5 CHAPTER

Algebra

EQUATIONS, POLYNOMIALS AND INEQUATIONS

1990

1. The value of $\frac{1}{(1-x)}+\frac{1}{(1+x)}+\frac{2}{\left(1+x^2\right)}+\frac{4}{\left(1+x^4\right)}$ is

(a) $\frac{8}{(1-x^8)}$ (b) $\frac{4x}{(1+x^2)}$

(c) $\frac{4}{(1-x^6)}$ (d) $\frac{4}{(1+x^4)}$

2. Let a, b be any positive integers and x = 0 or 1, then

(a) $a^x b^{(1-x)} = xa + (1-x)b$

(b) $a^x b^{(1-x)} = (1-x)a + xb$

(c) $a^x b^{(1-x)} = a^{(1-x)} bx$

(d) None of the above is necessarily true.

3. The value of $\frac{\left(1-d^3\right)}{(1-d)}$ is

(a) > 1 if d > –1 (b) > 3 if d > 1

(c) > 2 if 0 < d < 0.5 (d) < 2 if d < –2

4. The roots of the equation $ax^2 + 3x + 6 = 0$ will be reciprocal to each other if the value of *a* is

(a) 3 (b) 4

(c) 5 (d) 6

5. If xy + yz + zx = 0, then $(x + y + z)^2$ equals

(a) $(x + y)^2 + xz$ (b) $(x + z)^2 + xy$

(c) $x^2 + y^2 + z^2$ (d) 2(xy + yz + xz)

Directions for Questions 6 and 7: Each of the following questions is followed by two statements. MARK,

(a) if the question can be answered with the help of statement I alone

(b) if the question can be answered with the help of statement II alone

(c) if both, statement I and statement II are needed to answer the question, and

(d) if the statement cannot be answered even with the help of both the statements.

6. A man distributed 43 chocolates to his children. How many of his children are more than five years old?

I. A child older than five years gets 5 chocolates.

II. A child 5 years or younger in age gets 6 chocolates.

7. Is $\left[\frac{\left(x^{-1}-y^{-1}\right)}{\left(x^{-2}-y^{-2}\right)}\right] > 1$?

I. x + y > 0.

II. x and y are positive integers and each is greater than 2.

1991

8. What is the value of k for which the following system of equations has no solution:

2x – 8y = 3 and kx + 4y = 10

(a) –2 (b) 1

(c) –1 (d) 2

9. Iqbal dealt some cards to Mushtaq and himself from a full pack of playing cards and laid the rest aside. Iqbal then said to Mushtaq. "If you give me a certain number of your cards, I will have four times as many cards as you will have. If I give you the same number of cards, I will have thrice as many cards as you will have". Of the given choices, which could represent the number of cards with Iqbal?

(a) 9 (b) 31

(c) 12 (d) 35

10. The number of integers n satisfying $-n + 2 \geq 0$ and $2n \geq 4$ is

(a) 0 (b) 1

(c) 2 (d) 3

11. In Sivakasi, each boy's quota of match sticks to fill into boxes is not more than 200 per session. If he reduces the number of sticks per box by 25, he can fill 3 more boxes with the total number of sticks assigned to him. Which of the following is the possible number of sticks assigned to each boy?

(a) 200 (b) 150

(c) 125 (d) 175

1993

Directions for Questions 12 and 13 : Each of these items has a question followed by two statements. As the answer,

Mark (a) If the question can be answered with the help of statement I alone

Mark (b) If the question can be answered with the help of statement II, alone

Mark (c) If both, statement I and statement II are needed to answer the question, and

Mark (d) If the question cannot be answered even with the help of both the statements.

12. Given that X and Y are non-negative. What is the value of X?

I. $2X + 2Y \le 40$

II. $X - 2Y \ge 20$

13. What is the price of mangoes per kg?

I. Ten kg of mangoes and two dozens of oranges cost Rs.252.

II. Two kg of mangoes could be bought in exchange for one dozen oranges.

14. Two oranges, three bananas and four apples cost Rs.15. Three oranges, two bananas and one apple cost Rs 10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?

(a) Rs.10 (b) Rs.8

(c) Rs.15 (d) cannot be determined

15. John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange would be

(a) Re. 1 (b) Rs. 2

(c) Rs. 3 (d) Rs. 4

1994

16. If *a* + *b* + *c* = *0,* where $a \ne b \ne c$, then

$$\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}$$ is equal to

(a) zero (b) 1

(c) –1 (d) abc

17. If one root of $x^2 - 7 + 12 = 0$ is 4, while the equation $x^2 - 7x + q = 0$ has equal roots, then the value of *q* is

(a) $\frac{49}{4}$ (b) $\frac{4}{49}$

(c) 4 (d) $\frac{1}{4}$

Direction for Question 18: Data is provided followed by two statements – I and II – both resulting in a value, say I and II. As your answer,

Mark (a) if I > II.

Mark (b) if I < II.

Mark (c) if I = II.

Mark (d) if nothing can be said.

18. Nineteen year from now Jackson will be 3 times as old as Joseph is now. Johnson is three years younger than Jackson.

I. Johnson's age now.

II. Joseph's age now.

Direction for Question 19: The question is followed by two statements. As the answer,

Mark (a) If the question can be answered with the help of statement I alone

Mark (b) If the question can be answered with the help of statement II, alone

Mark (c) If both, statement I and statement II are needed to answer the question, and

Mark (d) If the question cannot be answered even with the help of both the statements.

19. If twenty sweets are distributed among some boys and girls such that each girl gets two sweets and each boy gets three sweets, what is the number of boys and girls?

I. The number of girls is not more than five.

II. If each girl gets 3 sweets and each boy gets 2 sweets, the number of sweets required for the children will still be the same.

1995

20. One root of $x^2 + kx - 8 = 0$ is square of the other. Then the value of k is

(a) 2 (b) 8

(c) –8 (d) –2

21. Two positive integers differ by 4 and sum of their reciprocals is $\frac{10}{21}$. Then one of the numbers is

(a) 3 (b) 1

(c) 5 (d) 21

22. What is the value of m which satisfies $3m^2 - 21m + 30 < 0$?

(a) m < 2 or m > 5

(b) m > 2

(c) 2 < m < 5

(d) Both (a) and (c)

Direction for Question 23: The question is followed by two statements, I and II. Mark the answer as

(a) if the question can be answered with the help of statement I alone.

(b) if the question can be answered with the help of statement II alone.

(c) if both statement I and statement II are needed to answer the question.

(d) if the question cannot be answered even with the help of both the statements.

23. What is the value of x, if x and y are consecutive positive even integers?

I. $(x - y)^2 = 4$

II. $(x + y)^2 < 100$

1996

24. Which of the following values of x do not satisfy the inequality $(x^2 - 3x + 2 > 0)$at all?

(a) $1 \le x \le 2$ (b) $-1 \ge x \ge -2$

(c) $0 \le x \le 2$ (d) $0 \ge x \ge -2$

25. Given the quadratic equation $x^2 - (A - 3)x - (A - 2) = 0$, for what value of A will the sum of the squares of the roots be zero?

(a) –2 (b) 3

(c) 6 (d) None of these

Direction for Question 26 : The question is followed by two statements, I and II. Mark the answer as:

(a) if the question cannot be answered even with the help of both the statements taken together.

(b) if the question can be answered by any one of the two statements.

(c) if each statement alone is sufficient to answer the question, but not the other one (e.g. statement I alone is required to answer the question, but not statement II and vice versa).

(d) if both statements I and II together are needed to answer the question.

26. If α and β are the roots of the equation $(ax^2 + bx + c = 0)$, then what is the value of $(\alpha^2 + \beta^2)$?

I. $\alpha + \beta = -\left(\frac{b}{a}\right)$

II. $2\alpha\beta = \left(\frac{c}{a}\right)$

27. Find the value of $\dfrac{1}{1+\dfrac{1}{3-\dfrac{4}{2+\dfrac{1}{3-\dfrac{1}{2}}}}} + \dfrac{3}{3-\dfrac{4}{3+\dfrac{1}{2-\dfrac{1}{2}}}}$

(a) $\frac{13}{7}$ (b) $\frac{15}{7}$

(c) $\frac{11}{21}$ (d) $\frac{17}{28}$

1997

28. A, B and C are defined as follows.

$$A = (2.000004) \div \left[(2.000004)^2 + (4.000008)\right]$$

$$B = (3.000003) \div \left[(3.000003)^2 + (9.000009)\right]$$

$$C = (4.000002) \div \left[(4.000002)^2 + (8.000004)\right]$$

Which of the following is true about the values of the above three expressions?

(a) All of them lie between 0.18 and 0.2

(b) A is twice of C

(c) C is the smallest

(d) B is the smallest

29. A student instead of finding the value of $\frac{7}{8}$ of a number, found the value of $\frac{7}{18}$ of the number. If his answer differed from the actual one by 770, find the number.

(a) 1584 (b) 2520

(c) 1728 (d) 1656

30. If the roots x_1 and x_2 of the quadratic equation $x^2 - 2x + c = 0$ also satisfy the equation $7x_2 - 4x_1 = 47$, then which of the following is true?

(a) c = –15

(b) $x_1 = -5, x_2 = 3$

(c) $x_1 = 4.5, x_2 = -2.5$

(d) None of these

Direction for Question 31: The question followed by two statements, I and II. Mark the answer

(a) if the question can be answered with the help of one statement alone.

(b) if the question can be answered with the help of any one statement independently.

(c) if the question can be answered with the help of both statements together.

(d) if the question cannot be answered even with the help of both statements together.

31. What is the value of x and y?

I. 3x + 2y = 45

II. 10.5x + 7y = 157.5

1998

Direction for Question 32: The question followed by two statements, I and II. Mark the answer

(a) if the question can be answered with the help of any one statement alone but not by the other statement.

(b) if the question can be answered with the help of either of the statements taken individually.

(c) if the question can be answered with the help of both statements together.

(d) if the question cannot be answered even with the help of both statements together.

32. In a group of 150 students, find the number of girls.

I. Each girl was given 50 paise, while each boy was given 25 paise to purchase goods totalling Rs. 49.

II. Girls and boys were given 30 paise each to buy goods totalling Rs. 45.

1999

33. The number of positive integer valued pairs (x, y) satisfying $4x - 17y = 1$ and $x \leq 1000$ is

(a) 59 (b) 57

(c) 55 (d) 58

Direction for Question 34: The question is followed by two statements I and II.

Mark:

(a) if the question can be answered by any one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both the statements together.

34. Find a pair of real numbers x and y that satisfy the following two equations simultaneously. It is known that the values of a, b, c, d, e and f are non-zero.

$ax + by = c$

$dx + ey = f$

I. $a = kd$ and $b = ke$, $c = kf$, $k \neq 0$

II. $a = b = 1$, $d = e = 2$, $f \neq 2c$

2000

35.

x	1	2	3	4	5	6
y	4	8	14	22	32	44

In the above table, for suitably chosen constants a, b and c, which one of the following best describes the relation between y and x?

(a) $y = a + bx$

(b) $y = a + bx + cx^2$

(c) $y = e^{a + bx}$

(d) None of these

36. If $x > 2$ and $y > -1$, then which of the following statements is necessarily true?

(a) $xy > -2$ (b) $-x < 2y$

(c) $xy < -2$ (d) $-x > 2y$

37. If the equation $x^3 - ax^2 + bx - a = 0$ has three real roots, then it must be the case that

(a) $b = 1$ (b) $b \neq 1$

(c) $a = 1$ (d) $a \neq 1$

38. There are three cities: A, B and C. Each of these cities is connected with the other two cities by at least one direct road. If a traveller wants to go from one city (origin)to another city (destination), she can do so either by traversing a road connecting the two cities directly, or by traversing two roads, the first connecting the origin to the third city and the second connecting the third city to the destination. In all there are 33 routes from A to B (including those via C). Similarly, there are 23 routes from B to C (including those via A). How many roads are there from A to C directly?

(a) 6 (b) 3

(c) 5 (d) 10

39. Two full tanks, one shaped like a cylinder and the other like a cone, contain jet fuel. The cylindrical tank holds 500 L more than the conical tank. After 200 L of fuel has been pumped out from each tank the cylindrical tank contains twice the amount of fuel in the conical tank. How many litres of fuel did the cylindrical tank have when it was full?

(a) 700 L (b) 1,000 L

(c) 1,100 L (d) 1,200 L

Direction for Question 40: The question is followed by two statements, I and II. Answer the question using the following instructions.

Mark the answer as:

(a) if the question can be answered by one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

40. Let X be a real number. Is the modulus of X necessarily less than 3?

I. $X(X + 3) < 0$

II. $X(X - 3) > 0$

2001

41. If $x > 5$ and $y < -1$, then which of the following statements is true?

(a) $(x + 4y) > 1$ (b) $x > -4y$

(c) $-4x < 5y$ (d) None of these

42. In a four-digit number, the sum of the first 2 digits is equal to that of the last 2 digits. The sum of the first and last digits is equal to the third digit. Finally, the sum of the second and fourth digits is twice the sum of the other 2 digits. What is the third digit of the number?

(a) 5 (b) 8
(c) 1 (d) 4

43. Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid their bill and noticed a bowl of mints at the front counter. Sita took one-third of the mints, but returned four because she had a momentary pang of guilt. Fatima then took one-fourth of what was left but returned three for similar reason. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl?

(a) 38 (b) 31
(c) 41 (d) None of these

44. Anita had to do a multiplication. In stead of taking 35 as one of the multipliers, she took 53. As a result, the product went up by 540. What is the new product?

(a) 1050 (b) 540
(c) 1440 (d) 1590

45. x and y are real numbers satisfying the conditions $2 < x < 3$ and $-8 < y < -7$. Which of the following expressions will have the least value?

(a) x^2y (b) xy^2
(c) $5xy$ (d) None of these

46. m is the smallest positive integer such that for any integer $n \geq m$, the quantity $n^3 - 7n^2 + 11n - 5$ is positive. What is the value of m?

(a) 4 (b) 5
(c) 8 (d) None of these

47. Every 10 years the Indian Government counts all the people living in the country. Suppose that the director of the census has reported the following data on two neighbouring villages Chota Hazri and Mota Hazri.

Chota Hazri has 4,522 fewer males than Mota Hazri.

Mota Hazri has 4,020 more females than males.

Chota Hazri has twice as many females as males.

Chota Hazri has 2,910 fewer females than Mota Hazri.

What is the total number of males in Chota Hazri?

(a) 11,264 (b) 14,174
(c) 5,632 (d) 10,154

48. At a certain fast food restaurant, Brian can buy 3 burgers, 7 shakes, and one order of fries for Rs. 120 exactly. At the same place it would cost Rs. 164.50 for 4 burgers, 10 shakes, and one order of fries. How much would it cost for an ordinary meal of one burger, one shake, and one order of fries?

(a) Rs. 31 (b) Rs. 41
(c) Rs. 21 (d) Cannot be determined

49. In some code, letters a, b, c, d and e represent numbers 2, 4, 5, 6 and 10. We just do not know which letter represents which number. Consider the following relationships:

I. $a + c = e$, II. $b - d = d$ and III. $e + a = b$

Which of the following statements is true?

(a) $b = 4, d = 2$ (b) $a = 4, e = 6$
(c) $b = 6, e = 2$ (d) $a = 4, c = 6$

50. Ujakar and Keshab attempted to solve a quadratic equation. Ujakar made a mistake in writing down the constant term. He ended up with the roots (4, 3). Keshab made a mistake in writing down the coefficient of x. He got the roots as (3, 2). What will be the exact roots of the original quadratic equation?

(a) (6, 1) (b) (–3, –4)
(c) (4, 3) (d) (–4, –3)

51. A change-making machine contains one-rupee, two-rupee and five-rupee coins. The total number of coins is 300. The amount is Rs. 960. If the numbers of one-rupee coins and two-rupee coins are interchanged, the value comes down by Rs. 40. The total number of five-rupee coins is

(a) 100 (b) 140
(c) 60 (d) 150

2002

52. If x, y and z are real numbers such that $x + y + z = 5$ and $xy + yz + zx = 3$, what is the largest value that x can have?

(a) $\frac{5}{3}$ (b) $\sqrt{19}$
(c) $\frac{13}{3}$ (d) None of these

53. A rich merchant had collected many gold coins. He did not want anybody to know about him. One day, his wife asked, " How many gold coins do we have?" After a brief pause, he replied, "Well! if I divide the coins into two unequal numbers, then 48 times the difference between the two numbers equals the difference between the squares of the two numbers." The wife looked puzzled. Can you help the merchant's wife by finding out how many gold coins the merchant has?

(a) 96 (b) 53
(c) 43 (d) None of these

54. Shyam visited Ram during his brief vacation. In the mornings they both would go for yoga. In the evenings they would play tennis. To have more fun, they indulge only in one activity per day, i.e. either they went for yoga or played tennis each day. There were days when they were lazy and stayed home all day long. There were 24 mornings when they did nothing, 14 evenings when they stayed at home, and a total of 22 days when they did yoga or played tennis. For how many days Shyam stayed with Ram?

(a) 32 (b) 24
(c) 30 (d) None of these

55. If $x^2 + 5y^2 + z^2 = 2y(2x + z)$, then which of the following statements is(are)necessarily true?

A. x = 2y B. x = 2z C. 2x = z

(a) Only A (b) B and C
(c) A and B (d) None of these

56. The number of real roots of the equation $\frac{A^2}{x} + \frac{B^2}{x-1} = 1$, where A and B are real numbers not equal to zero simultaneously, is

(a) None (b) 1
(c) 2 (d) 1 or 2

57. A piece of string is 40 cm long. It is cut into three pieces. The longest piece is three times as long as the middle-sized and the shortest piece is 23 cm shorter than the longest piece. Find the length of the shortest piece.

(a) 27 (b) 5
(c) 4 (d) 9

Directions for Questions 58 and 59: Each item is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (d) if the question cannot be answered even by using both statements together.

58. Is x = y?

A. $(x + y)\left(\frac{1}{x} + \frac{1}{y}\right) = 4$

B. $(x - 50)^2 = (y - 50)^2$

59. Is |x – 2| < 1?

A. |x| < 1

B. |x – 1| < 2

2003(R)

60. If both a and b belong to the set {1, 2, 3, 4}, then the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots is

(a) 10 (b) 7
(c) 6 (d) 12

61. If x and y are integers, then the equation 5x + 19y = 64 has

(a) no solution for x < 300 and y < 0
(b) no solution for x > 250 and y > –100
(c) a solution for 250 < x < 300
(d) a solution for –59 < y < –56

62. The number of roots common between the two equations $x^3 + 3x^2 + 4x + 5 = 0$ and $x^3 + 2x^2 + 7x + 3 = 0$ is

(a) 0 (b) 1
(c) 2 (d) 3

63. A real number x satisfying $1 - \frac{1}{n} < x \le 3 + \frac{1}{n}$, for every positive integer n, is best described by

(a) $1 < x < 4$ (b) $1 < x \le 3$
(c) $0 < x \le 4$ (d) $1 \le x \le 3$

64. If n is such that $36 \le n \le 72$, then $x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4}$ satisfies

(a) 20 < x < 54 (b) 23 < x < 58
(c) 25 < x < 64 (d) 28 < x < 60

65. If 13x + 1 < 2z and $z + 3 = 5y^2$, then

(a) x is necessarily less than y
(b) x is necessarily greater than y
(c) x is necessarily equal to y
(d) None of the above is necessarily true

Directions for Question 66: The question is followed by two statements, A and B. Answer the question using the following instructions:

Choose (a) if the question can be answered by using statement A alone but not by using B alone.

Choose (b) if the question can be answered by using statement B alone but not by using A alone.

Choose (c) if the question can be answered by using either statement alone and

Choose (d) if the question can be answered using both the statements together but not by either statement alone.

66. A family has only one kid. The father says, "After 'n' years, my age will be 4 times the age of my kid." The mother says, "After 'n' years, my age will be 3 times that of my kid." What will be the combined ages of the parents after 'n' years?

A. The age difference between the parents is 10 years.

B. After 'n' years the kid is going to be twice as old as she is now.

2003(L)

67. The number of non-negative real roots of $2^x - x - 1 = 0$ equals

(a) 0 (b) 1
(c) 2 (d) 3

68. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p + q + r \neq 0$?

$$x + 2y - 3z = p$$
$$2x + 6y - 11z = q$$
$$x - 2y + 7z = r$$

(a) $5p - 2q - r = 0$ (b) $5p + 2q + r = 0$
(c) $5p + 2q - r = 0$ (d) $5p - 2q + r = 0$

69. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs. 20 on a standard bag and Rs. 30 on a deluxe bag. Every bag must be processed on machine A and on Machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours/bag)	
	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

(a) Standard 75 bags, Deluxe 80 bags
(b) Standard 100 bags, Deluxe 60 bags
(c) Standard 50 bags, Deluxe 100 bags
(d) Standard 60 bags, Deluxe 90 bags

70. A test has 50 questions. A student scores 1 mark for a correct answer, –1/3 for a wrong answer, and –1/6 for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by that student cannot be less than

(a) 6 (b) 12
(c) 3 (d) 9

71. Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

(a) 0 (b) 3
(c) 4 (d) 5

72. Let a, b, c, d be four integers such that a + b + c + d = 4m + 1 where m is a positive integer. Given m, which one of the following is necessarily true?

