8. Area of Quadrilaterals

Questions Pg-149

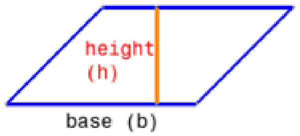
1. Question

Draw a parallelogra of sides 5 centimetres, 6 centimetres and area 25 square centimetres and area 25 square centimetres.

Answer

Area of the parallelogram:

The product of base of the parallelogram and the height of the parallelogram.



∴ area of the parallelogram = base × height

= b × h

Given that

Sides of the parallelogram are 5cm & 6cm

Area of the parallelogram = 25 cm^2

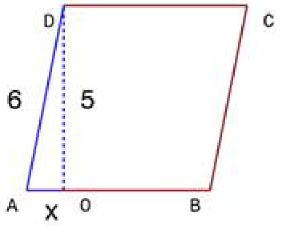
We can write that as

Area = 5 cm × 5 cm

That means base of the parallelogram (b) = 5 cm

height of the parallelogram (h) = 5 cm

With height and base and side we will draw a parallelogram.



We need to find the value of X

In the above fig. triangle AOD is a right- angled triangle with AD as hypotenuse.

∴ $AD^2 = A0^2 + OD^2$

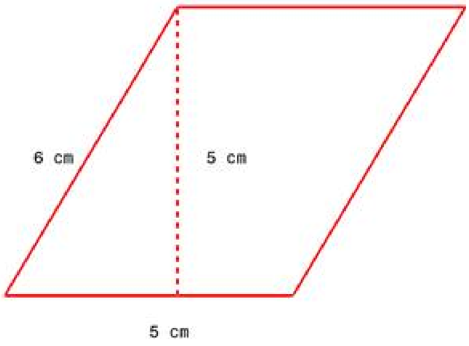
$6^2 = X^2 + 5^2$

$X^2 = 36 – 25$

$X = \sqrt{9}$

X = 3 cm

Using height, sides & X we can draw the parallelogram.



2. Question

Draw a parallelogram of area 25 square centimetres and perimeter 24 centimetres.

Answer

Given

Area = 25 cm^2

Area = 5 cm × 5 cm

That means base of the parallelogram (b) = 5 cm

height of the parallelogram (h) = 5 cm

Perimeter = 24 cm

2 (A + B) = 24 cm

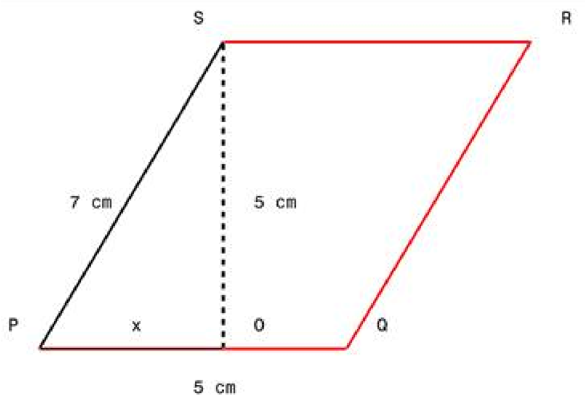
A + B = 12 cm

where A & B are sides of a parallelogram

since base is also a side of a parallelogram.

∴ B = 7 cm

With height and base and side we will draw a parallelogram.



We need to find the value of X

In the above fig. triangle POS is a right- angled triangle with PS as hypotenuse.

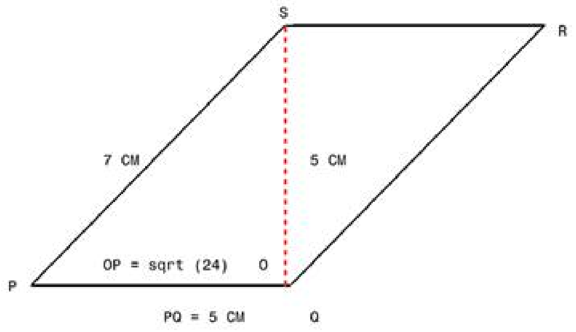
$PS^2 = PO^2 + OS^2$

$7^2 = X^2 + 5^2$

X = √(49-25)

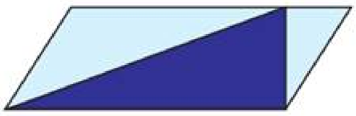
= √24

Using height, sides & X we can draw the parallelogram.



