two (D) exactly one

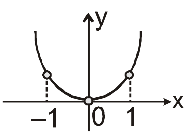
F-7 If the solution of the equation $\log_x(125x) \cdot \log_{25}^2 x = 1$ are α and β ($\alpha < \beta$), then the value of $1/\alpha\beta$ is:

(A) 5 (B) 25 (C) 125 (D) 625

F-8 The positive integral solution of the equation $\log_x\sqrt{5} + \log_x 5x = \frac{9}{4} + \log_x^2\sqrt{5}$ is :

(A) Composite number (B) Prime number (C) Even number (D) Divisible by 3

F-9 The correct graph of $y = x^{\log_x x^2}$ is

(A)


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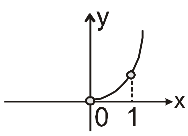

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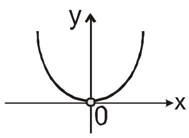

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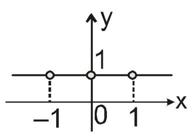


Section (G) : Logarithmic & Exponential inequalities

G-1 The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2-3x+2) \geq 2$ is

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(1, \frac{5}{2}\right)$ (C) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (D) None of these

G-2 If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(A) $(2, \infty)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) none of these

G-3 Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is :

(A) $[-4, 1]$ (B) $[-4, -3) \cup (0, 1]$

(C) $(-\infty, -3) \cup (1, \infty)$ (D) $(-\infty, -4) \cup [1, \infty)$

G-4 If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5}1$, then 'x' lies in the interval

(A) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$ (B) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$

(C) $(\sqrt{5}, 3\sqrt{5})$ (D) ϕ

G-5 The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set

(A) $(-\infty, 0)$ (B) $(-\infty, 1)$ (C) $(1, \infty)$ (D) $(3, \infty)$

G-6 The set of all the solutions of the inequality $\log_{1-x}(x-2) \geq -1$ is

(A) $(-\infty, 0)$ (B) $(2, \infty)$ (C) $(-\infty, 1)$ (D) ϕ

PART-III : MATCH THE COLUMN

1. Match the set P in column-I with its super set Q in column-II

Column-I		Column-II	
(A)	$\{3^{2n} - 8n - 1 : n \in N\}$	(p)	$\{49(n-1) : n \in N\}$
(B)	$\{2^{3n} - 1 : n \in N\}$	(q)	$\{64(n-1) : n \in N\}$
(C)	$\{3^{2n} - 1 : n \in N\}$	(r)	$\{7n : n \in N\}$
(D)	$\{2^{3n} - 7n - 1 : n \in N\}$	(s)	$\{8n : n \in N\}$

2.

Column-I		Column-II	
(A)	If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	(p)	-1
(B)	If $a = \sqrt{7+\sqrt{7^2-1}}$, $b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	(q)	1
(C)	The number of zeroes at the end of the product of first 20 prime numbers, is	(r)	2
(D)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is	(s)	$\frac{3}{2}$
		(t)	None

3.

Column-I		Column-II	
(A)	When the repeating decimal 0.363636............ is written as a rational fraction in the simplest form, the sum of the numerator and denominator is	(p)	4
(B)	Given positive integer p, q and r with $p = 3^q \cdot 2^r$ and $100 < p < 1000$. The difference between maximum and minimum values of $(q + r)$, is	(q)	5
(C)	If $\log_8 a + \log_8 b = (\log_8 a)(\log_8 b)$ and $\log_a b = 3$, then the value of 'a' is	(r)	15
(D)	The value of b satisfying the equation, $\log_e 2 \cdot \log_b 625 = \log_{10}16 \cdot \log_e 10$ is	(s)	16

Exercise # 2

PART-I : OBJECTIVE

1. Let A, B, C be distinct subsets of a universal set U. For a subset X of U, let X' denote the complement of X in U. Consider the following statements

S1: $((A \cap B) \cup C)' \cap B')' = B \cap C$ S2: $(A' \cap B') \cap (A \cup B \cup C') = (A \cup (B \cup C))'$

Which of the above statements is/are correct ?

(A) S1 is True and S2 is False (B) S1 is True and S2 is True

(C) S1 is False and S2 is False (D) S1 is False and S2 is True

2. Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct ?

