ALL INDIA TEST SERIES

TEST – 34

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

General Instructions:

- The test consists of total **54** questions.
- Each subject (PCM) has **18** questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section – B & Section-C.**

__Section-A (01 – 06, 19 – 24, 37– 42)__ this section contains __18 multiple__ choice questions.
Each question has FOUR options. __ONE OR MORE THAN ONE__ of these four option(s) is (are) correct answer(s).
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:

Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **: –2** In all other cases

__Section-B (07 – 12, 25 – 30, 43– 48)__ contains __18 Numerical__ based questions with __Single digit integer__ as answer, ranging from __0 to 9__ and each question carries __+3 marks__ for correct answer and __–1 mark__ for wrong answer.

__Section-C (13 – 18, 31 – 36, 49– 54)__ contains __18 Numerical__ answer type questions with answer XXXXX.XX and each question carries __+4 marks__ for correct answer and __0 marks__ for wrong answer.

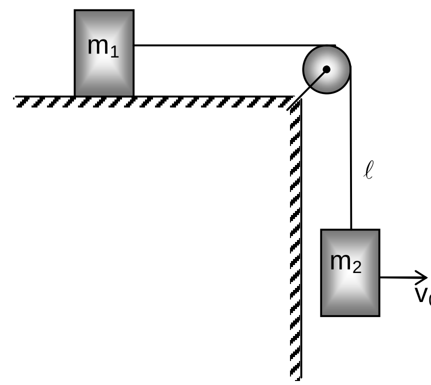
Physics

PART – I

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. In the figure shown, initially the block of mass m_1 is in limiting equilibrium. The coefficient of friction between the block of mass m_1 and the horizotnal surface is $\mu = 0.5$. Now the block of mass m_2 is given a horizontal velocity v_0 initially. The initial distance of the block of mass m_2 from the pulley is 'ℓ'. The masses of the smooth pulley and the thread are negligible. Then choose the correct option(s).



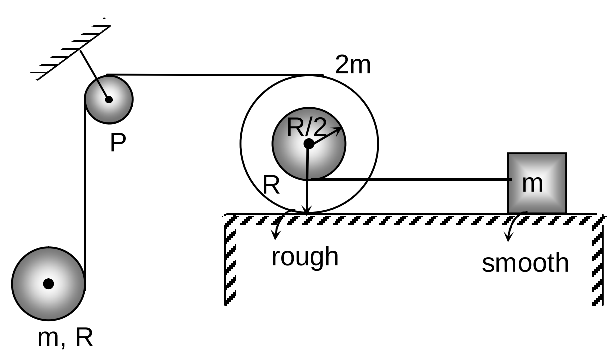
(A) The initial acceleration of the block of mass m_1 is $\frac{v_0^2}{\ell}$

(B) The initial acceleration of the block of mass m_1 is $\frac{v_0^2}{3\ell}$

(C) The initial acceleration of the block of mass m_2 is $\frac{2v_0^2}{3\ell}$

(D) The initial radius of curvature of trajectory followed by the block of mass m_2 is $\frac{3\ell}{2}$

2. Consider the arrangement shown in the figure. The smooth pulley P and the threads are massless. The mass of the spool is 2m and its moment of inertia is mR^2. The mass of the disc of radius R is m. The surface below the spool is rough to ensure pure rolling of the spool. The mass of the block is m and there is no friction between the block and the horizontal surface. Then choose the correct option(s).



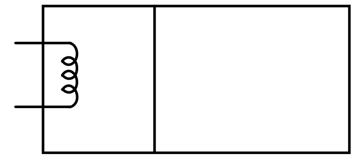
(A) The acceleration of the spool is $\frac{8g}{55}$

(B) The acceleration of the disc is $\frac{42g}{55}$

(C) The acceleration of the block is $\frac{4g}{55}$

(D) The frictional force acting on the spool due to the horizontal surface is $\frac{7mg}{55}$

3. A cylinder of volume V = 20 liter made by adiabatic walls containing 2 moles of helium gas is divided into two parts by a thin fixed rigid membrane. The volume of the right part is η = 2 times the volume of the left part. An electric heater of constant power installed in the left part is switched on.



Heat transfer rate through the membrane is H = 0.3 W per 1°C temperature difference across the membrane.

Initially the heater was switched off and both the parts were in thermal as well as mechanical equilibrium. The membrane can withstand a maximum pressure difference of $\Delta P = 10^3 Pa$. Then choose the correct option(s). (Take $R = \frac{25}{3}$ Joule K^{-1} mol^{-1})

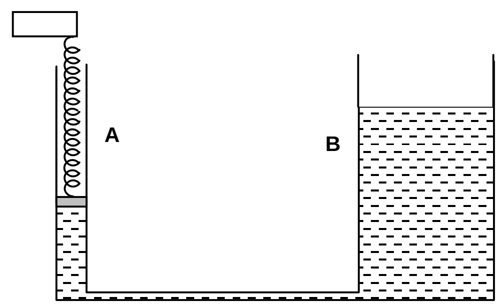
(A) The maximum power of the heater so that the membrane remains intact after long time of heating is P_{max} = 0.36 watt

(B) The maximum power of the heater so that the membrane remains intact after long time of heating is P_{max} = 0.54 watt

(C) The temperature difference of the two parts of the cylinder in the steady state is 0.8°C.

(D) The temperature difference of the two parts of the cylinder in the steady state is 1.2°C

4. Two cylindrical vessels A and B connected by a thin tube at their bottom containing a liquid of density ρ = 2×10^3 kg/m^3 are placed on a horizontal floor. The cross sectional area of vessel B is S = 1 m^2. A light piston that can slide without friction in the vessel A is connected at one end of a light spring, the other end of which is attached to a movable support. If the movable support is shifted downwards, the piston shifts down a distance that is a = 0.5 times of shift in the movable support and the liquid level in the vessel B moves up a distance that is b = 0.1 times of shift in the movable support. Then choose the correct option(s). (Take g = 10 m/s^2)



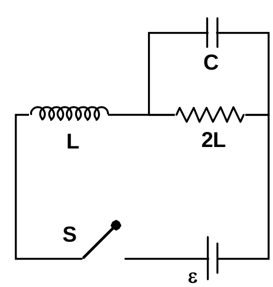
(A) The cross-sectional area of vessel A is 0.2 m^2

(B) The cross-sectional area of vessel A is 0.5 m^2

(C) The stiffness of the spring is 6 kN/m

(D) The stiffness of the spring is 4.8 kN/m

5. In the circuit shown, initially the current through each inductor is zero and capacitor is uncharged. Now the switch 'S' is closed at t = 0. Then choose the correct option(s).



(A) The charge on the capacitor at $t = \frac{\pi}{2}\sqrt{\frac{2LC}{3}}$ is $\frac{2C\varepsilon}{3}$

(B) The charge on the capacitor at $t = \frac{\pi}{2}\sqrt{\frac{2LC}{3}}$ is $\frac{C\varepsilon}{3}$

(C) The maximum current through the capacitor is $\varepsilon\sqrt{\frac{2C}{3L}}$

(D) The maximum potential drop across the capacitor is $\frac{4\varepsilon}{3}$

6. A string of length ℓ, cross sectional area S and density ρ is fixed at both ends. A standing wave $y = A \sin kx \cos \omega t$ is sustained by the string. Then choose the correct option(s).

