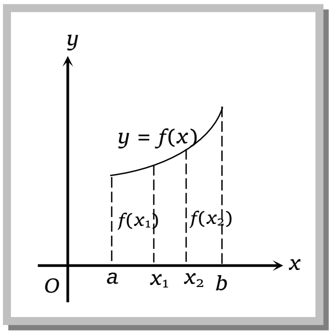
4.4 Increasing and Decreasing Function

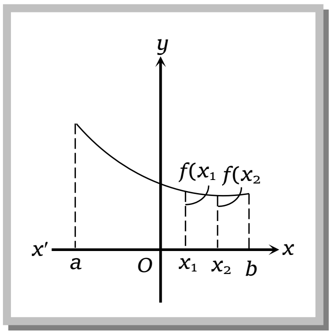
4.4.1 Definition

(1) **Strictly increasing function :** A function $f(x)$ is said to be a strictly increasing function on (a, b), if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ for all x_1, $x_2 \in (a, b)$.



Thus, $f(x)$ is strictly increasing on (a, b), if the values of $f(x)$ increase with the increase in the values of x.

(2) **Strictly decreasing function :** A function $f(x)$ is said to be a strictly decreasing function on (a, b), if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a, b)$. Thus, $f(x)$ is strictly decreasing on (a, b), if the values of $f(x)$ decrease with the increase in the values of x.



Example: 1 On the interval $(1, 3)$ the function $f(x) = 3x + \frac{2}{x}$ is **[AMU 1999]**

(a) Strictly decreasing (b) Strictly increasing

(c) Decreasing in (2, 3) only (d) Neither increasing nor decreasing

Solution: (b) $f(x) = 3x + \frac{2}{x} \Rightarrow f'(x) = 3 - \frac{2}{x^2}$

Clearly $f'(x) > 0$ on the interval (1, 3)

$\therefore$ $f(x)$ is strictly increasing.

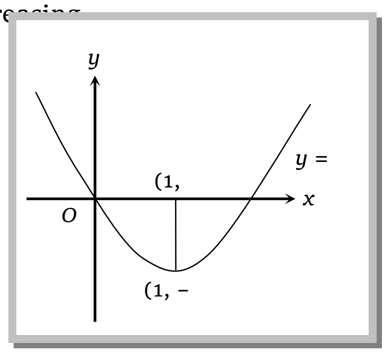
Example: 2 For which value of x, the function $f(x) = x^2 - 2x$ is decre...

(a) $x > 1$ (b) $x > 2$

(c) $x < 1$ (d) $x < 2$

Solution: (c) $f(x) = (x-1)^2 - 1$

Hence decreasing in $x < 1$



Alternative method: $f'(x) = 2x - 2 = 2(x-1)$

To be decreasing, $2(x-1) < 0 \Rightarrow (x-1) < 0 \Rightarrow x < 1$.

Example: 3 $2x^3 + 18x^2 - 96x + 45 = 0$ is an increasing function when

(a) $x \le -8, x \ge 2$ (b) $x < -2, x \ge 8$ (c) $x \le -2, x \ge 8$ (d) $0 < x \le -2$

Solution: (a) $f'(x) = 6x^2 + 36x - 96 > 0$, for increasing

$\Rightarrow f'(x) = 6(x+8)(x-2) \ge 0 \Rightarrow x \ge 2, x \le -8$.

Example: 4 The function x^x is increasing, when **[MP PET 2003]**

(a) $x > \frac{1}{e}$ (b) $x < \frac{1}{e}$ (c) $x < 0$ (d) For all real x

Solution: (a) Let $y = x^x \Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$; For $\frac{dy}{dx} > 0$

$x^x(1 + \log x) > 0 \Rightarrow 1 + \log x > 0 \Rightarrow \log_e x > \log_e \frac{1}{e}$

For this to be positive, x should be greater than $\frac{1}{e}$.

4.4.2 Monotonic Function

A function $f(x)$ is said to be monotonic on an interval (a, b) if it is either increasing or decreasing on (a, b).

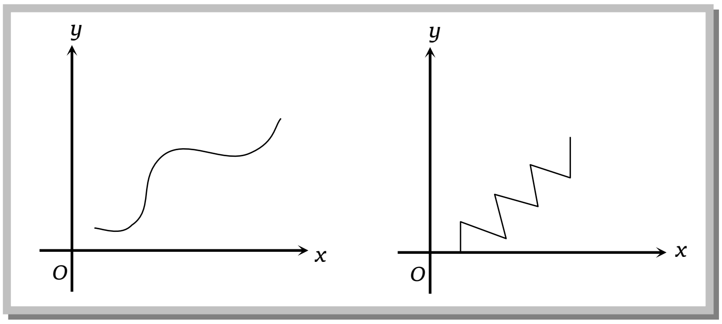
(1) **Monotonic increasing function :** A function is said to be a monotonic increasing function in defined interval if,

