Wave Motion and Waves on a String

Exercise Solutions

Solution 1:
Given: A wave pulse passing on a string with a speed of 40 cm s^{-1} in the negative x-direction has its maximum at x = 0 at t = 0.

We know, V = x/t

or x = vt

=> x = 0.4 x 5 m =2 m

or x = 200 cm along the negative x-axis.

Solution 2:

(a) dimensions of A, a and T

[A] = [L]
[a] = [L] and
[T] = [T]

(b) Find the wave speed:
Wave speed = v = λ/T = a/T

(c)

The whole structure depends upon the exponent.

$$\text{Let } Y = -\left(\frac{x}{a}+\frac{t}{T}\right)^2 = -\left(\frac{1}{T^2}\right)\left(\frac{xT}{a}+t\right)^2$$

$$\text{or } Y = f\left(t+\frac{x}{V}\right)$$

Now,

Case 1: If $y = f(t-x/v)$, then wave is travelling in positive direction.

Case 2: If $y = f(t + x/v)$, $x/v)$, then wave is travelling in negative direction.

(d) wave speed = $v = a/T$

The maximum pulse at $t = T$ is $(a/T) \times T = a$ -> negative x-axis
and maximum pulse at $t = 2T$ is $(a/T) \times 2T = 2a$ -> along negative x-axis

So, the wave travels in negative x-axis direction.

Solution 3:

Using relation: $x = vt$

At t = 1 s the pulse will be at 10 cm.
At t = 2 s the pulse will be, x = 2(10) cm = 20 cm
At t = 3 s the pulse will be, x = 3(10)cm = 30 cm

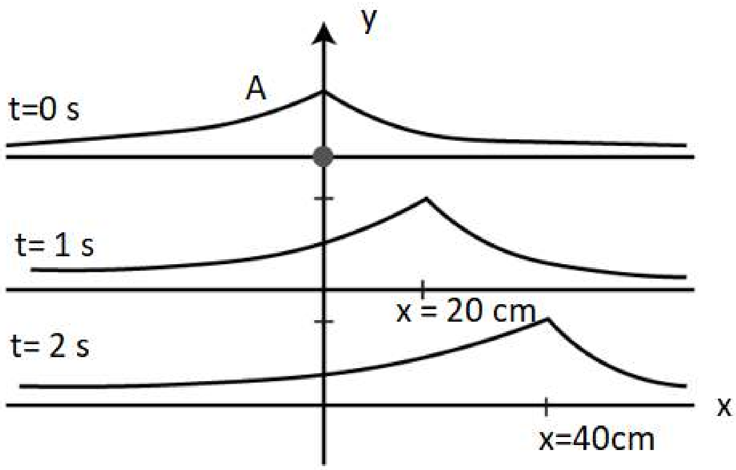
Solution 4:

$y = a^3/(x^2+a^2)$
For maximum, $dy/dx = 0$
=> $x = vt$
again $dx/dt = v$

At t = 0 s, x = 0 cm

At t = 1 s, x = 20 cm

At t = 2 s, x = 40 cm



Solution 5: AT x = 0, f(t) = a sin(t/T)

=> wavelength = λ = vT
So, general equation of wave,
Y = A sin((t/T) - (x/vT))

Solution 6:

(a) dimensions of A and a

[A]=[L] and [a]=[L]

(b) wave velocity is v (given). So, the time period will be,

T = λ/v

Here λ = a
=> T = a/v

Therefore, Y = sin(x/λ - t/T)

= A sin (x/a - vt/a)

= A sin [(x-vt)/a]

Solution 7:
The general equation:

$Y = A \sin(x/\lambda - t/T)$

Here $\lambda = a$ also $T = a/v$

$Y = A \sin(x/a - vt/a)$

=> $A \sin(x/a) = A \sin(x/a + vt_0/a)$

To sustain equality, the equation must be,

$Y = A \sin(x/a - vt/a + vt_0/a)$

Solution 8:
The equation of a wave travelling on a string:

$y = (0.10 \text{ mm}) \sin[(31.4 \text{ m}^{-1}) x + (314 \text{ s}^{-1}) t]$.

We know, The structure of the equation is $y = A \sin(kx + \omega t)$

(a) Negative x -direction.