(A) The kinetic energy of the string at time $t = \frac{\pi}{3\omega}$ is $\frac{3}{16}\rho\omega^2A^2S\ell$

(B) The kinetic energy of the string at time $t = \frac{\pi}{4\omega}$ is $\frac{1}{8}\rho\omega^2A^2S\ell$

(C) The elastic potential energy of the string at time $t = \frac{\pi}{4\omega}$ is $\frac{1}{4}\rho\omega^2A^2S\ell$

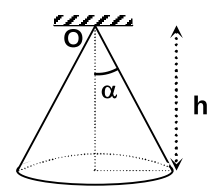
(D) The elastic potential energy of the string at time $t = \frac{\pi}{3\omega}$ is $\frac{1}{16}\rho\omega^2A^2S\ell$

SECTION – B
(Single Digit Integer Type)

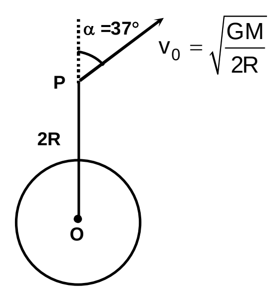
This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

7. A small cylindrical magnet is placed at the centre of a thin coil of radius 'a' containing N turns. The coil is connected to a ballistic galvanometer. The resistance of the circuit is R. After the magnet had been rapidly removed from the coil, a charge q passed through the galvanometer. If the magnetic moment of the magnet is $\left(\frac{kaRq}{\mu_0 N}\right)$. Find the value of k.

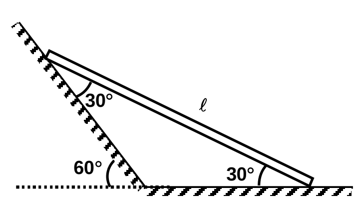
8. A solid cone of semi-vertical angle $\alpha = 30^\circ$ and height 'h' is suspended vertically from its vertex as shown in the figure. The cone is slightly rotated from its vertical position and then released. If the time period of small oscillations of the cone is $2\pi\sqrt{k\left(\frac{13h}{45g}\right)}$. Find the value of k.



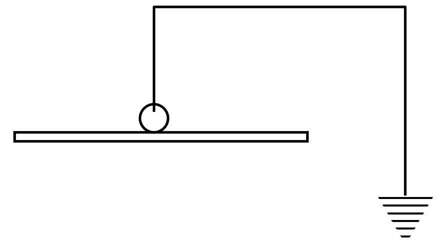
9. A satellite is projected into space from a point P at a distance 2R from the centre of the earth at an angle $\alpha = 37^\circ$ with the vertical as shown in the figure. The velocity of projection is $v_0 = \sqrt{\frac{GM}{2R}}$. It is found that the satellite is propelled into an elliptical orbit. If the eccentricity of the elliptical orbit is $e = \left(\frac{n}{10}\right)$. Find the value of 'n'. (Where M = mass of the earth, R = Radius of the earth)



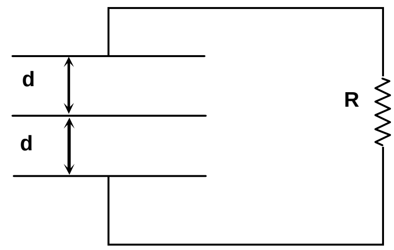
10. A thin uniform rod of length ℓ is released from rest from the position shown in the figure. All contact surfaces are smooth. If the initial angular acceleration of the rod is $\left(\frac{k\sqrt{3}g}{20\ell}\right)$. Find the value of k.



11. A grounded metallic ball of radius 'a' is placed on the centre of a uniformly charged thin insulating disc of radius R(R >> a). The total charge on the disc is Q. If the electrostatic force on the ball due to the uniformly charged disc is $\left(\frac{nQ^2a}{4\pi\varepsilon_0 R^3}\right)$. Find the value of n.



12. Three identical thin metal plates each of area A are arranged parallel to each other with separation 'd' between the adjacent plates. The outer plates are connected through a high resistance 'R' as shown in the figure. A charge Q is given to the middle plate and then the middle plate is quickly shifted towards the upper plate by a distance d/2. If heat dissipated in the resistor after this shift is $n\left(\frac{Q^2d}{32\varepsilon_0 A}\right)$. Find the value of n. (Neglect gravity and assume free space conditions)



SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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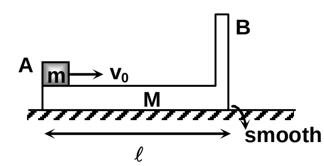
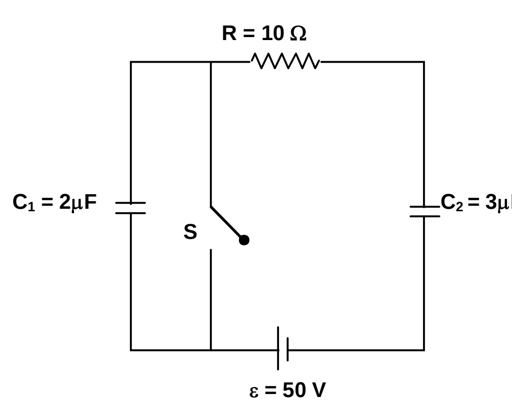
13. A small block A of mass m = 2 kg is placed on a plank B of mass M = 8 kg which is placed on a smooth horizontal surface. The distance between the left end of the plank and its vertical wall is ℓ = 4m. The coefficient of friction between the block and the plank is μ = 0.5 and collision between the block and the vertical wall of plank is perfectly elastic. An initial velocity v_0 is imparted to the block so that the block stops on the plank at its left end after collision with the vertical wall. Find the time (in sec) for which the block has moved relative to the plank. (Take g = 10 m/s^2)

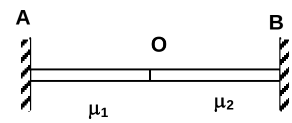


14. In a setup of displacement method experiment, the distance between the screen and a light source is 100 cm and the lens used has a small aperture. By moving the lens between the source and the screen, sharp images are obtained on the screen for two different positions of the lens. The ratio of sizes of these two images is 9 : 4. Find the focal length (in cm) of the lens.

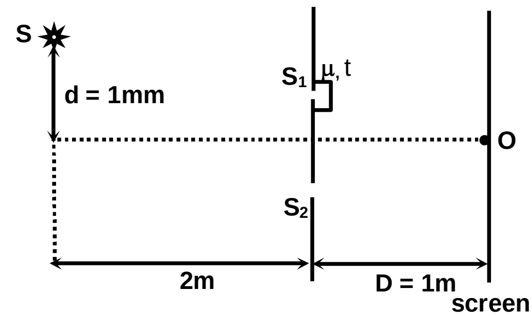
15. Consider the circuit shown in the figure. Initially the switch S is open and the capacitors are fully charged. Find the total heat (in mJ) dissipated in the circuit after the switch 'S' is closed.



16. Two strings of mass per unit length μ_1 = 0.09 kg/m and μ_2 = 0.04 kg/m are joined together and stretched between two rigid supports as shown in the figure. A sinusoidal wave of amplitude 'A' is incident from the side AO and transmission and reflection of wave take place at the joint 'O'. Find the percentage of incident power transmitted to the second string.



17. In YDSE, a monochromatic source 'S' of wavelength 4000 Å is placed at a distance d = 1mm from the central axis as shown in the figure. Where d = 1 mm is the separation between the two slits S_1 and S_2. Find the minimum thickness (in μm) of the thin film of refractive index μ = 1.5 which is placed in front of S_1 so that the intensity at point 'O' becomes maximum.



18. When light of wavelength 500 nm is incident on a metal surface of work function 1.83 eV, photoelectrons are emitted. A fastest photoelectron combines with a He^{2+} atom to form He^{+} atom. The He^{+} atom thus formed is in its first excited state. Find the energy (in eV) of the photon emitted during the combination.

