Chapter 2

Analog Modulation

CHAPTER HIGHLIGHTS

- *Amplitude Modulation*
- *Switching Modulator*
- *Envelope Detector*
- *Double Sideband Suppressed Carrier (DSB SC) Modulation*
- *Ring Modulator*
- *Coherent Detection*
- *Quadrature Carrier Multiplexing*
- *Single Side Band Suppressed Carrier Modulation (SSB SC)*
- *Vestigial Sideband Modulation (VSBSC)*
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- *Power Density Spectrum of Shot Noise in Diodes*
- *Noise Bandwidth*

In Analog modulation, one of the characteristics of the carrier-like amplitude, frequency, or phase is varied in accordance with the message signal.

Amplitude Modulation

In amplitude modulation (AM), the amplitude of the carrier is varied in accordance with the message signal $m(t)$. The amplitude-modulated signal is given as follows:

$$S(t) = A_c(1 + K_a\, m(t)) \cos(2\pi f_c t)$$

Where K_a is the amplitude sensitivity of the modulator.

In AM, K_a should be chosen such that

$$|K_a m(t)| \le 1$$

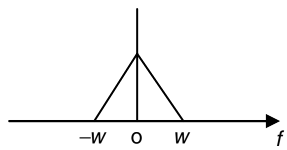
for all values of $m(t)$.

If $|K_a m(t)| > 1$ for any value $m(t)$, this scenario is called phase reversal or over modulation max $|K_a m(t)|$ indicates the percentage of modulation.

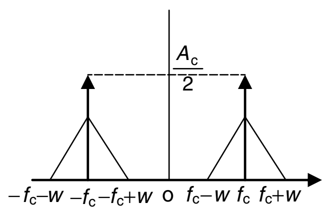
If we convert the modulated signal $s(t)$ into frequency domain

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c K_a}{2}[M(f - f_c) + M(f + f_c)]$$

If $M(f)$ is



$S(f)$ will be



The transmission bandwidth required for AM is 2W from $f_c - w$ to $f_c + w$.

If the power of $m(t)$ is p watt, the power of AM wave is

$$\underbrace{\frac{A_c^2}{2}}_{\text{Carrier power}} + \underbrace{\frac{A_c^2 K_a^2 p}{2}}_{\text{Side band power}}$$

The carrier component in AM does not carry any information about $m(t)$. The information is completely carried by side bands.

$$\text{Power efficiency of AM} = \frac{\text{side band power}}{\text{Total power}} = \frac{K_a^2 p}{1 + k_a^2 p}$$

Single-Tone AM

If $m(t) = A_m \cos(2\pi f_m t)$

$$s(t) = A_c(1 + K_a A_m \cos(2\pi f_m t)) \cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t) \cos(2\pi f_c t)$$

Where $K_a A_m = \mu$ is called modulation index for AM.

$$= \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{\frac{A_c \mu}{2} \cos\left(2\pi\left(f_c + f_m\right)t\right)}_{\text{Upper side band}}$$

$$+ \underbrace{\frac{A_c \mu}{2} \cos\left(2\pi\left(f_c - f_m\right)t\right)}_{\text{Lower side band}}$$

The power of AM signal is

$$\underbrace{\frac{A_c^2}{2}}_{\text{Carrier power}} + \underbrace{\frac{A_c^2 \mu}{8} + \frac{A_c^2 \mu^2}{8}}_{\text{Side band power}}$$

Power efficiency of AM for single

$$\text{tone} = \frac{\textit{side band power}}{\textit{Total power}} = \frac{\mu^2}{2 + \mu^2}$$

If $\mu = 1$, i.e., for 100% modulation.

$$\% \text{ efficiency} = \frac{1}{1+2} \times 100 = 33.3\%$$

Being $\mu \leq 1$, efficiency $\leq 33.3\%$

$$s(t) = A_c(1 + \mu \cos(2\pi f_m t).\cos(2\pi f_c t)$$

Maximum value of modulated signal:

$$S_{max} = A_c(1 + \mu)$$

Minimum value of modulated signal:

$$S_{min} = A_c(1 - \mu)$$

$$\therefore \quad \mu = \frac{S_{max} - S_{min}}{S_{max} + S_{min}}$$

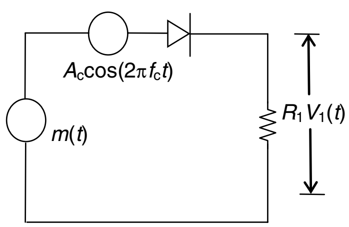
If $\mu = 1$

$$S_{max} = 2A_c$$

$$S_{min} = 0$$

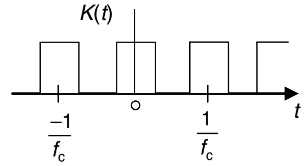
Switching Modulator

This modulator is used to generate AM signal



If the diode is ideal, it works as a perfect switch.

If we assume $A_c \gg |m(t)|$, the diode works as ON/OFF switch in accordance with the polarity of the carrier wave as mentioned in the diagram below.



$k(t)$ can be expressed as Fourier series

$$k(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\left(2\pi f_c t\right) - \frac{\cos\left(6\pi f_c t\right)}{3} + ...\right)$$

The output across 'R' is given by

$$v(t) = (m(t) + A_c \cos(2\pi f_c t))\, K(t) = (m(t) + A_c \cos(2\pi f_c t))$$

$$\left(\frac{1}{2} + \frac{2}{\pi}\cos\left(2\pi f_c t\right) - \frac{2}{3\pi}\cos\left(6\pi f_c t + ...\right)\right)$$

$$= \underbrace{m(t)\frac{1}{2}}_{-w \text{ to } w} + \underbrace{m(t)\frac{2}{\pi}\cos\left(2\pi f_c t\right)}_{f_c - w \text{ to } f_c + w} - \underbrace{m(t)\frac{2}{3\pi}\cos\left(6\pi f_c t\right)}_{3f_c - w \text{ to } 3f_c + w} +$$

$$\underbrace{A_c \cos\left(2\pi f_c t\right)\frac{1}{2}}_{\text{fc}} + \underbrace{\frac{2A_c}{\pi}\cos\left(2\pi f_c t\right)}_{\text{2fc}}$$

If we pass the $v(t)$ through BPF around f_c and with BW 2w

$$s(t) = m(t)\frac{2}{\pi}\cos\left(2\pi f_c t\right) + \frac{A_c}{2}\cos\left(2\pi f_c t\right)$$

$$= \frac{A_c}{2}\cos\left(2\pi f_c t\right)\left(1 + \frac{4}{\pi A_c} m(t)\right)$$

Thus, the amplitude sensitivity of switching modulator is

$$K_a = \frac{4}{\pi A_c}$$

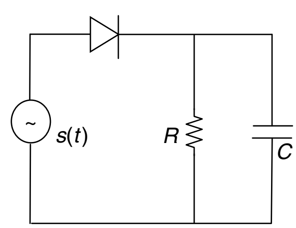
To generate AM without any distortion, the undesired terms in the above multiplication (terms 1, 3, 5…) should not interfere with the desired terms (2 and 4). From the first two terms of $v(t)$,

$$f_c - w > w$$

i.e., $\therefore f_c > 2$ w in switching modulator.

Envelope Detector

Envelope detector is used to demodulate AM wave



If R_f is the forward resistance of diode and R_s is internal resistance of source.

The charging time constant of the envelope detector is $(R_s + R_f).C$.

The discharging time constant is RC.

The envelope detector should track all the variations of envelope and it should not track the carrier variation.

Thus, the condition that the envelope detector must satisfy is

$$\frac{1}{f_c} << RC << \frac{1}{W}$$

The purpose of the diode is to suppress the envelope variation in the negative amplitude side. For this, the charging time constant must satisfy

$$(R_s + R_f)C \ll \frac{1}{f_c}.$$

Double–Side Band Suppressed Carrier (DSB-SC) Modulation

The disadvantage of AM is more than 66.6% of power in AM does not carry any information about message signal.

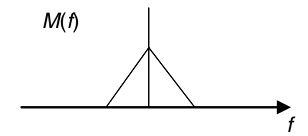
DSB-SC eliminates this disadvantage of AM.

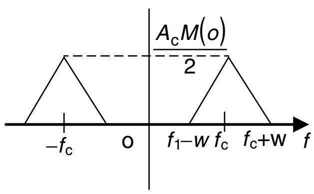
In DSB-SC, the modulated signal is represented by

$$S(t) = A_c \cos(2\pi f_c t)m(t)$$

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

i.e., the spectrum of $m(t)$ is shifted to f_c and $-f_c$.

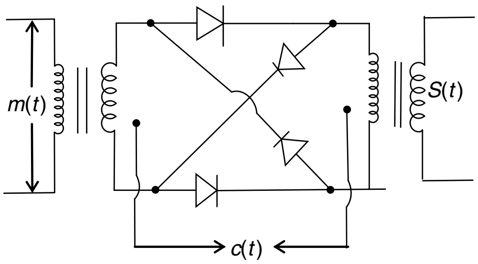




The bandwidth required to transmit DSB-SC is 2W from $f_c - w$ to $f_c + w$.

Ring Modulator

Ring modulator is used to generate DSB-SC



Where $c(t)$ is a square wave with fundamental frequency f_c and duty cycle 50%.

$c(t)$ can be represented in Fourier series as

$$c(t) = \frac{4}{\pi}\left(\cos(2\pi f_c t) - \frac{\cos(6\pi f_c t)}{3} + \frac{\cos(10\pi f_c t)}{5} + \ldots\right)$$

The centre-tapped transformer performs the functioning of multiplier.

$$s(t) = m(t)\ c(t)$$

$$= m(t).\frac{4}{\pi}\left(\cos(2\pi f_c t) - \frac{\cos(6\pi f_c t)}{3} + \ldots.\right)$$

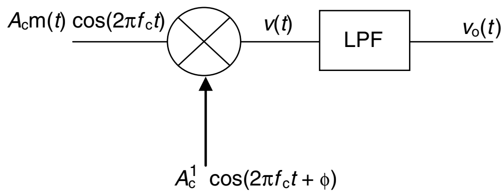
$$= \underbrace{\frac{4}{\pi}m(t)\cos(2\pi f_c t)}_{f_c - w \text{ to } f_c + w} - \underbrace{\frac{4m(t)}{3\pi}\cos(6\pi f_c t)}_{3f_c - w \text{ to } 3f_c + w} +$$

If we pass the output through a BPF with centre frequency f_c and BW 2W, we get the DSB-SC wave.

To avoid interference from the unwanted components of ring modulator, $3f_c - w \geq f_c + w$ i.e., $f_c \geq w$.

Coherent Detection

DSB-SC-modulated wave can be demodulated by using coherent detection.



$$v(t) = A_c m(t) \cos(2\pi f_c t).A_c{}^1 \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c A_c^1}{2} m(t)\cos(\varphi) + \frac{A_c A_c^1}{2} m(t)\cos(4\pi f_c t + \phi)$$

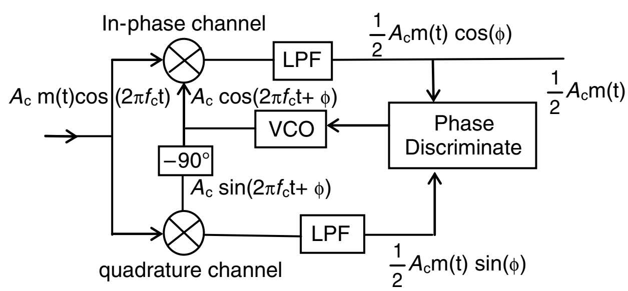
after LPF with bandwidth W, we get

$$v_0(t) = \frac{A_c A_c^1}{2} m(t)\cos(\varphi)$$

If the phase error ϕ in the local oscillator is $\frac{\pi}{2}$, the demodulated output is zero. This is called quadrature null effect.

Costas Receiver

The effect of phase error in coherent detection of DSB-SC can be eliminated by using costas receiver.



Costas receiver has a feedback, which adjusts its phase error to zero by comparing the outputs at in-phase channel and quadrature channel.

Quadrature Carrier Multiplexing

The carriers $A_c \cos(2\pi f_c t)$ and $A_c \sin(2\pi f_c t)$ are orthogonal in nature. Even though both carriers have same frequency f_c, they can be separated by using phase discriminator. This property can be used to multiplex two different message signals at the same frequency f_c by using two orthogonal carriers. If $m_1(t)$ and $m_2(t)$ are message signals, quadrature carrier multiplexing by DSB-SC is given by

$$S(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

If max frequency of $m_1(t)$ and $m_2(t)$ is w, the BW required to transmit both $m_1(t)$ and $m_2(t)$ is 2w.