(b) $k = 31.4 \text{ m}^{-1}$

=> $2\pi/\lambda = 3.14$

or $\lambda = 20$ cm

Again, $\omega = 314 \text{ s}^{-1}$

=> $2\pi f = 314$

=> $f = 50 \text{ sec}^{-1}$

Therefore, wave speed = $v = \lambda f = 20 \times 50 = 1000$ cm/s

(c) Max displacement = 0.10 mm
Max. velocity = $a\omega = (0.1) \times 10^{-1} \times 314 = 3.14$ cm/sec

Solution 9:
Here, $\lambda = 2$ cm, $V = 2.0$ m/s and $A = 0.20$ cm

(a) Equation of wave along the x-axis

$y = A \sin(kx - wt)$

$k = 2\pi/\lambda = \pi \text{ cm}^{-1}$

and $T = \lambda/v = 2/2000 = 10^{-3}$ sec

This implies, $\omega = 2\pi/T = 2\pi\ 10^{3}$ sec

So, the wave equation is,

$y = (0.2) \sin\pi\ x - (2\pi\ 10^{3})t$

(b) At $x = 2$ cm and $t = 0$

$y = (0.2)\sin(\pi/2) = 0$

Therefore, particle velocity, $v = r\omega \cos(\pi x) = 0.2 \times 2000\pi \times \cos 2\pi = 400\pi$

$= 400 \times 3.14$

$= 4\pi$ m/s

Solution 10:
(a) $T = 2\text{x}0.01$ sec $= 20$ min
$\lambda = 2\text{x}2 = 4$ cm

(b) $v = dy/dt = d/dt\ [\sin 2\pi(x/4 - t/0.02)]$

$= - \cos 2\pi\ (x/4 - t/0.02) \times 1/0.02$

$\Rightarrow v = -50 \cos 2\pi\ (x/4 - t/0.02)$

At $x = 1$ and $t = 0.01$ sec, $v = -50 \cos 2(1/4-1/2) = 0$

(c)
At $x = 3$ cm, $t = 0.01$ sec
$v = -50 \cos 2\pi(3/4 - 1/2) = 0$

At $x = 5$ cm, $t = 0.01$ sec, $v = 0$

At $x = 7$ cm and $t = 0.011$ sec, $v = 0$

At $x = 1$ cm, and $t = 0.011$ sec

$v = -50 \cos 2\pi[1/4 - (0.011/0.02)]$

$= -9.7$ cm/sec

Solution 11:
frequency of vibration = $f = 1/T = 50$ Hz

Any two neighbouring mean positions always remain at half of the wave length, $\lambda = 4$ cm

Now, wave speed = $v = \lambda f = 2$ m/s

Solution 12:

$V = 200$ m/s (given)

(a) amplitude = $A = 1$ mm

(b) the wavelength = $\lambda = 4$ cm

(c) the wave number = $n = 2\pi/\lambda = 1.57\ cm^{-1}$

(d) the frequency of the wave = $f = 1/T = (26/\lambda)/20 = 5$ Hz
Where $T = \lambda/v$

Solution 13:

(a) $v = \lambda/T$

or $\lambda = vt = 20$ cm

(b) Phase shift difference = $(2\pi/\lambda)x = 2\pi/20 \times 10 = \pi$ rad

$y_1 = a \sin(\omega t - kx)$

=> $1.5 = a \sin(\omega t - kx)$

the displacement of a particle at x = 10 cm

$y_2 = a \sin(\omega t - kx + \pi)$

=> $-a \sin(\omega t - kx) = -1.5$ mm

Therefore, displacement of a particle is -1.5 mm

Solution 14:
Mass = 5g, Length = 64cm and Force = 8 N (given)

So, density= $\rho = (5/64)$ g/cm

Now, $F = \rho v^2$

or $v^2 = 8 \times (64 \text{ cm}/5\text{g})$

or v = 32 m/s

Solution 15:

(a) velocity of wave = $v^2 = T/m = (16\times10^5)/0.4$

or v = 2000 cm/s

Time taken to reach other end = 20/2000 = 0.01 sec

Time taken to see the pulse again in the original position = 0.01 x 2 = 0.02 s

(b) At t = 0.01 s, there will be a though at the right end as it is reflected.

