APPLICATIONS OF TRIGONOMETERY

TOPIC 1

SOME APPLICATIONS OF TRIGONOMETRY

Trigonometry is used for finding the heights and distances of various objects.

Some terms used to find height and distances are:

Line of Sight

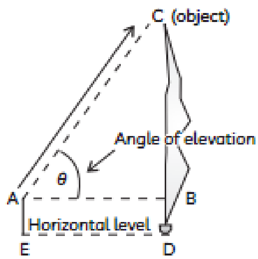
It is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Angle of Elevation

The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal, when the point being viewed is above the horizontal level.

This angle is formed when the viewer raise his head to look at the object.

For example: Look at the figure given:



The line AC drawn from the eye (A) of the observer to the top of the object is called the **line of sight**. The observer is looking at the top of the object. The angle BAC, so formed, by the line of sight with the horizontal, is called the **angle of elevation** of the top of the object from the eye of the observer.

Important

➥ *A plane level parallel to Earth's surface is horizontal plane level and line parallel to horizontal plane is called a horizontal line.*

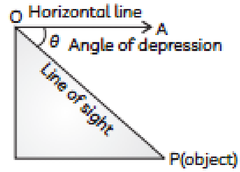
➥ *If an observer moves towards the object, angle of elevation increases and if moves away then angle of elevation decreases.*

➥ *If height of the object is doubled and distance between observer and foot (or base) of the object is also doubled, then angle of elevation remains same.*

Angle of Depression

The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal level, when the point being viewed is below the horizontal level.

It is formed when the viewer lower his head to look at the object.

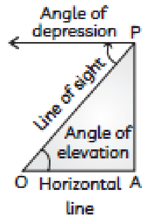


Let P be the position of the object being viewed, lying below the horizontal level OA and O be the observer's eye.

Then, OP is the line of sight and ∠AOP is the angle of depression.

Important

➥ *Angle of elevation of a point P as seen from a point O is always equal to the angle of depression of point O as seen from point P.*



➥ Angle of elevation and angle of depression are always acute angles.

Important Points For Solving Word Problems

(1) While solving problems, observer is represented by a point if his height is not given.

(2) The angle of elevation and angle of depression are always acute angles.

(3) When in problems, the angle of elevation of an object is given, then we conclude that the object is at higher altitude than observer.

(4) When in problems, the angle of depression of an object is given, then we conclude that the observer is at higher altitude than object.

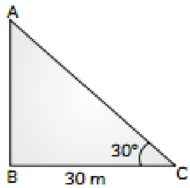
(5) In solving problems, we shall make use of trigonometric ratios of standard angles only namely 30°, 45° and 60°.

MOST LIKELY Questions

Very Short Answer Type Questions

[1 mark]

1. In the figure, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.



Ans. Let the height of the tower BA be 'h' metres.

In right ΔABC, we have:

$$\frac{AB}{BC} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{30} = \sqrt{3}$$

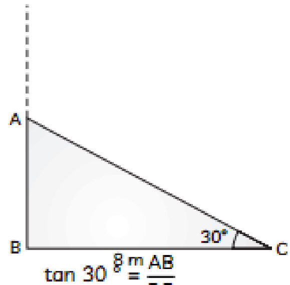
$$\Rightarrow \quad h = 30\sqrt{3}$$

Hence, the height of the tower is $30\sqrt{3}$ metres.

2. India is one of the most vulnerable countries to getting hit by tropical cyclones in the basin, from the east or from the west.On average, 2-3 tropical cyclones make landfall in India each year, with about one being a severe tropical cyclone or greater.

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground where it makes an angle 30°. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree from where it is broken.

Ans.



$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$AB = \frac{8}{\sqrt{3}} \text{ metres}$$

Height from where it is broken is $\frac{8}{\sqrt{3}}$ metres

3. Raju, a painter, has to use a ladder to paint the high walls and ceiling of homes. When Raghu was observing Raju paint his house, he told his friend that he can calculate the height of the wall upto the point where the ladder reaches by using his knowledge of trigonometry.

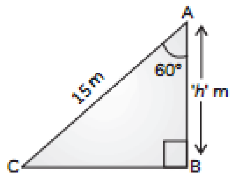
A ladder 15 m long makes an angle of 60° with the wall. Find the height of the point where the ladder touches the wall.

Ans. Let AC be the ladder of length 15 m, which is at the height AB *i.e.*, 'h' m from the ground.

The ladder makes an angle of 60° with the wall.