(A) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3

(B) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $A_2 \cup A_3$ but not both

(C) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1 and $A_2 \cup A_3$ only if $A_2 = A_3$

(D) None of these

3. In a recent survey (conducted by HLL) of 1,000 houses, washing machine, vacuum cleaners and refrigerators were counted. Each house had at least one of these products. 400 had no refrigerators, 380 had no vacuum cleaners, 542 had no washing machines. 294 had both a vacuum cleaner and washing machines, 277 had both a vacuum cleaner and a refrigerator, and 120 had both a refrigerator and a washing machine. How many had only a vacuum cleaner ?

(A) 132 (B) 234 (C) 342 (D) 62

4. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is -

(A) 11 (B) 12 (C) 13 (D) 14

5. In an examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects ?

(A) 9% (B) 7%

(C) 15% (D) Cannot be determined due to insufficient data

6. If $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$, then the least and the highest values of $4x^2$ are:

(A) 0 & 81 (B) 9 & 81 (C) 36 & 81 (D) none of these

7. If $\frac{4\alpha}{\alpha^2 + 1} \ge 1$ and $\alpha + \frac{1}{\alpha}$ is an odd integer then number of possible values of α is

(A) 1 (B) 2 (C) 3 (D) 4

8. If $\log_a b = 2$; $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$ then (a + b + c) equals

(A) 90 (B) 93 (C) 102 (D) 243

9. The sum of the solutions of the equation $9^x - 6 \cdot 3^x + 8 = 0$ is

(A) $\log_3 2$ (B) $\log_3 6$ (C) $\log_3 8$ (D) $\log_3 4$

10. The expression: $\dfrac{\left(\dfrac{x^2+3x+2}{x+2}\right)+3x-\dfrac{x(x^3+1)}{(x+1)(x^2-x+1)}\log_2 8}{(x-1)(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 2)}$ reduces to

(A) $\dfrac{x+1}{x-1}$ (B) $\dfrac{x^2+3x+2}{(\log_2 5)x-1}$ (C) $\dfrac{3x}{x-1}$ (D) x

11. If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(A) 489 (B) 469 (C) 464 (D) 400

12. If $\sum_{r=0}^{n-1} \log_2\left(\dfrac{r+2}{r+1}\right) = \prod_{r=10}^{99} \log_r(r+1)$, then 'n' is equal to

(A) 4 (B) 3 (C) 5 (D) 6

13. The solution set of the inequality $\dfrac{(5^x-6^x)\cdot \ell n(x+2)}{x^2-3x-4} \le 0$ is

(A) $(-\infty, 0] \cup (4, \infty)$ (B) $(-2, 0] \cup (4, \infty)$
(C) $(-1, 0] \cup (4, \infty)$ (D) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

14. Number of integers for which $f(x) = \sqrt{\dfrac{1}{\log_{(3x-2)}(2x+3)} - \log_{(2x+3)}(x^2-x+1)}$ is defined is equal to-

(A) 1 (B) 2 (C) 3 (D) 4

15. Let W,X,Y and Z be positive real numbers such that

log(W.Z) + log(W.Y) = 2; log(Y.Z) + log(Y.X) = 3; log(X.W) + log(X.Z) = 4.

The value of the product (WXYZ) equals (base of the log is 10)

(A) 10^2 (B) 10^3 (C) 10^4 (D) 10^9

16. If $\log_{1/3}\left(\dfrac{3x-1}{x+2}\right)$ is less than unity then x must lie in the interval -

(A) $(-\infty, -2) \cup (5/8, \infty)$ (B) $(-2, 5/8)$

(C) $(-\infty, -2) \cup (1/3, 5/8)$ (D) $(-2, 1/3)$

17. The set of values of x satisfying simultaneously the inequalities $\dfrac{\sqrt{(x-9)\,(3-x)}}{\log_{0.4}\left(\dfrac{5}{4}\,(\log_2 5-1)\right)} \ge 0$ and

$2^{x-3} - 31 > 0$ is :

(A) a unit set (B) { } (C) {3, 9} (D) $(-\infty, 3] \cup [9, \infty)$

PART-II : NUMERICAL QUESTIONS

1. Let Z be the set of all integers and $A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$ and $B = \{(a, b) : a > b, a, b \in Z\}$. Then, the number of elements in $A \cap B$, is -

2. If class with n students is organized into four groups keeping the following conditions :
Each student belongs to exactly two groups and
Each pair of groups has exactly one student in common.
What is the value of n ?

