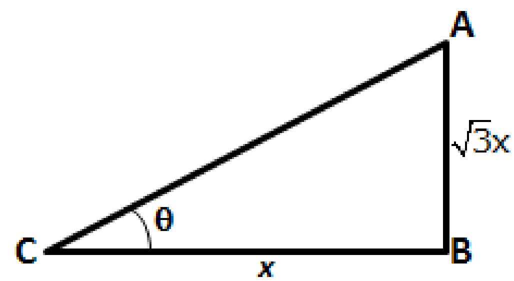
# Heights and Distances

## Exercise 22 A

**Question 1.**
The height of a tree is √3 times the length of its shadow. Find the angle of elevation of the sun.

**Solution:**




Let the length of the shadow of the tree be x m.

∴ Height of the tree = $\sqrt{3} \times$ m

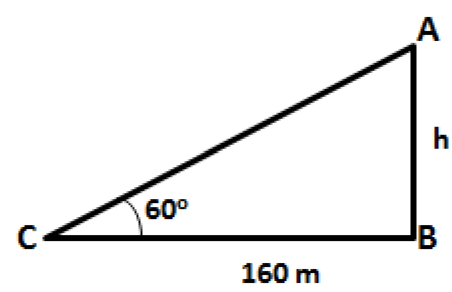
If θ is the angle of elevation of the sun, then

$$\tan\theta = \frac{\sqrt{3}\,x}{x} = \sqrt{3} = \tan 60^0$$

$$\therefore \quad \theta = 60^0$$

**Question 2.**
The angle of elevation of the top of a tower from a point on the ground and at a distance of 160 m from its foot, is found to be 60°. Find the height of the tower.

**Solution:**



Let the height of the tower be h m.
Given that angle of elevation is $60^{o}$

$$\tan 60^{o} = \frac{h}{160}$$

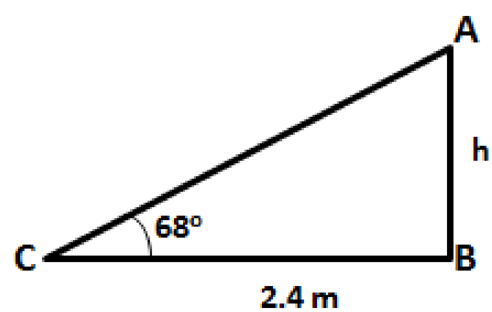
$$\Rightarrow \sqrt{3} = \frac{h}{160}$$

$$\therefore \quad h = 160\sqrt{3} = 277.12 \text{ m}$$

So, height of the tower is 277.12 m.

**Question 3.**
A ladder is placed along a wall such that its upper end is resting against a vertical wall. The foot of the ladder is 2.4 m from the wall and the ladder is making an angle of $68^{o}$ with the ground. Find the height, upto which the ladder reaches.

**Solution:**




Let the height upto which the ladder reaches be h m.
Given that angle of elevation is $68^{o}$

$$\tan 68^{o} = \frac{h}{2.4}$$

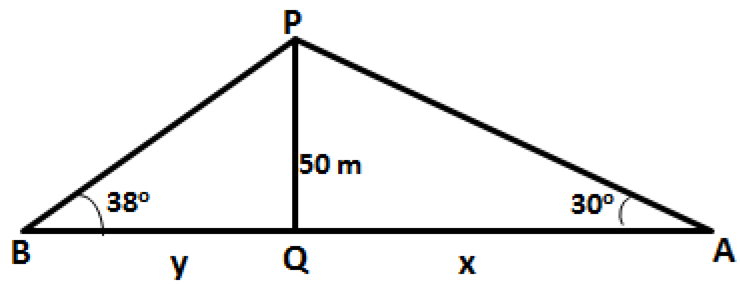
$$\Rightarrow 2.475 = \frac{h}{2.4}$$

$$\therefore \quad h = 2.475 \times 2.4 = 5.94 \text{ m}$$

So, the ladder reaches upto a height of 5.94 m.

**Question 4.**
Two persons are standing on the opposite sides of a tower. They observe the angles of elevation of the top of the tower to be $30^{o}$ and $38^{o}$ respectively. Find the distance between them, if the height of the tower is 50 m.

**Solution:**




Let one person A be at a distance x and the second person B be at a distance of y from the foot of the tower.
Given that angle of elevation of A is $30^0$

$$\tan 30^0 = \frac{50}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$$

$$\therefore \quad x = 50\sqrt{3} = 86.60 \text{ m}$$

The angle of elevation of B is $38^0$

$$\tan 38^0 = \frac{50}{y}$$

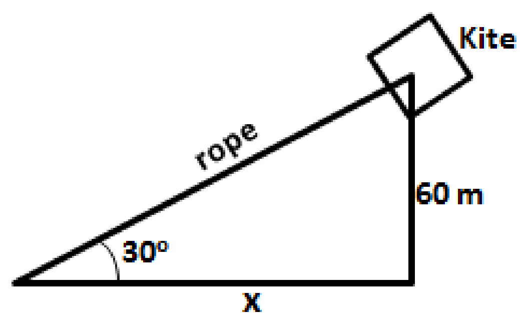
$$\Rightarrow 0.7813 = \frac{50}{y}$$

$$\therefore \quad y \approx 64 \text{ m}$$

So, distance between A and B is x + y = 150.6 m

**Question 5.**
A kite is attached to a string. Find the length of the string, when the height of the kite is 60 m and the string makes an angle 30° with the ground.

**Solution:**




Let the length of the rope be x m.
Now,

$$\sin 30^{o} = \frac{60}{x}$$

$$\Rightarrow \frac{1}{2} = \frac{60}{x}$$

$$\therefore \quad x = 120 \text{ m}$$

So, the length of the rope is 120m.

**Question 6.**
A boy, 1.6 m tall, is 20 m away from a tower and observes the angle of elevation of the top of the tower to be (i) 45°, (ii) 60°. Find the height of the tower in each case.

**Solution:**

Let the height of the tower be h m.

(i) Here $\theta = 45^{o}$

$$\therefore \quad \tan 45^{o} = \frac{h-1.6}{20}$$

$$\Rightarrow \quad 1 = \frac{h-1.6}{20}$$

$$\therefore \quad h = 21.6 \text{ m}$$

So, height of the tower is 21.6 m.

(ii) Here $\theta = 60^{o}$

$$\therefore \quad \tan 60^{o} = \frac{h-1.6}{20}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h-1.6}{20}$$

$$\therefore \quad h = 20 \times 1.732 + 1.6 = 36.24 \text{ m}$$

So, height of the tower is 36.24 m.

**Question 7.**
The upper part of a tree, broken over by the wind, makes an angle of 45° with the ground and the distance from the root to the point where the top of the tree touches the ground is 15 m. What was the height of the tree before it was broken?

**Solution:**

Let the height of the tree after breaking be h m.

Here $\theta = 45^0$

$$\therefore \quad \tan 45^0 = \frac{h}{15}$$

$$\Rightarrow \quad 1 = \frac{h}{15}$$

$$\therefore \quad h = 15 \text{ m}$$

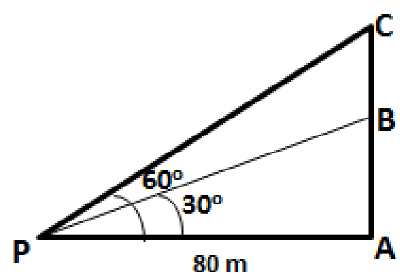
Now, length of the tree broken by the wind = $\dfrac{15}{\sin 45^0} = 15\sqrt{2} = 21.21 \text{ m}$

So, height of the tree before it was broken is (15 + 21.21) m = 36.21 m.

**Question 8.**

The angle of elevation of the top of an unfinished tower at a point distance 80 m from its base is 30°. How much higher must the tower be raised so that its angle of elevation at the same point may be 60°?

**Solution:**




Let AB be the unfinished tower and C be the top of the tower when finished. Let P be a point 80 m from the foot A.
In $\Delta$ BAP,

$$\tan 30^0 = \frac{AB}{AP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{80}$$

$$\Rightarrow \quad AB = \frac{80}{\sqrt{3}} = 46.19 \text{m}$$

In $\Delta$ CAP,

$$\tan 60^0 = \frac{AC}{AP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AC}{80}$$

$$\Rightarrow \quad AC = 80\sqrt{3} = 138.56 \text{ m}$$

Therefore, the tower must be raised by (138.56 - 46.19)m = 92.37 m

**Question 9.**
At a particular time, when the sun's altitude is 30°, the length of the shadow of a vertical tower is 45 m. Calculate
(i) the length of the tower.
(ii) the length of the shadow of the same tower, when the sun's altitude is
(a) 45° (b) 60°

**Solution:**

Let the length of the tower be h m.
(i) Here $\theta = 30^0$

$$\tan 30^0 = \frac{h}{45}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{45}$$

$$\Rightarrow \quad h = 25.98 \text{ m}$$

Hence the length of the tower is 25.98 m.
(ii) Let the length of the shadow be x m.
(a) Here, $\theta = 45^0$

$$\tan 45^0 = \frac{25.98}{x}$$

$$\Rightarrow \quad 1 = \frac{25.98}{x}$$

$$\Rightarrow \quad x = 25.98 \text{ m}$$

Hence the length of the shadow is 25.98 m
(b) Here, $\theta = 60^0$

$$\tan 60^0 = \frac{25.98}{x}$$

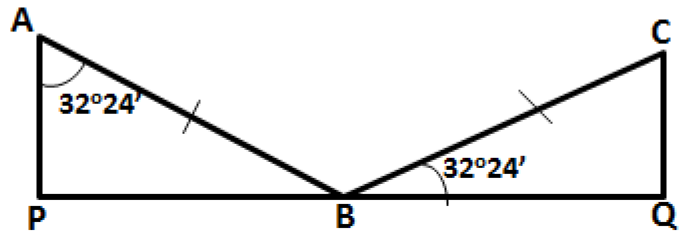
$$\Rightarrow \quad \sqrt{3} = \frac{25.98}{x}$$

$$\Rightarrow \quad x = \frac{25.98}{\sqrt{3}} = 15 \text{ m}$$

Hence the length of the shadow is 15 m.