$\therefore \angle CAB = 60°$

Now, in ΔABC,



$$\cos 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{h}{15}$$

$$\Rightarrow \quad h = 7.5$$

Hence, the height of the point where the ladder touches the wall is 7.5 m.

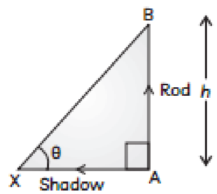
Short Answer Type-I Questions (SA-I)

[2 marks]

4. The ratio of the length of a vertical rod and the length of its shadow is $1:\sqrt{3}$. Find the angle of elevation of the sun at that moment.

Ans. Let AB be the rod and AX be its shadow when the angle of elevation of the sun is θ.

Let h be the length of the rod.



Then, its shadow is $\sqrt{3}h$.

Now, in ΔABX

$$\tan\theta = \frac{AB}{AX} = \frac{h}{\sqrt{3}h}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°.$$

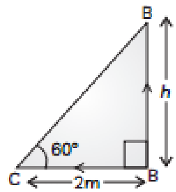
Hence, the angle of elevation of the Sun is 30°.

5. A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder makes an angle of 60° with the level of the ground. Find the height of the wall.

Ans. Let, AC be a ladder placed along a wall AB.

Also, let 'h' be the height of the wall.

In ΔABC



$$\tan 60° = \frac{AB}{BC}$$

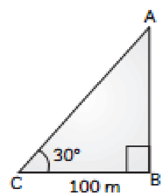
$$\Rightarrow \quad \sqrt{3} = \frac{h}{2}$$

$$\Rightarrow \quad h = 2\sqrt{3}$$

Hence, the height of the wall is $2\sqrt{3}$ m.

6. A vertical flagstaff stands on a horizontal plane. From a point 100 m from its foot, the angle of elevation of its top is 30°. Find the height of the flagstaff. **[CBSE 2016]**

Ans. Let AB be the vertical flagstaff, point C be the point of observation at the distance of 100 m from the foot of the tower flagstaff.



$\therefore \angle ACB = 30°$ and $BC = 100$ m

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{100}$$

$$\Rightarrow \quad AB = \frac{100}{\sqrt{3}} = \frac{100}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

Hence, the height of the flagstaff is $\frac{100\sqrt{3}}{3}$ m.

7. The shadow of a 5 m long stick is 2 m long. At the same time, find the length of the shadow of a 12.5 m high tree.

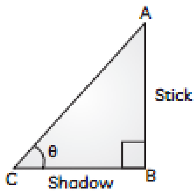
Ans. Let AB be the stick and BC be its shadow.

$\therefore$ AB = 5 m and BC = 2m

Let angle of elevation of the sun at that moment be θ.

$\therefore$ In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

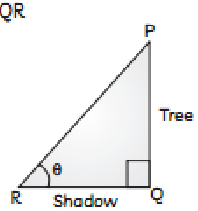


$$\Rightarrow \quad \tan\theta = \frac{5}{2} \quad ...(i)$$

Now, let PQ be the tree, 12.5 m high and QR be its shadow, x m long.

Since, angle of eleveation of sun in both the cases is same (as both shadows form at the same time),

So, in ΔPQR



$$\tan\theta = \frac{PQ}{QR}$$

$$\Rightarrow \quad \frac{5}{2} = \frac{12.5}{QR}$$

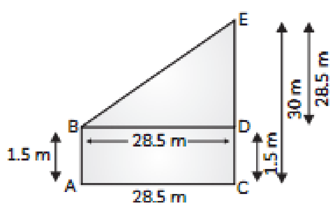
$$\Rightarrow \quad QR = \frac{2 \times 12.5}{5}$$

$$= 5$$

Hence, the length of shadow of the tree is 5 m.

8. An observer, 1.5 m tall, is 28.5 m away from a 30 m high tower. Determine the angle of elevation of the top of the tower from the eye of the observer.

Ans. Let, the angle of elevation be θ, AB be the observer, EC be the tower and AC be the straight line between the tower and observer.



So, AB = 1.5 m, EC = 30 m and AC = 28.5 m.

Then, $\quad$ ED = EC – DC

$= 30 - 1.5 = 28.5$ m $\quad$ [$\because$ AB = CD]

and $\quad$ BD = AC = 28.5 m

Now, in ΔBDE

$$\tan\theta = \frac{ED}{BD} = \frac{28.5}{28.5} = 1$$

$$\tan\theta = \tan 45° \quad [\because \tan 45° = 1]$$

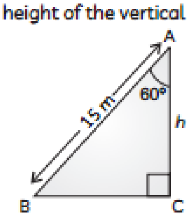
$$\Rightarrow \quad \theta = 45°$$

Hence, the angle of elevation is 45°.