$x_1 > x_2 \Rightarrow f(x_1) \ge f(x_2)$

or $x_1 > x_2 \Rightarrow f(x_1) \nless f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) \ngtr f(x_2)$

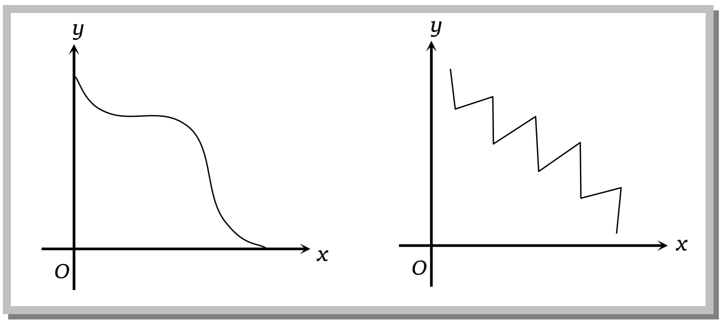


(2) **Monotonic decreasing function:** A function is said to be a monotonic decreasing function in defined interval, if $x_1 > x_2 \Rightarrow f(x_1)$

or $x_1 > x_2 \Rightarrow f(x_1) \ngtr f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$

or $x_1 < x_2 \nRightarrow f(x_1) < f(x_2)$



Example: 5 The function $f(x) = \cos x - 2px$ is monotonically decreasing for **[MP PET 2002]**

(a) $p < \frac{1}{2}$ (b) $p > \frac{1}{2}$ (c) $p < 2$ (d) $p > 2$

Solution: (b) $f(x)$ will be monotonically decreasing, if $f'(x) < 0$.

$\Rightarrow f'(x) = -\sin x - 2p < 0 \Rightarrow \frac{1}{2}\sin x + p > 0 \Rightarrow p > \frac{1}{2}$ $[\because -1 \le \sin x \le 1]$

Example: 6 If $f(x) = x^5 - 20x^3 + 240x$, then $f(x)$ satisfies which of the following **[Kurukshetra CEE 1996]**

(a) It is monotonically decreasing everywhere (b) It is monotonically decreasing only in $(0, \infty)$

(c) It is monotonically increasing every where (d) It is monotonically increasing only in $(-\infty, 0)$

Solution: (c) $f'(x) = 5x^4 - 60x^2 + 240 = 5(x^4 - 12x^2 + 48) = 5[(x^2 - 6)^2 + 12]$

$\Rightarrow f'(x) > 0 \; \forall \; x \in R$

i.e., $f(x)$ is monotonically increasing everywhere.

Example: 7 The value of a for which the function $(a+2)x^3 - 3ax^2 + 9ax - 1$ decrease monotonically throughout for all real *x*, are

[Kurukshetra CEE 2002]

(a) $a < -2$ (b) $a > -2$ (c) $-3 < a < 0$ (d) $-\infty < a \le -3$

Solution: (d) If $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically for all $x \in R$, then $f'(x) \le 0$ for all $x \in R$

$\Rightarrow 3(a+2)x^2 - 6ax + 9a \le 0$ for all $x \in R$ $\Rightarrow (a+2)x^2 - 2ax + 3a \le 0$ for all $x \in R$

$\Rightarrow a + 2 < 0$ and discriminant ≤ 0 $\Rightarrow a < -2$ and $-8a^2 - 24a \le 0$

$\Rightarrow a < -2$ and $a(a+3) \ge 0 \Rightarrow a < -2$ and $a \le -3$ or $a \ge 0 \Rightarrow a \le -3 \Rightarrow -\infty < a \le -3$

Example: 8 Function $f(x) = \frac{\lambda \sin x + 6\cos x}{2\sin x + 3\cos x}$ is monotonic increasing if

(a) $\lambda > 1$ (b) $\lambda < 1$ (c) $\lambda < 4$ (d) $\lambda > 4$

Solution: (d) The function is monotonic increasing if, $f'(x) > 0$

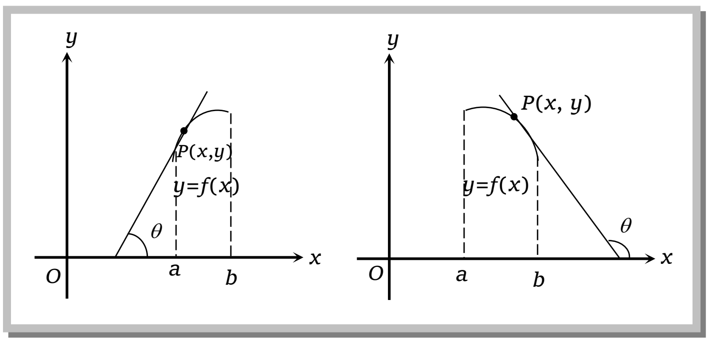
$$\Rightarrow \frac{(2\sin x + 3\cos x)(\lambda\cos x - 6\sin x)}{(2\sin x + 3\cos x)^2} - \frac{(\lambda\sin x + 6\cos x)(2\cos x - 3\sin x)}{(2\sin x + 3\cos x)^2} > 0$$

$\Rightarrow 3\lambda(\sin^2 x + \cos^2 x) - 12(\sin^2 x + \cos^2 x) > 0 \Rightarrow 3\lambda - 12 > 0 \Rightarrow \lambda > 4$.