3. In a class of 25 students, at least one of mathematics or statistics is taken by everybody. 12 have taken mathematics, 8 have taken mathematics but not statistics. Find the difference in the number of students who have taken mathematics and statistics and those who have taken statistics but not maths ?

4. Find the sum of all the real solutions of the inequality $\frac{(x^6+2)(\sqrt{x^2-16})}{(x^8+2)(x^2-9)} \le 0$

5. Find the value of $(\log_3 12)(\log_3 72) - \log_3(192).\log_3 6$

6. Let $x = \left(\log_{1/3} 5\right)\left(\log_{125} 343\right)\left(\log_{49} 729\right)$ and y = $25^{3\log_{289} 11 \, \log_{28} \sqrt{17} \, \log_{1331} 784}$, then find the value of $\frac{x^2}{y}$ is

7. If $c(a-b) = a(b-c)$ then find the value of $\frac{\log(a-c)}{\log(a+c)+\log(a-2b+c)}$

(Assume all terms are defined)

8. If $\log_b a. \log_c a + \log_a b. \log_c b + \log_a c. \log_b c = 3$ (where a, b, c are different positive real numbers $\neq 1$), then find the value of a b c.

9. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

10. Find the value of x satisfying the equation $\log_{\frac{1}{2}}(x-1) + \log_{\frac{1}{2}}(x+1) - \log_{\frac{1}{\sqrt{2}}}(7-x) = 1$

11. Find the sum of solutions of the equation $\log^2_{10} x + \log_{10} x^2 = \log^2_{10} 2 - 1$

12. If $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ then find the value of |x|

13. Let a, b, c, d are positive integers such that $\log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$. If $(a - c) = 9$, find the value of $\left(\frac{b}{2d}\right)^{1/3}$

14. If the product of all solutions of the equation $\frac{(2019)x}{2020} = (2019)^{\log_x(2020)}$ can be expressed in the lowest form as $\frac{m}{n}$ then the value of $(m - n)$ is

15. Find the sum of the roots of the equation
$\log_3 x.\log_4 x.\log_5 x = \log_3 x.\log_4 x + \log_4 x.\log_5 x + \log_5 x .\log_3 x$.

16. If $x,y > 0$, $\log_y x + \log_x y = \frac{10}{3}$ and $xy = 144$, then $\frac{x+y}{2}$ is equal to

17. Let the product of the positive roots of the equation $\sqrt{(2020)}(x)^{\log_{2020} x} = x^2$ is k^2 ($k \in N$), then $\frac{k}{100}$ is equal to

18. If the complete solution set of the inequality $(\log_{10} x)^2 \geq \log_{10} x + 2$ is $(0, a] \cup [b, \infty)$ then find the value of (a + b).

19. The complete solution set of the inequality $\frac{1}{\log_4 \frac{x+1}{x+2}} < \frac{1}{\log_4 (x+3)}$, is $(-a, \infty)$, then determine 'a'.

20. Find the number of integers which do not satisfy the inequality $\log_{1/2} (x + 5)^2 > \log_{1/2} (3x - 1)^2$.

21. Complete solution set of the inequality $(2+\sqrt{3})^{x^2-x} + (2-\sqrt{3})^{x^2-x} \geq 14$ is $(-\infty, a] \cup [b, \infty)$ then find $\left|\frac{a}{b}\right|$

PART - III : ONE OR MORE THAN ONE CORRECT

1. Let U be set with number of elements in U is 2020.

Consider the following statements :

I If A, B are subsets of U with $n(A \cup B) = 291$, then $n(A' \cap B') = x_1^3 + x_2^3 = y_1^3 + y_2^3$ for some positive integers x_1, $x_2 y_1$, y_2

II If A is a subset of U with n (A) = 1681 and out of these 1681 elements, exactly 1075 elements belong to a subset B of U , then $n(A - B) = m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes p_1, p_2, p_3

Which of the statements given above is / are correct ?

(A) S1 is True (B) S1 is False (C) S2 is False (D) S2 is True

2. In a class of 200 students, 70 played cricket, 60 played hockey and 80 played football. Thirty played cricket and football, 30 played hockey and football, 40 played cricket and hockey.

Let the maximum number of people playing all the three games is m and also the minimum number of people playing at least one game is n, then

(A) m = 100 (B) n = 110 (C) m = 30 (D) n = 120

3. Let $a > 2$, $a \in N$ be a constant. If there are just 18 positive integers satisfying the inequality $(x - a)(x - 2a)(x - a^2) < 0$ then which of the option(s) is/are correct?