9. A ladder 15 metres long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.

Ans. Given: length of ladder, AB = 15 m

Let h be the height of the vertical wall, AC.



The ladder makes an angle of 60° with the wall.

$\therefore \quad \angle$BAC = 60°

$\therefore$ In ΔABC,

$$\cos 60° = \frac{AC}{AB} = \frac{h}{15}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{h}{15}$$

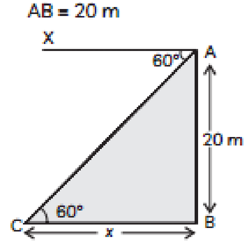
$$\Rightarrow \quad h = \frac{15}{2} = 7.5$$

Hence, the required height of the wall is 7.5 m.

10. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. [Take $\sqrt{3} = 1.732$.]

Ans. Let AB be the tower and C be the position of the ball on the ground.

$\therefore \quad$ AB = 20 m



In this figure,

Due to property of alternate angles, we obtain,

$$\angle XAC = \angle ACB = 60°$$

Let BC = x m.

Now, in ΔABC,

$$\tan 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20}{x}$$

$$\Rightarrow \quad x = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$$

$$\Rightarrow \quad = 20 \times \frac{1.732}{3} = 11.55$$

Hence, distance between the ball and the foot of the tower is 11.55 m.

Long Answer Type Questions (LA)

[5 marks]

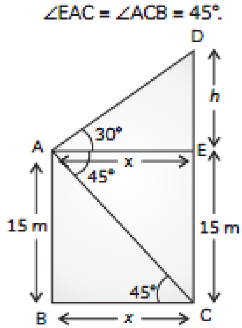
11. From a window, 15 m high above the ground, the angles of elevation and depression of the top and the foot of a house on the opposite side of the street are 30° and 45°, respectively. Find the height of the opposite house. (Use $\sqrt{3}$ = 1.732)

Ans. Let, A be the window, 15 m above the ground and CD be the house on the opposite side of the street at a distance BC.

$\therefore$ AB = 15 m,

$\angle$ADE = 30° and

$\angle$EAC = $\angle$ACB = 45°.



Let the length of DE = 'h' m

Then, the height of house, DC = (h + 15) m

[$\because$ AB = EC = 15 m]

and BC = AE = x m (say)

Now, in ΔABC

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{15}{x} \Rightarrow x = 15$$

In ΔDEA

$$\tan 30° = \frac{DE}{EA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad h = \frac{x}{\sqrt{3}} = \frac{15}{\sqrt{3}}$$

$$= \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3}$$

Then, DC = DE + EC = h + 15

$$= 5\sqrt{3} + 15$$

$$= 5(\sqrt{3} + 3) \text{ m}$$

$$= 5 \times 4.732 = 23.66$$

Hence, the height of the opposite house is 23.66 m.

12. A moving boat is observed from the top of a 150 m high cliff, moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/hr.

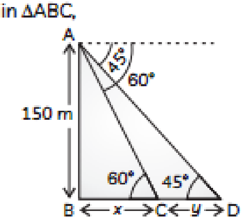
Ans. Here, AB is the cliff of height 150 m, C and D are the two positions of a boat.

$\therefore$ AB = 15 m,

$\angle$ACB = 60° and $\angle$ADB = 45°

Let, the distance BC be 'x' m and CD be 'y' m.

Now, in ΔABC,



$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{150}{x}$$

$$\Rightarrow \quad x = \frac{150}{\sqrt{3}} \quad \text{...(i)}$$

and in ΔABD,

$$\tan 45° = \frac{AB}{BD}$$

$$\Rightarrow \quad 1 = \frac{150}{x + y}$$

[$\because$ BD = BC + CD]

$$\Rightarrow \quad 150 = x + y$$

$$\Rightarrow \quad y = 150 - x$$

Using (i), we get:

$$y = 150 - \frac{150}{\sqrt{3}}$$

$$= \frac{150(\sqrt{3}-1)}{\sqrt{3}}$$

But, the time taken to cover distance 'y' or CD is 2 minutes *i.e.* $\frac{2}{60}$ hr or, $\frac{1}{30}$ hr

Then, $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$$= \frac{\frac{150(\sqrt{3}-1)}{\sqrt{3}}}{\frac{1}{30}}$$

$$= 150 \times 30 \frac{(\sqrt{3}-1)}{\sqrt{3}}$$

$$= 4500 \times \frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 1500\sqrt{3}(\sqrt{3}-1)$$

Hence, the speed of the boat is $1500\sqrt{3}(\sqrt{3}-1)$ m/hr.