(a) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$
(b) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$
(c) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$
(d) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

73. Given that $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$ and $w = vz/u$, then which of the following is necessarily true?

(a) $-0.5 \le w \le 2$ (b) $-4 \le w \le 4$
(c) $-4 \le w \le 2$ (d) $-2 \le w \le -0.5$

74. If x, y, z are distinct positive real numbers the $\dfrac{x^2(y+z)+y^2(x+z)+z^2(x+y)}{xyz}$ would be

(a) greater than 4. (b) greater than 5.
(c) greater than 6 (d) None of the above.

Direction for Question 75: In the question there are two statements: A and B.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together but cannot be answered using either statement alone.

Choose (d) if the question cannot be answered even by using both the statements A and B.

75. Each packet of SOAP costs Rs. 10. Inside each packet is a gift coupon labelled with one of the letters S, O, A and P. If a customer submits four such coupons that make up the word SOAP, the customer gets a free SOAP packets. Ms. X kept buying packet after packet of SOAP till she could get one set of coupons that formed the word SOAP. How many coupons with label P did she get in the above process?

A. The last label obtained by her was S and the total amount spent was Rs. 210.

B. The total number of vowels obtained was 18.

Direction for Question 76: The question is followed by two statements, A and B. Answer the question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (d) if the question cannot be answered even by using both the statements together.

76. What are the unique values of b and c in the equation $4x^2 + bx + c = 0$ if one of the roots of the equation is (–1/2)?

A. The second root is 1/2.

B. The ratio of c and b is 1.

2004

77. Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$

What is the value of y?

(a) $\frac{\sqrt{11}+3}{2}$ (b) $\frac{\sqrt{11}-3}{2}$

(c) $\frac{\sqrt{15}+3}{2}$ (d) $\frac{\sqrt{15}-3}{2}$

78. Each family in a locality has at most two adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families, Then the minimum possible number of families in the locality is

(a) 4 (b) 5

(c) 2 (d) 3

79. The total number of integers pairs (x, y) satisfying the equation x + y = xy is

(a) 0 (b) 1

(c) 2 (d) None of the above

Directions for Questions 80 and 81: Answer the questions on the basis of the information given below.

In an examination, there are 100 questions divided into three groups A, B and C such that each group contains at least one question. Each question in group a carries 1 mark, each question in group B carries 2 marks and each question in group C carries 3 marks. It is known that the questions in group A together carry at least 60% of the total marks.

80. If group B contains 23 questions, then how many questions are there in Group C?

(a) 1

(b) 2

(c) 3

(d) Cannot be determined

81. If group C contains 8 questions and group B carries at least 20% of the total marks, which of the following best describes the number of questions in group B?

(a) 11 or 12 (b) 12 or 13

(c) 13 or 14 (d) 14 or 15

Directions for Questions 82 and 83: Each question is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a) if the question can be answered by using one of the statements alone but not by using the other statement alone.

Choose (b) if the question can be answered by using either of the statements alone.

Choose (c) if the question can be answered by using both statements together but not by either statement alone.

Choose (d) if the question cannot be answered on the basis of the two statements.

82. Ravi spent less than Rs. 75 to buy one kilogram each of potato, onion, and gourd. Which one of the three vegetables bought was the costliest?

A. 2 kgs potato and 1 kg gourd cost less than 1 kg potato and 2 kg gourd.

B. 1 kg potato and 2 kgs onion together cost the same as 1 kg onion and 2 kgs gourd.

83. Nandini paid for an article using currency notes of denominations Re. 1, Rs. 2, Rs. 5, and Rs. 10 using at least one note of each denomination. The total number of five and ten rupee notes used was one more than the total number of one and two rupee notes used. What was the price of the article?

A. Nandini used a total of 13 currency notes.

B. The price of the article was a multiple of Rs. 10.

2005

84. For which value of k does the following pair of equations yield a unique solution of x such that the solution is positive?

$x^2 - y^2 = 0$

$(x-k)^2 + y^2 = 1$

(a) 2 (b) 0

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

85. Let $x = \sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\ldots\text{to infinity}}}}}$. Then x equals

(a) 3 (b) $\left(\frac{\sqrt{13}-1}{2}\right)$

(c) $\left(\frac{\sqrt{13}+1}{2}\right)$ (d) $\sqrt{13}$

86. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs. 250 and Rs. 300 per day respectively. In addition, a male operator gets Rs. 15 per call he answers and female operator gets Rs. 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

(a) 15 (b) 14

(c) 12 (d) 10

2006

Directions for Questions 87 and 88: Answer questions on the basis of the information given below:

An airline has a certain free luggage allowance and charges for excess luggage at a fixed rate per kg. Two passengers, Raja and Praja have 60 kg of luggage between them, and are charged Rs. 1,200 and Rs. 2,400 respectively for excess luggage. Had the entire luggage belonged to one of them, the excess luggage charge would have been Rs. 5,400.

87. What is the weight of Praja's luggage?

(a) 20kg (b) 25 kg

(c) 30 kg (d) 35 kg

(e) 40 kg

88. What is the free luggage allowance?

(a) 10 kg (b) 15 kg

(c) 20 kg (d) 25 kg

(e) 30 kg

89. What values of x satisfy $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 \le 0$ ('x' is a real number)?

(a) $-8 \le x \le 1$ (b) $-1 \le x \le 8$

(c) $1 < x < 8$ (d) $1 \le x \le 8$

(e) $-8 \le x \le 8$

90. The number of solutions of the equation 2x + y = 40 where both x and y are positive integers and $x \le y$ is:

(a) 7 (b) 13

(c) 14 (d) 18

(e) 20

91. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed?

(a) 5 (b) 6

(c) 7 (d) 8

(e) 10

2007

92. Suppose you have a currency, named Miso, in three denominations: 1 Miso, 10 Misos and 50 Misos. In how many ways can you pay a bill of 107 Misos?

(a) 17 (b) 16

(c) 18 (d) 15

(e) 19

93. A confused bank teller transposed the rupees and paise when he cashed a cheque for Shailaja giving her rupees instead of paise and paise instead of rupees. After buying a toffee for 50 paise, Shailaja noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount?

(a) Over Rupees 13 but less than Rupees 14

(b) Over Rupees 7 but less than Rupees 8

(c) Over Rupees 22 but less than Rupees 23

(d) Over Rupees 18 but less than Rupees 19

(e) Over Rupees 4 but less than Rupees 5

94. How many pairs of positive integers m, n satisfy $\frac{1}{m}+\frac{4}{n}=\frac{1}{12}$, where, 'n' is an odd integer less than 60?

(a) 6 (b) 4

(c) 7 (d) 5

(e) 3

95. A quadratic function $f(x)$ attains a maximum of 3 at x = 1. The value of the function at x = 0 is 1. What is the value $f(x)$ at x = 10?

(a) –119 (b) –159

(c) –110 (d) –180

(e) –105

Direction for Question 96: The question is followed by two statements A and B. Indicate your response based on the following directives.

Mark (a) if the questions can be answered using A alone but not using B alone.

Mark (b) if the question can be answered using B alone but not using A alone.

Mark (c) if the question can be answered using A and B together, but not using either A or B alone.

Mark (d) if the question cannot be answered even using A and B together.

96. Consider integers x, y, z. What is the minimum possible value of $x^2 + y^2 + z^2$?

A: x + y + z = 89.

B: Among x, y, z two are equal.

2008

97. If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b?

(a) $-\frac{1}{\sqrt{3}}$ (b) –1

(c) 0 (d) 1

(e) $\frac{1}{\sqrt{3}}$

98. Three consecutive positive integers are raised to the first, second and third powers respectively and then added. The sum so obtained is perfect square whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers?

(a) $1 \le m \le 3$ (b) $4 \le m \le 6$

(c) $7 \le m \le 9$ (d) $10 \le m \le 12$

(e) $13 \le m \le 15$

99. A shop stores x kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x?

(a) $2 \le x \le 6$ (b) $5 \le x \le 8$

(c) $9 \le x \le 12$ (d) $11 \le x \le 14$

(e) $13 \le x \le 18$

MEMORY BASED QUESTIONS

2009

100. $x^2y^3 = 8$, where x, y > 0.

What is the minimum value of 4x + 3y?

(a) 8 (b) 11

(c) 9 (d) 10

101. If a, b are integers then how many ordered pairs (a, b) satisfy the equation $a^2 + ab + b^2 = 1$?

(a) 6 (b) 4

(c) 2 (d) 8

102. Champak takes a test called RAT which comprises 28 questions. In RAT three marks are awarded for each correct response, one mark is deducted for each incorrect response and there are no marks for unattempted questions. If he scores more than 22 marks in RAT, then what is the maximum possible number of incorrect responses that he could have marked?

(a) 14 (b) 15

(c) 16 (d) 17

103. If a, b, c are the roots of the equation

$$3x^3 + 42x + 93 = 0,$$

then what is the value of $a^3 + b^3 + c^3$?

(a) 81 (b) 92

(c) –85 (d) –93

2010

104. $P + \frac{1}{Q} = Q + \frac{1}{R} = 1$, where P, Q and R are real numbers. What is the value of $PQR + R + \frac{1}{P}$?

(a) –2 (b) –1

(c) 0 (d) Cannot be determined

105. Anant purchased x chocolates for Rs. y, where y is a natural number. The shopkeeper had offered to give him x+10 chocolates for Rs. 2. Anant declined the offer though it would have resulted in a saving of 80 paise per dozen chocolates for him. Which of the following can be the number of chocolates purchased by Anant?

(a) 1 (b) 3

(c) 5 (d) 15

106. If $n = \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$, where a, b and c are the three sides of a triangle, then which of the following best describes the range in which n lies?

(a) $\left[\frac{3}{2}, 2\right)$ (b) $\left(\frac{2}{3}, \frac{3}{2}\right)$

(c) $\left(1, \frac{5}{2}\right]$ (d) $\left(\frac{4}{3}, \frac{5}{3}\right)$

107. What is the equation of the straight line which passes through the point of intersection of the straight lines 3x + 4y – 11 = 0 and x + y – 3 = 0 and is parallel to the line 2x + 5y = 0?

(a) 5x – 2y – 12 = 0 (b) 2x + 5y – 12 = 0

(c) 2x + 5y – 9 = 0 (d) 5x + 2y – 9 = 0

2011

108. The route taken by a bus from Delhi to Jaipur has 'n' stops, including the source and the destination. When 'm' new stops are added on the route of the bus (where m > 1) the number of different tickets that can be issued between two stops on the route increases by 11. What is the value of 'n'?

(a) 6 (b) 4

(c) 7 (d) 5

2012

109. If $\frac{bx - ay}{bc} = \frac{ay - cz}{ac} = \frac{cz - bx}{ab}$, then, given that $(bx \neq ay \neq cz)$

(a) ab + bc + ca = 0
(b) ab – bc + ca = 0
(c) ab + bc – ca = 0
(d) ab – bc – ca = 0

110. There are five consecutive integers a, b, c, d and e such that a < b < c < d < e and $a^2 + b^2 + c^2 = d^2 + e^2$. What is/are the possible value(s) of b?

(a) 0
(b) 11
(c) 0 and –11
(d) –1 and 11.

111. $ax^2 + bx + c = 0$ is a quadratic equation with rational coefficients such that a + b + c = 0, then which of the following is necessarily true?

(a) Both the roots of this equation are less than 1.
(b) One of the roots of the equation is c.
(c) One of the roots of the equation is $\frac{c}{a}$.
(d) Exactly one of the roots is 1.

112. If x + y = 1, then what is the value of $(x^3 + y^3 + 3xy)$?

(a) 1 (b) 3
(c) 9 (d) –1

2013

113. Let $f(x) = ax^2 + bx + c$, where a, b and c are real numbers and $a \neq 0$. If f(x) attains its maximum value at x = 2, then what is the sum of the roots of f(x) = 0?

(a) 4 (b) –2
(c) 8 (d) –4

2014

114. A shopkeeper sells four qualities of rice A, B, C and D having cost price Rs. 40/kg, Rs. 55/kg, Rs. 50/kg and Rs. 65/kg respectively. Ankit purchased 'a' kg of A and 'b' kg of B to make 'a + b' kg of a new quality 'E' of rice worth Rs. 50/kg. Then he purchased 'c' kg of C and 'd' kg of D to make 'c + d' kg of a new quality 'F' of rice worth Rs. 60/kg.

Finally he took 'x' kg of rice and 'y' kg of rice from 'E' quality of rice and 'F' quality of rice respectively to make 'x + y' kg of rice worth Rs. 53/kg. Ensuring that a, b, c, d, x and y are all integers then what is the minimum value of a + b + c + d + x + y in kg?

(a) 28 (b) 22
(c) 16 (d) 26

115. What is the sum of the roots of all the quadratic equations that can be formed such that both the roots of the quadratic equation are common with the roots of equation (x – a)(x – b)(x – c) = 0?

(a) 3(a + b + c)
(b) 2(a + b + c)
(c) (a + b + c)
(d) 4(a + b + c)

116. If x(x – 3) = –1, then the value of $x^3(x^3 - 18)$ is

(a) 1 (b) 0
(c) –1 (d) 2

117. The sum of the coefficients of the polynomial $(x - 1)^9 (x - 2)^4 (x - 4)$ is

(a) 0 (b) 16
(c) –20 (d) None of these

2015

118. Both the roots of the quadratic equation $x^2 + rx + s = 0$ are real and greater than 1. If $R = \left(\frac{r + s + 1}{s - r}\right)$, then which of the following is definitely true?

(a) R = 0
(b) R < 0
(c) R > 0
(d) Cannot be determined

119. A certain sum of money is made up of Re. 1, 50 paise and 25 paise coins. The ratio of the number of these coins is 5 : 6 : 8. Then, 3/5th of the Re. 1 coins are changed to 50 paise and 25 paise coins, such that the ratio of the total number of these coins in the same order became 1: 2. Now, half of the 50 paise coins are changed to Re. 1 coins and all the 25 paise coins are changed to Re. 1 and 50 paise coins in the ratio 7: 4. What is the ratio of the Re. 1 and 50 paise coins at the end of the conversions? (Note:- If you change a Re.1 coin into 50 paise coins, then you will get two coins of 50 paise for a Re. 1 coin.)

(a) 11 : 23 (b) 16 : 13
(c) 54 : 71 (d) None of these

120. Let x, y, z and t be the positive numbers which satisfy the following conditions:

I. If x > y, then z > t and
II. If x > z, then y < t

Which of the following is necessarily true?

(a) If x < y, then z < t
(b) If x > z then x – y < z + t
(c) If x > y + z, then z > y
(d) None of these

2016

121. If 'a' and 'b' are the roots of the equation $x^2 + 7x + 4 = 0$, where a > b, then find the value of

$$\left(\frac{2}{7a}\right)+\left(\frac{1}{3.5b}\right)+ab.$$

(a) $\frac{7}{2}$ (b) $\frac{14}{3}$

(c) $\frac{-7}{2}$ (d) $\frac{2}{7}$

122. If $a^2 + 6a - 1 = 0$, then what is the value of $\left(a+\frac{1}{a}\right)^2 - 5\left(a-\frac{1}{a}\right)$?

123. Find the minimum possible value of 'y' if $\frac{4}{7} < \frac{x}{y} < \frac{12}{13}$, where x and y are natural numbers.

(a) 4 (b) 5

(c) 3 (d) 2

124. If $f(x) = \max(x^2 - 4, x, -1)$, then what is the minimum value of f(x)?

(a) – 4 (b) $\frac{1-\sqrt{17}}{2}$

(c) –1 (d) 0

125. $x+\frac{1}{y+\frac{1}{z}}=\frac{68}{21}$, where x, y and z are natural numbers. Find the value of x + y + z.

(a) 9 (b) 12

(c) 11 (d) 14

2017

126. The minimum possible value of the sum of the squares of the roots of the equation

$$x^2 + (a + 3)x - (a + 5) = 0 \text{ is}$$

(a) 1 (b) 2

(c) 3 (d) 4

127. If $9^{x-\frac{1}{2}} - 2^{2x-2} = 4^x - 3^{2x-3}$, then X is

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{3}{4}$ (d) $\frac{4}{9}$

2018 Slot 1

128. If $u^2 + (u - 2v - 1)^2 = -4v(u + v)$, then what is the value of u + 3v?

(a) –1/4 (b) 1/4 (c) 1/2 (d) 0

2018 Slot 2

129. The smallest integer n such that $n^3 - 11n^2 + 32n - 28 > 0$ is

130. If N and x are positive integers such that $N^N = 2^{160}$ and $N^2 + 2^N$ is an integral multiple of 2^x, then the largest possible x is

131. If $A = \{6^{2n} - 35n - 1 : n = 1, 2, 3,...\}$ and $B = \{35(n - 1) : n = 1, 2, 3,...\}$ then which of the following is true?

(a) Every member of A is in B and at least one member of B is not in A

(b) At least one member of A is not in B

(c) Neither every member of A is in B nor every member of B is in A

(d) Every member of B is in A.

132. If a and b are integers such that $2x^2 - ax + 2 > 0$ and $x^2 - bx + 8 \geq 0$ for all real numbers x, then the largest possible value of 2a – 6b is

133. Let $t_1, t_2,\ldots$ be real numbers such that $t_1 + t_2 + \ldots + t_n = 2n^2 + 9n + 13$, for every positive integer $n \geq 2$. If $t_k = 103$, then k equals

FUNCTIONS AND GRAPHS

1990

1. From any two numbers x and y, we define x × y = x + 0.5y – xy. Suppose that both x and y are greater than 0.5. Then x × x < y × y if

(a) 1 > x > y (b) x > 1 > y

(c) 1 > y > x (d) y > 1 > x

2. Consider a function f(k)defined for positive integers k = 1, 2, …. ; the function satisfies the condition

$$f(1) + f(2) + \ldots = \frac{p}{(p-1)}.$$

Where p is fraction i.e. 0 < p < 1.

Then f(k) is given by

(a) $p(-p)^{k-1}$ (b) $p(1-p)^{k-1}$

(c) $\{p(1-p)\}^{k-1}$ (d) None of these

1991

3. A function can sometimes reflect on itself, i.e. if y = f(x), then x = f(y). Both of them retain the same structure and form. Which of the following functions has this property?

(a) $y=\frac{2x+3}{3x+4}$ (b) $y=\frac{2x+3}{3x-2}$

(c) $y=\frac{3x+4}{4x-5}$ (d) None of the above.

4. If y = f(x) and f(x) = (1–x)/ (1 + x), which of the following is true?

(a) f(2x) = f(x) – 1 (b) x = f(2y) – 1

(c) f(1/x) = f(x) (d) x = f(y)

5. Let Y = minimum of {(x + 2), (3 – x)}. What is the maximum value of Y for $0 \leq x \leq 1$?

(a) 1.0 (b) 1.5

(c) 3.1 (d) 2.5

1993

Directions for Questions 6 and 7: Answer the questions based on the following information.

A function f(x)is said to be even if f(–x)= f(x), and odd if f(–x) = –f(x). Thus, for example, the function given by $f(x) = x^2$ is even, while the function given by $f(x) = x^3$ is odd. Using this definition, answer the following questions.

6. The function given by $f(x) = |x|^3$ is

(a) even (b) odd
(c) neither (d) both

7. The sum of two odd functions

(a) is always an even function
(b) is always an odd function
(c) is sometimes odd and sometimes even
(d) may be neither odd nor even

8. The maximum possible value of y = min ($1/2 - 3x^2/4$, $5x^2/4$)for the range 0 < x < 1 is

(a) 1/3 (b) 1/2
(c) 5/27 (d) 5/16

1994

Directions for Questions 9 and 10: Answer the questions based on the following information.

If $md(x) = |x|$,

$mn(x, y) =$ minimum of *x* and *y* and

$Ma(a, b, c, ...) =$ maximum of *a, b, c, ...*

9. Value of

$Ma\left[md(a), mn(md(b), a), mn(ab, md(ac))\right]$ where *a* = –2, *b* = –3, *c* = 4 is

(a) 2 (b) 6
(c) 8 (d) –2

10. Given that a > b then the relation

Ma[md(a),mn(a, b)] = mn[a, md(Ma(a, b))] does not hold if

(a) a < 0, b < 0
(b) a > 0, b > 0
(c) a > 0, b < 0, $|a| < |b|$
(d) a > 0, b < 0, $|a| > |b|$

Directions for Questions 11 to 14: Answer the questions based on the following information.

If $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$, then

11. *fog*(x) is equal to

(a) 1 (b) gof(x)
(c) $\frac{15x+9}{16x-5}$ (d) $\frac{1}{x}$

12. For what value of *x*; $f(x) = g(x-3)$?

(a) –3 (b) $\frac{1}{4}$
(c) –4 (d) None of these

13. What is the value of (gofofogogof) (x) (fogofog)(x)?

(a) x (b) x^2
(c) $\frac{5x+3}{4x-1}$ (d) $\frac{(x+3)(5x+3)}{(4x-5)(4x-1)}$

14. What is the value of fo(fog)o(gof)(x)?

(a) x (b) x^2
(c) 2x + 3 (d) $\frac{x+3}{4x-5}$

1995

15. Largest value of min($2 + x^2$, 6 – 3x), when x > 0, is

(a) 1 (b) 2
(c) 3 (d) 4

Directions for Questions 16 to 19: Answer the questions based on the following information.

le(x, y) = Least of (x, y)

mo(x) = |x|

me(x, y) = Maximum of (x, y)

16. Find the value of me(a + mo(le(a, b)), mo(a + me(mo(a), mo(b))), at a = –2 and b = –3.

(a) 1 (b) 0
(c) 5 (d) 3

17. Which of the following must always be correct for a, b > 0?

(a) mo(le(a, b))≥ (me(mo(a), mo(b))
(b) mo(le(a, b))> (me(mo(a), mo(b))
(c) mo(le(a, b))< (le(mo(a), mo(b))
(d) mo(le(a,b))= le(mo(a), mo(b))

18. For what values of 'a' is me(a^2 – 3a, a – 3) < 0?

(a) a > 3 (b) 0 < a < 3
(c) a < 0 (d) a = 3

19. For what values of 'a' is le(a^2 – 3a, a – 3) < 0?

(a) a > 3 (b) 0 < a < 3
(c) a < 0 (d) Both b and c

1996

Directions for Questions 20 and 21: Answer the questions based on the following information.

