

Chapter –11

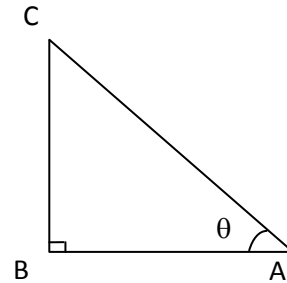
Trigonometry

In a right triangle ABC as show in the figure

AC is called hypotenuse.

BC is called “Opposite side of $\angle A$ ”

AB is called “Adjacent side of the $\angle A$ ”.



Ratios in A Right Angle Triangle:

$$\sin A = \frac{\text{opposite side of } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{adjacent side of } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan A = \frac{\text{opposite side of } \angle A}{\text{Adjacent side of } \angle A} = \frac{BC}{AB}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent side of } \angle A} = \frac{AC}{AB} = \frac{1}{\cos A}$$

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{opposite side of } \angle A} = \frac{AC}{BC} = \frac{1}{\sin A}$$

$$\cot A = \frac{\text{Adjacent side of } \angle A}{\text{opposite side of } \angle A} = \frac{AB}{BC} = \frac{1}{\tan A}$$

Note: We observe that $\tan A = \frac{\sin A}{\cos A}$ and $\cot A = \frac{\cos A}{\sin A}$

Trigonometric ratios of some specific angles:

$\angle A$	0°	30°	45°	60°	90°
Sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

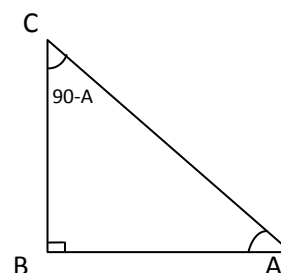
- The value of $\sin A$ will be increased from 0° to 90°
- The value of $\cos A$ will decreased from 0° to 90°
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be determined
- The value of $\sin A$ or $\cos A$ never exceeds, where as the value of $\sec A$ or $\operatorname{Cosec} A$ is always greater than or equal to 1.

Trigonometric ratios of complementary angles:

$$\sin (90^\circ - A) = \cos A \quad \operatorname{Cosec} (90^\circ - A) = \sec A$$

$$\cos (90^\circ - A) = \sin A \quad \sec (90^\circ - A) = \operatorname{Cosec} A$$

$$\tan (90^\circ - A) = \cot A \quad \cot (90^\circ - A) = \tan A$$



Exercise 11.1

1. In a right angle triangle ABC, 8cm, 15cm and 17cm are the lengths of AB, BC and CA respectively. Then, find out sinA, cosA and tanA.

A. In $\triangle ABC$, $AB = 8\text{cm}$

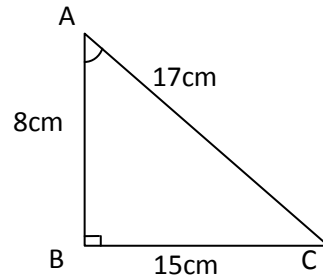
$$BC = 15\text{ cm}$$

$$AC = 17\text{ cm}$$

$$\sin A = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{15}{17}$$

$$\cos A = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8}{17}$$

$$\tan A = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{15}{8}$$



2. The sides of a right angle triangle PQR are PQ = 7cm, QR = 25cm

$\angle P = 90^\circ$ respectively then find tanQ – tanR.

Sol: Given that in PQR,

$$PQ = 7\text{cm and } QR = 25\text{ cm.}$$

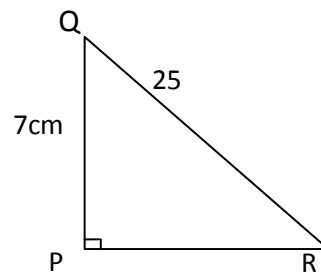
$\triangle PQR$ is a right angle triangle

By using Pythagoras theorem

$$PR = \sqrt{QR^2 - PQ^2}$$

$$= \sqrt{(25)^2 - (7)^2} = \sqrt{625 - 49}$$

$$= \sqrt{576} = 24\text{cm.}$$



$$\therefore \tan Q = \frac{PR}{PQ} = \frac{24}{7}$$

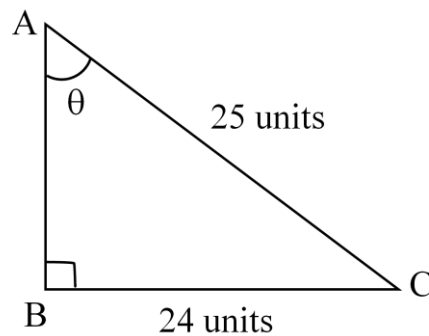
$$\tan R = \frac{PQ}{PR} = \frac{7}{24}$$

$$\therefore \tan Q - \tan R = \frac{24}{7} - \frac{7}{24} = \frac{576 - 49}{168} = \frac{527}{168}.$$

3. In a right angle triangle ABC with right angle at B, in which a = 24 units, b = 25 units and $\angle BAC = \theta$. Then find $\cos \theta$ and $\tan \theta$.

A. In a right angle triangle ABC $\angle B = 90^\circ$, $\angle BAC = \theta$.

a = BC = 24 units and b = AC = 25 units.