13. Two men on either side of a 75 m high building and in line with the base of the building, observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3}$ = 1.73)

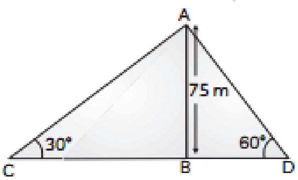
Ans. Here: AB is a building of height 75 m. Two men on either side of it are at the positions C and D.

∴ AB = 75 m,

∠ACB = 30° and

∠ADB = 60°.

The distance between the two men, CD = BC + BD



In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{BC}$$

$$\Rightarrow \quad BC = 75\sqrt{3}$$

In ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{75}{BD}$$

$$\Rightarrow \quad BD = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{75\sqrt{3}}{3}$$

$$= 25\sqrt{3}$$

Now, $CD = BC + BD$

$$= 75\sqrt{3} + 25\sqrt{3}$$

$$= 100 \times \sqrt{3}$$

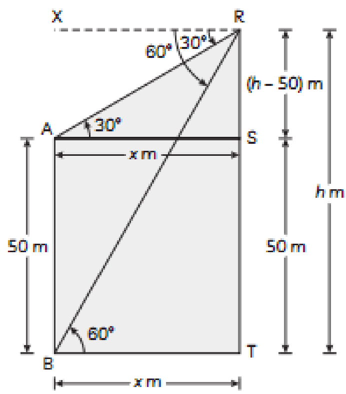
$$= 100 \times 1.73$$

$$= 173$$

Hence, the distance between the two men is 173 m.

14. The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower, and also the horizontal distance between the building and the tower.

Ans.



Let AB = Building of height 50 m

RT = tower of height = h m

BT = AS = x m

AB = ST = 50 m

RS = TR – TS = (h – 50) m

In ΔARS,

$$\tan 30° = \frac{RS}{AS}$$

$$\frac{1}{\sqrt{3}} = \frac{(h-50)}{x} \qquad ...(1)$$

$$\frac{1}{\sqrt{3}} = \frac{87}{y}$$

$$y = 87\sqrt{3} \quad ...(ii)$$

Subtracting (i) and (ii)

$$y - x = 87\sqrt{3} - \frac{87}{\sqrt{3}}$$

$$y - x = \frac{174\sqrt{3}}{\sqrt{3}.\sqrt{3}}$$

$$y - x = 58\sqrt{3}\text{ m}$$

Hence, the distance travelled by the balloon is equal to BD

$$y - x = 58\sqrt{3}\text{ m.}$$

17. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ metres, find the speed of the aeroplane.

Ans. Let C and E be the two positions of the aeroplane and let A be the point of observation.

Let AX be the horizontal ground.

Draw BC ⊥ AX and ED ⊥ AX.

Then, $\angle CAB = 60°, \angle EAD = 30°$

and $BC = ED = 3600\sqrt{3}$ m

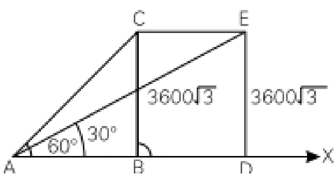
Let AB = 'x' m and BD = 'y' m

From right ΔACB, we have:

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{3600\sqrt{3}}{x}$$

$$\Rightarrow \quad x = 3600$$



From right ΔAED, we have:

$$\tan 30° = \frac{ED}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{x+y}$$

$$\Rightarrow \quad x + y = 3600 \times 3 = 10800$$

$$\Rightarrow \quad y = 10800 - x$$

$$= 10800 - 3600$$

$$= 7200$$

In 30 seconds, the distance covered by the aeroplane is 7200 m.

Then, speed of aeroplane = $\frac{\text{Distance}}{\text{Time}}$

$$= \frac{7200}{30}$$

$$= 240 \text{ m/s}$$

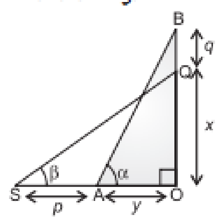
Hence, the speed of the aeroplane is 240 m/s.

18. A ladder rests against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p so that its upper end slides a distance q down the wall and then the ladder makes an angle β to the horizontal. Show that $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

Ans. Let AB be the ladder at an inclination α to the horizontal and SQ be its position when it makes an angle β to the horizontal.

So, SA = p, BQ = q, ∠BAO = α, ∠QSO = β.

Let, OQ = x and OA = y.