A, S, M and D are functions of x and y, and they are defined as follows.

A(x, y) = x + y

S(x, y) = x – y

M(x, y) = xy

$D(x, y) = \frac{x}{y}$, $y \neq 0$

20. What is the value of M(M(A(M(x, y), S(y, x)), x), A(y, x)) for x = 2, y = 3?

(a) 60 (b) 140

(c) 25 (d) 70

21. What is the value of S[M(D(A(a, b), 2), D(A(a, b), 2)), M(D(S(a, b), 2), D(S(a, b), 2))]?

(a) $a^2 + b^2$ (b) ab

(c) $a^2 - b^2$ (d) $\frac{a}{b}$

1997

Directions for Questions 22 to 24: Answer the questions based on the following information.

For these questions the following functions have been defined. $la(x, y, z) = \min(x + y, y + z)$

$$le(x, y, z) = \max(x - y, y - z)$$

$$ma(x, y, z) = \frac{1}{2}[le(x, y, z) + la(x, y, z)]$$

22. Given that $x > y > z > 0$. Which of the following is necessarily true?

(a) $la(x, y, z) < le(x, y, z)$

(b) $ma(x, y, z) < la(x, y, z)$

(c) $ma(x, y, z) < le(x, y, z)$

(d) None of these

23. What is the value of ma(10,4, le(la(10, 5, 3), 5, 3))?

(a) 7 (b) 6.5

(c) 8 (d) 7.5

24. For $x = 15$, $y = 10$ and $z = 9$, find the value of $le(x, \min(y, x - z), le(9, 8, ma(x, y, z)))$.

(a) 5 (b) 12

(c) 9 (d) 4

1998

Directions for Questions 25 to 27: Answer the questions based on the following information.

The following operations are defined for real numbers.

a # b = a + b, if a and b both are positive else a # b = 1

a ∇ b = $(a \times b)^{a+b}$ if a × b is positive else a ∇ b = 1.

25. $\frac{(2\#1)}{(1\nabla 2)} =$

(a) $\frac{1}{8}$ (b) 1

(c) $\frac{3}{8}$ (d) 3

26. $\frac{\{((1\#1)\#2) - (10^{1.3}\nabla \log_{10} 0.1)\}}{(1\nabla 2)} =$

(a) $\frac{3}{8}$ (b) $\frac{4.\log_{10} 0.1}{8}$

(c) $\frac{(4+10^{13})}{8}$ (d) None of these

27. $\left(\frac{(X\#-Y)}{(-X\nabla Y)}\right) = \frac{3}{8}$, then which of the following must be true?

(a) X = 2, Y = 1 (b) X > 0, Y< 0

(c) X, Y both positive (d) X, Y both negative

1999

28. If $|r - 6| = 11$ and $|2q - 12| = 8$, what is the minimum possible value of $\frac{q}{r}$?

(a) $-\frac{2}{5}$ (b) $\frac{2}{17}$

(c) $\frac{10}{17}$ (d) None of these

Directions for Questions 29 to 31: Answer the questions based on the following information.

Let x and y be real numbers and let $f(x,y) = |x + y|$, $F(f(x,y)) = -f(x,y)$ and $G(f(x, y)) = -F(f(x, y))$

29. Which of the following statements is true?

(a) F(f(x, y)) · G(f(x, y))= –F(f(x, y)) · G(f(x, y))

(b) F(f(x, y)) · G(f(x, y))> –F(f(x, y)) · G(f(x, y))

(c) F(f(x, y)) · G(f(x, y)) ≠ G(f(x, y)) · G(f(x, y))

(d) F(f(x, y)) + G(f(x, y))+ f(x, y) = f(–x, –y)

30. What is the value of f(G(f(1, 0)), f(F(f(1, 2)), G(f(1, 2))))?

(a) 3 (b) 2

(c) 1 (d) 0

31. Which of the following expressions yields x^2 as its result?

(a) $F(f(x, -x)) \cdot G(f(x, -x))$

(b) $F(f(x, x)) \cdot G(f(x, x)) \cdot 4$

(c) $-F(f(x, x)) \cdot G(f(x, x)) \div \log_2 16$

(d) $f(x, x) \cdot f(x, x)$

Directions for Questions 32 to 35: Answer the questions based on the following information.

In each of the following questions, a pair of graphs F(x) and F1(x) is given. These are composed of straight-line segments, shown as solid lines, in the domain $x \in (-2, 2)$.

Choose the answer as

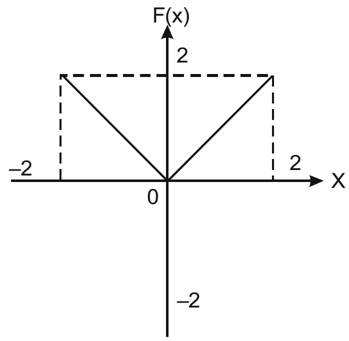
(a) if F1(x) = –F(x)

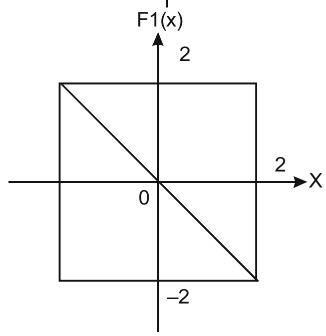
(b) if F1(x) = F(–x)

(c) if F1(x) = –F(–x)

(d) if none of the above is true

32.





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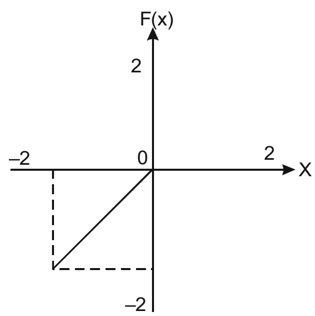

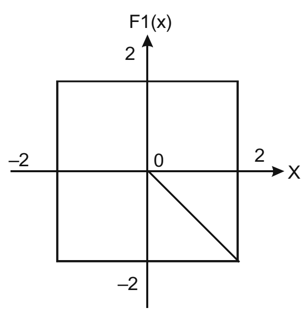

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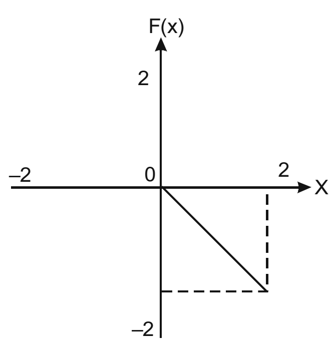

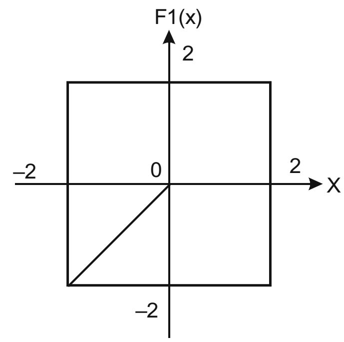

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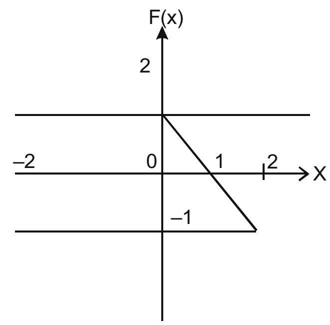
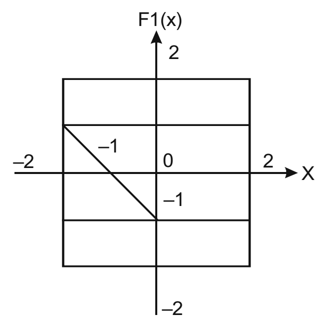




2000

Directions for Questions 36 and 37: Answer the questions based on the following information.

For real numbers x and y, let

$$f(x, y) = \begin{cases} \text{Positive square root of } (x + y), & \text{if } (x + y)^{0.5} \text{ is real} \\ (x + y)^2, & \text{otherwise} \end{cases}$$

$$g(x, y) = \begin{cases} (x + y)^2, & \text{if } (x + y)^{0.5} \text{ is real} \\ -(x + y), & \text{otherwise} \end{cases}$$

36. Which of the following expressions yields a positive value for every pair of non-zero real numbers (x, y)?

(a) f(x, y) – g(x, y) (b) f(x, y) – $(g(x, y))^2$

(c) g(x, y) – $(f(x, y))^2$ (d) f(x, y) + g(x, y)

37. Under which of the following conditions is f(x, y) necessarily greater than g(x, y)?

(a) Both x and y are less than –1

(b) Both x and y are positive

(c) Both x and y are negative

(d) y > x

Directions for Questions 38 to 40: Answer the questions based on the following information.

For three distinct real numbers x, y and z, let

f(x, y, z)= Min(Max(x, y), Max(y, z), Max(z, x))

g(x, y, z)= Max(Min(x, y), Min(y, z), Min(z, x))

h(x, y, z)= Max(Max(x, y), Max(y, z), Max(z, x))

j(x, y, z)= Min(Min(x, y), Min(y, z), Min(z, x))

m(x, y, z) = Max(x, y, z)

n(x, y, z) = Min(x, y, z)

38. Which of the following is necessarily greater than 1?
(a) [h(x, y, z)– f(x, y, z)] / j(x, y, z)
(b) j(x, y, z) / h(x, y, z)
(c) f(x, y, z) / g(x, y, z)
(d) [f(x, y, z) + h(x, y, z)– g(x, y, z)] / j(x, y, z)

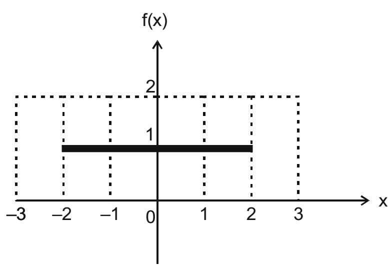
39. Which of the following expressions is necessarily equal to 1?
(a) [f(x, y, z)– m(x, y, z)] / [g(x, y, z) – h(x, y, z)]
(b) [m(x, y, z)– f(x, y, z)] / [g(x, y, z) – n(x, y, z)]
(c) [j(x, y, z)– g(x, y, z)] / h(x, y, z)
(d) [f(x, y, z)– h(x, y ,z)] / f(x, y, z)

40. Which of the following expressions is indeterminate?
(a) [f(x, y, z)– h(x, y, z)] / [g(x, y, z) – j(x, y, z)]
(b) [f(x, y, z) + h(x, y, z)+ g(x, y, z) + j(x, y, z)] / [j(x, y, z) + h(x, y, z)– m(x, y, z) – n(x, y, z)]
(c) [g(x, y, z)– j(x, y, z)] / [f(x, y, z) – h(x, y, z)]
(d) [h(x, y, z)– f(x, y, z)] / [n(x, y, z) – g(x, y, z)

Directions for Questions 41 to 43: Given below are three graphs made up of straight line segments shown as thick lines. In each case choose the answer as

(a) if f(x) = 3 f(–x) (b) if f(x) = –f(–x)
(c) if f(x) = f(–x) (d) if 3f(x) = 6 f(–x), for $x \geq 0$

41.



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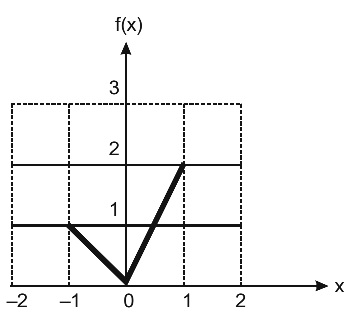

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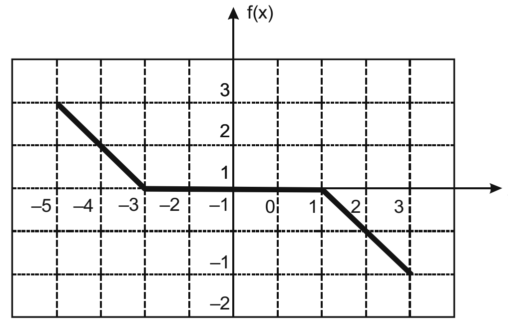


Directions for Questions 44 and 45: Answer the questions based on the following information.

For a real number x, let

$f(x) = \frac{1}{1+x}$, if x is non-negative

$= 1 + x$ if x is negative

$f^n(x)= f(f^{n-1}(x))$, n = 2, 3, ...

44. What is the value of the product $f(2)\, f^2(2)\, f^3(2)\, f^4(2)\, f^5(2)$?
(a) $\frac{1}{3}$ (b) 3
(c) $\frac{1}{18}$ (d) None of these

45. r is an integer ≥ 2. Then what is the value of $f^{r-1}(-r) + f^r(-r) + f^{r+1}(-r)$?
(a) –1 (b) 0
(c) 1 (d) None of these

46. If $x^2 + y^2 = 0.1$ and $|x – y| = 0.2$, then $|x| + |y|$ is equal to
(a) 0.3 (b) 0.4
(c) 0.2 (d) 0.6

47. The area bounded by the three curves |x + y| = 1, |x| = 1, and |y| = 1, is equal to
(a) 4 (b) 3
(c) 2 (d) 1

48. The set of all positive integers is the union of two disjoint subsets:
{f(1), f(2), ..., f(n), ...} and {g(1), g(2), ..., g(n), ...}, where
f(1) < f(2) < ... < f(n)..., and g(1) < g(2) < ... < g(n) ..., and
g(n) = f(f(n)) + 1 for all $n \geq 1$.
What is the value of g(1)?
(a) 0
(b) 2
(c) 1
(d) Cannot be determined

49. For all non-negative integers x and y, f(x, y) is defined as below.

$f(0, y) = y + 1$

$f(x + 1, 0) = f(x, 1)$

$f(x + 1, y + 1)= f(x, f(x + 1, y))$

Then what is the value of f(1, 2)?

(a) 2 (b) 4

(c) 3 (d) Cannot be determined

Direction for Question 50: The question is followed by two statements, I and II. Answer the question using the following instructions.

Mark the answer as:

(a) if the question can be answered by one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

50. For any two real numbers:

$a \oplus b = 1$ if both a and b are positive or both a and b are negative.

$= -1$ if one of the two numbers a and b is positive and the other negative.

What is $(2 \oplus 0) \oplus (-5 \oplus -6)$?

I. $a \oplus b$ is zero if a is zero

II. $a \oplus b = b \oplus a$

2001

51. If a, b, c and d are four positive real numbers such that abcd = 1, what is the minimum value of (1 + a)(1 + b)(1 + c)(1 + d)?

(a) 4 (b) 1

(c) 16 (d) 18

52. Let x and y be two positive numbers such that x + y = 1.

Then the minimum value of $\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$ is

(a) 12 (b) 20

(c) 12.5 (d) 13.3

Directions for Questions 53 and 54: Answer the questions based on the following information.

The batting average (BA)of a Test batsman is computed from runs scored and innings played — completed innings and incomplete innings (not out)in the following manner:

r_1 = Number of runs scored in completed innings

n_1 = Number of completed innings

r_2 = Number of runs scored in incomplete innings

n_2 = Number of incomplete innings $BA = \frac{r_1 + r_2}{n_1}$

To better assess a batsman's accomplishments, the ICC is considering two other measures MBA_1 and MBA_2 defined as follows:

$$MBA_1 = \frac{r_1}{n_1} + \frac{n_2}{n_1}\max\left[0, \left(\frac{r_2}{n_2} - \frac{r_1}{n_1}\right)\right]$$

$$MBA_2 = \frac{r_1 + r_2}{n_1 + n_2}$$

53. Based on the above information which of the following is true?

(a) $MBA_1 \le BA \le MBA_2$

(b) $BA \le MBA_2 \le MBA_1$

(c) $MBA_2 \le BA \le MBA_1$

(d) None of these

54. An experienced cricketer with no incomplete innings has BA of 50. The next time he bats, the innings is incomplete and he scores 45 runs. It can be inferred that

(a) BA and MBA_1 will both increase

(b) BA will increase and MBA_2 will decrease

(c) BA will increase and not enough data is available to assess change in MBA_1 and MBA_2

(d) None of these

2002

55. If $f(x) = \log\left\{\frac{(1+x)}{(1-x)}\right\}$, then f(x) + f(y) is

(a) f(x + y) (b) $f\left\{\frac{(x+y)}{(1+xy)}\right\}$

(c) $(x+y)f\left\{\frac{1}{(1+xy)}\right\}$ (d) $\frac{f(x)+f(y)}{(1+xy)}$

56. Suppose for any real number x, [x] denotes the greatest integer less than or equal to x. Let L(x, y) = [x] + [y] + [x + y] and R(x, y)= [2x] + [2y]. Then it is impossible to find any two positive real numbers x and y for which

(a) L(x, y) = R(x, y) (b) L(x, y) ≠ R(x, y)

(c) L(x, y) < R(x, y) (d) L(x, y) > R(x, y)

57. Davji Shop sells samosas in boxes of different sizes. The samosas are priced at Rs. 2 per samosa up to 200 samosas. For every additional 20 samosas, the price of the whole lot goes down by 10 paise per samosa. What should be the maximum size of the box that would maximise the revenue?

(a) 240 (b) 300

(c) 400 (d) None of these

2003(R)

58. If three positive real numbers x, y and z satisfy $y - x = z - y$ and $x\,y\,z = 4$, then what is the minimum possible value of y?

(a) $2^{1/3}$ (b) $2^{2/3}$

(c) $2^{1/4}$ (d) $2^{3/4}$

59. If $|b| \ge 1$ and $x = -|a|\,b$, then which one of the following is necessarily true?

(a) $a - xb < 0$ (b) $a - xb \ge 0$

(c) $a - xb > 0$ (d) $a - xb \le 0$

Directions for Questions 60 to 62: Answer the questions on the basis of the tables given below.

Two binary operations $\oplus$ and $*$ are defined over the set {a, e, f, g, h} as per the following tables:

$\oplus$	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

*	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	f	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first table $f \oplus g = a$, while according to the second table $g * h = f$, and so on. Also, let $f^2 = f * f$, $g^3 = g * g * g$, and so on.

60. What is the smallest positive integer n such that $g^n = e$?

(a) 4 (b) 5

(c) 2 (d) 3

61. Upon simplification, $f \oplus [f * \{f \oplus (f * f)\}]$ equals

(a) e (b) f

(c) g (d) h

62. Upon simplification, $\{a^{10} * (f^{10} \oplus g^9)\} \oplus e^8$ equals

(a) e (b) f

(c) g (d) h

2003(L)

63. Let g(x)= max(5 – x, x + 2). The smallest possible value of g(x) is

(a) 4.0

(b) 4.5

(c) 1.5

(d) None of the above

64. The function f(x)= |x – 2| + |2.5 – x| + |3.6 – x|, where x is a real number, attains a minimum at

(a) x = 2.3

(b) x = 2.5

(c) x = 2.7

(d) None of the above

65. Consider the following two curves in the x-y plane:

$$y = x^3 + x^2 + 5$$
$$y = x^2 + x + 5$$

Which of following statements is true for $-2 \le x \le 2$?

(a) The two curves intersect once.

(b) The two curves intersect twice.

(c) The two curves do not intersect

(d) The two curves intersect thrice.

2004

66. If f(x) = $x^3 - 4x + p$, and f(0) and f(1)are of opposite sings, then which of the following is necessarily true

(a) $-1 < p < 2$ (b) $0 < p < 3$

(c) $-2 < p < 1$ (d) $-3 < p < 0$

67. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at x = 0, f(x) is

(a) maximized whenever a > 0, b > 0

(b) maximized whenever a > 0, b < 0

(c) minimized whenever a > 0, b > 0

(d) minimized whenever a > 0, b < 0

Directions for Questions 68 and 69: Answer the questions on the basis of the information given below.

$f_1(x) = x$	$0 \le x \le 1$
$= 1$	$x \ge 1$
$= 0$	Otherwise
$f_2(x) = f_1(-x)$	for all x
$f_3(x) = -f_2(x)$	for all x
$f_4(x) = f_3(-x)$	for all x

68. How many of the following products are necessarily zero for every x?

$f_1(x)f_2(x)$, $f_2(x)f_3(x)$, $f_2(x)f_4(x)$

(a) 0 (b) 1

(c) 2 (d) 3

69. Which of the following is necessarily true?

(a) $f_4(x) = f_1(x)$ for all x

(b) $f_1(x) = -f_3(-x)$ for all x

(c) $f_2(-x) = f_4(x)$ for all x

(d) $f_1(x) = f_3(x) = 0$ for all x

2005

70. If $R = \dfrac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then

(a) $0 < R \le 0.1$ (b) $0.1 < R \le 0.5$

(c) $0.5 < R \le 1.0$ (d) $R > 1.0$

71. In the X-Y plane, the area of the region bounded by the graph $|x + y| + |x - y| = 4$ is

(a) 8 (b) 12

(c) 16 (d) 20

72. Let g(x)be a function such that g(x + 1) + g(x – 1) = g(x)for every real x. Then for what value of p is the relation g(x + p) = g(x)necessarily true for every real x?