Solution 16:
(a) The distance travelled by the wave = 20+20 = 40 cm

time = t = x/v = 40/20 = 2 sec

(b) The string regains its original shape after completing a periodic distance i.e. (30+30) cm = 60cm.

time period = 60/20 = 3 sec

(c) frequency = n = (1/3) sec^{-1}

n = (1/2l)√(T/m)

m = mass per unit length = 0.5 g/cm

=> (1/3) = 1/(2x30) x √(T/0.5)

=> T = 2 x 10^{-3} N

Solution 17: Let v_1 and v_2 are the velocities of wires. ρ_1 and ρ_2 are the respective densities.

Therefore, $T_1 = T_2$

=> $\rho_1 v_1^2 = \rho_2 v_2^2$

or $\rho_1/\rho_2 = v_2^2/v_1^2$

Given that, $V_1 = 2V_2$

$\rho_1/\rho_2 = v_2^2/4v_2^2 = 1/4$

or $\rho_1/\rho_2 = 0.25$

Solution 18: A transverse wave described by
$y = (0.02 \text{ m}) \sin(1.0 \text{ m}^{-1}) x + (30 \text{ s}^{-1})t]$

Speed = $v = \omega/k$

here, $\omega = 30 \text{ sec}^{-1}$ and $k = 1 \text{ m}^{-1}$

=> $v = 30$ m/s

But $T = \rho v^2$

$= 1.2 \times 90 \times 10^{-4}$ N

or $T = 0.108$ N

Solution 19:
Amplitude = A = 1 cm, tension = T = 90N, frequency = f = 200/2 = 100 Hz and mass=m = 0.1 kg/m

(a)
$v = \sqrt{(T/\rho)}$

$= \sqrt{(90/0.1)} = 30$ m/s

Again, $v = \lambda f = 30$ cm

(b) $y = 10 \cos 2\pi[x/30 - t/0.01]$

At t = 0 and x = 0, it has maxima, consists a phase of $\pi/2$

=> $y = (1) \sin[2\pi x/30 - 2\pi t/0.01 + \pi/2]$

the required equation is, $y = (1) \cos[2\pi x/30 - 2\pi t/0.01]$

(c) The velocity of the particle,

$y' = -(1) (2\pi/0.01) \sin[2\pi x/30 - 2\pi t/0.01]$

At x = 50 cm and t = 10 s

$y' = -5.4$ m/s

Now the acceleration is :

$y'' = -(1)(2\pi/0.01)^2 \cos[2\pi x/30 - 2\pi t/0.01]$

At x = 50 cm and t = 10 s

$y'' = 2 \text{ km/s}^2$

Solution 20:

Here l = 40 cm , spring constant = k = 160 N/m and mass = 10 g

Mass per unit length = m = 10/40 = (1/4) g/cm

Now, deflection = x = 1 cm = 0.01 m

=> $T = kx = 1.6 \text{ N} = 16 \times 10^4$ dyne

Also, $v = \sqrt{(T/m)} = 8 \times 10^2$ cm/s = 800 cm/s

Therefore, time taken by the pulse to reach the spring:

t = 40/800 = 0.05 sec

Solution 21:

Force due to gravity on AB:

$T_{AB} = (3.2 + 3.2)9.8 \text{ N} = 62.72 \text{ N}$

Force due to gravity on CD:

$T_{AB} = 3.2 \times 9.8 = 31.36 \text{ N}$

The velocities are:

$v_{AB} = \sqrt{(T_{AB}/\rho_{AB})} = 79 \text{ m/s}$

and

$v_{CD} = \sqrt{(T_{CD}/\rho_{CD})} = 63 \text{ m/s}$

Solution 22:

Mass density = ρ = (0.0045/2.25) kg/m

force on the string = T = 20 N

Now, speed of the wave = v = $\sqrt{(T/\rho)}$ = 100 m/s

Therefore, time taken = t = 2.25/100 = 0.02 sec

Solution 23:

T = ma + mg = (4x2 + 4x10) = 48 N

And, speed = v = $\sqrt{(T/\rho)}$ = 50 m/s

Solution 24:
Tension = T = mg
Speed = $v_1 = \sqrt{(mg/\rho)}$

At motion:

Tension = T = $m\sqrt{(a^2+g^2)}$ and
Speed = $v_2 = \sqrt{[(m\sqrt{(a^2+g^2)})/\rho]}$