4.4.3 Necessary and Sufficient Condition for Monotonic Function

In this section we intend to see how we can use derivative of a function to determine where it is increasing and where it is decreasing

(1) **Necessary condition :** From figure we observe that if $f(x)$ is an increasing function on (*a*, *b*), then tangent at every point on the curve $y = f(x)$ makes an acute angle θ with the positive direction of *x*-axis.



$\therefore \tan\theta > 0 \Rightarrow \frac{dy}{dx} > 0$ or $f'(x) > 0$ for all $x \in (a, b)$

It is evident from figure that if $f(x)$ is a decreasing function on (a, b), then tangent at every point on the curve $y = f(x)$ makes an obtuse angle θ with the positive direction of x-axis.

$\therefore \quad \tan\theta < 0 \Rightarrow \frac{dy}{dx} < 0$ or $f'(x) < 0$ for all $x \in (a, b)$.

Thus, $f'(x) > 0 (< 0)$ for all $x \in (a, b)$ is the necessary condition for a function $f(x)$ to be increasing (decreasing) on a given interval (a, b). In other words, if it is given that $f(x)$ is increasing (decreasing) on (a, b), then we can say that $f'(x) > 0 (< 0)$.

(2) **Sufficient condition : Theorem :** Let f be a differentiable real function defined on an open interval (a, b).

(a) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b).

(b) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b).

Corollary : Let $f(x)$ be a function defined on (a, b).

(a) If $f'(x) > 0$ for all $x \in (a, b)$, except for a finite number of points, where $f'(x) = 0$, then $f(x)$ is increasing on (a, b).

(b) If $f'(x) < 0$ for all $x \in (a, b)$, except for a finite number of points, where $f'(x) = 0$, then $f(x)$ is decreasing on (a, b).

Example: 9 The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is

(a) Increasing on $[0, \infty)$ (b) Decreasing on $[0, \infty)$

(c) Decreasing on $\left[0, \frac{\pi}{e}\right)$ and increasing on $\left[\frac{\pi}{e}, \infty\right)$ (d) Increasing on $\left[0, \frac{\pi}{e}\right)$ and decreasing on $\left[\frac{\pi}{e}, \infty\right)$

Solution: (b) Let $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$

$$\therefore \quad f'(x) = \frac{\ln(e+x) \times \frac{1}{\pi + x} - \ln(\pi + x)\frac{1}{e + x}}{\{\ln(e+x)\}^2} = \frac{(e+x)\ln(e+x) - (\pi + x)\ln(\pi + x)}{\{\ln(e+x)\}^2 \times (e+x)(\pi + x)}$$

$\Rightarrow f'(x) < 0$ for all $x \ge 0$ $\{\because \pi > e\}$. Hence, $f(x)$ is decreasing in $[0, \infty)$.

Example: 10 Which of the following is not a decreasing function on the interval $\left(0, \frac{\pi}{2}\right)$

(a) $\cos x$ (b) $\cos 2x$ (c) $\cos 3x$ (d) $\cot x$

Solution: (c) Obviously, here $\cos 3x$ in not decreasing in $\left(0, \frac{\pi}{2}\right)$ because $\frac{d}{dx}\cos 3x = -3\sin 3x$.

But at $x = 75^o$, $-3\sin 3x > 0$. Hence the result.

Example: 11 The interval of increase of the function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(1, \infty)$ (d) $(-\infty, -1)$

Solution: (b, d) We have $f(x)=x-e^x+\tan\left(\frac{2\pi}{7}\right) \Rightarrow f'(x)=1-e^x$

For $f(x)$ to be increasing, we must have $f'(x)>0 \Rightarrow 1-e^x>0 \Rightarrow e^x<1 \Rightarrow x<0 \Rightarrow x\in(-\infty,0) \Rightarrow (-\infty,-1)\subseteq(-\infty,0)$

4.4.4 Test for Monotonicity

(1) **At a point :** (i) Function $f(x)$ will be monotonic increasing in domain at a point if and only if, $f'(a)>0$

(ii) Function $f(x)$ will be monotonic decreasing in domain at a point if and only if, $f'(a)<0$.