(A) 'a' is composite (B) 'a' is odd

(C) 'a' is greater than 8 (D) 'a' lies in the interval (3, 11)

4. Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is :

(A) a natural number (B) a prime number (C) a rational number (D) an integer

5. Values of x satisfying the equation $\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are

(A) 1 (B) 5 (C) $\frac{1}{25}$ (D) 3

6. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has :

(A) one irrational solution (B) no prime solution

(C) two real solutions (D) one integral solution

7. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution (B) at least one real solution

(C) exactly one irrational solution (D) complex roots.

8. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is :

(A) {6, 3} (B) {3, 6} (C) {6, 12} (D) {12, 6}

9. Consider the quadratic equation, $(\log_{10} 8)x^2 - (\log_{10} 5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.

(A) sum of the roots (B) product of the roots

(C) sum of the coefficients (D) discriminant

10. If $\log_a x = b$ for permissible values of a and x then identify the statement(s) which can be correct?

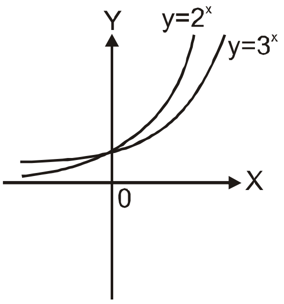
(A) If a and b are two irrational numbers then x can be rational.

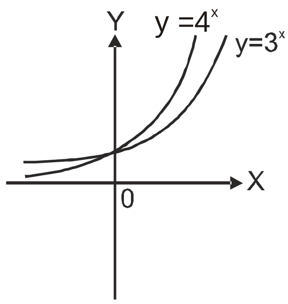
(B) If a rational and b irrational then x can be rational.

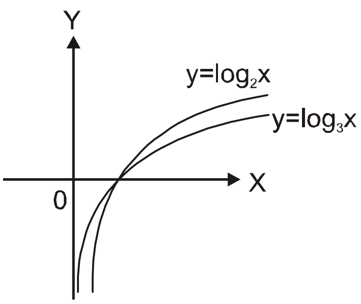
(C) If a irrational and b rational then x can be rational.

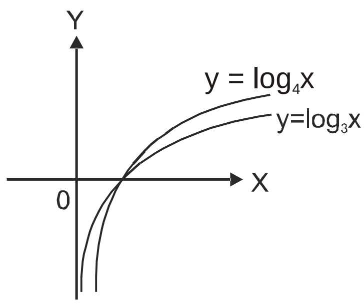
(D) If a rational and b rational then x can be rational.

11. Which of the following is correct :

(A) 

(B) 

(C) 

(D) 

12. Which of the following statements are true

(A) $\log_2 3 < \log_{12} 10$ (B) $\log_6 5 < \log_7 8$

(C) $\log_3 26 < \log_2 9$ (D) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

13. If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(A) maximum value of x is $\frac{1}{10}$

(B) x lies in interval $\left[\frac{1}{100}, \frac{1}{\sqrt{10}}\right]$

(C) minimum value of x is $\frac{1}{10}$

(D) minimum value of x is $\frac{1}{100}$

14. If $\log_a x \log_a(xyz) = 48$, $\log_a y \log_a(xyz) = 12$, $\log_a z \log_a(xyz) = 84$, $a > 0$, $a \ne 1$, then triplet (x, y, z) can be equal to

(A) (a^4, a, a^7) (B) (a^7, a, a^4) (C) $\left(\frac{1}{a^7}, \frac{1}{a}, \frac{1}{a^4}\right)$ (D) $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$

PART - IV : COMPREHENSION

Comprehension # 1

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

1. Difference of number of people who can speak Hindi only and number of people who can speak Bengali only is

(A) 300 (B) 400 (C) 500 (D) 350

2. Number of people who can speak both Hindi and Bengali is

(A) 50 (B) 100 (C) 150 (D) 200

Comprehension # 2

Let A denotes the sum of the roots of the equation $\frac{1}{5-4\log_4 x} + \frac{4}{1+\log_4 x} = 3$.

B denotes the value of the product of m and n, if $2^m = 3$ and $3^n = 4$.

C denotes the sum of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right) + (\log_3 x)^2 = 1$.

