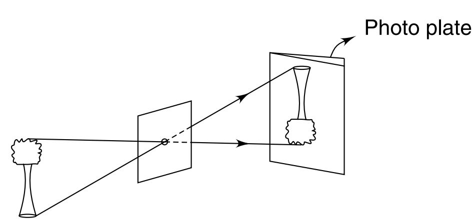
CHAPTER 13

Geometrical Optics

Level 1

Q. 1: A man of height 1.8 m is standing in front of a wall. The sun is exactly behind him. His shadow has a length 1.5 m on the ground and 0.75 m on the wall. Find the length of his shadow on the ground if the wall is removed.

Q. 2: Light travels in a straight line. This principle is illustrated in a pinhole camera. In this simple device the image of an object is formed on a photographic plate by light passing through a small hole.



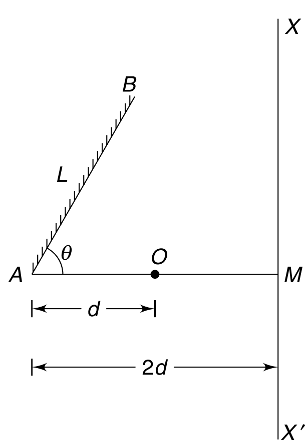
In one experiment, 5 cm high image of a tree was obtained on a photo plate placed at a distance of 15 cm from the pin hole. Actual height of the tree is 20 m.

(a) Find the distance of the pinhole from the tree.

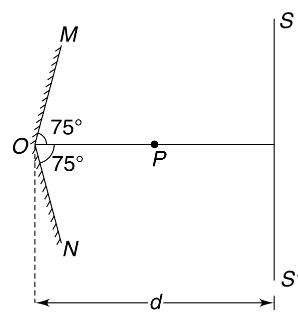
(b) How is the size of image affected if the photo plate is moved away from the pinhole?

(c) What will happen if a large hole is made in place of a pin hole?

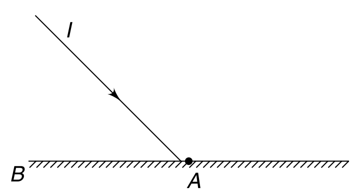
Q. 3: A point object O is kept in front of a plane mirror AB having length L = 2 m. The line AOM makes an angle θ = 60° with the mirror. An observer is walking along the line XMX' (perpendicular to AOM). Find the length of his path along XMX' in which he can see the image of the object. Given $AO = d$ = 1 m and $AM = 2d$.



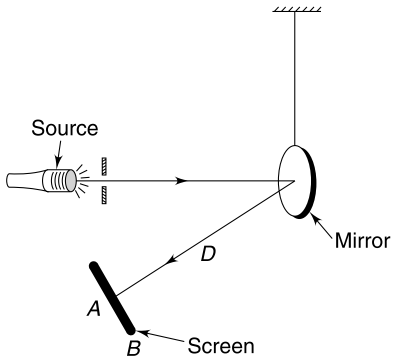
Q. 4: Two large plane mirrors OM and ON are arranged at 150° as shown in the figure. P is a point object and SS' is a long line perpendicular to the line OP. Find the length of the part of the line SS' on which two images of the point object P can be seen.



Q. 5: I is a ray incident on a plane mirror. Keeping the incident ray fixed, the mirror is rotated by an angle θ about an axis passing through A perpendicular to the plane of the Fig. show that the refracted ray rotates through an angle 2θ. Does your answer differ if the mirror is rotated about an axis passing through B?



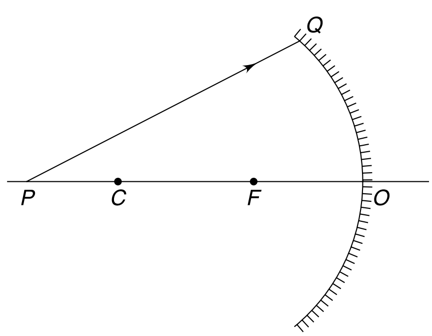
Q. 6: The Fig. shows a device used to measure small twist in a thread. A plane mirror is suspended from a twist free thread. A light ray striking the mirror is reflected on to a screen placed at a distance D = 1 m from the mirror.



As the thread is twisted by an angle θ (so that mirror rotates by θ), the light spot on the screen moves from A to B such that $AB = 0.5$ cm. Find θ.

Q. 7: While looking at her face in a mirror, Hema notes that her face is highly magnified when she is close to the mirror. As she backs away from the mirror, her image first gets blurry, then disappears when she is at a distance of 45 cm from the mirror. Explain the happenings? What will happen if she moves beyond 45 cm distance from the mirror?

Q. 8: We know that parallel light rays which are inclined to the principal axis of a spherical mirror, after reflection converge at a point in the focal plane of the mirror. With this knowledge explain how you will trace the reflected ray for incident ray PQ shown in the Fig. F is focus and C is centre of curvature of the mirror.



Q. 9: The inner surface of the wall of a sphere is perfectly reflecting. Radius of the sphere is R. A point source S is placed at a distance $R/2$ from the centre of the sphere. Consider the reflection of light from the farthest wall followed by reflection from the nearest wall. Where is the image of the source? Consider paraxial rays only.

Q. 10: A concave mirror forms a real image, on a screen of thrice the linear dimension of a real object placed on its principal axis. The mirror is moved by 10 cm along its principal axis and once again a sharp image of the object is obtained on the screen. This time the image is twice as large as the object. Find the focal length of the mirror.

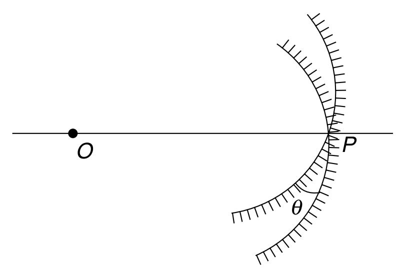
Q. 11: Plot the graphs of $\left|\frac{1}{v}\right|$ vs $\left|\frac{1}{u}\right|$ where v is image distance and u is object distance for the conditions given below:

(a) for concave mirror when image is real
(b) for concave mirror when image is virtual
(c) for convex mirror when image is virtual
(d) for convex mirror when image is real.

Q. 12: A point object O is placed at a distance of 60 cm from a concave mirror of radius of curvature 80 cm.

(a) At what distance from the concave mirror should a plane mirror be kept so that rays converge at O itself after getting reflected from the concave mirror and then from the plane mirror?
(b) Will the position of the point where the rays meet change if they are first reflected from the plane mirror?

Q. 13: A point object (O) is placed at the centre of curvature of a concave mirror. The mirror is rotated by a small angle θ about its plole (P). Find the approximate distance between the object and its image. Focal length of the mirror is f.



Q. 14: A one eyed demon has a circular face of radius $a_0 = 10$ cm. The eye is located at the centre of the face. At what distance from his face he must hold a convex mirror of 5 cm aperture diameter so as to see his complete face? Focal length of the mirror is 10 cm.

Q. 15: A small insect starts walking away from a concave mirror along its principal axis. At a point (P) 20 cm from the mirror the image flips upside down.

(a) What can you say about the size of the image at the instant it flips upside down – it is very large, very small or of the size similar to the insect?
(b) Find the distance of the insect from point (P) where its image is thrice as large as the insect.

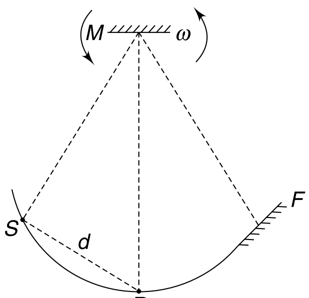
Q. 16: A piece of metal is cut from a hollow metal sphere of radius R and is polished on both sides. A boy looks at the metal piece and finds his image 13 cm behind the metal piece. His friend flips the mirror, keeping its position unchanged and now the boy finds his image to be 52 cm behind the mirror. Find R.

Q. 17: The aperture diameter of a spherical mirror is $D = \eta R$ where η is a positive number less than 2 and R is radius of curvature of the mirror. Consider a wide parallel beam of light incident on the mirror parallel to its principal axis.

(a) Find minimum value of η for which marginal rays start getting reflected twice.

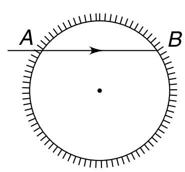
(b) Find minimum value of η for which marginal rays undergo three reflections.

Q. 18: A source of laser (S), a receiver (R) and a fixed mirror (F) – all lie on an arc of a circle of radius R = 0.5 km. The distance between the source and the receiver is d = 0.5 m. At the centre of the circle there is a small mirror M which is rotating with angular speed ω (see figure). Find smallest value of ω is if it is seen that the source shoots a laser pulse which gets reflected at M, then gets reflected at F and finally gets reflected at M to be received by the receiver.



Q. 19: A real object is kept at a distance a from the focus of a concave mirror on the principal axis. A real image is formed at a distance b from the focus. Plot the variation of b with a. If b versus a graph is given to you, how will you find the focal length of the mirror? Explain.

Q. 20: A light ray enters horizontally into a vertical cylinder through a small hole at A. The ray is initially travelling along a chord (AB) whose length is $\left(\frac{\sqrt{5}+1}{2}\right) R$ where R is the radius of the cylinder. After how many reflections on the inner wall of the cylinder the light ray will be incident at point A?

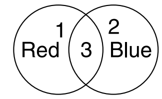


$$\left[\text{Given } \cos 36° = \frac{\sqrt{5}+1}{4}\right]$$

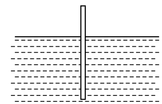
Q. 21: A ship has a green light (λ = 510 nm) on its mast. What colour would be observed for this light by a diver deep inside water. Refractive index for water is $\mu = \frac{4}{3}$.

Q. 22: A vertical rod is partially submerged in an aquarium. You look at the aquarium from some distance. Does the underwater part of the rod appear to be closer than, farther than or the same distance as the top of the rod.

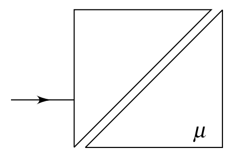
Q. 23: Two transparent plastic sheets of red and blue colour overlap as shown in the Fig. An observer looks at a clear sky through the sheets. What can you say about the colour and brightness of light coming through sections 1, 2 and 3 (see Fig.)



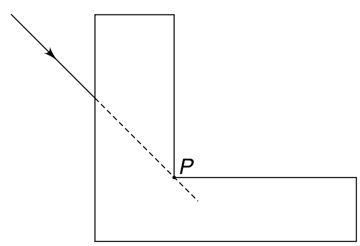
Q. 24: A wooden stick of length 100 cm is floating in water while remaining vertical. The relative density of the wood is 0.7. Calculate the apparent length of the stick when viewed from top (close to the vertical line along the stick) Refractive index of water = $\frac{4}{3}$.



Q. 25: A glass cube is cut symmetrically into two halves and the two parts are kept at a small separation between them. Calculate the angular deviation suffered by a light ray incident normally on one of the faces of the cube.



Q. 26: A glass block of refractive index μ = 1.5 has an L cross section with both arms identical. A light ray enters the block from left at an angle of incidence of 45°, as shown in the figure. If the block was absent the ray would pass through the point P. Calculate the angle at which the ray will emerge from the bottom face after refraction through the block.



Q. 27: A ray of light passes through a rectangular glass block placed in air. Which diagram shows a possible path of a ray?

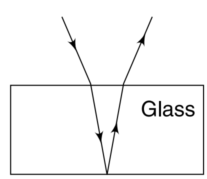


Fig. (a)

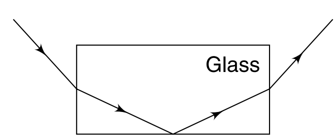
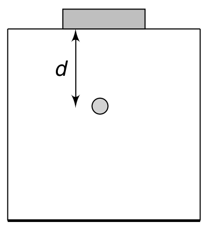


Fig. (b)

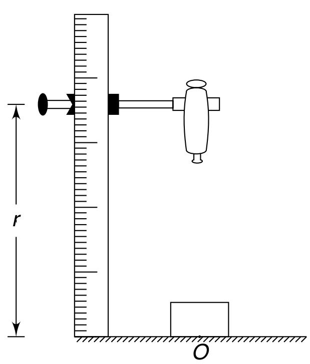
Q. 28: A ray of light travelling in air is incident on a composite transparent slab at an angle of incidence i = 45°. The composite slab consist of 100 parallel faced slices of equal thickness. The refractive index of the n^{th} slice (counting from the one on which light is incident) is given by $\mu_n = 1.0 + 0.01n$. The medium on the other side of the slab has refractive index of $\sqrt{2}$. Calculate the angular deviation suffered by the ray as it comes out of the slab.

Q. 29: Prove that it is impossible to see through adjacent sides of a square block of glass with index of refraction 1.5.

Q. 30: A large transparent cube (refractive index = 1.5) has a small air bubble inside it. When a coin (diameter 2 cm) is placed symmetrically above the bubble on the top surface of the cube, the bubble cannot be seen by looking down into the cube at any angle. However, when a smaller coin (diameter 1.5 cm) is placed directly over it, the bubble can be seen by looking down into the cube. What is the range of the possible depths d of the air bubble beneath the top surface?



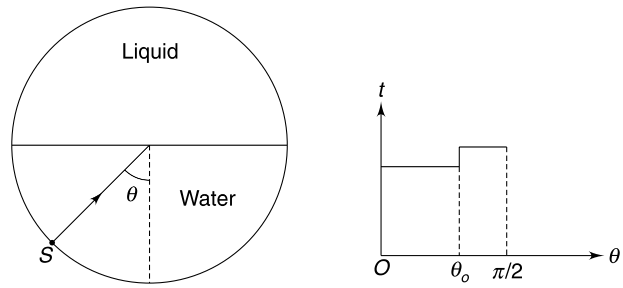
Q. 31: A travelling microscope can move vertically along a scale. It is focused at a mark O on the table and the reading on the vertical scale is r_1. Now a glass slab is placed over mark O and the microscope has to be moved up to bring the mark in focus again. This time the scale reads r_2. Lycopodium powder is spread over the top of the glass slab and the microscope is moved up once again to bring the powder particles in sharp focus. This time the vertical scale reads r_3. Find the refractive index of the material of the glass slab.



Q. 32: A vertical beam of light of cross sectional radius $\frac{R}{2}$ is incident symmetrically on the curved surface of a glass hemisphere of refractive index $\mu = \frac{3}{2}$. Radius of the hemisphere is R and its base is on a horizontal table. Find the radius of luminous spot formed on the table.

$\sin 20° = \frac{1}{3}$ and $\sin 80° = 0.98$

Q. 33: A horizontal cylindrical tank is half full of water $\left(\text{refractive index} = \frac{4}{3}\right)$. The space above the water is filled with a light liquid of unknown refractive index (μ). A small laser source (s) can move along the curved bottom of the cylinder and aims a light beam towards the centre of the cylinder. The time needed by the laser beam to travel from the source to the rim of the cylinder depends on position (θ) of the source as shown in the graph. Find μ, it is given that $\sin \theta_0 = \frac{5}{6}$.



Q. 34: A man standing on sea-shore sees an elongated image (shown by dashed line) of a floating object AB. In fact he finds the image to be oscillating due to air turbulence. Figure (ii) gives three plots (a, b and c) of height from the water surface vs air temperature. Which one best illustrates the air-temperature condition that can create this image? [Many people have seen sea monsters due to this phenomena!]

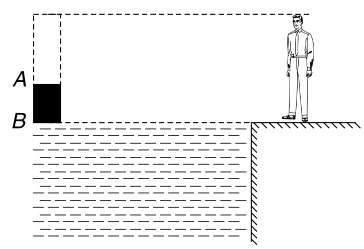


Fig. (a)

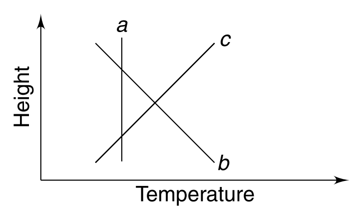
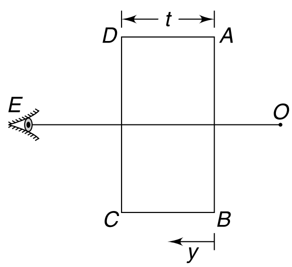


Fig. (b)

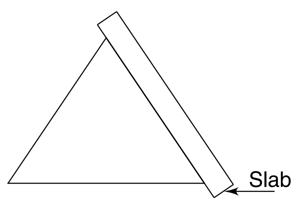
Q. 35: The atmosphere of earth extends upto height H and its refractive index varies with depth y from the top as $\mu = 1 + \frac{y}{H}$. Calculate the apparent thickness of the atmosphere as seen by an observer in space.

Q. 36: A glass slab is placed between an object (O) and an observer (E) with its refracting surfaces AB and CD perpendicular to the line OE. The refractive index of the glass slab changes with distance (y) from the face AB as $\mu = \mu_0(1 + y)$. Thickness of the slab is t. Find how much closer (compared to original distance) the object appears to the observer. Consider near normal incidence only.

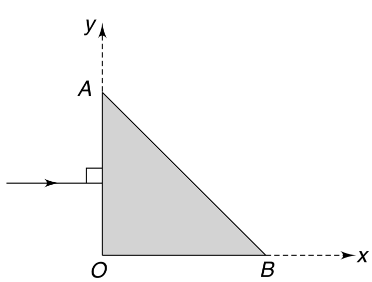


Q. 37: An equilateral prism deviates a ray through 40° for two angles of incidence. The two incidence angels differ by 20°. Find their values.

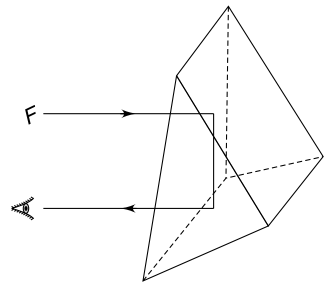
Q. 38: An equilateral glass prism can produce a minimum deviation of 30° to the path of an incident ray. A transparent slab of refractive index 1.5 is placed in contact with one of the refracting faces of the prism. Thickness of the slab is 3 cm. Now calculate the minimum possible deviation which can be produced by this prism-slab combination.



Q. 39: A triangular medium has varying refracting index $\mu = \mu_0 + ax$ where x is the distance (in cm) along x-axis from origin and $\mu_0 = 4/3$. A ray is incident normally on face OA at the mid-point of OA. Find the range of value of a so that light does not escape through face AB when it falls first time on the face AB.



Q. 40: Letter F is kept in front of a right triangular psim. The light rays enter perpendicular to the large rectangular face, is reflected twice by small rectangular faces and exits perpendicularly to the large rectangular face (see Fig.). Draw the image of the letter seen by the eye.



Q. 41: A prism (apex angle $A \leq 90°$) produces minimum deviation that is equal to the apex angle. What can be said about the refractive index of the material of the prism.

Q. 42: Limiting angle of a prism is defined as the largest angle of the prism (A) for which no emergent ray is obtained.

(a) Find the limiting angle (A_0) for a glass prism having refractive index μ.

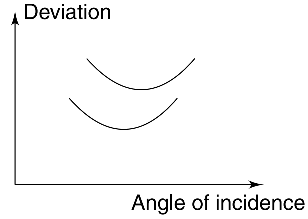
(b) A prism has limiting angle for light of wavelength λ_0. Can there be any emergent ray for light of wavelength $\lambda < \lambda_0$.

Q. 43: An isosceles glass prism has one of its faces silvered. A light ray is incident normally on the other face which is identical in size to the silvered face. The light ray is reflected twice on the same sized faces and emerges through the base of the prism perpendicularly. Find the minimum value of refractive index of the material of the prism.

Q. 44: A parallel beam of light falls normally on the first face of a prism of small refracting angle. At the second refracting face it is partly reflected and partly transmitted. The reflected light strikes the first face again and emerges from it making an angle of 4° with the reversed direction of incident beam. The deviation suffered by refracted ray is 1° from original direction of incident ray. Find the refractive index of glass of the prism and the angle of the prism.

Q. 45: A ray of light is incident upon one face of a prism in a direction perpendicular to the other refracting face. The critical angle for glass – air interface is 30°. Find the angle of the prism (assuming it to be less than 90°) if the ray fails to emerge from the other face.

Q. 46: The plot of deviation versus angle of incidence for two prisms made of same material has been shown in the Figure. Which of the two graph corresponds to the prism of higher refracting angle (A)?



Q. 47: A large rectangular glass block of refractive index μ is lying on a horizontal surface as shown in Figure. Find the minimum value of μ so that the spot S on the surface cannot be seen through top plane '$ABCD$' of the block.

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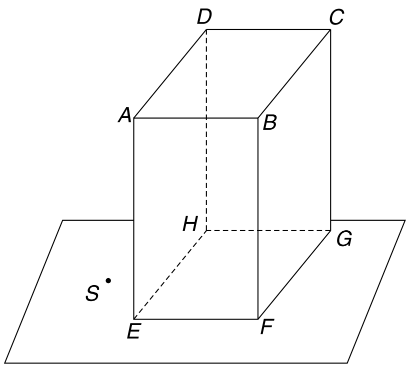
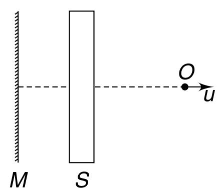


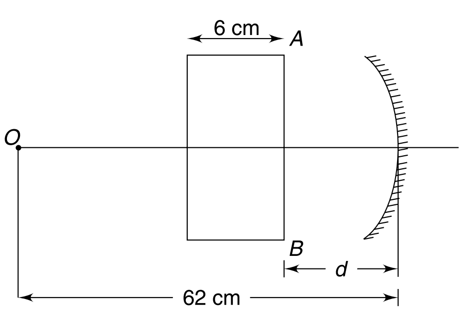
Q. 48: Object O is placed in front of a plane mirror M. A glass slab S having thickness $t = 3$ cm and refractive index $\mu = 1.8$ is placed between the object and the mirror. The refracting faces of the slab are parallel to the mirror surface. The object is made to move with a velocity of $u = 2$ m/s perpendicular to the mirror surface. Find the speed of the image formed after



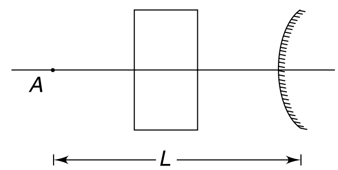
(a) refraction from the slab followed by reflection from the mirror.

(b) refraction from the slab followed by reflection from the mirror followed by the refraction from the slab.

Q. 49: A point object O is placed at a distance of 62 cm in front of a concave mirror of focal length $f = 20$ cm. A glass slab of refractive index $\mu = \frac{3}{2}$ and thickness 6 cm is inserted between the object and the mirror. Let's define final image as image formed after the light ray originating from O passes through the slab, gets reflected from the mirror and then again passes through the slab. At what distance d from the mirror, the face AB of the slab can be placed so that the final image is formed inside the slab itself?

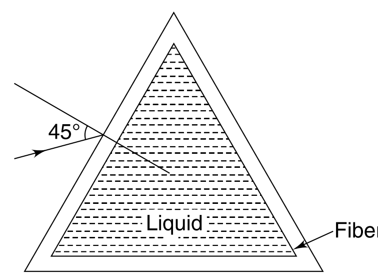


Q. 50: A point object (A) is kept at a distance (L) from a convex mirror on its principal axis. A glass slab is inserted between the object and the mirror with its refracting surfaces perpendicular to the principal axis of the mirror. The thickness of the slab is 6 cm and its refractive index is $\frac{3}{2}$. The image formed after refraction through the slab, reflection from the mirror followed by refraction through the slab is a virtual image at a distance of 10 cm from the pole of the mirror (on its principal axis). Consider paraxial rays only and calculate the distance (L) of the object from the mirror. Focal length of the mirror is $f = 20$ cm.

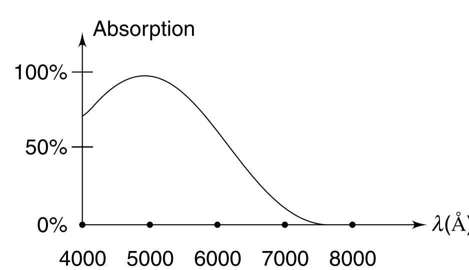


Q. 51: The two perpendicular faces of a right angled isosceles prism are silvered. Prove that a light ray incident on the third face (hypotenuse face) will emerge from the prism parallel to the initial direction.

Q. 52: An equilateral prism has its faces made of a transparent fiber sheet (having refractive index = 1.25) having thickness of 1 mm. The fibre prism is filled with a liquid of refractive index $\sqrt{2}$. Find the deviation of a light ray incident on one face of such a prism at an angle of 45°.



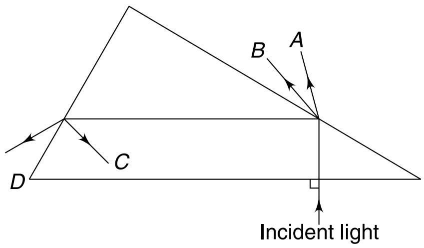
Q. 53: The Figure shows the absorption spectrum for a body. What is the colour of the body?



Q. 54: Two thin prisms are combined such that they neither produce any average deviation nor do they cause any dispersion when white light is incident on the combination. Angle of one prism is $A = 2°$ and refractive index of its glass for red, yellow and violet lights are 1.49, 1.50 and 1.51 respectively. Find the dispersive power of the glass of the other prism.

Q. 55: White light is incident on a glass prism as shown. Four easily identifiable colors – red, green, yellow and blue get separated as A, B, C and D.

Which of the rays (A, B, C and D) correspond to which color?

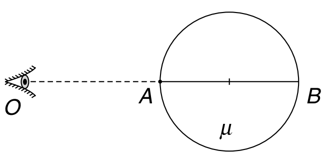


Q. 56: A concave spherical surface of radius of curvature R = 20 cm separates two media A and B having refractive indices $\mu_A = \dfrac{4}{3}$ and $\mu_B = \dfrac{3}{2}$ respectively. A point object is placed on the principal axis. Find the distance of the object from the surface so that its image is virtual when

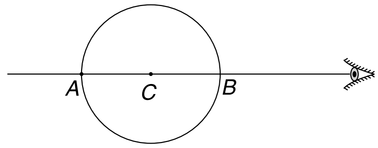
(a) the object is in medium A.

(b) the object is in medium B.

Q. 57: A transparent ball of radius R is viewed by an observe O along its diameter AB. The observe O sees the distance AB to be infinitely large. Find the refractive index of the material of the ball.



Q. 58: A glass (μ = 1.5) sphere of radius R is viewed from outside along a diameter. Calculate the distance between two points (say P and Q) lying on the line AB whose images are seen at centre C and point A respectively.



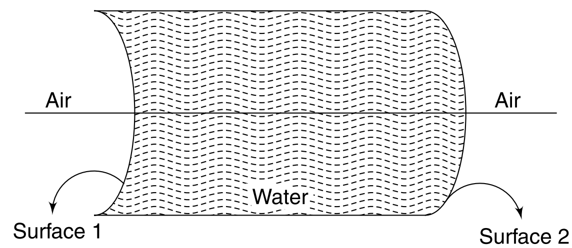
Q. 59: A concave spherical surface of radius of curvature 10 cm separates two mediums X and Y of refractive indices 4/3 and 3/2 respectively. Centre of curvature of the surface lies in the medium X. An object is placed in medium X. Will the image be real or virtual?

Q. 60: A region bounding water has air on two sides. Tell the nature (real or virtual) of the image for following cases- (The object is real and lies on the principal axis (see fig) in all cases.)

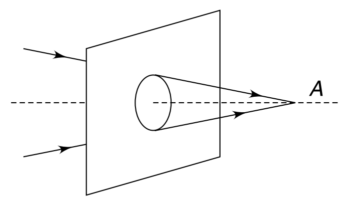
(a) The object is to the left of surface 1 and the image to be considered is formed after the first refraction.

(b) The object is to the left of surface 1 and the image to be considered is formed after two refractions.

(c) The object is to the right of second surface and the image to be considered is formed after one refraction.

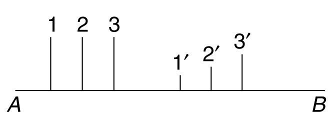


Q. 61: A converging beam of light rays passes through a round opening in a screen. The beam converges at a point A which is at a perpendicular distance of 15 cm from the screen and lies slightly above the central axis of the circular opening. A convex lens of focal length 30 cm is inserted in the opening. At what distance from the screen do the rays converge now?

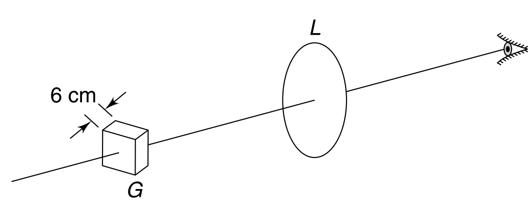


Q. 62: In the Figure AB is the principal axis of an optical element (a lens or a mirror). For position 1, 2 and 3 of a real object, the corresponding position of images are 1′, 2′ and 3′ respectively. Size of image at 3′ is largest and that at 1′ is smallest.

Identify the optical element and indicate its position.



Q. 63: A transparent glass slab (G) of thickness 6 cm is held perpendicular to the principal axis of a convex lens (L) as shown in the Figure. The refractive index of the material of the glass is $\dfrac{3}{2}$ and its nearer face is at a distance 40 cm from the lens. Focal length of the lens is 20 cm. Find the thickness of the glass slab as observed through the lens.

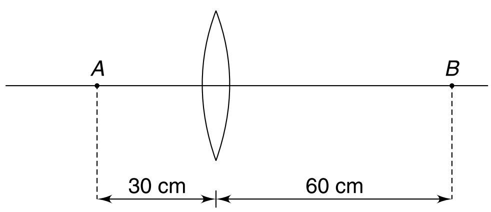


Q. 64: An object 240 cm in front of a lens forms a sharp image on a screen 12 cm behind the lens. A glass slab 1 cm thick, having refractive index 1.50 is placed between the lens and the screen with its refracting faces perpendicular to the principal axis of the lens.

(a) By how much distance the object must be moved so as to again cast a sharp image on the screen?

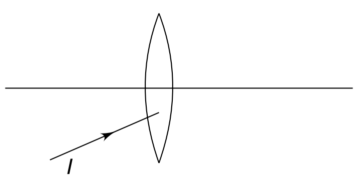
(b) Another identical glass slab is interposed between the object and the lens. How much further the object shall be moved so as to form a sharp image on the screen.