Chemistry

PART – II

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

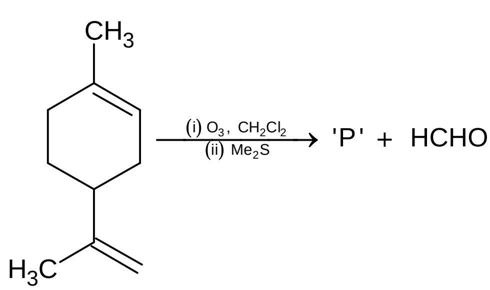
19. Which of the following statement(s) is/are incorrect?
(A) Monoclinic sulphur is stable at room temperature
(B) Rhombic sulphur is soluble in CS_2 where as monoclinic sulphur is insoluble in CS_2
(C) Plastic sulphur is insoluble in CS_2
(D) Colloidal sulphur is formed on passing H_2S through solution of HNO_3

20. Select the incorrect statements of the following:
(A) In a reversible reaction, $aA_{(g)} + bB_{(g)} \rightleftharpoons cC_{(g)} + dD_{(g)}$ n_1 and n_2 are the orders of the reaction with respect to A and B respectively, then rate of forward reaction in quasistate is $(\text{Rate})_f = k_f[A]^{n_1}[B]^{n_2}$
(B) For a hypothetical gaseous reaction $A_{(g)} \longrightarrow 3B_{(g)}$, at 10 Kelvin, $k_p > k_c$
(P in terms of atm and C in terms of mol/litre)
(C) Freezing point of water at a pressure of 5 atm is greater than its normal freezing point.
(D) Position of equilibrium in a reaction, $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$ at a given temperature remains unchanged on addition of more of $NH_2COONH_4(s)$.

21. Select the correct statement(s):
(A) Calcium Cyanamide on treatment with steam under pressure gives NH_3 and $CaCO_3$
(B) Ammonium nitrite on heating gives ammonia and nitrous acid.
(C) Phosphine gas is formed when red phosphorous is heated with NaOH
(D) PCl_5 is kept in well stoppered bottle because it reacts readily with moisture

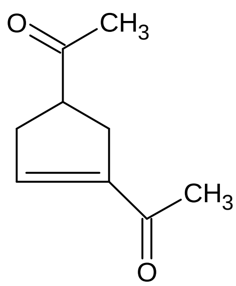
22. 0.1 M solution of Glucose in water (in Beaker I) at 0°C and saturated aqueous solution of a compound dissolving endothermically at 25°C (in Beaker II) are continuously cooled with stirring. Which of the following observation(s) will be correct with passage of time?
(A) Concentration in Beaker I will decrease.
(B) Concentration in Beaker I will increase
(C) Concentration in Beaker II will increase
(D) Concentration in Beaker II will decrease

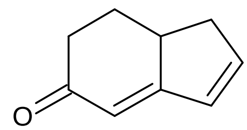
23.

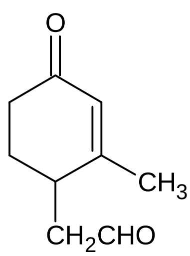


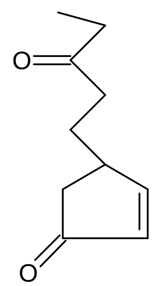
$'P' \xrightarrow[\Delta]{\text{dil } OH^-}$ Products

For given reaction which of the following is/are products

(A) 

(B) 

(C) 

(D) 

24. Which of the following combination of reagents can give 3-methyl hexan-3-ol?
(A) n-C_3H_7MgBr + butanone followed by hydrolysis
(B) C_2H_5MgBr + 2-pentanone followed by hydrolysis
(C) CH_3MgBr + 3-hexanone followed by hydrolysis
(D) $C_5H_{11}MgBr$ + 3-pentanone followed by hydrolysis

SECTION – B
(Single Digit Integer Type)

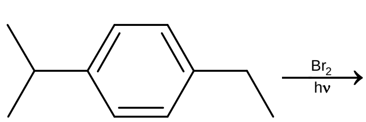
This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

25. Which of the following are co-polymers?
Nylon-6,6, Polyvinyl chloride, Teflon, Glyptal, Bakelite, Polyurethane, Polyacrylonitrile, Polyethylene, Buna-S

26. Which of the following are carbonate ores?
Siderite, Anglesite, Cerrusite, Magnesite, Limestone, Calamine, Argentite, Cassiterite, Dolomite.

27. In a compound A_2BC_2, having CCP structure, 'A' atoms constitute cubic close packing and atom 'B' and 'C' occupy 50% of octahedral and tetrahedral voids respectively. The effective number of vacant voids per unit cell is

28. At 127°C and pressure of 4.1 atm,, the compressibility factor of a real gas is 1.125. The molar volume of the gas (in dm^3) under the given condition of temperature and pressure is

29. 

In above reaction how many monobrominated products are possible?

30. How many of the acid radicals on treatment with $AgNO_3$ gives white precipitate?
SO_3^{2-} , S^{2-} , $S_2O_3^{2-}$, CH_3COO^- , $C_2O_4^{2-}$, Cl^- , Br^- , I^- , CrO_4^{2-} , $Cr_2O_7^{2-}$

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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31. Tollen's reagent is used to detect –CHO group in a compound when ammonical solution of $AgNO_3$ is added to Glucose, gluconic acid is formed according to the following reaction:

$$2Ag^+ + C_6H_{12}O_6 + H_2O \longrightarrow 2Ag(s) + C_6H_{12}O_7 + 2H^+$$

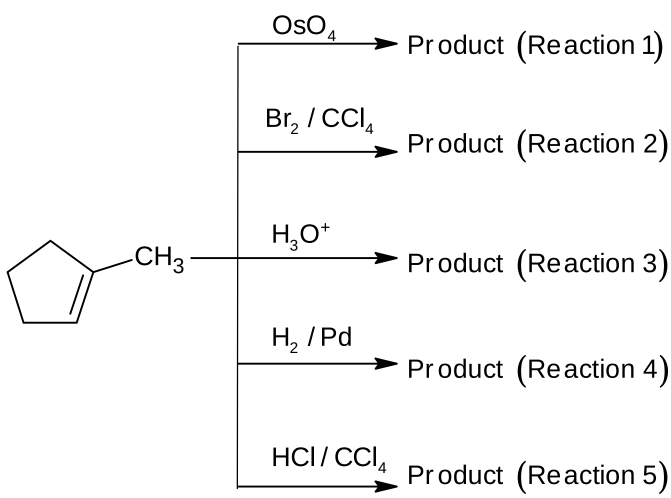
Find lnK for the above reaction, given that

$$Ag^+ + e^- \longrightarrow Ag(s) \quad E^o = 0.80V$$

$$C_6H_{12}O_6 + H_2O \longrightarrow C_6H_{12}O_7 + 2H^+ + 2e^- \quad E^o = -0.05V$$

and $\frac{F}{RT} = 38.92$ at 298 K.

32.



If 'X' is the sum of products formed (including stereoisomers) from reaction 1, 2, 3, 4, 5, then what is the value of $\frac{'x'}{2}$?

33. When 7.45 g of KCl was dissolved in 10 moles of water contained in a calorimeter, temperature of water fell from 30°C to 27°C. Enthalpy of solution of KCl in $KJmol^{-1}$ will be:
[Given: specific heat of water = 4.2 $JK^{-1}g^{-1}$ and water equivalent of calorimeter is 30g]

34. Specific rotation of pure laevorotatory 2 butanol is (–)21°. A sample of liquid 2-butanol has a specific rotation of (+)12° under identical condition. What is mole % of dextrarotary enantiomer in the sample.

35. 45.10 g of $SnCl_2.2H_2O$ and 26.9 g of $CuCl_2$ were mixed in aqueous solution. The contents were filtered and filtrate was treated quantitatively with excess of $HgCl_2(aq)$. What is the total weight (in gram) of white precipitate formed?
[Sn = 118.5, Hg = 200.6, Cu = 63.5, Cl = 35.5]

36. 0.5 moles of mixture of pure aluminium carbide and calcium carbide in the molar ratio of 1 : 4 was allowed to react with excess of water. What volume (litres) of oxygen at NTP would be needed to burn completely the gaseous mixture evolved.