**Question 10.**
Two vertical poles are on either side of a road. A 30 m long ladder is placed between the two poles. When the ladder rests against one pole, it makes angle 32°24' with the pole and when it is turned to rest against another pole, it makes angle 32°24' with the road. Calculate the width of the road.

**Solution:**




Let AB be the ladder and $\angle ABP = 32^o24'$.

$\therefore \quad \frac{BP}{AB} = \sin 32^o24'$

$\Rightarrow \quad BP = 30 \times 0.536 = 16.08$

When rotated, let the ladder be AC and $\angle CAQ = 32^o24'$.

$\therefore \quad \frac{BQ}{BC} = \cos 32^o24'$

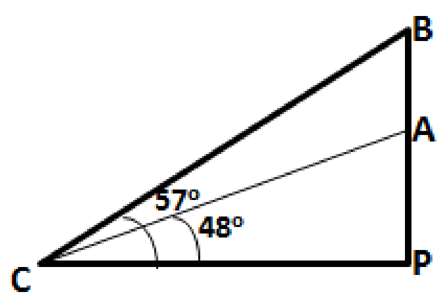
$\Rightarrow \quad BQ = 30 \times 0.844 = 25.32$

Hence, width of the road = (16.08 + 25.32) = 41.4 m

**Question 11.**

Two climbers are at points A and B on a vertical cliff face. To an observer C, 40m from the foot of the cliff, on the level ground, A is at an elevation of 48° and B of 57°. What is the distance between the climbers?

**Solution:**




Let P be the foot of the cliff on level ground.

Then, $\angle ACP = 48^o$ and $\angle BCP = 57^o$

$\therefore \quad \frac{BP}{PC} = \tan 57^o$

$\Rightarrow \quad BP = 40 \times 1.539 = 61.57$ m

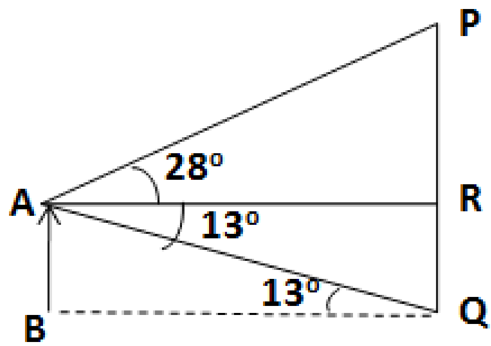
Also, $\frac{AP}{PC} = \tan 48^o$

$\Rightarrow \quad AP = 40 \times 1.110 = 44.4$m

Hence, distance between the climbers = AB = BP - AP = 17.17 m

**Question 12.**
A man stands 9 m away from a flag-pole. He observes that angle of elevation of the top of the pole is 28º and the angle of depression of the bottom of the pole is 13º. Calculate the height of the pole.

**Solution:**




Let AB be the man and PQ be the flag-pole.
Given, AR = 9 m.
Also, $\angle PAR = 28^\circ$ and $\angle QAR = 13^\circ$

$$\therefore \quad \frac{PR}{AR} = \tan 28^\circ$$

$$\Rightarrow \quad PR = 9 \times 0.532 = 4.788 \text{ m}$$

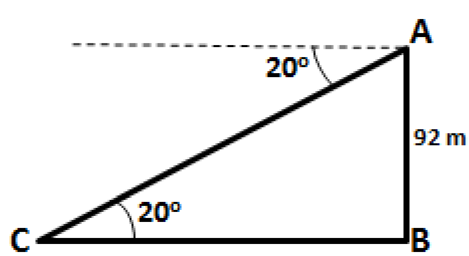
$$\text{Also,} \quad \frac{RQ}{AR} = \tan 13^\circ$$

$$\Rightarrow \quad RQ = 9 \times 0.231 = 2.079 \text{ m}$$

Hence, height of the pole = PR + RQ = 6.867 m

**Question 13.**
From the top of a cliff 92 m high, the angle of depression of a buoy is 20º. Calculate, to the nearest metre, the distance of the buoy from the foot of the cliff.

**Solution:**




Let AB be the cliff and C be the buoy.
Given, AB = 92 m.
Also, $\angle ACB = 20^\circ$

$$\therefore \quad \frac{AB}{BC} = \tan 20^o$$

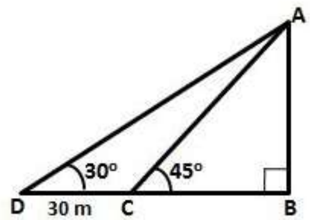
$$\Rightarrow \quad BC = \frac{92}{0.3640} = 252.7m \approx 253m$$

Hence, the buoy is at a distance of 253 m from the foot of the cliff.

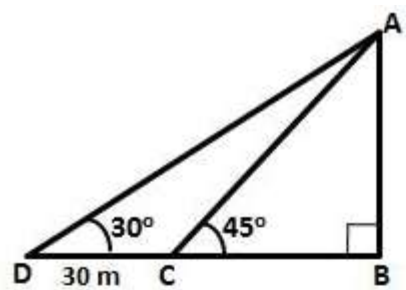
# Exercise 22 B

**Question 1.**

In the figure, given below, it is given that AB is perpendicular to BD and is of length X metres. DC = 30 m, $\angle$ ADB = $30^o$ and $\angle$ ACB = $45^o$. Without using tables, find X.




**Solution:**




In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 45^o = 1$$

$$\Rightarrow \quad BC = AB = X$$

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan 30^o$$

$$\Rightarrow \quad \frac{X}{30 + X} = \frac{1}{\sqrt{3}}$$

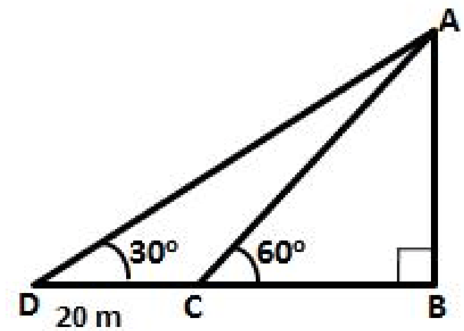
$$\Rightarrow \quad 30 = \left(\sqrt{3} - 1\right)X$$

$$\therefore \quad X = \frac{30}{1.732 - 1} = 40.98m$$

**Question 2.**

Find the height of a tree when it is found that on walking away from it 20 m, in a horizontal line through its base, the elevation of its top changes from $60^o$ to $30^o$.

**Solution:**




Let AB be the tree of height h m.

Let the two points be C and D such that CD = 20 m, $\angle ADB = 30^o$ and $\angle ACB = 60^o$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 60^o = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}$$

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan 30^o$$

$$\Rightarrow \quad \frac{h}{20 + BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h = 20 + \frac{h}{\sqrt{3}}$$

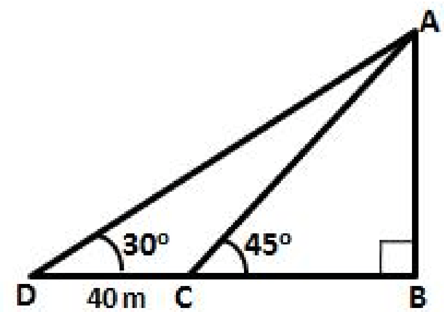
$$\therefore \quad h = \frac{20}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{20}{1.154} = 17.32\text{ m}$$

Hence, height of the tree is 17.32 m.

**Question 3.**

Find the height of a building, when it is found that on walking towards it 40 m in a horizontal line through its base the angular elevation of its top changes from 30° to 45°.

**Solution:**




Let AB be the building of height h m.

Let the two points be C and D such that CD = 40 m, $\angle ADB = 30^\circ$ and $\angle ACB = 45^\circ$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 45^\circ = 1$$

$\Rightarrow \quad BC = h$

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\Rightarrow \quad \frac{h}{40+h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h = 40 + h$$

$$\therefore \quad h = \frac{40}{\sqrt{3}-1} = \frac{40}{0.732} = 54.64\,m$$

Hence, height of the building is 54.64 m.