Q. 65: Two point objects A and B are kept on the principal axis of a convex lens as shown. Image of both the objects is formed at same position. Find the focal length of the lens.

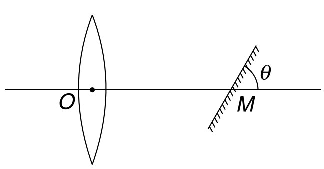


Q. 66: In which case does a light ray pass through the centre of a thin lens without deviation?

Q. 67: The medium on both sides of a convex lens is same. A light ray (I) is incident on it as shown. Draw the path of the ray after it emerges from the lens. Write each step of your construction. The focal length of the lens is known.

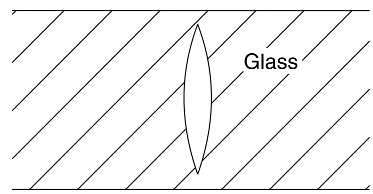


Q. 68: A horizontal parallel beam of light passes though a vertical convex lens of focal length 40 cm. Behind the lens there is a plane mirror making an angle θ with the principal axis of the lens. The mirror intersects the principal axis at M. Distance between the optical centre of the lens and point M is OM = 20 cm. The light beam reflected by the mirror converges at a point P. Distance OP is 20 cm. Find θ.

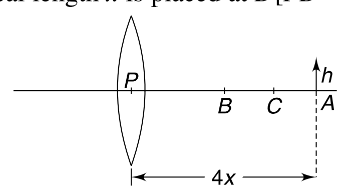


Q. 69: A cylindrical tube has a length of 60 cm. Three identical convex lenses, each of focal length f = 10 cm are fixed inside the tube; one at each of the ends and one at the centre. One end of the tube is placed 10 cm away from a point source. The device casts an image of the object. How much does the image shift when the tube is moved away from the source by 10 cm.

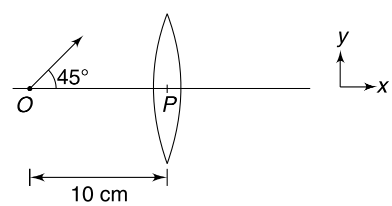
Q. 70: There is an air lens in an extended glass medium. The radius of curvature of both the curved surfaces is R, and refractive index of the glass is $\frac{3}{2}$. Power of this air lens is P. Find the refractive index of the material to be filled inside the lens so that its power becomes $-P$.



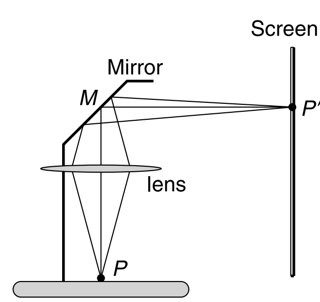
Q. 71: A thin converging lens forms a real image of an object located far away from the lens. The image is formed at A at a distance $4x$ from the lens and height of the image is h. A thin diverging lens of focal length x is placed at B [$PB = 2x$] and a converging lens of focal length $2x$ is placed at C [$PC = 3x$]. The principal axes of all lenses coincide. Find the height of the final image formed.



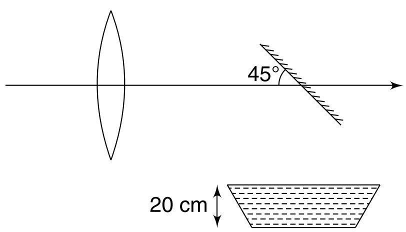
Q. 72. A point object (O) lies at a distance of 20 cm on the principal axis of a convex lens of focal length f = 10 cm. The object begins to move in a direction making an angle of 45° with the principal axis. At what angle with the principal axis does the image beings to move?



Q. 73: The lens in an overhead projector forms an image P' of a point P on a transparency. If the screen is moved away from the projector, how should we move the lens to keep the image on the screen in focus?



Q. 74: A small object is at the bottom of a container which has water filled up to a height of 20 cm. There is a plane mirror inclined at 45° to the horizontal above the container. A convex lens having focal length 15 cm is at a distance of 50 cm from the mirror. The horizontal principal axis of the lens is at a distance of 45 cm from the bottom of the container. Find the distance of the image (from the lens) of the object as seen by an observer to the left of the lens. Light rays from the object hit the lens only after they are reflected from the mirror. (use μ_{water} = 4/3)

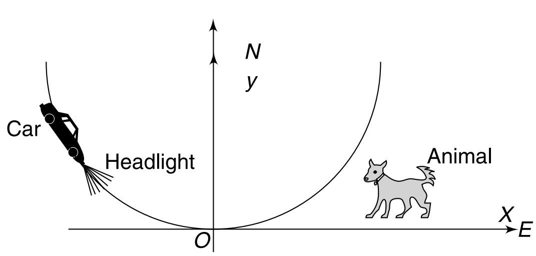


Q. 75: The principal axis of a thin equi-convex lens is the x-axis. The co-ordinate of a point object and its image are (–20 cm, 1 mm) and (25 cm, 2 mm) respectively. Find the focal length of the lens.

Level 2

Q. 76: Sunrays pass through a pinhole in the roof of a hut and produce an elliptical spot on the floor. The minor and major axes of the spot are 6 cm and 12 cm respectively. The angle subtended by the diameter of the sun at our eye is 0.5°. Calculate the height of the roof.

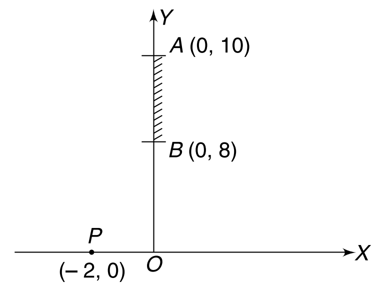
Q. 77: A car is travelling at night along a highway shaped like a parabola with its vertex at the origin of the co-ordinate system. The car starts at a point 200 m West and 200 m North of the origin and travels in easterly direction. There is an animal standing 200 m East and 100 m North of the origin. At what point on the highway will the car's headlight illuminate the animal?



Q. 78: Consider the situation shown in the Figure. The mirror *AB* forms image of point object *P*. Co-ordinates of *A*, *B* and *P* are (0, 10) m, (0, 8) m and (–2, 0) m respectively. Two observers O_1 and O_2 are located at (–2, 10) m and (–1, 13) m respectively.

(a) One of the two observers cannot see the image of point *P*. Identify the observer.

(b) To ensure that both the observers are able to see the image, it was decided to use a longer mirror. Keeping the upper end of the mirror fixed at *A* (0, 10) m, what is the minimum length of mirror required so that both observers can see the image of point *P*?



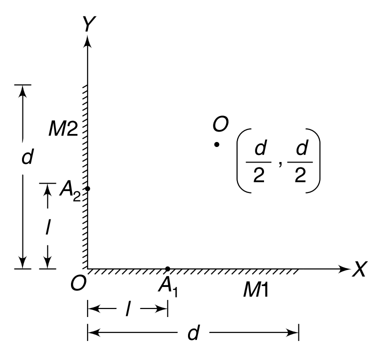
Q. 79: Two plane mirrors *M*1 and *M*2 of length *d* each are placed at right angle as shown. A point object *O* is placed symmetrically with respect to the mirrors at co-ordinates $\left(\frac{d}{2}, \frac{d}{2}\right)$

(a) How many images of *O* will be seen?

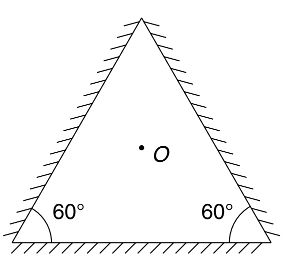
(b) Show that all the images lie on a circle.

(c) Length *l* (= OA_1 = OA_2) of both the mirrors is cut and removed.

Find least value of *l* such that only two images of the object are formed.



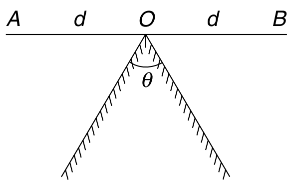
Q. 80: Three plane mirrors are kept as shown in the Figure A point object (*O*) is kept at the centroid of the triangle seen in the Figure. How many images will be formed?



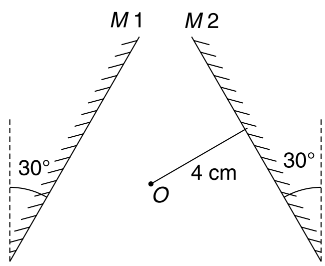
Q. 81: Two plane mirrors are joined together as shown. Two point objects *A* and *B* are placed symmetrically such that *OA* = *OB* = *d*. [*AOB* is a straight line]

(a) If the images of *A* and *B* coincide find θ (call it θ_0).

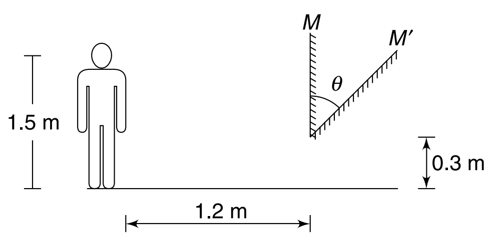
(b) Keeping the position of objects unchanged the angle between the two mirrors is increased to $\theta = \frac{4}{3}\theta_0$. Now find the distance between the images of A and B.



Q. 82: Two plane mirrors $M1$ and $M2$ are inclined at 30° to the vertical. A point object (O) is placed symmetrically between them at a distance of 4 cm from each mirror. Find the distance of the object from the second image formed in mirror $M1$.



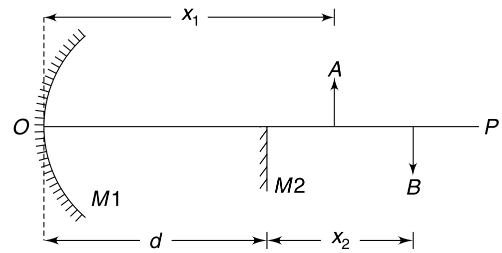
Q. 83: The distance between the eye and the feet of a boy is 1.5 m. He is standing on a flat ground and *a* vertical plane mirror M is placed at a distance of 1.2 m from the boy, with its lower edge at a height of 0.3 m from the ground. Now the mirror is tilted about is lower edge as shown in the Figure. Find maximum value of angle θ for which the image of feet remains visible to the boy. $\left[\text{Take } \tan 15° \simeq \frac{1}{4}\right]$



Q. 84: OP is the principal axis of a concave mirror $M1$. Just below the axis a plane mirror $M2$ is placed at a distance d from the concave mirror. Two small pins A and B are placed on the principal axis as shown. By moving $M2$ and changing d, the virtual image of A formed in mirror $M1$ and the virtual image of B formed in mirror $M2$ were made to coincide.

(a) Calculate the focal length of the concave mirror if it was found that the images coincide when separation between the mirrors was d_0.

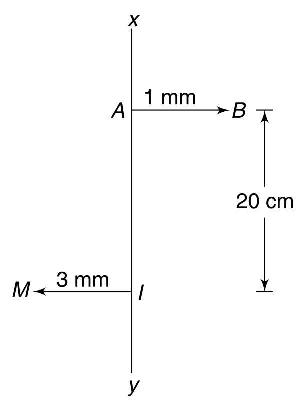
(b) Can the two virtual images be observed by the eye simultaneously?



Q. 85: A real object AB has its image as IM when placed in front of a spherical mirror. XY is the principal axis of the mirror.

(a) Draw a ray diagram to locate the position of the mirror and its focus.

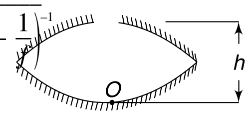
(b) Find the focal length of the mirror.



Q. 86: Two spherical concave mirrors of equal focal length are put against each other with their reflecting surfaces facing each other. The upper mirror has an opening at its centre. A small object (O) is kept at the bottom of the cavity so formed (see Figure). The top mirror produces a virtual image of the object and the lower mirror then creates a real image of the virtual image. The second image is created just outside the cavity mouth. This creates an optical illusion as if the object is raised above its original position.

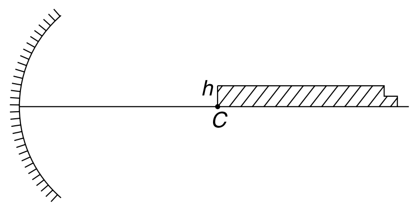
(a) Prove that the height h shown in the Figure is related to focal length of the mirror as

$$\frac{1}{f} = \frac{1}{h} + \frac{1}{h + \left(\frac{1}{h} - \frac{1}{f}\right)^{-1}}$$

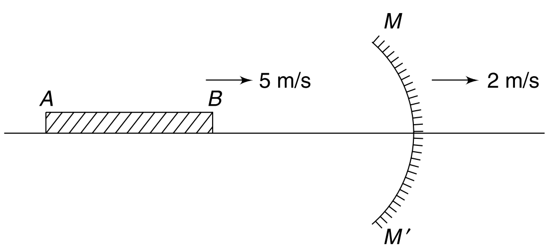


(b) Rewrite the above equation in terms of $x = \frac{h}{f}$ and solve it for x.

Q. 87: A long rectangular strip is placed on the principal axis of a concave mirror with its one end coinciding with the centre of curvature of the mirror (see Figure). The width (h) of the object is very small compared to the focal length (f) of the mirror. Calculate the area of the image formed.



Q. 88: A pencil (AB) of length 20 cm is moving along the principal axis of a concave mirror MM', with a velocity 5 m/s approaching the mirror. The mirror itself is moving away from the pencil at a speed of 2 m/s. Find the rate of change of length of the image of the pencil at the instant end A is at a distance of 60 cm from the mirror.



Q. 89: A small object of height h is placed perpendicular to the principal axis of a convex mirror of focal length f at a distance x from the pole of the mirror. An observer is located on the principal axis at a distance L from the pole of the mirror. (Take $x, f, L >> h$].

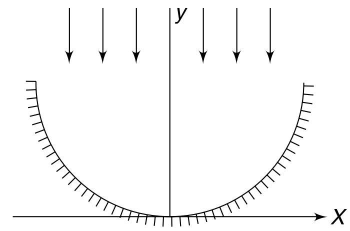
(a) Calculate the angle α formed by the image at the eye of the observer

(b) If the convex mirror is replaced by a plane mirror, with all other things remaining unchanged, calculate the angle β formed by the image at the eye.

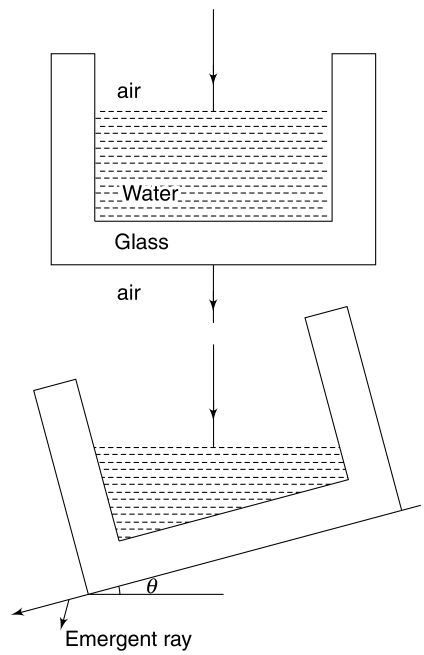
(c) Justify the statement "objects are closer than they appear" written on the rear view mirror of your car.

Q. 90: Consider a large parabolic mirror whose section can be represented by $y = kx^2$, where k is a positive number. Show that a parallel beam of light travelling in negative y direction, after reflection from the mirror, gets focused at a point $\left(0, \frac{1}{4k}\right)$.

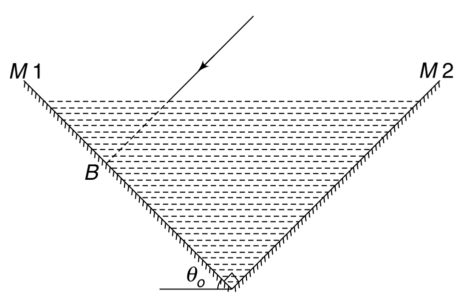
If you need a parallel beam of light, will you prefer a parabolic reflector over a spherical one?



Q. 91: Figure shows a glass ($\mu_g = 1.5$) vessel, partly filled with water $\left(\mu_w = \frac{4}{3}\right)$. A ray of light is incident normally on the water surface and passes straight through. The vessel is tilted slowly till angle θ such that the light ray is emergent grazing the lower surface of the glass. Find θ.



Q. 92: Two plane mirrors $M1$ and $M2$, placed at right angles, form two sides of a container. Mirror $M1$ is inclined at an angle θ_0 to the horizontal. A light ray AB is incident normally on $M1$. Now the container is filled with a liquid of refractive index μ so that the ray AB is first refracted, then reflected at $M1$. The ray is next reflected at $M2$ and then comes out of the liquid surface making an angle θ with the normal to the liquid surface. Find θ.

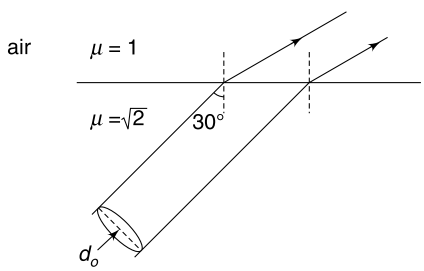


Q. 93: When the sun appears to be just on horizon, it is in fact below the horizon. This is because the light from the sun bends when it enters the earth's atmosphere. Let us assume that atmosphere is uniform and has index of refraction equal to μ. It extends upto a height h ($<< R$ = radius of earth) above the earth's surface. In absence of atmosphere how late would we see the sunrise compared to what we see now? Take time period of rotation of earth to be T.

Calculate this time for following data

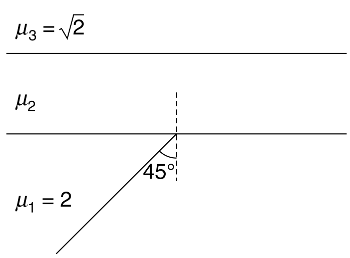
$R = 6400$ km; $\mu = 1.0003$; $h = 20$ km; $T = 24$ hr.

Q. 94: Intensity of a light beam can be defined as amount of light energy incident in unit time on a unit area held normal to the direction of beam. A light beam of intensity I_0 has a circular cross section of diameter d_0. This beam is travelling in a medium of refractive index $\mu = \sqrt{2}$ and gets incident on the medium – air boundary at an angle of incidence $i = 30°$. Assume that entire light energy gets transmitted into air. Find the intensity (I) of transmitted light beam.



Q. 95: In the diagram shown, a light ray is incident on the lower medium boundary at an angle 45° with the normal. Find the deviation suffered by the light ray if–

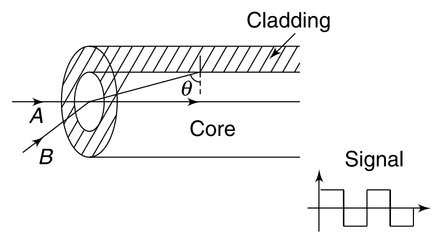
(a) $\mu_2 > \sqrt{2}$ (b) $\mu_2 < \sqrt{2}$



Q. 96: A fibre optic cable has a transparent core of refractive index 1.6 and the cladding has a refractive index of 1.5. An optical signal travels along path A and another signal travels along path B such that it strikes the core – cladding interface at an angle of incidence θ that is just greater than the critical angle. Length of the cable is 1500 m.

(a) Find the time difference between the two signals reaching the other end of the cable.

(b) A digital signal shown in the Figure. is transmitted through the cable. Find maximum frequency so that the crest from path A never arrives with a trough from path B at the receiving end of the cable.



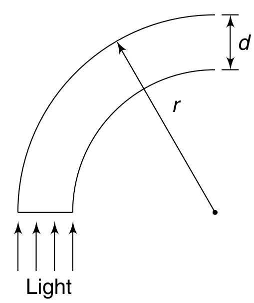
Q. 97: An optical fibre has diameter d and is made of material of refractive indeed μ. It is surrounded by air. Light is made to enter through one end of the fibre as shown. The fiber is in the shape of a circular bend of outer radius r.

(a) Find least value of $r (= r_o)$ for which no light can escape out of the fibre.

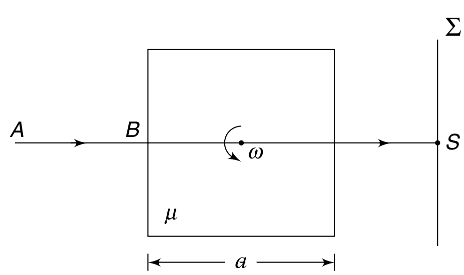
Calculate r_o for $d = 200$ μm and $\mu = 1.4$.

(b) How is value of r_o affected as d is made smaller?

(c) For sharper bends, shall we have higher μ or smaller μ?

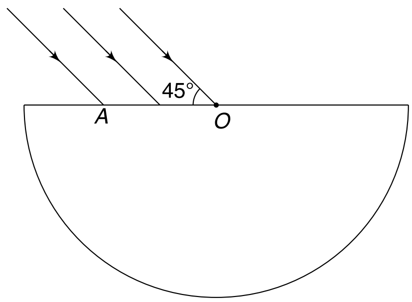


Q. 98. A glass cube has side length a and its refractive index is $\mu = \frac{3}{2}$. A ray of light (AB) is incident normally on one of its face and after passing through the cube it forms a spot S on screen Σ. The cube begins to rotate with angular speed ω about its central axis as shown in the Figure. Immediately after the cube begins to rotate, find the speed of the spot S on the screen.

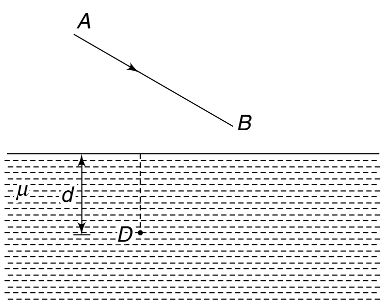


Q. 99: A single ray traverses a glass plate (thickness = t) with plane surfaces that are parallel to each other. The emergent ray is parallel to the incident ray but suffers a lateral displacement d. Assuming that glass plate (refractive index μ) is placed in air, find the dependence of d on angle of incidence i. Plot the variation of d with i (changing from 0° to 90°)

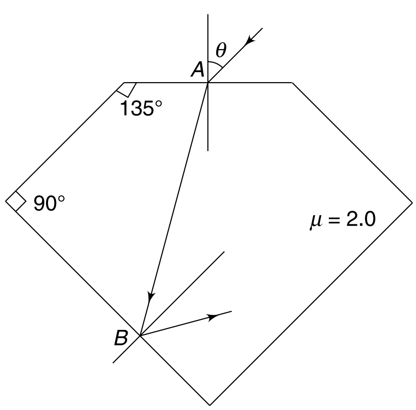
Q. 100: A transparent semicylinder has refractive index $\mu = \sqrt{2}$. A parallel beam of light is incident on its plane surface making an angle of 45° with the surface. The incident beam extends from O to A on the plane surface. Find the maximum width OA (in terms of radius R of the cylinder) so that no ray suffers total internal reflection at the curved surface. [O is the centre of the circular cross section of the cylinder]



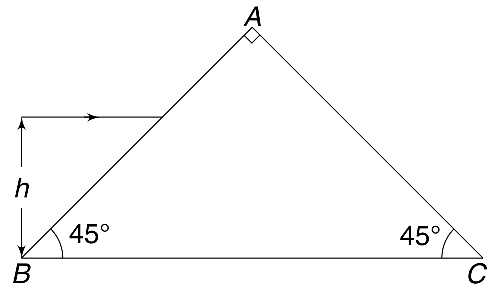
Q. 101: A diver D is still under water $\left(\mu = \frac{4}{3}\right)$ at a depth $d = 10$ m. A bird is diving along line AB at a constant velocity in air. When the bird is exactly above the diver he sees it at a height of 50 m from himself and velocity of the bird appears to be inclined at 45° to the horizontal. At what distance from the diver the bird actually hits the water surface.



Q. 102: Light is incident at point A on one of the faces of a diamond crystal ($\mu = 2.0$). Find the maximum allowed value of angle of incidence θ so that light suffers total internal reflection at point B.



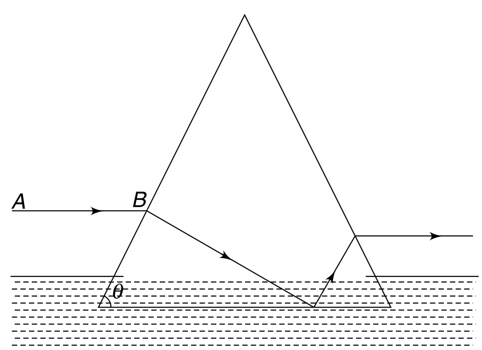
Q. 103: ABC is a glass prism with $\angle A = 90°$ and other two angles 45° each.



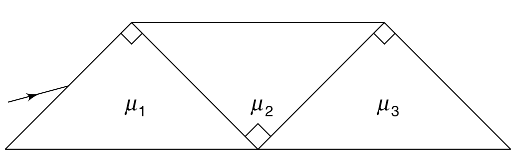
(a) Prove that any light ray that enters the prism through face AB will emerge out through the face AC if refractive index of the glass of the prism is $\mu \geq \sqrt{2}$.

(b) A ray of light is incident parallel to BC at a height $h = 3.0$ cm from BC. Find the height above BC at which the emergent ray leaves the surface AC. It is given that $\mu = \sqrt{2}$ and length $BC = 20$ cm.
[Take $\tan 15° \simeq 0.25$]

Q. 104: An isosceles glass prime $\left(\text{refractive index} = \frac{3}{2}\right)$ has its base just submerged in water $\left(\text{refractive index} = \frac{4}{3}\right)$. The base of the prism is horizontal. A horizontal light ray AB is incident on the prism and takes a path shown in figure to emerge out of the prism. Find the maximum value of base angle θ of the prism for which total internal reflection can take place at the base.



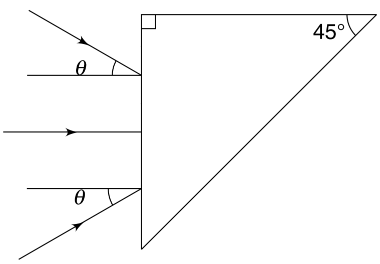
Q. 105: Three right angled prisms are glued as shown in the figure. An incident ray passes undeviated through the system. Express the refractive index (μ_2) of the middle prism in terms of μ_1 and μ_3.



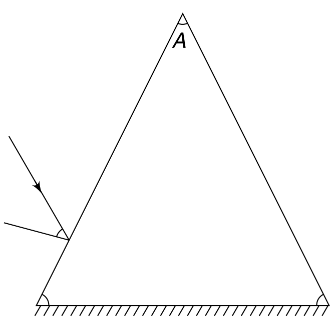
Q. 106: A prism has refracting angle of 60° and its material has refractive index 1.5 and 1.6 for red and violet light respectively. A parallel beam of white light is incident on one face of the prism such that the red light undergoes minimum deviation. Find the angle of incidence (i) and the angular width (θ) of the spectrum obtained.

Given: $\sin(49°) = 0.75°$; $\sin(28°) = 0.47$; $\sin(32°) = 0.53$; $\sin(58°) = 0.85$

Q. 107: A converging beam of light is incident on a right angled isosceles prism as shown in the Figure. The marginal rays in the beam are incident at angle $\pm\theta$. The refractive index for the glass of the prism is $\mu = 1.49 \left(= \dfrac{1}{\sin 42°}\right)$. Find the maximum value of θ for which no light comes out of the hypotenuse surface.



Q. 108: The isosceles prism shown in the Figure has one of its face silvered. A light ray is incident on the prism as shown. Prove that the deviation suffered by the ray is independent of the wavelength of the light.



Q. 109: An equilateral prism is made of glass whose refractive index for red and violet light is 1.510 and 1.550 respectively. White light is incident at an angle of incidence i and the prism is set to give minimum deviation for red light. Find

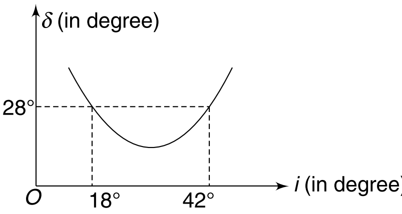
(a) angle of incidence

(b) angular dispersion (i.e., angular width of the spectrum).

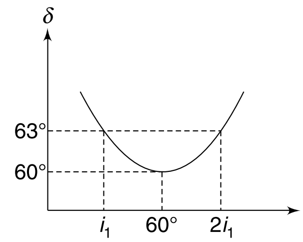
Given

$\theta =$	28°	32°	50°	55°
$\sin\theta =$	0.487	0.529	0.755	0.819

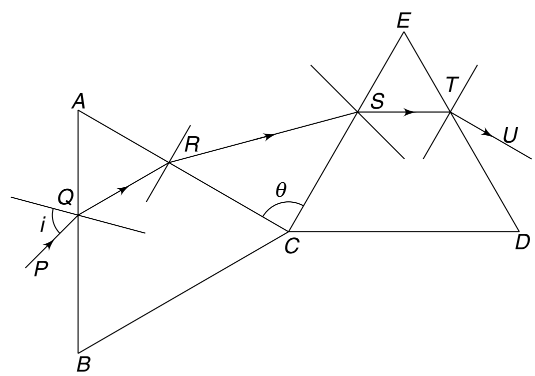
Q. 110: The plot of deviation (δ) *vs* angle of incidence (i) for a prism is as shown in the figure. Find the angle of the prism (A).



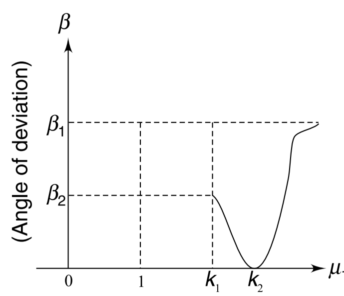
Q. 111: For a glass prism the plot of deviation (δ) vs angle of incidence (i) is as shown. Find the refractive index of the glass and value of angle i_1.



Q. 112: Two identical equilateral glass (refractive index = $\sqrt{2}$) prisms ABC and CDE are kept such that the angle between faces AC and CE is θ. A ray of light is incident at an angle i at the face AB and traverses through the two prisms along the path $PQRSTU$. Find the value of angle i and θ such that angle between the incident ray PQ and emergent ray TU is minimum.



Q. 113: An equilateral prism of refractive index $\mu = \dfrac{4}{\sqrt{3}}$ is kept in a medium of refractive index μ_1. Consider a light ray to be normally incident on one of the refracting faces. The diagram shows variation of magnitude of angle of deviation (β) with respect to μ_1.