(a) 5 (b) 3

(c) 2 (d) 6

2006

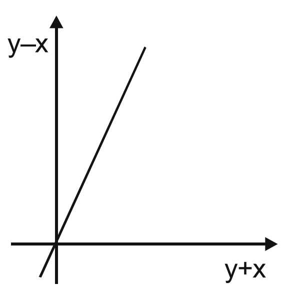
73. If x = – 0.5, then which of the following has the smallest value?

(a) $2^{\frac{1}{x}}$ (b) $\dfrac{1}{x}$

(c) $\dfrac{1}{x^2}$ (d) 2^x

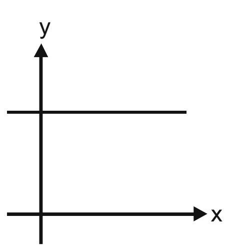
(e) $\dfrac{1}{\sqrt{-x}}$

74. The graph of y – x against y + x is as shown below. (All graphs in this question are drawn to scale and the same scale has been used on each axis.)

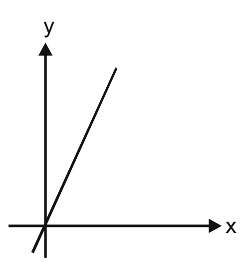


Which of the following shows the graph of y against x?

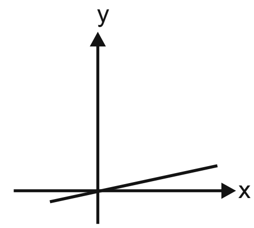
(a)



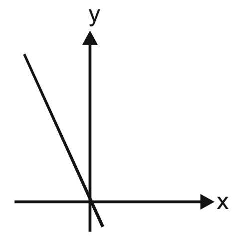
(b)



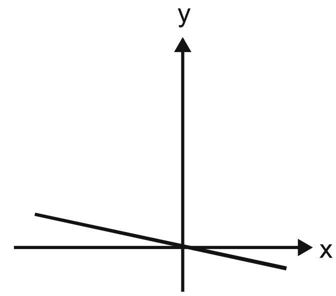
(c)



(d)



(e)



75. Let f(x)= max(2x + 1, 3 – 4x), where x is any real number. Then the minimum possible value of f(x) is:

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{2}{3}$ (d) $\dfrac{4}{3}$

(e) $\dfrac{5}{3}$

2007

76. A function $f(x)$ satisfies $f(1) = 3600$ and $f(1) + f(2) + \ldots + f(n) = n^2 f(n)$, for all positive integers $n > 1$. What is the value of $f(9)$?

(a) 80 (b) 240

(c) 200 (d) 100

(e) 120

Directions for Questions 77 and 78: Answer the following questions based on the information given below:

Mr. David manufactures and sells a single product at a fixed price in a niche market. The selling price of each unit is Rs. 30. On the other hand, the cost, in rupees, of producing 'x' units is $240 + bx + cx^2$, where 'b' and 'c' are some constants. Mr. David noticed that doubling the daily production from 20 to 40 units increases the daily production cost by $66\frac{2}{3}\%$. However, an increase in daily production from 40 to 60 units results in an increase of only 50% in the daily production cost. Assume that demand is unlimited and that Mr. David can sell as much as he can produce. His objective is to maximize the profit.

77. How many units should Mr. David produce daily?

(a) 130 (b) 100

(c) 70 (d) 150

(e) Cannot be determined

78. What is the maximum daily profit, in rupees, that Mr. David can realize from his business?

(a) 620 (b) 920
(c) 840 (d) 760
(e) Cannot be determined

2008

Directions for Questions 79 and 80:

Let $f(x) = ax^2 + bx + c$, where a, b and c are certain constants and $a \neq 0$ It is known that $f(5) = -3f(2)$ and that 3 is a root of $f(x) = 0$.

79. What is the other root of $f(x) = 0$?

(a) –7 (b) – 4
(c) 2 (d) 6
(e) cannot be determined

80. What is the value of a + b + c?

(a) 9 (b) 14
(c) 13 (d) 37
(e) cannot be determined

81. Let f(x)be a function satisfying $f(x)f(y) = f(xy)$ for all real x, y. If f(2)= 4, then what is the value of $f\left(\frac{1}{2}\right)$?

(a) 0 (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 1
(e) cannot be determined

MEMORY BASED QUESTIONS

2009

82.

$$f(m, n) = \begin{cases} n + 1 \text{ if } m = 0 \\ f(m-1, 1) \text{ if } m > 0, n = 0 \\ f(0, f(m, n-1)) \text{ if } m > 0, n > 0 \end{cases}$$

What is the value of 20[f(1, 2) + f(1, 1) + 15]?

(a) 440 (b) 480
(c) 520 (d) 560

2010

83. A function f(x) is defined for all real values of x as $f(x) = \frac{x-1}{x+1}$. If $y_1 = f(x)$, $y_2 = f(f(x))$, $y_3 = f(f(f(x)))$ and so on, then what is the value of y_{501}?

(a) $\frac{-1}{x}$ (b) $\frac{x+1}{x-1}$
(c) $501x - 1$ (d) $\frac{x-1}{x+1}$

2011

84. A function f(x) is defined for all real values of x as $f(x) = ax^2 + bx + c$. If $f(1) = f(-1)$, $f(0) = 10$ and $f(2) = 14$, then what is the value of f(10)?

(a) 100
(b) 110
(c) 64
(d) None of these

85. Two operations, for real numbers x and y, are defined as given below.

(i) $x \square y = (x + y)^2$

(ii) $x \Delta y = (x - y)^2$

If $x^2 \square y^2 = 169$ and $x^2 \Delta y^2 = 25$, then what is the value of x^2y^2?

(a) 81 (b) 36
(c) 64 (d) None of these

2012

86. A function f(x)is defined for real values of x as:

$$f(x) = \frac{1}{\log_{5-|x|}\sqrt{x^3 - 7x^2 + 14x - 8}}$$

What is the domain of f(x)?

(a) $x \in (0, \infty)$
(b) $x \in (-5, -4) \cup (-4, 4) \cup (4, 5)$
(c) $x \in (1, 2) \cup (4, 5)$
(d) $x \in (1, 2) \cup (4, \infty)$

87. A function F(n)is defined as $F(n-1) = \frac{1}{(2 - F(n))}$ for all natural numbers 'n'. If F(1) = 2, then what is the value of [F(1)] + [F(2)] +………..+ [F(50)]? (Here, [x] is equal to the greatest integer less than or equal to 'x')

(a) 51 (b) 55
(c) 54 (d) None of these

2013

88. If $f(x) = (\sec x + \operatorname{cosec} x)(\tan x - \cot x)$ and $\frac{\pi}{4} < x < \frac{\pi}{2}$, then f(x) lies in the range of

(a) $[0, \infty)$ (b) $(0, \infty)$
(c) $(-\infty, 0]$ (d) None of these

89. At how many points do the graphs of $y = \frac{1}{x}$ and $y = x^2 - 4$ intersect each other?

(a) 0 (b) 1
(c) 2 (d) 3

2014

90. If $f(n) = 1^4 + 2^4 + 3^4 + \dots + n^4$, then how can $1^4 + 3^4 + 5^4 + \dots + (2n-1)^4$ be expressed?

(a) $f(2n-1) - 16 \times f(n)$ (b) $f(2n-1) - 8 \times f(n)$

(c) $f(2n) - 16 \times f(n)$ (d) $f(2n) - 8 \times f(n)$

91. Find the range of 'x' if $\frac{1}{|x|-2} < 0.5$

(a) $x < -4$

(b) $(x > 4) \cup (x < -4)$

(c) $(x < -4) \cup (-2 < x < 2) \cup (x > 4)$

(d) $-2 < x < 2$

92. 'f' is a real function such that $f(x + y) = f(xy)$ for all real values of x and y. If $f(-7) = 7$, then the value of $f(-49) + f(49)$ is

(a) 7 (b) 14

(c) 0 (d) 49

93. Find the domain of the function

$$f(x) = \frac{3}{9-x^2} + \log_{10}(x^3 - x).$$

(a) (–1, 0) U (1, 3)

(b) (–1, 0) U (1, 3) U (3, ∞)

(c) (–3, 0) U (3, ∞)

(d) (–∞,–3) U (–3, 3) U (3, ∞)

94. A ray of light along the line $\sqrt{3}x + y = \sqrt{3}$ gets reflected on the x-axis to become a ray along the line

(a) $y = x + \sqrt{3}$ (b) $\sqrt{3}y = x - 1$

(c) $y = \sqrt{3}x - \sqrt{3}$ (d) $\sqrt{3}y = x - \sqrt{3}$

2015

95. Find the solution set for [x] + [2x] + [3x] = 8, where x is a real number and [x] is the greatest integer less than or equal to x.

(a) $1 \le x < \frac{4}{3}$ (b) $x < \frac{5}{3}$

(c) $\frac{3}{2} \le x < \frac{5}{3}$ (d) None of these

2016

96. A function f(x) is defined as $(x + 1) \times f(x + 1) + x \times f(x) + (x - 1) \times f(x - 1) = 0$ for $x \ge 2$. If f(a) = 40 and f(6) = 180, find the value of f(14).

(a) –80 (b) –160

(c) –1120 (d) Cannot be determined

2017

97. Let $f(x) = x^2$ and $g(x) = 2^x$, for all real x. Then the value of f(f(g(x)) + g(f(x))) at x = 1 is

(a) 16 (b) 18 (c) 36 (d) 40

98. If f(ab) = f(a)f(b) for all positive integers a and b, then the largest possible value of f(a) is

99. Let f(x) = 2x – 5 and g(x) = 7 – 2x. Then |f(x) + g(x)| = |f(x)| + |g(x)| if and only if

(a) $\frac{5}{2} < x < \frac{7}{2}$ (b) $x \le \frac{5}{2}$ or $x \ge \frac{7}{2}$

(c) $x < \frac{5}{2}$ or $\ge \frac{7}{2}$ (d) $\frac{5}{2} \le x \le \frac{7}{2}$

2018 Slot 1

100. If f(x + 2) = f(x) + f(x + 1) for all positive integers x, and f(11) = 91, f(15) = 617, then f(10) equals

101. Let f(x) = min {$2x^2$, 52 – 5x}, where x is any positive real number. Then the maximum possible value of f(x) is

2018 Slot 2

102. Let f(x) = max{5x, 52 – $2x^2$}, where x is any positive real number. Then the minimum possible value of f(x) is

LOGARITHM AND EXPONENTS

1994

1. If $\log_7 \log_5\left(\sqrt{x+5} + \sqrt{x}\right) = 0$, find the value of *x*.

(a) 1 (b) 0

(c) 2 (d) None of these

2. $\log_6 216\sqrt{6}$ is

(a) 3 (b) $\frac{3}{2}$

(c) $\frac{7}{2}$ (d) None of these

1997

3. If $\log_2\left[\log_7\left(x^2 - x + 37\right)\right] = 1$, then what could be the value of 'x'?

(a) 3 (b) 5

(c) 4 (d) None of these

1999

Direction for Question 4: The question followed by two statements, I and II. Mark the answer

(a) if the question can be answered by any one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both the statements together.

4. What is the distance x between two cities A and B in integral number of kilometres?

I. x satisfies the equation $\log_2 x = \sqrt{x}$

II. $x \le 10$ km

2003(R)

5. If $\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$, then the possible value of x is given by

(a) 10 (b) $\frac{1}{100}$

(c) $\frac{1}{1000}$ (d) None of these

6. If $\frac{1}{3}\log_3 M + 3\log_3 N = 1 + \log_{0.008} 5$, then

(a) $M^9 = \frac{9}{N}$ (b) $N^9 = \frac{9}{M}$

(c) $M^3 = \frac{3}{N}$ (d) $N^9 = \frac{3}{M}$

7. What is the sum of 'n' terms in the series $\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3}+\right) + ...?$

(a) $\log\left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{\frac{n}{2}}$ (b) $\log\left[\frac{m^m}{n^n}\right]^{\frac{n}{2}}$

(c) $\log\left[\frac{m^{(1-n)}}{n^{(1-m)}}\right]^{\frac{n}{2}}$ (d) $\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{\frac{n}{2}}$

2003(L)

8. When the curves $y = \log_{10}x$ and $y = x^{-1}$ are drawn in the x-y plane, how many times do they intersect for values $x \ge 1$?

(a) Never

(b) Once

(c) Twice

(d) More than twice

9. If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in arithmetic progression, then the value of x is equal to

(a) 5 (b) 4

(c) 2 (d) 3

2004

10. Let $u = (\log_2 x)^2 - 6\log_2 x + 12$ where x is a real number. Then the equation $x^u = 256$, has

(a) no solution for x

(b) exactly one solution for x

(c) exactly two distinct solutions for x

(d) exactly three distinct solutions for x

2005

11. If $x \ge y$ and $y > 1$, then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be

(a) –1 (b) –0.5

(c) 0 (d) 1

2006

12. If $\log_y x = (a \cdot \log_z y) = (b \cdot \log_x z) = ab$, then which of the following pairs of values for (a, b) is not possible?

(a) $\left(-2, \frac{1}{2}\right)$ (b) (1, 1)

(c) (0.4, 2.5) (d) $\left(\pi, \frac{1}{\pi}\right)$

(e) (2, 2)

13. What are the values of x and y that satisfy both the equations?

$$2^{0.7x} . 3^{-1.25y} = \frac{8\sqrt{6}}{27}$$

$$4^{0.3x} . 9^{0.2y} = 8.(81)^{\frac{1}{5}}$$

(a) x = 2, y = 5

(b) x = 2.5, y = 6

(c) x = 3, y = 5

(d) x = 3, y = 4

(e) x = 5, y = 2

MEMORY BASED QUESTIONS

2010

14. If a and b are integers such that $\log_2(a+b) + \log_2(a-b) = 3$, then how many different pairs (a, b) are possible?

(a) 0 (b) 1

(c) 2 (d) 3

2011

15. If $\log_{16}5 = m$ and $\log_5 3 = n$, then what is the value of $\log_3 6$ in terms of 'm' and 'n'?

(a) $\frac{1+4mn}{4mn}$

(b) $\frac{4mn}{1+4mn}$

(c) $\frac{1}{1+4mn}$

(d) Cannot be determined

2014

16. If $X=\sum_{i=1}^{i=n}\log_{10}(i)-\sum_{j=1}^{j=p}\log_{10}(j)-\sum_{k=1}^{k=(n-p)}\log_{10}(k)$, where $p \le n$, then the maximum value of X for n = 8 is:
(a) $1 + \log_{10} 24$
(b) $\log_{10} 56$
(c) $1 + \log_{10} 7$
(d) $1 + \log_{10} 48$

17. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in Arithmetic Progression, then x is equal to
(a) 2 (b) 3
(c) 2 or 4 (d) 2 or 3

2015

18. If log 2x = 2 log (x + 1), find the number of real values of x?.

2016

19. If $\log_c a = \frac{1}{2}$, $\log_d b = \frac{1}{3}$, where a, b, c and d are distinct natural numbers and a < b < c < d, what is the minimum possible value of a + b + c + d?
(a) 10 (b) 22 (c) 34 (d) 36

2017

20. If x is a real number such that $\log_3 5 = \log_5 (2 + x)$, then which of the following is true?
(a) 0 < x < 3 (b) 23 < x < 30
(c) x > 30 (d) 3 < x < 23

21. If log $(2^a \times 3^b \times 5^c)$ is the arithmetic mean of log $(2^2 \times 3^3 \times 5)$, log$(2^6 \times 3 \times 5^7)$, and log $(2 \times 3^2 \times 5^4)$, then a equals

2018 Slot 1

22. If $\log_2(5 + \log_3 a) = 3$ and $\log_5(4a + 12 + \log_2 b) = 3$, then a + b is equal to
(a) 40 (b) 32 (c) 67 (d) 59

23. If $\log_{12} 81 = p$, the $3\left(\frac{4-p}{4+p}\right)$ is equal to
(a) $\log_4 16$ (b) $\log_2 8$
(c) $\log_6 8$ (d) $\log_6 16$

24. If x is a positive quantity such that $2^x = 3^{\log_5 2}$, then x is equal to
(a) $\log_5 9$ (b) $\log_5 8$
(c) $1+\log_3 \frac{5}{3}$ (d) $1 + \log_5 \frac{3}{5}$

25. The number of integers x such that $0.25 \le 2^x \le 200$, and $2^x + 2$ is perfectly divisible by either 3 or 4, is

26. Given that $x^{2018}y^{2017} = 1/2$ and $x^{2016}y^{2019} = 8$, the value of $x^2 + y^3$ is
(a) $\frac{33}{4}$ (b) $\frac{35}{4}$
(c) $\frac{37}{4}$ (d) $\frac{31}{4}$

2018 Slot 2

27. If $p^3 = q^4 = r^5 = s^6$, then the value of $\log_s(pqr)$ is equal to
(a) $\frac{24}{5}$ (b) 1
(c) $\frac{16}{5}$ (d) $\frac{47}{10}$

28. $\frac{1}{\log_2 100}-\frac{1}{\log_4 100}+\frac{1}{\log_5 100}-\frac{1}{\log_{10} 100}+\frac{1}{\log_{20} 100}-\frac{1}{\log_{25} 100}+\frac{1}{\log_{50} 100}=?$
(a) 0 (b) 10
(c) –4 (d) $\frac{1}{2}$

PROGRESSIONS

1990

1. What is the sum of the series:
$\frac{1}{(1\times 2)}+\frac{1}{(2\times 3)}+\frac{1}{(3\times 4)}+.......+\frac{1}{(100\times 101)}$?
(a) $\frac{99}{100}$ (b) $\frac{1}{100}$
(c) $\frac{100}{101}$ (d) $\frac{101}{102}$

2. N the set of natural numbers is partitioned into subsets $S_1 = (1)$, $S_2 = (2, 3)$, $S_3 = \{4, 5, 6)$, $S_4 = \{7, 8, 9, 10\}$ and so on. The sum of the elements of the subset S_{50} is
(a) 61250 (b) 65525
(c) 42455 (d) 62525

1993

3. Let $u_{n+1} = 2u_n + 1 (n = 0,1, 2,....)$ and $u_0 = 0$. Then u_{10} nearest to
(a) 1023 (b) 2047
(c) 4095 (d) 8195

1994

4. If the harmonic mean between two positive numbers is to their geometric mean as 12 : 13; then the numbers could be in the ratio

(a) 12 : 13 (b) 1/12 : 1/13

(c) 4 : 9 (d) 2 : 3

5. Fourth term of an arithmetic progression is 8. What is the sum of the first 7 terms of the arithmetic progression?

(a) 7

(b) 64

(c) 56

(d) Cannot be determined

6. Along a road lie an odd number of stones placed at intervals of 10m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(a) 35

(b) 15

(c) 29

(d) 31

Direction for Question 7: The question is followed by two statements. As the answer:

Mark (a) if the question can be answered with the help of statement I alone

Mark (b) if the question can be answered with the help of statement II, alone

Mark (c) if both, statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

7. Three boys had a few coffee Bite toffees with them. The number of toffees with the second were four more than those with the first and the number of toffees with the third were four more than those with the second. How many toffees were there in all?

I. The number of toffees with each of them is a multiple of 2.

II. The first boy ate up four toffees from what he had and the second boy ate up six toffees from what had and the third boy gave them two toffees each from what he had and the number of toffees remaining with each of them formed a geometric progression.

1995

Direction for Question 8: The question is followed by two statements, I and II. Mark the answer as:

(a) if the question can be answered with the help of statement I alone.

(b) if the question can be answered with the help of statement II alone.

(c) if both statement I and statement II are needed to answer the question.

(d) if the question cannot be answered even with the help of both the statements.

8. What is the first term of an arithmetic progression of positive integers?

I. Sum of the squares of the first and the second term is 116.

II. The fifth term is divisible by 7.

1996

Directions for Questions 9 to 13: Answer the questions based on the following information.

A series S_1 of five positive integers is such that the third term is half the first term and the fifth term is 20 more than the first term. In series S_2, the nth term defined as the difference between the (n+1)th term and the nth term of series S_1, is an arithmetic progression with a common difference of 30.

9. First term of S_1 is

(a) 80 (b) 90

(c) 100 (d) 120

10. Second term of S_2 is

(a) 50 (b) 60

(c) 70 (d) None of these

11. What is the difference between second and fourth terms of S_1?

(a) 10 (b) 20

(c) 30 (d) 60

12. What is the average value of the terms of series S_1?

(a) 60

(b) 70

(c) 80

(d) Average is not an integer

13. What is the sum of series S_2?

(a) 10 (b) 20

(c) 30 (d) 40

1998

14. One bacterium splits into eight bacteria of the next generation. But due to environmental condition only 50% survives and remaining 50% dies after producing next generation. If the seventh generation number is 4,096 million, what is the number in first generation?