In ΔBAO,

$$\cos\alpha = \frac{OA}{AB}$$

$$\Rightarrow \quad OA = AB\cos\alpha \quad ...(i)$$

and
$$\sin\alpha = \frac{OB}{AB}$$

$$\Rightarrow \quad OB = AB\sin\alpha \quad ...(ii)$$

In ΔQSO,

$$\cos\beta = \frac{OS}{SQ}$$

$$\Rightarrow \quad OS = SQ\cos = AB\cos \quad ...(iii)$$

and
$$\sin\beta = \frac{OQ}{SQ}$$

$$\Rightarrow \quad OQ = SQ\sin\beta = AB\sin\beta \quad ...(iv)$$

Now, $\quad SA = OS - AO$

In ΔRBT,

$$\tan 60° = \frac{RT}{BT}$$

$$\sqrt{3} = \frac{h}{x} \quad ...(2)$$

Solving (1) and (2), we get

$h = 75$ from (2)

$$x = \frac{h}{\sqrt{3}}$$

$$= \frac{75}{\sqrt{3}}$$

$$= 25\sqrt{3}$$

Hence, height of the tower = h = 75 m

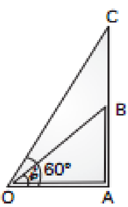
Distance between the building and the tower = $25\sqrt{3}$ = 43.25 m

15. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°.

Find the height of the pedestal. (Use $\sqrt{3}$ = 1.73).

Ans. Let BC be the statue, standing on a pedestal AB and O be the point of observation on the ground.

$\therefore$ BC = 1.6 m, $\angle AOC = 60°$ and $\angle AOB = 45°$.



In right ΔOAB,

$$\frac{AB}{OA} = \tan 45° = 1$$

$$\Rightarrow \quad OA = AB \quad ...(i)$$

In right ΔOAC,

$$\frac{AC}{OA} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad OA = \frac{AC}{\sqrt{3}} = \frac{AB + 1.6}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we have:

$$AB = \frac{AB + 1.6}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}\,AB = AB + 1.6$$

$$\Rightarrow \quad AB(\sqrt{3} - 1) = 1.6$$

or

$$AB = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

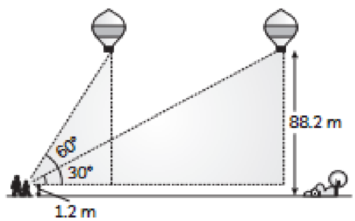
$$= \frac{16(\sqrt{3} + 1)}{2}$$

$$= 0.8\,(1.73 + 1)$$

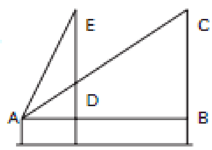
$$= 0.8 \times 2.73 = 2.184$$

Thus, the height of the pedestal is 2.184 m.

16. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Ans.



From the figure, the angle of elevation for the first position of the balloon $\angle EAD = 60°$ and for second position $\angle BAC = 30°$. The vertical distance

$$ED = CB = 88.2 - 1.2$$

$$= 87 \text{ m.}$$

Let AD = x m and AB = y m.

Then in right ΔADE,

$$\tan 60° = \frac{DE}{AD}$$

$$\sqrt{3} = \frac{87}{x}$$

$$x = \frac{87}{\sqrt{3}} \quad ...(i)$$

In right ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

$\Rightarrow \quad p = AB \cos\beta - AB \cos\alpha$

$= AB (\cos\beta - \cos\alpha) \quad ...(v)$

and $\quad BQ = BO - QO$

$= AB \sin\alpha - AB \sin\beta$

$\Rightarrow \quad q = AB (\sin\alpha - \sin\beta) \quad ...(vi)$

Dividing eqⁿ (v) by eqⁿ (vi), we get

$$\frac{p}{q} = \frac{AB(\cos\beta - \cos\alpha)}{AB(\sin\alpha - \sin\beta)}$$

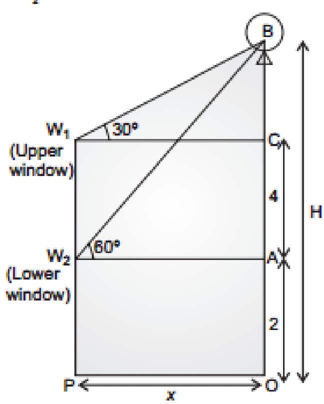
$$\therefore \quad \frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

Hence, proved.

19. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain distance the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground.

Ans. Let B be a balloon at a height of H metres from the ground, W_1 be the lower window and W_2 be the upper window.