3. The value of A + B equals

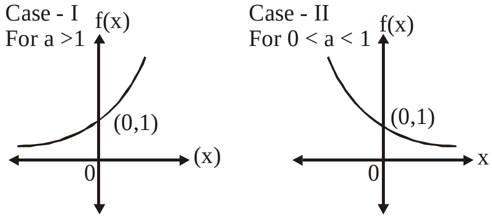
(A) 10 (B) 6 (C) 8 (D) 4

4. The value of A + C ÷ B equals

(A) 5 (B) 8 (C) 7 (D) 4

Comprehension # 3

A function $f(x) = a^x (a > 0, a \ne 1, x \in R)$ is called exponential function. Graph of exponential function can be as follows :



5. Number of solutions of $3^x + x - 2 = 0$ is/are:

(A) 1 (B) 2 (C) 3 (D) 4

6. The number of positive solutions of $\log_{1/2} x = 7^x$ is/are :

(A) 0 (B) 1 (C) 2 (D) 3

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. The number of solution(s) of $\log_4(x-1) = \log_2(x-3)$ is/are **[IIT-JEE-2002, Scr., (1, 0)/35]**

(A) 3 (B) 1 (C) 2 (D) 0

2. Let $f(x) = \dfrac{x^2-6x+5}{x^2-5x+6}$ **[IIT-JEE 2007, Paper-2, (6, 0), 81]**

Column – I	**Column – II**
(A) If $-1 < x < 1$, then f(x) satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then f(x) satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then f(x) satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then f(x) satisfies	(s) $f(x) < 1$

3 . Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ell n2} = (3y)^{\ell n3}$ $3^{\ell nx} = 2^{\ell ny}$.

Then x_0 is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

4. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$ is **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

5.* If $3^x = 4^{x-1}$, then x = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\dfrac{2\log_3 2}{2\log_3 2-1}$ (B) $\dfrac{2}{2-\log_2 3}$ (C) $\dfrac{1}{1-\log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3-1}$

6. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is ______ **[JEE(Advanced)-2018, 3(0)]**

PART - I : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE - 2009]**

(1) A = C (2) B = C (3) $A \cap B = \phi$ (4) A = B

2. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE- 2012]**

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

3. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE(Main) 2014]**

(1) X (2) Y (3) N (4) Y – X

4. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is :- **[JEE(Main) 2016]**

(1) –4 (2) 6 (3) 5 (4) 3

5. In a class 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whosw number is divisible by 3 opted Physics course and those whose number is divisible 5 opted Chemistry course. Then the number of student who did not opt for any of the three course is :- **[JEE(Main) 2019]**

(1) 38 (2) 42 (3) 102 (4) 1

6. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :- **[JEE(Main) 2019]**

(1) 12.8 (2) 13.5 (3) 13.9 (4) 13

Answers

Exercise # 1

PART - I

SECTION-(A)

A-1 (ii), (iv)

A-2 (i) {2, 3, 5, 7} (ii) {3, 4, 5, 6, 7, 8, 9}

(iii) {2, 4, 8, 12} (ii) $\{x : x = \lambda^2 + 1, \lambda \in N\}$

A-3 (i) $\{x : x = \frac{p}{q}, p \in I, q \in N\}$

(ii) $\{x : x = \lambda^2 + 1, \lambda \in N\}$

A-4 (ii), (iii)

A-5 (i) finite (ii) infinite

(iii) finite

A-6 31

A-7 $\{\phi, \{\phi\}, \{0\}, \{\{\phi\}\}, \{\phi, 0\}, \{0, \{\phi\}\}, \{\phi, \{\phi\}\}, \{\phi, 0, \{\phi\}\}\}$

SECTION-(B)

B-1 {3, 4, 10} **B-2** {9, 21}

B-3 $\{x : 1 \le x < 2\}$ **B-4** {1,2,5}

SECTION-(C)

C-1 300 **C-2** 5, 8

C-3 3300 **C-4** 43

C-5 22 **C-6** 60

SECTION-(D)

D-1 (i) $x \in (-\infty, -1] \cup \{1\} \cup [4, \infty)$

(ii) $(-\infty, -2) \cup (-2, -1/2) \cup (1, \infty)$

(iii) $[-\sqrt{2}, -1) \cup (-1, \sqrt{2}] \cup [3, 4)$

(iv) $(-\infty, -3] \cup (-1, 4)$

(v) $(-\infty, -5) \cup (1, 2) \cup (6, \infty)$

(vi) $(-\infty, -1) \cup (5, \infty)$

D-2 (i) $(-17/25, -3/8)$

(ii) $(-\infty, -20) \cup (23, \infty)$

(iii) $\left(\frac{1}{2}, 3\right)$

(iv) $x \in (-2, -1) \cup (-2/3, -1/2)$

(v) $(-\infty, -7) \cup (-4, -2)$

D-3 (i) 4 (ii) 7

(iii) 9 (iv) 2

(v) 0

D-4 (i) 1 (ii) 7

(iii) 0

D-5 3

D-6 84

SECTION-(E)