(a) 1 million (b) 2 million

(c) 4 million (d) 8 million

Direction for Question 15: The question is followed by two statements, I and II. Answer the question based on the statements and mark the answer as:

(a) if the question can be answered with the help of any one statement alone but not by the other statement.

(b) if the question can be answered with the help of either of the statements taken individually.

(c) if the question can be answered with the help of both statements together.

(d) if the question cannot be answered even with the help of both statements together.

15. Find the value of X in terms of 'a'.

I. Arithmetic mean of X and Y is 'a' while the geometric mean is also 'a'.

II. $\frac{X}{Y}$ = R; X – Y = D.

1999

Directions for Questions 16 to 18: Answer the questions based on the following information.

There are 50 integers a_1, a_2 , ..., a_{50}, not all of them necessarily different. Let the greatest integer of these 50 integers be referred to as G, and the smallest integer be referred to as L. The integers a_1 through a_{24} form sequence S1, and the rest form sequence S2. Each member of S1 is less than or equal to each member of S2.

16. All values in S1 are changed in sign, while those in S2 remain unchanged. Which of the following statements is true?

(a) Every member of S1 is greater than or equal to every member of S2.

(b) G is in S1.

(c) If all numbers originally in S1 and S2 had the same sign, then after the change of sign, the largest number of S1 and S2 is in S1.

(d) None of the above

17. Elements of S1 are in ascending order, and those of S2 are in descending order. a_{24} and a_{25} are interchanged. Then which of the following statements is true?

(a) S1 continues to be in ascending order.

(b) S2 continues to be in descending order.

(c) S1 continues to be in ascending order and S2 in descending order.

(d) None of the above

18. Every element of S1 is made greater than or equal to every element of S2 by adding to each element of S1 an integer x. Then x cannot be less than

(a) 2^{10}

(b) the smallest value of S2

(c) the largest value of S2

(d) (G – L)

2000

19. If a_1 = 1 and $a_{n+1} = 2a_n + 5$, n = 1, 2, ... , then a_{100} is equal to

(a) $(5 \times 2^{99} - 6)$ (b) $(5 \times 2^{99} + 6)$

(c) $(6 \times 2^{99} + 5)$ (d) $(6 \times 2^{99} - 5)$

20. What is the value of the following expression?

$$\left(\frac{1}{(2^2-1)}\right)+\left(\frac{1}{(4^2-1)}\right)+\left(\frac{1}{(6^2-1)}\right)+\cdots+\left(\frac{1}{(20^2-1)}\right)$$

(a) $\frac{9}{19}$ (b) $\frac{10}{19}$

(c) $\frac{10}{21}$ (d) $\frac{11}{21}$

21. Each of the numbers $x_1, x_2, \cdots, x_n$, $n \geq 4$, is equal to 1 or –1. Suppose

$$x_1x_2x_3x_4 + x_2x_3x_4x_5 + x_3x_4x_5x_6 + \cdots + x_{n-3}x_{n-2}x_{n-1}x_n + x_{n-2}x_{n-1}x_nx_1 + x_{n-1}x_nx_1x_2 + x_nx_1x_2x_3 = 0,$$ then

(a) n is even

(b) n is odd

(c) n is an odd multiple of 3

(d) n is prime

2001

22. Two men X and Y started working for a certain company at similar jobs on January 1, 1950. X asked for an initial salary of Rs. 300 with an annual increment of Rs. 30. Y asked for an initial salary of Rs. 200 with a rise of Rs. 15 every 6 months. Assume that the arrangements remained unaltered till December 31, 1959. Salary is paid on the last day of the month. What is the total amount paid to them as salary during the period?

(a) Rs. 93,300 (b) Rs. 93,200

(c) Rs. 93,100 (d) None of these

23. All the page numbers from a book are added, beginning at page 1. However, one page number was added twice by mistake. The sum obtained was 1000. Which page number was added twice?

(a) 44 (b) 45
(c) 10 (d) 12

24. For a Fibonacci sequence, from the third term onwards, each term in the sequence is the sum of the previous two terms in that sequence. If the difference in squares of 7th and 6th terms of this sequence is 517, what is the 10th term of this sequence?

(a) 147
(b) 76
(c) 123
(d) Cannot be determined

2002

25. The nth element of a series is represented as

$X_n = (-1)^n X_{n-1}$

If $X_0 = x$ and x > 0, then which of the following is always true?

(a) X_n is positive if n is even
(b) X_n is positive if n is odd
(c) X_n is negative if n is even
(d) None of these

26. Let S denotes the infinite sum

$2 + 5x + 9x^2 + 14x^3 + 20x^4 + \ldots$, where |x| < 1 and the coefficient of x^{n-1} is $\frac{1}{2}n(n+3), (n = 1, 2, \ldots)$. Then S equals:

(a) $\frac{2-x}{(1-x)^3}$ (b) $\frac{2-x}{(1+x)^3}$
(c) $\frac{2+x}{(1-x)^3}$ (d) $\frac{2+x}{(1+x)^3}$

27. A child was asked to add first few natural numbers (i.e. 1 + 2 + 3 + …) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered he had missed one number in the sequence during addition. The number he missed was

(a) less than 10
(b) 10
(c) 15
(d) more than 15

2003(R)

28. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio $\frac{P_1 + P_2 + P_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$ equals

(a) $\frac{2(1+\sqrt{2})}{a}$ (b) $\frac{2(2-\sqrt{2})}{a}$
(c) $\frac{2(2+\sqrt{2})}{a}$ (d) $\frac{2(1+2\sqrt{2})}{a}$

29. The infinite sum $1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \cdots$ equals

(a) $\frac{27}{14}$ (b) $\frac{21}{13}$
(c) $\frac{49}{27}$ (d) $\frac{256}{147}$

2003(L)

30. The sum of 3^{rd} and 15^{th} elements of an arithmetic progression is equal to the sum of 6^{th}, 11^{th} and 13^{th} elements of the same progression. Then which element of the series should necessarily be equal to zero?

(a) 1^{st} (b) 9^{th}
(c) 12^{th} (d) None of the above

31. The 288^{th} term of the series a,b,b,c,c,c,d,d,d,d,e,e,e,e,e,f,f,f,f,f,f,… is

(a) u (b) v
(c) w (d) x

32. There are 8436 steel balls, each with a radius of 1 centimeter, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is

(a) 34 (b) 38
(c) 36 (d) 32

33. If the product of n positive real numbers is unity, then their sum is necessarily

(a) a multiple of n (b) equal to $n + \frac{1}{n}$
(c) never less than n (d) a positive integer

34. Let T be the set of integers {3, 11, 19, 27, …, 451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is

(a) 32 (b) 28
(c) 29 (d) 30

Directions for Question 35: The question is followed by two statements, A and B. Answer the question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (d) if the question cannot be answered even by using both the statements together.

35. Is $\left(\frac{1}{a^2}+\frac{1}{a^4}+\frac{1}{a^6}+\cdots\right) > \left(\frac{1}{a}+\frac{1}{a^3}+\frac{1}{a^5}+\cdots\right)$?

A. $-3 \le a \le 3$

B. One of the roots of the equation $4x^2 - 4x + 1 = 0$ is a.

2004

36. If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

(a) 0 (b) –1
(c) 1 (d) Not unique

37. On January 1, 2004 two new societies s_1 and s_2 are formed, each n numbers. On the first day of each subsequent month, s_1 adds b members while s_2 multiples its current numbers by a constant factor r. Both the societies have the same number of members on July 2, 2004. If b = 10.5n, what is the value of r?

(a) 2.0 (b) 1.9
(c) 1.8 (d) 1.7

38. Consider the sequence of numbers a_1, a_2, a_3, ... to infinity where $a_1 = 81.33$ and $a_2 = -19$ and $a_j = a_{j-1} - a_{j-2}$ for $j \ge 3$. What is the sum of the first 6002 terms of this sequence?

(a) –100.33 (b) –30.00
(c) 62.33 (d) 119.33

2005

39. If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then a_{100} equals

(a) $3^{99} - 200$ (b) $3^{99} + 200$
(c) $3^{100} - 200$ (d) $3^{100} + 200$

2006

40. Consider a sequence where the nth term,

$t_n = \frac{n}{(n+2)}$, n = 1, 2, ...

The value of $t_3 \times t_4 \times t_5 \times \ldots \times t_{53}$ equals:

(a) $\frac{2}{495}$ (b) $\frac{2}{477}$
(c) $\frac{12}{55}$ (d) $\frac{1}{1485}$
(e) $\frac{1}{2970}$

41. A group of 630 children is arranged in rows for a group photograph session. Each row contains three fewer children than the row in front of it. What number of rows is not possible?

(a) 3 (b) 4
(c) 5 (d) 6
(e) 7

42. Consider the set S = {1, 2, 3, . . ., 1000}. How many arithmetic progressions can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements?

(a) 3 (b) 4
(c) 6 (d) 7
(e) 8

2007

43. Consider the set S = {2, 3, 4, ……, 2n + 1}, where 'n' is a positive integer larger than 2007. Define X as the average of the odd integers in S and Y as the average of the even integers in S. What is the value of X – Y?

(a) 0 (b) 1
(c) $\frac{1}{2}n$ (d) $\frac{n+1}{2n}$
(e) 2008

44. The price of Darjeeling tea (in rupees per kilogram) is 100 + 0.10n, on the n^{th} day of 2007 (n = 1, 2, ..., 100), and then remains constant. On the other hand, the price of Ooty tea (in rupees per kilogram) is 89 + 0.15n, on the n^{th} day of 2007 (n = 1, 2, ..., 365). On which date in 2007 will the prices of these two varieties of tea be equal?

(a) May 21 (b) April 11
(c) May 20 (d) April 10
(e) June 30

Directions for Questions 45 and 46: Answer the following questions based on the information given below:

Let $a_1 = p$ and $b_1 = q$, where p and q are positive quantities. Define

$a_n = pb_{n-1}$, $b_n = qb_{n-1}$, for even n > 1,

and $a_n = pa_{n-1}$, $b_n = qa_{n-1}$, for odd n > 1.

45. Which of the following best describes $a_n + b_n$ for even 'n'?

(a) $q(pq)^{\frac{1}{2}n-1}(p+q)$ (b) $qp^{\frac{1}{2}n-1}(p+q)$

(c) $q^{\frac{1}{2}n}(p+q)$ (d) $q^{\frac{1}{2}n}(p+q)^{\frac{1}{2}n}$

(e) $q(pq)^{\frac{1}{2}n-1}(p+q)^{\frac{1}{2}n}$

46. If $p = \frac{1}{3}$ and $q = \frac{2}{3}$, then what is the smallest odd 'n' such that $a_n + b_n < 0.01$?

(a) 7 (b) 13
(c) 11 (d) 9
(e) 15

2008

47. The integers 1, 2, …, 40 are written on a blackboard. The following operation is then repeated 39 times: In each repetition, any two numbers, say a and b, currently on the blackboard are erased and a new number a + b – 1 is written. What will be the number left on the board at the end?

(a) 820 (b) 821
(c) 781 (d) 819
(e) 780

48. The number of common terms in the two sequences 17, 21, 25,…, 417 and 16, 21, 26,…, 466 is

(a) 78 (b) 19
(c) 20 (d) 77
(e) 22

49. Find the sum

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\dots\dots+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$$

(a) $2008-\frac{1}{2008}$ (b) $2007-\frac{1}{2007}$

(c) $2007-\frac{1}{2008}$ (d) $2008-\frac{1}{2007}$

(e) $2008-\frac{1}{2009}$

MEMORY BASED QUESTIONS

2009

50. If the sum of the first 'n' terms of an Arithmetic Progression is 100 and the sum of the next 'n' terms of the Arithmetic Progression is 300, then what is the ratio of the first term and the common difference?

(a) 2 : 1
(b) 1 : 2
(c) 1 : 3
(d) Cannot be determined

2012

51. A sequence of terms is defined such that

$2a_n = a_{n+1} + a_{n-1}$; $a_0 = 1$; $a_1 = 3$.

What is the value of $a_0 + a_1 + a_2 + a_3 + \dots\dots + a_{50}$?

(a) 2551 (b) 2753
(c) 2601 (d) 2451

2013

52. If E = 3 + 8 + 15 + 24 + … + 195, then what is the sum of the prime factors of E?

(a) 29 (b) 31
(c) 33 (d) 23

2014

53. An equation with all positive roots is written as $x^n + a_n x^{n-1} + a_{n-1} x^{n-2} + \dots a_1 = 0$. Which of the following in necessarily true?

(a) $a_n^n \geq n^n \times a_1$ (b) $n^n \geq a_n^n \times a_1$
(c) $a_1^n \geq n^n \times a_n$ (d) None of these

54. The number of APs with 5 distinct terms that can be formed from the first 50 natural numbers is

(a) 325 (b) 300
(c) 375 (d) 288

55. $P = \frac{1}{1!+2!}+\frac{1}{2!+3!}+\frac{1}{3!+4!}+\dots+\frac{1}{9!+10!}$. Find the value of P.

(a) $\frac{1}{2!}-\frac{1}{11!}$ (b) $\frac{1}{2!}-\frac{1}{10!}$

(c) $\frac{1}{1!}-\frac{1}{10!}$ (d) $\frac{1}{1!}-\frac{1}{11!}$

56. A set 'P' is formed from the set of first 'N' natural numbers by deleting all the perfect squares and all the perfect cubes. If the numbers are arranged in an ascending order then, what is the 476th number of the set 'P'?

(a) 500 (b) 501
(c) 502 (d) 503

57. P1, P2, P3, ..., P11 are 11 friends. The number of balls with P1 through P11 in that order is in an Arithmetic Progression. If the sum of the number of balls with P1, P3, P5, P7, P9 and P11 is 72, what is the number of balls with P1, P6 and P11 put together?

(a) 24 (b) 48
(c) 36 (d) Cannot be determined

2017

58. An infinite geometric progression $a_1, a_2, a_3, \ldots$ has the property that $a_n = 3(a_{n+1} + a_{n+2} + \ldots)$ for every $n \geq 1$. If the sum $a_1 + a_2 + a_3 + \ldots = 32$, then a_5 is

(a) $\frac{1}{32}$ (b) $\frac{2}{32}$
(c) $\frac{3}{32}$ (d) $\frac{4}{32}$

59. If $a_1 = \frac{1}{2\times5}, a_2 = \frac{1}{5\times8}, a_3 = \frac{1}{8\times11}, \ldots,$ then $a_1 + a_2 + a_3 + \ldots + a_{100}$ is

(a) $\frac{25}{151}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{111}{55}$

2018 Slot 1

60. Let x, y, z be three positive real numbers in a geometric progression such that $x < y < z$. If 5x, 16y, and 12z are in an arithmetic progression then the common ratio of the geometric progression is

(a) $\frac{5}{2}$
(b) $\frac{3}{6}$
(c) $\frac{3}{2}$
(d) $\frac{1}{6}$

2018 Slot 2

61. Let $a_1, a_2, \ldots, a_{52}$ be positive integers such that $a_1 < a_2 < \ldots < a_{52}$. Suppose, their arithmetic mean is one less than the arithmetic mean of $a_2, a_3, \ldots, a_{52}$. If $a_{52} = 100$, then the largest possible value of a_1 is

(a) 20
(b) 23
(c) 45
(d) 48

ANSWERS

Equations, Polynomials and Inequations

1. (a)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	7. (b)	**8.** (c)	**9.** (b)	**10.** (b)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (d)
21. (a)	**22.** (d)	**23.** (d)	**24.** (a)	**25.** (d)	**26.** (d)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (a)
31. (d)	**32.** (a)	**33.** (a)	**34.** (d)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (a)
41. (d)	**42.** (a)	**43.** (d)	**44.** (d)	**45.** (c)	**46.** (d)	**47.** (c)	**48.** (a)	**49.** (b)	**50.** (a)
51. (b)	**52.** (c)	**53.** (d)	**54.** (c)	**55.** (c)	**56.** (d)	57. (c)	**58.** (a)	**59.** (b)	**60.** (b)
61. (c)	**62.** (a)	**63.** (c)	**64.** (d)	**65.** (d)	**66.** (a)	**67.** (c)	**68.** (a)	**69.** (a)	**70.** (c)
71. (d)	**72.** (b)	**73.** (b)	**74.** (c)	**75.** (c)	**76.** (b)	**77.** (d)	**78.** (d)	**79.** (c)	**80.** (a)
81. (c)	**82.** (c)	**83.** (d)	**84.** (c)	**85.** (c)	**86.** (d)	**87.** (d)	**88.** (b)	**89.** (a)	**90.** (b)
91. (b)	**92.** (c)	**93.** (d)	**94.** (e)	**95.** (b)	**96.** (a)	**97.** (b)	**98.** (a)	**99.** (b)	**100.** (d)
101. (a)	**102.** (b)	**103.** (d)	**104.** (c)	**105.** (c)	**106.** (a)	**107.** (b)	**108.** (d)	**109.** (a)	**110.** (d)
111. (c)	**112.** (a)	**113.** (a)	**114.** (b)	**115.** (d)	**116.** (c)	**117.** (a)	**118.** (c)	**119.** (b)	**120.** (c)
121. (a)	**122.** (70)	**123.** (c)	**124.** (c)	**125.** (b)	**126.** (c)	**127.** (a)	**128.** (a)	**129.** (8)	**130.** (10)
131. (a)	**132.** (36)	**133.** (24)							

Functions and Graphs

1. (b)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (a)
11. (b)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (d)
21. (b)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (c)
31. (c)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (d)	**37.** (a)	**38.** (d)	**39.** (a)	**40.** (b)
41. (c)	**42.** (d)	**43.** (b)	**44.** (c)	**45.** (b)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (c)
51. (c)	**52.** (c)	**53.** (d)	**54.** (b)	**55.** (b)	**56.** (d)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (a)
61. (d)	**62.** (a)	**63.** (d)	**64.** (b)	**65.** (d)	**66.** (b)	**67.** (d)	**68.** (c)	**69.** (b)	**70.** (d)
71. (c)	**72.** (d)	**73.** (b)	**74.** (d)	**75.** (e)	**76.** (a)	**77.** (b)	**78.** (d)	**79.** (b)	**80.** (e)
81. (b)	**82.** (a)	**83.** (d)	**84.** (b)	**85.** (b)	**86.** (c)	**87.** (a)	**88.** (b)	**89.** (d)	**90.** (c)
91. (c)	**92.** (b)	**93.** (b)	**94.** (c)	**95.** (c)	**96.** (a)	**97.** (c)	**98.** (a)	**99.** (d)	**100.** (54)
101. (32)	**102.** (20)								

Logarithm and Exponents

1. (d)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (b)
11. (d)	**12.** (e)	**13.** (e)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** 0	**19.** (d)	**20.** (d)
21. (c)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (5)	**26.** (a)	**27.** (d)	**28.** (d)		

Progressions

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (d)
11. (a)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (d)	**19.** (d)	**20.** (c)
21. (a)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (c)
31. (d)	**32.** (c)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (a)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (a)
41. (d)	**42.** (d)	**43.** (b)	**44.** (c)	**45.** (a)	**46.** (d)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (b)
51. (c)	**52.** (b)	**53.** (d)	**54.** (d)	**55.** (a)	**56.** (d)	**57.** (c)	**58.** (c)	**59.** (a)	**60.** (a)
61. (b)									

EXPLANATIONS

Equations, Polynomials and Inequations

1. a $\frac{1}{(1-x)}+\frac{1}{(1+x)}+\frac{2}{(1+x^2)}+\frac{4}{(1+x^4)}$

$=\frac{2}{(1-x^2)}+\frac{2}{(1+x^2)}+\frac{4}{(1+x^4)}=\frac{4}{(1-x^4)}+\frac{4}{(1+x^4)}$

$=\frac{8}{(1-x^8)}$.

2. a When x = 0, $a^x b^{(1-x)}=b$

When x = 1, $a^x b^{(1-x)}=a$

Only option (a)always satisfies the given constraints.

3. b $\left(\frac{1-d^3}{1-d}\right)=\frac{(1-d)(1+d+d^2)}{(1-d)}=(1+d+d^2)$

If d > 1, then $d^2 > 1$ and $(1 + d + d^2) > 3$.

Hence, (b)is the correct option.

4. d If the roots are reciprocal of each other their product = 1.

Product of roots of the equation $=\frac{6}{a}$

Since $\frac{6}{a}=1 \Rightarrow a=6$.

5. c $(x + y + z)^2$

$= x^2 + y^2 + z^2 + 2 (xy + yz + xz)$

$= x^2 + y^2 + z^2 + 2 \times 0$

$= x^2 + y^2 + z^2$.

6. c As 43 is neither a multiple of 5 nor 6, either statement alone is not sufficient to answer the question.

Using both statements together: Let the number of children older than 5 years be 'a' and that of 5 years or younger be 'b'.

As per the given information, 5a + 6b = 43.

As a and b are non-negative integer, only possible value: a = 5 and b = 3.

7. b $\left[\frac{(x^{-1}-y^{-1})}{(x^{-2}-y^{-2})}\right]=\frac{\left(\frac{1}{x}-\frac{1}{y}\right)}{\left(\frac{1}{x^2}-\frac{1}{y^2}\right)}$

$=\frac{\left(\frac{1}{x}-\frac{1}{y}\right)}{\left[\left(\frac{1}{x}-\frac{1}{y}\right)\left(\frac{1}{x}+\frac{1}{y}\right)\right]}=\frac{1}{\left(\frac{1}{x}+\frac{1}{y}\right)}$

For above expression to be > 1, $\left(\frac{1}{x}+\frac{1}{y}\right)$ has to be less than 1.