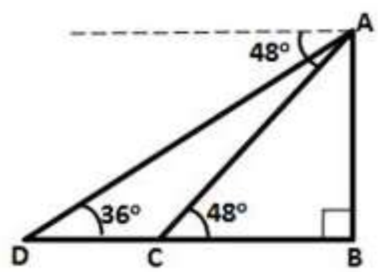
**Question 4.**

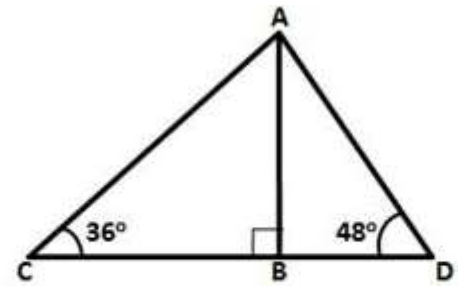
From the top of a light house 100 m high, the angles of depression of two ships are observed as 48° and 36° respectively. Find the distance between the two ships(in the nearest metre) if:

(i) the ships are on the same side of the light house.

(ii) the ships are on the opposite sides of the light house.

**Solution:**




Let AB be the lighthouse.

Let the two ships be C and D such that $\angle ADB = 36^{0}$ and $\angle ACB = 48^{0}$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 48^{0}$$

$$\Rightarrow \quad BC = \frac{100}{1.1106} = 90.04 \text{ m}$$

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan 36^{0}$$

$$\Rightarrow \quad BD = \frac{100}{0.7265} = 137.64 \text{ m}$$

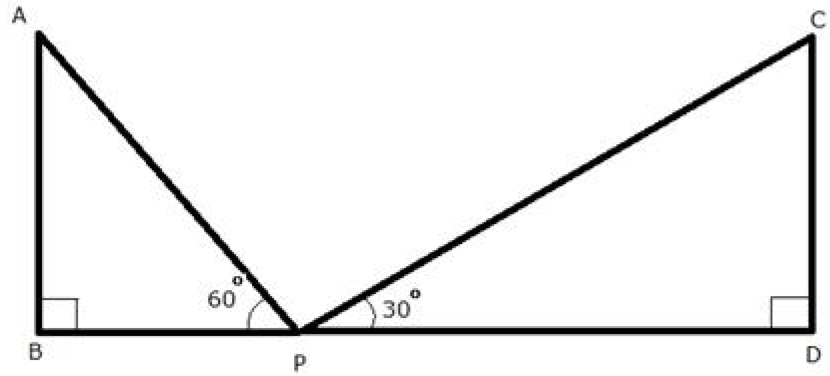
(i) If the ships are on the same side of the light house,
then distance between the two ships = BD - BC = 48 m

(ii) If the ships are on the opposite sides of the light house,
then distance between the two ships = BD + BC = 228 m

**Question 5.**

Two pillars of equal heights stand on either side of a roadway, which is 150 m wide. At a point in the roadway between the pillars the elevations of the tops of the pillars are $60^{o}$ and $30^{o}$ ; find the height of the pillars and the position of the point.

**Solution:**




Let AB and CD be the two towers of height h m.

Let P be a point in the roadway BD such that BD = 150 m, $\angle APB = 60^0$ and $\angle CPD = 30^0$

In $\Delta ABP$,

$$\frac{AB}{BP} = \tan 60^0$$

$$\Rightarrow \quad BP = \frac{h}{\tan 60^0} = \frac{h}{\sqrt{3}}$$

In $\Delta CDP$,

$$\frac{CD}{DP} = \tan 30^0$$

$$\Rightarrow \quad PD = \sqrt{3}\,h$$

Now, $150 = BP + PD$

$$\Rightarrow \quad 150 = \sqrt{3}h + \frac{h}{\sqrt{3}}$$

$$\therefore \quad h = \frac{150}{\sqrt{3} + \frac{1}{\sqrt{3}}} = \frac{150}{2.309} = 64.95\,m$$

Hence, height of the pillars is 64.95 m.

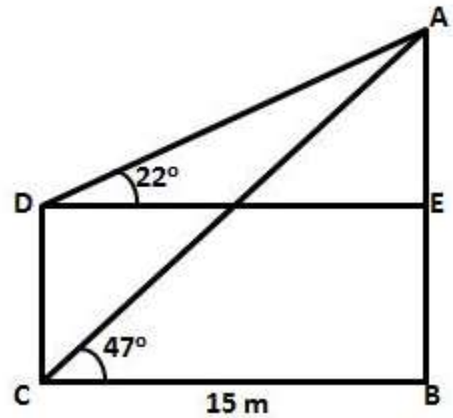
The point is $\frac{BP}{\sqrt{3}}$ from the first pillar.

That is the position of the point is $\frac{64.95}{\sqrt{3}}$ m from the first pillar.

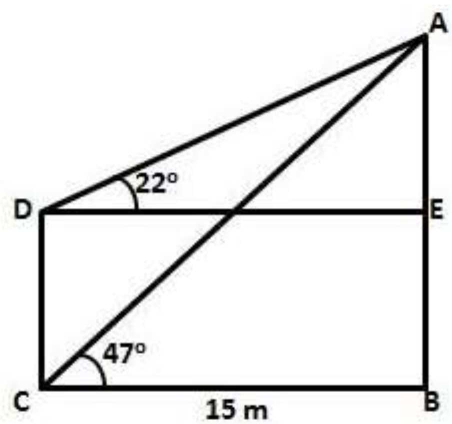
The position of the point is 37.5 m from the first pillar.

**Question 6.**

From the figure, given below, calculate the length of CD.




**Solution:**




In $\Delta AED$,

$$\frac{AE}{DE} = \tan 22^0$$

$$\Rightarrow \quad AE = DE \tan 22^0 = 15 \times 0.404 = 6.06 \text{ m}$$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 47^0$$

$$\Rightarrow \quad AB = BC \tan 47^0 = 15 \times 1.072 = 16.09 \text{m}$$

$$\therefore \quad CD = BE = AB - AE = 10.03 \text{ m}$$

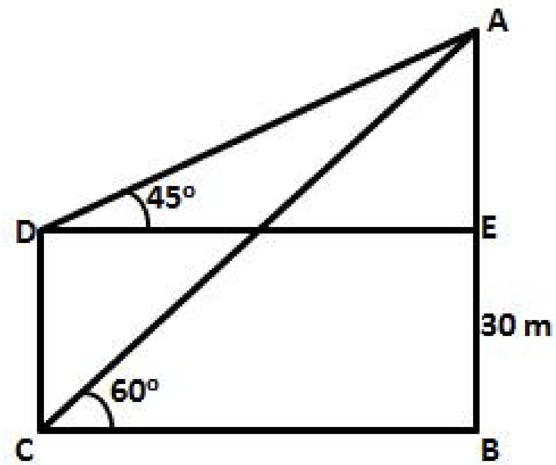
**Question 7.**

The angle of elevation of the top of a tower is observed to be 60°. At a point, 30 m vertically above the first point of observation, the elevation is found to be 45°. Find:

(i) the height of the tower,

(ii) its horizontal distance from the points of observation.

**Solution:**




Let AB be the tower of height h m.

Let the two points be C and D such that CD = 30 m, $\angle ADE = 45^{\circ}$ and $\angle ACB = 60^{\circ}$

In $\Delta ADE$,

$$\frac{AE}{DE} = \tan 45^{\circ}$$

$$\Rightarrow \quad AE = DE$$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 60^{\circ} = \sqrt{3}$$

$$\Rightarrow \quad AE + 30 = \sqrt{3}\ BC$$

$$\Rightarrow \quad BC + 30 = \sqrt{3}\ BC \qquad [\because AE = DE = BC]$$

$$\therefore \quad BC = \frac{30}{\sqrt{3}-1} = \frac{30}{0.732} = 40.98\ m$$

$$\therefore \quad AB = 30 + 40.98 = 70.98\ m$$

Hence, height of the tower is 70.98 m

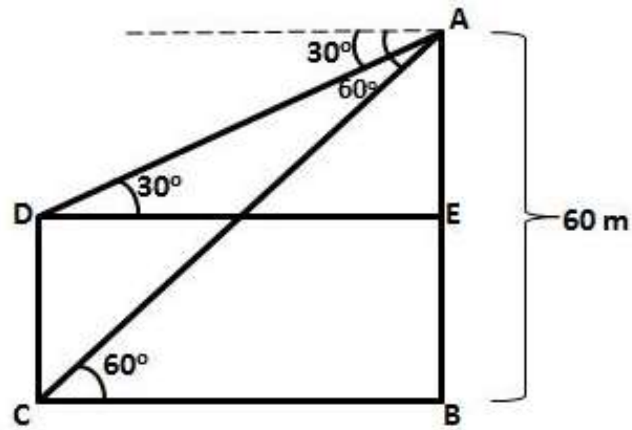
(ii)

The horizontal distance from the points of observation is BC = 40.98 m

**Question 8.**

From the top of a cliff, 60 metres high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

**Solution:**




Let AB be the cliff and CD be the tower.

Here AB = 60 m, $\angle ADE = 30^0$ and $\angle ACB = 60^0$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 60^0 = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{60}{\sqrt{3}}$$

In $\Delta ADE$,

$$\frac{AE}{DE} = \tan 30^0$$

$$\Rightarrow \quad AE = DE \tan 30^0$$

$$= \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \qquad [\because DE = BC]$$

$$= 20 \text{ m}$$

$$\therefore \quad CD = EB = AB - AE = (60 - 20) = 40 \text{ m}$$

Hence, height of the tower is 40 m.