Now,

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{m\sqrt{a^2+g^2}}{\rho}}}{\sqrt{\frac{mg}{\rho}}}$$

$$\left(\frac{v_2}{v_1}\right)^4 = \frac{a^2+g^2}{g^2}$$

$$g^2A = a^2 + g^2\ ;\ \left(\frac{v_2}{v_1}\right)^4 = A$$

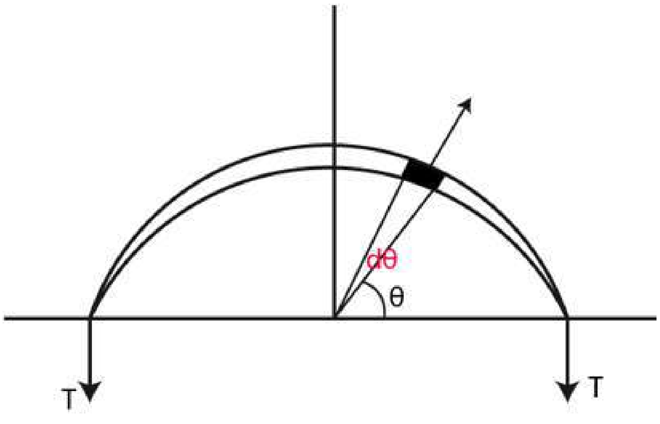
or $(A - 1)g^2 = a^2$

$a^2 = 0.140 \times 100$

or $a = 3.74\ m/s^2$

Solution 25:
R = Radius of the loop, m = mass per unit length of the string
ω = angular velocity, V = linear velocity of the string

The force for a small potion in the ring:
$dF = (mRd\theta)\omega^2 R$



$dF = 2 (mRd\theta)\omega^2 R \sin\theta$

$$F = \int_0^{\frac{\pi}{2}} 2(m\, Rd\theta)\omega^2 R \sin\theta$$

or $F = 2mR^2 \omega^2$

But whole of this process was for half of the ring:

$2T = 2mR^2 \omega^2$

Or $T = mR^2 \omega^2$

Now, velocity, $v = \sqrt{(T/m)} = R\omega$
Which is the speed of the disturbance.

Solution 26:
(a) Downward weight for the element = (mx)g = Tension in the string of upper part

velocity of transverse vibration = $v = \sqrt{(T/m)} = \sqrt{(mglm)} = \sqrt{gx}$

(b) For small displacement dx, $dt = dx/\sqrt{gx}$

Total time:

$$\int_0^T dt = \frac{1}{\sqrt{g}}\int_0^L \frac{dx}{\sqrt{x}}$$

= $\sqrt{(4L/g)}$

(c) Suppose, it will meet the pulse after y distance.
To get the in between time, we integrate,

$$\int_0^t dt = \frac{1}{\sqrt{g}}\int_0^y \frac{dx}{\sqrt{x}}$$

$$t = \sqrt{4\frac{y}{g}}$$

Therefore, distance travelled by the particle in this time is (L-y)

We know the relation, $S = ut + (1/2)gt^2$

=> $L – y(1/2)g \times \sqrt{(4y/g)}^2$ at u = 0
=> L – y = 2y
=> y = L/3, which shows that, the particle meet at distance L/3 from the lower end.

Solution 27:
Suppose v and v' are wave speeds in string A and B respectively.
T = 4.8 and m = 1.2×10^{-2} and T' = 7.5

Now,
v = $\sqrt{(T/m)}$ = 20 m/s and
v' = $\sqrt{(T'/m)}$ = 20 m/s = 25 m/s

t = 0 in string A,

t_1 = 0+20 ms = 20×10^{-3} = 0.02 sec

In 0.02 sec A has travelled 20 x 0.02 = 0.4 mt

relative speed between A and B = 25-20 = 5 m/s

Time taken for B for overtake A = s/v = 0.4/5 = 0.08 sec

Solution 28:

Average power of the source = P = $2\pi^2 mva^2f^2$...(1)

v = √(T/m) = 100 m/s and m = 0.01 kg/m
r = 0.5 x 10^{-3} and f = 100
(Given)

On substituting the values, (1)=> P = 49 mW

Solution 29:
Here A = 1 mm = 10^{-3} mA, m = 6 g/m = 6 x 10^{-3} kg/m
F = 200 Hz and T = 60 N

(a) Average power of the source = P = $2\pi^2 mvA^2f^2$ = 0.47 W
(b) Length of string = 2m
And t = 2/100 = 0.02 sec
So, Energy = $2\pi^2 mvtA^2f^2$ = 9.46 mJ

Solution 30: Given, m = 0.01 kg/m, T = 49N, r = 0.5 x 10^{-3} m and f = 440 Hz
(a) Let the wavelength of the wave be λ.