(2) In an interval : Function $f(x)$, defined in $[a, b]$ is

(i) Monotonic increasing in (a, b) if, $f'(x)\geq 0$, $a<x<b$

(ii) Monotonic increasing in $[a, b]$ if, $f'(x)\geq 0$, $a\leq x\leq b$

(iii) Strictly increasing in $[a, b]$, if, $f'(x)>0$, $a\leq x\leq b$

(iv) Monotonic decreasing in (a, b), if, $f'(x)\leq 0$, $a<x<b$

(v) Monotonic decreasing in $[a, b]$, if, $f'(x)\leq 0$, $a\leq x\leq b$

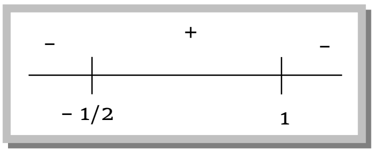
(vi) Strictly decreasing in $[a, b]$, if, $f'(x)<0$, $a\leq x\leq b$

Example: 12 $f(x)=xe^{x(1-x)}$ then $f(x)$ is **[IIT Screening 2001]**

(a) Increasing on $\left[\frac{-1}{2},1\right]$ (b) Decreasing on R (c) Increasing on R (d) Decreasing on $\left[\frac{-1}{2},1\right]$

Solution: (a) $f'(x)=e^{x(1-x)}+x.e^{x(1-x)}.(1-2x) = e^{x(1-x)}\{1+x(1-2x)\}=e^{x(1-x)}.(-2x^2+x+1)$

Now by the sign-scheme for $-2x^2+x+1$



$f'(x)\geq 0$, if $x\in\left[-\frac{1}{2},1\right]$, because $e^{x(1-x)}$ is always positive. So, $f(x)$ is increasing on $\left[-\frac{1}{2},1\right]$.

Example: 13 x tends 0 to π then the given function $f(x)=x\sin x+\cos x+\cos^2 x$ is

(a) Increasing (b) Decreasing

(c) Neither increasing nor decreasing (d) None of these

Solution: (b) $f(x)=x\sin x+\cos x+\cos^2 x$

$\therefore$ $f'(x)=\sin x+x\cos x-\sin x-2\cos x\sin x = \cos x(x-2\sin x)$

Hence $x\to 0$ to π, then $f'(x)\leq 0$, *i.e.*, $f(x)$ is decreasing function.

4.4.5 Properties of Monotonic Function

(1) If $f(x)$ is strictly increasing function on an interval [*a, b*], then f^{-1} exists and it is also a strictly increasing function.

(2) If $f(x)$ is strictly increasing function on an interval [*a, b*] such that it is continuous, then f^{-1} is continuous on $[f(a), f(b)]$

(3) If $f(x)$ is continuous on [*a, b*] such that $f'(c) \geq 0 (f'(c) > 0)$ for each $c \in (a,b)$, then $f(x)$ is monotonically (strictly) increasing function on [*a, b*].

(4) If $f(x)$ is continuous on [*a, b*] such that $f'(c) \leq 0 (f'(c) < 0)$ for each $c \in (a,b)$, then $f(x)$ is monotonically (strictly) decreasing function on [*a, b*]

(5) If $f(x)$ and $g(x)$ are monotonically (or strictly) increasing (or decreasing) functions on [*a, b*], then *gof*(*x*) is a monotonically (or strictly) increasing function on [*a, b*]

(6) If one of the two functions $f(x)$ and $g(x)$ is strictly (or monotonically) increasing and other a strictly (monotonically) decreasing, then *gof*(*x*) is strictly (monotonically) decreasing on [*a, b*].

Example: 14 The interval in which the function $x^2 e^{-x}$ is non decreasing, is

(a) $(-\infty, 2]$ (b) [0, 2] (c) $[2, \infty)$ (d) None of these

Solution: (b) Let $f(x) = x^2 e^{-x}$

$$\Rightarrow \frac{dy}{dx} = 2xe^{-x} - x^2 e^{-x} = e^{-x}(2x - x^2)$$

Hence $f'(x) \geq 0$ for every $x \in [0, 2]$, therefore it is non-decreasing in [0, 2].

Example: 15 The function $\sin^4 x + \cos^4 x$ increase if **[IIT 1999]**

(a) $0 < x < \frac{\pi}{8}$ (b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$ (c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Solution: (b) $f(x) = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$$= 1 - \frac{4\sin^2 x \cos^2 x}{2} = 1 - \frac{\sin^2 2x}{2} = 1 - \frac{1}{4}(2\sin^2 2x)$$

$$= 1 - \left(\frac{1 - \cos 4x}{4}\right) = \frac{3}{4} + \frac{1}{4}\cos 4x$$

Hence function $f(x)$ is increasing when $f'(x) > 0$

$f'(x) = -\sin 4x > 0 \Rightarrow \sin 4x < 0$

Hence $\pi < 4x < \frac{3\pi}{2}$ or $\frac{\pi}{4} < x < \frac{3\pi}{8}$.