By using Pythagoras theorem

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{(25)^2 - (24)^2} = \sqrt{625 - 576}$$

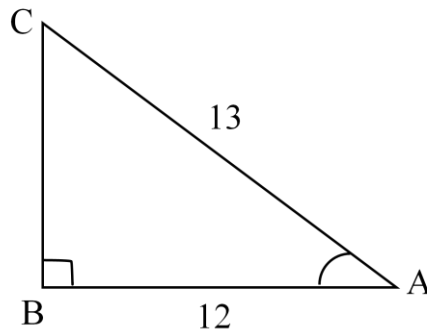
$$\therefore AB = \sqrt{49} = 7 \text{ cm.}$$

$$\text{Then } \cos \theta = \frac{AB}{AC} = \frac{7}{25}$$

$$\tan \theta = \frac{BC}{AB} = \frac{24}{7}$$

4. If $\cos A = \frac{12}{13}$, then find $\sin A$ and $\tan A$.

A. Given that $\cos A = \frac{12}{13} = \frac{BA}{AC}$



$$\frac{BA}{12} = \frac{AC}{13} = K (\text{say}).$$

Where k is a positive number $BA = 12k$

$$AC = 13K$$

By using Pythagoras theorem $BC = \sqrt{AC^2 - BA^2}$

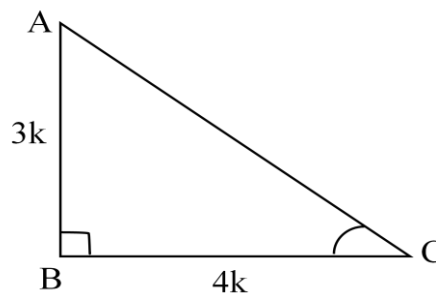
$$BC = \sqrt{(13k)^2 - (12k)^2} = \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\tan A = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

5. If $3 \tan A = 4$, Then find $\sin A$ and $\cos A$.

A. Given that $3 \tan A = 4$



$$\Rightarrow \tan A = \frac{4}{3} = \frac{BC}{AB}$$

$$\Rightarrow \frac{BC}{AB} = \frac{4}{3} \text{ say.}$$

$$\frac{BC}{4} = \frac{AB}{3} = k \text{ say. for any positive integer } k.$$

$$BC = 4k \text{ and } AB = 3k$$

By using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$= (3k)^2 + (4k)^2$$

$$= 9k^2 + 16K^2$$

$$AC^2 = 25k^2$$

$$\therefore AC = 5k.$$

$$\text{Now, } \sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}.$$

$$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}.$$

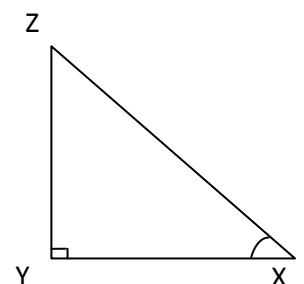
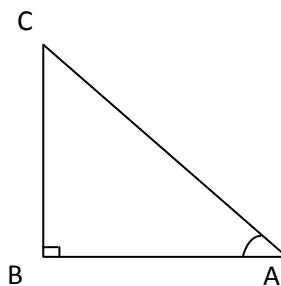
6. If $\angle A$ and $\angle X$ are acute angles such that $\cos A = \cos X$, then show that

$$\angle A = \angle X.$$

Sol: Let us consider two right angled triangles $\triangle ABC$ and $\triangle XYZ$ and right angles at $\angle B$ and $\angle Y$ respectively.

$$\text{Given that } \cos A = \cos x$$

From $\triangle ABC$



$$\cos A = \frac{AB}{AC} \longrightarrow (1)$$

From $\triangle XYZ$

$$\cos X = \frac{XY}{XZ} \longrightarrow (2)$$

From (1) & (2) $\frac{AB}{AC} = -\frac{XY}{XZ} \quad (\because \cos A = \cos X)$

$$\text{Let } \frac{AB}{AC} = \frac{XY}{XZ} = \frac{K}{1} \Rightarrow \frac{AB}{XY} = \frac{AC}{XZ} = k \longrightarrow (3)$$

$$\frac{BC}{YZ} = \frac{\sqrt{AC^2 - AB^2}}{\sqrt{XZ^2 - XY^2}} \quad (\because \text{By pythagoras theorem})$$

$$= \frac{\sqrt{K^2 XZ^2 - K^2 XY^2}}{\sqrt{XZ^2 - XY^2}} = \frac{K \sqrt{XZ^2 - XY^2}}{\sqrt{XZ^2 - XY^2}} = K$$

$$\therefore \frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}$$

$$\Rightarrow \triangle ABC \sim \triangle XYZ$$

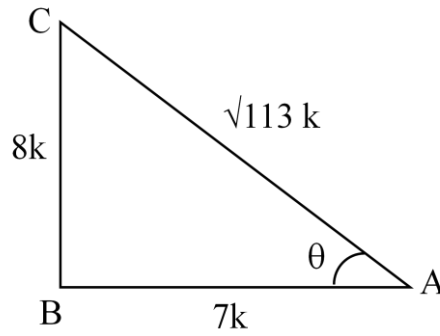
$$\Rightarrow \angle A = \angle X \text{ Proved.}$$