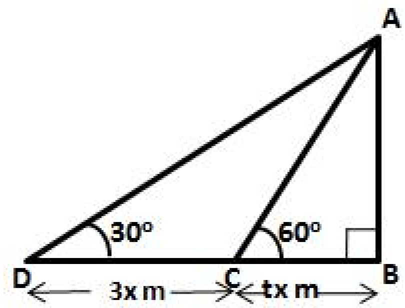
**Question 9.**

A man on a cliff observes a boat, at an angle of depression 30°, which is sailing towards the shore to the point immediately beneath him. Three minutes later, the angle of depression of the boat is found to be 60°. Assuming that the boat sails at a uniform speed, determine:

(i) how much more time it will take to reach the shore.

(ii) the speed of the boat in metre per second, if the height of the cliff is 500 m.

**Solution:**




Let AB be the cliff and C and D be the two positions of the boat such that $\angle ADE = 30^\circ$ and $\angle ACB = 60^\circ$
Let speed of the boat be x metre per minute and let the boat reach the shore after t minutes more.
Therefore, CD = 3x m ; BC = tx m

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \quad \frac{h}{tx} = \sqrt{3}$$

In $\Delta ADB$,

$$\frac{AB}{DB} = \tan 30^\circ$$

$$\Rightarrow \quad \frac{h}{3x + tx} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\sqrt{3}t}{3 + t} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 3t = 3 + t$$

$$\therefore \quad t = \frac{3}{2} = 1.5 \text{ minute}$$

Also, if h = 500m, then

$$\frac{500}{1.5x} = \sqrt{3}$$

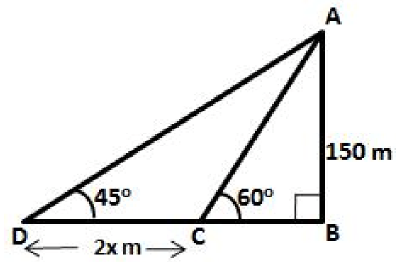
$$\Rightarrow \quad x = \frac{500}{1.5 \times 1.732} = 192.455 \text{ metre per minute}$$

$$= 3.21 \text{ m/sec}$$

Hence, the boat takes an extra 1.5 minutes to reach the shore.
And, if the height of cliff is 500 m, the speed of the boat is 3.21 m/sec

**Question 10.**
A man in a boat rowing away from a lighthouse 150 m high, takes 2 minutes to change the angle of elevation of the top of the lighthouse from 60° to 45°. Find the speed of the boat.

**Solution:**




Let AB be the lighthouse and C and D be the two positions of the boat such that AB = 150 m, $\angle$ ADB = 45° and $\angle$ ACB = 60°

Let speed of the boat be x metre per minute.

Therefore, CD = 2x m ;

In $\Delta ADB$,

$$\frac{AB}{DB} = \tan 45^{\circ}$$

$$\Rightarrow \quad BD = 150 \text{ m}$$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 60^{\circ} = \sqrt{3}$$

$$\Rightarrow \quad \frac{150}{BC} = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{150}{\sqrt{3}} = \frac{150}{1.732} = 86.605 \text{ m}$$

$$\therefore \quad CD = BD - BC$$

$$= 150 - 86.605 = 63.395 \text{ m}$$

$$\Rightarrow \quad 2x = 63.395$$

$$\Rightarrow \quad x = \frac{63.395}{2} = 31.6975 \text{ m/min}$$

$$= \frac{31.6975}{60} \text{ m/sec} = 0.53 \text{ m/sec}$$

Hence, the speed of the boat is 0.53 m/sec

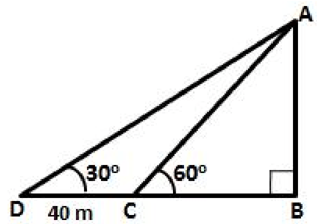
**Question 11.**

A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find:

(i) the height of the tree, correct to 2 decimal places,

(ii) the width of the river.

**Solution:**




Let AB be the tree of height 'h' m and BC be the width of the river. Let D be the point on the opposite bank of tree such that CD = 40 m. Here $\angle ADB = 30^o$ and $\angle ACB = 60^o$

Let speed of the boat be x metre per minute.

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 60^o = \sqrt{3}$$

$$\Rightarrow \quad \frac{h}{BC} = \sqrt{3}$$

$$\Rightarrow \quad h = BC\sqrt{3}$$

In $\Delta ADB$,

$$\frac{AB}{BD} = \tan 30^o$$

$$\Rightarrow \quad \frac{h}{40 + BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{BC\sqrt{3}}{40 + BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 3\ BC = 40 + BC$$

$$\Rightarrow \quad BC = 20\ m$$

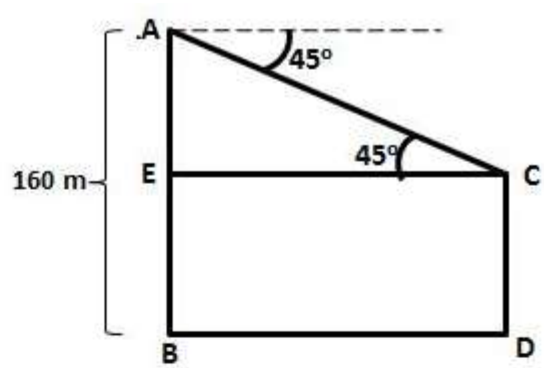
$$\therefore \quad h = 20 \times 1.732 = 34.64\ m$$

Hence, height of the tree is 34.64 m and width of the river is 20 m.

**Question 12.**

The horizontal distance between two towers is 75 m and the angular depression of the top of the first tower as seen from the top of the second, which is 160 m high, is 45°. Find the height of the first tower.

**Solution:**




Let AB and CD be the two towers

The height of the first tower is AB = 160 m

The horizontal distance between the two towers is BD = 75 m

And the angle of depression of the first tower as seen from the top of the second tower is $\angle ACE = 45^0$.

In $\Delta ACE$,

$$\frac{AE}{EC} = \tan 45^0 = 1$$

$\Rightarrow \quad AE = EC = BD = 75\,m$

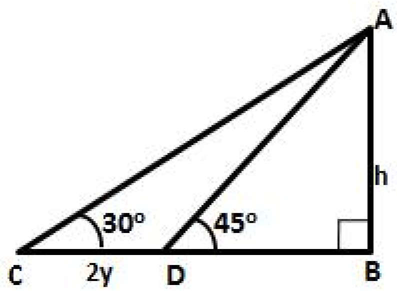
$\therefore \quad CD = EB = AB - AE = (160 - 75) = 85\,m$

Hence, height of the other tower is 85 m

**Question 13.**

The length of the shadow of a tower standing on level plane is found to be 2y metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of the tower is y(√3 + 1) metres.

**Solution:**




Let AB be the tower and C and D are two points such that CD = 2y m, $\angle ADB = 45^0$ and $\angle ACB = 30^0$

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan 45^0 = 1$$

$\Rightarrow \quad h = BD$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 30^0$$

$\Rightarrow \quad \dfrac{h}{BC} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad BC = \sqrt{3}h$

$\Rightarrow \quad CD + h = \sqrt{3}h$

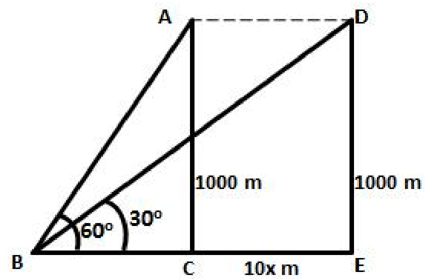
$\Rightarrow \quad 2y = \left(\sqrt{3} - 1\right)h$

$\therefore \quad h = \dfrac{2y}{\sqrt{3} - 1} = \dfrac{2y\left(\sqrt{3} + 1\right)}{\left(\sqrt{3}\right)^2 - 1} = y\left(\sqrt{3} + 1\right)\ m$

Hence, height of the tower is $y\left(\sqrt{3} + 1\right)$ m.

**Question 14.**
An aeroplane flying horizontally 1 km above the ground and going away from the observer is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°; find the uniform speed of the aeroplane in km per hour.

**Solution:**




Let A be the aeroplane and B be the observer on the ground. The vertical height will be AC = 1 km = 1000 m. After 10 seconds, let the aeroplane be at point D.
Let the speed of the aeroplane be x m/sec.
$\therefore$ CE = 10x
In $\Delta ABC$,

$$\frac{AC}{BC} = \tan 60^{\circ}$$

$$\Rightarrow \quad \frac{1000}{BC} = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{1000}{\sqrt{3}} \text{ m}$$

In $\Delta BDE$,

$$\frac{DE}{BE} = \tan 30^{\circ}$$

$$\Rightarrow \quad BE = 1000\sqrt{3}$$

$$\therefore \quad CE = BE - BC$$

$$\Rightarrow \quad 10x = 1000\sqrt{3} - \frac{1000}{\sqrt{3}}$$

$$\Rightarrow \quad x = 100\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 100 \times 1.154$$

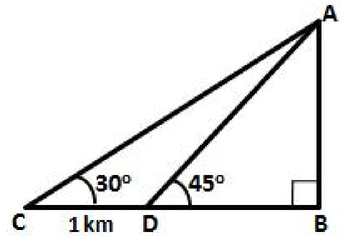
$$= 115.4 \text{ m/sec}$$

$$= 115.4 \times \frac{18}{5} \text{ km/hr} = 415.67 \text{ km/hr}$$

Hence, speed of the aeroplane is 415.67 km/hr.