Mathematics PART – III

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\hat{b}$, then possible value of $(\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ is
(A) –3
(B) 3
(C) 4
(D) –4

38. Internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of ΔABC, then
(A) AE is HM of b and c
(B) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$
(C) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$
(D) triangle AEF is isosceles

39. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2, ..., then
(A) $I_n = I_{n+2}$
(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$
(C) $\sum_{m=1}^{10} I_{2m} = 0$
(D) $I_n = I_{n+1}$

40. If f(x) be twice differentiable function such that f″(x) is positive in [0, 4], then which of the following is/are correct?
(A) f(0) + f(4) = 2f(c) for at least one c $\in$ (0, 4)
(B) f(0) + f(4) < 2f(2)
(C) f(0) + f(4) > 2f(2)
(D) 2f(0) + f(4) > $3f\left(\frac{4}{3}\right)$

41. Lines px + qy + r = 0, qx + ry + p = 0 and rx + py + q = 0 are concurrent if
(A) p + q + r = 0
(B) $p^2 + q^2 + r^2 = pr + rq + pq$
(C) $p^3 + q^3 + r^3 = 3pqr$
(D) none of these

42. Three distinct lines are drawn in a plane. Suppose there exist n circles in the plane tangent to all the three lines, then the possible values of n is/are
(A) 0
(B) 1
(C) 2
(D) 4

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

43. The value of integral $\sqrt{2}\times\int_{-\pi/4}^{3\pi/4}\frac{\sin x+\cos x}{e^{\left(x-\frac{\pi}{4}\right)}+1}dx$

44. If z satisfy $|z - 3 - 2i| \le 2$, then find the minimum value of $|2z - 6 + 5i|$

45. Number of real values of x for which the matrix $A=\begin{bmatrix}3-x & 2 & 2\\ 2 & 4-x & 1\\ -2 & -4 & -1-x\end{bmatrix}$ is singular is

46. A is a 2×2 matrix such that $A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$. The sum of the elements of A is

47. A six letter word is formed by using the letters of the word “GOOGLE”. If the probability that O’s remain between G’s is p, then the value of 6p is

48. If $\alpha=\frac{2\pi}{7}$ and $\tan\alpha\tan2\alpha+\tan2\alpha\tan4\alpha+\tan4\alpha\tan\alpha=-k$, then the value of k is

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

49. If $(x+iy)^{\frac{1}{3}}=(a+ib)$ and $\left(\frac{x}{a}+\frac{y}{b}\right)=\lambda\left(a^2-b^2\right)$, then λ is

50. A tangent to the circle $x^2 + y^2 = 4$ intersects the hyperbola $x^2 - 2y^2 = 2$ at P and Q. If locus of midpoint of PQ is $(x^2 - 2y^2)^2 = \lambda(x^2 + 4y^2)$, then λ is equal to

51. Number of z satisfying $\arg(z-1)=\frac{\pi}{4}$ and $\arg z=-\frac{\pi}{4}$

52. The value of $\left(\frac{pi+1}{pi-1}\right)^m\cdot e^{2mi\cot^{-1}p}$ is equal to

53. Given a cube with side length 6, a regular tetrahedron is constructed such that two vertices of the tetrahedron lie on the cube’s body diagonal and the other two vertices lie on the diagonal of one of the faces of the cube. The volume of this tetrahedron is $\frac{k}{\sqrt{6}}$, then k is equal to

54. If $\ln(1 - i) = \ln|a| + iB$, then a is equal to

ALL INDIA TEST SERIES

TEST – 34

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. B, C, D

Sol. $m_2g = \mu m_1 g \Rightarrow m_2 = \frac{m_1}{2}$

Now, $T - \mu m_1 g = m_1 a$ …(i)

$m_2 g + m_2\frac{v_0^2}{\ell} - T = m_2 a$ …(ii)

Solving equation (i) and (ii), we get

$\frac{m_2 v_0^2}{\ell} = (m_1 + m_2)a$

$a = \frac{v_0^2}{3\ell}$

The initial acceleration of block of mass m_1 is $a = \frac{v_0^2}{3\ell}$

The initial acceleration of block of mass m_2 is $a_2 = \frac{v_0^2}{\ell} - a = \frac{2v_0^2}{3\ell}$

The initial radius of curvature of trajectory followed by the block of mass m_2 is

$R = \frac{v_0^2}{a_2} = \frac{v_0^2}{2v_0^2/3\ell} = \frac{3\ell}{2}$

2. A, B, C, D

Sol. $a_1 = \alpha_1 R$ ...(i)

$a = a_1 - \frac{\alpha_1 R}{2} = \frac{a_1}{2}$ ...(ii)

$a_2 - \alpha_2 R = 2a_1$

$\alpha_2 R = a_2 - 2a_1$ ...(iii)

$T_1 = ma = \frac{ma_1}{2}$ ...(iv)

$T_2 + f_s - T_1 = 2ma_1$ ...(v)

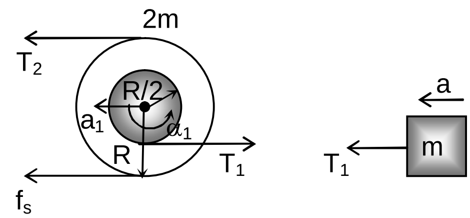
$T_2 R + T_1 \frac{R}{2} - f_s R = mR^2 \alpha_1$

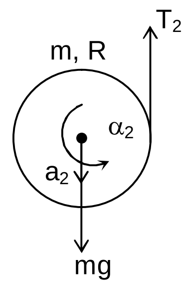
$T_2 + \frac{T_1}{2} - f_S = ma_1$ ...(vi)

$mg - T_2 = ma_2$ ...(vii)

$T_2 R = \frac{mR^2}{2}\alpha_2$

$T_2 = \frac{m\alpha_2 R}{2} \Rightarrow T_2 = \frac{m}{2}(a_2 - 2a_1)$ ...(viii)





Solving the above equations, we get

$a_1 = \frac{8g}{55}$, $a_2 = \frac{42g}{55}$, $a = \frac{4g}{55}$, and $f_s = \frac{7mg}{55}$

3. B, D

Sol. Let the temperature difference

$(T_1 - T_2) = T$

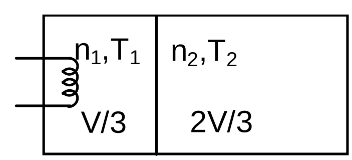
$n_1 C_{V_1} \frac{dT_1}{dt} = (P_{max} - HT)$

$\frac{2}{3} \times \frac{3R}{2} \frac{dT_1}{dt} = (P_{max} - HT)$

$\frac{dT_1}{dt} = \frac{(P_{max} - HT)}{R}$ ...(i)

$n_2 C_{V_2} \frac{dT_2}{dt} = HT$

$\frac{4}{3} \times \frac{3R}{2} \frac{dT_2}{dt} = HT \Rightarrow \frac{dT_2}{dt} = \frac{HT}{2R}$ ...(ii)



$n_1 = \frac{2}{3}$mole

$n_2 = \frac{4}{3}$mole

From (i) and (ii)

$\frac{d}{dt}(T_1 - T_2) = \frac{P_{max}}{R} - \frac{3HT}{2R}$

$\frac{dT}{dt} = \frac{3H}{2R}\left(\frac{2P_{max}}{3H} - T\right)$

In the steady state,

$\frac{dT}{dt} = 0 \Rightarrow T = T_0 = \frac{2P_{max}}{3H}$ ...(iii)

Now, $P_1 - P_2 = \left(\frac{n_1 RT_1}{V_1} - \frac{n_2 RT_2}{V_2}\right) = \frac{2R}{V}(T_1 - T_2)$

$\Rightarrow \Delta P = \frac{2RT_0}{V} \Rightarrow \Delta P = \frac{2R}{V}\left(\frac{2P_{max}}{3H}\right)$

$$\Rightarrow P_{max} = \frac{3VH\Delta P}{4R} = \frac{3\times 20\times 10^{-3}\times 0.3\times 10^{3}}{4\times \frac{25}{3}}$$

$P_{max} = 0.54$ watt

From equation (iii), $T_0 = \frac{2P_{max}}{3H} = \frac{2\times 0.54}{3\times 0.3} = 1.2\,°C$