Assignment

Increasing and Decreasing Function

Basic Level

1. The function $x+\frac{1}{x}$ ($x \neq 0$) is a non-increasing function in the interval

(a) [– 1, 1] (b) [0, 1] (c) [– 1, 0] (d) [–1, 2]

2. The interval for which the given function $f(x)=2x^3-3x^2-36x+7$ is decreasing, is

(a) (– 2, 3) (b) (2, 3) (c) (2, –3) (d) None of these

3. If $f(x)=\sin x-\frac{x}{2}$ is increasing function, then **[MP PET 1987]**

(a) $0<x<\frac{\pi}{3}$ (b) $-\frac{\pi}{3}<x<0$ (c) $-\frac{\pi}{3}<x<\frac{\pi}{3}$ (d) $x=\frac{\pi}{2}$

4. If the function $f:R\to R$ be defined by $f(x)=\tan x-x$, then $f(x)$

(a) Increases (b) Decreases (c) Remains constant (d) Becomes zero

5. $2x^3 - 6x + 5$ is an increasing function if **[UPSEAT 2003]**

(a) $0 < x < 1$ (b) $-1 < x < 1$ (c) $x < -1$ or $x > 1$ (d) $-1 < x < -1/2$

6. The function $f(x) = 1 - x^3 - x^5$ is decreasing for **[Kerala (Engg.) 2002]**

(a) $1 \le x \le 5$ (b) $x \le 1$ (c) $x \ge 1$ (d) All values of x

7. For which interval, the given function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is decreasing **[MP PET 1993]**

(a) $(-2, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -1)$ (d) $(-\infty, -2)$ and $(-1, \infty)$

8. The function $f(x) = \tan x - x$ **[MNR 1995]**

(a) Always increases (b) Always decreases

(c) Never decreases (d) Sometimes increases and sometimes decreases

9. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in each interval, then **[Rajasthan PET 1992; Kurukshetra CEE 2002]**

(a) $k < 3$ (b) $k \le 3$ (c) $k > 3$ (d) None of these

10. The least value of k for which the function $x^2 + kx + 1$ is an increasing function in the interval $1 < x < 2$ is

(a) – 4 (b) – 3 (c) – 1 (d) – 2

11. The function $f(x) = x + \cos x$ is

(a) Always increasing (b) Always decreasing

(c) Increasing for certain range of x (d) None of these

12. The function $f(x) = x^2$ is increasing in the interval

(a) $(-1, 1)$ (b) $(-\infty, \infty)$ (c) $(0, \infty)$ (d) $(-\infty, 0)$

13. Function $f(x) = x^4 - \frac{x^3}{3}$ is

(a) Increasing for $x > \frac{1}{4}$ and decreasing for $x < \frac{1}{4}$ (b) Increasing for every value of x

(c) Decreasing for every value of x (d) None of these

14. The function $y = 2x^3 - 9x^2 + 12x - 6$ is monotonic decreasing when **[MP PET 1994; Rajasthan PET 1996]**

(a) $1 < x < 2$ (b) $x > 2$ (c) $x < 1$ (d) None of these

15. The interval in which the $x^2 e^{-x}$ is non-decreasing, is

(a) $(-\infty, 2]$ (b) [0, 2] (c) $[2, \infty)$ (d) None of these

16. The function $\frac{1}{1 + x^2}$ is decreasing in the interval

(a) $(-\infty, -1]$ (b) $(-\infty, 0]$ (c) $[1, \infty)$ (d) $(0, \infty)$

17. The function $\sin x - bx + c$ will be increasing in the interval $(-\infty, \infty)$ if

(a) $b \le 1$ (b) $b \le 0$ (c) $b < -1$ (d) $b \ge 0$

18. In the interval [0, 1], the function $x^2 - x + 1$ is

(a) Increasing (b) Decreasing

(c) Neither increasing nor decreasing (d) None of these

19. $f(x)=x^3-27x+5$ is an increasing function, when **[MP PET 1995]**

(a) $x<-3$ (b) $|x|>3$ (c) $x\le -3$ (d) $|x|<3$

20. For the every value of *x* the function $f(x)=\frac{1}{5^x}$ is

(a) Decreasing (b) Increasing

(c) Neither increasing nor decreasing (d) Increasing for $x>0$ and decreasing for $x<0$

21. In which interval is the given function $f(x)=2x^3-15x^2+36x+1$ is monotonically decreasing

(a) [2, 3] (b) (2, 3) (c) $(-\infty,2)$ (d) $(3,\infty)$

22. The interval of the decreasing function $f(x)=x^3-x^2-x-4$ is

(a) $\left(\frac{1}{3},1\right)$ (b) $\left(-\frac{1}{3},1\right)$ (c) $\left(-\frac{1}{3},\frac{1}{3}\right)$ (d) $\left(-1,-\frac{1}{3}\right)$

23. Let $f(x)=x^3+bx^2+cx+d, 0<b^2<c$. Then *f* **[IIT JEE Screening 2004]**

(a) Is bounded (b) Has a local maxima (c) Has a local minima (d) Is strictly increasing