7. Given $\cos \theta = \frac{7}{8}$, then evaluate

(i) $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$

(ii) $\frac{1 + \sin \theta}{\cos \theta}.$

Sol: let us draw a right angle triangle ABC in which $\angle BAC = \theta$.



$$\cot \theta = \frac{7}{8} \text{ (Given)}$$

$$\Rightarrow \frac{AB}{BC} = \frac{7}{8}$$

$$\Rightarrow \frac{AB}{7} = \frac{BC}{8} = k \text{ (Say) where } k \text{ is a positive integer.}$$

$$\Rightarrow AB = 7k \text{ and } BC = 8k.$$

By using Pythagoras Theorem $AC = \sqrt{AB^2 + BC^2}$

$$\begin{aligned} &= \sqrt{(7k)^2 + (8k)^2} \\ &= \sqrt{49k^2 + 64k^2} = \sqrt{113}k. \end{aligned}$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

$$(i) \quad \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{(1)^2 - \sin^2 \theta}{(1)^2 - \cos^2 \theta} = \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\Rightarrow \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{113 - 64}{113 - 49} = \frac{49}{64}.$$

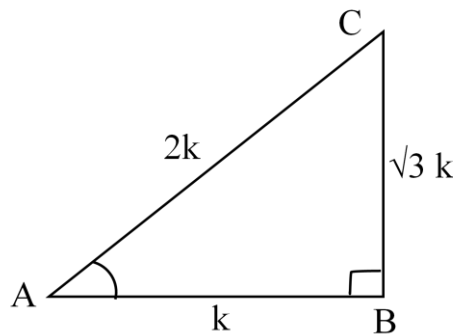
$$(ii) \quad \frac{1+\sin \theta}{\cos \theta}$$

$$\Rightarrow \frac{1+\frac{8}{\sqrt{113}}}{\frac{7}{\sqrt{113}}} = \frac{\sqrt{113}+8}{7}.$$

8. In a right angle triangle ABC, right angle at B, if $\tan A = \sqrt{3}$. Then find the value of

(i) $\sin A \cos C + \cos A \sin C$. (ii) $\cos A \cos C - \sin A \sin C$.

Sol: let us draw a right angled triangle ABC.



Given that $\tan A = \sqrt{3} = \frac{\sqrt{3}}{1}$, $B = 90^\circ$.

$$\Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow \frac{BC}{\sqrt{3}} = \frac{AB}{1} = k \text{ say.}$$

Where k is any positive integer

$$\Rightarrow BC = \sqrt{3}k, AB = k.$$

By using Pythagoras theorem $AC = \sqrt{AB^2 + BC^2}$

$$= \sqrt{k^2 + (\sqrt{3}k)^2}$$

$$\sqrt{4k^2} = 2k.$$

$$\therefore AC = 2k.$$

$$\text{There fore } \sin A = \frac{BC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{2k} = \frac{1}{2}.$$

$$\sin C = \frac{AB}{AC} = \frac{k}{2k} = \frac{1}{2}; \quad \cos C = \frac{BC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

Now (i) $\sin A \cos C + \cos A \sin C$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

(ii) $\cos A \cos C - \sin A \sin C$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = 0.$$

Exercise - 11.2

1. Evaluate the following.

(i) $\sin 45^\circ + \cos 45^\circ$

Sol: we know that $\sin 45^\circ = \frac{1}{\sqrt{2}}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$.

$$\therefore \sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

(ii) $\frac{\cos 45^\circ}{\sec 30^\circ + \csc 60^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2}$$

$$= \frac{\sqrt{3}}{4\sqrt{2}}.$$

(iii) $\frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\cot 45^\circ + \cos 60^\circ - \sec 30^\circ}$

We know that $\sin 30^\circ = \frac{1}{2}$, $\tan 45^\circ = 1$, $\csc 60^\circ = \frac{2}{\sqrt{3}}$

$$\cot 45^\circ = 1; \cos 60^\circ = \frac{1}{2}; \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\therefore \frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\cot 45^\circ + \cos 60^\circ - \sec 30^\circ} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{1 + \frac{1}{2} - \frac{2}{\sqrt{3}}} = 1$$

(iv) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

A. $\tan 45^\circ = 1$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sin 60^\circ = \frac{\sqrt{3}}{2}$.

$$\begin{aligned} &\therefore 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ \\ &= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = 2. \end{aligned}$$

$$(v) \quad \frac{\sec^2 60^\circ - \tan^2 60^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$A. \quad \sec 60^\circ = 2, \tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{\sec^2 60^\circ - \tan^2 60^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} = \frac{(2)^2 - (\sqrt{3})^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{4-3}{\frac{1}{4} + \frac{3}{4}} = \frac{1}{1} = 1.$$

2. Evaluate $\sin 60^\circ \cdot \cos 30^\circ + \sin 30^\circ \cos 60^\circ$. What is the value of $\sin (60^\circ + 30^\circ)$. What can you conclude?

$$A. \quad \text{We know that } \sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 60^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}.$$

$$\therefore \sin 60^\circ \cdot \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1.$$

$$\sin (60^\circ + 30^\circ) = \sin (90^\circ) = 1$$

$\therefore$ we can conclude that

$$\sin (60^\circ + 30^\circ) = \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$\Rightarrow \sin (A + B) = \sin A \cos B + \cos A \sin B.$$