If max frequency of $m_1(t)$ is w_1 and $m_2(t)$ is w_2 ($w_2 > w_1$), the BW required to transmit both $m_1(t)$ and $m_2(t)$ is $2w_2$.

Single-Side Band Suppressed Carrier Modulation (SSB-SC)

If $m(t)$ is real signal, the frequency spectrum of $m(t)$ satisfy

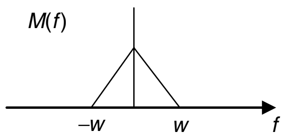
$$|M(-f)| = |M(f)|$$

i.e., The spectrum of $m(t)$ is symmetric with respect to $f = 0$. Thus, in DSB-SC, the spectrum at both the sides of f_c is equal and carries the same information. This redundancy is eliminated by using SSB-SC modulation.

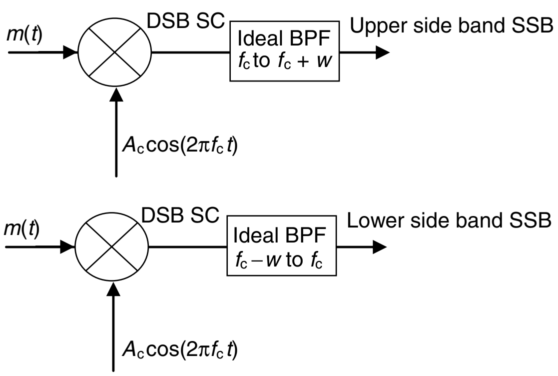
In the SSB-SC modulation, we transmit the spectrum from either f_c to $f_c + w$ or $f_c - w$ to f_c.

f_c to $f_c + w$: upper side band SSB-SC

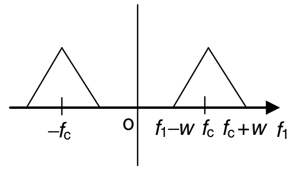
$f_c - w$ to f_c: Lower side band SSB-SC



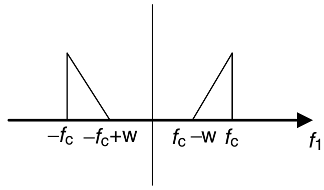
Generation of SSB-SC



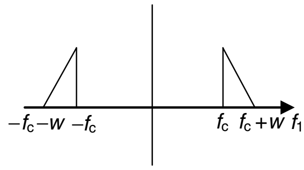
DSB-SC



Lower side band SSB- SC



Upper side band SSB- SC



The transmission bandwidth required in SSB is W from f_c to $f_c + w$ or $f_c - w$ to f_c.

The modulated signal of SSB is given by

For upper side band

$$s(t) = m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)$$

Where $\hat{m}(t)$ is the Hilbert transform of $m(t)$

For lower side band

$$s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)$$

SSB for single tone

$$s(t) = m(t)\cos(2\pi f(t) \pm \hat{m}(t)\sin(2\pi f_c t)$$

If $m(t) = A_m \cos(2\pi f_m t)$

$$s(t) = A_m \cos(2\pi(f_c + f_m)\, t)$$

For lower side band

$$s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)$$

if $m(t) = A_m \cos(2\pi f_m t)$

$$s(t) = A_m \cos 2\pi(f_c - f_m)t$$

Vestigial Side Band Modulation (VSBSC)

The disadvantage of SSB-SC is the requirement of ideal band pass filters. If the BPF is not ideal, the SSB signal gets distorted at and around carrier frequency f_c. Vestigial side band modulation (VSBSC) eliminates this drawback. For VSBSC, ideal BPF is not essential, but BPF should satisfy the condition

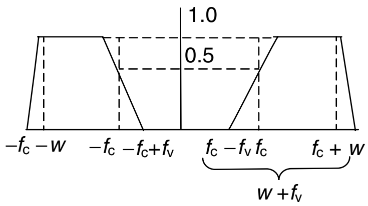
$$H(f - f_c) + H(f + f_c) = 2H(f_c)$$

If we take $H(f_c) = \frac{1}{2}$

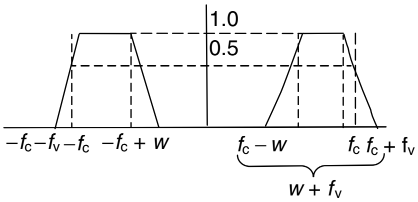
The condition for VSBSC filter is $H(f - f_c) + H(f + f_c) = 1$

i.e., the filter will have all vestigial of small frequency f_v in the other side band.

Upper Side Band $H(f)$

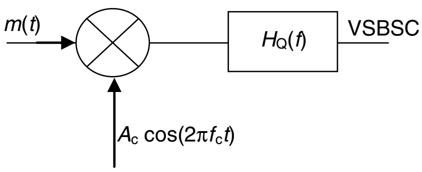


Lower Side Band $H(f)$



The transmission bandwidth required in VSBSC is $w + f_v$

Generation of VSBSC



VSBSC can be represented by

$$s(t) = m(t)\cos(2\pi f_c t) \pm m_Q(t)\sin(2\pi f_c t)$$

Where $m_Q(t)$ is the output of the filter $H_Q(f)$ to the input is $m(t)$. $H_Q(f)$ is low-pass envelope of $H(f)$.

If $f_v = 0$, VSBSC modulation becomes SSBSC modulation.

If a single-tone AM with $\mu = 1$ requires 100 W, DSB-SC requires 33.3 W and SSB-SC requires 16.6 W only.

Solved Examples

Example 1

Consider the AM signal

$$s(t) = A_C\cos(w_c t) + 5\cos(w_m t)\cos(w_c t)$$

The minimum value of A_c to avoid over modulation is

(A) 1 (B) 5 (C) 0 (D) 2.5

Solution

$$s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t)$$

To avoid over modulation, the condition is

$$|K_a m(t)| < 1$$

In the given AM signal

$$S(t) = A_c\cos(w(t))\left[1 + \frac{5}{A_c}\cos(w_m t)\right]$$

To avoid over modulation

$$\frac{5}{A_c} < 1$$

$$\therefore \quad A_C > 5$$

Example 2

An AM signal is detected using a envelope detector. The carrier frequency and modulating signal frequency are 100 MHz and 10 kHz. An approximate value of time constant of envelope detector is

(A) 5 ns (B) 50 µs (C) 2 µs (D) 100 µs

Solution

For envelope detector

$$\frac{1}{f_c} << RC << \frac{1}{W}$$

$$10 \text{ ns} \ll RC \ll 100\ \mu s$$

The best value of RC in the given multiple choice is RC = 2 µs.

Example 3

A modulated signal is given by $s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$. The maximum frequency in $m_1(t)$ and $m_2(t)$ is 20 kHz and 30 kHz, respectively. The bandwidth of the modulated signal is

(A) 100 kHz (B) 50 kHz (C) 40 kHz (D) 60 kHz

Solution

The given $s(t)$ is a quadrature multiplexed signal.

The BW required for $m(t)\cos(2\pi f_c t)$ is $f_c \pm 20$ kHz.

The BW required for $m_2(t)\sin(2\pi f_c(t)$ is $f_c \pm 30$ kHz

The BW required for $S(t)$ is

Max $[f_c \pm 20\text{ kHz}, f_c \pm 30\text{ kHz}] = f_c \pm 30$ kHz

= 60 kHz BW is required.

Example 4

A message signal

$$m(t) = \frac{1}{3}\cos(w_1 t) - \frac{1}{2}\cos(w_2 t)$$

is amplitude modulated with a carrier of frequency w_c to generate.

$$s(t) = [1 + m(t)]\cos(w_c t)$$

The power efficiency achieved by this AM scheme is

(A) 8% (B) 12% (C) 16% (D) 25%

Solution

$$s(t) = (1 + m(t))\cos(w_c t) = \cos(w_c t) + m(t)\cos(w_c t)$$

$$\cos(w_c t) + \frac{1}{6}\cos((w_c + w_1)t) + \frac{1}{6}\cos((w_c - w_1)t)$$

$$-\frac{1}{4}\cos((w_c + w_2)t) - \frac{1}{4}\cos((w_c - w_2)t)$$

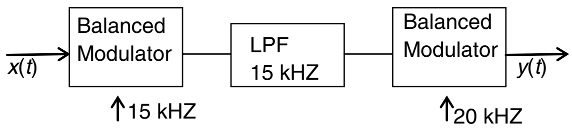
Out of above five terms, first term is the carrier component and all remaining terms are side band components.

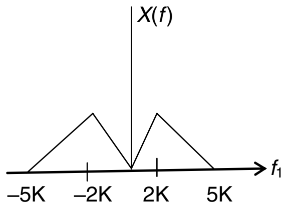
$$\text{Power efficiency of AM} = \frac{\text{side band power}}{\text{total power}}$$

$$= \frac{\frac{1}{36}+\frac{1}{16}}{1+\frac{1}{16}+\frac{1}{36}} = 0.08 = 8\%$$

Example 5

Consider the system shown below. Let $x(f)$ and $Y(f)$ denote the Fourier transform of $x(t)$ and $y(t)$, respectively. The ideal LPF has the cut-off frequency 15 kHz.





The positive frequencies where $Y(f)$ has spectrum peaks are

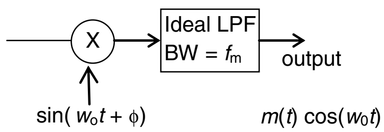
(A) 11K and 13K (B) 11K and 33K
(C) 7K and 31K (D) 7K and 33K

Solution

$X(f)$ has the spectral peaks at –2K and 2K,
After first balanced modulator, the spectral, peaks shifts to $15 \pm 2 = 13\text{K}$ and 17K,
After 15K LPF, the spectral peak exist at 13K.
After second balanced modulator, the spectral peak shifts to $20 \pm 13\text{K} = 7\text{K}$ and 33K

Example 6

A message $m(t)$ band limited to the frequency f_m has a power of P. The power of the output signal in the below figure is



(A) $P \cos 2\phi$ (B) $\frac{P\cos^2\phi}{4}$

(C) $p \sin^2\phi$ (D) $\frac{P\sin^2\phi}{4}$

Solution

The output of product modulator is

$$m(t)\cos(w_0 t)\,.\,\sin(w_o t+\phi)$$

$$\frac{1}{2}m(t)\sin(2w_0 t+\phi)+\frac{1}{2}m(t)\sin(\phi)$$

The output of LPF is $\frac{1}{2}m(t)\sin\phi$

The power of the output is $\frac{1}{4}P\cdot\sin^2\phi$

Example 7

A 1 MHz sinusoidal carrier is amplitude modulated by a symmetric square wave of period 50 μs. Which one of the following frequencies will be present in the modulated signal?

(A) 1,010 kHz (B) 1,020 kHz
(C) 1,030 kHz (D) 1,040 kHz

Solution

50 μs symmetrical square wave has the fundamental frequency $f_0 = \frac{1}{50\mu s} = 20$ kHz.

The square wave can be represented as follows:

$$m(t) = \frac{4A}{\pi}\left[\cos(2\pi f_0 t) - \frac{\cos(6\pi f_0 t)}{3} + \cdots\right]$$

i.e., $m(t)$ have the frequencies f_0, $3f_0$, $5f_0$,…….

20K, 60K, 100K,……

If we amplitude modulate $m(t)$, the modulated signal will have

1 MHz ± 20K, 1 MHz ± 60K…

Frequency components

Frequency Modulation

In frequency modulation (FM), the instantaneous frequency of modulated signal is proportional to the message signal $m(t)$.

$$f_i(t) = f_c + k_f m(t)$$

where k_f is the frequency sensitivity of FM.

The relation between instantaneous frequency and phase is

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi_i(t)}{dt}$$

$$\therefore \phi_i(t) = 2\pi\int_0^t f_i(t)\,dt$$

For frequency modulation

$$\therefore \phi_i(t) = 2\pi\int_0^t \left(f_c + k_f m(t)\right)dt$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt$$

The modulated wave

$$s(t) = A_c\cos(\varphi_i(t)) = A_c\cos(2\pi f_c t + 2\pi k_f \int_o^t m(t)\,dt)$$

The power of frequency modulated wave is $\frac{A_c^2}{2}$.