4. A, D

Sol. Let the movable support is shifted downwards by a distance ‘x’

$axS_0 = bxS$

$$S_0 = \left(\frac{b}{a}\right)S = \frac{0.1}{0.5}\times 1 = 0.2\ m^2$$

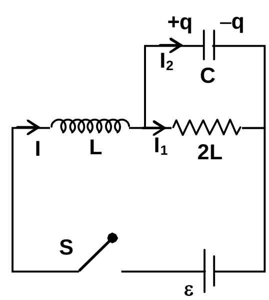
Now, $k(1-a)x = \rho g(a+b)x\left(\frac{b}{a}S\right)$

$$K = \frac{2\times 10^3\times 10\times 0.6\times 0.1\times 1}{0.5\times 0.5}$$

k = 4800 N/m

k = 4.8 kN/m

5. A, C, D



Sol. $L\frac{dI}{dt} + 2L\frac{dI_1}{dt} = \varepsilon$

$$\frac{dI}{dt} = \frac{\varepsilon}{L} - 2\frac{dI_1}{dt} \quad \ldots(i)$$

$$q = 2LC\frac{dI_1}{dt} \quad \ldots(ii)$$

Now, $I = I_1 + I_2$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$\frac{\varepsilon}{L} - 2\frac{dI_1}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \frac{\varepsilon}{L} - 3\frac{dI_1}{dt}$$

$$\frac{d^2q}{dt^2} = \frac{\varepsilon}{L} - \frac{3q}{2LC} \Rightarrow \frac{d^2q}{dt^2} = -\frac{3}{2LC}\left(q - \frac{2C\varepsilon}{3}\right)$$

$$\omega = \sqrt{\frac{3}{2LC}}$$

Now, $q - \frac{2C\varepsilon}{3} = A\sin(\omega t + \alpha)$ …(iii)

$$I_2 = \frac{dq}{dt} = \omega A\cos(\omega t + \alpha) \ldots(iv)$$

At t = 0, q = 0, $I_2 = 0$

From (iii) and (iv), we get

$$\alpha = \frac{3\pi}{2},\ A = \frac{2C\varepsilon}{3}$$

Hence $q = \frac{2C\varepsilon}{3}\left[1-\cos\left(t\sqrt{\frac{3}{2LC}}\right)\right]$

$I_2 = \varepsilon\sqrt{\frac{2C}{3L}}\sin\left(t\sqrt{\frac{3}{2LC}}\right)$

6. A, B, D

Sol. The equation of standing wave is

y = A sin kx cos ωt

The kinetic energy of the string at time 't' is

$$K = \int_0^{\ell}\frac{1}{2}\rho\left(\frac{\partial y}{\partial t}\right)^2 S dx$$

$$K = \frac{1}{2}\rho\omega^2 A^2 S\sin^2\omega t\int_0^{\ell}\sin^2 kx dx$$

$$K = \frac{1}{2}\rho\omega^2 A^2 S\sin^2\omega t\int_0^{\ell}\frac{\left[1-\cos(2kx)\right]}{2}dx$$

$$K = \frac{1}{4}\rho\omega^2 A^2 S\ell\sin^2\omega t \quad \ldots(i)$$

At $t = \frac{\pi}{3\omega}$

$$K = \frac{3}{16}\rho\omega^2 A^2 S\ell$$

The elastic potential energy stored in the string at time 't' is

$$U = \int_0^{\ell}\frac{1}{2}\rho v^2\left(\frac{\partial y}{\partial x}\right)^2 S dx$$

$$U = \int_0^{\ell}\frac{1}{2}\rho v^2 k^2 A^2\cos^2 kx\cos^2\omega t S dx$$

$$U = \int_0^{\ell}\frac{1}{2}\rho\omega^2 A^2 S\cos^2\omega t\cos^2 kx dx$$

$$U = \frac{1}{2}\rho\omega^2 A^2 S\cos^2\omega t\int_0^{\ell}\frac{\left(1+\cos(2kx)\right)}{2}dx$$

$$U = \frac{1}{4}\rho\omega^2 A^2 S\ell\cos^2\omega t \quad \ldots(ii)$$

At $t = \frac{\pi}{3\omega}$

$$U = \frac{1}{16}\rho\omega^2 A^2 S\ell$$

SECTION – B

7. 2

Sol. $q = \frac{|\Delta\phi|}{R} = \frac{\phi}{R}$ …(i)

We mentally replace the magnet by a small current loop. If the area of the loop is S and the current is I, the magnetic moment of the magnet is

$M = IS$ ...(ii)

If current flows through the coil is I_0.

The magnetic flux through the small loop is

$$\phi_0 = \frac{\mu_0 N I_0 S}{2a}$$

Mutual inductance is

$$M_0 = \frac{\mu_0 NS}{2a} \quad \text{...(iii)}$$

The magnetic flux through the coil is

$$\phi = M_0 I = \frac{\mu_0 NSI}{2a} \quad \text{...(iv)}$$

From (i) and (ii)

$$\phi = qR$$

$$\frac{\mu_0 NIS}{2a} = qR$$

$$IS = \frac{2aRq}{\mu_0 N} \Rightarrow M = \frac{2aRq}{\mu_0 N}$$

Hence k = 2

8. 3

Sol. $$I\frac{d^2\theta}{dt^2} = -\left(mg\frac{3h}{4}\right)\theta$$

$$\frac{3mh^2}{20}(4+\tan^2\alpha)\frac{d^2\theta}{dt^2} = -\left(\frac{3mgh}{4}\right)\theta$$

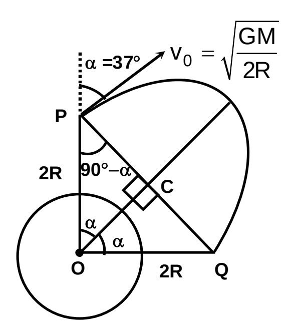
$$\frac{d^2\theta}{dt^2} = -\left(\frac{5g}{h(4+\tan^2\alpha)}\right)\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{15g}{13h}\right)\theta$$

Time period, $T = 2\pi\sqrt{\frac{13h}{15g}}$

9. 8

Sol. Total mechanical energy of the satellite



$$\varepsilon = -\frac{GMm}{2R} + \frac{1}{2}mv_0^2$$

$$\varepsilon = \frac{-GMm}{2R} + \frac{GMm}{4R}$$

$$\varepsilon = -\frac{GMm}{4R}$$

Semi-major axis, a = 2R

a cos α = ae

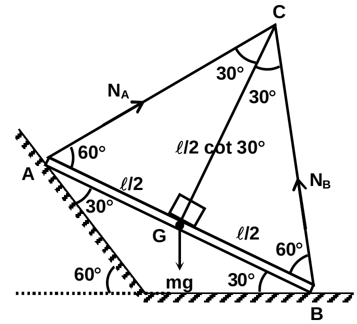
$$e = \cos 37^\circ = \frac{4}{5}$$

e = 0.8

n = 8

10. 6

Sol. 'C' is the point through which the instantaneous axis of rotation passes and G is the centre of mass of the rod.