**Question 15.**
From the top of a hill, the angles of depression of two consecutive kilometer stones, due east, are found to be 30° and 45° respectively. Find the distances of the two stones from the foot of the hill.

**Solution:**




Let AB be the hill of height 'h' km and C and D be the two consecutive stones such that CD = 1 km, $\angle ACB = 30^o$ and $\angle ADB = 45^o$.

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan\ 45^o = 1$$

$\Rightarrow \quad BD = h$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 30^o$$

$$\Rightarrow \quad \frac{h}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{h+1} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2} = \frac{2.732}{2} = 1.366 km$$

$\therefore \quad BD = 1.366\ km$

$\therefore \quad BC = BD + DC = 1.366 + 1 = 2.366 km$

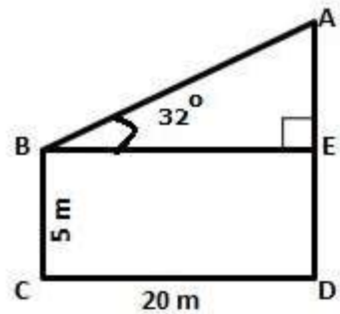
Hence, the two stones are at a distance of 1.366 km and 2.366 km from the foot of the hill.

# Exercise 22 C

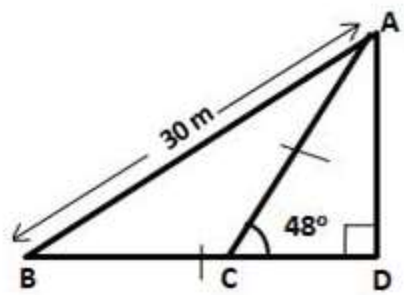
**Question 1.**

Find AD:

(i)




(ii)




**Solution:**

(i) In $\Delta AEB$,

$$\frac{AE}{BE} = \tan 32^o$$

$\Rightarrow$ $AE = 20 \times 0.6249 = 12.50 \text{ m}$

$\therefore$ $AD = AE + ED = 12.50 + 5 = 17.50 \text{m}$

(ii) In $\Delta ABC$,

$\angle ACD = \angle ABC + \angle BAC$

and $\angle ABC = \angle BAC$ $\quad (\because AC = BC)$

$\therefore$ $\angle ABC = \angle BAC = \dfrac{48^o}{2} = 24^o$

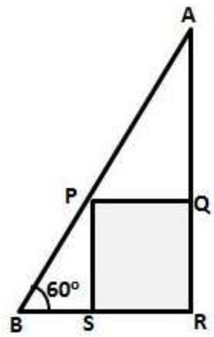
Now,

$$\frac{AD}{AB} = \sin 24^o$$

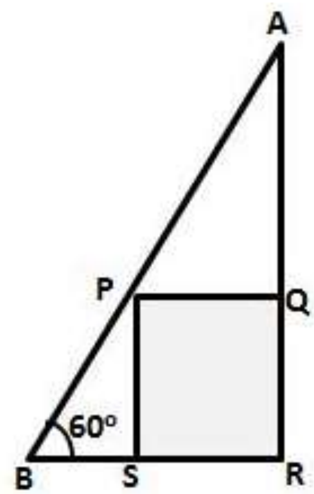
$\Rightarrow$ $AD = 30 \times 0.4067 = 12.20 \text{ m}$

**Question 2.**

In the following diagram, AB is a floor-board; PQRS is a cubical box with each edge = 1 m and $\angle B = 60^0$. Calculate the length of the board AB.




**Solution:**




In $\Delta PSB$,

$$\frac{PS}{PB} = \sin 60^0$$

$$\Rightarrow \quad PB = \frac{2}{\sqrt{3}} = 1.155 \text{ m}$$

In $\Delta APQ$,

$$\angle APQ = 60^0$$

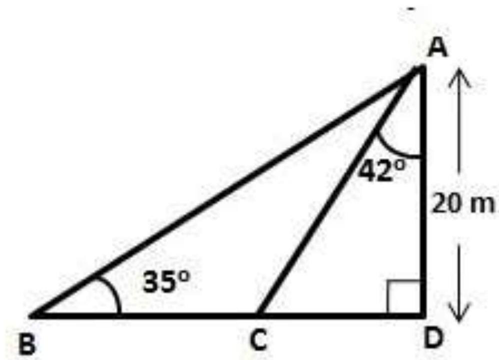
$$\therefore \quad \frac{PQ}{AP} = \cos 60^0$$

$$\Rightarrow \quad AP = \frac{1}{1/2} = 2 \text{ m}$$

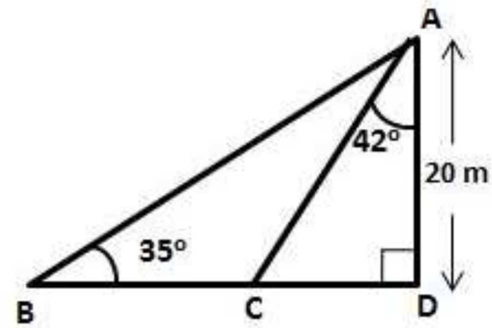
$$\therefore \quad AB = AP + PB = 2 + 1.155 = 3.155 \text{ m}$$

**Question 3.**

Calculate BC.




**Solution:**




In $\Delta ADC$,

$$\frac{CD}{AD} = \tan 42^0$$

$$\Rightarrow \quad CD = 20 \times 0.9004 = 18.008\,m$$

In $\Delta ADB$,

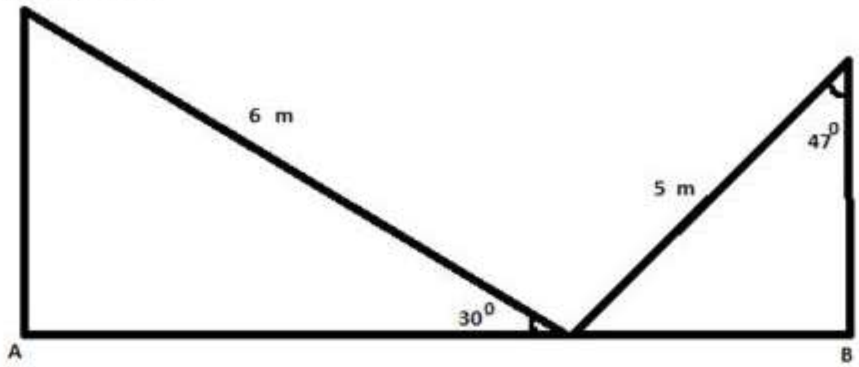
$$\frac{AD}{BD} = \tan 35^0$$

$$\Rightarrow \quad BD = \frac{AD}{\tan 35^0} = \frac{20}{0.7002} = 28.563\ m$$

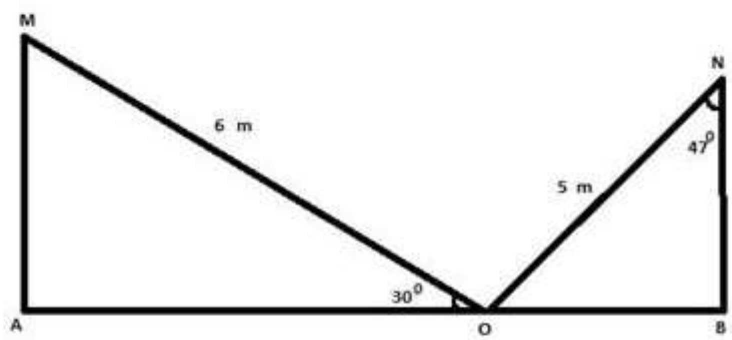
$$\therefore \quad BC = BD - CD = 10.55\ m$$

**Question 4.**

Calculate AB.




**Solution:**




In $\Delta AMOB$

$\cos 30° = \frac{AO}{MO}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AO}{6}$

$\Rightarrow AO = 5.20\ m$

In $\Delta BNO$

$\sin 47° = \frac{OB}{NO}$

$\Rightarrow 0.73 = \frac{OB}{5}$

$\Rightarrow OB = 3.65\ m$

$AB = OA + OB$

$\Rightarrow AB = 5.20 + 3.65$

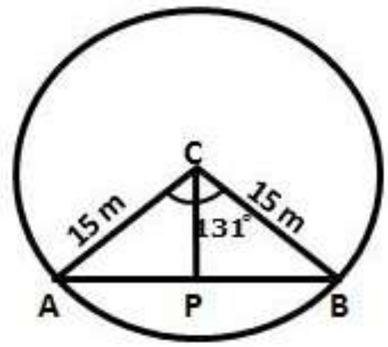
$\Rightarrow AB = 8.85m$

**Question 5.**

The radius of a circle is given as 15 cm and chord AB subtends an angle of 131° at the

centre C of the circle. Using trigonometry, calculate:
(i) the length of AB;
(ii) the distance of AB from the centre C.

**Solution:**




Given, CA = CB = 15 cm, $\angle ACB = 131^\circ$
Drop a perpendicular CP from centre C to the chord AB.
Then CP bisects $\angle ACB$ as well as chord AB.