If $m(t)$ is single tone, $m(t) = A_m\cos(2\pi f_m t)$

$$f_i(t) = f_c + k_f A_m\cos(2\pi f_m t)$$

$k_f A_m = \Delta f$ is called maximum frequency deviation.

$$s(t) = A_c\cos(2\pi f_c t + \frac{2\pi k_f A_m}{2\pi f_m}\sin 2\pi f_m t)$$

$\frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \beta$ is called modulation index for FM

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

β is the maximum phase change in the FM signal.

Narrow Band FM

If $\beta \ll 1$,

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$
$$= A_c \cos(2\pi f_c t) \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t)).$$

This can be approximated as

$$s(t) \cong A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$
$$= A_c \cos(2\pi f_c t) - \frac{\beta A_c}{2} \cos(2\pi(f_c - f_m)t) + \frac{\beta A_c}{2} \cos(2\pi(f_c + f_m)t)$$

The narrow band FM signal appears like AM signal.

The only difference in NBFM is that the sign of lower side band is negative, whereas in AM, the sign of lower side band is positive.

The bandwidth required for the transmission of NBFM is $2f_m$.

Wide Band FM ($\alpha > 0.5$)

If β is large, the FM signal can be expressed in terms of Bessel function $J_n(\beta)$.

$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ can be expressed as follows:

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

i.e., $s(t)$ consists of frequencies $f_c \pm n f_m$

The properties of Bessel function are as follows:

(i) $J_n(\beta) = (-1)^n J_{-n}(\beta)$

(ii) $J_o(\beta) \cong 1$

(iii) $J_1(\beta) \cong \frac{\beta}{2}$

(iv) $J_n(\beta)$ decreases as n increases and $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

For a given β, from $J_n(\beta)$ table, we can calculate the magnitude of each frequency component in the $s(t)$ spectrum.

Bandwidth of FM

The bandwidth required in FM is given by Carson rule.

$$\text{BW} = 2(\Delta f + w)$$

Where Δf is the maximum frequency deviation and w is the maximum frequency of $m(t)$.

$$\Delta f = \frac{k_f (m_{\max} - m_{\min})}{2}$$

For single tone

$$\text{BW} = 2(\Delta f + f_m) = 2f_m(1 + \beta)$$

If modulation index β increases, the BW required will increase.

In the frequency modulation of a modulating signal $m(t)$

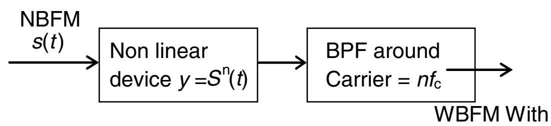
$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_o^t m(t)dt)$$

The maximum phase deviation is given by $2\pi k_f$ max $\left(\int_o^t m(t)dt \right)$

Maximum frequency deviation is given by $K_f.\max(m(t))$.

FM Modulators

There are two ways of generating FM signal, namely indirect method and direct method. In the indirect method, first narrow band FM is generated and then by using non-linear devices wide band FM is generated from NBFM.

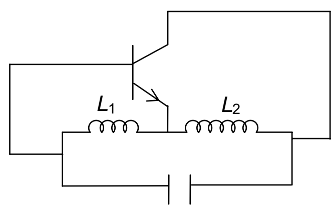


freq deviation : Δf freq deviation : n Δf

Modulation index : β Modulation index : nβ

By using non-linear devices, we can increase the modulation index by integer multiples. The carrier frequency of NBFM can be shifted from nf_c to the required value by using mixers.

In the direct method of generating FM, we use Hartley oscillator with voltage-controlled capacitors, such as varactor or varicap diode.



The capacitance offered by $c(t)$ varies according to voltage applied across it.

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)c(t)}}$$

If we assume sinusoidal modulating signal with frequency f_m, the $c(t)$ can be given as follows:

$$c(t) = c_o + \Delta c \cos(2\pi f_m t)$$

The instantaneous frequency can be given as follows:

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C_o.\left(1 + \frac{\Delta c}{c_o}\cos(2\pi f_m t)\right)}}$$
$$= f_0 \left(1 + \frac{\Delta c}{c_o}\cos(2\pi f_m t)\right)^{-1/2}$$
$$= f_o - \frac{\Delta c.f_o}{2c_o}\cos(2\pi f_m t)$$

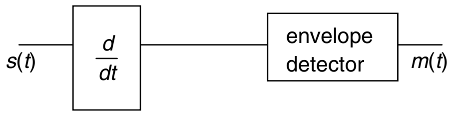
Where $f_o = \dfrac{1}{2\pi\sqrt{(L_1+L_2)c_o}}$

For Hartely oscillator,

$$\Delta f = \frac{-\Delta c.f_o}{2c_o}$$

Demodulation of FM

Any frequency discriminator circuit whose transfer function $|H(f)| = j2\pi f + k_o$ demodulates FM. If we choose $k_o = 0$, $|H(f)| = j2\pi f$. That is, a simple differentiator circuit followed by envelope detector demodulates FM.



$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_o^t m(t)\,dt)$$

$$\frac{d}{dt}s(t) = A_c sin\left(2\pi f_c t + 2\pi k_f \int_o^t m(t)\,dt\right)[2\pi f_c + 2\pi k_f m(t)]$$

The envelope detector eliminates high frequency.

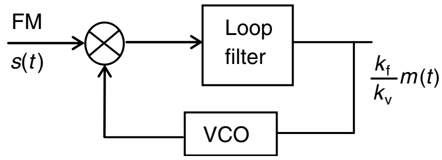
The output of envelope detector is $2\pi f_c + 2\pi k_f m(t)$).

A simple high-pass filter can be used as differentiator.

$$H(f) = \frac{j2\pi fRC}{1 + j2\pi fRC}.\text{if } 1 >> j2\pi fRC, H(f) \cong j2\pi f\text{RC}$$

Another method of demodulating FM is by phase-locked loop.

PLL consists of a multiplier, loop filter, and a voltage control oscillator



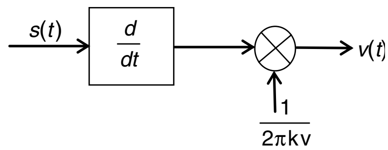
The loop filter works as a low-pass filter.

If $s(t) = A_c\cos(2\pi f_c t + \phi_1(t))$

Then, the output of PLL can be given as

$$v(t) = \frac{1}{2\pi k_v}\frac{d\phi_1(t)}{dt}$$

i.e., The PLL can be approximated as



PLL can also be used for phase synchronization, frequency multiplication, division etc.

Demodulation of FM Waves

FM discriminators can be divided into two types.

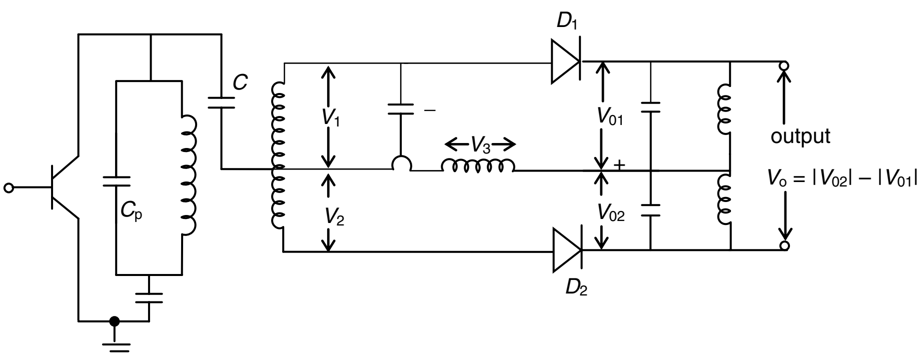
1. Slope detectors

(a) Simple slope detector
(b) Balanced slope detector

2. Phase difference discriminators

(a) Foster–Seeley discrimination.
(b) Ratio detector.

Foster–Seeley Discriminator



Foster–Seeley discriminator circuit shown in figure. It is also known as phase shift discriminator. Nowadays, for the detection of FM wave, their uses are diminishing because of new techniques having ICs in the circuit.

In this circuit, the tank circuit is tuned exactly at the carrier frequency.

Foster–Seeley discriminators are sensitive to both frequency and amplitude variations. So a limiter stage is used before the detector to remove amplitude variations in the signal.

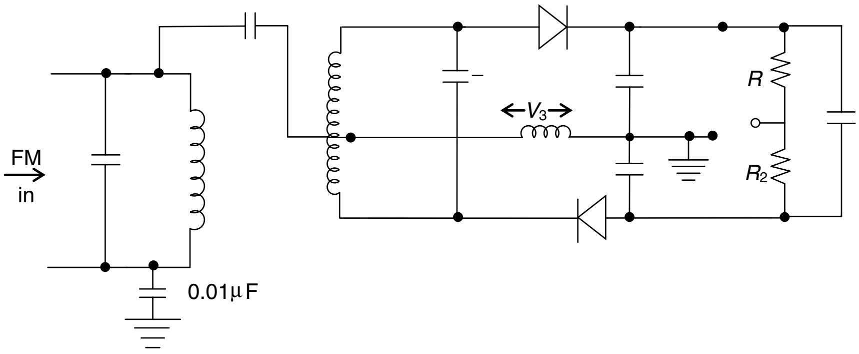
For lower amplitudes, limiter acts as a class A amplifier.

For higher amplitudes, limiter acts as a class C amplifier.

It gives excellent liner response to wide band FM signals.

Ratio Detector

Ratio detector is an improved version of Foster–Seeley discriminator. The ratio detector does not respond to amplitude variations. So no need to use limiter before detection.



Ratio detector responds to only frequency changes of the input signal.

The circuit is almost same as Foster–Seeley discriminator except one of the diode has place in reverse order.

Disadvantage: The major disadvantage of this design is that it requires transformers for coupling and balance.

NOTE

Foster–Seeley or ratio detector may drift out alignment over time. The best alignment is done by sweep generator.

Phase Modulation

If the phase of carrier is proportional to $m(t)$, the signal is called phase modulated.

$$\phi_i(t) = \phi_o(t) + k_p m(t) = 2\pi f_c t + k_p m(t)$$

For PM

$$\therefore \quad s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

The maximum phase deviation is given by $k_p.\max(m(t))$.

$$\phi_i(t) = 2\pi f_c t + k_p m(t)$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi_i(t)}{dt} = f_c + \frac{Kp}{2\pi}.\frac{dm(t)}{dt}$$

$\therefore$ Maximum frequency deviation of PM is given by

$$\frac{Kp}{2\pi}.\max\frac{dm(t)}{dt}$$

The bandwidth required to transmit PM signal is $= 2\Delta f + 2f_m$

$$= \frac{Kp}{\pi}\max\frac{dm(t)}{dt} + 2f_m$$

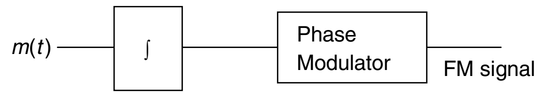
For FM

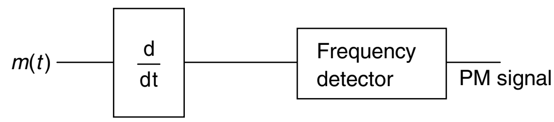
$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t)\,dt\,)$$

For PM

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

The modulated signals for FM and PM are similar except the integration of $m(t)$.



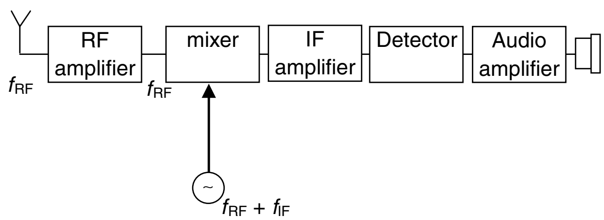


Thus, frequency modulator can be used as a modulator for PM. The demodulation techniques of FM can be used to demodulate PM.

Frequency modulation and phase modulation are called angle modulation.

Superheterodyne Receiver

Superheterodyne receiver converts the RF signal into IF (intermediate frequency) signal before converting it to audio frequency. The advantage of converting IF is to improve frequency selectivity and to provide better amplification.



The frequency of local oscillator for tuning of f_{RF} is $f_{RF} + f_{IF}$

The f_{LO} frequency is higher than RF frequency and IF frequency. For the RF frequency f_{RF}, $f_{RF} + 2f_{IF}$ is called the image frequency.

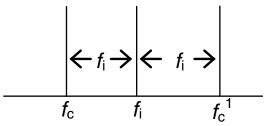
Stages of Superheterodyne Receiver

RF amplifier: This is class C-tuned voltage amplifier.