3. Is it right to say $\cos(60^\circ + 30^\circ) = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$.

Sol. L.H.S $\cos(60^\circ + 30^\circ) = \cos 90^\circ = 0$

We know that $\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

$\therefore$ R.H.S = $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

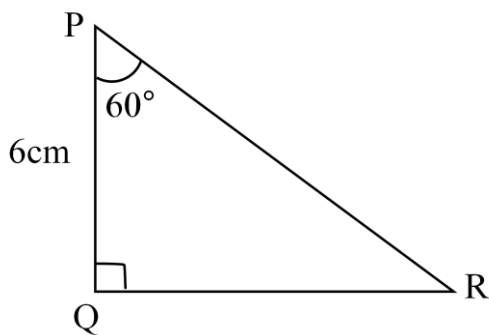
$\therefore$ It is right to say that

$$\cos(60^\circ + 30^\circ) = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ.$$

4. In right angled triangle PQR, right angle is at Q and PQ = 6cm, $\angle RPQ = 60^\circ$.

Determine the lengths of QR and PR.

Sol: In $\triangle PQR$ $\angle Q = 90^\circ$



$$PQ = 6\text{cm.}$$

Then $\cos 60^\circ = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$$\frac{1}{2} = \frac{PQ}{PR} \quad \left(\because \cos 60^\circ = \frac{1}{2} \right)$$

$$\therefore \frac{PQ}{PR} = \frac{1}{2} \Rightarrow PR = 2PQ = 2 \times 6\text{cm} = 12\text{cm}.$$

Similliarly

$$\sin 60^\circ = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{QR}{PR}$$

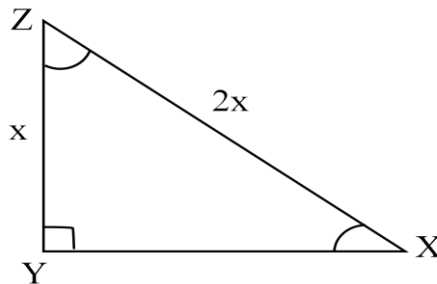
$$\frac{\sqrt{3}}{2} = \frac{QR}{PR} \Rightarrow QR = \frac{\sqrt{3}.PR}{2}$$

$$\Rightarrow QR = \frac{\sqrt{3}(12)}{2} = 6\sqrt{3}\text{cm}.$$

$$\therefore QR = 6\sqrt{3}\text{cm}, PR = 12\text{cm}.$$

5.In $\triangle XYZ$, right angle is at y, $yz = x$, and $XZ = 2x$ then determine $\angle YXZ$ and $\angle YZX$.

$$\text{Sol: } \sin X = \frac{\text{opposite side of } \angle X}{\text{hypotenuse}} = \frac{x}{2x} = \frac{1}{2}$$



$$\sin X = \frac{1}{2} = \sin 30^\circ$$

$$\Rightarrow X = 30^\circ \text{ or } \angle YXZ = 30^\circ$$

$$\cos Z = \frac{\text{Adjacent side of } \angle Z}{\text{hypotenuse}} = \frac{x}{2x} = \frac{1}{2}$$

$$\cos Z = \frac{1}{2} = \cos 60^\circ$$

$$Z = 60^\circ \text{ or } \angle YZX = 60^\circ$$

Exercise – 11.3

1) Evaluate

(i) $\frac{\tan 36^\circ}{\cot 54^\circ}$ We can write $\tan \theta = \cot (90 - \theta)$.

$$\cot \theta = \tan (90 - \theta)$$

$$\therefore \tan 36^\circ = \cot (90^\circ - 36^\circ)$$

$$\Rightarrow \frac{\tan 36^\circ}{\cot 54^\circ} = \frac{\cot (90^\circ - 36^\circ)}{\cot 54^\circ} = \frac{\cot 54^\circ}{\cot 54^\circ} = 1$$

ii) $\cos 12^\circ - \sin 78^\circ$

$$\Rightarrow \cos (90 - 78^\circ) - \sin 78^\circ \quad (\because \cos(90^\circ - \theta) = \sin \theta)$$

$$\Rightarrow \sin 78^\circ - \sin 78^\circ = 0.$$

iii) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

$$\Rightarrow \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ$$

$$\Rightarrow \sec 59^\circ - \sec 59^\circ = 0. \quad (\sec \theta = \operatorname{cosec} (90^\circ - \theta))$$

iv) $\sin 15^\circ \sec 75^\circ$

$$\sin 15^\circ \cdot \sec (90 - 15^\circ) \quad (\because \sec(90^\circ - \theta) = \operatorname{cosec} \theta)$$

$$= \sin 15^\circ \cdot \operatorname{Cosec} 15^\circ \quad \left(\because \sin \theta = \frac{1}{\operatorname{cosec} \theta} \right)$$

$$= \sin 15^\circ \cdot \frac{1}{\sin 15^\circ} = 1$$

v) $\tan 26^\circ \cdot \tan 64^\circ$

$$\tan 26^\circ \cdot \tan 64^\circ = \tan 26^\circ \cdot \tan (90^\circ - 26^\circ) \quad \left(\begin{array}{l} \because \tan (90^\circ - \theta) = \cot \theta \\ = \cot \theta \cdot \frac{1}{\tan \theta} \end{array} \right)$$

$$= \tan 26^\circ \cdot \cot 26^\circ$$

$$= \tan 26^\circ \cdot \frac{1}{\tan 26^\circ} = 1$$

2. Show that

(i) $\tan 48^\circ \cdot \tan 16^\circ \cdot \tan 42^\circ \cdot \tan 74^\circ = 1$