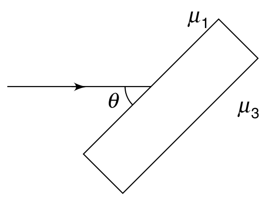
(a) Find value of k_2.

(b) Find value of k_1.

Q. 114: A rectangular glass block ($\mu = 1.5$) is on top of a sheet of paper on which there on which there is a small dot. There is a layer of liquid between the paper and the glass block. The dot is visible through a vertical face up to a point where the angle of emergence of light (starting from the dot) is 30°. Find the refractive index (μ_0) of the liquid. Can we see the dot through a vertical face if the liquid layer is replaced with air?

$$\sin^{-1}\frac{2}{3} = 42°$$

Q. 115: A light ray travelling in a medium of refractive index μ_1 is incident on a parallel faced glass slab making an angle of θ with the glass surface. The refractive index of the medium on the other side of the glass slab is $\mu_3 (> \mu_1)$. Find the angular deviation suffered by the light ray.

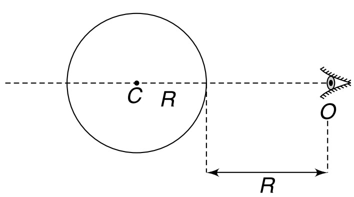


Q. 116: The Figure shows the equatorial circle of a glass sphere of radius R having centre at C. The eye of an observer is located in the plane of the circle at a distance R from the surface. A small insect is crawling along the equatorial circle.

(a) Calculate the length (L) of the are on the circle where the insect lies where its image is visible to the observer.

(b) Calculate the value of L when the eye is brought very close to the sphere.

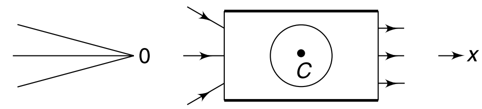
Refractive index of the glass is $\mu = \dfrac{1}{\sqrt{2}}$



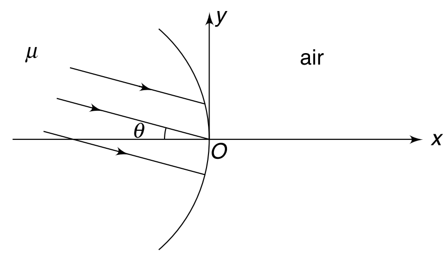
Q. 117: On a hot summer day in a desert the refractive index of the atmosphere changes with height (y) above the surface of the earth as $\mu = \mu_0(1 + by)^{1/2}$ where μ_0 is the refractive index at the surface and $b = 6 \times 10^{-4}\ \mathrm{m}^{-1}$. A man of height 1.5 m is standing on a straight level road. Calculate the distance beyond which he cannot see a point on the road.

Q. 118: The angle of minimum deviation caused by a prism is equal to the angle of the prism. What are the possible values of refractive index of the material of the prism?

Q. 119: A beam of light rays converges to a point O on x-axis as shown. The angle of convergence is small. A cube of glass of refractive index $\mu = 1.5$ and side length 40 cm containing a concentric spherical air cavity of radius 10 cm is to be placed in the path of the converging beam so that the beam emerging from the cube is parallel to x-axis. At what point C on x-axis should the centre of the cube be placed to achieve this? Give the x coordinate of C taking O as origin.

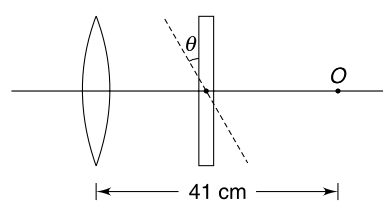


Q. 120: A spherical surface of radius R separates air from a medium of refractive index μ. Parallel beam of light is incident, from medium side, making a small angle θ with the principal axis of the spherical surface. Find the co-ordinates of the point where the rays will focus in air.



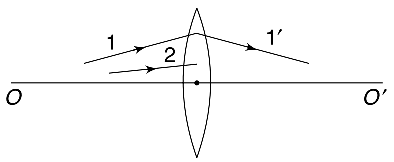
Q. 121: A point object O is placed at a distance of 41 cm from a convex lens of focal length $f = 20$ cm on its principal axis. A glass slab of thickness 3 cm and refractive index $\mu = 1.5$ is placed between the lens and the object with its faces perpendicular to the principal axis of the lens. Image of the object is formed at point I_1. Now the glass slab is tilted by an angle of $\theta = 1°$ (as shown in the Figure) and the final image is formed at I_2. Calculate the distance between points I_1 and I_2.

Consider only paraxial rays for the lens and near normal incidence for the glass slab.



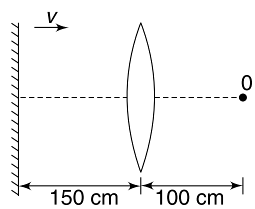
Q. 122: A man is Standing on the peak of a mountain and finds that evening sun rays are nearly horizontal. At a horizontal distance of 6 km from him, its raining and he sees a beautiful rainbow. The sun rays entering water drop get refracted, reflected and refracted to form a rainbow. The red light is emitted from a drop upto a maximum angle of 42° with respect to the incident sunlight. In front of the man there is a flat valley at a depth of 0.5 km from the mountain peak. What fraction of the complete circular arc of the rainbow is visible to the man?

Q. 123: A light ray 1 after passing through a lens kept in air, goes along path 1′. OO' is the principal axis. Draw the refracted path of light ray 2. Write all steps used in construction.



Q. 124: An observer holds in front of himself a thin equi convex lens. R is the radius of curvature of each face. He sees two images of his nose, one erect and the other inverted. Explain the formation of these images and assuming the refractive index of glass to be 1.50 prove that he will see two erect images if the distance of the lens is less than $0.25R$ from his nose.

Q. 125: An observer is standing at a point O, at a distance of 100 cm from a convex lens of focal length 50 cm. A plane mirror is placed behind the lens at a distance of 150 cm from the lens. The mirror now starts moving towards the right with a velocity of 10 cm/s. What will be the magnitude of velocity (in m/s) of her own image as seen by the observer, at the moment when the mirror just starts moving?

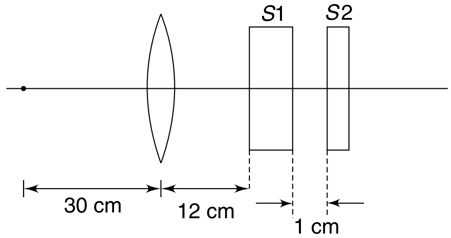


Q. 126: A small object (A) is placed on the principal axis of an equiconvex lens at a distance of 30 cm. The refractive index of the glass of the lens is 1.5 and its surfaces have radius of curvature R = 20 cm. Two glass slabs $S1$ and $S2$ have been placed behind the lens as shown in Figure. Thickness of the two slabs is 6 cm and 4 cm respectively and their refractive indices are $\frac{3}{2}$ and 2 respectively.

(a) Find the distance of the final image measured from the lens. Also find the magnification.

(b) How does the position of the image change if the slab $S2$ is moved to left so as to put it in contact with $S1$.

(c) How does the position of the image change if the two slabs in contact are together moved to right by a distance of 100 cm.



Q. 127: A virtual image is formed by a lens for a real object.

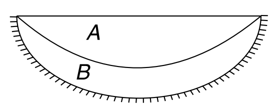
Take $\left|\frac{v}{f}\right| = y$ and $\left|\frac{u}{f}\right| = x$ and draw y vs x graph if

(a) lens is diverging

(b) lens is converging.

Q. 128: An equiconvex lens of refractive index 1.5 has its two surfaces having radius of curvature of 30 cm. A point object has been placed on the principal axis at a distance of 60 cm from the lens. Find the distance of image from the lens formed by the rays which suffer refraction at first surface, reflection at second surface, again a reflection on the first surface and finally a refraction from the second surface.

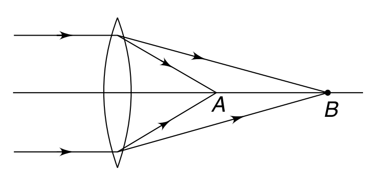
Q. 129: A thin plano convex lens A has material of refractive index μ_A = 1.8 and its curved surface has radius of curvature R. A thin layer of transparent material B is laid over the curved surface of A. The refractive index of B is μ_B = 1.2 and the curved surface of B that is not touching a has radius of curvature $\frac{R}{2}$. This surface of B is silvered. A point object is kept at a distance of 10 cm on the principal axis of the system (above the plane surface) and its image is formed at a distance of 40 cm above the plane surface. Find R.



Q. 130: The refractive index of light in glass varies with its wavelength according to equation

$$\mu(\lambda) = a + \frac{b}{\lambda^2}$$

where a and b are positive constants.

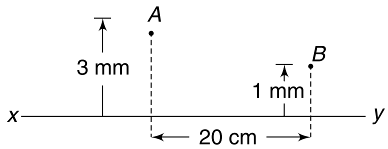


A nearly monochromatic parallel beam of light is incident on a thin convex lens as shown. The wavelength of incident light is $\lambda_0 \pm \Delta\lambda$ where $\Delta\lambda << \lambda_0$. The light gets focused on the principal axis of the lens over a region AB. If the focal length of the lens for a light of wavelength λ_0 is f_0, find the spread AB.

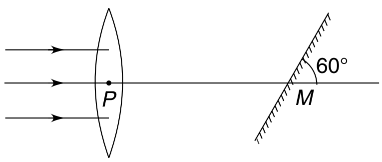
Q. 131: An image B is formed of a real point object A by a lens whose optic axis is XY.

(a) Draw a ray diagram to locate the lens and its focus point.

(b) If A and B are separated by 20 cm along the axis, find the focal length of the lens.



Q. 132: A horizontal parallel beam of light passes through a vertical convex lens of focal length f. The optical centre of the lens is P. A small plane mirror is placed at point M inclined at 60° to the axis of the lens. Distance $PM = f/2$. The mirror reflects the light passing through the lens and forms an image at point I. Find distance PI.



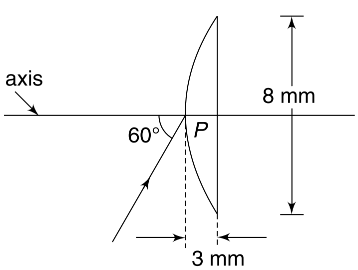
Q. 133: A plane convex lens has aperture diameter of 8 mm and thickness of the lens at the centre is 3 mm. The refractive index of the material of the lens is $\sqrt{3}$. A light ray is incident at mid point P of the curved surface at an angle of incidence of 60°.

(a) Calculate the angular deviation suffered by the ray as it passes through the lens.

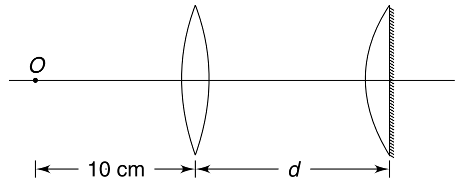
(b) Find the lateral shift in the path of the ray as it passes through the lens.

(c) Find the radius of curvature of the curved surface.

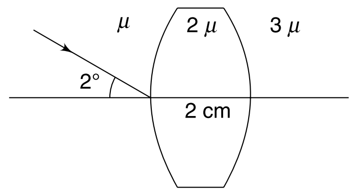
(d) If a narrow beam of light is incident at P parallel to the axis shown, where will it get focused. Take $2\sqrt{3}(\sqrt{3} - 1) \simeq 2.5$



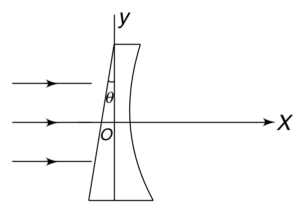
Q. 134: A convex lens of focal length 20 cm and another planocovex lens of foal length 40 cm are placed co-axially. The plane surface of the plano convex lens is silvered. An object O is kept on the principal axis at a distance of 10 cm from the convex lens (see Figure). Find the distance d between the two lenses so that final image is formed on the object itself.



Q. 135: An equiconvex lens has its two surfaces of radius of curvature R = 10 cm. Thickness of the lens at its centre is 2 cm. A light ray is incident making an angle of 2° with the optic axis of the lens. Find the angle that the emergent ray will make with the optic axis. The refractive index of all media is as marked in the Figure.



Q. 136: Due to manufacturing defect, the plane surface of a thin plano-concave lens has been made tilted at a small angle θ outwards from its usual place. The spherical surface has radius of curvature R and refractive index of the material of the lens is μ. A parallel beam of light is incident as shown. In the co-ordinate system shown find the co-ordinates of the point where the rays will focus or appear to be diverging from.



Q. 137: Two lenses L_1 and L_2 are used to make a telescope. The larger lens L_1 is a convex lens with both surfaces having radius of curvature equal to 0.5 m. The smaller lens L_2 has two surfaces with radius of curvature 4 cm. Both the lenses are made of glass having refractive index 1.5. The two lenses are mounted in a tube with separation between them equal to 1 cm less than the sum of their focal length.

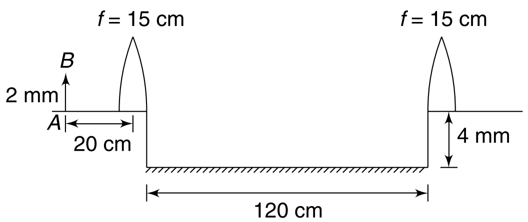
(a) Find the position of the image formed by such a telescope for an object at a distance of 100 m from the objective lens L_1.

(b) What is the size of the image if object is 1 m high? Do you think that lateral magnification is a useful way to characterize a telescope?

(c) Angular magnification is defined as ratio of angle subtended by the image at the eyepiece to the angle subtended by the distant object at an unaided eye. Find the angular magnification of the above mentioned telescope.

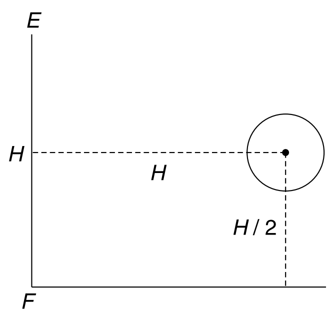
Q. 138: A convex lens of focal length 15 cm is split into two halves and the two halves are placed at a separation of 120 cm. Between the two halves of convex lens a plane mirror is placed horizontally and at a distance of 4 mm below the principal axis of the lens halves. An object AB of length

2 mm is placed at a distance of 20 cm from one half lens as shown in figure. Find the distance of final image of the point A from the principal axis.



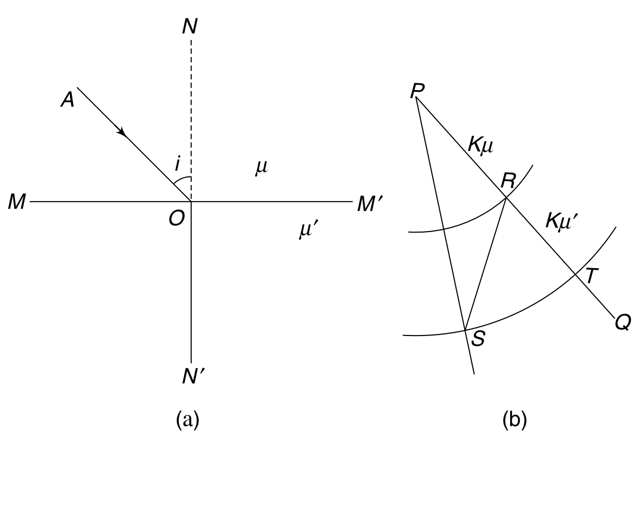
Level 3

Q. 139: In the Figure FE is a man of height H standing on a floor. E is eye of the man and F is his foot. The distance between eye and the head is negligible. A steel ball of radius r is suspended in front of him. The distance of the ball from the man is H and height of the centre of the ball from the floor is $\frac{H}{2}$. It is given that $r << H$. The surface of the ball acts like a mirror and the man sees his image in it. Calculate the angle subtended by the image at the eye of the man.



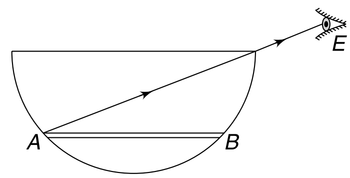
Q. 140: An observer views his own image in a convex mirror of radius of curvature R. If the least distance of distinct vision for the observer is d, calculate the maximum possible magnification.

Q. 141: The Figure (a) shows two media having refractive index μ and μ' with MM' as boundary. A ray of light AO is incident at the boundary and gets refracted.

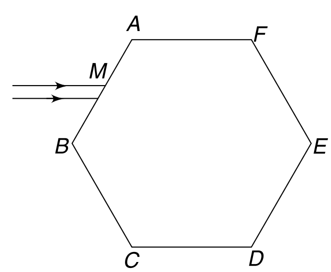


In order to construct the refracted ray graphically a teacher suggests to draw a line PQ parallel to the incident ray AO. With P as centre, two circular arcs are drawn having radii $k\mu$ and $k\mu'$ where k is a positive number. The arcs intersect the line PQ at R and T respectively. From R, a line is drawn parallel to normal NN' and it intersects the other arc at S. Prove that the refracted ray is parallel to line PS.

Q. 142: A stick is placed inside a hemispherical bowl as shown in Figure. The stick is horizontal and has a length of $2a$. Eye of an observer is located at E such that it can just see the end A of the stick. A liquid is filled upto edge of the bowl and the end B of the stick becomes visible to the observer. Radius of the bowl is R. Find the refractive index (μ) of the liquid.

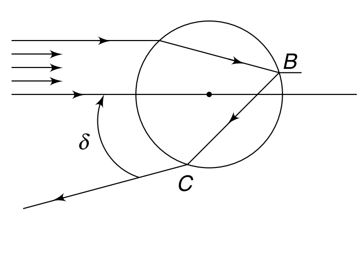


Q. 143: The cross section of a prism is a regular hexagon. A narrow beam of light strikes a face of the prism just below the midpoint (M) of the edge AB. The beam is parallel to the top and bottom faces of the prism. Final the minimum value of refractive index of the material of the prism for which the emergent beam will be parallel to the incident beam.

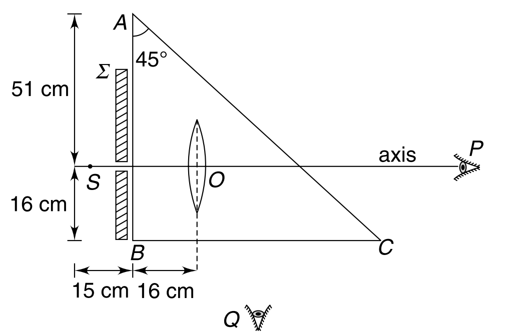


Q. 144: A parallel beam of light is incident on a spherical drop of water (μ = 4/3). Consider refraction of light at air-water interface, then reflection at water-air boundary (of course there will be refracted light energy in air as well), followed by refraction at water-air interface. Path of a typical ray refracted at A, then reflected at B and finally refracted at C has been shown in the Figure. Find the maximum value of angle δ.

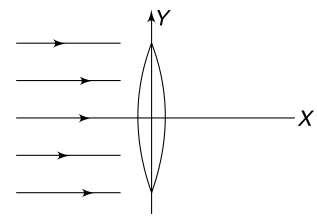
use $\sin^{-1}\sqrt{\frac{20}{27}} = 60°$ and $\sin^{-1}\sqrt{\frac{5}{12}} = 40°$.



Q. 145: An isosceles right angled triangular glass (μ = 1.6) prism has a cavity inside it in the shape of a thin convex lens whose both surfaces have radius of curvature equal to 20 cm. The cavity has been filled with a transparent liquid of refractive index 2.4. S is a point source and Σ is an opaque sheet having a small hole such that the source and the hole both lie on the principal axis of the lens. The small hole in the opaque sheet is just to ensure that only paraxial rays are incident on the optical system. However, the size of hole is large enough to neglect diffraction effects. Will the observers at P and Q be able to see the image of source S? Where is the image located?



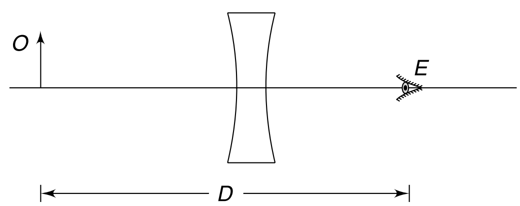
Q. 146: Monochromatic light rays parallel to the principal axis (the x axis) are incident on a convex lens of focal length f. If the lens oscillates such that it tilts up to a small angle θ on either side of the y axis, then find the distance between the extreme positions of the image. Take $\sec\theta \simeq 1 + \frac{\theta^2}{2}$ for small θ.



Q. 147: O is a small object placed at a distance D from the eye E of an observer. A concave lens of focal length $f(\angle D)$ is placed near to the eye and image of the object is viewed. Now the lens is moved towards the object O, away from the eye.

(a) Show that the angle subtended by the image at the eye first decreases and then increases as the lens is moved away from the eye.

(b) Find the distance of the lens from the object when the apparent size of image is smallest.



ANSWERS

1. 2.57 m

2. (a) 60 m (b) Size of image increases

(c) Image will get blurred.

3. $\frac{10}{\sqrt{3}}$ m

4. $\frac{2d}{\sqrt{3}}$

5. No

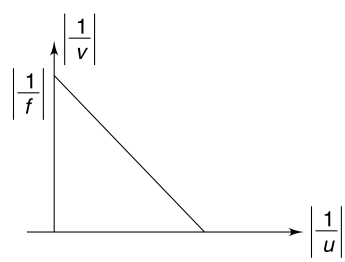
6. 0.14°

7. Image gets inverted beyond 45 cm.

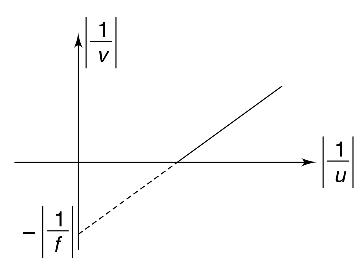
9. At a distance $\frac{5R}{6}$ from the nearest wall

10. 60 cm

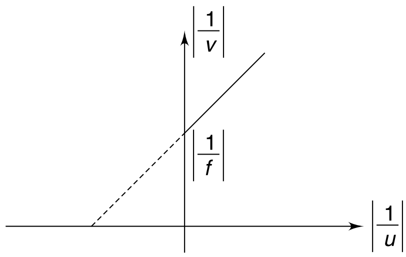
11. (a)



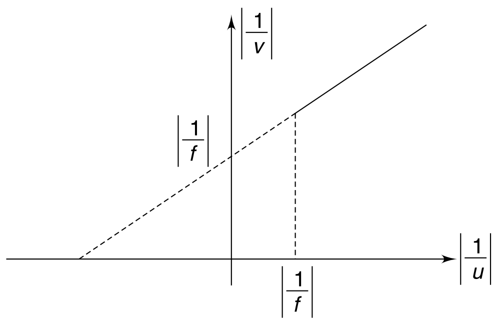
(b)



(c)



(d)



12. (a) 90 cm (b) No.

13. $4f\theta$

14. 20 cm

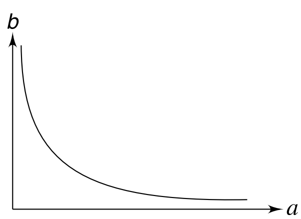
15. (a) very large (b) 6.67 cm on both sides of P.

16. 69.33 cm.

17. (a) $\sqrt{2}$ (b) $\sqrt{3}$

18. 150 rad s^{-1}

19.



20. 9

21. green

22. Underwater part is closer

23. Section 1: Red colour, less bright than usual

Section 2: Blue colour, less bright than usual

Section 3: Dark, almost no light.

24. 82.5 cm

25. Zero.

26. 45°

27. Fig. (b)

28. 15°

30. $\frac{3\sqrt{5}}{8} < d < \frac{\sqrt{5}}{2}$ cm

31. $\mu = \frac{r_3 - r_1}{r_3 - r_2}$

32. 0.34 R

33. $\mu = \frac{10}{9}$

34. C

35. $H \ln 2$

36. $t - \frac{1}{\mu_0} \ln(1 + t)$

37. 60° and 40°

38. 30°

39. $a > \frac{2}{9}$

40. E

41. $\sqrt{2} < \mu < 2$

42. (a) $A_0 = 2\sin^{-1}\left(\frac{1}{\mu}\right)$ (b) No

43. $\mu > \frac{1}{\sin 72°}$

44. $\mu = 2$, $A = 1°$

45. $\cot A < (\sqrt{3} - 1)$ where A is angle of the prism.

46. Upper graph

47. $\mu_{\text{min}} = \sqrt{2}$

48. (a) 2 m/s (b) 2 m/s

49. 26 cm < d < 32 cm

50. L = 32 cm

52. 30°

53. Red

54. 0.04

55. $A \to$ red, $B \to$ yellow, $C \to$ Blue, $D \to$ green.

56. (a) For all distance

(b) For all distance

57. $\mu = 2$

58. $\frac{R}{2}$

59. Image is always virtual

60. (a) Virtual

(b) May be real or virtual

(c) May be real or virtual

61. 10 cm

62. Concave lens. Lens is to the right of 3′

63. 3.3 cm

64. (a) 320 cm (b) $\frac{1}{3}$ cm

65. $f = 40$ cm

66. when media on both sides of the lens is same.

68. 60°

69. The image does not move.

70. 2

71. $2h$

72. 45° downward

73. Moved down

74. 18 cm

75. 90 cm

76. 3.44 m

77. $X = 58.6$ m; $Y = 17.17$ m.

78. (a) O_1 will not see the image (b) 5 m.

79. (a) 3 (c) $l = \frac{d}{3}$

80. 12

81. (a) $\theta_0 = 90°$ (b) d

82. $8\sqrt{3}$ cm

83. 15°

84. (a) $|f| = \frac{(x_2 - d_0)\, x_1}{x_2 - (d_0 + x_1)}$ (b) No.

85. Mirror is at a distance of 10 cm from A. Concave mirror, $f = 7.5$ cm

86. (b) $x = 1$

87. $\frac{1}{2} fh$

88. $\frac{9}{4}$ m/s

89. (a) $\alpha = \frac{h}{L + x + \frac{Lx}{f}}$ (b) $\beta = \frac{h}{L + x}$

91. $\theta = \sin^{-1}\left(\frac{3}{4}\right)$

92. θ_0

93. $\Delta t = \frac{T}{2\pi}\left[\sin^{-1}\left(\frac{\mu R}{R + h}\right) - \sin^{-1}\left(\frac{R}{R + h}\right)\right]$

= 0.9 minute.

94. $I = \frac{3}{2} I_0$

95. (a) 45° (b) 90°

96. (a) 0.53 μs (b) 0.94 MHz

97. (a) $r_0 = \frac{\mu d}{\mu - 1}$

0.7 mm

(b) r_0 gets smaller

(c) higher μ.

98. $\frac{a\omega}{3}$

99. $d = t \sin i\left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right]$

100. $\sqrt{\frac{2}{3}}\, R$

101. 24.62 m

102. $\theta_{max} = \sin^{-1}(2 \sin 15°)$

103. (b) 1 cm

104. $\theta_{max} = \cos^{-1}\left(\sqrt{\frac{17}{21}}\right)$

105. $\mu_2 = \sqrt{\mu_1^2 + \mu_3^2 - 1}$

106. $i = 49°$; $\theta = 9°$

107. 4.47°

109. 5°

110. 32°

111. $\mu = \sqrt{3}$ and $i_1 = 41°$

112. $i = 45°$, $\theta = 90°$

113. (a) $\dfrac{4}{\sqrt{3}}$ (b) $\dfrac{8}{3}$

114. $\mu_0 = \sqrt{2}$, No

115. $\delta = \dfrac{\pi}{2} - \theta - \sin^{-1}\left(\dfrac{\mu_1}{\mu_3}\cos\theta\right)$

116. (a) $\dfrac{\pi R}{3}$ (b) πR

117. 100 m

118. $\sqrt{2} < \mu < 2$

119. 0

120. $x = \dfrac{R}{\mu - 1}$; $y = R\left(\dfrac{\mu}{\mu - 1}\right)$

121. $\dfrac{\pi}{180}$ cm

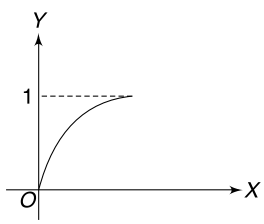
122. 0.53

124. Reflection from the two surfaces results in formation of two images.

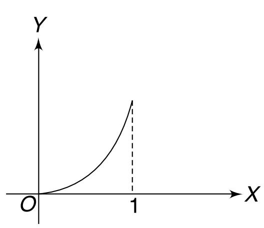
125. 20/9 = 2.22 cm/s

126. (a) 60 cm; $m = -2$ (b) No change (c) No change

127. (a)



(b)



128. $\dfrac{60}{13}$ cm

129. 48 cm.

130. $f_0\left[\dfrac{4b\Delta\lambda}{\lambda_0(a\lambda_0^2 + b - \lambda_0^2)}\right]$

131. (b) concave lens, focal length = 15 cm

132. $f/2$

133. (a) 0 (b) $\sqrt{3}$ mm

(c) $\dfrac{25}{6}$ mm

(d) At a distance $\left(3 + \dfrac{7}{\sqrt{3}}\right)$ mm from P.

134. $d = 20$ cm

135. $\left(\dfrac{8}{15}\right)^{\circ}$

136. $\left(\dfrac{-R}{\mu - 1}, R\theta\right)$

137. (a) 8.8 cm from L_2 in between the two lenses (b) 1.6 cm, No (c) 18.2

138. 8/3 mm

139. $\theta = \dfrac{2r}{5H}$

140. $\dfrac{R}{\sqrt{R^2 + d^2} + d}$

142. $\mu = \sqrt{\dfrac{R + a}{R - a}}$

143. $\mu_{\min} = \dfrac{\sqrt{13}}{2}$

144. 40°

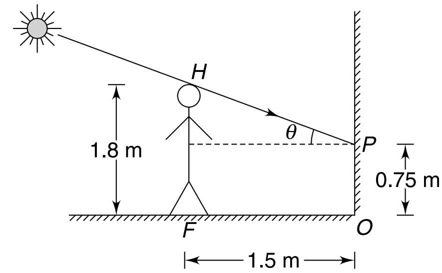
145. Observer at Q sees the image at a distance 6.88 cm from face BC, inside the prism.