E-1 (i) +ve; (ii) – ve; (iii) +ve; (iv) +ve; (v) + ve; (vi) – ve; (vii) +ve; (viii) – ve; (ix) – ve

E-2 (i) 1 (ii) –72

(iii) 2 (iv) 1

(v) $7 + \frac{1}{196}$ (vi) 0

(vii) 9 (viii) 1

(ix) $\frac{25}{2}$ (x) 1/6

E-3 (i) b – 2a (ii) a + 3b

(iii) $\frac{2b^2 + 3a^2}{ab}$ (iv) $\frac{4(2a + b)}{1 - a + 2b}$

E-4 (i) 1 (ii) 89

E-6 –1

SECTION-(F)

F-1 (i) 3 (ii) 3

(iii) 2

(iv) no root (v) x = 16 or x = – 4

(vi) ± 2 (vii) 8

(viii) {1/3} (ix) {– 4}

(xi) x = 5 (xii) x = 10

F-2 (i) $\frac{1}{20}, \frac{1}{5}$ (ii) x = 16

(iii) x = 5 (iv) $x = 2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$

(v) (2) (vi) 10 or $\frac{1}{100}$

(vii) $\{10^{-5}, 10^{3}\}$

F-3 (i) 9 (ii) $\log_2 6$

SECTION-(G)

G-1 (i) $\left[-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$ (ii) $(1, 2) \cup (3, 4)$

(iii) $\left(-\infty, \frac{1}{2}\right)$ (iv) $(-\infty, 2)$

(v) $[-4, -3) \cup (0, 1]$ (vi) $(-1, 0) \cup (1, 2)$

(vii) $(-\infty, -5) \cup (-5, -1) \cup (3, \infty)$

(viii) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$

(ix) $[2, \infty)$ (x) $\left(0, \frac{1}{10}\right] \cup (10, \infty)$

G-2 (i) $\left(\frac{3}{4}, 1\right) \cup (1, 3]$ (ii) $(-\infty, -1) \cup (1, \infty)$

(iii) $x \in (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (2, \infty)$

G-3 0 **G-4** 8

G-5 1 **G-6** 2

SECTION-(H)

H-1 (i) 2.6336 (ii) 2.3096

(iii) $\bar{3}.3802$

H-2 (i) 199.5 (ii) 0.205

(iii) 0.0154

H-3 343

H-4 (a) 12 (b) 47 **H-5** 186.5

PART - II

SECTION-(A)

A-1 (B) **A-2** (C)
A-3 (C) **A-4** (4)
A-5 (A, B)

SECTION-(B)

B-1 (A) **B-2** (B)
B-3 (B) **B-4** (D)
B-5 (C) **B-6** (B)
B-7 (D)

SECTION-(C)

C-1 (C) **C-2** (C)
C-3 (B) **C-4** (B)
C-5 (C) **C-6** (B)

SECTION-(D)

D-1 (A) **D-2** (D)
D-3 (D) **D-4** (D)
D-5 (B) **D-6** (D)

SECTION-(E)

E-1 (B) **E-2** (A)
E-3 (D) **E-4** (C)
E-5 (A) **E-6** (D)
E-7 (B) **E-8** (D)
E-9 (D) **E-10** (A)
E-11 (D) **E-12** (C)

SECTION-(F)

F-1 (D) **F-2** (D)
F-3 (C) **F-4** (C)
F-5 (C) **F-6** (B)
F-7 (C) **F-8** (B)
F-9 (B)

SECTION-(G)

G-1 (C) **G-2** (A)
G-3 (B) **G-4** (A)
G-5 (D) **G-6** (D)

PART - III

1. (A) → (q), (B) → (r), (C) → (s), (D) → (p)
2. A - r B -p C-q D-q
3. (A) r; (B) p; (C) s; (D) q

Exercise # 2

PART - I

1.	(D)	**2.**	(A)
3.	(D)	**4.**	(D)
5.	(B)	**6.**	(A)
7.	(B)	**8.**	(B)
9.	(C)	**10.**	(A)
11.	(B)	**12.**	(B)
13.	(D)	**14.**	(B)
15.	(B)	**16.**	(A)
17.	(A)		