(ii) $\cos 36^\circ \cdot \cos 54^\circ - \sin 36^\circ \cdot \sin 54^\circ = 0$

A. (i) $\tan 48^\circ \cdot \tan 16^\circ \cdot \tan 42^\circ \cdot \tan 74^\circ$

We know that $\tan (90 - \theta) = \cot \theta$.

Re write $\tan 48^\circ = \tan (90^\circ - 42^\circ)$ and $\tan 16^\circ = \tan (90 - 74^\circ)$

$$\Rightarrow \tan 48^\circ \cdot \tan 16^\circ \cdot \tan 42^\circ \cdot \tan 74^\circ$$

$$= \tan (90 - 42^\circ) \cdot \tan (90 - 74^\circ) \cdot \tan 42^\circ \cdot \tan 74^\circ$$

$$= \cot 42^\circ \cdot \cot 74^\circ \cdot \tan 42^\circ \cdot \tan 74^\circ.$$

$$= (\cot 42^\circ \cdot \tan 42^\circ) (\cot 74^\circ \cdot \tan 74^\circ)$$

$$= 1 \times 1$$

$$\left(\begin{array}{l} \because \cot \theta \cdot \tan \theta = 1 \\ or = \tan \theta \cdot \frac{1}{\cot \theta} \end{array} \right)$$

$$= 1$$

(ii) $\cos 36^\circ \cdot \cos 54^\circ - \sin 36^\circ \sin 54^\circ = 0.$

A. take L.H.S. $\cos 36^\circ \cdot \cos 54^\circ - \sin 36^\circ \sin 54^\circ$

$$= \cos 36^\circ \cdot \cos (90 - 36^\circ) - \sin 36^\circ (90 - 36^\circ)$$

$$= \cos 36^\circ \cdot \sin 36^\circ - \sin 36^\circ \cdot \cos 36^\circ.$$

$$\left(\begin{array}{l} \because (\cos (90^\circ - \theta) = \sin \theta) \\ \sin (90^\circ - \theta) = \cos \theta \end{array} \right)$$

$$= 0. \text{ R.H.S.}$$

3. If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle. Find the value of A .

A. Given that $\tan 2A = \cot (A - 18^\circ)$

$$\Rightarrow \cot (90^\circ - 2A) = \cot (A - 18^\circ) \quad (\because \cot(90^\circ - 2A) = \tan 2A)$$

$$\Rightarrow 90^\circ - 2A = A - 18^\circ \quad \left(\begin{array}{l} \because 90^\circ - 2A \text{ and } A - 18^\circ \\ \text{both are acute angles} \end{array} \right)$$

$$\Rightarrow -2A - A = -18 - 90^\circ$$

$$-3A = -108^\circ$$

$$\Rightarrow A = \frac{-108}{-3} = 36^\circ \Rightarrow A = 36^\circ.$$

4. If $\tan A = \cot B$, where A & B are acute angles, prove that $A + B = 90^\circ$.

Sol: Given that $\tan A = \cot B$.

$$\Rightarrow \tan A = \tan (90^\circ - B) \quad (\because \tan(90^\circ - \theta) = \cot \theta)$$

$$\Rightarrow A = 90^\circ - B \quad \left(\begin{array}{l} \because A \text{ and } (90^\circ - B) \\ \text{both are acute angle.} \end{array} \right)$$

$$\Rightarrow A + B = 90^\circ.$$

5. If A, B and C are interior angle of a triangle ABC, then show that

$$\tan\left(\frac{A+B}{2}\right) = \cot \frac{C}{2}.$$

Sol: The sum of the interior angles in ABC is 180

$$\Rightarrow A + B + C = 180$$

$$\frac{A+B}{2} + \frac{C}{2} = \frac{180^\circ}{2} = 90^\circ$$

$$\Rightarrow \frac{A+B}{2} = 90^\circ - \frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right) \quad \left(\because \tan(90^\circ - \theta) = \cot \theta\right)$$

$$\therefore \tan\left(\frac{A+B}{2}\right) = \cot \frac{C}{2}.$$

6. Express $\sin 75^\circ + \cos 65^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

$$\text{A. } \sin 75^\circ = \sin (90^\circ - 15^\circ) = \cos 15^\circ \quad \left(\because \sin(90^\circ - \theta) = \cos \theta\right)$$

$$\cos 65^\circ = \cos (90^\circ - 25^\circ) = \sin 25^\circ. \quad \left(\because \cos(90^\circ - \theta) = \sin \theta\right)$$

$$\therefore \sin 75^\circ + \cos 65^\circ = \cos 15^\circ + \sin 25^\circ.$$

Exercise – 11.4

1. Evaluating the following.

(i) $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$.