146. $\dfrac{f\theta^2}{2}$

147. (b) $\dfrac{D}{2}$

SOLUTIONS

1. If sun rays make angle θ with horizontal

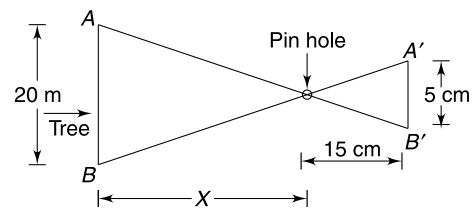


$$\tan\theta = \frac{1.05}{1.5} = \frac{7}{10}$$

When there is no wall, length of shadow (L) is given as

$$\frac{1.8}{L} = \tan\theta$$

$$L = \frac{1.8 \times 10}{7} = \frac{18}{7} = 2.57 \text{ m}$$



2. (a) $$\frac{x}{15 \text{ cm}} = \frac{20 \text{ m}}{5 \text{ cm}}$$

$\Rightarrow$ $$x = 60 \text{ m}$$

3. One extreme ray OA, after reflection reaches P on the line XMX'

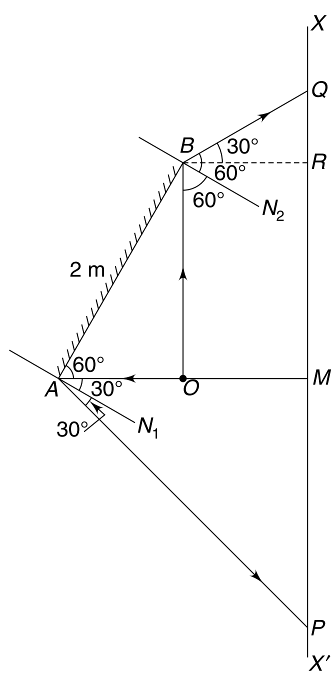
$$MP = AM \cdot \tan 60° = 2\sqrt{3} \text{ m.}$$

The other extreme ray OB, after reflection reaches the observe at Q.

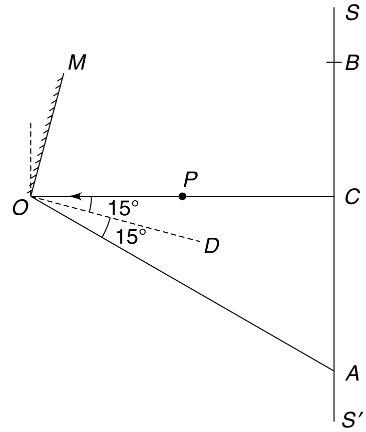
$$QR = BR \cdot \tan 30° = \frac{1}{\sqrt{3}} \text{ m.}$$

$\therefore$ $$PQ = 2\sqrt{3} + 2\sin 60° + \frac{1}{\sqrt{3}}$$

$$= 2\sqrt{3} + \sqrt{3} + \frac{1}{\sqrt{3}} = 3\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{10}{\sqrt{3}} \text{ m.}$$



4. OD is normal to mirror OM at O. The ray PO is reflected along OA.



$$\frac{AC}{d} = \tan 30° \Rightarrow AC = \frac{d}{\sqrt{3}}$$

Various rays starting from P and incident on the mirror OM, after getting reflected from OM, reach the line SS' at all points except below point A.
Similarly rays incident on ON will reach all points on line SS' except above B, where

$$CB = \frac{d}{\sqrt{3}}$$

$\therefore$ An observe located between A and B will be able to sea two images of P.

6. When mirror rotates by θ, the reflected ray rotated by 2θ.

$$\therefore \quad D(2\theta) = AB$$

$$\theta = \frac{0.5 \text{ cm}}{2 \times 100 \text{ cm}} = 0.0025 \text{ radian}$$

$$= 0.14°$$

7. Since image is magnified, the mirror must be concave.

Image is magnified and erect when the object is between pole and focus of the mirror. As the object moves away, the magnification increases sharply and becomes infinite near the focus. Highly magnified image will appear to be blurred and when the object is at focus the image disappear. In fact image is at great distance behind the mirror when object is just about to cross the focus and moves to a great distance on the other side as the object crosses the focus.

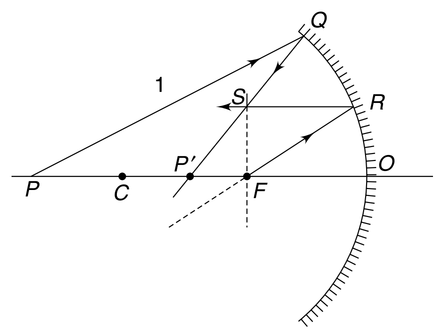
Focal length of mirror = 45 cm.

Beyond 45 cm, the image gets inverted.

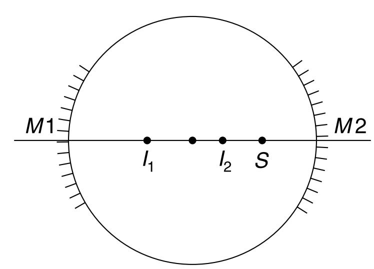
8. Draw a ray FR parallel to PQ. This ray will get parallel to principal axis after reflection. This reflected ray intersects the focal plane at S. Reflect ray at Q will also pass through S.

Hence QSP' is the reflected ray.

P' is image of point P.



9. For mirror M1 (The farthest wall)



$$u = -\frac{3R}{2}; \quad f = \frac{-R}{2}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{2}{R} + \frac{2}{3R} = \frac{-4}{3R}$$

$$v = -\frac{3R}{4} \text{ (Image at } I_1\text{)}$$

This image acts as an object for mirror $M2$

$$u = -\left(2R - \frac{3R}{4}\right) = \frac{-5R}{4}$$

$$\therefore \quad \frac{1}{v} = \frac{-2}{R} + \frac{4}{5R}$$

$$v = -\frac{5R}{6} \text{ (image at } I_2\text{)}$$

$\therefore$ Distance of image from the nearest wall is $\frac{5R}{6}$.

10. Let x = object distance and

y = image distance, initially.

$$y = 3x$$

$$-\frac{1}{f} = -\frac{1}{x} - \frac{1}{3x} \quad \Rightarrow \quad f = \frac{3x}{4} \qquad ...(1)$$

The mirror must be moved away from the object so that the object position moves towards centre of curvature and the size of image decreases.

Now $x' = x + 10$ = object distance

$y' = 2x'$ = image distance

$$\therefore \quad \frac{1}{x'} + \frac{1}{2x'} = \frac{1}{f}$$

$$\Rightarrow \quad f = \frac{2x'}{3} \qquad ...(2)$$

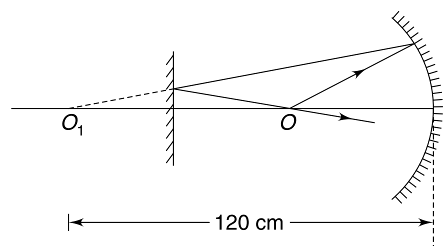
From (1) and (2) $\frac{3}{4}x = \frac{2x'}{3}$

$$\Rightarrow \quad \frac{3}{4}x = \frac{2}{3}(x + 10)$$

$$\Rightarrow \quad 9x = 8x + 80 \quad \Rightarrow \quad x = 80 \text{ cm.}$$

From (1) $f = 60$ cm.

12. (a) For concave mirror



$$u = -60 \text{ cm}; \quad f = -\frac{R}{2} = -40 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{40} + \frac{1}{60} = \frac{-3 + 2}{120}$$

$$v = -120 \text{ cm}$$

If O_1 is the image of O formed by the concave mirror, distance between O and O_1 is 60 cm.

The plane mirror shall be placed exactly midway between O and O_1.

It means the required distance is 90 cm.

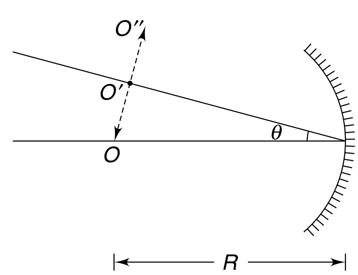
(b) Position will not change. It follows from the principle of reversibility of the optical path. (See Figure)

13. $$OO' = R\theta = 2f\theta$$

If $00'$ is an object then its image is $O'O''$ with

$$O'O'' = OO'$$

$$\therefore \quad OO'' = 4f\theta$$



14. AB is face and E is eye

$$AB = 2a_0 = 20 \text{ cm}$$

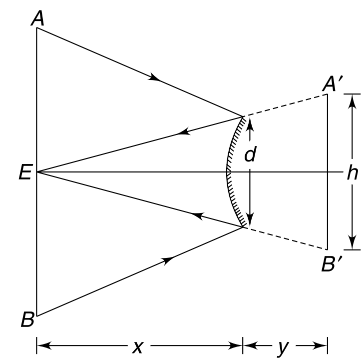
$$d = 5 \text{ cm}$$

$A'B'$ is the image.

From similarity of triangles

$$\frac{x + y}{h} = \frac{x}{d} \qquad ...(1)$$

From mirror equation



$$\frac{1}{y} + \frac{1}{-x} = \frac{1}{f}$$

$$\Rightarrow \quad y = \frac{xf}{x+f} \qquad \text{...(2)}$$

and magnification

$$\frac{h}{2a_0} = \frac{y}{x}$$

$$\Rightarrow \quad h = \frac{y \cdot 2a_0}{x} = 2a_0 \frac{f}{f+x} \qquad \text{...(3)}$$

Put (2) and (3) in (1)

$$\frac{x + \dfrac{xf}{f+x}}{2a_0 \dfrac{f}{f+x}} = \frac{x}{d}$$

$$\Rightarrow \quad \frac{2f+x}{2a_0 f} = \frac{1}{d}$$

$$\Rightarrow \quad \frac{2 \times 10 + x}{2 \times 10 \times 10} = \frac{1}{5}$$

$$20 + x = 40$$

$$x = 20 \text{ cm}$$

15. (a) The image flips at focus of the mirror. Between the pole and the focus the image is erect and as soon as the insect crosses the focus its image gets inverted.

For an object near the focus, magnification is large. Hence, image size is very large.

(b) There are two positions of the object for which we get magnified image of same size. Once when object is between focus and pole and other when the object is between focus and the centre of curvature.

For object between focus and the pole

Let distance of insect from pole be x.

$$u = -x, \quad v = +3x, \quad f = -20 \text{ cm}$$

$$\therefore \quad \frac{1}{+3x} + \frac{1}{-x} = \frac{1}{-20}$$

$$\frac{-2}{3x} = -\frac{1}{20}$$

$$\Rightarrow \quad x = \frac{40}{3} \text{ cm} = 13.33 \text{ cm.}$$

Distance of object from focus (P) is = 20 – 13.33

= 6.67 cm

When the insect is between F and C

$$\frac{1}{-3x} + \frac{1}{-x} = \frac{1}{-20}$$

$$\Rightarrow \quad x = \frac{80}{3} = 26.67 \text{ cm}$$

Distance from focus = 6.67 cm

16. In both cases image is virtual. Virtual image in a concave mirror is magnified. Hence, in the context of the problem it is clear that image distance is 52 cm when the boy is looking at the concave side. And image distance is 13 cm when he is looking into the convex side.

For concave mirror we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{52} + \frac{1}{u} = \frac{2}{-R} \qquad ...(1)$$

For convex mirror

$$\frac{1}{13} + \frac{1}{u} = \frac{2}{R} \qquad ...(2)$$

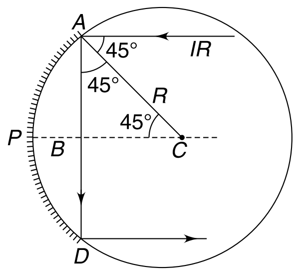
(2) – (1) gives

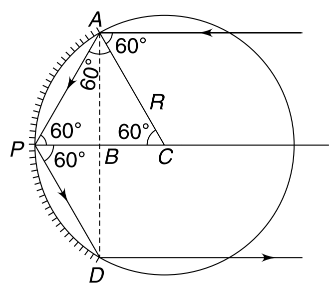
$$\frac{1}{13} - \frac{1}{52} = \frac{4}{R}$$

$$\Rightarrow \quad \frac{3}{52} = \frac{4}{R}$$

$$\Rightarrow \quad R = \frac{52 \times 4}{3} = 69.33 \text{ cm}$$

17.





$AB = \frac{R}{\sqrt{2}} \quad \therefore \quad D = AD = \sqrt{2}R$ $\qquad$ $AB = \frac{\sqrt{3}}{2}R \quad \therefore \quad D = AD = \sqrt{3}R$

18. $\angle SMR = \theta \simeq \frac{d}{R} = \frac{0.5}{0.5 \times 10^3} = 10^{-3}$ rad. The laser light after getting reflected at M, takes $\Delta t = \frac{2R}{c}$ time to return back to M. If the mirror M was at rest, the light would have been reflected back along MS. For the incident light FM to be reflected along MR, the mirror M must rotate through an angle equal to $\frac{\theta}{2}$. Recall that if the mirror rotates by an angle α, the reflected ray rotates by 2α.

$$\Delta t = \frac{2R}{c} = \frac{2 \times 0.5 \times 10^3}{3 \times 10^8} = \frac{1}{3} \times 10^{-5} \text{ s.}$$

In time Δt, mirror rotates by $\Delta\theta = \frac{\theta}{2}$

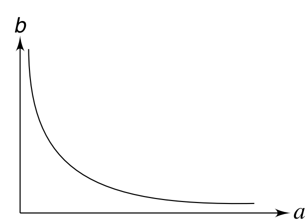
$$= \frac{1}{2} \times 10^{-3} \text{ rad.}$$

$$\therefore \quad \omega = \frac{\Delta\theta}{\Delta t} = \frac{3}{2} \times 10^2 = 150 \text{ rad s}^{-1}.$$

19. $u = f + a$ and $v = f + b$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{f+b} + \frac{1}{f+a} = \frac{1}{f}$$



Simplifying this gives $ab = f^2$

This is known as Newton's formula.

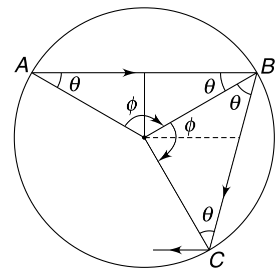
Since ab = a constant the graph is a rectangular hyperbola.

Draw a line through the origin inclined at 45° to the x-axis. This line intersects the graph at P. x or y co-ordinate of P gives the focal length of the mirror.

20.

$$\cos\theta = \frac{\frac{(\sqrt{5}+1)}{4}R}{R}$$

$$= \frac{\sqrt{5}+1}{4}$$



$\therefore$ $\theta = 36°$

Clearly $\phi = 2 \times 54° = 108°$

Ray will be incident at A when

$$n\phi = m \times 360° \quad [m, n \text{ are integers}]$$

Obviously, $m = 3, \quad n = 10$

$\therefore$ light ray will suffer 9 reflections before reaching A again.

22. An object in a denser medium appears to be closer when viewed from a rarer medium.

23. A red sheet transmits red component and absorbs all other colours. Similarly, blue sheet absorbs everything apart from blue.

24. 30% length of the stick will lie outside water.

The lower end will be at a distance 70 cm from the surface of the water.

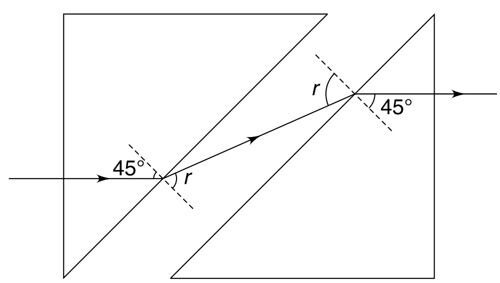
Apparent depth of the lower end is

$$h = \frac{70}{\frac{4}{3}} \text{ cm} = 52.5 \text{ cm}$$

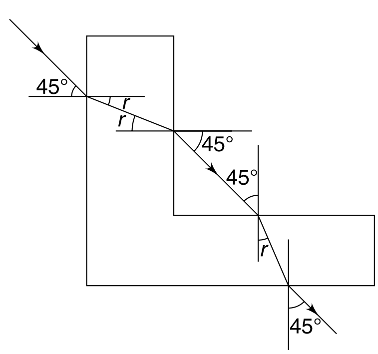
$\therefore$ Apparent length of the stick = 30 cm + 52.5

= 82.5 cm.

25. Path of the ray is as shown in the Figure.



26. The path of the light ray is as shown.



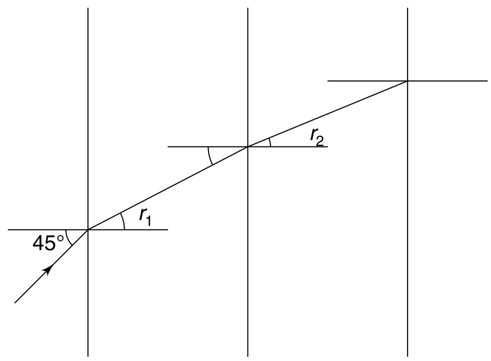
28. Since all refracting surfaces are parallel to each other.

$$1 \cdot \sin 45° = \mu_1 \sin r_1 = \mu_2 \sin r_2 \; ... = \sqrt{2} \sin e$$

$\therefore$ $$\sin e = \frac{1}{2}$$

$\Rightarrow$ $e = 30°$ where e = angle of emergence into medium of refractive index $\sqrt{2}$

Deviation $$\delta = i - e = 15°.$$



29. We are basically supposed to prove that no light ray entering the face AB can leave the glass block through face BC.

Consider a light ray incident on face AB at an angle of incidence $i \to 90°$.

$$\mu \sin\theta = 1 \cdot \sin 90°$$

$$\sin\theta = \frac{2}{3}$$

Now $$\sin\theta' = \sin(90 - \theta)$$

$$= \cos\theta$$

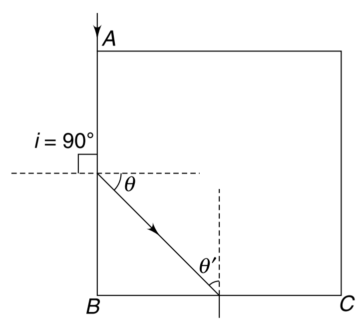
$\therefore$ $$\sin\theta' = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

Critical angle is $$C = \sin^{-1}\left(\frac{2}{3}\right)$$

But $$\frac{\sqrt{5}}{3} > \frac{2}{3}$$

$\therefore$ $$\sin\theta' > \sin C$$

$\therefore$ $$\theta' > C$$



$\therefore$ Light ray will suffer total internal reflection at face BC and no light will pass through BC.

If i is made smaller, θ will become smaller. This will make θ' even larger.

Hence no light ray incident on face AB can come out of face BC.

30. The depth of the bubble shall not be more than that shown in the figure , when the coin of diameter 2 cm is placed above it. A larger depth will mean that that the angle of incidence at points not covered by the coin can be lesser than the critical angle (C).

$$\mu \sin C = 1 \quad \Rightarrow \quad \tan C = \frac{1}{\sqrt{\mu^2 - 1}} \quad \Rightarrow \quad \tan C = \frac{2}{\sqrt{5}}$$

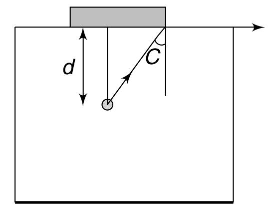
$\Rightarrow$ $$\frac{1}{d} = \frac{2}{\sqrt{5}} \quad \Rightarrow \quad d = \frac{\sqrt{5}}{2}$$

The depth at which the bubble will be just visible if the smaller coin is placed can be similarly calculated as –

$$\tan C = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \qquad \frac{3/4}{d} = \frac{2}{\sqrt{5}} \quad \Rightarrow \quad d = \frac{3\sqrt{5}}{8}$$

If the bubble happens to be below this depth it will be certainly visible.



Therefore, $\frac{3\sqrt{5}}{8} < d < \frac{\sqrt{5}}{2}$ cm

31. Actual thickness of the glass slab is

$$t = r_3 - r_1$$

Apparent thickness of the slab is

$$t' = r_3 - r_2$$

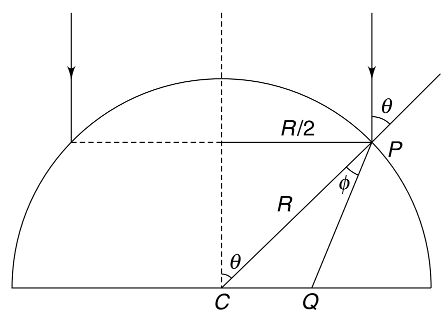
$$\therefore \qquad \mu = \frac{t}{t'} = \frac{r_3 - r_1}{r_3 - r_2}$$

32. Apply snell's law to refraction of extreme rays.

$$1 \cdot \sin\theta = \mu \sin\phi$$

$$\Rightarrow \qquad \frac{1}{2} = \frac{3}{2}\sin\phi \quad \left[\because \quad \sin\theta = \frac{1}{2}\right]$$

$$\Rightarrow \qquad \sin\phi = \frac{1}{3} \quad \Rightarrow \quad \phi = 20°$$



Using sine rule in ΔCPQ

$$\frac{R}{\sin[180° - 60° - 20°]} = \frac{CQ}{\sin 20°}$$

$$CQ = \frac{\sin 20°}{\sin 80°} \cdot R$$

$$= \frac{0.33}{0.98} R$$

$$= 0.34\ R.$$

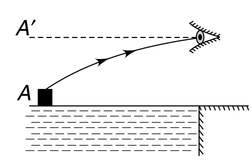
33. At angle of incidence > θ_0, total internal refraction takes place.
When light travels all throughout in water it takes more time in covering a distance = $2R$ (R = radius).
When light gets refracted into the liquid, it takes lesser time to travel through a distance $2R$.

$$\theta_0 = \text{critical angle}$$

$$\mu_\omega \sin\theta_0 = \mu \cdot \sin 90°$$

$$\frac{4}{3} \times \frac{5}{6} = \mu$$

$$\Rightarrow \qquad \mu = \frac{10}{9}.$$



34. With rise in temperature density (and hence refractive index) of air decreases. Light ray from A takes a curved path as shown and appears to come from A'.

35. If we have multiple layers of different media with parallel surfaces, the apparent depth is given by (see Figure (a))–

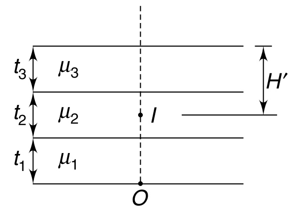


Fig. (a)

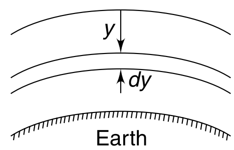


Fig. (b)

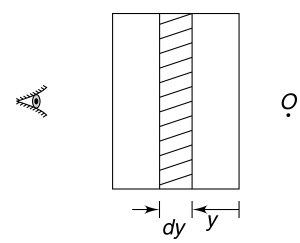
$$H' = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \frac{t_3}{\mu_3} + \ldots$$

The atmosphere is like infinite number of thin layers. Hence apparent depth is given by

$$H' = \int_0^H \frac{dy}{\mu} = \int_0^H \frac{dy}{1 + \frac{y}{H}}$$

$\Rightarrow$ $$H' = H \ln 2.$$

36. Shift in position of the object caused due to thickness dy of the slab is



$$ds = dy\left(1 - \frac{1}{\mu}\right)$$

$$= dy\left(1 - \frac{1}{\mu_0(1 + y)}\right)$$

Total shift due to all such layers is

$$S = \int ds = \int_o^t dy - \frac{1}{\mu_0} \int_o^t \frac{dy}{1 + y}$$

$$= t - \frac{1}{\mu_o}[\ln(1 + y)]_o^t$$

$$= t - \frac{\ln(1 + t)}{\mu_o}$$

37. $$\delta = i + e - A$$

$$40° = i + e - 60°$$

$$i + e = 100° \quad \ldots(1)$$

If $$i - e = 20° \quad \ldots(2)$$

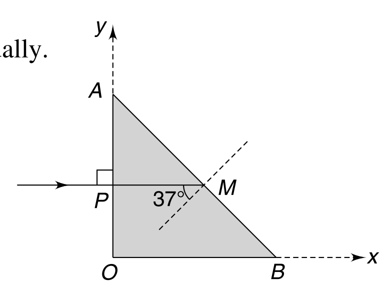
Solving (1) and (2) $$i = 60°$$

$$e = 40°.$$

38. Glass slab causes no deviation.

The net deviation is the sum of deviations produced by the two elements individually.

39. $$PM = \frac{3}{2} \text{ cm}$$



$$37° > C$$

$\Rightarrow$ $$37° > \sin^{-1}\left(\frac{1}{\mu_0 + a\left(\frac{3}{2}\right)}\right)$$

$$\Rightarrow \quad \frac{3}{5} > \frac{1}{\mu_0 + \frac{3}{2}a}$$

$$\Rightarrow \quad 3\mu_o + \frac{9a}{2} > 5$$

$$\frac{9a}{2} > 5 - 3 \times \frac{4}{3}$$

$$a > \frac{2}{9}$$

41. The deviation is minimum when angle of incidence (i) = angle of emergence (e)

$$\delta = i + e - A$$

For minimum deviation ($\delta_{\min} = A$)

$$A = i + i - A$$

$$\Rightarrow \quad i = A.$$

For angle of incidence $i = A$, Snell's law at first refracting surface gives.

$$1 \cdot \sin A = \mu \sin\left(\frac{A}{2}\right) \quad \left[\because \quad r = \frac{A}{2} \text{ for minimum deviation}\right]$$

$$\therefore \quad \mu = 2\cos\left(\frac{A}{2}\right)$$

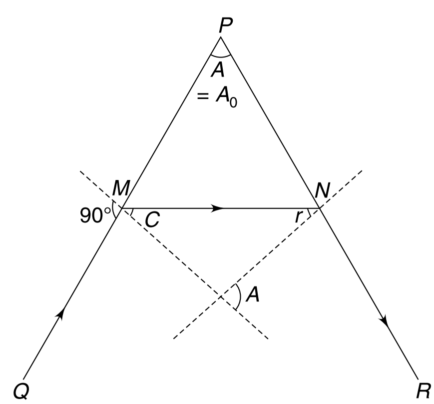
When $A \to 0$

$\mu \to 2.$

When $i = A \to 90°$

$\mu \to \sqrt{2}$

42. Ray QM is at grazing incidence. If angle of prism (A) is increased keeping ray QM at grazing incidence, the angle of incidence on the other face (r) will go on increasing. At a certain value of $A\,(= A_0)$ $\angle r$ will become equal to critical angle (C) and the emergent ray will be NR. If A is increased beyond this, total internal reflection will occur at N and no emergent ray will be there.



From simple geometry $A_o = 2c = 2\sin^{-1}\left(\frac{1}{\mu}\right)$

43. Angle of incidence on silvered face is $r_2 = A = 180 - 2\theta$

$\angle PRQ = 180° - 90° - 2r_2 = 90 - (360° - 4\theta) = 4\theta - 270°.$

Now $\quad r_3 = 90° - \angle PRQ = 360° - 4\theta \quad$ ...(1)

Also $\quad \angle BRD = 90° - r_3$

$\therefore \quad 90 - \theta = 90 - r_3$

$\Rightarrow \quad r_3 = \theta$

From (1) $\quad 5\theta = 360° \quad \Rightarrow \quad \theta = 72°$

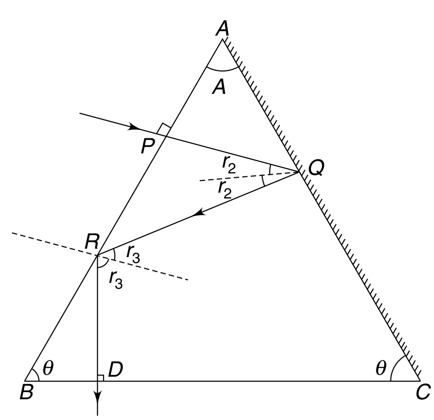
For total internal reflection at R

$$r_3 > C$$

$$\Rightarrow \quad \sin 72° > \sin C$$

$$\Rightarrow \quad \sin 72° > \frac{1}{\mu}$$

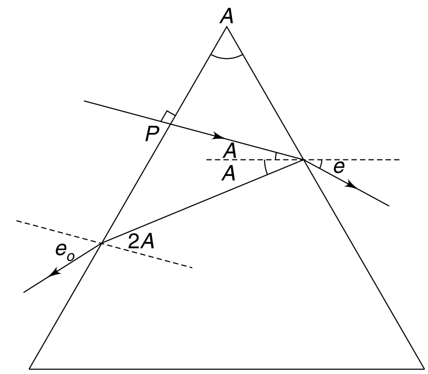
$$\Rightarrow \quad \frac{1}{\sin 72°} < \mu$$



44. $e = A + 1°$ and $e_0 = 4°$.

Using snell's law for two refractions.

$$\mu = \frac{\sin(A + 1°)}{\sin A} = \frac{\sin 4°}{\sin(2A)}$$



All angles are small therefore [$\sin\theta \simeq \theta$]

$$\frac{A + 1°}{A} = \frac{4°}{2A}$$

$$\Rightarrow \quad A = 1°$$

and $$\mu = \frac{4°}{2°} = 2$$

45. From geometry angle of incidence $i = A$.

Light will not emerge out of the second face if

$$r_2 > C$$

$$\Rightarrow \quad A - r_1 > C$$

$$\Rightarrow \quad r_1 < A - C$$

$$\Rightarrow \quad \sin r_1 < \sin(A - C) \qquad ...(1)$$

Using Snell's law

$$\mu \sin r_1 = 1 \cdot \sin A$$

$$\Rightarrow \quad \sin r_1 = \frac{1}{\mu}\sin A$$

$$\Rightarrow \quad \sin r_1 = \sin C \cdot \sin A \qquad ...(2)$$

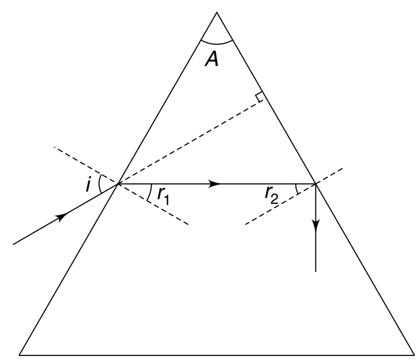
From (1) and (2)

$$\sin A \sin C < \sin A \cos C - \sin C \cos A$$

$$\Rightarrow \quad 1 < \cot C - \cot A$$

$$\Rightarrow \quad \cot A < \cot C - 1$$

$$\Rightarrow \quad \cot A < \sqrt{3} - 1$$



47. The spot S will not be visible if any ray of light entering through face $AEHD$ of the slab fails to emerge out of face $ABCD$.