PART - II

1.	6	**2.**	6
3.	9	**4.**	**0**
5.	1	**6.**	1.8
7.	0.5	**8.**	1
9.	10	**10.**	x = 3
11.	0.25	**12.**	0.25
13.	1.25	**14.**	1
15.	61	**16.**	22.51 or 22.52
17.	20.20	**18.**	100.10
19.	1	**20.**	6
21.	0.50		

PART - III

1.	(A, D)	**2.**	(C, D)
3.	(B, D)	**4.**	(A, B, C, D)
5.	(A, B, C)	**6.**	(A, B, C, D)
7.	(A, B, C, D)	**8.**	(A, B)
9.	(C, D)	**10.**	(A, B, C, D)
11.	(B, C)	**12.**	(B, C)
13.	(B, D)	**14.**	(A, D)

PART - IV

1.	(D)	2.	(C)
3.	(C)	**4.**	(B)
5.	(A)	6.	(B)

Exercise # 3

PART - I

1. (B)
2. (A) → (p), (r), (s) ; (B) → (q), (s) ; (C) → (q), (s); (D) → (p), (r), (s)
3. (C)
4. (4) **5. (A, B, C)** **6.** 8

PART - II

1.	(2)	**2.**	(2)
3.	(2)	**4.**	(4)
5.	(1)	**6.**	(3)

Self Assessment Paper (SAP)

JEE ADVANCED

Maximum Marks : 62 **Total Time : 1:00 Hr**

SECTION-1 : ONE OPTION CORRECT (Marks - 12)

1. If $\log_{105}7 = a$, $\log_7 5 = b$ then $\log_{35} 105$ is equal to

(A) ab (B) (b + 1) a (C) $\frac{1}{ab}$ (D) $\frac{1}{a(b+1)}$

2. In a class of 80 student numbered 1 to 80, all odd numbered student opt for Cricket, student whose numbers are divisible by 5 opt for Football and those whose numbers are divisible by 7 opt for Hockey. The number of students who do not opt any of the three games, is

(A) 13 (B) 24 (C) 28 (D) 52

3. Interval containing all the solution of the equation $7^{x+2} - 21.7^{x-1} + 2.7^{x} = 48$ is

(A) [1, 3] (B) [–1, 3] (C) [–4, –1] (D) [1, 4]

4. The solution set of the inequation $3 + \log_{\frac{1}{7}}(x^2 + x + 1) > 0$ contains the number of integers

(A) 35 (B) 18 (C) 36 (D) 17

SECTION-2 : ONE OR MORE THAN ONE CORRECT (Marks - 32)

5. Let $f(x) = \frac{x^2 - 10x + 9}{x^2 - 7x + 12}$ then choose the correct option :-

(A) if $5 < x < 7$ then $f(x) < 0$ (B) if $5 < x < 7$ then $f(x) < -1$

(C) if $16 < x < 20$ then $-1 < f(x) < 0$ (D) if $-1 < x < 1$ then $0 < f(x) < 1$

6. Which of the following set of values of 'x' satisfy the inequality.

$x^{16} - x^7 + x^8 - x^3 + 1 > 0$

(A) $-5 < x \leq 7$ (B) $x \leq -5$ or $x \geq 7$

(C) $x \leq 0$ or $x \geq 7$ (D) $0 \leq x < 7$

7. Let $a = \log_5 \log_5 2$ an integer k satisfying $1 < 4^{(-k+5^{-a})} < 2$ must be less than :-

(A) 1 (B) 3 (C) 2 (D) 4

8. The number of integral values of x satisfying the inequality $5x - 1 < (x + 1)^2 < 7x - 3$ is

(A) 1 (B) 2 (C) 3 (D) 4

9. In a particular day of "Kota" having 1,00, 000 citizens every citizen used exactly 5 public tarnsports and every public transport is used by exactly 1,000 citizens. If number of public transports in "Kota" on that day is 'n' then 'n' is divisible by

(A) 5^2 (B) 200 (C) 5^3 (D) 100

10. If $5^{x-2} = 9^{x-3}$ then x is equal to

(A) $\frac{6\log_5 3-2}{2\log_5 3-1}$ (B) $\frac{3-\log_9 25}{1-\log_9 5}$ (C) $\frac{6-2\log_3 5}{2-\log_3 5}$ (D) $\frac{1-\log_3 5^{1/3}}{\frac{1}{3}-\log_9 5^{1/3}}$