3. Question

In the figure, the two bottom corners of a parallelogram are joined to a point on the top side.



The area of the dark triangle in the figure is 5 square centimeters. What is the area of the parallelogram?

Answer

From the given picture we can see that a triangle is drawn inside the parallelogram.

The area of the triangle is 5 cm^2

We can see that the base and height of both the triangle and parallelogram is same

The area of the triangle = $\frac{1}{2}$ base × height

5 = $\frac{1}{2}$(base × height)

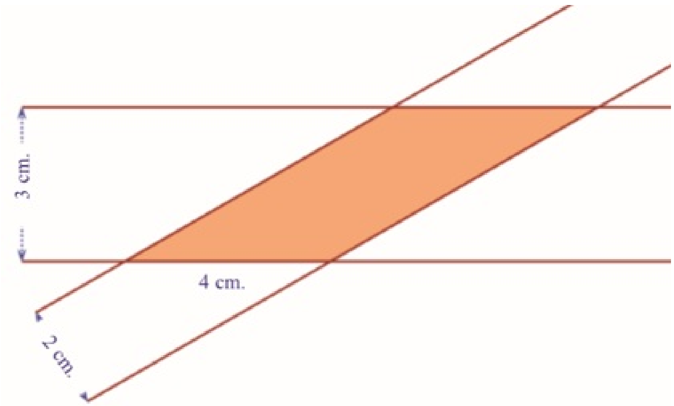
10 cm^2 = base × height

The area of the parallelogram = base × height

= 10 cm^2

4. Question

The picture below shows the parallelogram formed by the intersection of two pairs of parallel lines?



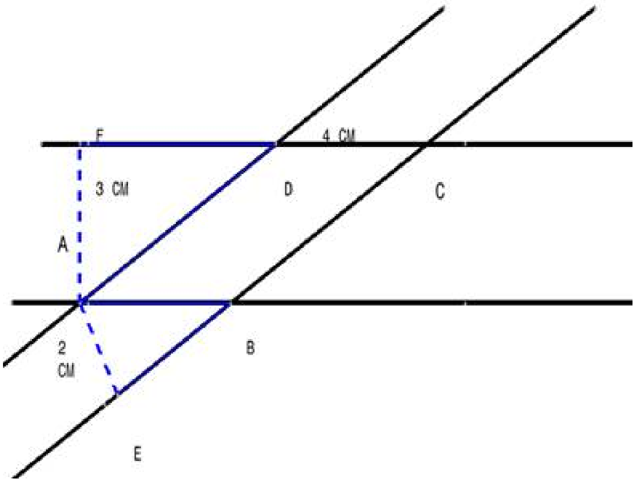
What is the area of this parallelogram? And the perimeter?

Answer

Let ABCD is a parallelogram is formed by two set of parallel lines.

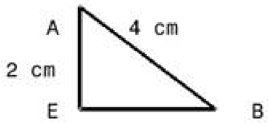
The distance between the two horizontal lines is 3 cm,

The distance between the vertically slanted lines is 2 cm.



Connect the two vertically slanted lines a right-angle triangle is formed.