The main functions of this stage are as follows:

1. Amplification of radio signal for better sensitivity and improved SNR.
2. Rejection of image signal.
3. Rejection of unwanted signals and improved channel selectivity.

Image signal: This is the signal whose frequency is twice the intermediate frequency. Intermediate frequency is always set as 455 kHz in super heterodyne receiver.



where f_c^1 = frequency of image signal.

f_c = frequency of desired signal.

f_i = Intermediate frequency.

Mixer: Mixer performs the job of frequency changers. Mixer mixes the incoming signal frequency (f_c) with local oscillator frequency (f_L) and generates an output voltage of intermediate frequency ($f_L - f_C$).

The mixer stage is also known as first detector.

Local Oscillator

Superheterodyne receivers up to 36 MHz use mostly Armstrong or Hartley oscillator.

In superheterodyne receivers, the local oscillator frequency is always kept higher than the signal frequency by an amount equal to the intermediate frequency.

IF Amplifier: Most of the gain is provided by this stage and used to get a good sensitivity.

Detector or Second Detector

We obtain automatic volume control (AVC) bias from this stage to keep receiver output almost constant with time for any variations in receiver input voltage.

Audio amplifier: The fidelity of receiver is determined by the frequency response of this stage. Fidelity is directly proportional to the bandwidth of the stage.

Example 8

Consider the angle-modulated signal

$$S(t) = 10\cos(2\pi \times 10^8 t + 1{,}000\cos(2{,}000\pi t))$$

The average power of $s(t)$ is

(A) 100 W (B) 1,000 W (C) 50 W (D) 500 W

Solution

The envelope of angle modulated signal is constant = 10 volt.
The average power of sinusoidal signal with amplitude

$$\text{A is } \frac{A^2}{2}$$

$\therefore$ The power of $s(t)$ is 50 W

Example 9

A signal $m(t) = 10\cos(2{,}000\pi t)$ is applied to an FM modulator with the sensitivity constant of 10 kHz/V. the modulation index of FM wave is

(A) 100 (B) 50 (C) 200 (D) 20

Solution

Max amplitude of $m(t)$ is 10
Frequency deviation

$$\Delta f = K_f A_m$$

$$f_M = 1{,}000$$

$$\therefore \quad \beta = \frac{100\text{KHZ}}{1000} = 100$$

Example 10

In a FM system, a carrier of 10 MHz is modulated by a sinusoidal signal of 2 kHz. The bandwidth required by Carson approximation is 0.1 MHz. if $y(t)$ = (Modulated wave form)^3. Then, by using Carson's approximation, the BW of $y(t)$ around 30 MHz is

(A) 0.1 MHz (B) 0.2 MHz
(C) 0.3 MHz (D) 0.5 MHz

Solution

FM signal can be given by

$$s(t) = A_C\cos(2\pi f_c t + \beta\sin(2\pi f_m t))$$

Where f_c = 10 MHz

$$f_m = 2 \text{ kHz}$$

Given

$$2f_m(1+\beta) = 1.1 \text{ MHz}$$

$$\beta = 24$$

$$y(t) = s^3(t)$$

$$= A_C^3\cos^3(2\pi f_c t + \beta\sin(2\pi f_m t))$$

$$= \frac{A_C^3}{4}\cos(6\pi f_c t + 3\beta\sin(2\pi f_m t))$$

In the above FM wave carrier = $3f_c$.
Modulation index = 3β
Modulating frequency = f_m

$$\text{BW} = 2f_m(1+3\beta) \sim 0.3 \text{ MHz}$$

Example 11

An angle-modulated signal is given by

$$s(t) = \cos(2\pi(2\times 10^6 t + 20\sin(200t) + 50\sin(250t))$$

The maximum frequency and phase deviations of $s(t)$ are

(A) 70, 15 kHz (B) 140π, 15 kHz
(C) 70, 16.5 kHz (D) 140π, 16.5 kHz

Solution

Maximum phase deviation

$$= 2\pi \times 20 + 2\pi \times 50 = 140\pi$$

Maximum frequency deviation

$$= \frac{1}{2\pi}\frac{d\phi_i(t)}{dt}\bigg|_{max}$$

$$= \max\left(\frac{d}{dt}(20\sin(200t)+50\sin(250t))\right)$$

$$= 4{,}000 + 12{,}500 = 16{,}500 = 16.5 \text{ kHz}$$

Example 12

$c(t)$ and $m(t)$ are used to generate an FM signal. If the peak frequency deviation of the generated FM signal is five times the maximum frequency $f_M = 2$ kHz, then the coefficient of the term $\cos(2\pi(1006\times 10^3)t)$ in the FM signal is (consider $f_c = 1$ MHz)

(A) $J_6(3)$ (B) $J_3(5)$ (C) $J_3(6)$ (D) $J_5(3)$

Solution

Peak frequency deviation = $5f_m$

$$\Delta f = 5f_M$$

$$\beta = 5$$

We can express the single-tone FM signal as

$$s(t) = A\sum_{n=-\infty}^{\infty}\cos(2\pi(f_c + nf_m)t)\cdot J_n(\beta)$$

The coefficient of

$$\cos(2\pi(10^8 + 3\times 2\times 10^3)t) \text{ is}$$

$$J_3(5)$$

Example 13

A carrier is phase modulated with a frequency deviation of 20 kHz by a single-tone frequency of 1 kHz. If the single-tone frequency is increased to 3 kHz, assuming that the phase deviation remains unchanged, the bandwidth of the PM signal is

(A) 42 kHz (B) 66 kHz (C) 126 kHz (D) 46 kHz

Solution

$$\Delta_f = 20 \text{ kHz}$$

$$f_m = 1 \text{ kHz}$$

$$\beta = \frac{\Delta_f}{f_m} = 20$$

BW required = $2f_m(1 + \beta) = 2 \times 3\text{K}(1 + 20) = 126$ kHz

Example 14

Consider the frequency-modulated signal

$$s(t) = 10\cos(2\pi\times 2\times 10^5 t + 10\cos(2\pi\times 1{,}500t) + 15\sin(2\pi\times 2{,}500\pi t))$$

The modulation index of $s(t)$ is

(A) 10 rad (B) 15 rad (C) 25 rad (D) 30π rad

Solution

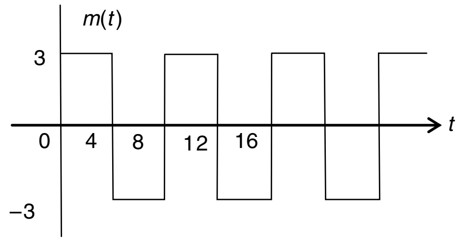
Modulation index = max phase change

$$= \max(10\cos(2\pi\times 1{,}500t) + 15\sin(2\pi\times 2{,}500t))$$

$$= 25 \text{ rad}$$

Example 15

The signal $m(t)$ as shown is applied to a phase modulator with k_p as phase constant and to a frequency modulator with K_f as frequency constant. Both the modulators have the same carrier frequency. The ratio $\frac{K_p}{K_f}$ for the same maximum phase deviation in both modu-lations is



(A) 2π (B) 4π (C) 8π (D) 16π

Solution

Max phase deviation in

$$\text{PM} = K_p \cdot \max(m(t)) = 3\text{Kp}$$

Max phase deviation in

$$\text{FM} = 2\pi K_f \max\left(\int m(t)\,dt\right) = 2\pi k_f \cdot 12$$

Given both the max phase deviations are equal.

$$3K_p = 2\pi K_f \times 12 = \frac{K_p}{K_f} = 8\pi$$

Example 16

A message signal with band width 10 kHz is lower side band SSB modulated with a carrier frequency of $f_c = 200$ kHz. The resulting signal is then passed through a NBFM with carrier frequency 100 MHz. The bandwidth of output of NBFM would be

(A) 20 kHz (B) 200 Hz (C) 10 kHz (D) 380 kHz

Solution

The frequency components in the lower side band SSB would be $f_c - f_m = 190$ kHz

The BW of NBFM is $2f_M = 2 \times 190$ kHz = 380 kHz

Example 17

A 100 MHz carrier of 1 V amplitude and a 1 MHz modulating signal of 1 V are fed to a balanced modulator.

The output of the modulator is passed through an ideal low-pass filter with cut-off frequency 100 MHz. The output of the filter is added with 100 MHz signal of 1 V amplitude and 90° phase shift. The envelope of the resultant signal is

(A) $\sqrt{\frac{5}{4} - \sin(2\pi\times 10^6 t)}$ (B) $\sqrt{\frac{5}{4} + \sin(2\pi\times 10^6 t)}$

(C) $\frac{1}{4} + \sin(2\pi\times 10^6 t)$ (D) constant

Solution

The output of balanced modulator is

$\cos(2\pi\, 10^6 t)\, .\, \cos(2\pi\, .\, 10^8 t)$

$$= \frac{1}{2}\cos(2\pi\times 99\times 10^6 t) + \frac{1}{2}\cos(2\pi\times 101\times 10^6 t)$$

The o/p of LPF would be

$$\frac{1}{2}\cos(2\pi\times 99\times 10^6 t)$$

The O/P after adder =

$$\frac{1}{2}\cos(2\pi\times 99\times 10^6 t)t + \sin(100\times 2\pi\times 10^6 t)$$

$$= \frac{1}{2}\cos(2\pi\times(100-1)\times 10^6 t)t + \sin(100\times 2\pi\times 10^6 t)$$

$$= \frac{1}{2}\cos(2\pi\times 100\times 10^6 t)\cos(2\pi\times(10^6 t)$$

$$+ \frac{1}{2}\sin(2\pi\times 100\times 10^6 t)\sin(2\pi\times 10^6 t)$$

$$+ \sin(2\pi\times 100\times 10^6 t)$$

$$= \cos(2\pi\times 100\times 10^6 t)\left[\frac{1}{2}\cos(2\pi\times 10^6 t)\right]$$

$$+ \sin(2\pi\times 100\times 10^6 t)\left[1 + \frac{1}{2}\cos(2\pi\times 10^6 t)\right]$$

The output of envelope detector is

$$\left[\frac{1}{2}\cos(2\pi\times 10^6 t)\right]^2 \left[1 + \frac{1}{2}\sin(2\pi\times 10^6 t)^2\right]^{1/2}$$

$$= \left[\frac{5}{4}\sin(2\pi\times 10^6 t)\right]^{1/2}$$

Example 18

A super heterodyne receiver is to operate in the frequency range of 800 kHz, – 1,600 kHz, with the intermediate frequency of 400 kHz. Let R = $\frac{C_{\text{max}}}{C_{\text{min}}}$ denote the required capacitance ratio of local oscillator, then the ratio of R is

(A) 2.0 (B) 4.0 (C) 2.8 (D) 4.8

Solution

$$f_{\text{Lo}} = f_{\text{RF}} + f_{\text{IF}}$$

$$\max f_{\text{LO}} = 1{,}600 + 400 = 2{,}000 \text{ kHz}$$

$$\min f_{\text{LO}} = 800 + 400 = 1{,}200 \text{ kHz}$$

$$f_{\text{LO}} = \frac{1}{2\pi\sqrt{LC}}$$

$$\sqrt{\frac{C_{\text{max}}}{C_{\text{min}}}} = \frac{f_{\text{LO}} \max}{f_{\text{LO}} \min}$$

$$R = \frac{C_{\text{max}}}{C_{\text{min}}} = \left(\frac{20}{12}\right)^2$$

$$= \frac{400}{144} = 2.8$$

Noise in Analog Modulations

Figure of Merit

Figure of merit of a receiver indicates the ability of a receiver to suppress the received noise.

$$\text{Figure of merit} = \frac{(SNR)_o}{(SNR)_c}$$

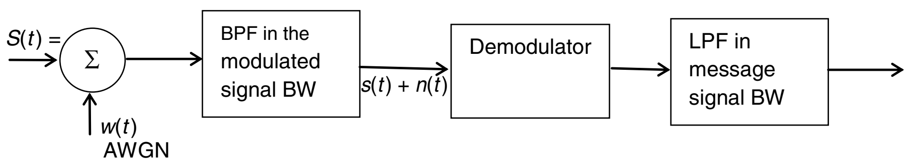
$(\text{SNR})_{\text{o}}$ = signal-to-noise ratio at the output of receiver

$$= \frac{\text{Average power of demodulated message signal}}{\text{average power of noise at receiver output}}$$

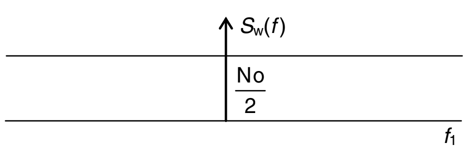
$(\text{SNR})_{\text{c}}$ = signal-to-channel noise ratio at the input of receiver.

$$= \frac{\text{Average power of modulated signal}}{\text{average power of noise in the message BW at output of receiver}}$$

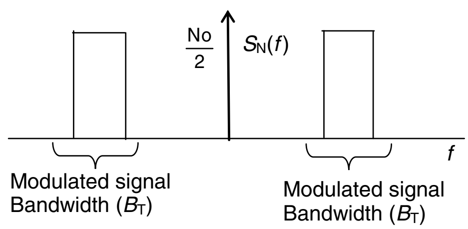
The noise at the input of the receiver is assumed to be AWGN with power spectrum density $\frac{No}{2}$. The receiver model is as mentioned below.