A. $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

$$= (1 + \tan \theta + \sec \theta) \left(1 + \frac{1}{\tan \theta} - \operatorname{cosec} \theta \right)$$

$$= (1 + \tan \theta + \sec \theta) \left(\frac{\tan \theta + 1 - \tan \theta \cdot \operatorname{cosec} \theta}{\tan \theta} \right)$$

$$\left(\frac{(1 + \tan \theta + \sec \theta)(\tan \theta + 1 - \sec \theta)}{\tan \theta} \right) \left(\begin{array}{l} \because \tan \theta \operatorname{cosec} \theta \\ = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \\ \frac{1}{\cos \theta} = \sec \theta \end{array} \right)$$

$$\frac{(1 + \tan \theta)^2 - \sec^2 \theta}{\tan \theta}$$

$$\frac{1 + \tan^2 \theta + 2 \tan \theta - \sec^2 \theta}{\tan \theta}$$

$$\frac{1 + 2 \tan \theta - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta}$$

$$= \frac{1 + 2 \tan \theta - 1}{\tan \theta} = \frac{2 \tan \theta}{\tan \theta} = 2.$$

ii) $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$.

A. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$$

$$= 2\sin^2 \theta + 2\cos^2 \theta = 2(\sin^2 \theta + \cos^2 \theta) \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= 2(1) = 2.$$

iii) $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1)$

A. $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) \quad \left(\begin{array}{l} \because \sec^2 \theta - 1 = \tan^2 \theta \\ \operatorname{cosec}^2 \theta - 1 = \cot^2 \theta \end{array} \right)$

$$= \tan^2 \theta \cdot \cot^2 \theta$$

$$= 1.$$

2) Show that $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

A. L.H.S $(\operatorname{cosec} \theta - \cot \theta)^2$

$$= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 = \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} = R.H.S$$

3) Show that $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

A. L.H.S

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \quad (\because 1 - \sin^2 A = \cos^2 A)$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A. \text{ R.H.S.}$$

4. Show that $\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A.$

A. L.H.S

$$\frac{1 - \tan^2 A}{\cot^2 A - 1} = \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1} = \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A. \quad \text{R.H.S}$$

II Method:

$$\frac{1 - \tan^2 A}{\cot^2 A - 1} = \frac{1 - \tan^2 A}{\frac{1}{\tan^2 A} - 1} = \frac{1 - \tan^2 A}{\frac{1 - \tan^2 A}{\tan^2 A}}$$

$$= \tan^2 A = \text{R.H.S.}$$

5. Show that $\frac{1}{\cos \theta} - \cos \theta = \tan \theta. \sin \theta.$

A. L.H.S.

$$\frac{1}{\cos \theta} - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta = \tan \theta \cdot \sin \theta = \text{R.H.S.}$$

$\therefore$

$$\text{L.H.S.} = \text{R.H.S.}$$

6. Simplify $\sec A \cdot (1 - \sin A) \cdot (\sec A + \tan A)$.

A. $\sec A (1 - \sin A) \cdot (\sec A + \tan A)$

$$= (\sec A - \sec A \sin A) (\sec A + \tan A)$$

$$= \left(\sec A - \frac{1}{\cos A} \cdot \sin A \right) (\sec A + \tan A) \left(\because \sec A = \frac{1}{\cos A} \right)$$

$$= (\sec A - \tan A) (\sec A + \tan A) \left(\because \tan A = \frac{\sin A}{\cos A} \right)$$

$$= \sec^2 A - \tan^2 A \left(\because \sec^2 A - \tan^2 A = 1 \right)$$

$$= 1.$$

7. Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A \cot^2 A$

A. $\text{L.H.S } (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \cdot \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \sec A \cdot \cos A$$

$$= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 \sin A \operatorname{cosec} A + 2 \sec A \cos A$$

$$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 2 \sin A \cdot \frac{1}{\sin A} + 2 \cdot \frac{1}{\cos A} \cdot \cos A$$

$$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$$

$$= 7 + \cot^2 A + \tan^2 A$$

$$= \text{R.H.S.}$$

8. Simplify $(1 - \cos \theta) (1 + \cos \theta) (1 + \cot^2 \theta)$.

A. $(1 - \cos \theta) (1 + \cos \theta) (1 + \cot^2 \theta)$

$$= (1 - \cos^2 \theta) (1 + \cot^2 \theta) \quad \left(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \right)$$

$$= \sin^2 \theta (1 + \cot^2 \theta) = \sin^2 \theta \cdot \operatorname{Cosec}^2 \theta.$$

$$= 1.$$

9. If $\sec \theta + \tan \theta = P$; then what is the value of $\sec \theta - \tan \theta$?

A. $\sec \theta + \tan \theta = P$. (Given)

$$\text{We know that } \sec^2 \theta - \tan^2 \theta = 1 \quad \left(a^2 - b^2 = (a+b)(a-b) \right)$$

$$\Rightarrow (\sec \theta + \tan \theta) (\sec \theta - \tan \theta) = 1$$

$$P (\sec \theta - \tan \theta) = 1$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{P}$$

$$\therefore \text{The value of } \sec \theta - \tan \theta = \frac{1}{P}.$$

10. If $\operatorname{cosec} \theta + \cot \theta = k$, then show that $\cos \theta = \frac{k^2 - 1}{k^2 + 1}$.

A. Given that $\operatorname{cosec} \theta + \cot \theta = k \rightarrow (1)$

$$\text{Then } \operatorname{cosec} \theta - \cot \theta = \frac{1}{k} \rightarrow (2)$$

$$\left[\begin{array}{l} \because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \\ (\operatorname{cosec} \theta + \cot \theta) \\ (\operatorname{cosec} \theta - \cot \theta) = 1 \\ \operatorname{cosec} \theta - \cot \theta = \frac{1}{\operatorname{cosec} \theta + \cot \theta} \end{array} \right]$$