Speed of the transverse wave = v = √(T/m) = √(49/0.01) = 70 m/s

Also, v = f/λ

or λ = f/v = 70/440 = 16 cm

(b) y = A sin(ωt - kx)

Therefore, v = dy/dt = Aω cos(ωt - kx)

Now,
v_{max} = (dy/dt) = Aw

= (0.50) x 10^{-3} x 2π x 440

$= 1.381$ m/s

and

$a = d^2y/dt^2$

$\Rightarrow a = -A\omega^2 \sin(\omega t - kx)$

$a_{max} = -A\omega^2$

$= 0.50 \times 10^{-3} \times 4\pi^2 \times 440^2$

$= 3.8$ km/s^2

(c) Average rate = $p = 2\pi^2 vA^2f^2$

$= 2 \times 10 \times 0.01 \times 70 \times (0.50\times10^{-3})^2 \times 440^2$

$= 0.67$ W

Solution 31: Consider equation of waves:

$y' = r \sin \omega t$ and $y'' = r \sin(\omega t + \pi/2)$

Now, $y = y' + y''$

$$y = r[2(\sin\frac{2\omega t + \frac{\pi}{2}}{2})(\cos -\frac{\pi}{\omega})]$$

Or $y = \sqrt{2}\, r \sin(\omega t + \pi/4)$

The amplitude is $4\sqrt{2}$ mm

Solution 32:

Distance travelled by any classical wave = $s = vt$
At $t = 4 \Rightarrow s = 4 \times 10^{-3} \times 50 \times 10 = 2$ mm
At $t = 6 \Rightarrow s = 6 \times 10^{-3} \times 50 \times 10 = 3$ mm
At $t = 8 \Rightarrow s = 8 \times 10^{-3} \times 50 \times 10 = 4$ mm
At $t = 12 \Rightarrow s = 12 \times 10^{-3} \times 50 \times 10 = 6$ mm

Solution 33:

(a) Wave speed = wave length × wave frequency

$v = 100 \times 0.02 = 2$ m/s

In 0.015 sec, the path travelled by wave,
$x = 0.015 \times 2 = 0.03$ m

The phase difference:

$\phi = (2\pi x)/\lambda = (2\pi)/0.02 \times 0.03 = 3\pi$

(b) Again, for path difference, $x = 0.04$ m

$\phi = 4\pi$

(c) Two waves have same amplitude if their frequency and wavelength are same.

Now, consider two wave equations, $y' = r \sin \omega t$ and $y'' = r \sin (\omega t + \phi)$

=> $y = y' + y''$

$= 2r \sin (\omega t + \phi/2)\cos (\phi/2)$

Therefore, resultant amplitude, $A = 2r \cos(\phi/2)$

For $A = 0$, $\phi = 3\pi$

For $A = 4$, $\phi = 4\pi$

Solution 34:

Fundamental frequency = $f = v/2L = 30$ Hz

Solution 35:

Fundamental frequency = $f = v/2L = 1/2L \times \sqrt{(T/m)} = 1$ g/m

Solution 36:

Fundamental frequency = $f = v/2L = 1/2L \times \sqrt{(T/m)} = 62.5$ Hz

frequency of 4th harmonic = $F_4 = 4 \times 62.5 = 250$ Hz

Now, $v = F_4 \lambda_4$

or $\lambda_4 = 250/v = 40$ cm

Solution 37:

Fundamental frequency = $f = (1/2L)\sqrt{(T/m)}$

or $f = \sqrt{150\ T}$

Also, f = 261.63 Hz (given)

=> $261.63 = \sqrt{150\ T}$

=> T = 1480 N (approx.)