$\therefore PW_1 = 2$ m, $PW_2 = 4$ m, $\angle AW_1B = 60°$ and $\angle CW_2B = 30°$.



So, $\quad OA = PW_1 = 2m$

and $\quad OC = PW_2 = 4m$

$\therefore \quad BC = OB - (AC + AO)$

$= H - (4 + 2)$

$= H - 6$

Let $\quad OP = AW_1 = CW_2 = x$ m.

In ΔBW_1A,

$$\tan 60° = \frac{BA}{W_1A} = \frac{BC + CA}{W_1A}$$

$$\Rightarrow \quad \sqrt{3} = \frac{(H-6)+4}{x}$$

$$\Rightarrow \quad x = \frac{H-2}{\sqrt{3}} \quad ...(i)$$

In ΔBW_2C,

$$\tan 30° = \frac{BC}{W_1C}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{H-6}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(H-6) \quad ...(ii)$$

From eqn (i) and (ii), we get

$$\sqrt{3}(H-6) = \frac{(H-2)}{\sqrt{3}}$$

$\Rightarrow \quad 3(H-6) = (H-2)$

$\Rightarrow \quad 3H - 18 = H - 2$

$\Rightarrow \quad 2H = 16$

$\Rightarrow \quad H = 8$

Hence, the height of the balloon is 8 m from the ground.

20. A man in a boat rowing away from a light house, 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in meters per minute. [Use $\sqrt{3}$ = 1.732]

Ans. Consider AB as the light house of height 100 m, C and D be two distance positions of the boat at an instant, when the angle changes from 60° to 30°.

Let the speed of the boat be 'y' m/min.

Time taken by the boat to reach from D to C = 2 min

$\therefore \quad$ Distance CD = Time × Speed

$= 2y$

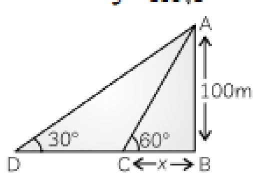
Let: $\quad BC = x$ m

Now, in right ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{x + 2y}$$

$$\Rightarrow \quad x + 2y = 100\sqrt{3} \quad ...(i)$$



In right ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100}{x}$$

$$\Rightarrow \quad x = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

Put the value of x in equation (i), we get:

$$2y = 100\sqrt{3} - \frac{100\sqrt{3}}{3}$$

$$\Rightarrow \quad 2y = 100\sqrt{3} \times \left(\frac{2}{3}\right)$$

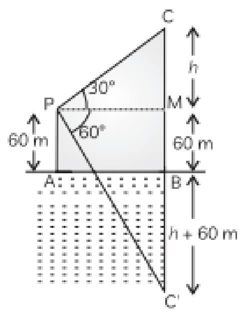
$$\Rightarrow \quad y = \frac{100\sqrt{3}}{3} = \frac{100 \times 1.732}{3}$$

$$= 57.73$$

Hence, the speed of the boat is 57.73 m/min.

21. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

Ans. Let AB be the surface of the lake and P be the point of observation such that AP = 60 m.



Let C be the position of the cloud and C' be its reflection in the lake.

Let CM = h m.

Draw PM ⊥ CC'.

Then, CB = $(h + 60)$ m

and C'B = $(h + 60)$ m

[∵ CB = C'B, as reflection of C is C']

Now, in ΔCPM

$$\tan 30^\circ = \frac{CM}{PM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{PM}$$

$$\Rightarrow \quad PM = \sqrt{3}h \qquad ...(i)$$

Similarly, in $\Delta PMC'$

$$\tan 60^\circ = \frac{C'B + BM}{PM}$$

$$\Rightarrow \quad \sqrt{3} = \frac{(h+60)+60}{PM}$$

$$\Rightarrow \quad PM = \frac{h + 120}{\sqrt{3}} \qquad ...(ii)$$

From (i) and (ii), we get

$$\sqrt{3}h = \frac{h+120}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = h + 120$$

$$\Rightarrow \quad 2h = 120$$

$$\Rightarrow \quad h = 60$$

So, $\quad CB = h + 60 = 60 + 60$

$$= 120$$

Hence, the height of the cloud from the surface of the lake is 120 m.

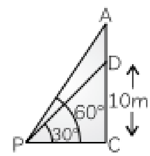
22. From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, at some height vertically over the top the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

Ans. Let CD be a building, A be the position of the helicopter above the building and P be the point of observation on the ground.

∴ CD = 10 m, ∠DPC = 30° and ∠APC = 60°.

Let the height of helicopter above the ground be h m.