We can apply the condition of no emergence for a prism of angle $A = 90°$.

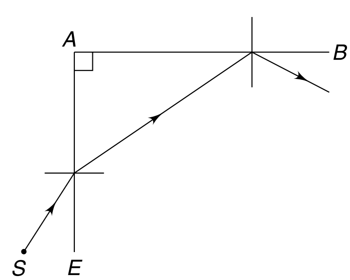
$$\Rightarrow \quad \frac{A}{2} > C$$

$$45° > C$$

$$\sin 45° > \sin C$$

$$\frac{1}{\sqrt{2}} > \frac{1}{\mu}$$

$$\boxed{\sqrt{2} < \mu}$$



48. The glass slab causes a shift but the shift does not depend on the distance between the slab and the object. It means, even if the distance of the object changes from the slab the shift caused will remain same.

Presence of glass slab makes no difference to the velocity of the image.

If we are looking at traffic through a thick glass door, we see the cars moving at their true speed.

49. The action of slab is to cause a shift in the direction of incident ray.

$$\text{shift } S = t\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{2}{3}\right)$$

$$= 2 \text{ cm}$$

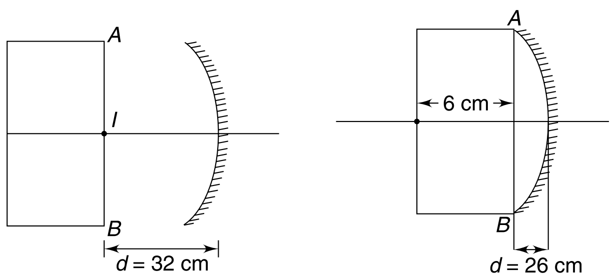
After refraction from the slab, image is formed at distance of 60 cm from the mirror. This acts as object for mirror.

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{-20}$$

$$\Rightarrow \quad v = -30 \text{ cm}$$

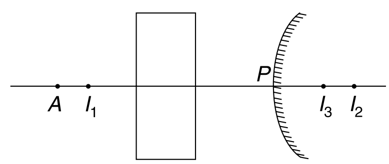
After reflection from the mirror, image is formed at a distance of 30 cm to the left of the mirror. This act as an object for the slab and its image is shifted by 2 cm. Final image is at a distance of 32 cm from the mirror. For this image to be inside the slab.

$$26 < d < 32$$



50. Shift caused by the slab; $\quad S = t\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{2}{3}\right)$

$$= 2 \text{ cm}$$



I_1, I_2 and I_3 are images formed after the light rays get refracted from the slab, reflected from the mirror, and finally after refraction through the slab respectively.

$$I_2 I_3 = AI_1 = 2 \text{ cm}$$

Given $\quad PI_3 = 10$ cm

$\therefore \quad PI_2 = 12$ cm = image distance for the mirror.

PI_1 = object distance from the mirror.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{12} + \frac{1}{u} = \frac{1}{20}$$

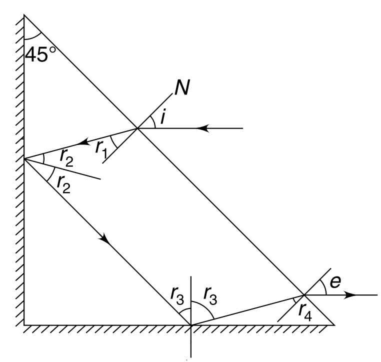
$$\therefore \quad \frac{1}{u} = \frac{1}{20} - \frac{1}{12} = \frac{3-5}{60}$$

$$u = -30 \text{ cm}$$

$$\therefore \quad PI_1 = 30 \text{ cm}$$

$$\therefore \quad PA = 32 \text{ cm}$$

51. The path of the ray is shown is the Figure.



$$r_2 + r_1 = 45°$$

$$\therefore \quad r_2 = 45° - r_1$$

$$r_3 = 90° - r_2 = 45° + r_1$$

$$(90 - r_3) + 45^\circ + (90^\circ + r_4) = 180^\circ$$

$$\therefore \quad r_4 = r_3 - 45^\circ = 45^\circ + r_1 - 45^\circ = r_1$$

From Snell's law (since $r_1 = r_4$) we can easily prove that $i = e$.

Hence incident and emergent rays are parallel.

52. The parallel faced sheet will not result in any deviation.

$$\therefore \quad 1 \cdot \sin 45^\circ = \sqrt{2} \cdot \sin r_1$$

$$r_1 = 30^\circ$$

$$r_2 = A - r_1 = 30^\circ$$

$$\therefore \quad e = 45^\circ$$

$$\therefore \quad \delta = i + e - A = 45^\circ + 45^\circ - 60^\circ = 30^\circ.$$

54. The two prisms must be combined with their vertex placed opposite to each other. For no deviation we must have

$$A(\mu_y - 1) = A'(\mu'_y - 1) \quad ...(1)$$

For no dispersion

$$(\delta_v - \delta_R) = (\delta'_v - \delta'_R)$$

$$\Rightarrow \quad A(\mu_v - \mu_R) = A'(\mu'_v - \mu'_R)$$

$$\Rightarrow \quad A\omega(\mu_y - 1) = A'\omega'(\mu'_y - 1)$$

using (1) we get $\omega = \omega'$

$$\therefore \quad \omega' = \omega = \frac{\mu_v - \mu_R}{\mu_y - 1} = \frac{1.51 - 1.49}{1.5 - 1} = \frac{0.02}{0.5}$$

$$= 0.04$$

55.

$$\lambda_R > \lambda_Y > \lambda_G > \lambda_B$$

$$\mu_R < \mu_Y < \mu_G < \mu_B$$

$$C_R > C_Y > C_G > C_B \quad [C = \text{Critical Angle}]$$

$\therefore$ Red and yellow are least likely to suffer TIR.

Since $\mu_y > \mu_R$, the yellow color suffers more deviation.

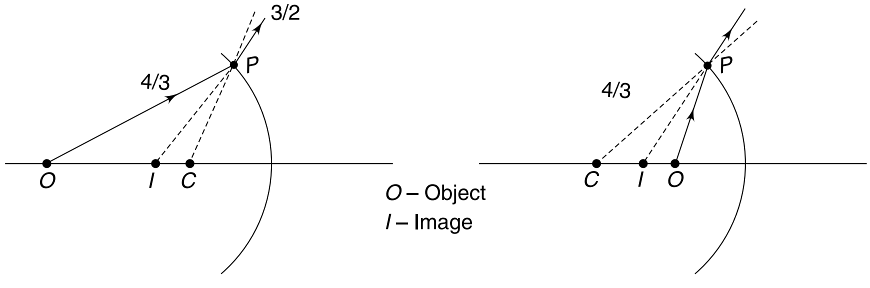
A is red

B is yellow.

Out of Blue and green, critical angel for green is higher. It is less likely to suffer TIR.

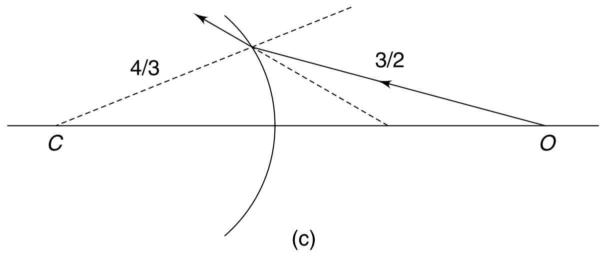
$\therefore$ D is green.

56. (a) When object is in medium A, the image is always virtual as shown in Figure (a) and (b)



(a) (b)

(b) As shown in Figure (c), the image is virtual for all positions of O.



(c)

57. Image of B is formed at infinity

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{\infty} - \frac{\mu}{(-2R)} = \frac{1-\mu}{(-R)}$$

$$\frac{\mu}{2} = -1 + \mu$$

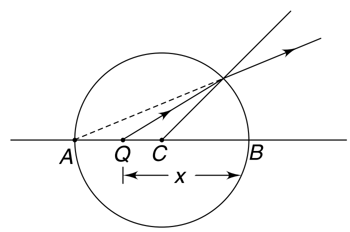
$$\Rightarrow \quad \frac{\mu}{2} = 1$$

$$\Rightarrow \quad \mu = 2$$

58. Image of point C is formed at C itself as light rays diverging from C will not suffer any deviation at the spherical refracting surface.

$\therefore$ Point P is same as point C.

Let point Q be at a distance x from B as shown.



using $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{-2R} - \frac{1.5}{-x} = \frac{1-1.5}{-R}$$

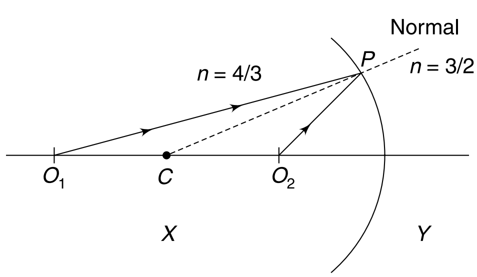
$$\Rightarrow \quad \frac{1}{2R} - \frac{3}{2x} = -\frac{1}{2R}$$

$$\Rightarrow \quad \frac{3}{2x} = \frac{1}{R}$$

$$\Rightarrow \quad x = \frac{3R}{2}$$

Distance between C and Q is $\frac{R}{2}$.

59. Consider two point objects O_1 and O_2. Incident rays from O_1 and O_2 at point P shall both bend towards normal. The corresponding refracted rays will intersect the principal axis, when extended, in the left medium. Therefore image formed under given condition shall always be virtual.



60. (a) Consider object on left side of spherical surface 1 separating two media.

If real object is in rarer medium i.e., $\mu_2 > \mu_1$

Then
$$\frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{(-R)} + \frac{\mu_1}{(-u)}$$

v_1 is definitely negative. Hence image shall be virtual.

(b) For second refraction

$$\frac{\mu_1}{v_2} = \frac{\mu_1 - \mu_2}{(-R)} + \frac{\mu_2}{(-u_2)} = \frac{\mu_2 - \mu_1}{R} - \frac{\mu_2}{u_2}$$

Thus v_2 can be positive or negative. It means the image can be real or virtual.

(c) It can be concluded as above that the image can be real or virtual.

61. Point A will act like a virtual object for the lens. It is off the principal axis.

$$u = +15 \text{ cm}, f = +30 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{15} + \frac{1}{30} = \frac{2+1}{30}$$

$$\therefore \quad v = +10 \text{ cm}$$

Therefore, the rays converge at 10 cm from the screen.

63. Image of the front face is formed at a distance of 40 cm on the other side of the lens.

The rear face of the glass slab appears to be shifted by $s = t\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{1}{3/2}\right) = 2$ cm nearer to the lens.

For lens the distance of rear face of the slab is 40 + 6 – 2 = 44 cm.

Its image is formed at a distance v from lens given by

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{-44} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{44} = \frac{11-5}{220} = \frac{3}{110}$$

$$\therefore \quad v = \frac{110}{3} = 36.7 \text{ cm.}$$

$\therefore$ Thickness of slab in the image formed by the lens is = 40 – 36.7 = 3.3 cm.

64. (a)
$$\frac{1}{f} = \frac{1}{12} - \frac{1}{-240} = \frac{21}{240}$$

Shift caused by glass slab
$$S = t\left(1 - \frac{1}{\mu}\right)$$

$$= 1\left(1 - \frac{2}{3}\right) = \frac{1}{3} \text{ cm.}$$

$\therefore$ Image must be formed at a distance of $12 - \frac{1}{3} = \frac{35}{3}$ cm from the lens

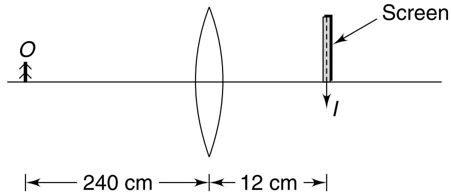
$$\frac{3}{35} - \frac{1}{u} = \frac{21}{240}$$

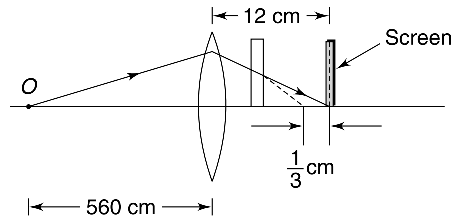
$$\Rightarrow \quad \frac{1}{u} = -\frac{3}{1680}$$

$$\therefore \quad u = -560 \text{ cm}$$

∴ Object shall be moved further away by

$$560 - 240 = 320 \text{ cm}$$

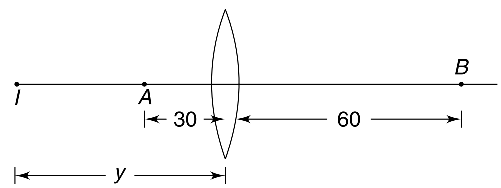




(b) The object shall be moved further away from the lens by $\frac{1}{3}$ cm.

65. It shall be noted that $30 < f < 60$ cm. This will ensure that image of A is to the left of the lens (virtual image) which can coincide with the real image of B.

Let distance of image from the lens be y.



For A: $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

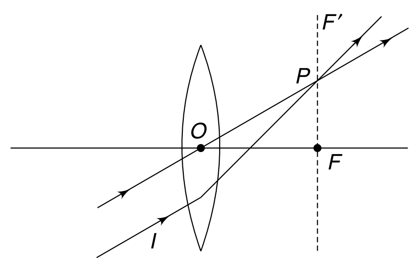
$\Rightarrow$ $$\frac{1}{-y} - \frac{1}{-30} = \frac{1}{f}; \quad \Rightarrow \quad \frac{1}{y} - \frac{1}{30} = -\frac{1}{f} \qquad ...(1)$$

For B: $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{y} - \frac{1}{-60} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{y} + \frac{1}{60} = \frac{1}{f} \qquad ...(2)$$

(2) – (1) gives $$\frac{1}{60} + \frac{1}{30} = \frac{2}{f}$$

$\Rightarrow$ $$f = 40 \text{ cm}$$



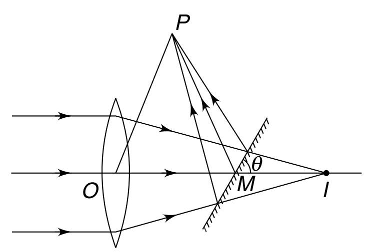
67. Consider a ray passing through optical centre and parallel to the given ray. This ray will pass undeviated. Draw the focal plane FF'. Let the ray intersect this plane at P.

The incident ray, after refraction will pass through point P.

68. $OI = 40$ cm; $MI = MP = 20$ cm.

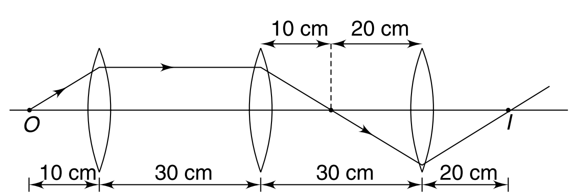
ΔOPM is equilateral

$$\theta = 60°.$$



69. The diagram explains the position of image (I).

If the tube is moved away by 10 cm, the object distance for the first lens becomes 20 cm. From the principle of reversibility of path, we can conclude that final image will be formed at a distance of 10 cm from the third lens. Hence image position does not change.



70. For air lens $$P = \left(\frac{\mu_a}{\mu_g} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{1}{3/2} - 1\right)\left(\frac{2}{R}\right)$$

$$= -\frac{2}{3R}$$

If a material of refractive index μ is filled in the cavity

$$P' = \left(\frac{\mu}{3/2} - 1\right)\left(\frac{2}{R}\right)$$

$$\frac{2}{3R} = \left(\frac{2\mu}{3} - 1\right)\frac{2}{R}$$

$$\Rightarrow \quad 1 = 2\mu - 3$$

$$\Rightarrow \quad \mu = 2$$

71. For, lens at B, A is object.

$$\therefore \quad \frac{1}{v_B} - \frac{1}{2x} = \frac{1}{-x}$$

$$\frac{1}{v_B} = -\frac{1}{2x} \quad \Rightarrow \quad v_B = -2x$$

$\therefore$ Image is formed at P and magnification is $m_1 = \left(\frac{-2x}{2x}\right) = -1$

The image at P acts as object for the lens at C.

$$\frac{1}{v_C} - \frac{1}{-3x} = \frac{1}{2x}$$

$$\frac{1}{v_C} = \frac{1}{2x} - \frac{1}{3x} = \frac{1}{6x}$$

$$v_C = 6x$$

Find image is at a distance $6x$ from C.

Magnification $\quad m_2 = \left(\frac{6x}{-3x}\right) = -2$

$$\therefore \quad M = m_1 m_2 = 2$$

$\therefore$ Find height of image = $2h$.

72. The image of the object is formed on the other side of the lens at a distance of 20 cm.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad -\frac{1}{v^2}\frac{dv}{dt} = -\frac{1}{u^2}\frac{du}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \left(\frac{v}{u}\right)^2 \frac{du}{dt}$$

When $\quad v = u; \quad \frac{dv}{dt} = \frac{du}{dt}$

$$\therefore \quad V_{IX} = V_{OX}$$

Magnification $\quad m = \frac{v}{u}$

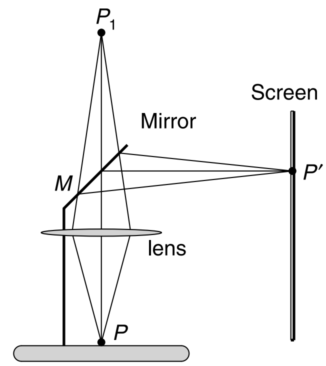
$$\therefore \quad \frac{y_I}{y_o} = \frac{v}{u}$$

$$\frac{dy_I}{dt} = \frac{v}{u}\frac{dy_0}{dt} + y_0 \frac{d}{dt}\left(\frac{v}{u}\right).$$

In given position $\quad y_0 = 0$

$$\therefore \quad V_{YI} = \left(\frac{v}{u}\right) V_{Y0} = \left(\frac{20}{-20}\right) V_{Y0} = -V_{Y0}$$

$\therefore$ Velocity of image will also make 45° with the principal axis (in downward direction)



73. In absence of mirror the image formed by mirror would have been at P1 with distance MP1 equal to MP'. If screen is moved away the distance MP1 must also increase. For this to happen the object must come closer to the lens. Thus the lens must be moved down.

74. Depth of image from water surface formed due to refraction = $\frac{3}{4} \times 20 = 15$cm. This image acts as an object at a distance of 40 cm from the mirror.

The mirror forms a virtual image at a distance of 40 cm that lies on the principal axis of the lens. This acts as an object for the lens at a distance of 90 cm.

Applying lens formula–

$$\frac{1}{v} - \frac{1}{-90} = \frac{1}{15}$$

$$\therefore \qquad v = 18 \text{ cm}$$

Thus final image is to the left of the lens at a distance of 18 cm.

75. Since magnification is positive, the object is at a distance less than its focal length and the image is virtual. The X co–ordinate of the lens is less than -20 cm. Let it be $-x_0$.

$$u = -(x_0 - 20),\ v = -(x_0 + 25)$$

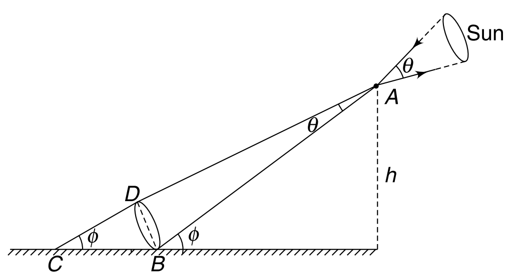
Also $$\left|\frac{v}{u}\right| = 2 \quad \Rightarrow \quad x_0 + 25 = 2(x_0 - 20) \quad \Rightarrow \quad x_0 = 65 \text{ cm}$$

$$\therefore \qquad u = -(65 - 20) = -45\text{cm and } v = -(65 + 25) = -90 \text{ cm}$$

Lens formula gives $$\frac{1}{f} = \frac{1}{-90} - \frac{1}{-45} \quad \Rightarrow \quad f = 90 \text{ cm}$$

76. The cone of rays passing through the hole at A produce an elliptical spot on the floor.

The circular base having diameter BD will get projected on the floor as an ellipse.



CB = major axis = 12 cm

DB = Minor axis = 6 cm

$$DB \simeq AB(\theta) = \frac{h\theta}{\sin\phi} \quad \Rightarrow \quad h = (6 \text{ cm})\,\frac{\sin\phi}{\theta}$$

But $$\sin\phi = \frac{DB}{CB} = \frac{6}{12} = \frac{1}{2}$$

$$\therefore \qquad h = \frac{(6 \text{ cm})}{2 \times \theta}$$

$$= \frac{3 \text{ cm} \times 180}{0.5 \times \pi} \qquad \left[0.5° = \frac{0.5 \times \pi}{180} \text{ radian}\right]$$

$$= 344 \text{ cm}$$
$$= 3.44 \text{ m}$$

77. For light to fall on the animal, tangent to the parabola must pass through the animal (200, 100)

Equation of parabola $y = ax^2$

because (–200, 200) lies on this parabola

$$\therefore \quad 200 = a(-200)^2 \Rightarrow a = \frac{1}{200}$$

$$\therefore \quad y = \frac{x^2}{200}$$

Let the required point has co-ordinates (x_0, y_0). Slope of line passing through (x_0, y_0) and (200, 100) is

$$m = \frac{y_0 - 100}{x_0 - 200} = \frac{ax_0^2 - 100}{x_0 - 200} = \frac{\frac{x_0^2}{200} - 100}{x_0 - 200}$$

This should be equal to $\left.\dfrac{dy}{dx}\right|_{x_0, y_0} = 2ax_0$

$$= \frac{x_0}{100}$$

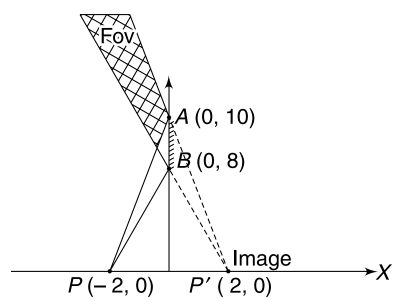
$$\therefore \quad \frac{\frac{x_0^2}{200} - 100}{x_0 - 200} = \frac{x_0}{100}$$

$$\left(\frac{x_0^2}{200} - 100\right) \times 100 = x_0^2 - 200x_0$$

$$\Rightarrow \quad x_0^2 - 400x_0 + 20000 = 0$$

Solving $x_0 = 58.6$ m [other value of x_0 is not acceptable. Why?]

$$\therefore \quad y_0 = \frac{(58.6)^2}{200} = 17.17 \text{ m.}$$



78. (a) Image is visible only if the observer lies in the field of view (FOV) region

Equation of line BP' is $\quad y_1 = -4x + 8$

equation of line AP' is $\quad y_2 = -5x + 10$

Point O_1 lies outside the FOV whereas O_2 lies inside it. [x – co-ordinate for O_1 is –2. Putting in equation the two lines gives $y_1 = 16$ and $y_2 = 20$. The y co-ordinate of O_1 does not lie in this range. Similarly for O_2 – $y_1 = 12$; $y_2 = 15$ and y co-ordinate of O_2 lies in this range $12 < 13 < 15$. Hence, O_2 is in FOV. He will see the image].

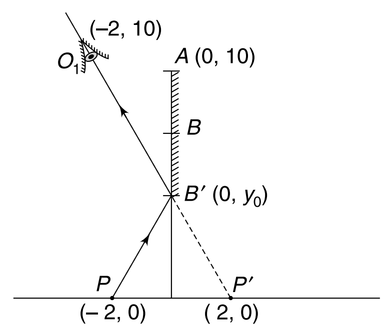
(b) Let the mirror be extended upto B' $(0, y_0)$.

O_1 will be just able to see the image if ray reflected at B' passes through $O_1(-2, 10)$. Equation of line O_1P' is

$$y = -2.5\,x + 5$$

$$\therefore \quad y \text{ intercept} = 5 = y_0$$

Hence length of mirror $\quad AB' = 5$ m



79. (a) Ray OA_1 incident on $M1$ just misses mirror $M2$. In this case third image will not be formed. Similarly, if OA_2 $(= OA_1)$ is removed, no ray incident on $M2$ can hit mirror $M1$.

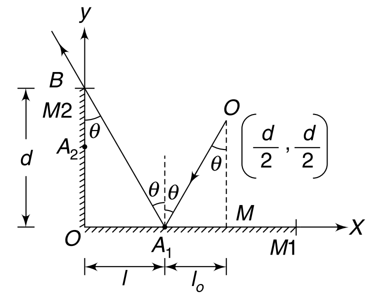
From geometry

$$\tan\theta = \frac{l}{d} = \frac{l_0}{d/2}$$

$$\Rightarrow \qquad l = 2l_0$$

$$l + l_0 = \frac{d}{2}$$

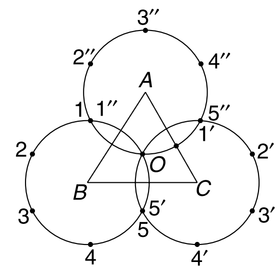
$$\therefore \qquad l = \frac{2}{3}\frac{d}{2} = \frac{d}{3}$$



80. Images formed due to multiple reflections in mirrors AB and BC will lie on a circle centred at B. Position of images are 1, 2, 3, 4, 5. Images due to reflections in BC and CA are 1′, 2′, 3′, 4′ and 5′. Images due to reflections in AB and AC are 1″, 2″, 3″ 4″ and 5″.

1 and 1″, 1′ and 5″ and 5 and 5′ coincide.

$\therefore$ Total number of images = 12.



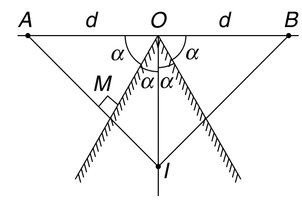
81. (a) Line joining the object and the image is perpendicular to the mirror, and object and the image are at equal distance from the mirror.

$$\Delta AOM \cong \Delta IOM$$

$\therefore$ All angles marked as α in the figure are equal.

$$4\alpha = 180° \quad \Rightarrow \quad \alpha = 45°$$

$$\therefore \qquad \theta_0 = 90°$$

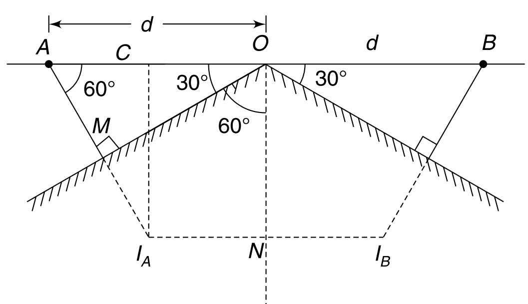


(b)
$$\theta = \frac{4}{3}\theta_0 = 120°$$

$$AM = d\sin 30° = \frac{d}{2} \quad \Rightarrow \quad AI_A = 2AM = d$$

$$\therefore \qquad AC = (AI_A)\cos 60° = \frac{d}{2} \quad \Rightarrow \quad OC = \frac{d}{2}\ (= NI_A)$$

$$\therefore \qquad I_AI_B = d.$$

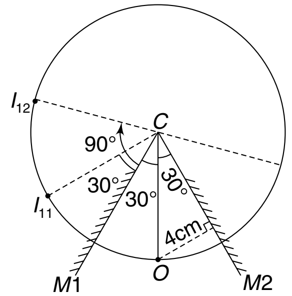


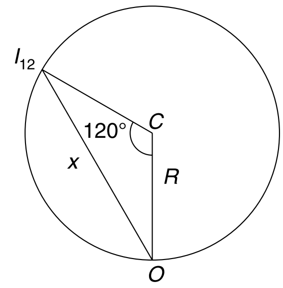
82. All the images lie on a circle of radius

$$R = OC = \frac{4}{\sin 30°} = 8 \text{ cm.}$$

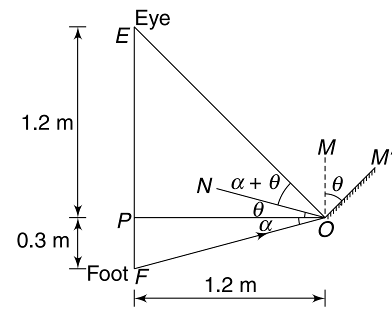
Position of second image (I_{12}) formed in $M1$ has been shown in the Figure. It is actually the image of first image (I_{21}) formed in $M2$. Required distance is

$$x = 2R\sin 60° = 2 \times 8 \times \frac{\sqrt{3}}{2} = 8\sqrt{3} \text{ cm.}$$





83. When mirror is in position OM, normal to it is OP. When mirror is rotated, the normal to it rotates to position ON.



FO = incident ray from foot to the lower edge.

OE = reflected ray.

$$\angle FON = \text{angle of incidence} = \alpha + \theta$$

$$= \text{angle of reflection} = \angle NOE$$

From Figure $\quad EP = PO = 1.2$ m

$$\therefore \quad \angle EOP = 45°$$

$$\therefore \quad 2\theta + \alpha = 45°$$

But

$$\tan\alpha = \frac{0.3}{1.2} = \frac{1}{4} \quad \Rightarrow \quad \alpha = 15°$$

$$\therefore \quad \theta = 15°$$

If mirror is rotated beyond this angle, reflected rays will pass above the eye.

84. (a) Image formed by $M2$ is x_2 behind it. It means the image is $x_2 - d_0$ behind the mirror $M1$.

For concave mirror (with A as object)

$$u = -x_1; \quad v = x_2 - d_0$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

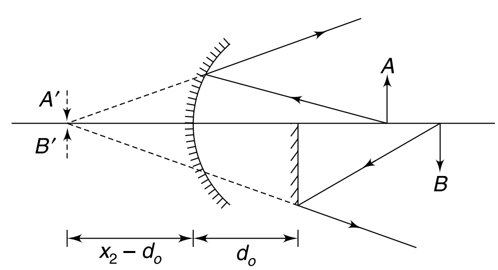
$$\frac{1}{f} = -\frac{1}{x_1} + \frac{1}{x_2 - d_0}$$

$$f = \frac{(x_2 - d_0)\, x_1}{-x_2 + d_0 + x_1}$$

$$= -\frac{(x_2 - d_0)\, x_1}{x_2 - (d_0 + x_1)}$$

$$\therefore \quad |f| = \frac{(x_2 - d_0)\, x_1}{x_2 - (d_0 + x_1)}$$

(b) Rays incident on $M1$ will be reflected above the principal axis and rays reflected from $M2$ will be found only below the axis. There is no region where both set of reflected rays are present. An observer cannot see both the images simultaneously. To compare the position of the images the observer should move his eye in the vertical plane and view the images in turn.