$CG = \frac{\ell}{2}\cot 30° = \frac{\ell\sqrt{3}}{2}$

The moment of inertia abut the instantaneous axis of rotation is

$I = \frac{m\ell^2}{12} + m\left(\frac{\ell\sqrt{3}}{2}\right)^2$

$I = \frac{m\ell^2}{12} + \frac{3m\ell^2}{4}$

$I = \frac{5m\ell^2}{6}$

Now, $mg\frac{\ell\sqrt{3}}{4} = \left(\frac{5m\ell^2}{6}\right)\alpha$

$\alpha = \frac{3\sqrt{3}g}{10\ell}$

Hence k = 6

11. 4

Sol. $\sigma = \frac{Q}{\pi R^2}$

Let the charge on the ball be 'q'

$\frac{\sigma R}{2\varepsilon_0} + \frac{q}{4\pi\varepsilon_0 a} = 0$

$q = -\sigma R 2\pi a$ …(i)

The electrostatic force on the ball due to the uniformly charged disc is

$F = \frac{\sigma}{2\varepsilon_0}|q| = \frac{\sigma}{2\varepsilon_0}(\sigma R 2\pi a) = \frac{\sigma^2 \pi a R}{\varepsilon_0}$

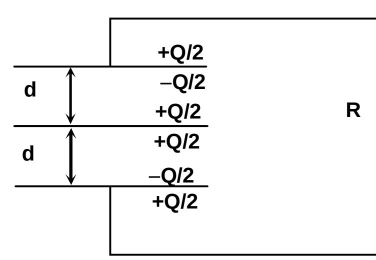
$F = \left(\frac{Q}{\pi R^2}\right)^2 \frac{\pi a R}{\varepsilon_0}$

$F = \frac{Q^2 a}{\pi\varepsilon_0 R^3}$

Hence n = 4

12. 2

Sol. When charge Q is given to the middle plate before shifting



$C = \frac{\varepsilon_0 A}{d}$, $C_{eq} = 2C$

$U_i = \frac{Q^2}{2C_{eq}} = \frac{Q^2}{4C}$ …(i)

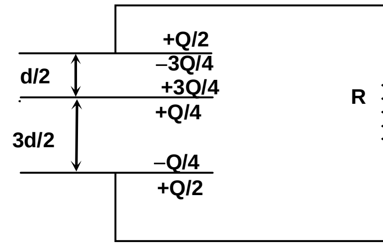
When the middle pate is shifted towards the upper plate by a distance d/2,

$C_1 = \frac{2\varepsilon_0 A}{d} = 2C$

$$C_2 = \frac{2\varepsilon_0 A}{3d} = \frac{2C}{3}$$

$$C'_{eq} = (C_1 + C_2) = 2C + \frac{2C}{3}$$

$$C'_{eq} = \frac{8C}{3}$$



$$U_f = \frac{Q^2}{2C'_{eq}} = \frac{3Q^2}{2\times 8C} = \frac{3Q^2}{16C} \quad \ldots(ii)$$

Heat dissipated in the resistor after this shift is

$$\Delta H = U_i - U_f = \frac{Q^2}{4C} - \frac{3Q^2}{16C} = \frac{Q^2}{16C} = \frac{Q^2 d}{16\varepsilon_0 A}$$

Hence n = 2

SECTION – C

13. 00001.60

Sol. Consider the motion of the block relative to the plank.
The retardation of the block relative to the plank is

$$a = \mu g + \frac{\mu m g}{M}$$

$$a = \mu g\left(\frac{M+m}{M}\right) \quad \ldots(i)$$

$$v_0^2 - 2a2\ell = 0 \Rightarrow v_0 = \sqrt{4a\ell} \quad \ldots(ii)$$

Now,

$$v_0 - at = 0$$

$$t = \frac{v_0}{a} = \frac{\sqrt{4a\ell}}{a}$$

$$t = \sqrt{\frac{4\ell}{a}} = \sqrt{\frac{4M\ell}{\mu g(M+m)}}$$

$$t = \sqrt{\frac{4\times 8\times 4}{0.5\times 10\times 10}} = 1.60 \text{ sec}$$

t = 1.60 sec

14. 00024.00

Sol. $$\left(\frac{D+x}{D-x}\right)^2 = \frac{9}{4}$$

$$\frac{D+x}{D-x} = \frac{3}{2}$$

$$2D + 2x = 3D - 3x$$

5x = D

$$x = \frac{D}{5} = \frac{100}{5} = 20\text{ cm}$$

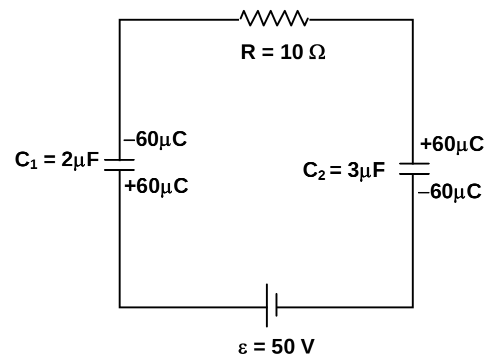
x = 20 cm

Now, $$f = \frac{D^2 - x^2}{4D} = \frac{(100\times 100) - (20\times 20)}{4\times 100}$$

f = 24 cm

15. 00002.25

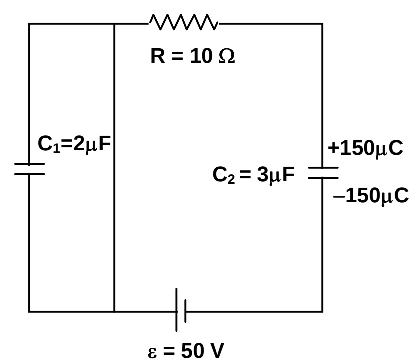
Sol. Before switch 'S' is closed



$$C_{eq} = \frac{6}{5}\mu F$$

$$Q = C_{eq}\varepsilon = \frac{6}{5}\times 50 = 60\mu C$$

After switch 'S' is closed



$\Delta Q = 150 - 60 = 90\ \mu C$

Now,

$\Delta W_b = \Delta U + \Delta H$

$$90\times 10^{-6}\times 50 = \frac{1}{2}\times 3\times 10^{-6}\times (50)^2 - \frac{1}{2}\times\frac{6}{5}\times 10^{-6}\times (50)^2 + \Delta H$$

$$4500\times 10^{-6} = (3750 - 1500)\times 10^{-6} + \Delta H$$

$\Delta H = 2250 \times 10^{-6}$ J

$\Delta H = 2.25 \times 10^{-3}$ J

16. 00096.00

Sol. $$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)Ai$$

$$A_r = \left(\frac{\sqrt{\frac{F}{\mu_2}} - \sqrt{\frac{F}{\mu_1}}}{\sqrt{\frac{F}{\mu_2}} + \sqrt{\frac{F}{\mu_1}}}\right)A$$

$$A_r = \left(\frac{1-\sqrt{\frac{\mu_2}{\mu_1}}}{1+\sqrt{\frac{\mu_2}{\mu_1}}}\right)A$$

$$A_r = \left(\frac{1-\sqrt{\frac{4}{9}}}{1+\sqrt{\frac{4}{9}}}\right)A = \left(\frac{1-\frac{2}{3}}{1+\frac{2}{3}}\right)A = \frac{A}{5}$$

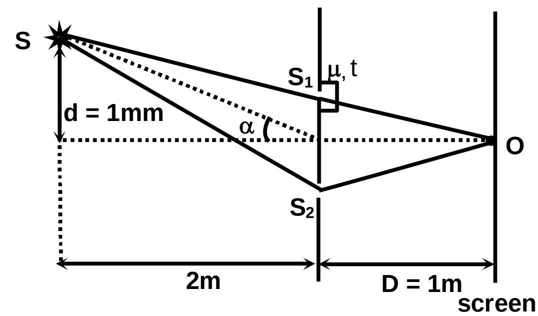
$$A_r = \frac{A}{5}$$

The percentage of incident power reflected from the joint O

$$\frac{P_r}{P_i}\times 100 = \left(\frac{A_r}{A_i}\right)^2\times 100 = \left(\frac{1}{5}\right)^2\times 100 = \frac{1}{25}\times 100 = 4\%$$

The percentage of incident power transmitted to the second string = 100 – 4 = 96%

17. 00000.20



Sol. $\tan\alpha = \frac{1\times10^{-3}}{2} = 5\times10^{-4}$

For the intensity at point 'O' to be maximum

$d \sin \alpha - (\mu - 1)t = \lambda$ (for $t = t_{min}$)