For this both 'x' and 'y' have to be greater than 2. Statement I doesn't give any information about this, but statement II clearly specifies this. Hence, only statement II is required to answer the given question.

8. c For no solution, lines must be parallel and not over lapping

$\Rightarrow \frac{2}{k}=\frac{-8}{4}\neq\frac{3}{10}$

$\therefore k=-1$

9. b Let Mushtaq has M cards while Iqbal has got I cards with him.

Let number of cards exchanged be x.

Case 1: I + x = 4(M – x) ... (i)

Case 2: I – x = 3(M + x) ... (ii)

From (i) and (ii),

I = 31x.

Only possible value for I could be 31.

10. b The two equations can be simplified into $n \le 2$ and $n \ge 2$. The only value that satisfies both these conditions is n = 2.

11. b Let x be the total number of sticks assigned to each boy and y be the number of boxes in which he has to fill them. If he reduces number of sticks per box by 25, he would fill $\left(\frac{x}{y}-25\right)$ in each box and hence he would now fill (y + 3) boxes.

$\therefore\ x=\left(\frac{x}{y}-25\right)(y+3) = x + \frac{3x}{y} - 25y - 75.$

$\Rightarrow\ 3x = (25y + 75)y$

$\Rightarrow\ x=\frac{(25y^2+75y)}{3}=\frac{25y(y+3)}{3}$

For x to have an integer value, y(y + 3)has to be a multiple of 3. This is possible only when y is a multiple of 3. Subsequently, (y + 3) will be a multiple of 3. Therefore, $\frac{25y(y+3)}{3}=x$ will be a multiple of 3. Hence, the correct choice is (b).

12. c **From statement I:** $2X+2Y\le 40$ or $X+Y\le 20$

This statement alone cannot give the value of X.

From statement II: $X-2Y\ge 20$

This statement also alone cannot give the value of X.

On combining statements I and II:

Multiplying the second statement by –1 and adding both the statements, we get

$3Y \le 0$ i.e., $Y \le 0$, but it is given that Y is non negative.

$\therefore$ Y = 0 and X = 20

Hence, using both statements together we can answer the question.

13. c **From statement I:** Let price per kg of mangoes be Rs.x and price per dozen of oranges be Rs.y.

$\therefore 10x + 2y = 252$

From this statement, we cannot find x.

From statement II: $2x = y$

From this statement also, we cannot find the price per kg of mangoes.

On combining statements I and II: $14x = 252$ i.e., $x = 18$

Hence, using both statements together we can answer the question.

14. c The two equations are : $2o + 3b + 4a = 15$ and $3o + 2b + a = 10$.

Adding the two equations, we get

$5o + 5b + 5a = 25$

$\Rightarrow o + b + a = 5$

$\therefore 3o + 3b + 3a = 15$.

15. b The price of 1 mango is equal to the price of 2 oranges. Hence, 5 mangoes will be equivalent to 10 oranges. So 20 oranges cost Rs.40, therefore one orange will cost Rs.2.

16. b Assume some values of a, b and c such that sum of a, b and c is 0 where $a \ne b \ne c$, and find the value of the given expression.

Let a = 1, b = –1 and c = 0.

$$\Rightarrow \frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}$$

$$= \frac{1}{2}+\frac{1}{2}+0 = 1.$$

17. a If one root of $x^2 + px + 12 = 0$ is 4, then $4^2 + 4p + 12 = 0$, i.e. $p = –7$.

$x^2 – 7x + q = 0$ has equal roots.

If the roots are α each, $2\alpha = -\frac{(-7)}{1} = 7$, i.e. $\alpha = \frac{7}{2}$, and $q = \alpha^2 \Rightarrow q = \frac{49}{4}$.

18. d Since the ages of none of them is mentioned and we have two equations and three unknowns.

Hence, we cannot say anything about the ages of any of them.

19. b $2g + 3b = 20$.

Since b & g should be integers the values that satisfy this equation are (g = 10 & b = 0), (g = 7 and b = 2), (g = 4 & b = 4), and (g = 1 and b = 6).

From the statement I, we can shortlist the last two possibilities i.e. g =4 or g = 1, but cannot get a unique answer.

The statement II suggests that the number of girls and boys have to be equal. Hence, we get a unique answer viz. g = 4 & b = 4. Only statement II is required to answer the question.

20. d If the roots are a and a^2, the product of roots = $a^3 = –8$.

$\therefore a = –2$.

Hence, sum of the roots = $k = –(a + a^2) = –(–2 + 4) = – 2$.

21. a Let one of the numbers be x. So the other number would be (x + 4).

According to the question, we have

$\frac{1}{x}+\frac{1}{(x+4)} = 21$ or $x = 3$.

Hint: Please note that the sum of reciprocals is basically = $\frac{(\text{Sum of the integers})}{(\text{Product of the integers})}$. So we have to find two integers whose sum is 10 and whose product is 21.

So x + (x + 4)= 10 or x = 3.

22. d $3m^2 – 21m + 30 < 0$

$\therefore m^2 – 7m + 10 < 0 \Rightarrow (m – 5)(m – 2) < 0$.

So either (m – 5)< 0 and (m – 2) > 0 or (m – 2) < 0 and (m – 5) > 0.

Hence, either m < 5 and m > 2, i.e., 2 < m < 5 or m < 2 and m > 5.

23. d Statement I is useless as it only tells that if x and y are consecutive positive even integers, then $(x – y)^2$ has to be equal to 4.

Statement II suggests the possibility that the numbers could be 2 and 4. But it does not suggest which is x and which is y.

Hence, we cannot find the value of x using either statements.

24. a If we simplify the expression $x^2 – 3x + 2 > 0$, we get $(x – 1)(x – 2) > 0$. For this product to be greater than zero, either both the factors should be greater than zero or both of them should be less than zero. Therefore, (x – 1)> 0 and (x – 2) > 0 or (x – 1) < 0 and (x – 2)< 0. Hence, x > 1 and x > 2 or x < 1 and x < 2. If we were to club the ranges, we would get either x > 2 or x < 1. So for any value of x equal to or between 1 and 2, the above equation does not follow.

25. d If we write the given equation in the conventional form, i.e. $ax^2 + bx + c = 0$, a = 1, b = – (A – 3), i.e. (3 – A) and c = –(A – 2), i.e. (2 – A). Let the roots of this equation be α and β. So the sum of the squares of the roots = $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

Now $(\alpha + \beta)$= Sum of the roots = $\frac{-b}{a} = \frac{(A-3)}{1}$

= (A – 3) and $\alpha\beta$ = Product of the roots = $\frac{c}{a}$

$= \frac{(2-A)}{1} = (2-A)$. Hence, $\alpha^2 + \beta^2 = (A-3)^2 - 2(2-A)$

$= A^2 - 4A + 5 = 0$. None of the answer choices matches this.

26. d Using statement II, $2\alpha\beta = \frac{c}{a} = \alpha\beta$

$\Rightarrow \alpha = 0$ or $\beta = 0$ or α and $\beta = 0$

Hence, the question cannot be answered.

27. b $$\frac{1}{1+\frac{1}{3-\frac{4}{2+\frac{1}{3-\frac{1}{2}}}}} + \frac{3}{3-\frac{4}{3+\frac{1}{2-\frac{1}{2}}}}$$

$$= \frac{1}{1+\frac{1}{3-\frac{4}{2+\frac{2}{5}}}} + \frac{3}{3-\frac{4}{3+\frac{1}{\frac{3}{2}}}}$$

$$= \frac{1}{1+\frac{1}{3-\frac{4}{\frac{12}{5}}}} + \frac{3}{3-\frac{4}{3+\frac{1}{\frac{3}{2}}}}$$

$$= \frac{1}{1+\frac{1}{3-\frac{5}{3}}} + \frac{3}{3-\frac{4}{3+\frac{2}{3}}}$$

$$= \frac{1}{1+\frac{1}{\frac{4}{3}}} + \frac{3}{3-\frac{4}{\frac{11}{3}}} = \frac{1}{1+\frac{3}{4}} + \frac{3}{3-\frac{12}{11}}$$

$$= \frac{1}{1+\frac{3}{4}} + \frac{3}{3-\frac{12}{11}} = \frac{1}{\frac{7}{4}} + \frac{3}{\frac{21}{11}}$$

$$= \frac{4}{7} + \frac{11}{7} = \frac{15}{7}$$

28. d $$A = \frac{2.000004}{[(2.000004)^2 + 2(2.000004)]}$$

$$= \frac{2.000004}{2.000004[(2.000004) + 2]}$$

$$= \frac{1}{[(2.000004) + 2]}$$

$$= \frac{1}{4.000004}$$

$$= \frac{1}{4} = 0.25 \text{ (Approximately)}$$

$$B = \frac{3.000003}{[(3.000003)^2 + 3(3.000003)]}$$

$$= \frac{3.000003}{3.000003[(3.000003) + 3]}$$

$$= \frac{1}{[(3.000003) + 3]} = \frac{1}{6.000003}$$

$$C = \frac{4.000002}{[(4.000002)^2 + 2(4.000002)]}$$

$$= \frac{4.000002}{4.000002[(4.000002) + 2]}$$

$$= \frac{1}{[(4.000002) + 2]} = \frac{1}{6.000002}$$

Looking at the answer choices, we can see that the only (d)satisfies the relationship, viz. B is the smallest.

29. a This equation is very straightforward. If the number is 'x', then $\frac{7x}{8} - \frac{7x}{18}$ = 770. On solving this equation, we get x = 1584.

Hint: Students please note that if the difference in $\frac{7}{8}$ and $\frac{7}{18}$ of a number is 770, then the difference in $\frac{1}{8}$ and $\frac{1}{18}$ of the number should be 110. If we express this as an equation, we get

$$\frac{x}{8} - \frac{x}{18} = 110$$

or $10x = 110 \times 18 \times 8$

or $x = 11 \times 18 \times 8$

You can further proceed from here in two ways: (i) the last digit of the required answer should be (1 × 8 × 8) = 4, (ii)number should be divisible by 11.

In both cases, the answer that is obtained from the given choices is 1584.

30. a We know that the sum of the roots = $-\frac{b}{a}$.

Hence, $x_1 + x_2 = 2$. Now we have two equations, viz.

$$x_1 + x_2 = 2 \quad ...(i)$$

and $$7x_2 - 4x_1 = 47 \quad ...(ii)$$

Solving these two equations, we get $x_1 = -3$ and $x_2 = 5$. Since it does not satisfy options (b) and (c), we will verify it for option (a). The product of the roots = $(-3) \times 5 = -15$, $\frac{c}{a}$ in our case is c. Hence, $c = -15$.

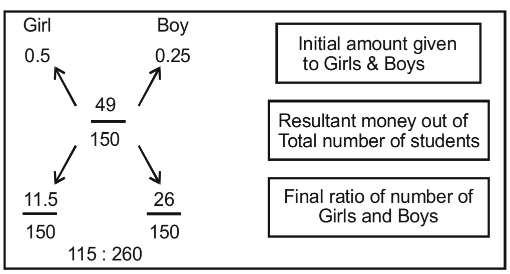
Alternative method:

Put values of x_1, x_2 in equation (ii). Do not match. So put $c = -15$ in equation (i) to get the roots of equation. After finding the roots of equation (i), check whether they satisfy equation (ii) or not. The roots (5, –3)satisfy the equation (ii) so answer is (a).

31. d Students! be careful. Generally, as we see two unknowns (i.e. x and y in this case) and two equations, we tempt to mark the answer as (c), i.e. combining two statements, we can easily find the values of x and y.

But have a look at the equations $3x + 2y = 45$ and $10.5x + 7y = 157.5$. Multiplying 1st equation by 3.5, we get 2nd equation. Hence, these are not really two different equations. Hence, data is insufficient to answer the question. In general, remember the following rule. If we have two equations $Ax + By = k_1$ and $Cx + Dy = k_2$, and $A \times D = B \times C$, then the equations cannot be solved.

32. a



Using the first statement alone, we can alligate and find the ratio of boys to girls and hence the number of girls, i.e. as shown in the adjacent diagram, 150 students when divided in the ratio 115 : 260, give 46 girls and 104 boys. The second statement, however, does not throw any further light on the data given in the question as it simply suggests $0.3B + 0.3G = 45$ or $B + G = 150$, which is already known. Hence, only statement I is required to answer the question.

33. a The difference between two integers will be 1, only if one is even and the other one is odd. 4x will always be even, so 17y has to be odd and hence, y has to be odd.

Moreover, the number 17y should be such a number that is 1 less than a multiple of 4. In other words, we have to find all such multiples of 17, which are 1 less than a multiple of 4. The first such multiple is 51. Now you will find that as the multiples of 17 goes on increasing, the difference between it and its closest higher multiple of 4 is in the following pattern, 0, 3, 2, 1, E.g. 52 – 51 = 1,

68 – 68 = 0, 88 – 85 = 3, 104 – 102 = 2, 120 – 119 = 1, 136 – 136 = 0

So the multiples of 17 that we are interested in are 3, 7, 11, 15.

Now since $x \le 1000,\ 4x \le 4000$. The multiple of 17 closest and less than 4000 is 3995 (17 × 235). And incidentally, 3996 is a multiple of 4, i.e. the difference is 4.

This means that in order to find the answer, we need to find the number of terms in the AP formed by 3, 7, 11, 15, … 235, where a = 3, d = 4.

Since we know that $T_n = a + (n - 1)d$

$\Rightarrow 235 = 3 + (n - 1) \times 4$

Hence, n = 59.

Alternate Solution:

$4x - 17y = 1$ and $x \le 1000$

so $17y + 1 \le 4000$ i.e. $y \le 235$ and moreover every 4th value of y with give value of x.

So number of values $= \frac{235}{4} \approx 58$

Hence, total number of terms will be 58 + 1 = 59

34. d Statement I when used to solve the sum gives us the same equation as the second substituted in to the first equation.

$$kdx + key = kf$$

$$\therefore \quad k(dx + ey) = kf$$

as $k \ne 0$, $dx + ey = f$ which is same as second equation.

So it is of no use as we get infinite solutions and not a unique one.

Statement II gives us the following equations.

$$x + y = c$$

$$2x + 2y = f.$$

These are two linear equations in x and y, such that $\frac{1}{2} = \frac{1}{2} \ne \frac{c}{f}$.

As $\frac{c}{f} \ne \frac{1}{2}$ (Given), the system will have no solution.

As the data given in both the statements is inconsistent, the question cannot be answered.

35. b The data is not linear. So check (b).

Let the equation be $y = a + bx + cx^2$.

Putting the values of x and y, we get the following result.

$\Rightarrow$ $4 = a + b + c$, $8 = a + 2b + 4c$ and $14 = a + 3b + 9c$. Solving these, we get a = 2, b = 1 and c = 1.

So the equation is $y = 2 + x + x^2$.

36. b Use choices. The answer is (b), because $-x < -2$ and $-2 < 2y \Rightarrow -x < 2y$.

37. b Use the choices. If b = 1, then the factors are $(x - a)(x^2 + 1)$. This cannot yield 3 real roots.

38. a Let the number of direct roads from A to B, B to C and C to A be x, y and z respectively. Then, $x + yz = 33$, $y + xz = 23$. Hence, by solving, we get z = 6.

39. d Work with options. If the cylinder has a capacity of 1,200 L, then the conical vessel shall have a capacity of 700 L. Once 200 L have been taken out from the same, the remaining volume in each of them shall be 1000 and 500.

Alternative method:

Let the volume of conical tank be x.

Then the volume of cylindrical tank = x + 500

$x + 300 = 2(x - 200)$

$\Rightarrow$ x = 700

Volume of cylindrical tank = 700 + 500 = 1200L.

40. a The first statement implies that X must lie between 0 and – 3. Hence, it gives the answer. But from the second statement, we have either X > 3 or X < 0. This does not gives us any information about the modulus of X.

41. d $x > 5$, $y < -1$

Use answer choices.

Take x = 6, y = –6. We see none of the statements (a), (b) and (c) is true. Hence, the correct option is (d).

42. a Let the four-digit number be abcd.

$a + b = c + d$ ... (i)

$b + d = 2(a + c)$ ... (ii)

$a + d = c$ ... (iii)

From (i) and (iii), b = 2d

From (i) and (ii), 3b = 4c + d

$\Rightarrow$ 3(2d) = 4c + d

$\Rightarrow$ 5d = 4c

$\Rightarrow$ $c = \frac{5}{4}d$

Now, d can be 4 or 8.

But if d = 8, then c = 10 not possible.

So d = 4 which gives c = 5.

43. d Let there be x mints originally in the bowl.

Sita took $\frac{1}{3}$, but returned 4. So now the bowl has $\frac{2}{3}x + 4$ mints.

Fatima took $\frac{1}{4}$ of the remainder, but returned 3.

So the bowl now has $\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3$ mints.

Eshwari took half of remainder that is

$\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3\right]$

She returns 2, so the bowl now has

$\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3\right] + 2 = 17 \Rightarrow x = 48$

Short cut:

Since Sita was the first person to pick and she picks up $\frac{1}{3}$ of the mint, but if you see the options, none of the option is a multiple of 3.

44. d Let the number be x.

Increase in product = 53x – 35x = 18x

$\Rightarrow$ 18x = 540 $\Rightarrow$ x = 30

Hence, new product = 53 × 30 = 1590.

45. c The value of y would be negative and the value of x would be positive from the inequalities given in the question.

Therefore, from (a), y becomes positive. The value of xy^2 would be positive and will not be the minimum.

From (b)and (c), x^2y and 5xy would give negative values but we do not know which would be the minimum.

On comparing (a)and (c), we find that

$x^2 < 5x$ in $2 < x < 3$.

$\therefore$ $x^2y > 5xy$ [Since y is negative.]

$\therefore$ 5xy would give the minimum value.

46. d Let $y = n^3 - 7n^2 + 11n - 5$

At n = 1, y = 0

$\therefore$ $(n - 1)(n^2 - 6n + 5) = (n - 1)^2(n - 5)$

Now $(n - 1)^2$ is always positive.

For n < 5, the expression gives a negative quantity. Therefore, the least value of n will be 6.

Hence, m = 6.

47. c Let 'x' be the number of males in Mota Hazri.

	Chota Hazri	Mota Hazri
Males	x – 4522	x
Females	2(x – 4522)	x + 4020

x + 4020 – 2(x – 4522)= 2910 $\Rightarrow$ x = 10154

$\therefore$ Number of males in Chota Hazri = 10154 – 4522 = 5632

48. a Let the cost of 1 burger, 1 shake and 1 fries be x, y and z.

Then,

3x + 7y + z = 120 ... (i)

4x + 10y + z = 164.50 ... (ii)

x + 3y = 44.50 ... (iii) (ii – i)

Multiplying (iii)by 4 and subtracting (ii)from it, we get

2y – z = 13.50 ...(iv)

Subtracting (iv)from (iii), we get x + y + z = 31.

49. b From II, b = 2d

Hence, b = 10, d = 5 or b = 4, d = 2

From III, e + a = 10 or e + a = 4

From I, a + c = e or e – a = c

From III and I, we get 2e = 10 + c or 2e = 4 + c

$\Rightarrow \quad e = 5 + \frac{c}{2}$ ... (i)

or $\quad e = 2 + \frac{c}{2}$ ... (ii)

From (i), we can take c = 2, 4, 6, 10.

For c = 2, e = 6

c = 4, e = 7 (Not possible)

c = 6, c = 8 (Not possible)

c = 10, e = 10 (Not possible since both c and e cannot be 10)

From (ii), we have c = 2, 4, 6, 10.

For c = 2, e = 3 (Not possible)

c = 4, e = 4 (Not possible)

c = 6, e = 5 (Possible)

c = 10, e = 7 (Not possible)

Considering the possibility from B that c = 6 and $\Rightarrow$ e = 5 means e + a = 4

$\Rightarrow$ a = –1 (Not possible)

Hence, only possibility is b = 10, d = 5, c = 2, e = 6.

e + a = 10 $\quad\Rightarrow\quad$ a = 4

50. a Quadratic equation having roots (4, 3) is

(x – 4) (x – 3) = 0

$\Rightarrow$ $x^2 - 7x + 12 = 0$ ... (i)

Quadratic equation having roots (3, 2) is

(x – 3) (x – 2) = 0

$\Rightarrow$ $x^2 - 5x + 6 = 0$ ... (ii)

Picking the coefficient of x from (i) and the constant term from (ii), we get the required equation

$x^2 - 7x + 6 = 0$

$\Rightarrow$ (x – 6) (x – 1) = 0

$\therefore$ x = 1, 6

Hence, actual roots are (6,1).

Alternate method:

Since constant = [3 × 2] and coefficient of

$\Rightarrow$ x = [–4x – 3x] = –7

Since quadratic equation is

x^2 – (Sum of roots)x + Product of roots = 0

or $x^2 - 7x + 6 = 0$

Solving the equation,

(x – 6)(x – 1)= 0 or x = (6, 1).

51. b Let the number of five-rupee, two-rupee and one-rupee coins be x, y and z respectively.

x + y + z = 300

5x + 2y + z = 960

5x + y + 2z = 920

y – z = 40

And x + 2y = 340

Using the answer choices, y = 140 satisfies all the given conditions.