Solution 38:

Fundamental frequency of First Harmonic = 256/2 = 128 Hz
Here, Second harmonic= 2 × First Harmonic

When the fundamental wave is produced, $\lambda/2 = 1.5$

=> $\lambda = 3$ m

speed of the wave = $f \lambda = 128 \times 3 = 384$ m/s

Solution 39:
Mass of the wire = 12 gm
Length of the wire between two pulleys (L) = 1.5 m

so, Mass per unit length = m = (12/1.5) g/m = 8×10^{-3} kg/m

Tension in the wire = T = 9g = 90 N

Now, Fundamental frequency = f' = (1/2L) √(T/m)

Second harmonic = 2(First Harmonic)

=> f = 2 f' = (1/1.5) x √(90/8 x 10^-3)

= 70.7 Hz

Solution 40:

Using relation, L = nλ/2

Here n = 4 and L = 1 m

=> λ = 0.5

Also, v = fλ = √(T/m)

=> $T = f^2\lambda^2 m = 128^2 \times 0.5^2 \times 40 \times 10^{-3}$

= 164 N

Solution 41:

(a) Two frequencies are v_1 = 240 Hz and v_2 =320 Hz

So, Maximum fundamental frequency = $v_2 - v_1$
= 320 − 240 = 80 Hz

(b) Given v = 40 m/s

=> L x 80 = 0.5 x 40

=> L = 0.25 m

Solution 42:

Let n be the frequency, L is length of the string and λ be the distance between two consecutive nodes.

Therefore, $L = n\lambda$

for next higher frequency, say (n+1) the distance between two consecutive nodes is λ', then

$L = (n+1)\lambda'$

Equating Equations, we get

$n\lambda = (n+1)\lambda'$

or $\lambda' = n(\lambda - \lambda')$

Here $\lambda = 2$ cm and $\lambda' = 1.6$ cm

On putting values,

$n = 4$

$\Rightarrow L = 4 \times 2 = 8$ cm

Solution 43:

f = 660 Hz and v = 220 m/s

Wave length = $\lambda = v/f = 1/3$ m

(a) Number of loops = n =3

Therefore, $L = (n\lambda)/2 = (3/2) \times (1/3) = 1/2$ m = 50 cm

(b) resultant wave equation

$$y = 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi ft}{\lambda}$$

$$y = (0.5)\sin[(0.06\pi\ \text{cm}^{-1})x]\cos[(1320\pi\ \text{s}^{-1})t]$$

[On Substituting the values of λ,f and A]

Solution 44:

We know that, $f \propto 1/l$ or $f = v/l$ (where v = constant for a medium)

l_1 = 30 cm or 0.3 m (given)

f_1 = 196 Hz and f_2 = 220 Hz (given)

Now, $f_1/f_2 = l_2/l_1$

=> l_2 = 26.7 cm

Again, f_3 = 247 Hz

so, $f_3/f_1 = l_1/l_3 = 0.3/l_3$

=> l_3 = 0.224 m = 22.4 cm
in same way, we have l_4 = 2(

Solution 45:
Fundamental frequency = f_o =200 Hz
nth harmonic = f' = n × fundamental frequency

and frequency of the highest harmonic = 14 kHz = 14000 Hz

Now, f'/f_o = 14000/200

nf_o/f_o = 70

=> n = 70
The highest audible to man is 70th harmonic.

Solution 46:

(a) gcd of 90,150 and 210 is 30

So, f = 30 Hz

(b) Let f_1, f_2 and f_3 are Three resonant frequencies of the string.

$f_1 = 3f$, $f_2 = 5f$ and $f_3 = 7f$

(c) nth overtone is (n+1)th frequency.

So, 3f is 2nd overtone and 3rd harmonic.
5f is 4th overtone and 6th harmonic.
and 7f is 6th overtone and 7th harmonic.

(d) We know, $f_1 = (3/2)v$

so, 90 = (3/2x80) k

=> k = 48 m/s

Solution 47: The ratio of mass per unit length of the wires:

$p_1/\rho_2 \times r_1^2/r_2^2 = (1/2) \times (9/1) = 9/2$

Fundamental frequency of wire = $(1/2l)\sqrt{(T/\mu)}$

Thus, $f_1/f_2 = 2:3$

Solution 48:

We know, $f = (1/2L)\sqrt{(T/m)}$

Given, $f_d = 2f_2$

SO, $\sqrt{(T_1/T_2)} = 2$

Now, $T_1 + T_2 = 48 + 12 = 60$ N

By replacing the relations, $T_1 = 48$ N and $T_2 = 12$ N

Taking moment about a point, $T_2 \times 0.4 = 48x + 12(0.2)$
on solving above equation, we have x = 5 cm

Therefore, mass should be placed at a distance 5cm from the left end.