$\therefore \quad \angle ACP = 65.5^\circ$

In $\Delta ACP$,

$$\frac{AP}{AC} = \sin(65.5^\circ)$$

$\Rightarrow \quad AP = 15 \times 0.91 = 13.65 \text{ cm}$

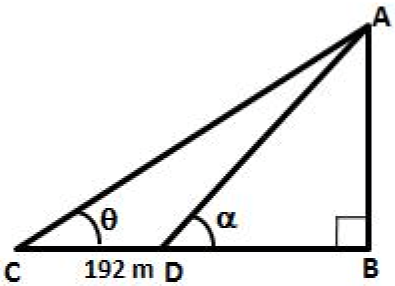
(i) $\quad AB = 2\,AP = 2 \times 13.65 = 27.30 \text{ cm.}$

(ii) $\quad CP = AP \cos(65.5^\circ)$

$\quad\quad = 15 \times 0.415 = 6.22 \text{ cm.}$

**Question 6.**
At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is 5/12. On walking 192 metres towards the tower, the tangent of the angle is found to be 3/4. Find the height of the tower.

**Solution:**




Let AB be the vertical tower and C and D be two points such that CD = 192 m. Let $\angle ACB = \theta$ and $\angle ADB = \alpha$.

Given, $\tan\theta = \frac{5}{12}$

$$\Rightarrow \quad \frac{AB}{BC} = \frac{5}{12}$$

$$\Rightarrow \quad AB = \frac{5}{12}BC \qquad --(i)$$

Also, $\tan\alpha = \frac{3}{4}$

$$\Rightarrow \quad \frac{AB}{BD} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\frac{5}{12}BC}{BD} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{192 + BD}{BD} = \frac{3}{4} \times \frac{12}{5}$$

$$\Rightarrow \quad BD = 240 \text{ m}$$

$$\therefore \quad BC = (192 + 240) = 432 \text{ m}$$

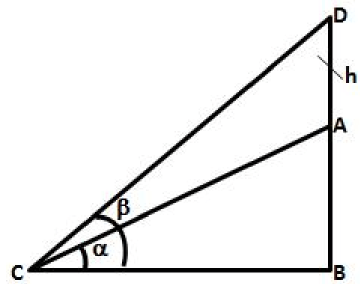
$$\therefore \quad \text{By (i)}, \quad AB = \frac{5}{12} \times 432 = 180 \text{ m}$$

Hence, the height of the tower is 180 m.

**Question 7.**

A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h metre. At a point on the plane, the angle of elevation of the bottom of the flagstaff is $\alpha$ and at the top of the flagstaff is $\beta$. Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta - \tan\alpha}$.

**Solution:**




Let AB be the tower of height x metre, surmounted by a vertical flagstaff AD. Let C be a point on the plane such that $\angle ACB = \alpha$, $\angle DCB = \beta$ and AD = h.

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan\alpha$$

$$\Rightarrow \quad BC = \frac{x}{\tan\alpha} \qquad ---(i)$$

In $\Delta DBC$,

$$\frac{BD}{BC} = \tan\beta$$

$$\Rightarrow \quad BD = \left(\frac{x}{\tan\alpha}\right)\tan\beta \quad [From\ (i)]$$

$$\Rightarrow \quad (h + x)\tan\alpha = x\tan\beta$$

$$\Rightarrow \quad x\tan\beta - x\tan\alpha = h\tan\alpha$$

$$\therefore \quad x = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, height of the tower is $\frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

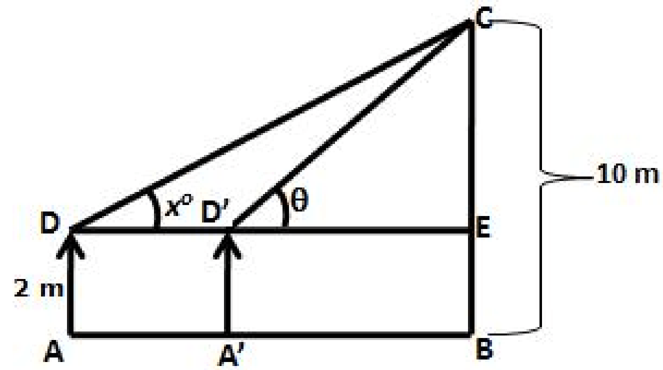
**Question 8.**

With reference to the given figure, a man stands on the ground at point A, which is on the same horizontal plane as B, the foot of the vertical pole BC. The height of the pole is 10 m. The man's eye s 2 m above the ground. He observes the angle of elevation of C, the top of the pole, as $x^o$ , where tan $x^o$ = 2/5. Calculate:

(i) the distance AB in metres;

(ii) angle of elevation of the top of the pole when he is standing 15 metres from the pole. Give your answer to the nearest degree.

**Solution:**




Let AD be the height of the man, AD = 2 m.

$\therefore \quad CE = (10 - 2) = 8 \text{ m}$

(i) In $\Delta CED$,

$$\frac{CE}{DE} = \tan x^o = \frac{2}{5}$$

$$\Rightarrow \quad \frac{8}{DE} = \frac{2}{5} \quad \Rightarrow \quad DE = 20 \text{ m}$$

Here AB=DE.

$\therefore$ AB=20 m

(ii) Let A'D' be the new position of the man and θ be the angle of elevation of the top of the tower.

So, D'E = 15 m

In $\Delta CED$,

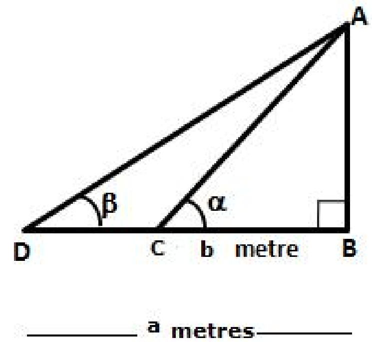
$$\tan\theta = \frac{CE}{D'E} = \frac{8}{15} = 0.533$$

$$\Rightarrow \quad \theta = 28^o$$

**Question 9.**

The angles of elevation of the top of a tower from two points on the ground at distances *a* and *b* metres from the base of the tower and in the same line are complementary. Prove that the height of the tower is √ab metre.

**Solution:**




Let AB be the tower of height h metres.
Let C and D be two points on the level ground such that BC = b metres, BD = a metres, $\angle ACB = \alpha$ and $\angle ADB = \beta$.

Given, $\alpha + \beta = 90^0$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan\alpha$$

$$\Rightarrow \quad \frac{h}{b} = \tan\alpha \qquad ---(i)$$

In $\Delta ABD$,

$$\frac{AB}{BD} = \tan\beta$$

$$\Rightarrow \quad \frac{h}{a} = \tan\left(90^0 - \alpha\right) = \cot\alpha --(ii)$$

Multiplying (i) by (ii), we get,

$$\left(\frac{h}{a}\right)\left(\frac{h}{b}\right) = 1$$

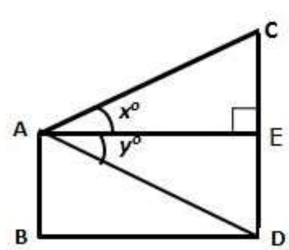
$$\Rightarrow \quad h^2 = ab$$

$$\therefore \quad h = \sqrt{ab} \text{ metre}$$

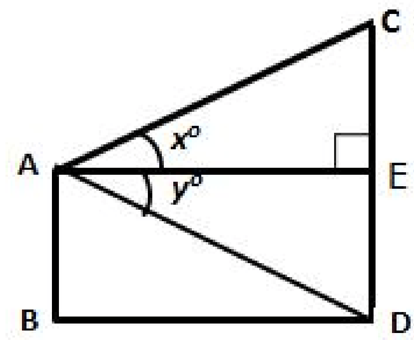
Hence, height of the tower is $\sqrt{ab}$ metre.

**Question 10.**

From a window A, 10 m above the ground the angle of elevation of the top C of a tower is $x^o$, where $\tan x^o = \frac{5}{2}$ and the angle of depression of the foot D of the tower is $y^o$, where $\tan y^o = \frac{1}{4}$. Calculate the height CD of the tower in metres.



**Solution:**




Here, AB = DE = 10 m

In $\Delta ADE$,

$$\frac{DE}{AE} = \tan y = \frac{1}{4}$$

$\Rightarrow$ $AE = 4DE = 4 \times 10 = 40$ m

In $\Delta AEC$,

$$\frac{CE}{AE} = \tan x = \frac{5}{2}$$

$\Rightarrow$ $CE = 40 \times \frac{5}{2} = 100$ m

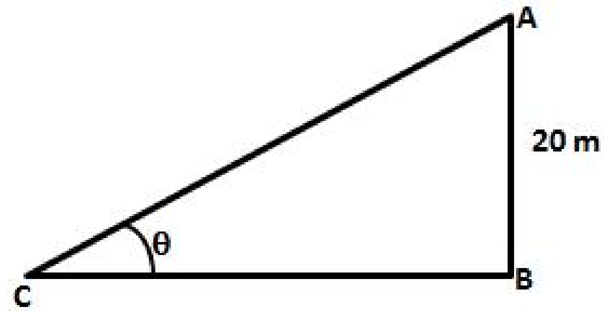
$\therefore$ $CD = DE + EC = (10 + 100) = 110$ m

Hence, height of the tower CD is 110 m.