24. The function $f(x)=x^3-3x^2-24x+5$ is an increasing function in the interval given below

(a) $(-\infty,-2)\cup(4,\infty)$ (b) $(-2,\infty)$ (c) $(-2,4)$ (d) $(-\infty,4)$

25. Which one is the correct statement about the function $f(x)=\sin 2x$

(a) $f(x)$ is increasing in $\left(0,\frac{\pi}{2}\right)$ and decreasing in $\left(\frac{\pi}{2},\pi\right)$

(b) $f(x)$ is decreasing in $\left(0,\frac{\pi}{2}\right)$ and increasing in $\left(\frac{\pi}{2},\pi\right)$

(c) $f(x)$ is increasing in $\left(0,\frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

(d) The statement (a), (b) and (c) are all correct

26. If $f(x)=x^3-10x^2+200x-10$, then **[Kurukshetra CEE 1998]**

(a) $f(x)$ is decreasing in $]-\infty,10]$ and increasing in $[10,\infty[$ (b) $f(x)$ is increasing in $]-\infty,10]$ and decreasing in $[10,\infty[$

(c) $f(x)$ is increasing throughout real line (d) $f(x)$ is decreasing throughout real line

27. If *f* is a strictly increasing function, then $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is equal to

(a) 0 (b) 1 (c) – 1 (d) 2

28. Function x^3-6x^2+9x+1 is monotonic decreasing when **[Rajasthan PET 1991]**

(a) $1<x<3$ (b) $x<3$ (c) $x>1$ (d) $x>3$ or $x<1$

29. The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-6x+8$ is decreasing in the interval

(a) $x<-3$ (b) $x>2$ (c) $-3<x<2$ (d) None of these

30. The function $f(x) = 2\log(x-2) - x^2 + 4x + 1$ increases in the interval

(a) (1, 2) (b) (2, 3) (c) $(-\infty, -1)$ (d) (2, 4)

31. The function $f(x) = \frac{|x|}{x}(x \neq 0), x > 0$ is

(a) Monotonically decreasing (b) Monotonically increasing (c) Constant function (d)

32. In the following decreasing function is

(a) ln x (b) $\frac{1}{|x|}$ (c) $e^{1/x}$ (d) None of these

33. If $f(x) = kx - \sin x$ is monotonically increasing, then

(a) $k > 1$ (b) $k > -1$ (c) $k < 1$ (d) $k < -1$

Advance Level

34. The function *f* defined by $f(x) = (x+2)e^{-x}$ is **[IIT Screening 1994]**

(a) Decreasing for all x

(b) Decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$

(c) Increasing for all x

(d) Decreasing in $(-1, \infty)$ and increasing in $(-\infty, -1)$

35. The value of a in order that $f(x) = \sqrt{3}\ \sin x\ - \cos x - 2ax + b$ decreases for all real values of x, is given by

(a) $a < 1$ (b) $a \geq 1$ (c) $a \geq \sqrt{2}$ (d) $a < \sqrt{2}$

36. The interval in which the function x^3 increases less rapidly then $6x^2 + 15x + 5$, is

(a) $(-\infty, -1)$ (b) $(-5, 1)$ (c) $(-1, 5)$ (d) $(5, \infty)$

37. Let $f(x) = \int e^x(x-1)(x-2)dx$. Then *f* decreases in the interval

(a) $(-\infty, -2)$ (b) $(-2, -1)$ (c) (1, 2) (d) $(2, +\infty)$

38. If $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1+x^2} - x)$, then $f(x)$

(a) Increases in $[0, \infty)$

(b) Decreases in $[0, \infty)$

(c) Neither increases nor decreases in $(0, \infty)$

(d) Increases in $(-\infty, \infty)$

39. The function $\frac{(e^{2x} - 1)}{(e^{2x} + 1)}$ is **[Roorkee 1998]**

(a) Increasing (b) Decreasing (c) Even (d) Odd

40. The function $\frac{a \sin x + b \cos x}{c \sin x + d \cos x}$ is decreasing if **[Rajasthan PET 1999]**

(a) $ad - bc > 0$ (b) $ad - bc < 0$ (c) $ab - cd > 0$ (d) $ab - cd < 0$

41. If $f(x) = \sin x - \cos x$, $0 \leq x \leq 2\pi$ the function decreasing in **[UPSEAT 2001]**

(a) $\left[\frac{5\pi}{6}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (c) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ (d) None of these

42. If $f(x)=\frac{1}{x+1}-\log(1+x), x>0$ then f is **[Rajasthan PET 2002]**

(a) An increasing function (b) A decreasing function

(c) Both increasing and decreasing function (d) None of these

43. The function $f(x)=x^{1/x}$ is **[AMU 2002]**

(a) Increasing in $(1,\infty)$ (b) Decreasing in $(1,\infty)$

(c) Increasing in $(1,e)$, decreasing in (e,∞) (d) Decreasing in $(1,e)$ increasing in (e,∞)