Adding (1) & (2) we get

$$2 \cos \theta = k + \frac{1}{k} \Rightarrow 2 \cos \theta = \frac{k^2 + 1}{k} \longrightarrow (3)$$

Subtracting (2) from (1) we get

$$2 \cot \theta = k - \frac{1}{k} \Rightarrow 2 \cot \theta = \frac{k^2 - 1}{k} \longrightarrow (4)$$

Dividing (4) by (3) we get

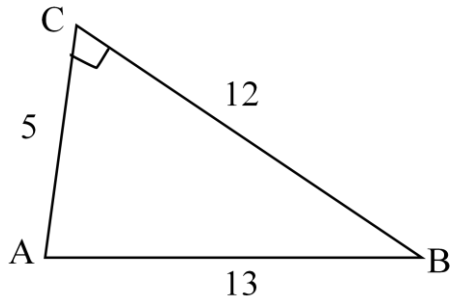
$$\frac{2 \cot \theta}{2 \cos \theta} = \frac{\frac{k^2 - 1}{k}}{\frac{k^2 + 1}{k}} = \frac{k^2 - 1}{k} \times \frac{k}{k^2 + 1}$$

$$\frac{\cos \theta}{\sin \theta} \cdot \sin \theta = \frac{k^2 - 1}{k^2 + 1}$$

$$\therefore \cos \theta = \frac{k^2 - 1}{k^2 + 1}$$

Objective Type Questions

1. In the following figure, the value of $\cot A$ is []



- a) $\frac{12}{13}$ b) $\frac{5}{12}$ c) $\frac{5}{13}$ d) $\frac{13}{5}$
2. If in $\triangle ABC$, $\angle B = 90^\circ$, $AB = 12$ cm and $BC = 5$ cm then the value of $\cos C$ is. []

- a) $\frac{5}{13}$ b) $\frac{5}{12}$ c) $\frac{12}{5}$ d) $\frac{13}{5}$

3. If $\cot \theta = \frac{b}{a}$ then the value of $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$ is []

- a) $\frac{b-a}{b+a}$ b) $b-a$ c) $b+a$ d) $\frac{b+a}{b-a}$

4. The maximum value of $\sin \theta$ is _____ []

- a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) 1 d) $\frac{1}{\sqrt{2}}$

5. If A is an acute angle of a $\triangle ABC$, right angled at B, then the value of $\sin A + \cos A$ is []

- a) Equal to one b) greater than two
c) Less than one d) equal to two

6. The value of $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$ is []
- a) $\sin 60^\circ$ b) $\cos 60^\circ$ c) $\tan 60^\circ$ d) $\sin 30^\circ$
7. If $\sin \theta = \frac{1}{2}$, then the value of $(\tan \theta + \cot \theta)^2$ is []
- a) $\frac{16}{3}$ b) $\frac{8}{3}$ c) $\frac{4}{3}$ d) $\frac{10}{3}$
8. If $\sin \theta - \cos \theta = 0$; then the value of $\sin^4 \theta + \cos^4 \theta$ is []
- a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) 1
9. If $\theta = 45^\circ$ then the value of $\frac{1 - \cos 2\theta}{\sin 2\theta}$ is []
- a) 0 b) 1 c) 2 d) ∞
10. If $\tan \theta = \cot \theta$, then the value of $\sec \theta$ is []
- a) 2 b) 1 c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{2}$
11. If $A + B = 90^\circ$, $\cot B = \frac{3}{4}$ then $\tan A$ is equal to []
- a) $\frac{5}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) $\frac{1}{4}$
12. If $\sin (x - 20^\circ) = \cos (3x - 10)^\circ$. Then x is []
- a) 60° b) 30° c) 45° d) 35.5
13. The value of $1 + \tan 5^\circ \cot 85^\circ$ is equal to []
- a) $\sin^2 5^\circ$ b) $\cos^2 5^\circ$ c) $\sec^2 5^\circ$ d) $\operatorname{cosec}^2 5^\circ$