Solution 49:

Calculate the mass per unit length of aluminium and steel wire using given values.

We know, $\rho = M/V$

$M/l = \rho A$

Here, $m = M/l$

so, $m = \rho A$

For aluminium:

Put the value into the formula

$m_a = 2.6 \times 3 \times 10^{-2} = 7.8 \times 10^{-2}$ g/cm

For steel:

$m_s = 7.8 \times 10^{-2}$ g/cm

Now, $v = \sqrt{(T/m)}$

Here, T = 40 N and $m = 7.8 \times 10^{-2}$ g/cm

=> v = 71.6 m/s

For minimum frequency, there would be maximum wavelength. And, for maximum wavelength, minimum number of loops are to be produced.

Wavelength = $\lambda = 2d = 2 \times 20 = 40$ cm

The minimum frequency of a tuning fork :
$f = v/\lambda$
Given v = 71.6 m/s and $\lambda = 0.4$ m

=> f = 179 Hz

Solution 50:
Let L be the length of string.

Velocity of wave = v = $\sqrt{(T/m)}$

(a) wavelength = λ = velocity/frequency

=> $\lambda = \sqrt{(T/m)} \times 1/[(1/2L)\sqrt{(T/m)}] = 2L$

Now, wave number = $k = 2\pi/\lambda$

$= 2\pi/2L = \pi/L$

(b) Equation of the stationary wave:

$y = A \cos (2\pi x/\lambda) \sin (2\pi Vt/\lambda)$

As, $v = V/2L$

=> $y = A \cos (\pi x/L) \sin (2\pi/vt)$

Solution 51:
(a) Vibrating in first overtone means, n=2

We know, $L = n\lambda/2$
here, $\lambda = L = 2$ m

Again, $f = v/\lambda = 100$ Hz

(b) Suppose, the stationary wave equation:

$y = 2A \cos (2\pi x/\lambda) \sin(2\pi vt/\lambda)$

$= 0.5 \cos(2\pi x/2) \sin(2\pi(200)t/\lambda)$

$= 0.5 \cos[(\pi m^{-1})x] \sin[(200)\pi s^{-1}t]$

Solution 52:

The stationary wave equation

$y = (0.4 \text{ cm}) \sin[(0.314 \text{ cm}^{-1})x] \cos[(600\pi \text{ s}^{-1})t]$

(a)frequency of vibration:

$\omega = 600\pi$

So, $2\pi f = 600\pi$

=> $f = 300$ Hz

(b) positions of the nodes:

$\sin(0.314x) = 0$

=> $0.314x = k\pi = k \times 3.14$

=> $x = 10$ K

Nodes are at 0, 10 cm, 20 cm and 30 cm.

(c) length of the string:

Length = $3\lambda/2 = 3 \times 10 = 30$ cm

(d) the wavelength and the speed of two travelling waves that can interface to give this vibration

wave equation = $y = (0.4 \text{ cm}) \sin[0.314\, x] \cos(600\pi t]$

=> $\lambda =$ 20 cm

So, $v = \omega/k$ =6000 m/s = 60 m/s

Solution 53: Equation of standing wave

$y = (0.4\ cm)\ sin(0.314\ cm^{-1})x]\ cos[(600\pi\ s^{-1})t]$.

Here, $K = 0.314 = \pi/10$

We know, $\lambda = 2\pi/K = 20$ cm

Now, $L = (n\lambda/2)$

For smallest length, n = 1

=> L = 20/2 = 10 cm

Solution 54:

Strain = $\Delta l/l = 0.125 \times 10^{-2}$ and $f = 1/2L\ \sqrt{(T/m)}$

=> T = 248.19 N

Now, stress = Tension/Area = 248.19×10^{6}

Therefore, Young modulus = stress/strain = 19852×10^{8} N/m^{2}