85. (a) **Steps:**

(1) Mirror is concave because image is enlarged and inverted.

Magnification = 3, *i.e.* greater than 1. It means image distance is larger than object distance.

Hence mirror is above AB in our diagram.

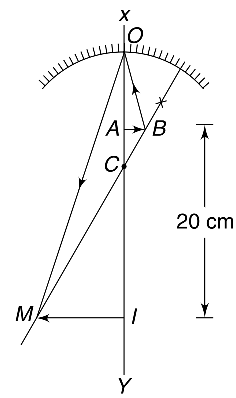
(2) $OI = 3\ OA$ where O is pole of the mirror.

$AI = 20$ cm

$\therefore \quad AO = 10$ cm

It means pole (O) of the mirror is 10 cm from A.

(3) To complete the ray diagram we can draw the incident ray BO which is reflected as OM.



(b) $\dfrac{MI}{AB} = \dfrac{CI}{CA}$

$3 = \dfrac{20 - CA}{CA}$

$\therefore \quad CA = 5$ cm

$\therefore \quad OC = 10 + 5 = 15$ cm

$\therefore \quad$ Focal length $f = \dfrac{OC}{2} = 7.5$ cm.

86. (a) The image formed by upper mirror is virtual. It means it is above the mirror. Say it is at a distance y from the mirror.

$$\frac{1}{y} + \frac{1}{-h} = \frac{1}{-f} \quad \Rightarrow \quad \frac{1}{y} = \frac{1}{h} - \frac{1}{f} \qquad \text{...(1)}$$

This image is at a distance $(y + h)$ from the lower miror and acts as an object. Image of this object is just above the mouth. This implies that image distance $\simeq h$

$$\therefore \qquad \frac{1}{-h} + \frac{1}{-(y+h)} = \frac{1}{-f}$$

$$\therefore \qquad \frac{1}{h} + \frac{1}{h + \left(\frac{1}{h} - \frac{1}{f}\right)^{-1}} = \frac{1}{f} \quad \text{Proved.}$$

(b) Multiply the above equation by f

$$\frac{f}{h} + \frac{1}{\frac{h}{f} + \left(\frac{f}{h} - 1\right)^{-1}} = 1$$

$$\Rightarrow \qquad \frac{1}{x} + \frac{1}{x + \left(\frac{1}{x} - 1\right)^{-1}} = 1$$

$$\Rightarrow \qquad \frac{1}{x} + \frac{1}{x + \frac{x}{1-x}} = 1$$

$$\Rightarrow \qquad \frac{1}{x} + \frac{1-x}{x - x^2 + x} = 1$$

$$\Rightarrow \qquad x - x^2 + x + x - x^2 = x^2 - x^3 + x^2$$

$$\Rightarrow \qquad x^3 - 4x^2 + 3x = 0$$

$$\Rightarrow \qquad x[x^2 - 4x + 3] = 0 \quad \text{But} \quad h \neq 0 \quad \Rightarrow \quad x \neq 0$$

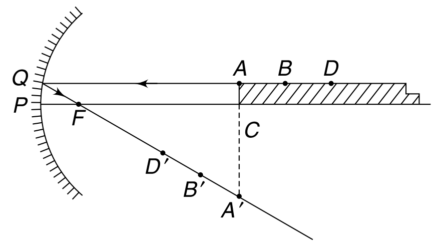
$\therefore \qquad x^2 - 4x + 3 = 0 \quad \Rightarrow \quad x = 1;\ x = 3$

$$x = 3 \quad \Rightarrow \quad h = 3f$$

This means that first image cannot be virtual.

Hence $x = 1$

87. AQ is an incident ray parallel to the principal axis which gets reflected as QF. [F is the focus]. Image of A is at A'. Image of B, D etc. will be nearer to the focus on the line QF.

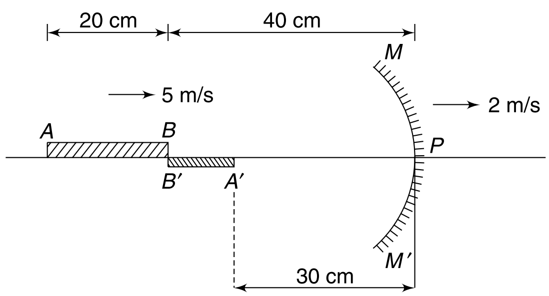


A point that is at infinite distance along ABD will have its image at F. Hence the image of the rectangular strip will be a triangle $A'CF$.

$$\text{Area of image} = \frac{1}{2} \times FC \times CA'$$

$$= \frac{1}{2} fh.$$

88. Point B is at the centre of curvature of the mirror and its image will be at the same position. Image of point A is formed at distance v_A given by mirror formula.



$$\frac{1}{v_A} + \frac{1}{-60} = \frac{1}{-20}$$

$$\frac{1}{v_A} = \frac{1}{60} - \frac{1}{20}; \quad \Rightarrow \quad v_A = -30 \text{ cm}$$

And $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\frac{dv}{dt} = -\left(\frac{v}{u}\right)^2 \frac{du}{dt}$$

$$V_{Im} = -\left(\frac{v}{u}\right)^2 \cdot V_{Om} \qquad \begin{bmatrix} V_{Im} = \text{velocity of image wrt mirror} \\ V_{Om} = \text{velocity of object wrt mirror} \end{bmatrix}$$

$$V_{B'} - V_m = -\left(\frac{40}{40}\right)^2 \cdot [V_B - V_m]$$

$$V'_B - 2 = -[5 - 2]$$

$$V'_B = -1 \text{ m/s} = 1 \text{ m/s } (\rightarrow)$$

For A' (Image of A)

$$V_{A'} - V_m = -\left(\frac{30}{60}\right)^2 [V_A - V_m]$$

$$V_{A'} - 2 = -\frac{1}{4}[5 - 2]$$

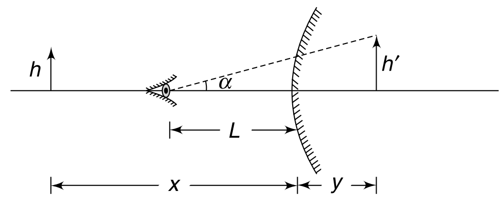
$$V_{A'} = +\frac{5}{4} \text{ m/s} = \frac{5}{4} \text{ m/s } (\rightarrow)$$

$\therefore$ length $A'B'$ is increasing at a rate of

$$1 + \frac{5}{4} = \frac{9}{4} \text{ m/s.}$$

89. (a) Let h and h' be the height of the object and image respectively. y is image distance.

From mirror formula



$$\frac{1}{y} + \frac{1}{-x} = \frac{1}{f}$$

$$\Rightarrow \quad y = \frac{fx}{f + x} \quad ...(1)$$

And $$\frac{h'}{h} = \frac{y}{x}$$

$$\therefore \quad \tan\alpha \simeq \alpha = \frac{h'}{L + y} = \frac{\frac{yh}{x}}{L + y}$$

$$= \frac{h}{x\left[\frac{L}{y} + 1\right]}$$

$$= \frac{h}{\frac{L(f + x)}{f} + x} = \frac{h}{L + x + \frac{Lx}{f}} \quad \text{[using (1)]}$$

(b) For a plane mirror $y = x$ and $h' = h$

$$\therefore \quad \beta = \frac{h'}{y + L} = \frac{h}{x + L}$$

(c) Clearly $\alpha < \beta$.

It means images in convex mirror subtend smaller angles at our eyes and hence appear to be smaller. We perceive them to be at larger distance.

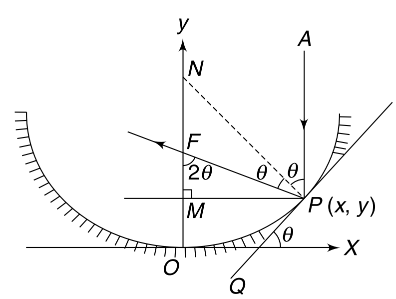
90. Consider a light ray incident on the mirror at point $P(x, y)$ on the mirror.

$AP \rightarrow$ incident ray, $PN \rightarrow$ normal

$PF \rightarrow$ reflected ray, $PQ \rightarrow$ tangent at P

$$y = kx^2$$

$$\therefore \quad \tan\theta = \frac{dy}{dx} = 2kx \quad \ldots(1)$$



In ΔFMP $$\tan 2\theta = \frac{PM}{FM} = \frac{x}{f - y}$$

$$[f = OF]$$

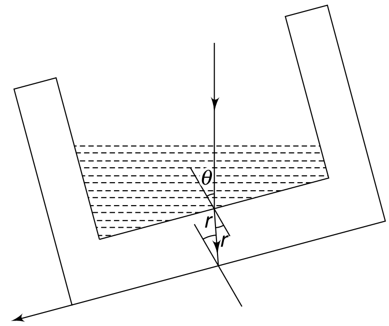
$$\Rightarrow \quad \frac{x}{f - y} = \tan 2\theta$$

$$\Rightarrow \quad f = y + \frac{x}{\tan 2\theta} = y + \frac{x[1 - \tan^2\theta]}{2\tan\theta}$$

$$= kx^2 + \frac{x[1 - (2kx)^2]}{2\cdot(2\cdot kx)} = \frac{1}{4k} \text{ (independent of } (x, y)!)$$

91. $$\mu_w \sin\theta = \mu_g \sin r$$

$$\frac{4}{3}\sin\theta = \frac{3}{2}\sin r \quad ...(1)$$



And $$\frac{3}{2}\sin r = 1\cdot\sin 90°$$

$$\sin r = \frac{2}{3} \quad ...(2)$$

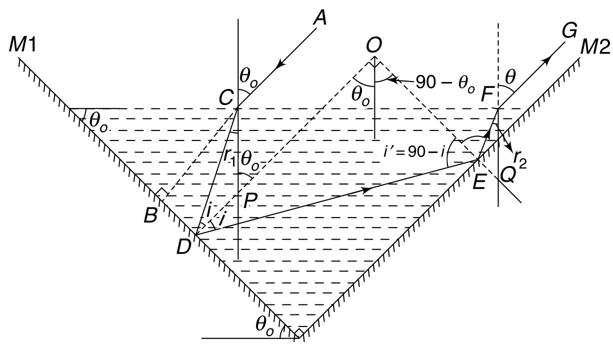
From (1) and (2) $$\sin\theta = \frac{3}{4}$$

92. Angle of incidence at liquid surface = θ_0

DO is normal to $M1$ and hence parallel to AC

EO is normal to $M2$

$$\angle DOE = 90°$$



In ΔCDP

$$i + r_1 = \theta_0 \quad ...(1)$$

Angle of incidence on $M2$

$$i' = 90 - i \quad [\because \text{ In } \Delta DOE - \angle DOE = 90°]$$

In ΔQEF $$\angle EQF = 90° - \theta_0$$

$$\angle FEQ = 180° - (90 - i) = 90 + i$$

$\therefore$ $$r_2 = 180° - [90 - \theta_0 + 90 + i] = \theta_0 - i$$

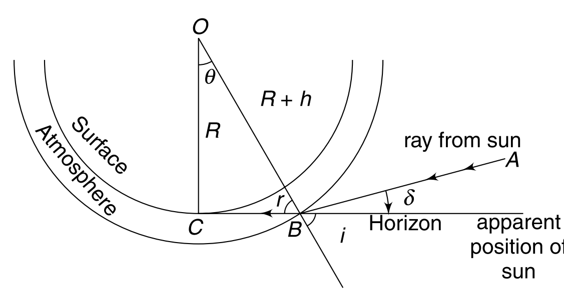
$$= r_1 \quad [\text{from (1)}]$$

Because $$r_2 = r_1$$

$\therefore$ $$\theta = \theta_0$$

93 In Figure AB is incident ray reaching the observe at C after undergoing refraction at B.

Snell's law



$$\sin i = \mu \sin r$$

$$\sin i = \left(\frac{\mu R}{R+h}\right)$$

$$i = \sin^{-1}\left(\frac{\mu R}{R+h}\right)$$

$$\delta + r = \sin^{-1}\left(\frac{\mu R}{R+h}\right)$$

$$\delta = \sin^{-1}\left(\frac{\mu R}{R+h}\right) - \sin^{-1}\left(\frac{R}{R+h}\right)$$

In presence of atmosphere the sun becomes visible when it is δ below the horizon. In absence of atmosphere, the observer at C will be able to see the sun only after the earth rotates further by an angle δ.

$$\therefore \qquad \Delta t = \frac{\delta}{\frac{2\pi}{T}} = \frac{T}{2\pi}\left[\sin^{-1}\left(\frac{\mu R}{R+h}\right) - \sin^{-1}\left(\frac{R}{R+h}\right)\right]$$

$$\simeq 0.9 \text{ min.}$$

94.
$$\sqrt{2}\sin 30° = 1\cdot\sin r \;\Rightarrow\; r = 45°$$

$$\frac{PR}{PQ} = \cos i \;\Rightarrow\; PQ = \frac{d_0}{\cos 30°} = \frac{2d_0}{\sqrt{3}}$$

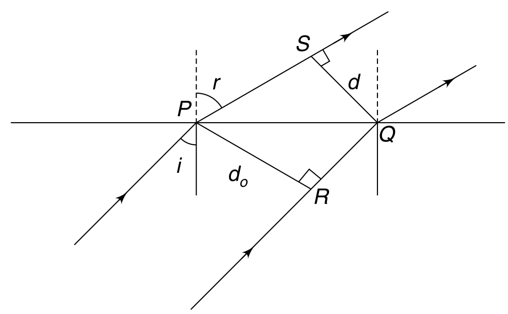
$$QS = d = \text{diameter of refracted light}$$

$$\frac{d}{PQ} = \cos r \;\Rightarrow\; PQ = \frac{d}{\cos 45°} = \sqrt{2}d.$$

$$\therefore \qquad \sqrt{2}d = \frac{2d_0}{\sqrt{3}} \;\Rightarrow\; d = \sqrt{\frac{2}{3}}\, d_0$$

The refracted beam has lesser diameter.

Intensity of refracted beam



$$I = \frac{I_0\pi\left(\frac{d_0}{2}\right)^2}{\pi\left(\frac{d}{2}\right)^2} = I_0\cdot\left(\frac{d_0}{d}\right)^2$$

$$\therefore \qquad I = \frac{3}{2} I_0$$

95. Angle of refraction at first refraction is given by

$$\mu_1 \sin 45° = \mu_2 \sin r \;\Rightarrow\; 2\frac{1}{\sqrt{2}} = \mu_2 \sin r$$

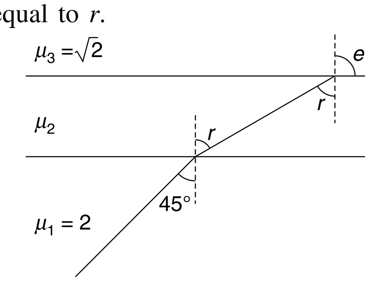
$$\Rightarrow \qquad \mu_2 \sin r = \sqrt{2} \qquad \text{...(i)}$$

For $\mu_2 < \sqrt{2}$, $\sin r > 1$ which is not possible. It means the ray will suffer total internal reflection.

Deviation in this case will be 90°.

For $\mu_2 > \sqrt{2}$, the ray will pass on to the second interface with angle of refraction equal to r.

Applying snell's law at second interface–



$$\Rightarrow \qquad \mu_2 \sin r = \sqrt{2}\sin e \qquad \text{...(ii)}$$

From (i) and (ii)

$$\sqrt{2}\sin e = \sqrt{2} \;\Rightarrow\; e = 90°$$

Deviation is clearly 45°

96. (a) $\theta \simeq C$

$$\therefore \qquad 1.6\sin\theta = 1.5\sin 90°$$

$$\sin\theta = \frac{15}{16}$$

A distance x along the length of the cable means the ray along B must have travelled a path length of $\frac{x}{\sin\theta} = \frac{16x}{15}$.

$$\therefore \quad \text{Total path length for } B \text{ is} \quad x_B = \frac{16}{15}\times 1500$$

$$= 1600 \text{ m.}$$

$$\therefore \qquad \Delta x = 100 \text{ m.}$$

$$\therefore \qquad \Delta t = \frac{100 \text{ m}}{V} = \frac{100 \text{ m}}{\dfrac{3 \times 10^8}{1.6} \text{ ms}^{-1}} = 0.53 \times 10^{-6} \text{ s}$$

$$= 0.53 \ \mu\text{s}.$$

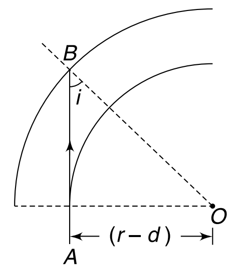
(b) Crest from path A will reach with the trough from path B if

Δt = Half the time period of the signal.

$\therefore$ Time period of signal $= 2\Delta t = 1.06 \ \mu$s.

$$\therefore \qquad f = \frac{1}{T} = \frac{1}{1.06 \times 10^{-6}} = 0.94 \text{ MHz}.$$

97. (a) The minimum radius r would be when the angle of incidence of the ray AB is such that it is totally reflected.



$$\Rightarrow \qquad i = C$$

$$\sin C = \frac{r_o - d}{r_o}$$

and

$$\mu \sin C = 1 \cdot \sin 90°$$

$$\Rightarrow \qquad \sin C = \frac{1}{\mu}$$

$$\therefore \qquad \frac{r_o - d}{r_o} = \frac{1}{\mu}$$

$$\Rightarrow \qquad r_o = \frac{\mu d}{(\mu - 1)}$$

for

$$d = 200 \ \mu\text{m} \quad \text{and} \quad \mu = 1.4$$

$$r_o = \frac{1.4 \times 200}{0.4} = 700 \ \mu\text{m} = 0.7 \text{ mm (very small !!)}$$

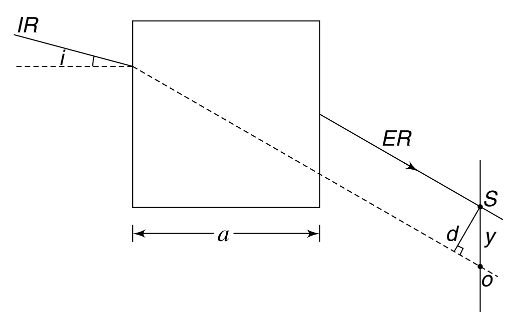
(b) as $d \to 0$; $\quad r_o \to 0$

(c)

$$r_o = \frac{d}{1 - \dfrac{1}{\mu}}$$

$\therefore$ for smaller r_o, μ shall be larger

98. For angle of incidence i, the incident ray (IR) and the emergent ray (ER) has been shown in the Figure. The lateral shift when i is small is given by



$$d = \left(1 - \frac{1}{\mu}\right) i$$

The spot at S is at a distance y from O (the point where incident ray will hit the screen if there is no shift).

$$y = \frac{d}{\cos i}$$

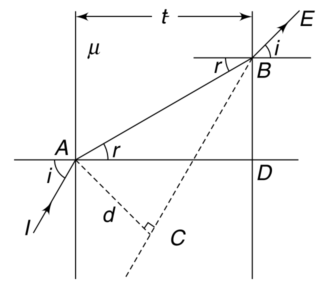
When $i \to 0$; $\quad y \to d$.

$$\therefore \qquad y = a\left(1 - \frac{1}{\mu}\right) i$$

$$\frac{dy}{dt} = a\left(1 - \frac{1}{\mu}\right) \frac{di}{dt}$$

$$V = a\left(1 - \frac{1}{\mu}\right) \omega = \frac{a\omega}{3}$$

99.



$$AB = \frac{t}{\cos r}$$

$$AC = AB\,\sin(i - r)$$

$$d = \frac{t}{\cos r}\cdot\sin(i - r) \qquad ...(1)$$

$$= \frac{t}{\cos r}[\sin i\,\cos r - \cos i\,\sin r]$$

$$= t\sin i\left[1 - \frac{1}{\mu}\,\frac{\cos i}{\cos r}\right] \qquad \left[\because\ \frac{\sin i}{\sin r} = \mu\right]$$

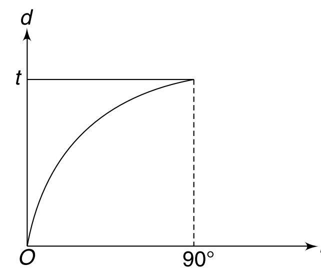
$$= t\sin i\left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right] \qquad ...(2)$$

$$\left[\because\ \sin r = \frac{\sin i}{\mu},\ \cos r = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}\mu,\ \cos r = \sqrt{\mu^2 - \sin^2 i}\right]$$

For $i \to 0$ $\quad d = t(i - r)$ [from (1)]

$$d \simeq ti\left(1 - \frac{r}{i}\right) = ti\left(1 - \frac{1}{\mu}\right)$$

[$\because$ for $i \to 0$ $\sin(i - r) = i - r$ and $\cos r \to 1$]



For $i \to \frac{\pi}{2}$, d is maximum

$$d_{\max} = t \quad \text{[from (2)]}$$

$\therefore$ graph is as shown.

100. Snell's law

$$\mu\sin r = 1\cdot\sin 45°$$

$$\therefore \quad \sin r = \frac{1}{2}$$

$$r = 30°.$$

Light ray incident at O will not suffer any deviation at the curved surface.

Consider another ray incident at point A on the plane surface. For this ray to not suffer a TIR at B

$$\alpha \le C$$

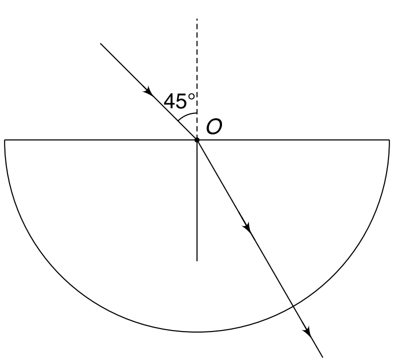
$$\alpha \le \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

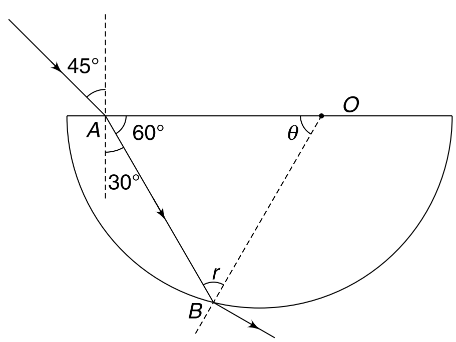
$$\alpha \le 45°$$

$$(180° - 60° - \theta) < 45°$$

$$-\theta \le -75°$$

$$\theta \ge 75°$$





For θ = 75°

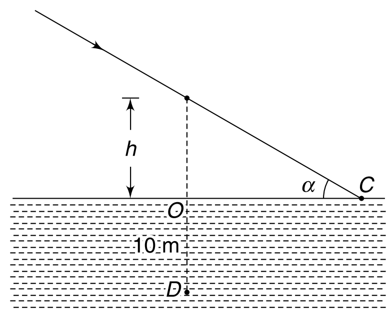
$$\frac{OA}{\sin 45°} = \frac{R}{\sin 60°}$$

$$OA = \frac{2R}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} = \sqrt{\frac{2}{3}} \cdot R$$

101.

$$h_{\text{app}} = \mu h$$

$$h_{\text{app}} = 50 - d = 40 \text{ m} \quad \text{and} \quad \mu = \frac{4}{3}$$



∴ $$h = 30 \text{ m}$$

Now, $$h_{\text{app}} = \mu h$$

$$V_{y\text{app}} = \mu V_y$$

$$V_{x\text{app}} = V_x$$

Given $$\frac{V_{y\text{app}}}{V_{x\text{app}}} = \tan 45° = 1$$

∴ $$\frac{\mu V_y}{V_x} = 1 \quad \Rightarrow \quad \tan\alpha = \frac{1}{\mu} = \frac{3}{4}$$

$$\alpha = 37°$$

α is the true angle that the line of motion of the bird makes with horizontal.

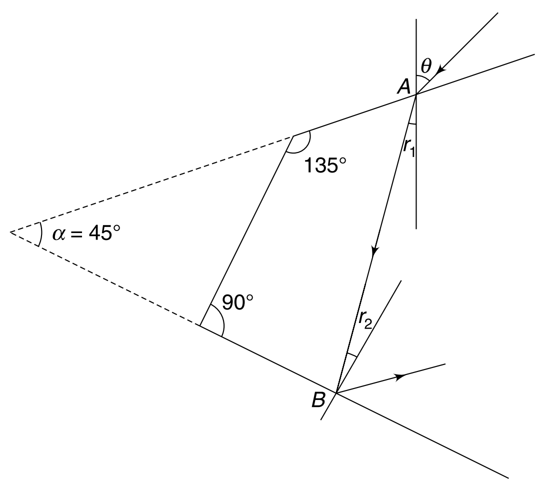
∴ $$OC = h\tan a = 30 \cdot \frac{3}{4}$$

$$= 22.5 \text{ m}$$

∴ $$DC = \sqrt{(22.5)^2 + 10^2}$$

$$= 24.62 \text{ m.}$$

102. For TIR at B, $r \geq C$



$$r_2 \geq \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$r_2 \geq \sin^{-1}\left(\frac{1}{2}\right)$$

$$r_2 \geq 30°$$

$$\alpha - r_1 \geq 30°$$

$$45° - r_1 \geq 30°$$

$$15° \geq r_1$$

$$\sin(15°) \geq \sin(r_1) \quad [\mu \sin r_1 = 1 \cdot \sin\theta]$$

$$\sin 15° \geq \frac{1}{2}\sin\theta$$

$$2\sin 15° \geq \sin\theta$$

$$\sin^{-1}(2\sin 15°) \geq \theta$$

103.(a) For any light ray $$i' = 90° - \alpha$$

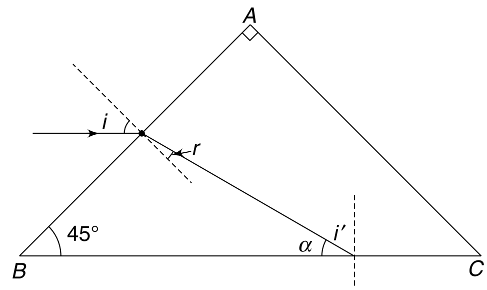
$$= 90° - [180° - 45° - (90° + r)]$$

$$= 45° + r$$

∴ Minimum value of $$i' = 45°$$

when $r = 0$, i.e., when $i = 0$

Total internal reflection will occur at BC if



$$45° \geq C$$

$$\Rightarrow \quad \sin 45° \geq \sin C$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} \geq \frac{1}{\mu}$$

$$\Rightarrow \quad \sqrt{2} \leq \mu.$$

(b) $PQ \rightarrow$ incident ray

$ST \rightarrow$ emergent ray

$$1 \cdot \sin 45° = \mu \cdot \sin r$$

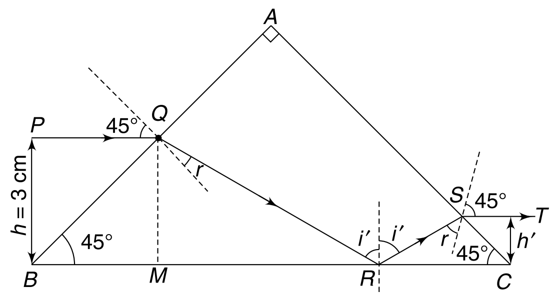
$$\Rightarrow \quad r = 30° \quad [\because \quad \mu = \sqrt{2}]$$

Path of light ray is as shown.

$$\Delta BQR \sim \Delta CSR$$

$$\therefore \quad \frac{BQ}{SC} = \frac{BR}{RC} \quad \Rightarrow \quad \frac{\sqrt{2}h}{\sqrt{2}h'} = \frac{BR}{RC}$$

$$\Rightarrow \quad h' = 3\left(\frac{RC}{BR}\right) \quad [\because \quad h = 3 \text{ cm}] \qquad ...(1)$$



$$i' = 45° + r = 75°$$

$$\angle QRM = 15°$$

$$\therefore \quad \frac{QM}{MR} = \tan 15° \quad \Rightarrow \quad MR = \frac{3}{0.25} = 12 \text{ cm}$$

$$BM = 3 \text{ cm}$$

$$\therefore \quad BR = 3 + 12 = 15 \text{ cm}$$

$$\therefore \quad RC = 20 - 15 = 5 \text{ cm}$$

$$\therefore \quad \text{from (1)} \quad h' = 3 \times \frac{5}{15} = 1 \text{ cm}$$

104. Angle of incidence at the first refracting surface is

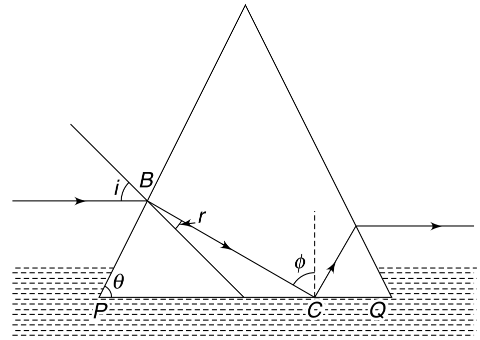
$$i = \frac{\pi}{2} - \theta$$

Snell's law gives

$$\mu_g \sin r = \sin i$$

$$\frac{3}{2} \sin r = \cos \theta \qquad ...(1)$$

In ΔPBC



$$\angle B + \angle P + \angle C = 180°$$

$$90 + r + \theta + 90 - \phi = 180°$$

$$\Rightarrow \quad \phi = r + \theta.$$

For total internal reflection at the base.