$1\times10^{-3}\times5\times10^{-4} - (1.5-1)t = 4\times10^{-7}$

$5\times10^{-7} - (1.5-1)t = 4\times10^{-7}$

$(1.5 - 1)t = 1 \times 10^{-7}$

$0.5t = 1 \times 10^{-7}$

$t_{min} = 0.20\ \mu m$

18. 00014.25

Range 14.24 to 14.26

Sol. Energy of each photon is

$E = \frac{hC}{\lambda} = \frac{1240}{500} = 2.48\text{ eV}$

Now,

$K_{max} = E - \phi$

$K_{max} = 2.48 - 1.83$

$K_{max} = 0.65\text{ eV}$

$E_n = -13.6\left(\frac{z^2}{n^2}\right)$

The energy of first excited state (n = 2) of He^+ atom,

$E_2 = -13.6\left(\frac{2^2}{2^2}\right) = -13.6\text{ eV}$

The energy of photon emitted during combination

$\Delta E = K_{max} - E_2$

$\Delta E = 0.65 - (-13.6)$

$\Delta E = 14.25\text{ eV}$

Chemistry PART – II

SECTION – A

19. A, B

Sol. (A) Monoclinic sulphur is stable above 95.6°C

(B) Both Rhombic and monoclinic sulphur are soluble in CS_2

20. A, B, C

Sol. (A) In quasistate i.e. at equilibrium no reaction occurs in either direction, forward or backward, then $(\text{Rate})_f = k_f[A]^a[B]^b$

(B) At 10 Kelvin, $(RT)^{\Delta n} = (0.0821 \times 10)^2 = 0.67$ hence $k_P < k_c$.

(C) Normal freezing point of water is freezing point at 1 atm pressure which is 0.0°C. Since specific volume of ice is greater than that of water, on applying pressure greater than 1 atm, ice at 0.0°C will change into water (le-Chatelier's Principle). Temperature will therefore has to be lowered below 0.0°C to get ice again.

(D) Active mass of NH_2COOHN_4(s) being solid, remains unity, despite the addition of more solid at equilibrium.

21. A, D

Sol. (A) $CaCN_2 + 3H_2O \longrightarrow 2NH_3 + CaCO_3$

(B) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(C) Red phosphorous is unreactive towards alkalies

(D) $PCl_5 + H_2O \longrightarrow POCl_3 + 2HCl$ (insufficient water)

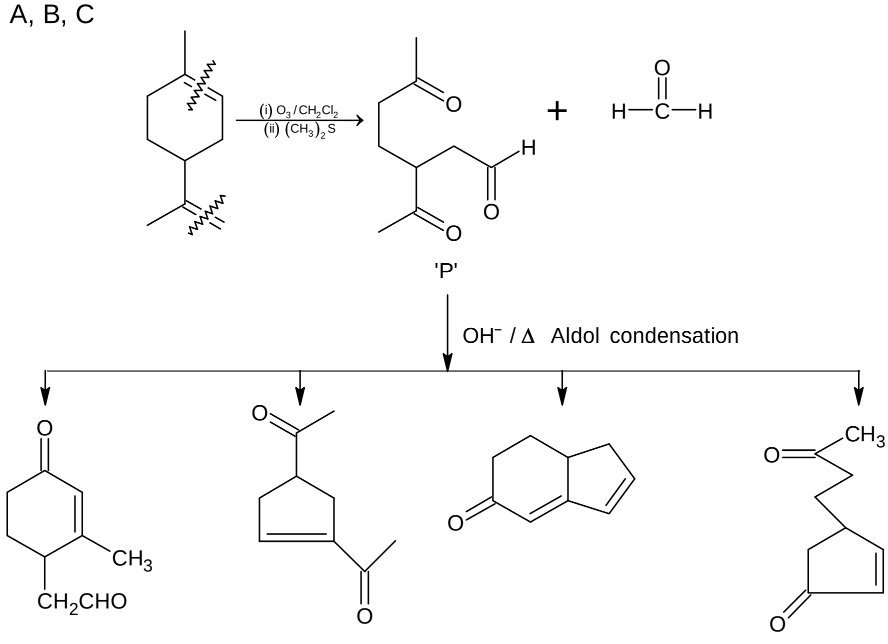
$PCl_5 + 4H_2O \longrightarrow H_3PO_4 + 5HCl$ (excess of water)

22. B, D

Sol. On cooling aqueous solution at 0°C or below water would be frozen out continuously resulting in decease of water content in the solution whereas the amount of glucose remains same.
In Beaker II, the dissolved solute continues to crystallize out with no change of water content, therefore concentration falls.

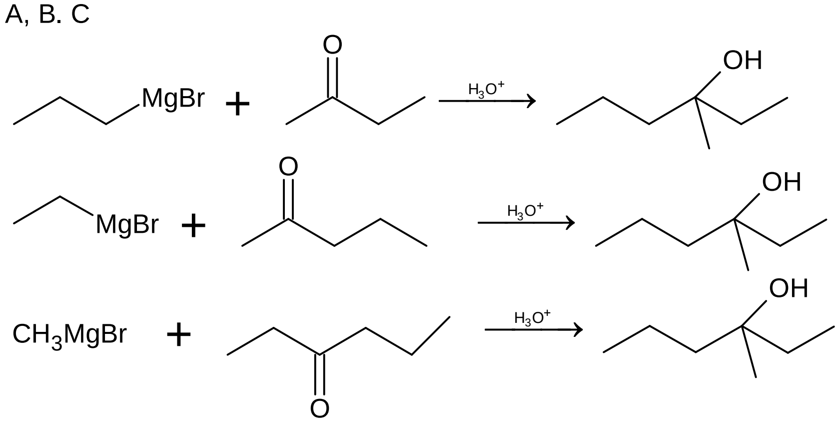
23. A, B, C

Sol.



24. A, B. C

Sol.



SECTION – B

25. 5

Sol. Nylon-6, 6, Glyptal, Bakelite, Polyurethane, Buna-S are co-polymers.

26. 6

Sol. Siderite – $FeCO_3$,
Cerrusite – $PbCO_3$
Magnesite – $MgCO_3$
Limestone – $CaCO_3$
Calamine – $ZnCO_3$
Dolomite – $MgCO_3.CaCO_3$

27. 6

Sol. In CCP number of 'A' atoms per unit cell = 4
No. of octahedral voids = 4
No. of tetrahedral voids = 8

Number of 'B' atoms present in O.V's = $\frac{4}{2}$ = 2

Number of 'C' atoms present in T.V's = $\frac{8}{2}$ = 4

No. of vacant voids per unit cell = $2 + 4 = 6$

28. 9

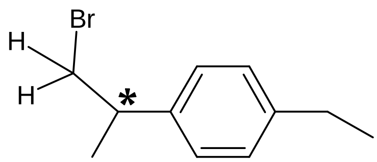
Sol. Compressibility factor of a non-ideal gas $Z = \frac{PV}{RT}$

or $1.125 = \frac{4.1 \times V\left(dm^3\right)}{0.082 \times 400}$

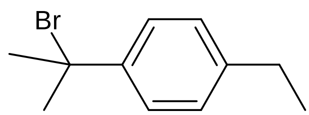
$V(dm^3) = 9$

29. 6

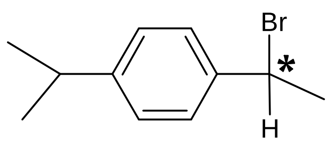
Sol.