Now, in ΔPDC,



$$\tan 30^\circ = \frac{CD}{PC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{PC}$$

$$\Rightarrow \quad PC = 10\sqrt{3} \qquad ...(i)$$

and in ΔPAC

$$\tan 60^\circ = \frac{AC}{PC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{10\sqrt{3}}$$

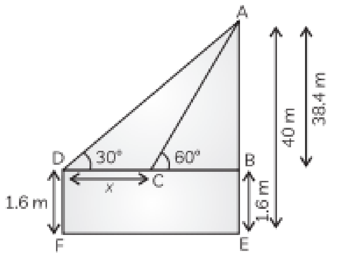
[∵ PC = $10\sqrt{3}$ from (i)]

$$\Rightarrow \quad h = 10\sqrt{3} \times \sqrt{3} = 30$$

Hence, the height of the helicopter above the ground is 30 m.

23. **A 1.6 m tall boy is standing at some distance from a 40 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.**

Ans. Let DF be the tall boy of height 1.6 m and AE be the building of height 40 m.



The angle of elevation changes from 30° to 60° when the boy moves from point D to point C.

Let the distance of CD be x m.

Here, $AE = 40$ m,

and $DF = BE = 1.6$ m

$\therefore$ $AB = AE - BE$

$= 40 - 1.6 = 38.4$ m

Now, inΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{38.4}{BD}$$

$$\Rightarrow \quad BD = 38.4\sqrt{3} \quad ...(i)$$

In Δ ABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{38.4}{BC}$$

$$\Rightarrow \quad BC = \frac{38.4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{38.4\sqrt{3}}{3} = 12.5\sqrt{3} \quad ...(ii)$$

$\therefore$ Distance between two positions, CD

$= BD - BC$

$= 38.4\sqrt{3} - 12.8\sqrt{3}$ [using (i) and (ii)]

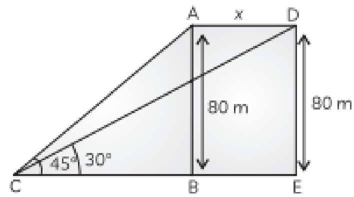
$= 25.6\sqrt{3}$

Hence, the distance the boy walked towards the building is $25.6\sqrt{3}$ m.

24. **A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of the bird. (Take $\sqrt{3}$ = 1.732)**

Ans. Let AB be a tree on which the bird is sitting at point A, D be its position after 2 seconds, and C be the point of observation on the ground.

CE ⊥ DE



$\therefore$ AB = DE = 80 m, ∠ACB = 45° and ∠DCE = 30°

Let $AD = x$ m

Then, $BE = AD = x$ m

Now, in ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{80}{BC}$$

$$\Rightarrow \quad BC = 80 \quad ...(i)$$

And in ΔDCE,

$$\tan 30° = \frac{DE}{CE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{CB + BE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{80 + x}$$

$$\Rightarrow \quad 80 + x = 80\sqrt{3}$$

$$\Rightarrow \quad x = 80(\sqrt{3} - 1)$$

$= 80 \times (1.732 - 1)$

$= 80 \times 0.732 = 58.56$

Then, speed of bird $= \dfrac{\text{Distance covered}}{\text{Time taken}}$

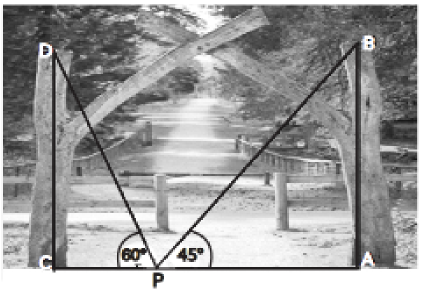
$= \dfrac{58.56}{2} = 29.28$ m/s

Hence, the speed of the bird is 29.28 m/s.

Case Based Questions (VSA Type)

[**4** & **5** marks]

25. *Two trees of equal heights are standing opposite to each other on either side of a road in a garden, which is 20 m wide. From a point between them on the road, the angles of elevation of their tops are 45° and 60°.*



(A) Find the distance of the point P from the base of the tree CD.

(B) Find the distance of the straight line joining the top of the tree CD to the point P.

(C) Find the distance of the point P from the base of the tree AB.

(D) Find the height of the two trees.

(E) Find the distance of the straight line joining the top of tree AB to the point P.

Ans. (A) It is given that the width of the road = 20 m.

Let PC = x m, then AP = $(20 - x)$ m.