The noise spectrum density of $w(t)$ is



The noise spectrum density after BPF is



$$x(t) = s(t) + n(t)$$

being the noise at output of BPF is a band-pass signal, $n(t)$ can be expressed as follows:

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$n(t) = n_I(t) + jn_Q(t)$$

$n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature components of $n(t)$. $n_I(t)$ and $n_Q(t)$ are low-frequency components.

The spectrum density of $n_I(t)$ and $n_Q(t)$ is

$$S_{NI}(f) = S_{NQ}(f) = N_o \text{ in } f \in \left(\frac{-B_T}{2}, \frac{B_T}{2}\right)$$

i.e., the spectrum density of in-phase and quadrature components is twice the spectrum density of $w(t)$ or $n(t)$ in the frequency range $\left(\frac{-B_T}{2}, \frac{B_T}{2}\right)$, where B_T is transmission bandwidth.

Figure of Merit of DSB-SC Receiver

This receiver consists of a BPF with BW 2W and coherent detector.

The received signal

$$x(t) = C\,A_c\cos(2\pi f_c t)\,m(t) + n(t)$$

The received signal power $= \frac{1}{2}C^2 A_c{}^2 P$

where p is the average power of $m(t)$.

The noise power at the input of receiver $= 2W \cdot \frac{No}{2} = \text{WN}_\text{O}$

$$(\text{SNR})_c = \frac{C^2 A_c{}^2 P}{2WNo}$$

$$x(t) = s(t) + n(t)$$

$= C\,A_c\cos(2\pi f_c t)\,m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$

$= (C\,A_c\,m(t) + n_I(t))\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$

$n(t)$ is applied to a coherent detector. The output $x(t)\cos(2\pi f_c t)$

$= (C\,A_c m(t) + n_I(t))\cos^2(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$

$$= \frac{1}{2}(CA_C m(t) + n_1(t)) + \frac{1}{2}(CA_C m(t) + n_1(t))\cos(4\pi f_c t) - \frac{1}{2}n_Q(t)\sin(4\pi f_c t)$$

If we pass the output through LPF

$$v(t) = \underbrace{\frac{1}{2}CA_C m(t)}_{\text{Required component}} + \underbrace{\frac{1}{2}n_1(t)}_{\text{unwanted component}}$$

Signal power at output of receiver $= \frac{C^2 A^2 P}{4}$

Noise power at output of receiver

$$= \frac{1}{4}\cdot 2WNo = \frac{WNo}{2}$$

$$(\text{SNR})_o = \frac{C^2 A_c^2 P}{2WNo}$$

Figure of merit of DSB-SC receiver $= \frac{(SNR)_o}{(SNR)_c} = 1$

Figure of Merit in SSB-SC Receiver

In the SSB SC

$$s(t) = \frac{1}{2}CA_c\cos(2\pi f_c t)m(t) \pm \frac{1}{2}CA_c\sin(2\pi f_c t)m(t)$$

If the power of $m(t)$ is P

The power of $s(t)$ is $\frac{C^2 A_c^2 P}{8} + \frac{C^2 A_c^2 P}{8}$

$$= \frac{C^2 A_c^2 P}{4}$$

The power of noise at receiver input

$$= \frac{2W.No}{2} = WNo$$

$$(SNR)_c = \frac{C^2 A_c^2 P}{4WNo}$$

The received signal =

$x(t) = s(t) + n(t)$

$$= \frac{1}{2}\left|A_c\cos(2\pi f_c t)m(t) \pm \frac{1}{2}\right|A_c\sin(2\pi f_c t)m(t)$$

$$+ n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$= \frac{1}{2}(A_c m(t) + n_1(t)\cos(2\pi f_c t) + \frac{1}{2}(A_c m(t) + n_Q(t))$$

$$\sin(2\pi f_c t)\left(\frac{1}{2}A_c m(t) + n_Q(t))\sin(2\pi f_c t)\right)$$

By using coherent detection, the output will be

$x(t)\cos(2\pi f_c t)$. If we pass the output of coherent detector through a LPF.

$$v(t) = \underbrace{\frac{1}{2}A_c m(t)}_{\text{Required signal}} + \underbrace{n_1(t)}_{\text{unwanted signal}}$$

Signal power at output receiver $\frac{1}{4}C^2 A_c^2 P$

Noise power at output of receiver $= W.N_o$

$$(\text{SNR})_o = \frac{C^2 A_c^2 P}{4WNo}$$

Figure of merit $= \frac{(SNR)_o}{(SNR)_1} = 1$

Figure of Merit in AM Receivers

In AM, the received signal is

$$x(t) = A_c[1 + k_a(m(t)]\cos(2\pi f_c t) + n(t)$$

Signal power at the input of receiver $= \frac{A_c^2}{2}\left[1+k_a^2 P\right]$

Noise power at the input of receiver $= \frac{2WNo}{2} = WNo$

$$(SNR)_c = \frac{A_c^2\left|1+k_a^2 P\right|}{2WNo}$$

The AM signal is detected by using envelope detector

$$x(t) = A_c[(1 + k_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$= (A_c(1 + k_a\, m(t) + n_I(t))\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

The output of envelope detector is

$$((A_c + A_c K_a m(t) + n_I(t))^2 + n_Q{}^2(t))^{1/2}$$

$$\cong \underbrace{A_c}_{\text{DC}} + \underbrace{A_c K_a m(t)}_{\text{Required signal}} + \underbrace{n_I(t)}_{\text{Unwanted signal}}$$

The signal power at output of receiver $= A_c{}^2\, k_a{}^2 p$
Noise power at output of receiver $= 2W.N_o$.

$$(SNR)_o = \frac{A_c^2 k_a^2 P}{2WNo}$$

Figure of merit $= \frac{(SNR)_o}{(SNR)_c} = \frac{k_a^2 P}{1+k_a^2 P}$

∴ Figure of merit of AM receiver is always less than unity.
For a single-tone signal, $m(t) = A_m\cos(2\pi f_m t)$

$$P = \frac{A_m^2}{2}$$

Figure of merit $= \frac{\frac{1}{2}k_a^2 A_m^2}{1+\frac{1}{2}k_a^2 A_m^2} = \frac{\mu^2}{2+\mu^2}$

Where μ is the modulation index in amplitude modulation for single tone $= k_a A_m$

Threshold Effect

If the carrier-to-noise ratio is small compared with unity, the noise term dominates and the performance of envelope detector completely effected by noise. The performance of envelope detector deteriorates much more rapidly than proportionately to the carrier-to-noise ratio. This effect is called threshold effect.

Every non-linear detector exhibits a threshold effect.

Linear detectors, such as coherent detectors, do not exhibit threshold effect.

Signal-to-noise ratio in AM should be more than 6dB to avoid threshold effect.

Figure of Merit of FM Receiver

The input to FM demodulator is given by

$$x(t) = A_C\cos\left(2\pi f_C t + 2\pi k_f\int_0^t m(t)dt\right) + n(t)$$

The signal power at input $= \frac{A_C^2}{2}$

Noise power at input $= 2w \times \frac{No}{2} = wN_0$

$$(\text{SNR})_C = \frac{A_C^2}{2WNo}$$

FM signal is demodulated by frequency discriminator.

The output of frequency discriminator is given by

$$v(t) = \underbrace{K_f m(t)}_{\text{required component}} + \underbrace{\frac{1}{2\pi A_C}\frac{dn_Q(t)}{dt}}_{\text{unwanted component}}$$

When $n_Q(t)$ is the quadrature component of noise.

Power spectral density of $n_Q(t) = N_O$

Power spectral density of $\frac{dn_Q(t)}{dt} = (2\pi f)^2 \cdot N_o$

∴ The spectral density of

$$\frac{1}{2\pi A_C}\frac{dn_Q(t)}{dt} = \frac{f^2}{A_C^2}N_o$$

Total noise power in the message bandwidth $= \int_{-w}^{w}\frac{f^2 N_o}{A_C^2}df = \frac{2N_o W^3}{3A_C^2}$

Total signal power $= K_f{}^2 P$

$$\therefore \quad (\text{SNR})_0 = \frac{3A_C^2 K_f{}^2 P}{2N_o W^3}$$

Figure of merit $= \frac{3K_f{}^2 P}{W^2}$

For single-tone modulation $P = \frac{A_m^2}{2}$

Figure of merit $= \frac{3K_f{}^2 A_C^2}{2W^3} = \frac{3}{2}\beta^2$

Where $\beta = \frac{K_f A_m}{W}$ is the modulation index for single tone.

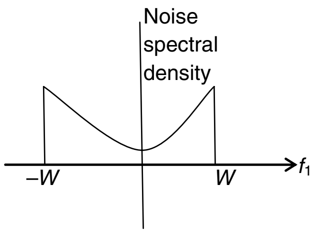
In FM, if β increases, the figure of merit of receiver increases. There is a clear trade-off between BW used in transmission and figure of merit of FM receiver.

In FM modulation, the signal power $\frac{A_C^2}{2} > 20B_T N_o$ to avoid threshold effect.

$(\text{SNR})_C > 13$ dB to avoid threshold effect in FM. There exist number of methods to decrease the FM threshold like phase-locked loop or FMFB (FM demodulator with negative feedback).

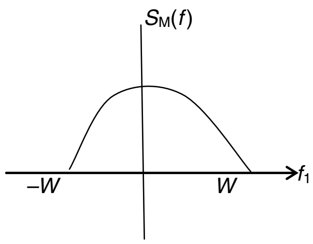
Pre-emphasis and De-emphasis

In FM demodulation, the spectral density of received noise power $= \frac{f^2}{A_C^2} N_o$



i.e., noise spectral density proportional f^2.

On the other hand, in a typical message signal, the low-frequency components dominate. The spectral density of a message signal can be given by



At the demodulator output,

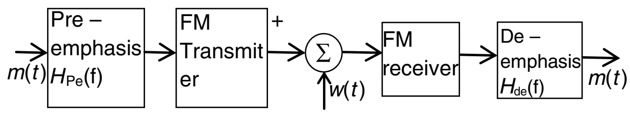
$(SNR)_0$ at low frequencies is very high.

But $(SNR)_0$ at high frequencies is very low.

To reduce the variation of SNR with respect to frequency, pre-emphasis technique is used at the transmitter to boost high frequencies of message signal.

De-emphasis is used at the FM receiver to perform reciprocal action of pre-emphasis.

$$H_{de}(f) = \frac{1}{H_{pe}(f)}$$

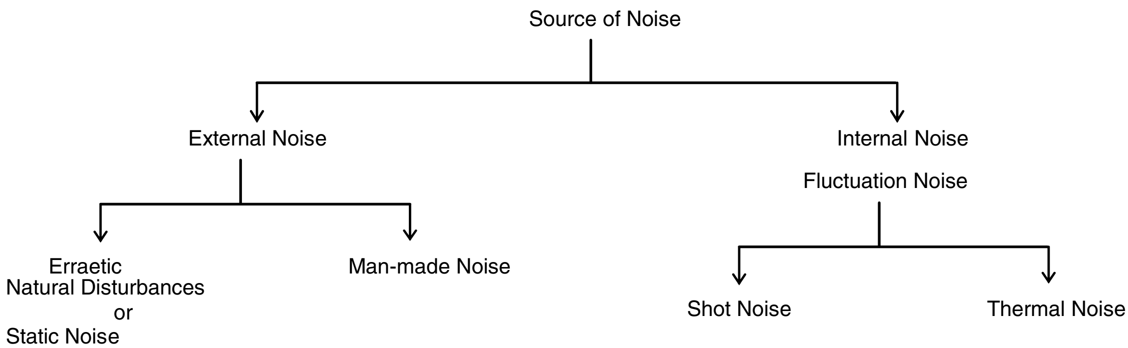


High-pass filters can be used as pre-emphasis filter and low-pass filter can be used as de-emphasis filter.