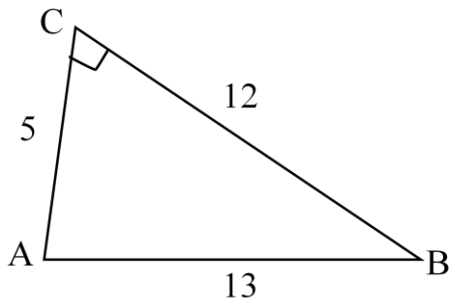
14. If any triangle ABC, the value of $\sin \frac{B+C}{2}$ is _____ []
- a) $\cos \frac{A}{2}$ b) $\sin \frac{A}{2}$ c) $-\sin \frac{A}{2}$ d) $-\cos \frac{A}{2}$
15. If $\cos \theta = \frac{a}{b}$, then cosec θ is equal to []
- a) $\frac{b}{a}$ b) $\frac{b}{\sqrt{b^2 - a^2}}$ c) $\frac{\sqrt{b^2 - a^2}}{b}$ d) $\frac{a}{\sqrt{b^2 - a^2}}$
16. The value of $\cos 20^\circ \cos 70^\circ - \sin 20^\circ \sin 70^\circ$ is equal to []
- a) 0 b) 1 c) ∞ d) $\cos 50^\circ$
17. The value of $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$ is _____ []
- a) 2 b) 3 c) 1 d) 4
18. If $\tan \theta + \cot \theta = 5$ then the value of $\tan^2 \theta + \cot^2 \theta$ is _____ []
- a) 23 b) 25 c) 27 d) 15
19. If $\operatorname{cosec} \theta = 2$ and $\cot \theta = \sqrt{3} p$ where θ is an acute angle, then the value of p is []
- a) 2 b) 1 c) 0 d) $\sqrt{3}$
20. $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$ is equal to []
- a) $\sin A + \cos A$ b) $\sec A + \tan A$
c) $\sec A - \tan A$ d) $\sec^2 A + \tan^2 A$

Key:

1. b; 2. a; 3. d; 4. c; 5. b; 6. a; 7. a; 8. a; 9. b; 10. d;
11. c; 12. b; 13. c; 14. a; 15. c; 16. b; 17. c; 18. a; 19. b; 20. b.

Fill in the Blanks

1. If $\operatorname{cosec}\theta - \cot\theta = \frac{1}{4}$ then the value of $\operatorname{cosec}\theta + \cot\theta$ is _____
2. $\sin 45^\circ + \cos 45^\circ =$ _____
3. $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ =$ _____
4. $\sin (90^\circ - A) =$ _____
5. If $\sin A = \cos B$ then, the value of $A + B =$ _____
6. If $\sec\theta = \frac{m+n}{2\sqrt{mn}}$ then $\sin\theta =$ _____
7. In the adjacent figure, the value of $\sec A$ is _____.



8. If $\sin A = \frac{1}{2} \tan^2 45^\circ$, where A is an acute angle then the value of A is _____
9. The maximum value of $\frac{1}{\sec\theta}$, $0^\circ < \theta < 90^\circ$ is _____
10. $\frac{\sin^2 \theta}{1 - \cos^2 \theta}$ is equal to _____
11. if $\cot\theta = 1$ then $\frac{1 + \sin\theta}{\cos\theta} =$ _____
12. $\sec^2\theta - 1 =$ _____
13. If $\sec\theta + \tan\theta = p$, then the value of $\sec\theta - \tan\theta =$ _____.

14. The value of $\sin A$ or $\cos A$ never exceeds _____.

15. $\sec (90^\circ - A) =$ _____

Key:

- 1) 4; 2) $\sqrt{2}$; 3) 2; 4) $\cos A$; 5) 90° ; 6) $\frac{m-n}{m+n}$;
7) $\frac{13}{5}$; 8) 15° ; 9) 1; 10) 1; 11) $\sqrt{2} + 1$; 12) $\tan^2 \theta$;
13) $\frac{1}{p}$; 14) 1; 15) $\operatorname{cosec} A$.

Trigonometric Identities

An identity equation having trigonometric ratios of an angle is called trigonometric identity. And it is true for all the values of the angles involved in it.

$$(1) \sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A, \cos^2 A = 1 - \sin^2 A$$

$$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{(1 + \cos A)(1 - \cos A)}$$

$$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{(1 + \sin A)(1 - \sin A)}$$

$$(2) 1 + \tan^2 A = \sec^2 A \text{ or } \sec^2 A - 1 = \tan^2 A$$

$$\sec^2 A - \tan^2 A = 1 \text{ or } \tan^2 A - \sec^2 A = -1$$

$$(\sec A + \tan A)(\sec A - \tan A) = 1$$

$$\sec A + \tan A = \frac{1}{\sec A - \tan A} \text{ (or) } \sec A - \tan A = \frac{1}{\sec A + \tan A}$$

$$(3) \quad 1 + \cot^2 A = \operatorname{cosec}^2 A \text{ (or) } \operatorname{cosec}^2 A - 1 = \cot^2 A$$

$$\operatorname{Cosec}^2 A - \cot^2 A = 1 \text{ (or) } \cot^2 A - \operatorname{cosec}^2 A = -1$$

$$(\operatorname{cosec} A + \cot A) (\operatorname{cosec} A - \cot A) = 1$$

$$\operatorname{cosec} A + \cot A = \frac{1}{\operatorname{cosec} A - \cot A} \text{ (or) } \operatorname{cosec} A - \cot A = \frac{1}{\operatorname{cosec} A + \cot A}$$

$$(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$(\sec^2 \theta - 1) (\operatorname{cosec}^2 \theta - 1) = 1.$$