From the right-angle triangle ABE



AB is the hypotenuse

AE is the opposite side

EB is the Adjacent side to θ

Sin θ = $\frac{\text{Opposite Side}}{\text{Hypotenus}} = \frac{2}{4}$

∴ θ = 30^o

∴ ∠ ABE is 30^o

From the fig. we can say that ∠ C = ∠ ABE = 30^o

In parallelogram opposite sides are equal

∴ ∠ BAD = ∠ C

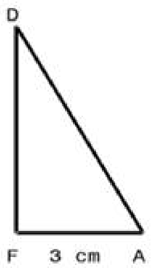
∠ BAD = 30^o

∵ ∠ BAF = 90^o

∠ BAF = ∠ BAD + ∠ DAF

$90^o = 30^o$ + ∠ DAF

∠ DAF = 60^o



From the diagram cos θ = $\frac{AF}{DA}$

COS $60^o = \frac{3}{DA}$

DA = 6 cm

We know that AD, AB are the sides of the parallelogram

Perimeter = 2 (AB + AD)

= 2 (4+6)

= 20 CM

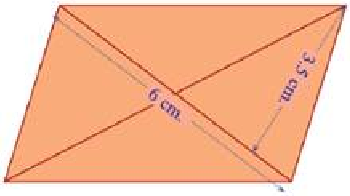
Area = side × perpendicular distance to the opposite side

= AB × AF

= 4 × 3 = 12 cm^2

5. Question

Compute the area of the parallelogram below:



Answer

Height of the triangle = 3.5 cm

Base of the triangle = 6 cm

The area of the triangle = $\frac{1}{2}$ base × height

= $\frac{1}{2}$ 6 × 3.5

= 10.5 cm^2

From the fig. we can say that the parallelogram is constituted of 2 triangles

∴ Area of the parallelogram = 2 × Area of triangle

= 2 × 10.5

= 21 cm^2

Questions Pg-153

1. Question

Draw a square of area $4\frac{1}{2}$ square centimetres.

Answer

Given area of the square = 4.$\frac{1}{2}$ cm^2

We know that area of the square = $\frac{1}{2}$ × d^2

d is the diagonal of the square

$\frac{1}{2}$ × d^2 = 4.$\frac{1}{2}$

d^2 = 9

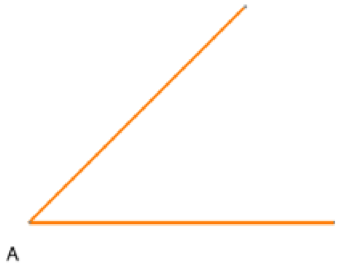
d = 3 cm

diagonal of the square = 3 cm

step – 1:

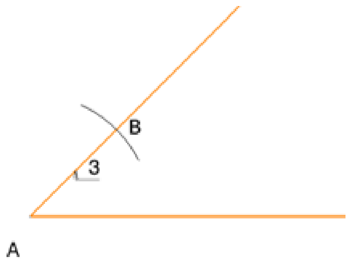
Draw the a horizonal line with the starting point as A and draw 45^{0} ray from point A.

(It is a square so, the angle between side and diagonal will be 90^{0})



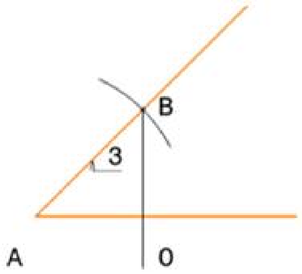
Step – 2:

We know the length of diagonal, so cut the ray with 3 cm arc



Step – 3:

Connect the intersecting point with horizontal line and name the intersection point as O



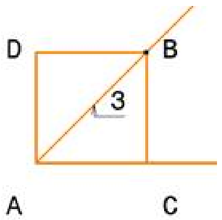
The above fig. forms a right-angle triangle

Sin $45^{0} = \frac{BO}{AB}$

$$\frac{1}{\sqrt{2}} = \frac{BO}{3}$$

BO = $\frac{3}{\sqrt{2}}$

In square all sides are equal we can draw rest of the fig. with the same measurement.



Thus, a square can be drawn when its area is given.

2. Question

Draw a non-square rhombus of area 9 square centimetres.

Answer

Area of the rhombus = 9

Area of the rhombus = $\frac{1}{2} \times d_1 \times d_2$

$\frac{1}{2} \times d_1 \times d_2 = 9$

$d_1 \times d_2 = 18$

we can divide the diagonal like this.