Sources of Noise

There are various sources of noise.

1. External noise
2. Internal noise.



External Noise

It is created outside the circuit.

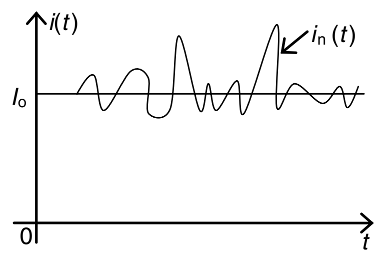
1. **Static noise:** This type of noise is unpredictable in nature. This is also called atmospheric noise. It is less severe above 30 MHz and does not come regularly.
2. **Man-made noise:** This type of noise is under human control and can be eliminated by removing the source of noise. It is effective in the frequency range of 1 MHz–500 MHz.

Internal Noise

Internal noise is generated by active and passive components present within the circuit itself.

Shot Noise

It occurs in the active devices due to random behaviour of charge carriers. In electronic devices, current appears to be continuous, and it is still a discrete phenomenon.



The current fluctuates about a mean value I_o and the current $i_n(t)$ moving around I_o is known as shot noise.

So the total current $i(t)$ may be expressed as follows:

$$i(t) = I_o + i_n(t)$$

where I_o = constant and $i_n(t)$ = shot noise current.

Power Density Spectrum of Shot Noise in Diodes

Shot noise current $i_n(t)$ is a random function and cannot be expressed as a function of time, but it can be expressed by its power density spectrum. Shot noise is Gaussian-distributed with zero mean.

Power density spectrum of shot noise current $i_n(t)$ is given by

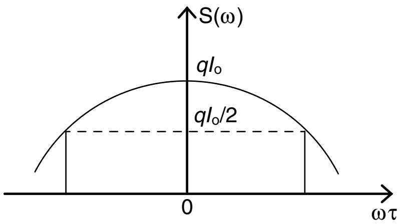
$$S(\omega) = q\, I_o \qquad (1)$$

Where q = electronic charge 1.6×10^{-19} coulombs

I_o = Mean current

From equation (1), we can see that PSD of shot noise current $i_n(t)$ is independent of frequency, but this independence to the frequency is up to a range which is decided by transit time.

After this range, PSD is dependent upon the frequency.



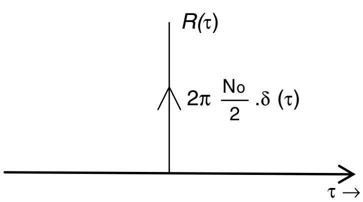
NOTE

Transit time is the time which electrons is taking to reach from anode to cathode.

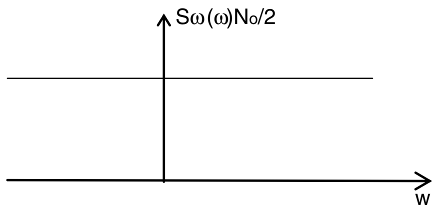
Additional Sources of Noise

1. **Partition noise**
   Shot noise is multigrid noise, and it is called partition noise.
2. **Flicker noise**
   This type of noise arises due to imperfections of cathode surfaces and around the junction surfaces, and this noise can be reduced by proper processing. It becomes significant at very low frequencies generally below few kHz.
3. **Resistor noise**
   Due to the random motion of free electron in a conducting media, noise arises called resistor noise or Johnson noise. The intensity of random motion is proportional to thermal energy supplied. So this noise is also called thermal noise and random motion is zero at a temperature of absolute zero.
4. **White noise**
   White noise contains all the frequencies. The PSD of white noise is constant for all frequencies, and it contains all the frequency components in equal amount. If probability of occurrence of a white noise known as White Gaussian noise, the PSD of white noise are

$$S_W(\omega) = \frac{N_o}{2}$$



Autocorrelation of white noise



PSD of white noise

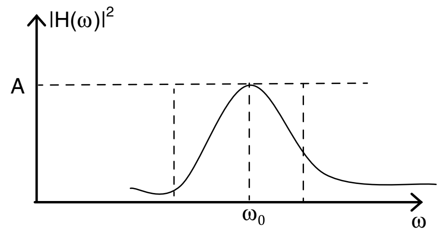
NOTE

Shot noise and thermal noise are considered as Gaussian white noise for practical purpose.

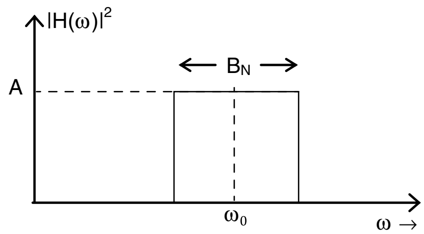
Noise Bandwidth

Equivalent noise bandwidth is the bandwidth of that ideal band-pass system that produces the same noise power as the actual system.

Let us consider an ideal band-pass system with rectangular characteristics of $|H(\omega)|^2$ such that the area under this curve is same as that of an actual system and the height of the ideal curve is equal to A. Which is the maximum value of $|H(\omega)|^2$ is actual system. Then, bandwidth of the ideal system is called as equivalent noise bandwidth denoted by B_N.



$|H(\omega)|^2$ of actual system



$|H(\omega)|^2$ of Ideal system

Equating the areas of actual and ideal systems

$$A.B_N = \int_0^{\infty} |H(\omega)|^2\, d\omega$$

Which gives the expression for noise bandwidth B_N as

$$B_N = \frac{1}{A}\int_0^{\infty} |H(\omega)|^2 d\omega$$

$$= \frac{\text{Area under the } |H(\omega)|^2 \text{ curve of actual system}}{\text{maximum value of the } |H(\omega)|^2 \text{ curve of actual system}}$$

Relation between Noise Bandwidth and 3-dB Bandwidth

3-dB bandwidth of a system is defined as the range of frequencies for which power does not fall below half of the maximum power.

$$B_N \geq B_{3\text{-dB}} \text{ for actual systems}$$

For ideal systems

$$B_N = \frac{\pi}{2} B_{3-dB}$$

Noise figure

It is defined as the ratio of total noise power spectral density S_{no} at the output of the two-port network to the noise power spectral density S^1_{no} at the output assuming the network to be entirely noiseless.

The noise figure F of a two-port network is –

$$F = \frac{(S_{no})_a}{(S^1_{no})_a}$$

When the power density at the output is only due to noise source at the input, the two-port network is assumed to be noiseless.

$$F = \frac{\text{Power density of the total noise at the o/p of network}}{\text{Power density at the output due to the source}}$$

Noise figure in terms of SNR ratio

$$F = \frac{S_i/N_i}{S_o/N_o}$$

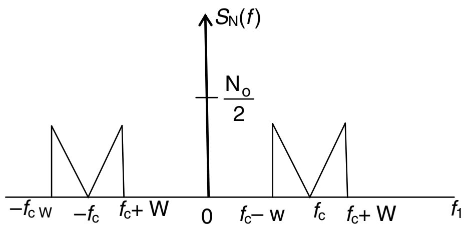
Where S_i/N_i is input signal to noise ratio and S_o/N_o is output signal to noise ratio.

Example 19

Consider the amplitude-modulated signal

$$s(t) = 40\cos(2\pi f_c t) + 20\sin(2\pi f_m t)\cos(2\pi f_c t)$$

The AM signal gets added to a noise with power spectral density $S_n(f)$ given below.
The ratio of average sideband power to mean noise power would be:



(A) $\frac{25}{WN_0}$ (B) $\frac{50}{WN_0}$ (C) $\frac{100}{WN_0}$ (D) $\frac{200}{WN_0}$

Solution

Total side band power is the power of

$$20\sin(2\pi f_m t)\cos(2\pi f_c t)$$

$$= 10\sin(2\pi(f_c + f_m)t) + 10\sin(2\pi(f_c - f_m)t)$$

$$= 50 + 50 = 100 \text{ W}$$

$$\text{noise power} = 4\left[\frac{1}{2}\cdot W \cdot \frac{N_o}{2}\right] = WN_0$$

$$\frac{\text{Side band power}}{\text{Noise power}} = \frac{100}{WN_o}$$

Example 20

A frequency-modulated signal is given by

$$s(t) = 10\cos(2\pi f_c t + 4\sin(2\pi f_m t))$$

The FM signal is demodulated with a frequency discriminator in the presence of AWG noise.
The figure of merit of the receiver is given by
(A) 1 (B) 4 (C) 16 (D) 24

Solution

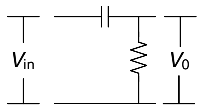
Maximum phase change of $s(t) = \beta = 4$ The figure of merit of FM receiver using frequency discrimination is given by

$$\frac{3}{2}\beta^2 = \frac{3}{2}\times 16 = 24$$

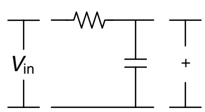
Example 21

Which of the following is a de-emphasis filter in FM receiver?

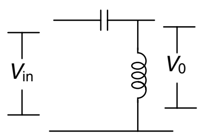
(A)



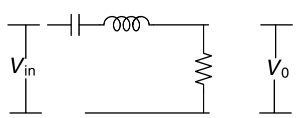
(B)



(C)



(D)



Solution

De-emphasis filter should be a low-pass filter. Out of four options, (B) is the only low-pass filter.

Exercises

Practice Problems I

Direction for questions 1 to 19: Select the correct alternative from the given choices.

1. A frequency-modulated signal is given by $s(t)$ given by $s(t) = 10 \sum_{n=-\infty}^{\infty} J_n(2)\cos(2\pi 10^6 + n \times 10^3)t)$ The power of this frequency-modulated signal is
(A) 100 W (B) 50 W
(C) 25 W (D) Depends on $J_n(2)$

2. A phase-modulated signal is given by $s(t) = 2\cos(2\pi \times 10^6 t) + 5\cos(1{,}000\pi t))$
The maximum frequency deviation of $s(t)$ is
(A) 5 Hz (B) 5,000 ‘’ Hz
(C) 500 Hz (D) 2,500 Hz

3. A frequency-modulated signal is given by $s(t) = 50 \cos(2\pi f_c t + 2\pi k_f \int m(t)dt)$. If this FM signal is transmitted in a AWGN environment with spectral density $\frac{N_O}{2}$ = 1MW/HZ, if the message signal is having a bandwidth of 1 kHz, the noise power at the output of frequency discriminator is
(A) 0.78 W (B) 0.86 W
(C) 0.53 W (D) 0.27 W

4. An Am transmitter is transmitting a power of 100 W. If the modulation index is 30%, the total power available in the side bands is
(A) 4.3 W (B) 8.6 W
(C) 2.15 W (D) 21.5 W

5. A frequency-modulated wave for single tone is given by $s(t) = 10\cos(2\pi \times 10^6 t + 4\cos(2\pi \times 10^4 t))$.If the above FM wave is demodulated by using frequency discriminator, the noise figure of FM receiver is
(A) 24 (B) 48 (C) 12 (D) 6

6. A coaxial/cable support band width range from 10 MHz–12 MHz. Speech signals of maximum frequency 5 kHz, are required to transmit in the coaxial cable with FDM. Maximum number of channels the coaxial cable supports are
(A) 100 (B) 200
(C) 400 (D) 1,000

7. An amplitude modulation scheme is working with a modulation index of 0.8. Power efficiency of this modulation scheme is
(A) 24% (B) 48% (C) 12% (D) 18%

8. A frequency modulator is having frequency sensitivity K_f = 1 kHz/volt. If the modulator is used modulate a single-tone signal of $m(t) = 10\cos(4{,}000\pi t)$ and the modulated signal is demodulated at receiver by using frequency discriminator, the figure of merit of receiver is
(A) 12.5 (B) 25 (C) 37.5 (D) 22.5

9. Figure of merit of SSB-SC receiver using coherent detection is
(A) 0.33
(B) 1
(C) 0.5
(D) depends on power of s(f)

10. The noise power in FM demodulation by using frequency discriminator is proportional to the message band width W by
(A) W (B) W^2 (C) W^3 (D) $\sqrt{W}$