In triangle BAP,

$$\tan 45° = \frac{AB}{AP}$$

$$\Rightarrow \quad 1 = \frac{AB}{20 - x}$$

$$\Rightarrow \quad AB = 20 - x$$

Also, in triangle DCP,

$$\tan 60° = \frac{CD}{PC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20 - x}{x}$$

$$\Rightarrow \quad 20 - x = \sqrt{3}x$$

$$\Rightarrow \quad x + \sqrt{3}x = 20$$

$$\Rightarrow \quad x(1 + \sqrt{3}) = 20$$

$$\Rightarrow \quad x = \frac{20}{(\sqrt{3}+1)}$$

$$= \frac{20(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{20(\sqrt{3}-1)}{2}$$

$$= 10(\sqrt{3} - 1) \text{ m}$$

Hence, distance of the point P from the base of the tree CD *i.e.* CP is $10(\sqrt{3} - 1)$ m.

(B) *(a)* $20(\sqrt{3} - 1)$*m*

Explanation: In ΔPCD,

$$\cos 60° = \frac{PC}{PD}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{10(\sqrt{3}-1)}{PD}$$

$$\Rightarrow \quad PD = 20(\sqrt{3}-1) \text{ m}$$

(C) The distance of the point P from the base of the tree AB = AP

$$= 20 - x$$

$$= 20 - 10(\sqrt{3} - 1)$$

$$= 20 - 10\sqrt{3} + 10$$

$$= 30 - 10\sqrt{3}$$

$$= 10\sqrt{3}(\sqrt{3} - 1)$$

Hence, distance of the point P from the base of the tree AB i.e. AP is $10\sqrt{3}(\sqrt{3} - 1)$ m.

(D) Height of the tree = CD = AB = AP = $10\sqrt{3}(\sqrt{3} - 1)$

(E) Distance of the straight line joining top of the free AB to the point P.

= BP

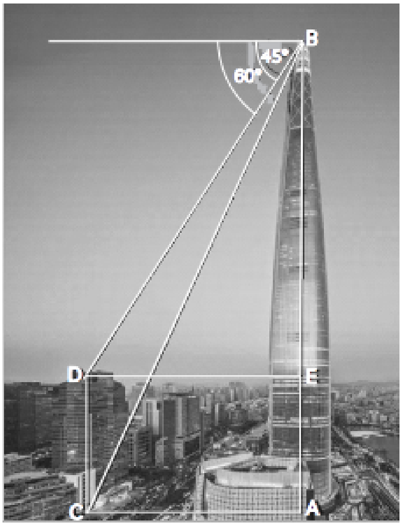
In Δ ABP

$$\sin 45° = \frac{AB}{BP}$$

$$\Rightarrow \quad BP = \frac{AB}{\sin 45°} = \frac{10\sqrt{3}(\sqrt{3}-1)}{1/\sqrt{2}}$$

$$= 10\sqrt{6}(\sqrt{3}-1)$$

26. *The horizontal distance between a towers AB and a building CD is 120 m. The angle of elevation of the top of the tower AB from the top and bottom of the building CD are 45° and 60° respectively.*



(A) Find the angle of depression of the bottom of the building CD as seen from the top of the tower AB?

(B) Find the height of the tower AB?

(C) Find the height of the building CD?

(D) What is the difference in height of the tower AB and building CD?

(E) What is the distance of the straight line joining the top of the tower AB and the bottom of the building CD?

Ans. (A) The angle of depression of the bottom of the building CD as seen from the top of the tower AB is equal to the angle of elevation of the top of the tower AB as seen from the bottom of the building CD *i.e.* 60°.

(B) Height of the tower = AB.

In ΔABC,

$$\tan 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{120}$$

$$\Rightarrow \quad AB = 120\sqrt{3} \text{ m}$$

(C) Height of the building CD = AE (since DEAC is a rectangle) = AB – BE

So, in ΔBDE,

$$\tan 45^\circ = \frac{BE}{DE}$$

$$\Rightarrow \quad 1 = \frac{BE}{120} \qquad [\because DE = AC]$$

$$\Rightarrow \quad BE = 120$$

So, $CD = 120\sqrt{3} - 120 = 120(\sqrt{3} - 1)$ m

(D) Difference in height of tower AB and building CD = AB – CD

= AB – AE = EB = 120 m

[Using part (C)]

(E) Hence, the height of the building CD is 120 $(\sqrt{3} - 1)$ m. The distance of the straight line joining the top of the tower AB and the bottom of the building CD = BC.

In ΔABC,

$$\cos 60^\circ = \frac{AC}{BC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{120}{BC}$$

$$\Rightarrow \quad BC = 240 \text{ m}$$

Hence, the required distance is 240 m.