$d_1 \times d_2 = 6 \times 3$

(you can also write it as 3 × 6 or 9 × 2)

∴ diagonal (d_1) = 6 cm

diagonal (d_2) = 3 cm

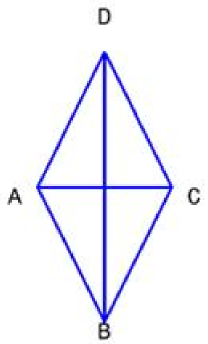
with these data we can draw a non- square rhombus.

Firstly, draw a diagonal and draw another diagonal passing through its mid points.

Note: the midpoint for both the diagonals should be same.

Connect all the adjacent points

Thus, a rhombus will be formed



3. Question

The area of a rhombus is 216 square centimetres and the length of one of its diagonals is 24 centimetres. Compute the following measurements of this rhombus.

i) Length of the second diagonal

ii) Length of a side

iii) Perimeter

iv) Distance between sides

Answer

Length of a diagonal (d_1) = 24 cm

Area of the rhombus = 216 cm^2

Area of the rhombus = $\frac{1}{2} \times d_1 \times d_2$

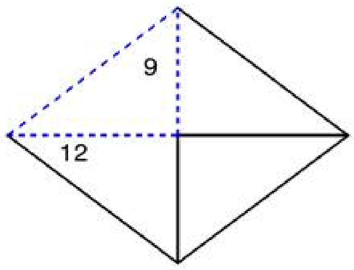
$\frac{1}{2} \times d_1 \times d_2 = 216$

$d_1 \times d_2 = 432$

$24 \times d_2 = 432$

$d_2 = 18$ cm

(i) length of another diagonal = 18 cm



We can see that some part of the rhombus constitutes a right-angle triangle.

From that

$H^2 = 9^2 + 12^2$

$H = \sqrt{225}$

= 15

(ii)∴ side of a rhombus is 15 cm

We know that sides are equal in a rhombus

(iii) So, perimeter = 4 × sides

= 4 × 15

= 60 cm

(iv) In rhombus distance between the side is also a side

∴ distance between the side is 15 cm

4. Question

A 68-centimeter-long rope is used to make a rhombus on the ground. The distance between a pair of opposite corners is 16 meters.

i) What is the distance between the other two corners?

ii) What is the area of the ground bounded by the rope?

Answer

68-centimeter-long rope is used to make a rhombus

The above statement states that perimeter of the rhombus is 68 cm.

And they have given that diagonal (d_1) is 16 cm

(In question they have given that as meter that should be a meter)

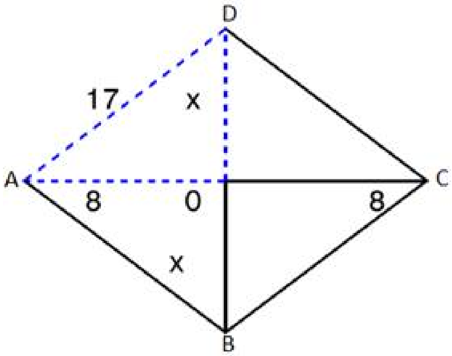
Perimeter = 4 × a

Where,

a = side of the rhombus.

68 = 4a

a = 17 cm



From the triangle AOD

$AD^2 = AO^2 + OD^2$

$17^2 = 8^2 + X^2$

$X = \sqrt{(289-64)}$

= 15

(i) The distance between the other two corners (d_2) = 2X

= 2 × 15

= 30 cm

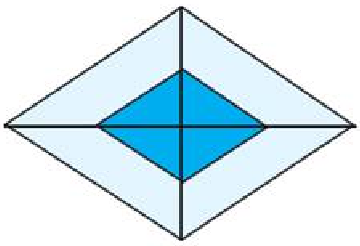
(ii)The area of the rhombus $= \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$

$= \frac{1}{2} \times \frac{16}{2} \times \frac{30}{2}$

= 60 cm^2

5. Question

In the figure, the midpoints of the diagonals of a rhombus are joined to form a small quadrilateral:

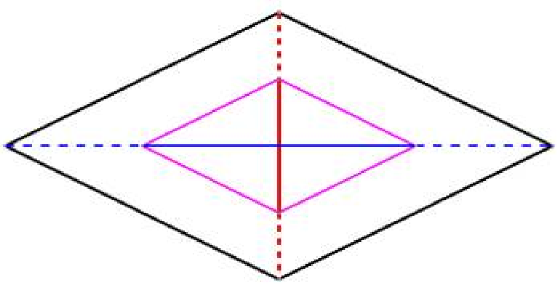


(i) Prove that this quadrilateral is a rhombus.