**Question 11.**

A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0.53. How far is he standing from the foot of the tower?

**Solution:**




Let AB be the tower of height 20 m.
Let $\theta$ be the angle of elevation of the top of the tower from point C.

Given, $\cos\theta = 0.53$

$\Rightarrow \quad \theta = 58^{0}$

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 58^{0}$$

$$\Rightarrow \quad \frac{20}{BC} = 1.6$$

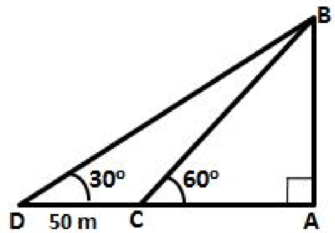
$$\therefore \quad BC = \frac{20}{1.6} = 12.5 \text{ m}$$

**Question 12.**

A man standing on the bank of a river observes that the angle of elevation of a tree on the opposite bank is 60°. When he moves 50 m away from the bank, he finds the angle of elevation to be 30°. Calculate:

(i) the width of the river;
(ii) the height of the tree.

**Solution:**




Let AB be the tree and AC be the width of the river. Let D be a point such that CD = 50 m. Given that $\angle BCA = 60^\circ$ and $\angle BDA = 30^\circ$.

In $\Delta BAD$,

$$\frac{BA}{AD} = \tan 30^0$$

$$\Rightarrow \quad BA = \frac{AD}{\sqrt{3}} \quad ---(i)$$

In $\Delta BAC$,

$$\frac{BA}{AC} = \tan 60^0$$

$$\Rightarrow \quad BA = AC\sqrt{3} \quad ---(ii)$$

From (i) and (ii), we get

$$\frac{AD}{\sqrt{3}} = AC\sqrt{3}$$

$$\Rightarrow \quad (50 + AC) = 3AC$$

$$\therefore \quad AC = 25 \text{ m}$$

Thus, width of the river is 25 m.

From (ii),

$$BA = 25 \times 1.732 = 43.3 \text{ m}$$

Hence, height of the tree is 43.3 m.

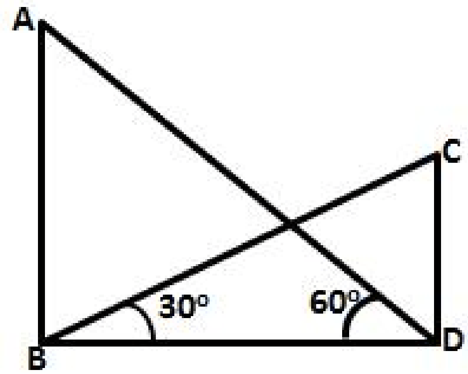
**Question 13.**

A 20 m high vertical pole and a vertical tower are on the same level ground in such a way that the angle of elevation of the top of the tower, as seen from the foot of the pole is 60° and the angle of elevation of the top of the pole, as seen from the foot of the tower is 30°. Find:

(i) the height of the tower ;

(ii) the horizontal distance between the pole and the tower.

**Solution:**




Let AB be the tower and CD be the pole.

Given, CD = 20 m, $\angle ADB = 60^0$ and

$\angle CBD = 30^0$

In $\Delta BDC$,

$$\frac{CD}{BD} = \tan 30^0$$

$\Rightarrow \quad BD = 20\sqrt{3}$ m

In $\Delta DBA$,

$$\frac{AB}{BD} = \tan 60^0 = \sqrt{3}$$

$\Rightarrow \quad AB = 20\sqrt{3} \times \sqrt{3} = 60$ m

Hence,

(i) height of the tower = 60 m

(ii) horizontal distance between the pole and tower

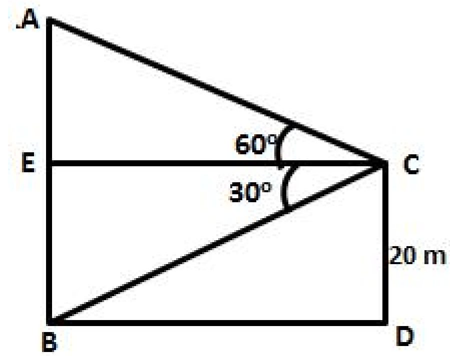
= 20 × 1.732 = 34.64 m

**Question 14.**

A vertical pole and a vertical tower are on the same level ground in such a way that from the top of the pole, the angle of elevation of the top of the tower is 60° and the angle of depression of the bottom of the tower is 30°. Find:

(i) the height of the tower, if the height of the pole is 20 m;
(ii) the height of the pole, if the height of the tower is 75 m.

**Solution:**




Let AB be the tower and CD be the pole.
Then $\angle ACE = 60^\circ$ and $\angle BCE = 30^\circ$.

(i) In $\Delta BEC$,

$$\frac{BE}{EC} = \tan 30^0$$

$$\Rightarrow \quad \frac{20}{EC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad EC = 20\sqrt{3}\ m$$

In $\Delta AEC$,

$$\frac{AE}{EC} = \tan 60^0$$

$$\Rightarrow \quad AE = 20\sqrt{3} \times \sqrt{3} = 60\ m$$

$\therefore$ Height of the tower $= AB = AE + EB$
$= (60 + 20) = 80\,m$

(ii) Let height of the pole be $x$ m.

$\therefore CD = BE = x$

In $\Delta BEC$,

$$\frac{BE}{EC} = \tan 30^0$$

$$\Rightarrow \quad EC = \sqrt{3}\,x$$

In $\Delta AEC$,

$$\frac{AE}{EC} = \tan 60^0$$

$$\Rightarrow \quad \frac{75 - x}{EC} = \sqrt{3}$$

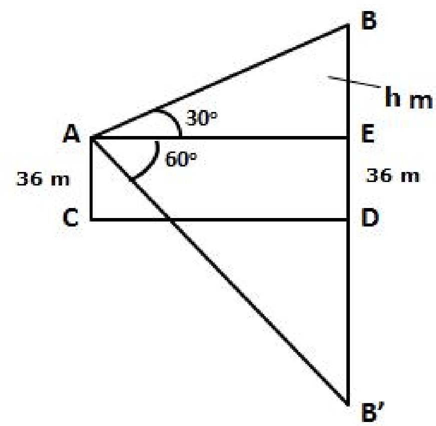
$$\Rightarrow \quad 75 - x = 3x$$

$$\therefore \quad x = \frac{75}{4} = 18.75\,m$$

$\therefore$ Height of the pole is $18.75\,m$.

**Question 15.**
From a point, 36 m above the surface of a lake, the angle of elevation of a bird is observed to be 30° and the angle of depression of its image in the water of the lake is observed to be 60°. Find the actual height of the bird above the surface of the lake.

**Solution:**




Let A be a point 36 m above the surface of the lake and B be the position of the bird. Let B' be the image of the bird in the water.

Here, $AC = DE = 36$ m, $\angle BAE = 30^0$ and

$\angle B'AE = 60^0$.

Let $BE = h$ m.

Then, $B'D = BD = 36 + h$

($\because$ B' is image of B about D)

$\therefore B'E = B'D + DE = 36 + 36 + h = 72 + h \quad ---(i)$

In $\Delta ABE$,

$$\frac{BE}{AE} = \tan 30^0$$

$$\Rightarrow \quad AE = \sqrt{3}\,h \quad ---(ii)$$

In $\Delta AB'E$,

$$\frac{B'E}{AE} = \tan 60^0$$

$$\Rightarrow \quad \frac{72 + h}{AE} = \sqrt{3} \quad [\text{from (i)}]$$

$$\Rightarrow \quad 72 + h = \left(\sqrt{3}h\right)\sqrt{3} \quad [\text{from (ii)}]$$

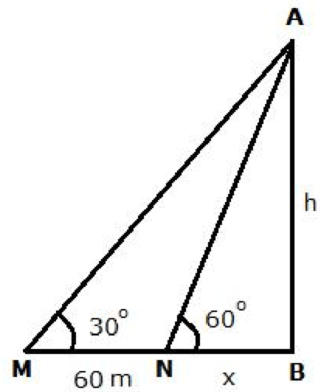
$$\Rightarrow \quad 72 + h = 3h$$

$$\therefore \quad h = 36 \text{ m}$$

Hence, the actual height of the bird above the surface of the lake = 36+36=72m

**Question 16.**
A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m, the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.