44. The length of the longest interval, in which the function $3\sin x-4\sin^3 x$ is increasing, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

45. The function $f(x)=1-e^{-x^2/2}$ is

(a) Decreasing for all x (b) Increasing for all x

(c) Decreasing for $x<0$ and increasing for $x>0$ (d) Increasing for $x<0$ and decreasing for $x>0$

46. The function $\sin x-\cos x$ is increasing in the interval

(a) $\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$ (b) $\left[0,\frac{3\pi}{4}\right)$ (c) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ (d) None of these

47. On the interval $\left(0,\frac{\pi}{2}\right)$, the function log $\sin x$ is

(a) Increasing (b) Decreasing

(c) Neither increasing nor decreasing (d) None of these

48. For all real values of x, increasing function $f(x)$ is **[MP PET 1996]**

(a) x^{-1} (b) x^2 (c) x^3 (d) x^4

49. The function which is neither decreasing nor increasing in $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ is

(a) $\text{cosec}x$ (b) $\tan x$ (c) x^2 (d) $|x-1|$

50. For every value of x, function $f(x)=e^x$ is

(a) Decreasing (b) Increasing

(c) Neither increasing nor decreasing (d) None of these

51. Consider the following statements S and R

S : Both $\sin x$ and $\cos x$ are decreasing functions in $\left(\frac{\pi}{2},\pi\right)$

R : If a differentiable function decreases in (a, b) then its derivative also decrease in (a,b)

Which of the following is true

(a) Both S and R are wrong

(b) Both S and R are correct but R is not the correct explanation for S

(c) S is correct and R is the correct explanation for S

(d) S is correct and R is wrong

52. If $f'(x)$ is zero in the interval (a,b) then in this interval it is

(a) Increasing function (b) Decreasing function

(c) Only for $a>0$ and $b>0$ is increasing function (d) None of these

53. The function $\frac{x-2}{x+1}$, $(x \neq -1)$ is increasing on the interval

(a) $(-\infty, 0]$ (b) $[0, \infty)$ (c) R (d) None of these

54. If *f* and *g* are two decreasing functions such that *fog* exists, then *fog*

(a) Is an increasing function (b) Is a decreasing function

(c) Is neither increasing nor decreasing (d) None of these

55. The function $f(x) = \cos(\pi / x)$ is increasing in the interval

(a) $(2n+1, 2n), n \in N$ (b) $\left(\frac{1}{2n+1}, 2n\right)$, $n \in N$ (c) $\left(\frac{1}{2n+2}, \frac{1}{2n+1}\right)$, $n \in N$ (d) None of these

56. The set of all values of *a* for which the function $f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right) x^5 - 3x + \log 5$ decreases for all real *x* is

(a) $(-\infty, \infty)$ (b) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$ (c) $\left(-3, 5 - \frac{\sqrt{27}}{2}\right) \cup (2, \infty)$ (d) $[1, \infty)$

57. The function $f(x) = x\sqrt{ax - x^2}, a > 0$

(a) Increases on the interval $\left(0, \frac{3a}{4}\right)$ (b) Decreases on the interval $\left(\frac{3a}{4}, a\right)$

(c) Decreases on the interval $\left(0, \frac{3a}{4}\right)$ (d) Increases on the interval $\left(\frac{3a}{4}, a\right)$

58. The function $f(x) = \frac{|x-1|}{x^2}$ is monotonically decreasing on

(a) $(-2, \infty)$ (b) (0, 1) (c) (0, 1) $\cup (2, \infty)$ (d) $(-\infty, \infty)$

59. The set of values of *a* for which the function $f(x) = x^2 + ax + 1$ is an increasing function on [1, 2] is

(a) $(-2, \infty)$ (b) $[-4, \infty]$ (c) $[-\infty, -2)$ (d) $(-\infty, 2]$

60. On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing

(a) $\left(0, \frac{\pi}{2}\right)$ (b) (0, 1) (c) $\left(\frac{\pi}{2}, \pi\right)$ (d) None of these

61. If $a < 0$ the function $f(x) = e^{ax} + e^{-ax}$ is a monotonically decreasing function for values of *x* given by

(a) $x > 0$ (b) $x < 0$ (c) $x > 1$ (d) $x < 1$

62. $y = [x(x-3)]^2$ increases for all values of *x* lying in the interval

(a) $0 < x < \frac{3}{2}$ (b) $0 < x < \infty$ (c) $-\infty < x < 0$ (d) $1 < x < 3$

63. The function $f(x) = \frac{\log x}{x}$ is increasing in the interval **[EAMCET 1994]**

(a) $(1, 2e)$ (b) $(0, e)$ (c) $(2, 2e)$ (d) $\left(\frac{1}{e}, 2e\right)$

64. The value of *a* for which the function $f(x) = \sin x - \cos x - ax + b$ decreases for all real values of *x*, is given by