52. c xy + yz + zx = 3

$\Rightarrow xy + (y + x)z = 3$

$\Rightarrow xy + (y + x)(5 - x - y) = 3$

$\Rightarrow x^2 + y^2 + xy - 5x - 5y + 3 = 0$

$\Rightarrow y^2 + (x - 5)\,y + x^2 - 5x + 3 = 0$

As it is given that y is a real number, the discriminant for above equation must be greater than or equal to zero.

Hence, $(x - 5)^2 - 4(x^2 - 5x + 3) \geq 0$

$\Rightarrow 3x^2 - 10x - 13 \leq 0$

$\Rightarrow 3x^2 - 13x + 3x - 13 \leq 0$

$\Rightarrow x \in \left[-1, \frac{13}{3}\right]$

Largest value that x can have is $\frac{13}{3}$.

53. d Let the number of gold coins = x + y

$\therefore$ $48(x - y) = x^2 - y^2$

$\Rightarrow$ 48(x – y)= (x – y)(x + y)

$\Rightarrow$ x + y = 48

Hence, the correct choice will be none of these.

54. c Let's assume that

p days : they played tennis

y days : they went for yoga

T days : total duration for which Ram and Shyam stayed together

$\Rightarrow$ p + y = 22

(T – y) = 24 and (T – p) = 14

Adding all of them,

2T = 22 + 24 + 14 $\Rightarrow$ T = 30 days.

55. c $x^2 + 5y^2 + z^2 = 4yx + 2yz$

$\Rightarrow \left(x^2 + 4y^2 - 4yx\right) + \left(z^2 + y^2 - 2yz\right) = 0$

$\Rightarrow (x - 2y)^2 + (z - y)^2 = 0$

It can be true only if x = 2y and z = y

56. d $\frac{A^2}{x} + \frac{B^2}{x-1} = 1 \Rightarrow A^2(x-1) + B^2x = x^2 - x$

When one of A or B is zero, it will be a linear equation which will have one real root. When both A and B are non-zero, it will be a quadratic equation which can have two real roots.

57. c Let the largest piece = 3x

Middle = x

Shortest = 3x – 23

$\therefore$ 3x + x + (3x – 23)= 40

$\Rightarrow$ x = 9

$\therefore$ the shortest piece = 3(9) – 23 = 4

58. a From (A), $(x + y\left(\frac{1}{x} + \frac{1}{y}\right)) = 4$ or $(x + y\left(\frac{y+x}{xy}\right)) = 4$

$\Rightarrow (x + y)^2 = 4xy$

$\Rightarrow (x - y)^2 = 0$

$\Rightarrow$ x = y ... (i)

From (B), $(x - 50)^2 = (y - 50)^2$

On solving,

x(x – 100) = y(y – 100) ... (ii)

This suggests that the values of x and y can either be 0 or 100.

59. b From statement A, we know that for all –1 < x < 1, we can determine |x – 2| < 1 is not true. Therefore, statement A alone is sufficient.

From statement B, –1 < x < 3, we cannot determinewhether |x – 2| < 1 or not. Therefore, statement B alone is sufficient.

60. b $ax^2 + bx + 1 = 0$

For real roots, $b^2 - 4ac \geq 0$

$\therefore b^2 - 4a(1) \geq 0$

$\Rightarrow b^2 \geq 4a$

For a = 1, 4a = 4 $\Rightarrow$ b = 2, 3, 4

a = 2, 4a = 8 $\Rightarrow$ b = 3, 4

a = 3, 4a = 12 $\Rightarrow$ b = 4

a = 4, 4a = 16 $\Rightarrow$ b = 4

$\therefore$ Number of equations possible = 7.

61. c 5x + 19y = 64

We see that if y =1, we get an integer solution for x = 9. Now, if y changes (increases or decreases) by 5x will change (decrease or increase) by 19.

Looking at the options, if x = 294, we get y = –74.

Using these values we see options (a), (b)and (d) are eliminated and also that there exists a solution for 250 < x $\leq$ 300.

62. a Let α be the common root.

$\therefore \alpha^3 + 3\alpha^2 + 4\alpha + 5 = 0$

$\Rightarrow \alpha^3 + 2\alpha^3 + 7\alpha + 3 = 0$

$\Rightarrow \alpha^2 - 3\alpha + 2 = 0$

$\alpha = 2, \ \alpha = 1$

But the above values of α do not satisfy any of the equations. Thus, no root is common.

63. c $1 - \frac{1}{n} < x \leq 3 + \frac{1}{n}$

Putting n = 1

$\therefore 0 < x \leq 4$

64. d $36 \leq n \leq 72$

$$x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4}$$

Putting n = 36, we get

$$x = \frac{(36)^2 + 2 \times 6 \times 40 + 16}{36 + 24 + 4} = 28$$

which is least value of x.

65. d 13x + 1 < 2z and z + 3 = $5y^2$

$\Rightarrow$ 13x + 1 < 2 $(5y^2 - 3)$

$\Rightarrow 13x + 1 < 10y^2 - 6$

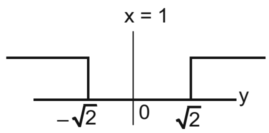
$\Rightarrow 13x + 7 < 10y^2$

Putting x = 1

$20 < 10y^2 \Rightarrow y^2 > 2$

$\Rightarrow\ (y^2 - 2) > 0\ \Rightarrow \left(y+\sqrt{2}\right)\left(y-\sqrt{2}\right) > 0.$

$\therefore y \in \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$



66. a F + n = 4 (k + n) … (i)

M + n = 3 (k + n) …(ii)

From the above equations

F – M = (k + n)

From statement A

F – M = 10 $\Rightarrow$ k + n = 10

F + n = 40

M + n = 30

$\Rightarrow$ F + M + 2n = 70

Hence, from statement A alone, we can get the answer.

67. c $2^x - x - 1 = 0$

$\Rightarrow\ 2^x - 1 = x$

If we put x = 0, then this is satisfied and if we put x = 1, then also this is satisfied.

Now, if we put x = 2, the equation this is not valid.

68. a It is given that $p+q+r \neq 0$. If we consider the option (a), and multiply the first equation by 5, second by –2 and third by –1, we see that the coefficients of x, y and z all add upto zero.

Thus, 5p – 2q – r = 0

No other option satisfies this.

69. a Let 'x' be the number of standard bags and 'y' be the number of deluxe bags.

Thus, $4x + 5y \leq 700$ and $6x + 10y \leq 1250$

Among the choices, (c)and (d) do not satisfy the second equation.

Choice (b)is eliminated as, in order to maximize profits, the number of deluxe bags should be higher than the number of standard bags because the profit margin is higher in a deluxe bag.

70. c Let the number of correct answers be 'x', number of wrong answers be 'y' and number of questions not attempted be 'z'.

Thus, x + y + z = 50 … (i)

And $x - \frac{y}{3} - \frac{z}{6} = 32$

The second equation can be written as,

6x – 2y – z = 192 … (ii)

Adding the two equations we get,

$7x - y = 242$ or $x = \frac{242}{7} + y$

Since x and y are both integers, y cannot be 1 or 2. The minimum value that y can have is 3.

71. d $p + q = \alpha - 2$ and $pq = -\alpha - 1$

$(p + q)^2 = p^2 + q^2 + 2pq$,

Thus $(\alpha - 2)^2 = p^2 + q^2 + 2(-\alpha - 1)$

$p^2 + q^2 = \alpha^2 - 4\alpha + 4 + 2\alpha + 2$

$p^2 + q^2 = \alpha^2 - 2\alpha + 6$

$p^2 + q^2 = \alpha^2 - 2\alpha + 1 + 5$

$p^2 + q^2 = (\alpha - 1)^2 + 5$

Thus, minimum value of $p^2 + q^2$ is 5.

72. b $(a + b + c + d)^2 = (4m + 1)^2$

Thus, $a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd) = 16m^2 + 8m + 1$

$a^2 + b^2 + c^2 + d^2$ will have the minimum value if (ab + ac + ad + bc + bd + cd) is the maximum.

This is possible if a = b = c = d = (m + 0.25) [since a + b + c + d = 4m + 1]

In that case, 2(ab + ac + ad + bc + bd + cd)

$= 12(m + 0.25)^2 = 12m^2 + 6m + 0.75$

Thus, the minimum value of $a^2 + b^2 + c^2 + d^2$

$= (16m^2 + 8m + 1) - 2(ab + ac + ad + bc + bd + cd)$

$= (16m^2 + 8m + 1) - (12m^2 + 6m + 0.75)$

$= 4m^2 + 2m + 0.25$

Since it is an integer, the actual minimum value

$= 4m^2 + 2m + 1$

73. b u is always negative. Hence, for us to have a minimum value of $\frac{vz}{u}$, vz should be positive. Also, for the least value, the numerator has to be the maximum positive value and the denominator has to be the smallest negative value. In other words, vz has to be 2 and u has to be –0.5.

Hence, the minimum value of $\frac{vz}{u} = \frac{2}{-0.5} = -4.$

To get the maximum value, vz has to be the smallest negative value and u has to be the highest negative value. Thus, vz has to be –2 and u has to be –0.5.

Hence, the maximum value of $\frac{vz}{u} = \frac{-2}{-0.5} = 4.$

74. c Here x, y, z are distinct positive real number

So $\frac{x^2(y+z)+y^2(x+2)+z^2(x+y)}{xyz}$

$= \frac{x}{y}+\frac{x}{z}+\frac{y}{x}+\frac{y}{z}+\frac{z}{x}+\frac{z}{y}$

$= \left(\frac{x}{y}+\frac{y}{x}\right)+\left(\frac{y}{z}+\frac{z}{y}\right)+\left(\frac{z}{x}+\frac{x}{z}\right)$

[We know that $\frac{a}{b}+\frac{b}{a}>2$ if a and b are distinct numbers]

> 2 + 2 + 2 i.e. > 6

75. c Since Ms. X bought 21 packets out of which there are 18 O's and A's in total. Since she got one S, there has to be 2 P's which she bought. Hence, both the statements are required to answer the question.

76. b Solution can be found using Statement A alone as we know both the roots for the equation (viz. $\frac{1}{2}$ and $-\frac{1}{2}$).

Also, statement B alone is sufficient.

Since ratio of c and b = 1, c = b.

Thus, the equation is $4x^2 + bx + b = 0$. Since $x = -\frac{1}{2}$ is one of the roots, substituting, we get $1 - \frac{b}{2} + b = 0$ or b = –2.

Thus, c = –2.

77. d $y = \frac{1}{2+\frac{1}{3+y}}$

$\Rightarrow y = \frac{3+y}{7+2y}$

$\Rightarrow 2y^2 + 6y - 3 = 0$

$\Rightarrow y = \frac{-6 \pm \sqrt{36+24}}{4}$

$= \frac{-6 \pm \sqrt{60}}{4} = \frac{-3 \pm \sqrt{15}}{2}$

Since 'y' is a +ve number, therefore:

$y = \frac{\sqrt{15} - 3}{2}$.

78. d

Family	Adults	Children
1	0, 1, 2	3, 4, 5, ...
II	0, 1, 2	3, 4, 5, ...
III	0, 1, 2	3, 4, 5, ...

As per the question, we need to satisfy three conditions namely:

1. Adults (A) > Boys (B)

2. Boys (B) > Girls (G)

3. Girls (G) > Families (F)

Clearly, if the number of families is 2, maximum number of adults can only be 4. Now, for the second condition to be satisfied, every family should have atleast two boys and one girl each. This will result in non-compliance with the first condition because adults will be equal to boys. If we consider the same conditions for 3 families, then all three conditions will be satisfied.

79. c Given equation is x + y = xy

$\Rightarrow xy - x - y + 1 = 1$

$\Rightarrow (x-1)(y-1) = 1$

$\therefore x-1=1$ and $y-1=1$ or $x-1=-1$ and $y-1=-1$

Clearly, (0, 0) and (2, 2)are the only pairs that will satisfy the equation.

80. a Since Group (B)contains 23 questions, the marks associated with this group are 46.

Now check for option (a). If Group (C) has one question, then marks associated with this group will be 3. This means that the cumulative marks for these two groups taken together will be 49. Since total number of questions are 100, Group (A) will have 76 questions, the corresponding weightage being 76 marks. This satisfies all conditions and hence is the correct option. It can be easily observed that no other option will fit the bill.

81. c Since Group (C)contains 8 questions, the corresponding weightage will be 24 marks. This figure should be less than or equal to 20% of the total marks. Check from the options . Option (c) provides 13 or 14 questions in Group (B), with a corresponding weightage of 26 or 28 marks. This means that number of questions in Group (A) will either be 79 or 78 and will satisfy the desired requirement.

82. c **Statement A:** 2 kg potato cost + 1 kg gourd cost < 1 kg potato cost + 1 kg gourd cost

$\Rightarrow$ 1 kg potato cost < 1 kg gourd cost.

Hence, statement A is not sufficient.

Statement B: 1 kg potato cost + 2 kg onion cost = 1 kg onion cost + 2 kg gourd cost

1 kg potato cost + 1 kg onion cost = 2 kg gourd cost.

Hence, statement B is also not sufficient.

Combining both statements, we get

1 kg potato cost < 1 kg gourd cost …(i)

1 kg potato cost + 1 kg onion cost

= 2 kg gourd cost …(ii)

Hence, onion is the costliest.

83. d **Statement A:** 13 currency notes will give different values.

Statement B: Multiple of 10 and by many.

Even if you combine the statement, we can have various values. Hence, the correct option is (d).

84. c $y^2 = x^2$

$2x^2 - 2kx + k^2 - 1 = 0$

$D = 0$

$\Rightarrow 4k^2 = 8k^2 - 8$

$\Rightarrow 4k^2 = 8$

$\Rightarrow k^2 = 2$

$\Rightarrow k = \pm\sqrt{2}$.

$k = +\sqrt{2}$ gives the equation $2x^2 - 2\sqrt{2}x + 1 = 0$;

Its root is $\frac{-b}{2a} = +\frac{1}{\sqrt{2}}$, $k = -\sqrt{2}$ gives the equation $2x^2 + 2\sqrt{2}x + 1 = 0$. Its root is $-\frac{1}{\sqrt{2}}$ this root is –ve, will reject $k = -\sqrt{2}$.

Only answer is $k = +\sqrt{2}$.

Alternate:

Graph based.

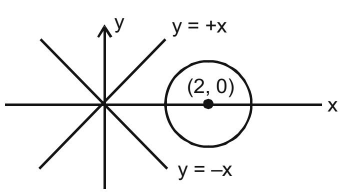
$x^2 - y^2 = 0$ & $(x - k)^2 + y^2 = 1$ are plotted below.

We are solving for a unique positive x.

$x^2 - y^2 = 0$ is a pair of straight lines

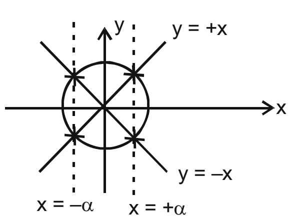
$y = x$ & $y = -x$

$(x - k)^2 + y^2 = 1$ is a circle with center (k, 0) & radius 1.



(a) k = 2; clearly, no solution

(b) k = 0

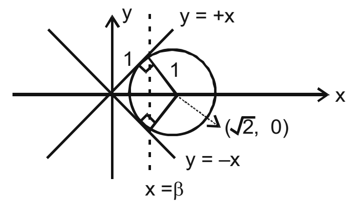


$x = \alpha, -\alpha$ are its two solutions.

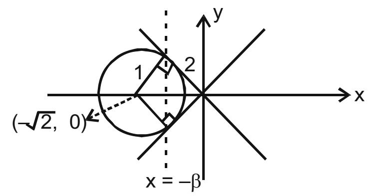
– rejected.

(c) $k = +\sqrt{2}$

unique value of x & a positive one as shown.



(d) $k = -\sqrt{2}$, also gives the unique value of x but it is negative one.



85. c $x = \sqrt{4 + \sqrt{4 - x}} \Rightarrow x^2 = 4 + \sqrt{4 - x}$

$\Rightarrow (x^2 - 4) = \sqrt{4 - x}$

Now putting the values from options, we find only option (c) satisfies the condition.

86. d There are two equations to be formed 40 m + 50 f = 1000

250 m + 300 f + 40 × 15 m + 50 × 10 × f = A

850 m + 8000 f = A

m and f are the number of males and females A is amount paid by the employer.

Then, the possible values of f = 8, 9, 10, 11, 12

If f = 8, m = 15.

If f = 9, 10, 11 then m will not be an integer while f = 12, then m will be 10.

By putting f = 8 and m = 15, A = 18800. When f = 12 and m = 10, then A = 18100

Therefore, the number of males will be 10.

For questions 87 and 88:

Let for Raja allowed luggage be A and excess luggage be E

∴ For Praja, his luggage must be A + 2E.

If all luggage belongs to one, (A + 3E) is the excess.

E corresponds to Rs. 1,200.

∴ A must correspond to (5400 – 3600) = Rs. 1,800

If E = 2x, A = 3x

So total weight = 2(A)+ 3E = 12x

$\Rightarrow x = 5$

Hence, Praja's luggage weight = 7x = 35 kg

Alternate method:

Let Raja = x kg, Free allowance = F kg and Praja = (60 – x) kg

According to the question,

$(x - F)V = 1200$ … (i)

{V = rate of levy on excess luggage}

$(60 - x - F)V = 2400$ … (ii)

$(60 - F)V = 5400$ … (iii)

Dividing equation (ii)by (i), we get $\frac{60 - x - F}{x - F} = 2$

$\Rightarrow 60 - x - F = 2x - 2F$

$\Rightarrow 3x - F = 60$ …(iv)

Dividing (iii) by (i), we get

$\Rightarrow \frac{60 - F}{x - F} = 4.5$

$\Rightarrow 60 - F = 4.5x - 4.5F$

$\Rightarrow 4.5x - 3.5F = 60$ …(v)

Multiplying equation (iv)by 1.5,

$4.5x - 1.5F = 90$

$4.5x - 3.5F = 60$

$2F = 30$

$\Rightarrow F = 15$

Putting value of F in (iv), we get

$3x = 75 \Rightarrow x = 25$

87. d Praja have 35 kg luggage

88. b 15 kg.

89. a $x^{2/3} + x^{1/3} - 2 \le 0$

$\Rightarrow x^{2/3} + 2x^{1/3} - x^{1/3} - 2 \le 0$

$\Rightarrow \left(x^{1/3} - 1\right)\left(x^{1/3} + 2\right) \le 0$

$\Rightarrow -2 \le x^{1/3} \le 1 \Rightarrow -8 \le x \le 1$

90. b $2x + y = 40$

$x \le y$

$\Rightarrow y = 40 - 2x$

Values of x and y that satisfy the equation

x	y
1	38
2	36
3	34
.	.
.	.
.	.
13	14

$\therefore$ 13 values of (x, y)satisfy the equation such that $x \le y$

91. b Let the number be 10x + y so when number is reversed the number because 10y + x. So, the number increases by 18

Hence, $(10y + x) - (10x + y) = 9(y - x) = 18$

$\Rightarrow y - x = 2$

So, the possible pairs of (x, y) are (3, 1) (4, 2)(5, 3) (6, 4), (7, 5) (8, 6) (9, 7)

But we want the number other than 13 so, there are 6 possible numbers, i.e. 24, 35, 46, 57, 68, 79.

So total possible numbers are 6.

92. c Let the number of currency 1 Miso, 10 Misos and 50 Misos be x, y and z respectively.

$\Rightarrow x + 10y + 50z = 107$

Now the possible values of z could be 0, 1 and 2.

For z = 0: $x + 10y = 107$

Number of integral pairs of values of x and y that satisfy the equation x + 10y = 107 will be 11. These values of x and y in that order are (7, 10); (17, 9); (27, 8); … (107, 0).

For z = 1: $x + 10y = 57$

Number of integral pairs of values of x and y that satisfy the equation x + 10y = 57 will be 6. These values of x and y in that order are (7, 5); (17, 4); (27, 3); (37, 2); (47, 1) and (57, 0).

For z = 2: $x + 10y = 7$

There is only one integer value of x and y that satisfies the equation x + 10y = 7 in that order is (7, 0).

Therefore, total number of ways in which you can pay a bill of 107 Misos = 11 + 6 + 1 = 18

93. d Suppose the cheque for Shailaja is of Rs. X and Y paise

As per the question,

$3 \times (100X + Y) = (100Y + X) - 50$

$\Rightarrow 299X = 97Y - 50$

$\Rightarrow Y = \frac{299X + 50}{97}$

Now the value of Y should be a integer.

Checking by options only for X = 18, Y is a integer and the value of Y = 56

94. e $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}, n < 60$

$\Rightarrow \frac{1}{m} = \frac{1}{12} - \frac{4}{n} = \frac{n - 48}{12n}$

$\Rightarrow m = \frac{12n}{n - 48}$

Positive integral values of m for odd integral values of n are for n = 49, 51 and 57.

Therefore, there are 3 integral pairs of values of m and n that satisfy the given equation.

95. b Let f(x) = $ax^2 + bx + c$

At x = 1, f(1)= a + b + c = 3

At x = 0, f(0) = c = 1

The maximum of the function f(x) is attained at

$$x = -\frac{b}{2a} = 1 = \frac{a-2}{2a}$$

$\Rightarrow$ a = –2 and b = 4

$\therefore$ f(x) = $-2x^2$ + 4x + 1

Therefore, f(10)= –159

96. a **Using statement A:** x = 30, y = 30 and z = 29 will give the minimum value.