(ii) The area of the small rhombus is 3 square centimetres. What is the area of the large rhombus?

Answer

(i) Given that the small quadrilateral is formed by the midpoints of the diagonals of a rhombus.



In the small one the diagonals cut the other to the half of it.

So, this is a rhombus.

(ii) The area of the small rhombus = 3 cm^2

$\frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2} = 3$

Let, d_1, d_2 are the diagonals of smaller rhombus

d_3, d_4 are the diagonals of larger rhombus

$d_1 \times d_2 = 24$

$d_1 \times d_2 = 4 \times 6$

$d_1 = 4$ cm

$d_2 = 6$ cm

They have given that diagonals of smaller rhombus is formed by midpoints of diagonals of larger rhombus.

$d_3 = 2\ d_1 = 2\ (4) = 8$ cm

$d_4 = 2\ d_2 = 2(6) = 12$ cm

∴ the area of larger rhombus $= \frac{1}{2} \times \frac{d_3}{2} \times \frac{d_4}{2}$

$= \frac{1}{2} \times \frac{8}{2} \times \frac{12}{2}$

= 12 cm^2

6. Question

What is the area of the largest rhombus that can be drawn inside a rectangle of sides 6 centimetres and 4 centimetres?

Answer

We have to draw largest rhombus inscribed in rectangle.

Rectangle breadth (b) = 6cm

Height (h) = 4 cm

We know that in rhombus is directly proportional to the diagonals of it.

In a rectangle we can't draw diagonals more than its height and breadth.

So, the largest diagonals are height, breadth of rectangle

∴ the area of rhombus $= \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$

$= \frac{1}{2} \times \frac{b}{2} \times \frac{h}{2}$

$= \frac{1}{2} \times \frac{6}{2} \times \frac{4}{2}$

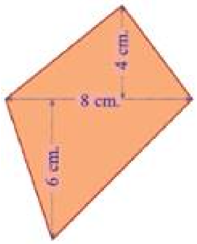
= 3 cm^2

A rhombus with area of 3 is largest possible in rectangle 6 × 4

Questions Pg-160

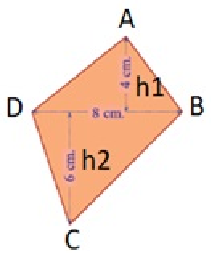
1. Question

What is the area of the quadrilateral shown below?



Answer

As shown in the figure below:



Area of the quadrilateral = area of ADB + area of DCB

$= \frac{1}{2} \times DB \times h_1 + \frac{1}{2} \times DB \times h_2$

(taking $\frac{1}{2} \times DB$ common)

$= \frac{1}{2} \times DB \times (h_1 + h_2)$

$= \frac{1}{2} \times 8 \times (4 + 6)$

$= 40\ sq.cm$

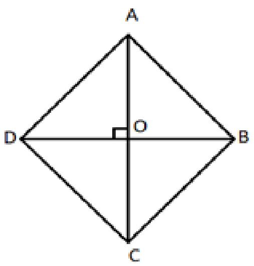
Hence, area of the quadrilateral is 40 square centimetres.

2. Question

Prove that for any quadrilateral with diagonals perpendicular, the area is half the product of the diagonals.

Answer

For this we choose a quadrilateral as shown below:



Clearly, diagonals of the quadrilateral are AC and BD, which are

perpendicular to each other.