$$\phi > C$$

$$r + \theta > C$$

$$\Rightarrow \quad \sin(r + \theta) > \sin C = \frac{4/3}{3/2} = \frac{8}{9}$$

$$\Rightarrow \quad \sin(r + \theta) > \frac{8}{9}$$

$$\Rightarrow \quad \cos\theta \cdot \sin r + \sin\theta \cos r > \frac{8}{9}$$

$$\frac{2}{3}\cos^2\theta + \sin\theta \cdot \sqrt{1 - \frac{4}{9}\cos^2\theta} > \frac{8}{9} \quad \text{[using (1)]}$$

$$\Rightarrow \quad \sin^2\theta\left(1 - \frac{4}{9}\cos^2\theta\right) > \left(\frac{8}{9} - \frac{2}{3}\cos^2\theta\right)^2$$

$$\cos^2\theta = x$$

$$\Rightarrow \quad (1 - x)\left(1 - \frac{4}{9}x\right) > \left(\frac{8}{9} - \frac{2}{3}x\right)^2$$

$$\Rightarrow \quad 9(1 - x)(9 - 4x) > (8 - 6x)^2$$

$$\Rightarrow \quad 9[9 - 13x + 4x^2] > (64 + 36x^2 - 96x)$$

$$\Rightarrow \quad \frac{17}{21} > x \quad \Rightarrow \quad \cos\theta < \sqrt{\frac{17}{21}}$$

$$\therefore \quad \theta_{\max} = \cos^{-1}\sqrt{\frac{17}{21}}$$

105. Using Snell's law for four successive refractions we get

$$\sin i = \mu_1 \sin r_1 \quad ...(1)$$

$$\mu_1 \cdot \sin(90 - r_1) = \mu_2 \sin r_3$$

$$\Rightarrow \quad \mu_2 \sin r_3 = \mu_1 \cos r_1 \quad ...(2)$$

$$\mu_2 \sin r_4 = \mu_3 \sin r_5$$

$$\Rightarrow \quad \mu_2 \sin(90 - r_3) = \mu_3 \sin r_5$$

$$\Rightarrow \quad \mu_2 \cos r_3 = \mu_3 \sin r_5 \quad ...(3)$$

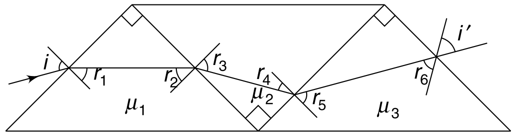
And $\quad \mu_3 \sin r_6 = \sin i'$

for emergent ray to be parallel to the incident ray

$$i' = 90 - i$$

$$\therefore \quad \mu_3 \sin(90 - r_5) = \sin(90 - i)$$

$$\Rightarrow \quad \cos i = \mu_3 \cos r_5 \quad ...(4)$$



Square all the equations -1, 2, 3 and 4 and add them

$$\sin^2 i + \mu_2^2 \sin^2 r_3 + \mu_2^2 \cos^2 r_3 + \cos^2 i = \mu_1^2 \sin^2 r_1 + \mu_1^2 \cos^2 r_1 + \mu_3^2 \sin^2 r_5 + \mu_3^2 \cos^2 r_5$$

$$\Rightarrow \quad 1 + \mu_2^2 = \mu_1^2 + \mu_3^2$$

$$\Rightarrow \quad \mu_2 = \sqrt{\mu_1^2 + \mu_3^2 - 1}$$

106. For minimum deviation of red light

$$r_1 = r_2 = \frac{A}{2} = 30°$$

Applying Snell's law at first refracting face

$$(1)\ \sin i = \mu \sin r_1$$

$$\sin i = 1.5 \cdot \sin 30° = 0.75$$

$$\therefore \quad i = \sin^{-1}(0.75) = 49°$$

Deviation of red light $\quad \delta_R = 2i - A$

$$= 2 \times 49° - 60° = 38°$$

For violet light

$$(1)\ \sin 49° = (1.6)\ \sin r_1$$

$$\therefore \quad \sin r_1 = \frac{0.75}{1.6} = 0.47$$

$$\therefore \quad r_1 = 28°$$

$$\therefore \quad r_2 = A - r_1 = 60 - 28 = 32°$$

For refraction at second face

$$1.6 \cdot \sin(32°) = (1)\ \sin(e)$$

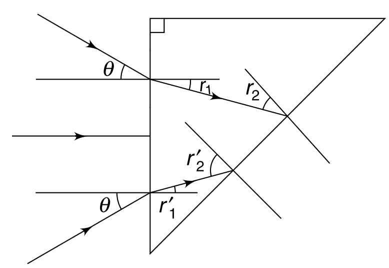
$$\therefore \quad \sin(e) = 1.6 \times 0.53 = 0.85$$

$$\therefore \quad e = 58°$$

$$\therefore \quad \delta_v = i + e - A = 49° + 58° - 60° = 47°$$

$\therefore$ Angular width $\quad (\theta) = \delta_v - \delta_R = 47° - 38° = 9°$

107. Critical angle $\quad C = \sin^{-1}\left(\frac{1}{\mu}\right) = 42°$



The Figure shows the extreme rays.
From geometry one can easily show that

$$r_2' = 45° + r_1' \quad [r_1' = r_1]$$

$$\therefore \quad r_2' > C$$

$\therefore$ Lower extreme ray will suffer TIR.
For upper extreme ray

$$r_2 = 45° - r_1$$

For TIR $\quad r_2 \geq C$

$$\therefore \quad 45° - r_1 \geq C$$

$$\therefore \quad 45° - C \geq r_1$$

$$3° \geq r_1$$

Snell's law

$$1 \cdot \sin\theta = \mu \sin r_1$$

For small angles $\quad 1(\theta) \simeq \mu(r_1)$

$$\therefore \quad \theta \leq 1.49 \times 3° = 4.47°$$

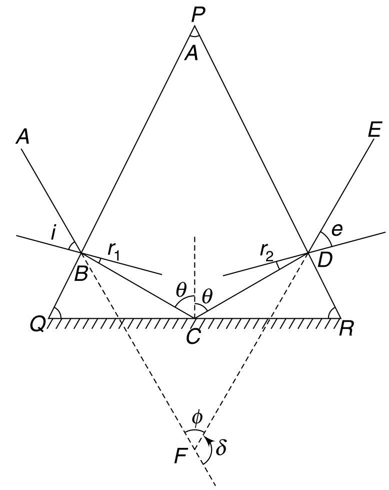
108. $$\delta = 180° - \phi \quad ...(1)$$

In polygon *PBFD*

$$A + (90 + i) + \phi + (90 + e) = 360°$$
$$\phi = 180° - i - e - A$$

From (1)

$$\delta = i + e + A$$



$\because \Delta QBC \sim \Delta CDR$ ‖ $\therefore \quad i = e$

$\therefore\ r_1 = r_2$ ‖ $\therefore \quad \delta = 2i + A$

This is independent of μ.

Note: Such prisms are known as reflecting prism which do not produce dispersion

109. (a) Minimum deviation for red

$$\frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \mu_r$$

$$\frac{\sin\left(\frac{60+\delta}{2}\right)}{\sin\left(\frac{60}{2}\right)} = 1.510$$

$$\therefore \quad \sin\left(30 + \frac{\delta}{2}\right) = 0.755$$

$$30 + \frac{\delta}{2} = 50$$

$$\delta = 40°$$

$$i + e - A = 40°$$

$$2i = 40 + 60 \quad \text{[In minimum deviation } i = e\text{]}$$

$$i = 50°$$

Refraction for violet

1st refraction $\quad \mu_v \sin r_1 = 1 \times \sin i$

$$\Rightarrow \quad \sin r_1 = \frac{\sin 50°}{1.550} = \frac{0.755}{1.550}$$

$$= 0.487$$

$$\therefore \quad r_1 = 28°$$

$$\therefore \quad r_2 = A - r_1 = 32°$$

2nd refraction $\quad 1.55 \sin 32° = 1 \times \sin e$

$$\sin e = 1.55 \times 0.529 = 0.819$$

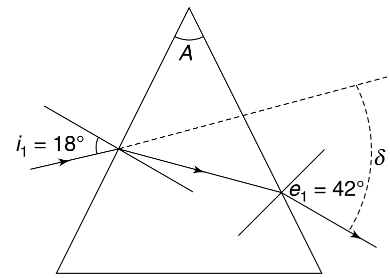
$$e = 55°$$

$\therefore$ Dispersion $\quad \theta = 55° - 50°$

$$= 5°$$

110. Path of the light ray is optically reversible. For two angle of incidence $i_1 = 18°$ and $i_3 = 42°$, the deviation suffered by the light ray is same. It means that for incidence angle, $i_1 = 18°$, the angle of emergence, $e_1 = 42°$ and vice–versa.

$$\delta = i + e - A$$
$\Rightarrow$ $$28° = 18° + 42° - A$$
$\Rightarrow$ $$A = 32°$$



111. $$\delta_m = 60° \quad \text{when} \quad i = 60°$$

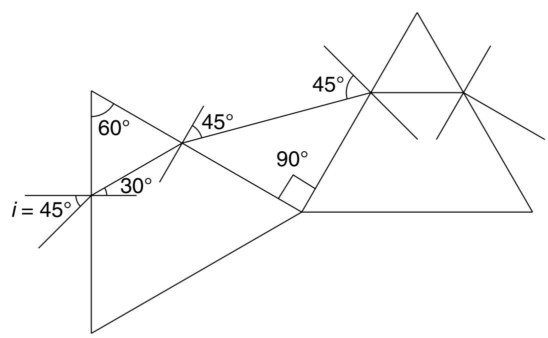
For minimum deviation $$i = e = 60°$$
$\therefore$ $$\delta_m = 2i - A$$
$\Rightarrow$ $$60° = 2 \times 60° - A \quad \Rightarrow \quad A = 60°$$
$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60 + 60}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$
$\therefore$ $$\mu = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

When angle of incidence is i_1, the angle of emergence is $e = 2i_1$.

$\therefore$ $$\delta = i_1 + 2i_1 - A$$
$$63° = 3i_1 - 60°$$
$\therefore$ $$i_1 = 41°$$

112. For overall deviation to be minimum, the light ray must suffer minimum deviations at both the prisms.

For minimum deviation at first prism
$$r_1 = r_2 = \frac{A}{2} = 30°$$



$\therefore$ Snell's law for first refraction gives $(1) \sin i = \sqrt{2} \sin 30°$

$\Rightarrow$ $$\sin i = \frac{1}{\sqrt{2}}$$
$\Rightarrow$ $$i = 45°$$

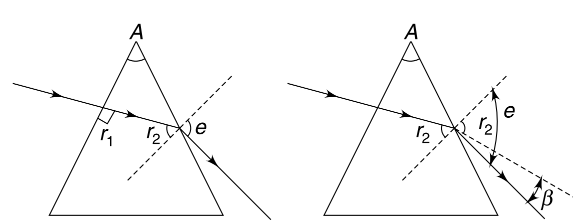
For angle of incidence to be 45° at the second prism, θ shall be 90° as shown in the diagram.

113.(a) There is no deviation when refractive index of surrounding medium is same as that of prism glass.

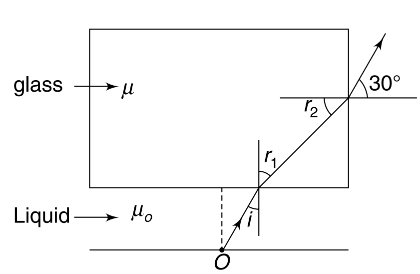
$\therefore$ $$\mu_1 = \frac{4}{\sqrt{3}} \quad \therefore \quad k_2 = \frac{4}{\sqrt{3}}$$

(b) It can be seen from graph when $\mu_1 < k_1$, light will not emerge.

$\therefore$ $$r_2 = C \quad \text{when} \quad \mu_1 = k_1$$



$$\sin r_2 = \sin C = \frac{\mu}{\mu_1}$$
$$\sin 60° = \frac{\mu}{\mu_1}$$
$$\frac{\sqrt{3}}{2} = \frac{4}{\sqrt{3}} = \times \frac{1}{\mu_1}$$
$$\mu_1 = \frac{8}{3}$$
$\therefore$ $$k_1 = \frac{8}{3}$$



114. $$i_{\max} = 90°$$
$\therefore$ $$\mu_0 \sin 90° = \mu \sin (r_{1\max})$$
$\Rightarrow$ $$\sin (r_{1\max}) = \frac{2\mu_0}{3} \quad \text{...(1)}$$

But $\qquad r_1 + r_2 = 90°$

$\therefore \qquad r_{2\,\min} = 90° - r_{1\,\max} \qquad ...(2)$

Using Snell's law at vertical face

$$\frac{3}{2}\sin r_2 = 1 \cdot \sin 30°$$

$$\Rightarrow \qquad \sin r_2 = \frac{1}{3}$$

$$\Rightarrow \qquad \sin(90 - r_1) = \frac{1}{3} \quad \Rightarrow \quad \cos r_1 = \frac{1}{3}$$

$$\Rightarrow \qquad \sqrt{1 - \frac{4\mu_0^2}{9}} = \frac{1}{3} \qquad \text{[using (i)]}$$

$$\Rightarrow \qquad \mu_0 = \sqrt{2}.$$

When there is no liquid layer; $\mu = 1$

$$r_{1\,\max} = \sin^{-1}\left(\frac{2}{3}\right) \simeq 42°$$

$$\therefore \qquad r_{2\,\min} = 48°$$

But critical angle $\qquad C = \sin^{-1}\left(\frac{2}{3}\right) = 42°$

Hence $r_{2\,\min} > C$ and light cannot come out of the vertical face. The dot cannot be seen.

115. Angle of incidence $i = 90 - \theta$.

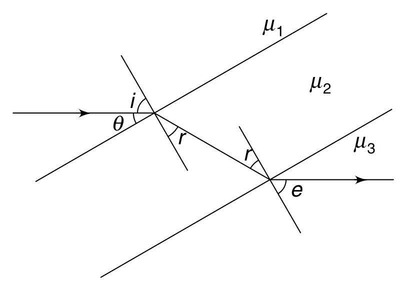
Snell's law for first refraction gives

$$\mu_2 \sin r = \mu_1 \sin i \quad \Rightarrow \quad r = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\sin i\right)$$

At the second face the angle of incidence is r.

$$\therefore \qquad \mu_2 \sin r = \mu_3 \sin e \quad \Rightarrow \quad \mu_1 \sin i = \mu_3 \sin e$$

$$\Rightarrow \qquad e = \sin^{-1}\left(\frac{\mu_1}{\mu_3}\sin i\right) \qquad ...(1)$$



Since i and e are angles made by incident ray with normal to the glass slab, the deviation must be

$$\delta = i - e$$

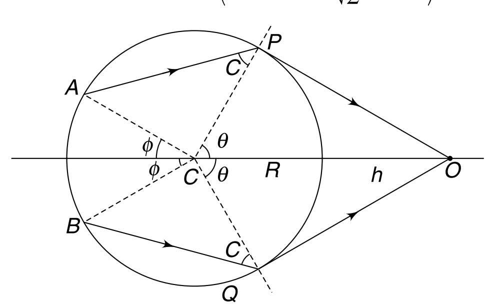
$$= i - \sin^{-1}\left(\frac{\mu_1}{\mu_3}\sin i\right)$$

(Note that equation (1) tells you that $e < i$ since $\mu_3 > \mu_1$]

putting $\qquad i = \frac{\pi}{2} - \theta$ gives

$$\delta = \frac{\pi}{2} - \theta - \sin^{-1}\left(\frac{\mu_1}{\mu_3}\cos\theta\right)$$

116. AB is the arc on which the insect is visible. Ray AP (and BQ) are incident at critical angle $\left(C = \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}\right)$ and come out tangentially to reach the observer at O. This follows from principle of reversibility of the path of light rays. Rays starting from O cannot reach the back surface of the sphere beyond arc AB.



Clearly $\qquad \cos\theta = \frac{CP}{CO} = \frac{R}{R + h} \qquad ...(1)$

(a) when $\qquad h = R; \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$

$$\therefore \qquad \phi = \pi - (\pi - 2C) - \theta = 2\cdot\frac{\pi}{4} - \frac{\pi}{3}$$

$$= \frac{\pi}{6}$$

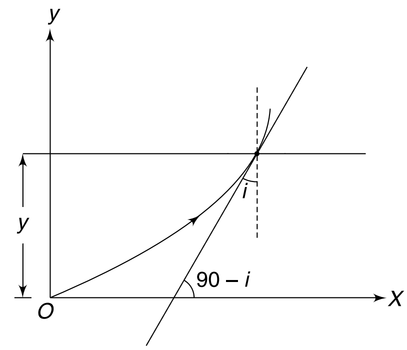
$$\therefore \qquad \text{arc } AB = 2 \times \frac{\pi R}{6} = \frac{\pi R}{3}$$

(b) When $h \to 0$; $\cos\theta \to 1$

$$\Rightarrow \qquad \theta \to 0$$

$$\therefore \qquad \phi = \pi - (\pi - 2C) = 2C = \frac{\pi}{2}$$

$$\therefore \qquad \text{arc } AB = R(2\phi) = \pi R.$$



117. A light ray starting from a point on the road will take a curved path to reach the eye of the man.

Consider a light ray starting from point O at grazing incidence and travelling up.

Let the angle of incidence at height y be i.

$$\mu_0 \sin 90° = \mu \sin i$$

$$\mu_0 = \mu_0 (1 + by)^{1/2} \sin i$$

$$\sin i = \frac{1}{(1 + by)^{1/2}}$$

$$\therefore \qquad \cot i = \frac{\sqrt{by}}{1}$$

But $$\frac{dy}{dx} = \tan(90 - i) = \cot i$$

$$\therefore \qquad \frac{dy}{dx} = \sqrt{by}$$

$$\int_0^y \frac{dy}{\sqrt{y}} = \sqrt{b} \int_0^x dx$$

$$2[\sqrt{y}]_0^y = \sqrt{b}\, x$$

$$\therefore \qquad x = \frac{2\sqrt{y}}{b}$$

Value of x when $y = 1.5$ m is

$$x = 2\sqrt{\frac{1.5}{6 \times 10^{-4}}}$$

$$= 100 \text{ m.}$$

This is maximum value of x as we have considered a ray starting at grazing incidence.

118. $$\mu = \frac{\sin\frac{(A + \delta_m)}{2}}{\sin\left(\frac{A}{2}\right)}$$

Since $$\delta_m = A$$

$$\therefore \qquad \mu = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2} \qquad \text{...(1)}$$

Also $A < 2C$, where C is critical angle for glass air interface. If this condition is not met there will be no emergent ray.

$$\therefore \qquad A < 2C \quad \Rightarrow \quad \frac{A}{2} < C$$

$$\Rightarrow \qquad \sin\left(\frac{A}{2}\right) < \sin C$$

$$\Rightarrow \qquad \sin\frac{A}{2} < \frac{1}{\mu} \qquad \text{...(2)}$$

From (1) and (2) $\quad \sin^2\dfrac{A}{2} + \cos^2\dfrac{A}{2} < \dfrac{1}{\mu^2} + \left(\dfrac{\mu}{2}\right)^2$

$$\Rightarrow \quad 1 < \frac{4+\mu^4}{4\mu^2}$$

$$\Rightarrow \quad \mu^4 - 4\mu^2 + 4 > 0$$

$$\Rightarrow \quad (\mu^2 - 2)^2 > 0$$

$$\Rightarrow \quad \mu^2 > 2$$

$$\Rightarrow \quad \mu > \sqrt{2}$$

119. Four refractions take place. The final emerging ray is perpendicular to the glass slab. Incident ray must also be perpendicular. It means that the third refraction at the back of the bubble must form image at infinity. Using the formula for refraction at curved surface gives the position of object for this refraction.

$$\frac{\mu}{\infty} - \frac{1}{u_3} = \frac{(1.5-1)}{(-10)} \Rightarrow u_3 = 20 \text{ cm}$$

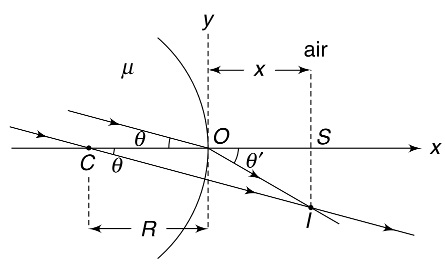
Therefore the image formed after second refraction is at a distance of 40 cm from the refracting surface. Using the formula again

$$\frac{1}{40} - \frac{1.5}{u_2} = \frac{1-1.5}{10} \Rightarrow u_2 = 20 \text{ cm}$$

This means that the image was formed at a distance of 30 cm from the first refracting surface after first refraction. The virtual object for this refraction must be at a distance $= \dfrac{30}{3/2} = 20$ cm from the first face.

Thus C must be at O.

120. Figure shows two rays – one through O and the other through the centre of curvature (C) of the surface. The second ray passes undeviated. The two rays intersect at I.



Snell's law gives

$$\sin\theta' = \mu\sin\theta$$

$$\Rightarrow \quad \theta' \simeq \mu\theta$$

$$SI = x\tan\theta' = (x+R)\tan\theta$$

$$\Rightarrow \quad x\theta' = (x+R)\,\theta$$

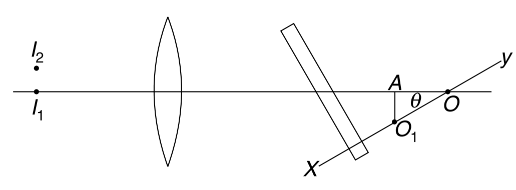
$$\Rightarrow \quad x\mu\theta = (x+R)\,\theta$$

$$\Rightarrow \quad (\mu-1)\,x = R$$

$$\Rightarrow \quad x = \frac{R}{\mu-1}$$

And $\quad SI \simeq (x+R)\,\theta = R\left(\dfrac{\mu}{\mu-1}\right)$

121. The glass slab shifts the position of the object (for the lens) by a distance of $t\left(1-\dfrac{1}{\mu}\right) = 3\left(1-\dfrac{2}{3}\right) = 1$ cm towards the lens.



since object is now at a distance of 40 cm (= $2f$) from the lens, its image is formed on the other side of the lens at a distance of 40 cm. When the glass slab is tilted slightly, the normal from the object on the slab is line XY.

Now the shift is OO_1

$$\therefore \qquad OO_1 = 1 \text{ cm}$$

$$OA = OO_1 \cos\theta \simeq OO_1 = 1 \text{ cm} \quad [\cos\theta \simeq 1]$$

$$AO_1 = OO_1 \sin\theta \simeq 1 \cdot \theta = \frac{\pi}{180} \text{ cm} \qquad \left[\because \quad 1^\circ = \frac{\pi}{180} \text{ radian}\right]$$

Distance of A from lens = 40 cm = $2f$.

$\therefore$ Final image is at 40 cm from lens.

Height of image = AO_1 [since magnification = 1]

$$\therefore \qquad I_1 I_2 = \frac{\pi}{180} \text{ cm.}$$

122. Radius of the ring of the rainbow

$$\frac{R}{6} = \tan 42^\circ$$

$$R = 0.9 \times 6 = 5.4 \text{ km}$$

Arc ABC is visible to the man

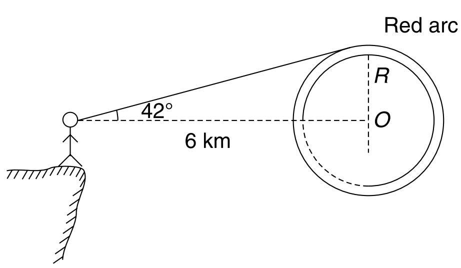
$$d = 0.5 \text{ km}$$

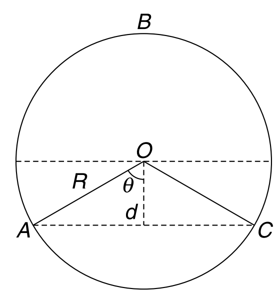
$$\cos\theta = \frac{0.5}{5.4} = 0.09$$

$$\theta = 85^\circ$$

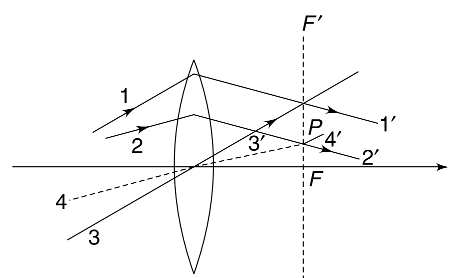
$$\therefore \qquad \text{Required fraction} = \frac{190}{360} = 0.53$$

$$= 53\%$$





123. Draw a ray 3 parallel to 1 and passing through the optical centre of the lens. This ray passes undeviated. Intersection of $1'$ and $3'$ occurs in focal plane of the lens. Mark the focal plane FF'. Draw another ray 4 through the optical centre that is parallel to ray 2. Ray 4 passes undeviated and intersects the focal plane at P. The refracted ray will pass through point P.



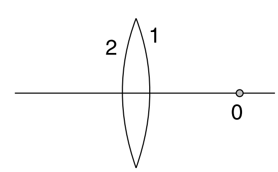
124. The light rays reflected from surface 1 form an erect image as the surface can be treated like a convex mirror.

The other image that he can see is formed by combination of three events – refraction at face 1, reflection at face 2 and refraction at face 2. We can treat the system as a convex lens whose one surface is silvered.

The system can be replaced with an equivalent mirror whose power is given by–

$$P_{\text{eqMirror}} = 2P_L + P_M$$

$$\Rightarrow \qquad \frac{1}{-f_{\text{eq}M}} = 2\,\frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{4}{R}$$



$$\Rightarrow \qquad f_{\text{eq}M} = -\frac{R}{4}$$

If the nose is kept at a distance less that focal length of the equivalent mirror, the image formed is virtual , erect and magnified.

125. Lens formula gives $\frac{1}{v} - \frac{1}{-100} = \frac{1}{50} \quad \Rightarrow \quad v = 100$ cm

The mirror forms an image 50 cm behind itself and the image is moving to right with a velocity of 20 m/s.

The image formed by the mirror will act as moving object for the lens.

Again applying the lens formula gives–

$$\frac{1}{v} - \frac{1}{-200} = \frac{1}{50} \quad \Rightarrow \quad v = \frac{200}{3} \text{ cm}$$

For lens $$\frac{dv}{dt} = \left(\frac{v}{u}\right)^2 \frac{du}{dt} = \left(\frac{200/3}{200}\right)^2 \cdot 20 = \frac{20}{9} \text{ cm/s}$$

126. For lens $$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) = (1.5 - 1)\left(\frac{2}{R}\right)$$

$\therefore$ $$f = R = 20 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60}$$

$\therefore$ $$v = 60 \text{ cm}$$

Shift due to $S1$ is $$t_1\left(1 - \frac{1}{\mu_1}\right) = 6\left(1 - \frac{2}{3}\right) = 2 \text{ cm}$$

Shift due to $S2$ is $$t_2\left(1 - \frac{1}{\mu_2}\right) = 4\left(1 - \frac{1}{2}\right) = 2 \text{ cm.}$$

$\therefore$ Final image will be at a distance of 60 + 2 + 2 = 64 cm from the lens.

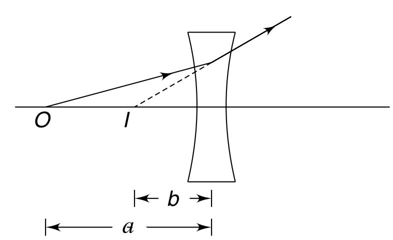
Position of image will remain unchanged in case (b) and (c).

glass slab does not produce any magnification.

$\therefore$ $$m = \frac{v}{u} \text{ (for lens)}$$

$$= \frac{60}{-30} = -2.$$

127.(a)



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

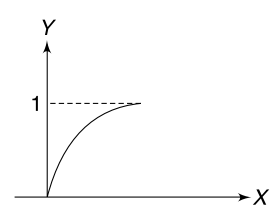
$$\frac{1}{-b} - \frac{1}{-a} = \frac{1}{-f}$$

$\therefore$ $$\frac{1}{b} - \frac{1}{a} = \frac{1}{f}$$

$$\frac{f}{b} - \frac{f}{a} = 1$$

$$\left|\frac{v}{f}\right| = \frac{b}{f} \quad \text{and} \quad \left|\frac{u}{f}\right| = \frac{a}{f}$$

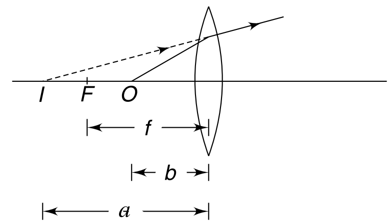
$\therefore$ $$\frac{1}{y} - \frac{1}{x} = 1 \quad \Rightarrow \quad y = \frac{x}{x+1}$$



When $x \to 0; \quad y \to 0$

$x \to \infty; \quad y \to 1$

(b) For converging lens



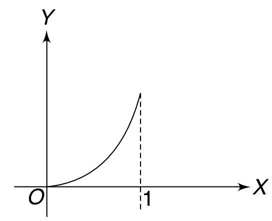
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-b} - \frac{1}{-a} = \frac{1}{f}$$

$$\frac{f}{b} - \frac{f}{a} = -1$$

$$\left|\frac{v}{f}\right| = \frac{b}{f}$$

$$\left|\frac{u}{f}\right| = \frac{a}{f}$$

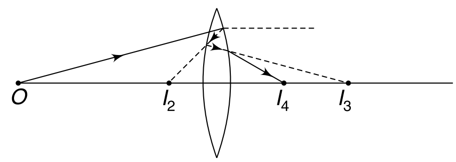


$\therefore$ $$\frac{1}{y} - \frac{1}{x} = -1 \Rightarrow y = \frac{x}{1-x} \quad [x < 1]$$

When $x \to 0$; $y \to 0$

$x \to 1$; $y \to \infty$

128. First refraction



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v_1} - \frac{1}{-60} = \frac{1.5-1}{30}$$

$\Rightarrow$ $$v_1 = \infty$$

Reflection at second surface

Mirror formula $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_2} + \frac{1}{\infty} = \frac{1}{-15} \quad \left(\because \ f = \frac{R}{2}\right)$$

$\therefore$ $$v_2 = -15 \text{ cm} \quad (\text{Image at } I_2)$$

Reflection at first surface

Object is I_2

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_3} + \frac{1}{15} = \frac{1}{-15} \Rightarrow v_3 = -\frac{15}{2} \text{ cm} \quad (\text{Image at } I_3)$$

Refraction at second surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v_4} - \frac{1.5}{15/2} = \frac{1-1.5}{-30}$$

$\Rightarrow$ $$v_4 = \frac{60}{13} \text{ cm. (Image at } I_4)$$

129. The system acts as a mirror of focal length f.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-40} + \frac{1}{-10} = \frac{1}{f} \Rightarrow \frac{1}{f} = -\frac{5}{40} \Rightarrow f = -8 \text{ cm}$$

The system is like a concave mirror of focal length 8 cm.