**Solution:**




Let AB be a building and M and N are the two positions of the man which makes angles of elevation of top of building as 30° and 60° respectively.
MN = 60 m
Let AB = h and NB = x m
Now in right ΔAMB,

$$\tan 30° = \frac{AB}{MB}$$

$$\Rightarrow \tan 30° = \frac{h}{60 + x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{60 + x}$$

$$\Rightarrow 60 + x = \sqrt{3}h$$

$$\Rightarrow x = \sqrt{3}h - 60 \ldots .(1)$$

Similaraly in right ΔANB,

$$\tan 60° = \frac{AB}{NB}$$

$$\tan 60° = \frac{h}{60 + x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \ldots .(2)$$

From (1) and (2), we have,

$$\sqrt{3}h - 60 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow 3h - 60\sqrt{3} = h$$

$$\Rightarrow 3h - h = 60\sqrt{3}$$

$$\Rightarrow 2h = 60\sqrt{3}$$

$$\Rightarrow h = \frac{60\sqrt{3}}{2}$$

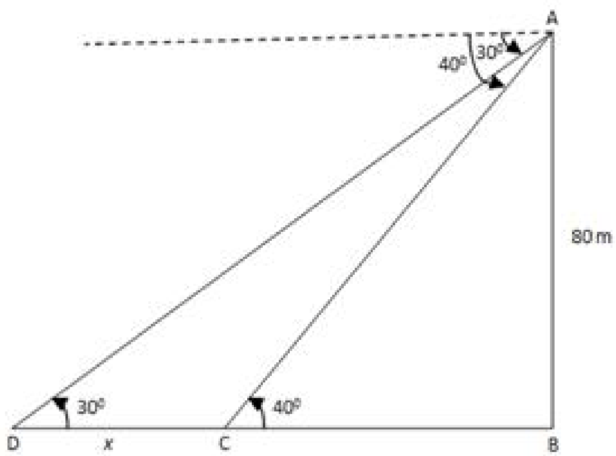
$$\Rightarrow h = 30\sqrt{3} = 30 \times 1.732$$

$$\Rightarrow h = 51.96\ m$$

$\therefore$ Height of the building = 51.96=52 m (approx)

**Question 17.**
As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships, on the same side of a light house in a horizontal line with its base, are 30° and 40° respectively. Find the distance between the two ships. Give your answer corrected to the nearest metre.

**Solution:**




Let AB represent the lighthouse.
Let the two ships be at points D and C having angle of depression 30° and 40° respectively.
Let x be the distance between the two ships.
Clearly, $m\angle ACB = 40°$ and $m\angle ADB = 30°$

In $\Delta ACB$

$$\tan 40° = \frac{80}{CB}$$

$$\Rightarrow CB = \frac{80}{0.84} = 95.24\text{m}$$

In $\Delta ADB$

$$\tan 30° = \frac{80}{DB}$$

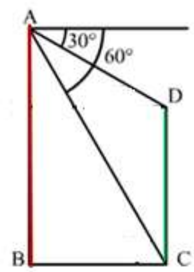
$$\Rightarrow DB = \frac{80}{0.58} = 137.93\text{m}$$

$DC = DB - CB$
$\Rightarrow x = 137.93 - 95.24$
$\Rightarrow x = 42.69 \approx 43\text{m}$

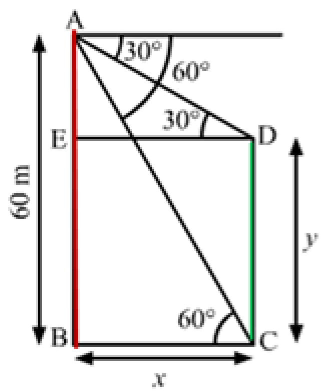
The distance between the two ships is 43 m.

**Question 18.**

In the given figure, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :
(i) the horizontal distance between AB and CD.
(ii) the height of the lamp post.




**Solution:**




Given that AB is a building that is 60 m, high.

Let BC = DE = x and CD = BE = y

$\Rightarrow$ AE = AB – BE = 60 – y

(i) In right$\Delta$AED,

$$\tan 30° = \frac{AE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60 - y}{x}$$

$$\Rightarrow x = 60\sqrt{3} - y\sqrt{3} \quad \ldots(1)$$

In right $\Delta$ABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60\sqrt{3}}{3}$$

$$\Rightarrow x = 20\sqrt{3}$$

$$\Rightarrow x = 20 \times 1.732 = 34.64 \text{ m}$$

Thus, the horizontal distance between AB and CD is 34.64 m.

(ii) From (i), we get the height of the lamp post = CD = y

$$x = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow 20\sqrt{3} = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow 20 = 60 - y$$

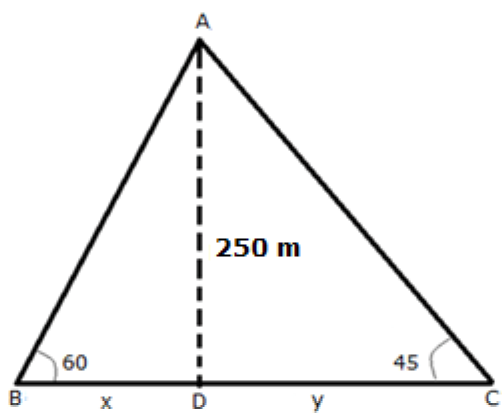
$$\Rightarrow y = 40 \text{ m}$$

Thus, the height of the lamp post is 40 m.

**Question 19.**

An aeroplane, at an altitude of 250 m, observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number.

**Solution:**




Let A be the position of the airplane and let BC be the river. Let D be the point in BC just below the airplane. B and C be two boats on the opposite banks of the river with angles of depression 60° and 45° from A.

In $\Delta ADC$,

$$\tan 45^\circ = \frac{AD}{DC}$$

$$\Rightarrow 1 = \frac{250}{y}$$

$$\Rightarrow y = 250 \text{ m} = DC$$

In $\Delta ADB$,

$$\tan 60^\circ = \frac{AD}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{250}{x}$$

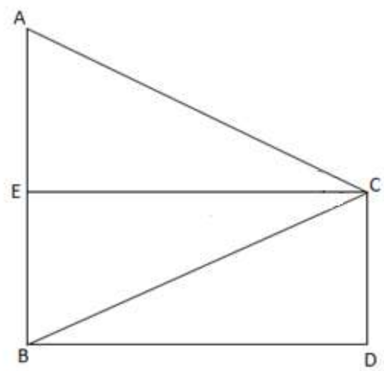
$$\Rightarrow x = \frac{250}{\sqrt{3}} = \frac{250\sqrt{3}}{3} = \frac{250 \times 1.732}{3} = 144.3 \text{ m} = BD$$

$$\therefore BC = BD + DC = 144.3 + 250 = 394.3 \approx 394 \text{ m}$$

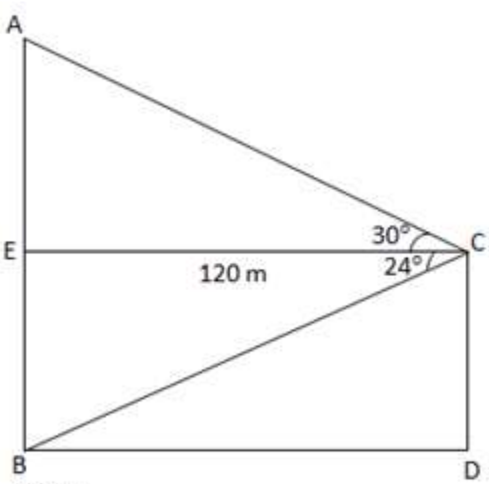
Thus, the width of the river is 394 m.

**Question 20.**

The horizontal distance between two towers is 120 m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the top of the second tower is 30° and 24° respectively. Find the height of the two towers. Give your answers. Give your answer correct to 3 significant figures.




**Solution:**




In $\Delta AEC$,

$$\tan 30^\circ = \frac{AE}{EC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{120}$$

$$\Rightarrow AE = \frac{120}{\sqrt{3}} = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3} = 40 \times 1.732 = 69.28 \text{ m}$$

In $\Delta BEC$,

$$\tan 24° = \frac{EB}{EC}$$

$$\Rightarrow 0.4452 = \frac{EB}{120}$$

$$\Rightarrow EB = 53.424 \text{ m}$$

Thus, height of first tower,

$AB = AE + EB = 69.28 + 53.424 = 122.704 = 123$ m (correct to 3 significant figures)

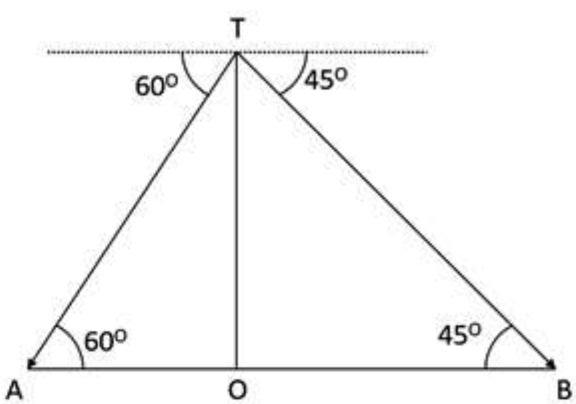
And, height of second tower,

$CD = EB = 53.424$ m $= 53.4$ m (correct to 3 significant figures)

**Question 21.**

The angles of depression of two ships A and B as observed from the top of a light house 60m high, are 60° and 45° respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships. Give your answer correct to the nearest whole number.

**Solution:**




In the above figure

OT=tower = 60m

A and B are the respective positions of ship

In $\Delta TAO$

$$\tan 60^\circ = \frac{TO}{AO}$$

$$\sqrt{3} = \frac{60}{AO}$$

$$AO = 20\sqrt{3} \text{ m}$$

In $\Delta TBO$

$$\tan 45^\circ = \frac{TO}{OB}$$

$$1 = \frac{60}{OB}$$

$$OB = 60 \text{ m}$$

$$AB = AO + OB = 20\sqrt{3} + 60 = 20(1.732) + 60 = 94.64 \approx 95 \text{ m}$$