Sum of areas of triangles ABD and CBD

Area of quadrilateral =

$= \frac{1}{2} \times DB \times OA + \frac{1}{2} \times DB \times OC$

$= \frac{1}{2} \times DB \times (OA + OC)$

OA+OC=AC

$= \frac{1}{2} \times DB \times AC$

Now,

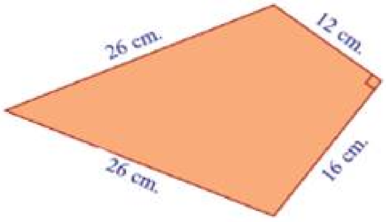
Since, DB × AC = product of diagonals

Therefore, Area of quadrilateral $= \frac{1}{2} \times \text{product of diagonals}$

Hence proved.

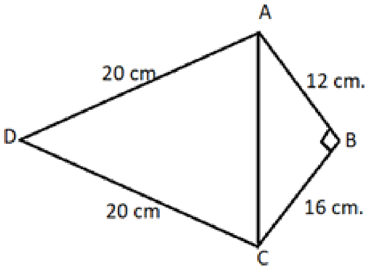
3. Question

Compute the area of the quadrilateral shown below:



Answer

More elaborated diagram is as shown below:



Area of quadrilateral = area(ABC) + area(ACD) .....(1)

Triangle ABC is a right triangle.

Therefore, area(ABC) = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 16 \times 12$

$= 96\ \text{sq.cm}$

Also, using Pythagoras theorem, we have:

$AB^2 + BC^2 = AC^2$

$12^2 + 16^2 = AC^2$

$\therefore AC = 20$

Now we find that all sides of triangle ACD are equal.

i.e. AC = CD = AD = 20 cm

Therefore, triangle ACD is an equilateral triangle.

∴ Area of equilateral triangle ACD $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (20)^2$

= 173.2 sq. cm

Therefore, from (1), we get:

Area of quadrilateral = area(ABC) + area(ACD)

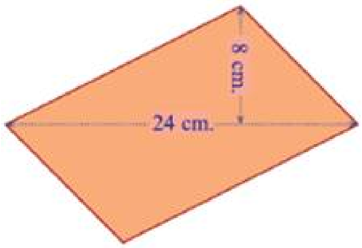
= 96 sq. cm+ 173.2 sq. cm

= 269.2 sq. cm

Hence, area of the quadrilateral is 269.2 square centimetres.

4. Question

Compute the area of the parallelogram shown below:



Answer

We know that a diagonal of a parallelogram divides it into two triangles of equal area.

Therefore, Area of quadrilateral = $2 \times (\text{area of one of the triangle})$

$= 2 \times (\frac{1}{2} \times \text{base} \times \text{height})$

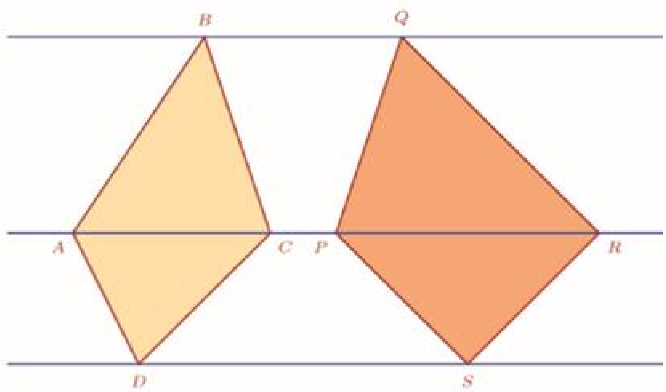
$= 2 \times (\frac{1}{2} \times 24 \times 8)$

= 192 sq. cm

Hence, area of the parallelogram is 192 square centimetres.

4. Question

The three blue lines in the picture below are parallel:



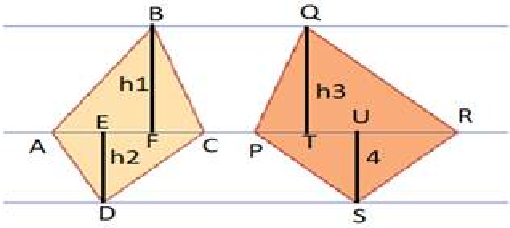
Prove that the areas of the quadrilaterals ABCD and PQRS are in the ratio of the lengths of the diagonals AC and PR.