For lens A, $\frac{1}{f_A} = (\mu_A - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{0.8}{R}$

$$\therefore \qquad f_A = \frac{R}{0.8}$$

For lens B, $\frac{1}{f_B} = (\mu_B - 1)\left(\frac{1}{-R} - \frac{1}{-R/2}\right) = \frac{0.2}{R}$

$$\therefore \qquad f_B = \frac{R}{0.2}$$

For the given system, power of equivalent mirror is given by.

$$P_{eqM} = 2P_A + 2P_B + P_M$$

$$\text{Power of mirror} = -\frac{1}{f}$$

$$\text{Power of lens} = \frac{1}{f}$$

$$\therefore \qquad -\frac{1}{(-8)} = \frac{2 \times 0.8}{R} + \frac{2 \times 0.2}{R} - \frac{1}{-R/4}$$

$$\frac{1}{8} = \frac{1.6}{R} + \frac{0.4}{R} + \frac{4}{R} = \frac{6}{R}$$

$$\therefore \qquad R = 48 \text{ cm}$$

130. Given $\qquad \mu = a + \frac{b}{\lambda^2}$

$$\Rightarrow \qquad \Delta\mu = \frac{-2b}{\lambda^3}\Delta\lambda$$

$\therefore$ Refractive index for incident light is

$$\left(\mu_0 - \frac{2b}{\lambda_0^3}\Delta\lambda\right) \le \mu_0 = a + \frac{b}{\lambda_0^2} \le \left(\mu_0 + \frac{2b}{\lambda_0^3}\Delta\lambda\right)$$

$$\therefore \qquad \text{Spread of refractive index} = \Delta\mu_0 = \frac{4b}{\lambda_0^3}\Delta\lambda$$

$$= \mu_A - \mu_B$$

Focal length of lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(1)}$$

A change in μ by $\Delta\mu$ will change the focal length by Δf, which is obtained by differentiating (1)

$$-\frac{\Delta f}{f^2} = \Delta\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{\Delta\mu}{\mu - 1}\right)\frac{1}{f}$$

$$\therefore \qquad \Delta f_0 = -f_0\left(\frac{\Delta\mu_0}{\mu_0 - 1}\right)$$

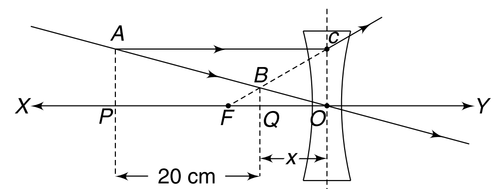
Which represents the spread

$$\therefore \qquad f_B - f_A = f_0\left(\frac{\mu_A - \mu_B}{\mu_0 - 1}\right)$$

$$= f_0\left(\frac{4b\,\Delta\lambda}{\lambda_0^3\left[a + \frac{b}{\lambda_0^2} - 1\right]}\right)$$

$$= f_0\left[\frac{4b\,\Delta\lambda}{\lambda_0(a\lambda_0^2 + b - \lambda_0^2)}\right]$$

131.(a) **Steps:**



(1) Join A to B. Extend the line so that it intersects XY at O. O is the optical centre of the lens.

(2) Since image is on same side of the principal axis and closer to it, the lens must be a diverging one.

(3) Draw AC parallel to XY.

(4) Joint C to B and extend. It meets XY at F. F is the focus.

(b) $\Delta AOP \sim \Delta BOQ$

$$\frac{AP}{BQ} = \frac{20 + x}{x}$$

$$\therefore \quad 3 = \frac{20 + x}{x}$$

$$\Rightarrow \quad x = 10 \text{ cm}$$

$$\therefore \quad u = -30 \text{ cm}; \quad v = -10 \text{ cm}; \quad f = ?$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{-10} + \frac{1}{30} = \frac{-3+1}{30}$$

$$f = -15 \text{ cm}$$

Alter: $\quad \Delta FBQ \sim \Delta FCO$

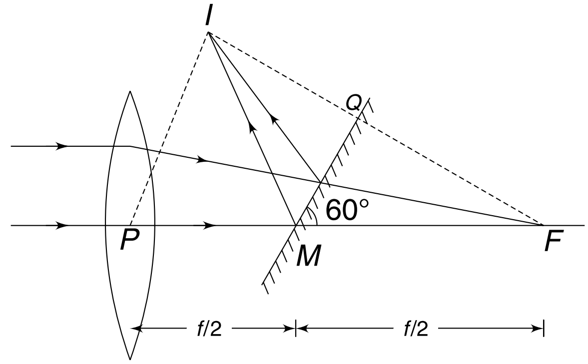
$$\frac{BQ}{CO} = \frac{FQ}{FO}$$

$$\frac{1}{3} = \frac{FQ}{FQ + 10}$$

$$\Rightarrow \quad FQ = 5 \text{ cm}$$

$$\Rightarrow \quad FO = 15 \text{ cm.}$$

132. Focus point F acts as object for the plane mirror. Its image is formed at I.



From geometry, $\angle IMQ = 60°$ and $MF = MI = f/2$

In ΔPIM $\quad IM = f/2$

$$PM = f/2$$

$$\therefore \quad IM = PM$$

and $\quad \angle PMI = 60°$

Hence, ΔPMI is equilateral.

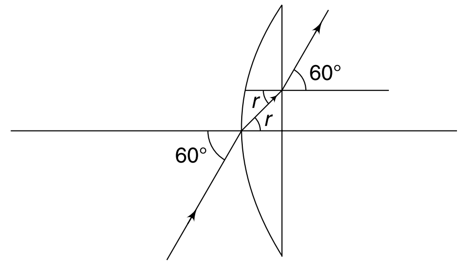
$$\therefore \quad PI = f/2$$

133.(a) The ray diagram makes it clear that the emergent ray will be parallel to the incident ray. Hence deviation angle = 0.

(b) Using Snell's law

$$1 \cdot \sin 60° = \sqrt{3}\,\sin r \quad \Rightarrow \quad r = 30°$$

The lateral shift produced will be same as deviation caused due to a parallel faced glass slab of thickness $t = 3$ mm.

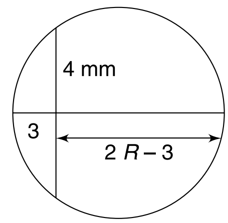


$$S = t\frac{\sin(i-r)}{\cos r} = 3\ \frac{\sin(60° - 30°)}{\cos 30°}$$

$$= \sqrt{3} \text{ mm}$$

(c) From geometry $\quad 3(2R - 3) = 4 \times 4$

$$\Rightarrow \quad R = \frac{25}{6} \text{ mm}$$



(d) Refraction at curved surface

$$\frac{\sqrt{3}}{v} - \frac{1}{\infty} = \frac{\sqrt{3}-1}{25/6}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{v} = \frac{6(\sqrt{3}-1)}{25}$$

$$\Rightarrow \quad v = \frac{25}{2\sqrt{3}(\sqrt{3}-1)} \simeq 10 \text{ mm}$$

For refraction at second (plane) surface the object distance will be

$$u = +7 \text{ mm}$$

$$\frac{1}{v} - \frac{\sqrt{3}}{7} = \frac{1-\sqrt{3}}{\infty} \quad [\because \quad R = \infty]$$

$$\frac{1}{v} = \frac{\sqrt{3}}{7} \quad \Rightarrow \quad v = \frac{7}{\sqrt{3}} \text{ mm}$$

$\therefore$ Rays will get focused at a distance $\left(3 + \frac{7}{\sqrt{3}}\right)$ mm from P.

134. Focal length of the plano convex lens is given by $\frac{1}{f} = \frac{\mu - 1}{R}$

where R is radius of curvature of its curved surface.

$$\therefore \quad \frac{1}{40} = \frac{\mu-1}{R} \qquad ...(1)$$

For the convex lens.

$$\frac{1}{v} - \frac{1}{-10} = \frac{1}{20} \quad \Rightarrow \quad V = -20 \text{ cm.}$$

Image formed by the lens (at a distance 20 cm from the lens) acts as an object for the curved surface of the plano convex lens.

Applying the formula for refraction at the curved surface, we get.

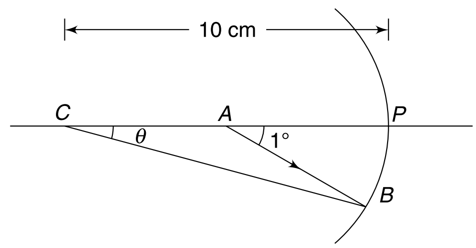
$$\frac{\mu}{v'} - \frac{1}{-(d+20)} = \frac{\mu - 1}{R}$$

If $v' = \infty$, the light rays will be incident normally on the silvered surface and after reflection they will retrace back their paths to O.

$$\therefore \quad \frac{\mu}{\infty} + \frac{1}{d+20} = \frac{1}{40}$$

$$\Rightarrow \quad d = 20 \text{ cm.}$$

135. Using Snell's law at first surface we get $\mu \sin 2° = 2\mu \sin r$.



For small angles $\sin\theta \simeq \theta$

$$\therefore \qquad r \simeq 1°.$$

The incident ray on the second surface and the normal to the surface has been shown in the Figure.

C is centre of curvature of the second surface and CB is normal at B. AB is the incident ray.

$$CB = CP = 10 \text{ cm}$$

and
$$AP = 2 \text{ cm}$$

$$\therefore \qquad \theta = \left(\frac{1}{5}\right)^{\circ}$$

$$\text{Angle of incidence} = \angle ABC = 1° - \left(\frac{1}{5}\right)^{\circ} = \left(\frac{4}{5}\right)^{\circ}$$

Using Snell's law

$$2\mu \sin\left(\frac{4°}{5}\right) = 3\mu \sin r'$$

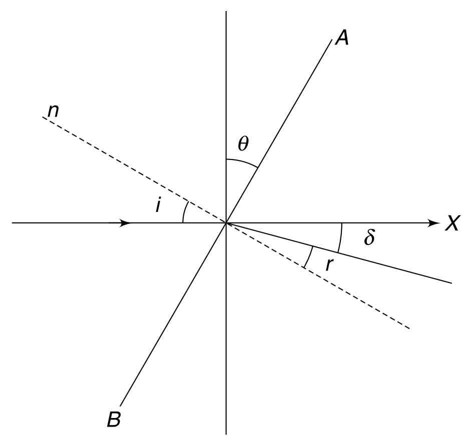
$$\Rightarrow \qquad r' = \frac{2}{3} \times \frac{4}{5} = \left(\frac{8}{15}\right)^{\circ}$$

136. The plane surface is AB.

n is normal to the surface.

Angle of incidence $i = \theta$

Snell's law $\sin\theta = \mu \sin r$



$$\Rightarrow \qquad \theta \simeq \mu r$$

$$\therefore \quad \text{Deviation} \qquad \delta = \theta - r = \theta\left(1 - \frac{1}{\mu}\right)$$

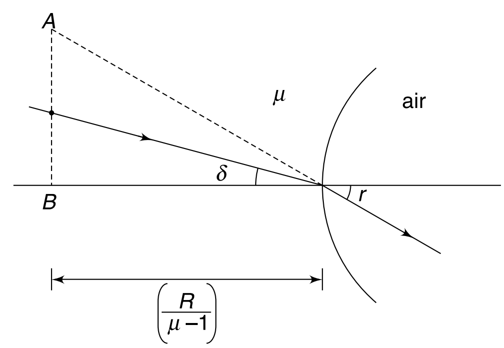
Now consider refraction at the curved surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{\mu} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{\mu}{-\infty} = \frac{1 - \mu}{R}$$

$$\therefore \qquad v = \frac{R}{1 - \mu} = -\left(\frac{R}{\mu - 1}\right)$$

$\therefore$ X co-ordinate of the image is $-\left(\frac{R}{\mu - 1}\right)$



Snell's law gives
$$\mu \sin\delta = 1 \cdot \sin r$$
$$\mu \cdot \delta = r$$
$$\Rightarrow \qquad r = (\mu - 1)\,\theta$$

Rays appear to be diverging from A.

$$BA = |v|\, r$$
$$= \left(\frac{R}{\mu - 1}\right)(\mu - 1)\,\theta = R\theta.$$

$\therefore$ Co-ordinates of point A are $\left(\frac{-R}{\mu - 1}, R\theta\right)$

137.(a) For L_1

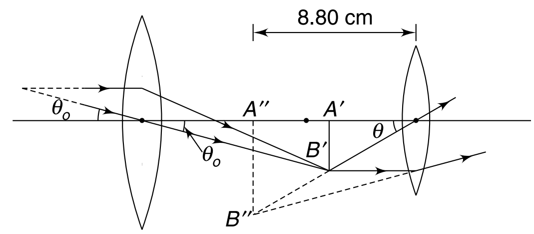
$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{50} - \frac{1}{-50}\right)$$

$\therefore \quad f_1 = 50 \text{ cm}$

Similarly, for L_2; $f_2 = 4$ cm

$\therefore$ Distance between the lenses = 53 cm.

For L_1



$$u_0 = -100{,}00 \text{ cm}; \; v_0 = \text{image distance}$$

$$f_0 = 50 \text{ cm}$$

$$\therefore \quad \frac{1}{v_0} - \frac{1}{-100{,}00} = \frac{1}{50}$$

$$\Rightarrow \quad v_0 = 50.25 \text{ cm}$$

For L_2

$$u_e = -2.75 \text{ cm}$$

[∵ distance between lenses = 53 cm]

$$\therefore \quad \frac{1}{v_e} - \frac{1}{-2.75} = \frac{1}{4}$$

$$v_e = -8.80 \text{ cm}$$

Final image is located between the two lenses at a distance of 8.80 from L_2.

(b) magnification $\quad m = m_1 \times m_2$

$$= -\frac{50.25}{100{,}00} \times \frac{8.80}{2.75} = -0.016$$

The image is 1.6 cm high, inverted (–sign) and virtual.

$$\text{Angular magnification} = \frac{\theta}{\theta_0} \simeq \frac{\tan\theta}{\tan\theta_0} = \frac{A''B''/8.80}{100/100{,}00}$$

$$= \frac{1.6 \times 100}{8.8} = 18.2$$

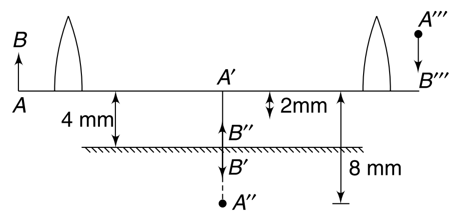
Linear magnification for a telescope is usually very small since objects are at large distances (near infinity). More useful parameter is angular magnification or magnifying power.

138. The first half–lens forms the image $A'B'$(= 6 mm).

$$\therefore \quad \frac{1}{v} - \frac{1}{-20} = \frac{1}{15} \Rightarrow v = +60 \text{ cm.}$$

$$\therefore \quad m_1 = -3.$$

The plane mirror forms the image $A''B''$ with A'' located 8 mm below the principal axis. The second half lens forms the image $A'''B'''$. Now $u = -60$ cm



$$\Rightarrow \quad v = +20 \text{ cm}$$

$$\therefore \quad m_2 = -\frac{1}{3}$$

$\therefore$ A is located at $\frac{8}{3}$ mm above the principal axis.

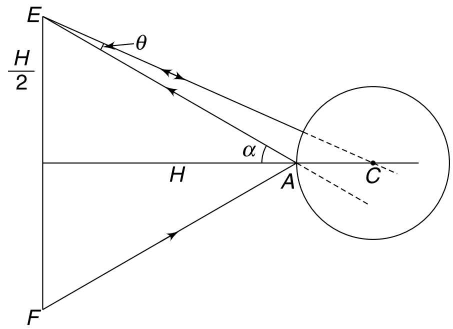
139. A light ray FA will get reflected and reach the eye of the man travelling along AE. It means image of F will be seen on line EA extended. Similarly a ray from E heading towards the centre of the ball will get reflected back along the same line (CE) and therefore image of E will lie somewhere on the line EC.

$\therefore$ Angle subtended by the image at the eye is $\angle AEC = \theta$ (say)

In ΔAEC: $\quad \angle C \simeq \alpha \quad$ [∵ $AC = r$ is very small]

$$AE = \sqrt{H^2 + \frac{H^2}{4}} = \frac{\sqrt{5}}{2} H$$

Applying sine rule in ΔAEC



$$\frac{\sin\theta}{AC} = \frac{\sin\alpha}{AE}$$

$$\Rightarrow \quad \sin\theta = \frac{r}{AE}\cdot\frac{H/2}{AE} = \frac{rH}{2(AE)^2}$$

$$\Rightarrow \quad \sin\theta = \frac{rH}{2\,\frac{5}{4}H^2} = \frac{2r}{5H}$$

Since θ is small $\sin\theta \simeq \theta$

$$\therefore \quad \theta \simeq \frac{2r}{5H}$$

140. A convex mirror will always form a virtual image on the other side of the mirror.

Let distance of observer from mirror be x and the distance of image from the mirror be y.

[Both x and y are positive numbers]

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{y} - \frac{1}{x} = \frac{2}{R}$$

$$\frac{x}{y} - 1 = \frac{2x}{R} \quad \Rightarrow \quad \frac{x}{y} = \frac{2x+R}{R}$$

magnification $$|m| = \frac{y}{x} = \frac{R}{2x+R} \qquad ...(1)$$

But $$y + x \geq d$$

$$\Rightarrow \quad \frac{Rx}{2x+R} + x \geq d$$

$$2x^2 + 2Rx \geq 2dx + Rd$$

$$2x^2 + 2(R-d)\,x - Rd \geq 0$$

Roots of $2x^2 + 2(R-d)\,x - Rd = 0$ are

$$x = \frac{-2(R-d) \pm \sqrt{4(R-d)^2 + 8Rd}}{4}$$

$$\therefore \quad x_1 = -\left[\frac{(R-d) + \sqrt{(R-d)^2 + 2Rd}}{2}\right]$$

and $$x_2 = \frac{\sqrt{(R-d)^2 + 2Rd} - (R-d)}{2}$$

Our x is positive. Hence for (2) to hold $x \geq x_2$

$$m = \frac{R}{2x+R}$$

M is maximum when x is minimum ($\Rightarrow \quad x = x_2$)

$$\therefore \quad m_{\max} = \frac{R}{\sqrt{(R-d)^2 + 2Rd} - (R-d) + R}$$

$$= \frac{R}{\sqrt{R^2 + d^2} + d}$$

141. RS is parallel to NN'

PQ is along incident ray

$$\therefore \quad \angle QRS = i$$

In ΔPSR
$$\frac{PR}{\sin(\angle PSR)} = \frac{PS}{\sin[\pi - i]}$$

$$\Rightarrow \quad \frac{k\mu}{\sin(\angle PSR)} = \frac{k\mu'}{\sin i}$$

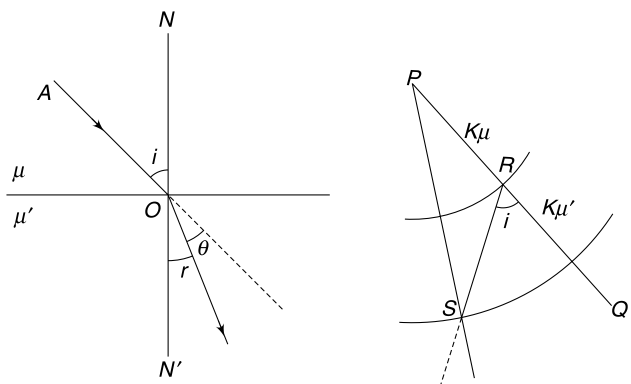
$$\Rightarrow \quad \mu' \sin(\angle PSR) = \mu \sin i$$

From Snell's law
$$\mu' \sin r = \mu \sin i$$

$$\therefore \quad \angle PSR = r$$

Angle between RS (which is parallel to NN') and PS is r.

$\therefore$ PS is in the direction of refracted ray.



142. $BP \rightarrow$ incident ray, $PE \rightarrow$ refracted ray.

$$\mu \sin i = 1 \cdot \sin r$$

$$\therefore \quad \mu = \frac{\sin r}{\sin i} = \frac{AQ/AP}{BQ/BP} = \frac{AQ}{AP} \times \frac{BP}{BQ}$$

$$\Rightarrow \quad \mu = \frac{AS + SQ}{\sqrt{(AS + SQ)^2 + (PQ)^2}} \times \frac{\sqrt{(PQ)^2 + (SQ - SB)^2}}{SQ - SB}$$

$$= \frac{a + R}{\sqrt{(a + R)^2 + (R - h)^2}} \times \frac{\sqrt{(R - h)^2 + (R - a)^2}}{R - a}$$

$$\mu = \left(\frac{R + a}{R - a}\right) \frac{\sqrt{(R - h)^2 + (R - a)^2}}{\sqrt{(R - h)^2 + (R + a)^2}}$$

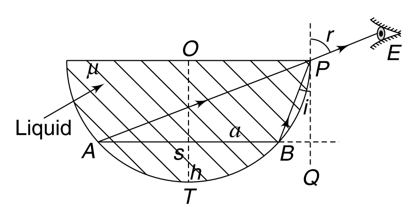
$$= \left(\frac{R + a}{R - a}\right) \sqrt{\frac{2R^2 + h^2 + a^2 - 2Rh - 2Ra}{2R^2 + h^2 + a^2 - 2Rh + 2Ra}}$$

But
$$(R - h)^2 + a^2 = R^2$$

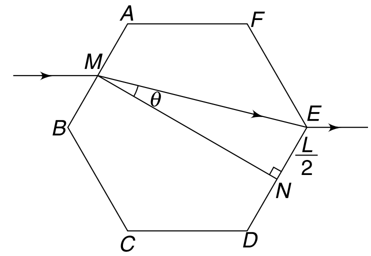
$$\Rightarrow \quad a^2 + h^2 = 2Rh$$

$$\therefore \quad \mu = \left(\frac{R + a}{R - a}\right) \sqrt{\frac{2R(R - a)}{2R(R + a)}}$$

$$\Rightarrow \quad \mu = \sqrt{\frac{R + a}{R - a}}$$



143. For light beam to emerge parallel to the incident direction it must strike the face DE which is parallel to face AB. It means incident ray at M must go to E (or below it).



Let side length of hexagon be b.

$$MN = \sqrt{3}b$$

$\therefore$

$$ME = \sqrt{\left(\frac{b}{2}\right)^2 + (\sqrt{3}b)^2} = \frac{\sqrt{13}}{2}\, b$$

Using Snell's law

$$1 \cdot \sin 30° = \mu \cdot \sin\theta$$

$\Rightarrow$
$$\frac{1}{2} = \mu \frac{1}{\sqrt{13}}$$

$\Rightarrow$
$$\mu = \frac{\sqrt{13}}{2}$$

144. The deviation suffered by the ray, incident at angle i is

$$\phi = (i - r) + (180° - 2r) + (i - r)$$
$$= 180° - 4r + 2i$$

$\therefore$
$$\delta = 180° - \phi = 4r - 2i$$

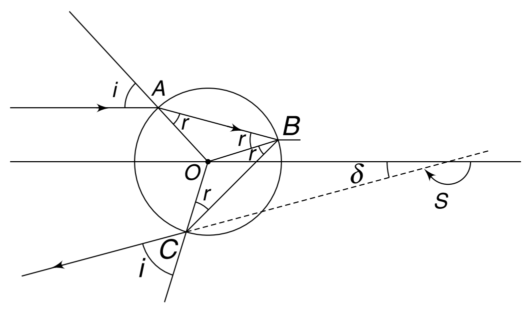
δ is maximum when $\dfrac{d\delta}{di} = 0$

$\Rightarrow$
$$4\frac{dr}{di} - 2 = 0$$

$\Rightarrow$
$$\frac{dr}{di} = \frac{1}{2} \qquad \text{...(1)}$$

Snell's law

$$1 \cdot \sin i = \mu \sin r$$



$$\sin i = \frac{4}{3}\sin r$$

$$\sin r = \frac{3}{4}\sin i \qquad \text{...(2)}$$

$\Rightarrow$
$$\cos r \frac{dr}{di} = \frac{3}{4}\cos i$$

$\therefore$ from (1)
$$\frac{3\cos i}{4\cos r} = \frac{1}{2}$$

$$3\cos i = 2\cos r$$

$$3\cos i = 2\sqrt{1 - \sin^2 r}$$

$$3\cos i = 2\sqrt{1 - \frac{9}{16}\sin^2 i} \qquad \text{[using (2)]}$$

$$(6\cos i)^2 = 16 - 9\sin^2 i$$

$$36(1 - \sin^2 i) = 16 - 9\sin^2 i$$

$$27\sin^2 i = 20$$

$\Rightarrow$
$$\sin i = \sqrt{\frac{20}{27}}; \quad i = 60°$$

from (2)
$$\sin r = \frac{3}{4}\sqrt{\frac{20}{27}}$$

$$= \sqrt{\frac{5}{12}} \quad \Rightarrow \quad r = 40°$$

$$\therefore \qquad \delta_{max} = 4r - 2i = 4 \times 40 - 2 \times 60$$

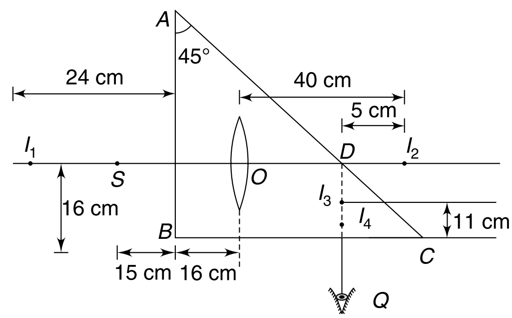
$$= 40°$$

145. Refraction at AB

Image is formed a I_1 at a distance $15 \times 1.6 = 24$ cm from AB

Refraction at lens

Focal length of the lens.



$$\frac{1}{f} = \left(\frac{2.4}{1.6} - 1\right)\left(\frac{2}{20}\right)$$

$$f = +20 \text{ cm}$$

Object distance = 24 + 16 = 40 cm

Lens formula $\quad \frac{1}{v} - \frac{1}{-40} = \frac{1}{+20}$

$$v = +40 \text{ cm}$$

Image is at I_2.

$$OD = 35 \text{ cm (from geometry of the Figure)}$$

$$\therefore \qquad DI_2 = 5 \text{ cm.}$$

Light ray after passing through the lens will suffer total internal reflection at face AC. This is because angle of incidence is close to 45°, as we are considering paraxial rays.

$$45° > \sin^{-1}\left(\frac{1}{1.6}\right)$$

Surface AC may be treated as a mirror for which object is at I_2.

Image is formed at I_3

$$DI_3 = 5 \text{ cm}$$

Observer at P will not be able to see image as light rays suffer TIR at surface AC. Observer at Q see the image at I_4. Image I_4 is above surface BC by a distance $= \frac{11}{1.6} = 6.88$ cm.

146. Consider the incident ray FA on the lens. F is focus point and angle θ is small. The emergent ray AB is parallel to x direction. Now rotate the entire figure by an angle θ about point O. The incident ray FA is now parallel to x direction but not parallel to the principal axis.

$$FA = f \sec\theta$$

Since $FA\ F'O$ is a parallelogram hence

$$OF' = FC = f\sec\theta = f'$$

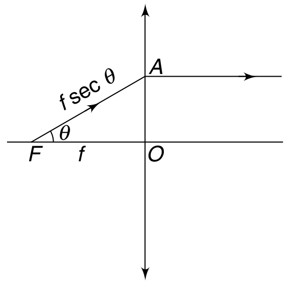
$\therefore$ shift in position of image is

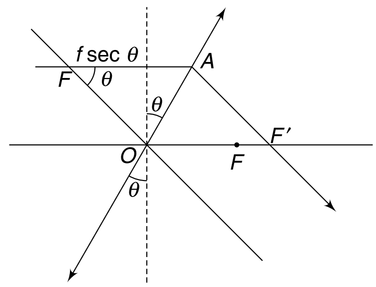
$$\Delta s = f' - f = f(\sec\theta - 1)$$

$$= f\left[1 + \frac{\theta^2}{2} - 1\right] = \frac{f\theta^2}{2}$$

If the whole Figure in (a) is rotated by θ in opposite direction, then also the image will be formed at F'.

Hence the image oscillates between F and F' and the required answer is $\frac{f\theta^2}{2}$





147.(a) When distance of object from lens is x, let the distance of image be y.

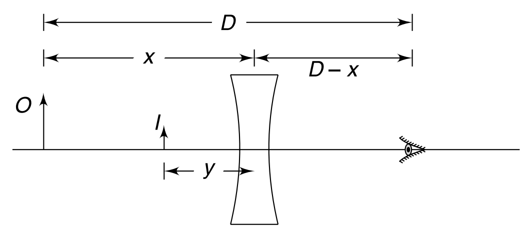
Lens formula,
$$\frac{1}{-y} - \frac{1}{-x} = \frac{1}{-f}$$
$$\frac{1}{y} = \left(\frac{1}{x} + \frac{1}{f}\right); \quad \Rightarrow \quad y = \frac{xf}{x+f}$$

Magnification
$$m = \frac{y}{x} = \frac{f}{x+f}$$

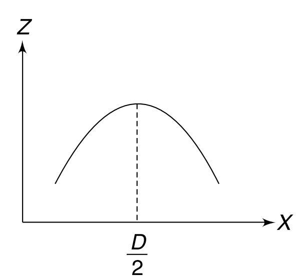
Height of image is
$$h_i = mh_0 \qquad [h_0 = \text{height of object}]$$
$$h_i = \frac{fh_0}{x+f}$$

Angle subtended by the image at the eye
$$\theta = \frac{h_i}{D-x+y} = \frac{fh_0}{(x+f)(D-x+y)}$$

Let
$$(x+f)\,(D-x+y) = Z$$
$$Z = \left(D - x + \frac{xf}{x+f}\right)(x+f)$$
$$Z = -x^2 + Dx + Df$$



'Z' is maximum when
$$\frac{dZ}{dx} = 0$$
$$x = \frac{D}{2}$$



When lens is moved away from the eye, x decreases. This means that Z first increases and then decreases. When Z increases 'θ' decreases and when Z decreases 'θ' increases.

(b) The apparent size of image depends on θ.

$\therefore$ Size of image will first decrease, become minimum and then will become larger and larger.

Apparent size is smallest when $\theta = \frac{D}{2}$.