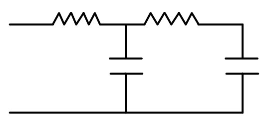
11. A frequency modulated wave is given by $s(t) = 10\cos(2\pi \times 10^6 t + 5\sin(2{,}000\pi t))$ Bandwidth required to transmit above FM wave is
(A) 10 kHz (B) 20 kHz
(C) 12 kHz (D) 24 kHz

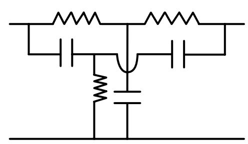
12. An amplitude-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^6 t)(1 + 0.5\sin(1{,}000\pi t))$. Figure of merit of AM receiver to demodulate the above AM wave is
(A) 0.33 (B) 0.25 (C) 1.0 (D) 0.11

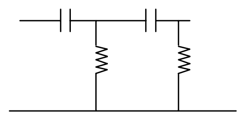
13. A phase-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^6 t + 2\cos(2{,}000\pi t))$. The bandwidth required to transmit this wave form is
(A) 6 K (B) 12 K (C) 10 K (D) 16 K

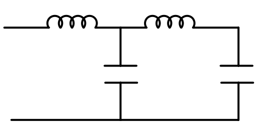
14. An amplitude-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^6 t)(1 + 0.5\cos(1{,}000\pi t) + 0.25\sin(2{,}000\pi t))$
The modulation index of above AM wave is
(A) 0.5 (B) 0.75 (C) 0.25 (D) 0.56

15. Which of the following is a pre-emphasis filter.

(A)


(B)


(C)


(D)


16. It two speech signals $m_1(t)$ and $m_2(t)$ are SSB–SC modulation by using carriers $A_C \cos 2\pi f_c t$ and $A_C \sin 2\Pi f_c t$. The max frequency of $m_1(t)$ and $m_2(t)$ is 4 kHz, the band width required to transmit both $m_1(t)$ and $m_2(t)$ is
(A) 4 kHz (B) 8 kHz
(C) 16 kHz (D) 12 kHz

17. The condition for the carrier frequency f_c in FM to avoid the non-linear effects of channel is

(A) $f_c > 2\Delta f + 2w$ (B) $f_c > 2\Delta f + 3w$
(C) $f_c > 4\Delta f + 3w$ (D) $f_c > 3\Delta f + 2w$

18. If a single-tone frequency 10 kHz is frequency modulated with modulation index 5 and suppose $J_n(5) \simeq 0$ for $n \geq 8$. The approximate bandwidth requisite this FM signal is
(A) 160 kHz (B) 20 kHz
(C) 140 kHz (D) Infinity

19. In an amplitude modulation, the carrier used is 1 MHz and the modulating signal is a single tone with 1 kHz, For the proper demodulation, the suggested value of RC for envelope detector is
(A) $RC = 10^{-3}$s (B) $RC = 10^{-6}$s
(C) $RC = 10^{-4}$s (D) $RC = 10^{-8}$s

Practice Problem 2

Direction for questions 1 to 16: Select the correct alternative from the given choices.

1. The approximate transfer function of a frequency modulation demodulator is

(A) $\dfrac{1}{j\pi f}$ (B) $\dfrac{1}{j2\pi f}$

(C) $j2\pi f$ (D) $\dfrac{1}{j2\pi f} + K$

For $K<<1$

2. A frequency-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^7 t + 5\cos(2\pi f_m t))$. If $J_5(5) \simeq 0$. The approximate bandwidth required to transmit $s(t)$ is
(A) $8f_m$ (B) $10f_m$ (C) $4f_m$ (D) $5f_m$

3. A superheterodyne receiver is working at an intermediate frequency of 10 MHz. If the receiver is tuned to an RF frequency of 150 MHz, the local oscillator frequency and image frequency, respectively, are
(A) 160 MHz, 150 MHz (B) 160 MHz, 170 MHz
(C) 140 MHz, 160 MHz (D) 140 MHz, 130 MHz

4. Two base band message signals $m_1(t)$ and $m_2(t)$ with maximum frequencies of 5 kHz and 6 kHz are quadrature amplitude-modulated with a carrier frequency of 1 MHz. The bandwidth required in this scheme is
(A) 10 K (B) 12 K (C) 22 K (D) 11 K

5. A single-tone signal $m(t) = 10\cos(5{,}000\ \pi\ t)$ is frequency modulated by using a modulator whose frequency sensitivity is 1 kHz/Volt. The BW required to transmit this signal is
(A) 15 kHz (B) 20 kHz
(C) 22 kHz (D) 25 kHz

6. A vestigial side band-modulated signal is generated with a filter whose characteristic satisfies $H(f - f_c) + H(f + f_c) = 1$ for $|f| \leq f_1 + 1$ kHz. If the above modulator is used to modulate a base band signal $m(t)$ of max frequency 5 kHz, the bandwidth required for transmission is
(A) 5 K (B) 10 K (C) 6 K (D) 11 K

7. A DSB-SC signal is demodulated by using coherent detection. Local carrier at the receiver is generated with a phase error of 10°. The figure of merit of this DSB-SC receiver is

(A) 1 (B) $\cos(10°)$

(C) $\cos^2(10°)$ (D) $\dfrac{1}{\cos(10°)}$

8. A switching modulator is used for amplitude modulation. If the carrier is $s(t) = 10\cos(2\pi \times 10^6 t)$, the amplitude sensitivity of the switching modulator is
(A) 0.127 (B) 0.202 (C) 0.5 (D) 0.68

9. A frequency-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^6 t + 2\sin(2{,}000\ \pi\ t)$. If the above FM signal is passed through a non-linear device $y = x^3$, the carrier frequency and modulation index for the FM wave at the output of non-linear device is
(A) 1 MHz, 6 (B) 3 MHz, 2
(C) 3 MHz, 6 (D) 1 MHz, 2

10. A frequency-modulated wave is given by $s(t) = 100\cos(2\pi \times 10^5 t + 0.2\sin(1{,}000\ \pi\ t))$. The bandwidth required to transmit this signal is
(A) 1 kHz (B) 2 kHz (C) 4 kHz (D) 8 kHz

11. A frequency-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^6 t + 3\cos(2{,}000\pi t) + 4\sin(2{,}000\pi t))$. The modulation index of above FM wave is
(A) 3 (B) 4 (C) 5 (D) 7

12. A frequency-modulated signal is given by $s(t) = 10\cos(2\pi \times 10^6 t + 3\sin(2{,}000\pi t) + 4\sin(4{,}000\pi t)$. The modulation index of given FM wave is
(A) 3 (B) 4 (C) 5 (D) 7

13. Bandwidth required to transmit the FM wave mentioned in the previous problem is
(A) 10 K (B) 14 K (C) 10 K (D) 32 K

14. If two random variables x and y are Gaussion and independent with mean zero and unity variance, Two new random variables defined as follows:

$U = x + y$
$V = x - y$

The correlation coefficient between u and v is
(A) 1 (B) 0 (C) 0.5 (D) 0.25

15. If a single-tone FM-modulated signal is passed through a square law device with characteristic output = $(\text{input})^2$. Which of the following is correct with respect to the frequency deviation of the output of the square law device?
(A) Doubled (B) No change
(C) Half (D) Increase four times

16. The transmission band width required to transmit a speech signal of highest frequency 4 kHz and modulation index $\beta = 10$ by using FM is

(A) 44 kHz (B) 80 kHz
(C) 88 kHz (D) 40 kHz

Previous Years' Questions

1. An AM signal is detected using an envelope detector. The carrier frequency and modulating signal frequency are 1 MHz and 2 KHz, respectively. An appropriate value of the time constant of the envelope detector is **[2004]**

(A) 500 μsec (B) 20 μsec
(C) 0.2 μsec (D) 1 μsec

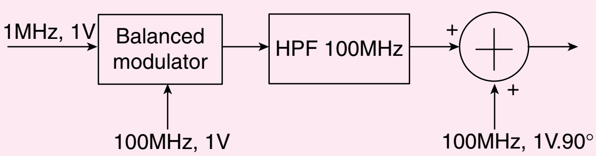
2. An AM signal and a narrowband FM signal with identical carriers, modulating signals, and modulation indices of 0.1 are added together. The resultant signal can be closely approximated by **[2004]**

(A) broad band FM (B) SSB with carrier
(C) DSB – SC (D) SSB without carrier

3. A 1 kHz sinusoidal signal is ideally sampled at 1,500 samples/sec and the sampled signal is passed through an ideal low -pass filter with cut-off frequency 800 Hz. The output signal has the frequency? **[2004]**

(A) 0 Hz (B) 0.75 kHz
(C) 0.5 kHz (D) 0.25 kHz

4. A 100 MHz carrier of 1 V amplitude and a 1 MHz modulating signal of 1 V amplitude are fed to a balanced modulator. The output of the modulator is passed through an ideal high-pass filter with cut-off frequency of 100 MHz, the output of the filter is added with 100 MHz signals of 1 V amplitude and 90° phase shift as shown in Figure. The envelope of the resultant signal is **[2004]**



(A) Constant

(B) $\sqrt{1+\sin\left(2\pi \times 10^{6} t\right)}$

(C) $\sqrt{\frac{5}{4}-\sin\left(2\pi \times 10^{6} t\right)}$

(D) $\sqrt{\frac{5}{4}+\cos\left(2\pi \times 10^{6} t\right)}$

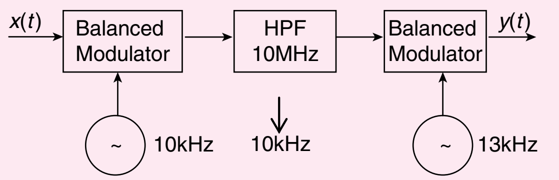
5. A 1 mW video signal having a bandwidth of 100 MHz is transmitted to a receiver through a cable that has 40 dB loss. If the effective one-sided noise spectral density at the receiver is 10^{-20}watt/Hz, then the signal to noise ratio at the receiver is **[2004]**

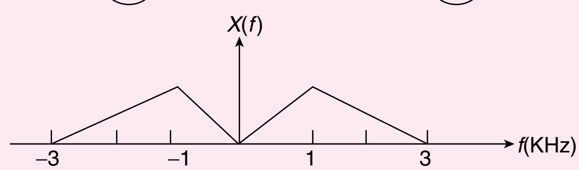
(A) 50 dB (B) 30 dB
(C) 40 dB (D) 60 dB

6. Two sinusoidal signals of same amplitude and frequencies 10 kHz and 10.1 kHz are added together. The combined signal is given to an ideal frequency detector. The output of the detector is **[2004]**

(A) 0.1 kHz sinusoid
(B) 20.1 kHz sinusoid
(C) a linear function of time
(D) a constant

7. Consider a system shown in Figure, let $X(f)$ and $Y(f)$ denote the Fourier transforms of $x(t)$ and $y(t)$, respectively. The ideal HPF has the cut off frequency 10 kHz. **[2004]**





The positive frequencies where $Y(f)$ has spectral peaks are

(A) 1 kHz and 24 kHz (B) 2 kHz and 24 kHz
(C) 1 kHz and 14 kHz (D) 2 kHz and 14 kHz

8. Find the correct match between Group 1 and group 2 **[2005]**

Group 1	Group 2
P $\{1 + km(t)\}\, A \sin(\omega_c t)$	*w* phase modulation
Q $km(t)\, A \sin(\omega_c t)$	*x* frequency modulation
R $A \sin\{\omega_c t + K_m(t)\}$	*y* amplitude modulation
S $A\sin\left(\omega_c t + k\int_{-\infty}^{t} m(\tau)d\tau\right)$	*z* DSB-SC modulation

(A) P – *z*, Q – *y*, R – *x*, S – *w*.
(B) P – *w*, Q – *x*, R – *y*, S – *z*.
(C) P – *x*, Q – *w*, R – *z*, S – *y*.
(D) P – *y*, Q – *z*, R – *w*, S – *x*.