(i) How should the diagonals be related for the quadrilaterals to have equal area?

(ii) Draw two quadrilaterals, neither parallelograms nor trapeziums, of area 15 square centimetres.

Answer

From the figure below:



Area(ABCD) = area(ACD) + area(ABC)

$= \frac{1}{2} \times AC \times h_2 + \frac{1}{2} \times AC \times h_1$

$= \frac{1}{2} \times AC \times (h_1 + h_2)$ ......(1)

Again, Area(PQRS) = area(PRS) + area(PQR)

$= \frac{1}{2} \times PR \times h_4 + \frac{1}{2} \times PR \times h_3$

$= \frac{1}{2} \times PR \times (h_3 + h_4)$ ......(2)

Also, we know that the perpendicular distance between two parallel line are same everywhere. So, we have:

$h_1 + h_2 = h_3 + h_4$ ......(3)

Now, using (3) and taking ratio of (1) and (2), we get:

$$\frac{\text{Area(ABCD)}}{\text{Area(PQRS)}} = \frac{\frac{1}{2} \times AC \times (h_1 + h_2)}{\frac{1}{2} \times PR \times (h_3 + h_4)}$$

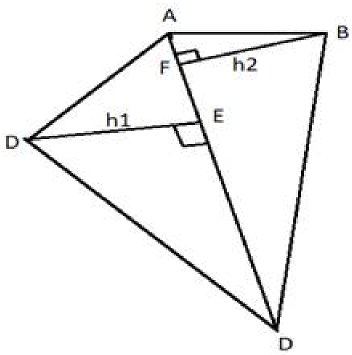
$\frac{\text{Area(ABCD)}}{\text{Area(PQRS)}} = \frac{AC}{PR}$ ......(4)

Thus, we see that areas of quadrilateral area in the ratio of the lengths of the diagonals AC and PR. Hence proved.

(i) For the condition of quadrilaterals to have equal area the ratio in (4) should be one. $\therefore 1 = \frac{AC}{PR} AC = PR$

Therefore, the diagonals should be equal in when the areas of the two quadrilaterals are equal.

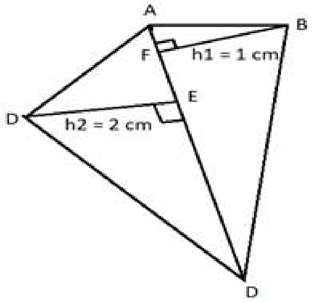
(ii) For this we draw a random quadrilateral which is neither a parallelogram nor trapezium, as shown in figure below:



Area of the above quadrilateral can be calculated to be

Area $= \frac{1}{2} \times AD(h_1 + h_2)$

Case 1: we take AD = 10 cm, $h1$ = 1 cm and $h2$ = 2 cm so that area of quadrilateral becomes 15 square centimetres, as shown:

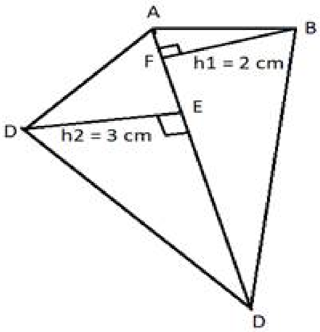


Area $= \frac{1}{2} \times \mathrm{AD}(\mathrm{h1} + \mathrm{h2})$

$= \frac{1}{2} \times 10 \times (1 + 2)$

$= 15\ \mathrm{sq.cm}$

Case 2: we take AD = 6 cm, $\mathrm{h1}$ = 2 cm and $\mathrm{h2}$ = 3 cm so that area of quadrilateral becomes 15 square centimetres, as shown:



Area $= \frac{1}{2} \times \mathrm{AD}(\mathrm{h}_1 + \mathrm{h}_2)$

$= \frac{1}{2} \times 6 \times (2 + 3)$

$= 15\ \mathrm{sq.cm}$