11. The value of

$10 + 2\log_{6/5}\left(\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}....}}}\right)$ is a proper divisor for :-

(A) 12 (B) 6 (C) 15 (D) 24

12. All the values of x satisfying the equation $5\cdot3^{\log_3 x}-2^{(1-\log_2 x)}=3$ lies in the set

(A) [1, 3] (B) [0, 3] (C) [–1, 5] (D) [–3, 5]

SECTION-3 : NUMERICAL VALUE TYPE (Marks - 18)

13. Let x, y, z be positive real numbers such that $x^{\log_2 7}=8$; $y^{\log_3 5}=9$ and $z^{\log_2 216}$ = $5^{1/3}$ then the value of $\frac{1}{10}\left(x^{(\log_2 7)^2}+y^{(\log_3 5)^2}+z^{(\log_5 216)^2}\right)$ is equal to

14. Number of integral values of x which satisfy the inequation $\frac{(x-1)(x-2)^3}{(x-3)^3(x-4)}$ + 1 < 0 is λ then $2^{\lambda} + \frac{\lambda}{5}$ is euqal to

15. Sides of a triangle are the characteristic of the logarithm of 325, 1603 and 10,000 to the base 3, 11 and 9 respectively then the area of the triangle is -

16. If $\sqrt{\log_{10} a}+\sqrt{\text{lob}_{10} b}+\log_{10}\sqrt{a}+\log_{10}\sqrt{b}=100$ and all four terms on the left are positive integers then value of $\frac{1}{10}(\log_{10} ab)$ is :-

17. If solution of inequation $\log_{(x-2)}(2x-3)>\log_{(x-2)}(24-6x)$ is (a, b) $\cup$ (c, d) then a + b + c + d is equal to :-

18. If inequality $\frac{(x-5)(x-7)^2}{(x-a)}$ < 0 (a $\in$Z) is satisfied by exaclty '3' integral values of x then ratio of sum of all possible values of 'a' to the product of all possible values of 'a' is

Answers

1. (D)	**2.** (C)	**3.** (B)	**4.** (C)
5. (A, B, D)	**6.** (A, B, C, D)	**7.** (B, D)	**8.** (A)
9. (A, D)	**10.** (A, B, C)	**11.** (A, D)	**12.** (A, B, C, D)
13. 37.40	**14.** 01.00	**15.** 06.00	**16.** 16.40
17. 12.37 or 12.38	**18.** 01.10		

LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170						5 9 13	17 21 26	30 34 38
						0212	0253	0294	0334	0374	4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569						4 8 12	16 20 23	27 31 35
						0607	0645	0682	0719	0755	4 7 11	15 18 22	26 29 33
12	0792	0828	0864	0899	0934						3 7 11	14 18 21	25 28 32
						0969	1004	1038	1072	1106	3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271						36 10	13 16 19	23 26 29
						1303	1335	1367	1399	1430	37 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584						36 9	12 15 19	22 25 28
						1614	1644	1673	1703	1732	36 9	12 14 17	22 25 26
15	1761	1790	1818	1847	1875						3 6 9	11 14 17	20 23 26
						1903	1931	1959	1987	2014	3 6 8	11 14 17	19 22 25
16	2041	2068	2095	2122	2148						3 6 8	11 14 16	19 22 24
						2175	2201	2227	2253	2279	3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405						3 5 8	10 13 15	18 20 23
						2430	2455	2480	2504	2529	3 5 8	10 12 15	17 20 22
18	2553	2577	2601	2625	2648						2 5 7	9 12 14	17 19 21
						2672	2695	2718	2742	2765	2 4 7	9 11 14	16 18 21
19	2788	2810	2833	2856	2878						2 4 7	9 11 13	16 18 20
						2900	2923	2945	2967	2989	2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	9766	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	9882	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	8699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 4 4

ANTILOGRITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1841	1845	0 1 1	2 2 3	3 4 4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2328	2234	1 1 2	2 3 3	4 4 5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
.38	2399	2404	2410	2415	2421	2432	2427	2432	2443	2449	1 1 2	2 3 3	4 4 5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

ANTILOGRITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 6 7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4156	1 2 3	4 5 6	7 8 9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
.99	9772	9795	9817	9849	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20