9. Which of the following Analog modulation scheme requires the minimum transmitted power and minimum channel bandwidth? **[2005]**

(A) VSB (B) DSB –SC
(C) SSB (D) AM

10. A device with input $x(t)$ and output $y(t)$ is characterized by:
$y(t) = x^2(t)$. An FM signal with frequency deviation of 90 kHz and modulating signal bandwidth of 5 kHz is applied to this device. The bandwidth of the output signal is **[2005]**

(A) 370 kHz (B) 190 kHz
(C) 380 kHz (D) 95 kHz

11. A carrier is phase modulated (PM) with frequency deviation of 10 kHz by a single tone frequency of 1 kHz. If the single tone frequency is increased to 2 kHz, assuming that phase deviation remains unchanged, the bandwidth of the PM signal is **[2005]**

(A) 21 kHz (B) 22 kHz
(C) 42 kHz (D) 44 kHz

12. A message signal with bandwidth 10 kHz is lowside band SSB modulated with carrier frequency $f_{c1} = 10^6$Hz. The resulting signal is then passed through a narrow-band frequency modulator with carrier frequency $f_{c2} = 10^9$Hz. The band width of the output would be: **[2006]**

(A) 4×10^4 Hz (B) 2×10^6Hz
(C) 2×10^9 Hz (D) 2×10^{10} Hz

13. The diagonal clipping in amplitude demodulation (using envelope detector) can be avoided if RC time -constant of the envelope detector satisfies the following condition, (here W is message band width and ω_c is carrier frequency both in rad/sec). **[2006]**

(A) $RC < \dfrac{1}{W}$ (B) $RC > \dfrac{1}{W}$

(C) $RC < \dfrac{1}{\omega_c}$ (D) $RC > \dfrac{1}{\omega_c}$

Direction for questions 14 and 15:

Let $g(t) = p(t) \times p(t)$, where $\times$ denotes convolution and $p(t) = u(t) - u(t-1)$ with $u(t)$ being the unit step function

14. The impulse response of filter matched to the signal $s(t) = g(t) - \delta(t-2) \times g(t)$ is given as follows: **[2006]**

(A) $s(1-t)$ (B) $-s(1-t)$
(C) $-s(t)$ (D) $s(t)$

15. An amplitude-modulated signal is given as
$x_{AM}(t) = 100(p(t) + 0.5g(t))\cos\omega_c t$.
In the interval $0 \le t \le 1$, one set of possible values of the modulating signal and modulation index would be **[2006]**

(A) t, 0.5 (B) t, 1.0
(C) t, 2.0 (D) t^2, 0.5

Direction for questions 16 and 17:

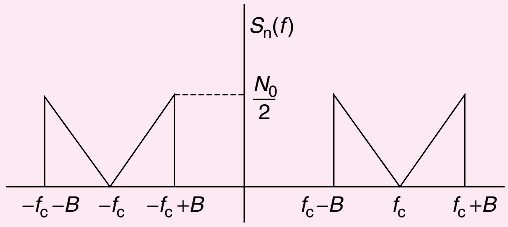
Consider the following amplitude-modulated (AM) signal, where $f_m < B$:

$$x_{AM}(t) = 10\,(1 + 0.5\sin 2\pi f_m t)\cos 2\pi f_c t$$

16. The average side band power for the AM signal given above is: **[2006]**

(A) 25 (B) 12.5
(C) 6.25 (D) 3.125

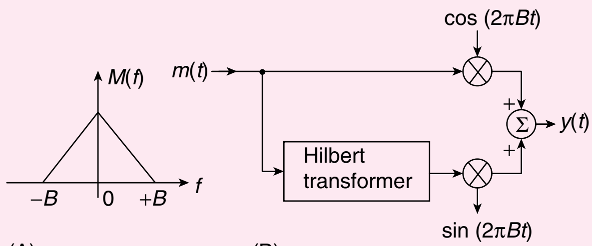
17. The AM signal gets added to a noise with power spectral density $S_n(f)$ given in the figure below. The ratio of average side band power to mean noise power would be: **[2006]**



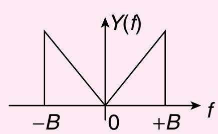
(A) $\dfrac{25}{8N_0B}$ (B) $\dfrac{25}{4N_0B}$

(C) $\dfrac{25}{2N_0B}$ (D) $\dfrac{25}{N_0B}$

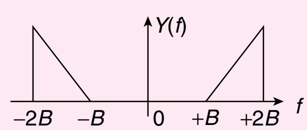
18. In the following scheme, if the spectrum $M(f)$ of $m(t)$ is as shown, then the spectrum $Y(f)$ of $y(t)$ will be: **[2007]**



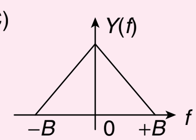
(A)



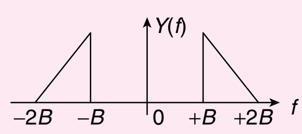
(B)



(C)



(D)



19. Consider the amplitude modulated (AM) signal $A_c \cos\omega_c t + 2\cos\omega_m t \cos\omega_c t$. For demodulating the signal using envelope detector, the minimum value of A_c should be **[2008]**

(A) 2 (B) 1 (C) 0.5 (D) 0

20. Noise with double-sided power spectral density of K over all frequencies is passed through a RC low-pass filter with 3 dB cut-off frequency of f_c. The noise power at the filter output is **[2008]**

(A) K (B) Kf_c (C) $K\pi f_c$ (D) ∞

21. Consider the frequency-modulated signal $10\cos[2\pi \times 10^5 t + 5\sin(2\pi \times 1{,}500t) + 7.5\sin(2\pi \times 1{,}000t)]$ with carrier frequency of 10^5 Hz. The modulation index is **[2008]**

(A) 12.5 (B) 10 (C) 7.5 (D) 5

22. The signal $\cos\omega_c t + 0.5\cos\omega_m t \sin\omega_c t$ is **[2008]**

(A) FM only
(B) AM only
(C) both AM and FM
(D) neither AM nor FM

23. For a message signal, $m(t) = \cos(2\pi f_m t)$ and carrier of frequency f_c, which of the following represents a single-side band (SSB) signal? **[2009]**

(A) $\cos(2\pi f_m t)\cos(2\pi f_c t)$
(B) $\cos(2\pi f_c t)$
(C) $\cos[2\pi(f_c + f_m)t]$
(D) $[1+\cos(2\pi f_m t)]\cos(2\pi f_c t)$

24. A message signal given by $m(t) = \left(\frac{1}{2}\right)\cos\omega_1 t - \left(\frac{1}{2}\right)\sin\omega_2$ is amplitude modulated with a carrier of frequency w_c to generate $s(t) = [1+m(t)]\cos w_c t$. What is power efficiency achieved by this modulation scheme? **[2009]**

(A) 8.33% (B) 11.11%
(C) 20% (D) 25%

25. Suppose that the modulating signal is $m(t) = 2\cos(2\pi f_m t)$ and the carrier signal is $x_C(t) = A_C\cos(2\pi f_c t)$, which one of the following is a conventional AM signal without over-modulation? **[2010]**

(A) $x(t) = A_c m(t)\cos(2\pi f_c t)$
(B) $x(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$
(C) $x(t) = A_c\cos(2\pi f_c t) + \frac{A_c}{4} m(t)\cos(2\pi f_c t)$
(D) $x(t) = A_c\cos(2\pi f_m t)\cos(2\pi f_c t) + A_c\sin(2\pi f_m t)\sin(2\pi f_c t)$

26. Consider an angle-modulated signal $x(t) = 6\cos[2\pi x 10^6 t + 2\sin(8{,}000\pi t) + 4\cos(8{,}000\pi t)]$ V. The average power of $x(t)$ is, **[2010]**

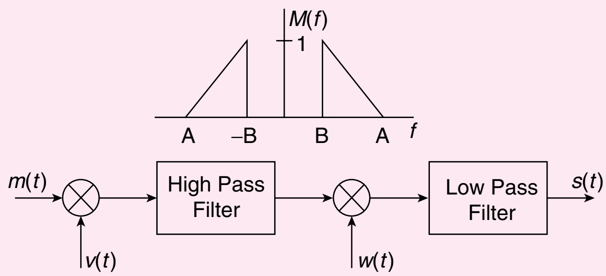
(A) 10 W (B) 18 W
(C) 20 W (D) 28 W

27. A message signal $m(t) = \cos 2{,}000\pi t + 4\cos 4{,}000\pi t$ modulates the carrier $c(t) = \cos 2\pi f_c t$ where $f_c = 1$ MHz to produce an AM signal. For demodulating the generated AM signal using an envelope detector, the constant RC of the detector circuit should satisfy. **[2011]**

(A) 0.5 ms < RC < 1 ms
(B) 1 ms ≪ RC < 0.5 ms
(C) RC ≪ 1 μs
(D) RC ≫ 0.5 ms

28. Consider sinusoidal modulation in an AM system. Assuming no overmodulation, the modulation index (μ) when the maximum and minimum values of the envelope, respectively, are 3 V and 1 V, is __________. **[2014]**

29. In the figure, $M(f)$ is the Fourier transform of the message signal $m(t)$ where $A = 100$ Hz and $B = 40$ Hz. Given $v(t) = \cos(2\pi f_c t)$ and $w(t) = \cos(2\pi(f_c + A)t)$, where $f_c > A$. The cut-off frequencies of both the filters are f_c. **[2014]**



The bandwidth of the signal at the output of the modulator (in Hz) is __________.

30. Consider an FM signal $f(t) = \cos[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t]$. The maximum deviation of the instantaneous frequency from the carrier frequency f_c is **[2014]**

(A) $\beta_1 f_1 + \beta_2 f_2$ (B) $\beta_1 f_2 + \beta_2 f_1$
(C) $\beta_1 + \beta_2$ (D) $f_1 + f_2$

31. The phase response of a pass band waveform at the receiver is given by

$$\Phi(f) = -2\pi\alpha(f - f_c) - 2\pi\beta f_c$$

where f_c is the centre frequency, and α and β are positive constants. The actual signal propagation delay from the transmitter to receiver is **[2014]**

(A) $\frac{\alpha - \beta}{\alpha + \beta}$ (B) $\frac{\alpha\beta}{\alpha + \beta}$
(C) α (D) β

32. In a double-side band (DSB) full-carrier AM transmission system, if the modulation index is doubled, then the ratio of total side band power to the carrier power increases by a factor of _____. **[2014]**

33. Consider the signal $s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c.t)$ where $\hat{m}(t)$ denotes the Hilbert transform of $m(t)$ and the bandwidth of $m(t)$ is very small compared to f_c. The signal $s(t)$ is a **[2015]**
(A) high-pass signal
(B) low-pass signal
(C) band-pass signal
(D) double-side band suppressed carrier signal

34. A message signal $m(t) = A_m \sin(2\pi f_c t)$ is used to modulate the phase of a carrier $A_c \cos(2\pi f_c t)$ to get the modulated signal $y(t) = A_c \cos(2\pi f_c t + m(t))$. The bandwidth of $y(t)$ **[2015]**
(A) depends on A_m but not on f_m
(B) depends on f_m but not A_m
(C) depends on both A_m and f_m
(D) does not depend on A_m or f_m

35. A superheterodyne receiver operates in the frequency range of 58 MHz – 68 MHz. The intermediate frequency f_{IF} and local oscillator frequency f_{LO} are chosen such that $f_{IF} \le f_{LO}$. It is required that the image frequencies fall outside the 58MHz – 68MHz band. The minimum required f_{IF} (in MHz) is ___________ . **[2016]**

36. The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal $s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t$. The value of the modulation index is ___________ . **[2016]**

37. For a superheterodyne receiver, the intermediate frequency is 15 MHz and the local oscillator frequency is 3.5 GHz. If the frequency of the received signal is greater than the local oscillator frequency, then the image frequency (in MHz) is ________ . **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** D	**3.** C	**4.** A	**5.** A	**6.** B	**7.** A	**8.** C	**9.** B	**10.** C
11. C	**12.** D	**13.** A	**14.** D	**15.** C	**16.** A	**17.** D	**18.** C	**19.** C	

Practice Problems 2

1. C	**2.** A	**3.** B	**4.** B	**5.** D	**6.** C	**7.** C	**8.** A	**9.** C	**10.** A
11. C	**12.** D	**13.** D	**14.** B	**15.** A	**16.** C				

Previous Years' Questions

1. C	**2.** B	**3.** C	**4.** C	**5.** A	**6.** A	**7.** B	**8.** D	**9.** C	**10.** A
11. D	**12.** B	**13.** A	**14.** C	**15.** A	**16.** C	**17.** B	**18.** A	**19.** A	**20.** C
21. B	**22.** C	**23.** C	**24.** C	**25.** C	**26.** B	**27.** B	**28.** 0.45 to 0.55		
29. 59.9 to 60.1		**30.** A	**31.** C	**32.** 4 times		**33.** C	**34.** C		**35.** 5
36. 0.5		**37.** 3485 MHZ							