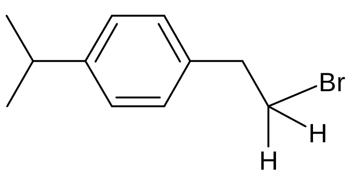
(2)



(1)



(2)



(1)

30. 5

Sol. Ag_2SO_3, $Ag_2S_2O_3$, CH_3COOAg, $Ag_2C_2O_4$, $AgCl$ = White

$\underset{(\text{Black})}{Ag_2S}$, $\underset{(\text{Pale yellow})}{AgBr}$, $\underset{(\text{Yellow})}{AgI}$, $\underset{(\text{Brick red})}{Ag_2CrO_4}$, $\underset{(\text{Reddish Brown})}{Ag_2Cr_2O_7}$

SECTION – C

31. 00058.38

Sol.

$$2Ag^+ + 2e^- \longrightarrow 2Ag(s) \qquad E^o = 0.80V$$

$$C_6H_{12}O_6 + H_2O \longrightarrow C_6H_{12}O_7 + 2H^+ + 2e^- \qquad E^o = -0.05V$$

The net reaction is

$$2Ag^+ + C_6H_{12}O_6 + H_2O \longrightarrow 2Ag(s) + C_6H_{12}O_7 + 2H^+ \qquad E^o = 0.75V$$

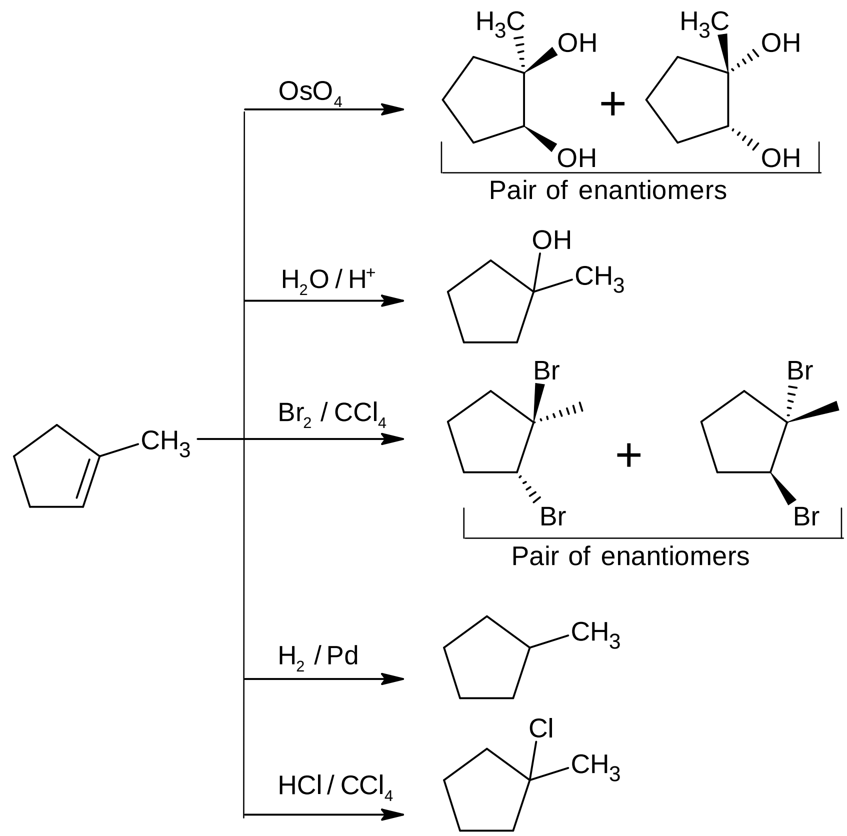
$$E_{cell} = E^o - \frac{RT}{nF}\ln K = 0$$

$$\text{or } \ln K = \frac{nFE^o}{RT} = 2 \times 0.75 \times 38.92$$

$= 58.38$

32. 00003.50

Sol.



33. 00026.46

Sol. Dissolution of KCl being endothermic, $\Delta H = +ve$

Moles of KCl dissolved = $\frac{7.45}{74.5} = 0.1$

Mass of water in calorimeter = 10 × 18 = 180 g

Heat absorbed = (180 + 30) × 4.2 × (30 – 27)

= 2.646 KJ

$\Delta H_{sol}\left(KJmol^{-1}\right) = \frac{2.646}{0.1} = 26.46$

34. 00078.58

Sol. Let fraction of laevo be ‘x’

$\Rightarrow -21x + (1 - x)21 = 12$

$x = \frac{9}{42} = 21.42$

% laevo = 21.42 and % dextro = 75.58

35. 00067.02

Sol. $n_{SnCl_2.2H_2O} = \frac{45.10}{225.5} = 0.2;$ $n_{CuCl_2} = \frac{26.9}{134.5} = 0.2$

$2CuCl_2 + SnCl_2 \longrightarrow Cu_2Cl_2 \downarrow + SnCl_4$

0.2 mole 0.1mole 0.1 mole

(white ppt.)

$$\underset{}{2HgCl_2} + \underset{0.1\,mole}{SnCl_2} \longrightarrow \underset{\substack{0.1\,mole \\ (white\ ppt.)}}{Hg_2Cl_2} + SnCl_4$$

Total weight of white ppt. $= \underset{(Cu_2Cl_2)}{19.8} + \underset{(Hg_2Cl_2)}{47.22}$

$= 67.02$ g

36. 00035.84

Sol. Moles of $Al_4C_3 = \frac{1}{5} \times 0.5 = 0.1$

Moles of $CaC_2 = \frac{4}{5} \times 0.5 = 0.4$

$$\underset{0.1}{Al_4C_3} + 12H_2O \longrightarrow 4Al(OH)_3 + \underset{0.3\ moles}{3CH_4} \uparrow$$

$$\underset{0.4}{CaC_2} + 2H_2O \longrightarrow Ca(OH)_2 + \underset{0.4\ moles}{C_2H_2} \uparrow$$

The gases require oxygen for burning as

$$\underset{0.3\ moles}{CH_4} + \underset{0.6\ moles}{2O_2} \longrightarrow CO_2 + 2H_2O$$

$$\underset{0.4}{C_2H_2} + \underset{0.4 \times \frac{5}{2} = 1.0\ mole}{\frac{5}{2}O_2} \longrightarrow 2CO_2 + H_2O$$

Total number of moles of oxygen needed = 0.6 + 1.0 = 1.6

$V_{O_2}(NTP) = 1.6 \times 22.4 = 35.84\ L$

Mathematics PART – III

SECTION – A

37. C, D

Sol. $(\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k}) = \pm(-14+6+12) = \pm 4$

38. A, B, C, D

Sol. $\Delta ABC = \Delta BD + \Delta ACD$

$EF = ED + DF = 2DE = 2 \times AD\tan\frac{A}{2}$

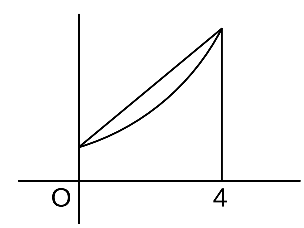
$AD \perp EF$, DE = DF, AD is bisector

39. A, B, C

Sol. Apply King's property and then calculate $(I_{n+2} - I_n)$

40. C, D

Sol. Concave up graph chord is above the graph



41. A, B, C

Sol. $\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

42. A, C, D

Sol. Draw the figure

SECTION – B

43. 2

Sol. $\left(x - \frac{\pi}{4}\right) = t$ and then King's property

44. 5

Sol. Draw the figure

45. 2

Sol. Expanding A = $-x(x-3)^2$

46. 5

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ solve, a + b + c + d = 5

a = –1, b = 0, c = 4, d = 2

47. 1

Sol. $n(A) = {}^5C_1 \times 2! + {}^5C_2 \times 2! = 30$

$p(E) = \frac{30}{180} = \frac{1}{6}$

48. 7

Sol. Open $\cos(A_1 + A_2 + A_3) = \cos 2\pi$

SECTION – C

49. 00004.00

Sol. Use $(x + iy) = (a + ib)^3$

50. 00004.00

Sol. $T = S_1$ and equation of tangent and compare $\lambda = 4$

51. 00000.00

Sol. Draw the figure

52. 00001.00

Sol. Let $\cot^{-1}p = \theta$

53. 00002.00

Sol. Volume of tetrahedron = $\frac{1}{6}$ volume of parallelepiped

54. 00001.41

Sol. $(1-i) = \left(\sqrt{2}e^{-\frac{i\pi}{4}}\right)$