(a) $a \ge \sqrt{2}$ (b) $a \ge 1$ (c) $a < \sqrt{2}$ (d) $a < 1$

65. If the function $f(x) = \cos|x| - 2ax + b$ increases along the entire number scale, the range of values of *a* is given by

(a) $a \le b$ (b) $a = \frac{b}{2}$ (c) $a \le -\frac{1}{2}$ (d) $a \ge -\frac{3}{2}$

66. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \le 1$, then in this interval

(a) Both $f(x)$ and $g(x)$ are increasing functions (b) Both $f(x)$ and $g(x)$ are decreasing function

(c) $f(x)$ is an increasing function (d) $g(x)$ is an increasing function

67. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number *x*, then

(a) *h* is increasing whenever *f* is increasing and decreasing whenever *f* is decreasing

(b) *h* is increasing whenever *f* is decreasing

(c) *h* is decreasing whenever *f* is increasing

(d) Nothing can be said in general

68. If $f(x) = \begin{cases} 3x^2 + 12x - 1 & , \ -1 \le x \le 2 \\ 37 - x & , \ 2 < x \le 3 \end{cases}$ then $f(x)$ is **[IIT 1993]**

(a) Increasing in [–1, 2] (b) Continuous in [–1, 3] (c) Greatest at $x = 2$ (d) All of these

69. If $f'(x) = g(x)(x - \lambda)^2$ where $g(\lambda) \ne 0$ and $g(x)$ is continuous at $x = \lambda$ then function $f(x)$

(a) Increasing near to λ if $g(\lambda) > 0$ (b) Decreasing near to λ if $g(\lambda) > 0$

(c) Increasing near to λ if $g(\lambda) < 0$ (d) Increasing near to λ for every value of $g(\lambda)$

70. Function $\cos^2 x + \cos^2\left(\frac{\pi}{3} + x\right) - \cos x \cos\left(\frac{\pi}{3} + x\right)$ for all real values of *x* will be

(a) Increasing (b) Constant (c) Decreasing (d) None of these

71. Let $Q(x) = f(x) + f(1 - x)$ and $f''(x) < 0$ whereas $0 \le x \le 1$ then function $Q(x)$ is decreasing in

(a) $\left[\frac{1}{2}, 1\right]$ (b) $\left[0, \frac{1}{2}\right]$ (c) $\left(\frac{1}{2}, 1\right)$ (d) (0, 1)

72. If $f(x) = \frac{x}{c} + \frac{c}{x}$ for $-5 \le x \le 5$, then $f(x)$ is increasing function in the interval

(a) [c, 5] (b) [0, c] (c) [c, 0] (d) [c, c]

73. If the domain of $f(x) = \sin x$ is $D = \{x : 0 \le x \le \pi\}$, then $f(x)$ is

(a) Increasing in *D* (b) Decreasing in *D*

(c) Decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$ (d) None of these

74. If $f(x) = (ab - b^2 - 1)x - \int_0^x (\cos^4\theta + \sin^4\theta)d\theta$ is a decreasing function of *x* for all $x \in R$ and $b \in R$, *b* being independent of *x*, then

(a) $a \in (0, \sqrt{6})$ (b) $a \in (-\sqrt{6}, \sqrt{6})$ (c) $a \in (-\sqrt{6}, 0)$ (d) None of these

75. If $f(x) = \frac{p^2 - 1}{p^2 + 1}x^3 - 3x + \log 2$ is a decreasing function of *x* in *R* then the set of possible values of *p* (independent of *x*) is

(a) [-1, 1] (b) $[1, \infty)$ (c) $(-\infty, -1]$ (d) None of these

76. Let f $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x$, where $a_i^{'}$s are real and $f(x) = 0$ has a positive root α_0. Then

(a) $f'(x) = 0$ has a root α_1 such that $0 < \alpha_1 < \alpha_0$ (b) $f'(x) = 0$ has at least two real root

(c) $f''(x) = 0$ has at least one real roots (d) None of these

77. If *a, b, c* are real, then $f(x) = \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ ac & bc & x + c^2 \end{vmatrix}$ is decreasing in

(a) $\left(-\frac{2}{3}(a^2 + b^2 + c^2), 0\right)$ (b) $\left(0, \frac{2}{3}(a^2 + b^2 + c^2)\right)$ (c) $\left(\frac{a^2 + b^2 + c^2}{3}, 0\right)$ (d) None of these

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Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	a	c	a	c	d	d	a	c	d	a	c	a	a	b	d	c	d	b	a
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	b	d	a	c	c	c	a	c	b	c	c	a	d	b	c	c	a,d	a,d	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
d	b	c	a	c	b	a	c	a	b	d	d	b	a	d	b	a,b	c	a	d
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**			
b	a	b	a	c	c	a	d	a	b	a	a	d	b	a	a,b,c	a			