Using statement B: Nothing specific can be said about the relation between x, y and z.

Hence, option (a)is the correct choice.

97. b $x^3 - ax^2 + bx - c = 0$

Let the roots of the above cubic equation be

$(\alpha - 1), \alpha, (\alpha + 1)$

$\Rightarrow \alpha(\alpha - 1) + \alpha(\alpha + 1) + (\alpha + 1)(\alpha - 1) = b$

$\Rightarrow \alpha^2 - \alpha + \alpha^2 + \alpha + \alpha^2 - 1 = b \Rightarrow 3\alpha^2 - 1 = b$

Thus, the minimum possible value of 'b' will be equal to – 1 and this value is attained at $\alpha = 0$.

98. a Let the three consecutive positive integers be equal to 'n – 1', 'n' and 'n + 1'.

$\Rightarrow n - 1 + n^2 + (n+1)^3 = (3n)^2$

$\Rightarrow n^3 + 4n^2 + 4n = 9n^2$

$\Rightarrow n^2 - 5n + 4 = 0$

$\therefore n = 1$ or $n = 4$

Since the three integers are positive, the value of 'n' cannot be equal to 1, therefore the value of 'n' = 4 or m = n – 1 = 3.

Hence, the three consecutive positive integers are 3, 4 and 5.

99. b Amount of rice bought by the first customer

$$= \left(\frac{x}{2} + \frac{1}{2}\right) \text{kgs}$$

Amount of rice remaining

$$= x - \left(\frac{x}{2} + \frac{1}{2}\right) = \frac{x-1}{2} \text{ kgs}$$

Amount of rice bought by the second customer

$$= \frac{1}{2} \times \left(\frac{x-1}{2}\right) + \frac{1}{2} = \frac{x+1}{4} \text{ kgs}$$

Amount of rice remaining

$$= \left(\frac{x-1}{2}\right) - \left(\frac{x+1}{4}\right) = \frac{x-3}{4} \text{ kgs}$$

Amount of rice bought by the third customer

$$= \frac{1}{2} \times \left(\frac{x-3}{4}\right) + \frac{1}{2} = \frac{x+1}{8} \text{ kgs}$$

As per the information given in the question $\frac{x+1}{8} = \frac{x-3}{4}$ because there is no rice left after the third customer has bought the rice.

Therefore, the value of 'x' = 7 kgs.

100. d $x^2.y^3 = 8$

$\Rightarrow$ (2x).(2x).y.y.y = 2.2.8 = 32

If the product of five variables is constant, then their sum would be minimum if the variables are equal.

For their sum, 4x + 3y, to be minimum all of them must be equal to 2.

$\Rightarrow$ 2x = y = 2

$\Rightarrow$ x = 1, y = 2

So the minimum value of 4x + 3y will be 10.

101. a $a^2 + ab + b^2 = 1$

So $a(a + b) + b^2 = 1$

and $b(a + b) + a^2 = 1$.

Adding the two equations we get:

$(a + b)^2 + a^2 + b^2 = 2$

The integer pairs (a, b) satisfying the equation are: (1, –1), (–1, 1), (1, 0), (0, 1), (–1, 0), (0, –1)

So 6 ordered pairs (a, b) are possible in all.

102. b To maximise the number of incorrect responses, the number of correct responses should also be maximised.

Let the number of correct responses be x.

So the number of incorrect responses = 28 – x

Total marks scored = 3x – (28 – x) > 22

$\Rightarrow$ 4x – 28 > 22

$\Rightarrow$ x > 12.5

The least possible value of x = 13

So the answer = 28 – 13 = 15

103. d Since the coefficient of x^2 is 0, the sum of the three roots of the equation is 0.

If a + b + c = 0, then

$$a^3 + b^3 + c^3 = 3abc = 3 \times \frac{-93}{3} = -93$$

104. c

$$P + \frac{1}{Q} = 1 \Rightarrow \frac{1}{P} = \frac{1}{1 - \frac{1}{Q}} = \frac{Q}{Q-1} \quad \text{...(i)}$$

$$Q + \frac{1}{R} = 1 \Rightarrow R = \frac{1}{1-Q} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$R+\frac{1}{P}=\frac{1}{1-Q}-\frac{Q}{1-Q}=1 \quad \text{...(iii)}$$

Also, $PQR=\left(\frac{Q-1}{Q}\right)Q\left(\frac{1}{1-Q}\right)=-1$ ...(iv)

From (iii) and (iv), we get $PQR+R+\frac{1}{P}=1-1=0.$

105. c Cost per dozen chocolates

$$=\frac{y}{\frac{x}{12}}=\text{Rs. }\frac{12y}{x}$$

Cost per dozen under the offer

$$=\frac{2}{\frac{x+10}{12}}=\text{Rs. }\frac{24}{x+10}$$

Saving per dozen = $\frac{12y}{x}-\frac{24}{x+10}=\frac{80}{100}$

$$\Rightarrow \frac{12y}{x}-\frac{24}{x+10}=\frac{4}{5}$$

$$\Rightarrow \frac{3y}{x}-\frac{6}{x+10}=\frac{1}{5}$$

$$\Rightarrow \frac{15y}{x}-\frac{30}{x+10}=1$$

The equation is satisfied for x = 5, y = 1.

106. a Assume that $a \le b \le c$.

So $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b} \ge \frac{a}{a+c}+\frac{b}{a+b}+\frac{c}{b+c}$,

and

$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b} \ge \frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a}.$$

Adding these two inequalities and dividing the resultant by 2, we get

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \ge \frac{3}{2}$$

$s=\frac{a+b+c}{2}$, where s is the semiperimeter of triangle.

a + b > c and a + b > s.

$$\Rightarrow \frac{c}{a+b}<\frac{c}{s}, \frac{a}{b+c}<\frac{a}{s} \text{ and } \frac{b}{a+c}<\frac{b}{s}$$

But $\frac{a}{s}+\frac{b}{s}+\frac{c}{s}=\frac{a+b+c}{s}=2.$

Hence $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}<2.$

107. b Solving the two linear equations 3x + 4y – 11 = 0 and x + y – 3 = 0, we get x = 1 and y = 2.

Hence, the two lines intersect at the point (1, 2).

Any line which is parallel to 2x + 5y = 0 should be of the form 2x + 5y – k = 0 …(i)

where k is a real number.

Putting x = 1 and y = 2 in (i), we get k = 12.

Hence, the equation of the straight line will be 2x + 5y – 12 = 0.

108. d $^{n+m}C_2 = {}^nC_2 + 11$

$$\frac{(n+m)(n+m-1)}{2}=\frac{n(n-1)}{2}+11$$

$$n^2 + 2nm - n + m^2 - m = n^2 - n + 22$$

$$m^2 + (2n - 1)m = 22$$

$$m(m + 2n - 1) = 22$$

Hence, m = 2 and n = 5.

109. a Suppose $\frac{bx-ay}{bc}=\frac{ay-cz}{ac}=\frac{cz-bx}{ab}=k.$

So

$$bx - ay = kbc$$

$$ay - cz = kac$$

$$cz - bx = kab$$

On adding, k (ab + bc + ca) = 0

or, ab + bc + ca = 0.

110. d Solve the equation to find the answer.

$$(x - 1)^2 + x^2 + (x + 1)^2 = (x + 2)^2 + (x + 3)^2.$$

111. c Quadratic equation $ax^2 + bx + c = 0$ must have two roots which may or may not be identical.

Let $f(x) = ax^2 + bx + c.$

$f(1) = a(1)^2 + b(1) + c$

$= a + b + c = 0$

So, x = 1 is definitely a root of $ax^2 + bx + c = 0$.

Product of roots of $ax^2 + bx + c = 0$ is $\frac{c}{a}$.

So if one of the roots is 1 then the other root must be $\frac{c}{a}$.

112. a Given that

$$x + y = 1$$

$$\Rightarrow x + y - 1 = 0$$

$$\Rightarrow x^3 + y^3 - 1 = -3xy$$

$(a^3 + b^3 + c^3 = 3abc \text{ if } a + b + c = 0)$

$$\Rightarrow x^3 + y^3 + 3xy = 1$$

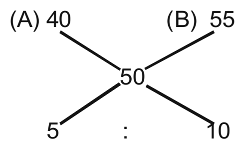
113. a The sum of the roots of $ax^2 + bx + c = 0$ is $\frac{-b}{a}$.

$ax^2 + bx + c$ attains its maximum value at $x = \frac{-b}{2a}$.

$\therefore \quad \frac{-b}{2a} = 2 \Rightarrow \frac{-b}{a} = 4$

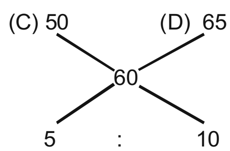
Hence, the sum of the roots = 4.

114. b Using allegation,



$\Rightarrow$ 1 : 2

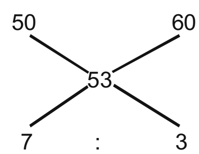
Hence a : b = 1 : 2



$\Rightarrow$ 1 : 2

Hence, c : d = 1 : 2

Also,



Hence, x : y = 7 : 3

To have minimum x (= 7 kg) and y (= 3 kg)

we need $(a + b) \geq 7$ and $(c + d) \geq 3$.

Hence, minimum a + b = 9 (as it has to be in the ratio of 1 : 2 it must be a multiple of 3)

Minimum c + d = 3.

Minimum a + b + c + d + x + y

= 9 + 3 + 7 + 3 = 22 kg.

115. d The equations formed by the roots of the equation $(x - a)(x - b)(x - c)$ can be as follows:

(i) $(x - a)(x - b) \Rightarrow$ Roots are a, b

(ii) $(x - b)(x - c) \Rightarrow$ Roots are b, c

(iii) $(x - c)(x - a) \Rightarrow$ Roots are c, a

(iv) $(x - a)^2 \Rightarrow$ Roots are a, a

(v) $(x - b)^2 \Rightarrow$ Roots are b, b

(vi) $(x - c)^2 \Rightarrow$ Roots are c, c

Adding all these roots, we get 4(a + b + c).

116. c By the question,

$x(x - 3) = -1$

$\Rightarrow x^3(x - 3)^3 = -1$

$\Rightarrow x^3(x^3 - 27 - 9x^2 + 27x) = -1$

$\Rightarrow x^3(x^3 - 18) + x^3(-9 - 9x^2 + 27x) = -1$

$\Rightarrow x^3(x^3 - 18) - 9x^3(x^2 - 3x + 1) = -1$

$\Rightarrow x^3(x^3 - 18) - 9x^3(-1 + 1) = -1$

$\Rightarrow x^3(x^3 - 18) = -1.$

117. a We obtain the sum of all the coefficients of a polynomial by equating all the variables to 1. Here by putting x = 1 in the polynomial, the required sum comes out to be zero.

118. c $x^2 + rx + s = 0$

r = – (sum of roots)   r = –ve

s = product of roots   s = +ve

s – r = +ve

r + s + 1

= – a – b + ab + 1 (where a & b are roots)

= (a – 1) (b – 1)

= +ve

$\Rightarrow \frac{+ve}{+ve} = +ve$

Alternative Method:

Take roots as 2, 2

$\Rightarrow r = -4 \;\&\; s = 4 \Rightarrow \frac{r + s + 1}{s - r} = \frac{1}{8} = +ve$.

119. b Let the number of Re. 1, 50 paise and 25 paise coins be 360, 432 and 576 respectively (ratio 5 : 6 : 8).

Re. 1	50 paise	25 paise
360	432	576

I transaction: 3/5th of Re. 1 coins changed

216 coins of Re. 1 would be changed with 144 coins of 50 paise and 576 coins of 25 paise (so that total 50 paise coins = 576 and total 25 paise coins = 1152 in the ratio 1:2)

Re. 1	50 paise	25 paise
144	576	1152

II transaction: Half of 50 paise coins to Re. 1 and all 25 paise coins to Re. 1 and 50 paise in the ratio 7:4

Half of 50 paise coins $\Rightarrow$ 144 coins of Re. 1

1152 coins of 25 paise $\Rightarrow$ 224 coins of Re. 1 and 128 coins of 50 paise

Re. 1	50 paise	25 paise
512	416	0

Ratio = 512 : 416 = 16 : 13.

120. c By assuming the values of x, y, z and t, (a) and (b) can be very easily ruled out.

Checking option (c), if x > y + z, then x > y and x > z(since all numbers are positive).

So, using statements I and II, x > z > t > y.

So, option (c) is correct.

121. As 'a' and 'b' are the roots of the given equation,

$$a + b = -7$$

$$ab = 4$$

The given expression can be rewritten as

$$\frac{2}{7}\left(\frac{a+b}{ab}\right)+ab = \frac{2}{7}\left(\frac{-7}{4}\right)+4=\frac{7}{2}.$$

122. $a^2 - 1 = -6a$

$$\Rightarrow \quad a - \frac{1}{a} = -6$$

Now, $\left(a+\frac{1}{a}\right)^2 - 5\left(a-\frac{1}{a}\right) = \left(\left(a-\frac{1}{a}\right)^2+4\right)-5\left(a-\frac{1}{a}\right)$

$$= \left((-6)^2+4\right)-5\times-6$$

$$= 40 + 30 = 70.$$

123. $\frac{4}{7}<\frac{x}{y}<\frac{12}{13}, \Rightarrow \frac{13x}{12}<y<\frac{7x}{4}$

As 'x' increases, the interval in which 'y' lies also increases.

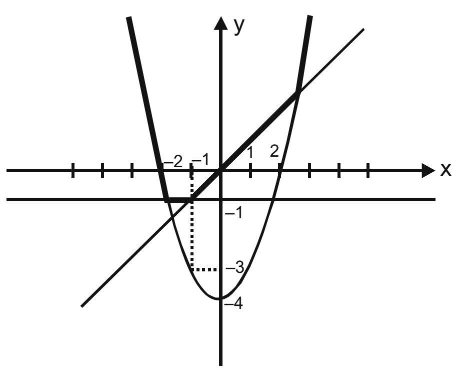
For x = 1, the intervals in which y lies is $\left(1\frac{1}{12}, 1\frac{3}{4}\right)$.

This interval does not contain any integer.

For x = 2, the interval in which y lies is $\left(2\frac{1}{6}, 3\frac{1}{2}\right)$ and 3 lies in the interval.

Hence, the minimum possible value of y is 3.

124. The graph of f(x) is shown below.



We can see from the graph that the minimum value of f(x) is –1.

125. Given, $x+\frac{1}{y+\frac{1}{z}}=\frac{68}{21}=3+\frac{5}{21}$.

As x is a natural number, the only possible value of x is 3.

$$\therefore \quad y+\frac{1}{z} = \frac{21}{5}=4+\frac{1}{5}$$

As y and z are natural numbers, the only possible values of y and z are 4 and 5 respectively.

$$\therefore \quad x + y + z = 3 + 4 + 5 = 12$$

126. $x^2 + (a + 3)x - (a + 5) = 0$

Let α and β are the roots of the above equation

So $\alpha + \beta = -\frac{(a+3)}{1}$

$\alpha\beta = -\frac{(a+5)}{1}$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - \alpha \times \beta$

$= (a + 3)^2 - 2(-(a + 5))$

$a = a^2 + 9 + 6a + 2a + 10$

$= a^2 + 8a + 19.$

$= (a + 4)^2 + 3$

At a = –4, minimum value = 3.

127. $9^{x-\frac{1}{2}} - 2^{2x-2} = 4^x - 3^{2x-3}$

$\Rightarrow 3^{2x-1} + 3^{2x-3} = 2^{2x} + 2^{2x-2}$

$\Rightarrow \frac{3^{2x}}{3}+\frac{3^{2x}}{27}=2^{2x}+\frac{2^{2x}}{4}$

$\Rightarrow \frac{210\times3}{27}=\frac{5\times2}{4}$

$\Rightarrow \left(\frac{3}{2}\right)^{2x}=\frac{27}{8}=\left(\frac{3}{2}\right)^3 \Rightarrow 2x = 3 \Rightarrow x = 3/2.$

128. $u^2+(u-2v-1)^2=-4v(u+v)$

Put $u=\frac{1}{2}$ and $v = -\frac{1}{4}$ in the above equation,

$$\text{L.H.S.}=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}+\frac{1}{2}-1\right)^2=\frac{1}{4}$$

$$\text{R.H.S} = -4\times-\frac{1}{4}\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{1}{4}$$

So, L.H.S = R.H.S.

$$\therefore \; u+3v=\frac{1}{2}+3\times\left(-\frac{1}{4}\right)=-\frac{1}{4}$$

129. Given equation is $n^3 - 11n^2 + 32n - 28$

$= (n - 4)^3 + n^2 - 16n + 36$

$= (n - 4)^3 + (n - 6)^2 - 4n$

We can straight away check the value for the given equation at n = 6, to quickly get – 16

Similarly, n = 7 gives the answer as zero thereby making the required answer as 8.

130. Given that: $N^N = 2^{160}$

$\Rightarrow N^N = 2^{5\times2\times16}$ or N = 32

Now, $N^2 + 2^N = 32^2 + 2^{32} = 2^{10} + 2^{32} = 2^{10}(1 + 2^{22})$

Hence, largest possible value of x = 10.

131. Given that, $A = \{6^{2n} - 35n - 1: n = 1, 2, 3....\}$ and $B = \{35(n - 1) : n = 1, 2, 3...\}$

Since, $B = 35(n - 1)$ so $B = 35k$

and in $A = 6^{2n} - 35n - 1$, 35n will always be a multiple of 35

$\Rightarrow 6^{2n} - 1 = (6n + 1)(6n - 1)$

Therefore, putting n = 1, 6... we get multiples of 35 in A.

However, smaller multiples of 35 are not present in A such as 70, 105 etc.

Hence, every member of A is in B and at least one member of B is not in A.

132. Given equations are:

$2x^2 - ax + 2 > 0$

(Upward parabola that doesn't touch the x - axis)

and $x^2 - bx + 8 \geq 0$ (Upward parabola)

Therefore, $D_1 < 0$ for $(2x^2 - ax + 2 > 0)$ and $D_2 = 0$ for $(x^2 - bx + 8 \geq 0)$

$\Rightarrow a^2 - 16 < 0$ and $b^2 - 32 \leq 0$

Now, to find out the largest possible value of 2a – 6b we will take largest possible value of 'a' which is 3 and smallest possible value of 'b' which is – 5.

Hence, required answer = 2 × 3 – (6 × – 5) = 36.

133. For n = 2, we have $t_1 + t_2 = 39$

For n = 3, we have $t_1 + t_2 + t_3 = 58$ and thus $t_3 = 19$

For n = 4, we have $t_1 + t_2 + t_3 + t_4 = 81$ and thus $t_4 = 23$

For n = 5, we have $t_1 + t_2 + t_3 + t_4 + t_5 = 108$ and thus $t_5 = 27$

Thus we can see that from t_3 onwards an AP has been formed with first term as 19 and common difference as 4.

$\Rightarrow 19 + (n - 1) \times 4 = 103$

$\Rightarrow 19 + 4n - 4 = 103$

Hence, n = 22 and k = 22 + 2 = 24.

Functions and Graphs

1. b $x \times x = 1.5x - x^2$ and $y \times y = 1.5y - y^2$.

For $x \times x < y \times y$ to be true, $1.5x - x^2 < 1.5y - y^2$

$\Rightarrow x(1.5 - x) < y(1.5 - y)$

Option I: 1 > x > y

Thus, x × x and y × y must be greater than 0.5.

If x = 0.6 and y = 0.9

In this case x × x = y × y

Thus, this condition is not always true.

Option II: x > 1 > y

Here, y × y must be greater than 0.5 and x × x must be less than 0.5.

This condition is always true.

Option III: 1 > y > x

Thus, x × x and y × y must be greater than 0.5.

If x = 0.6 and y = 0.9

In this case x × x = y × y

Thus, this condition is not always true.

Option IV: y > 1 > x

Here, x × x must be greater than 0.5 and y × y must be less than 0.5.

This condition can never be true.

2. a The summation of all terms = $\frac{p}{(p - 1)} = \frac{-p}{(1 - p)}$,

comparing this expression with sum of an infinite geometric progression with first term as 'a' and common ration = r, i.e. $\frac{a}{1-r}$ a = – p and r = p f(k) is the k^{th} term of geometric progression.

Hence, $f(k) = -p \times p^{(k-1)}$.

3. b For each of the given expressions, you may have to simplify and express x in terms of y and hence verify for which one does the form and structure remain the same. In general, any function of the form $y = \frac{(ax + b)}{(bx - a)}$ reflects on to itself as we arrange it can be found that $x = \frac{(ay + b)}{(by - a)}$.

Hence, the answer is (b).

4. d $y = f(x) = \frac{1-x}{1+x}$

$\Rightarrow y(1 + x) = 1 - x$

$\Rightarrow y + xy = 1 - x$

$\Rightarrow x + xy = 1 - y$

$\Rightarrow x(1 + y) = 1 - y$

$\Rightarrow x = \frac{1-y}{1+y} = f(y)$

5. d For Y to be maximum,

$x + 2 = 3 - x \Rightarrow x = 0.5$

Maximum value of Y = x + 2 = 2.5

6. a $f(x) = |x|^3$

$\therefore f(-x) = |-x|^3 = |x|^3 = f(x)$.

Hence, the given function is even.

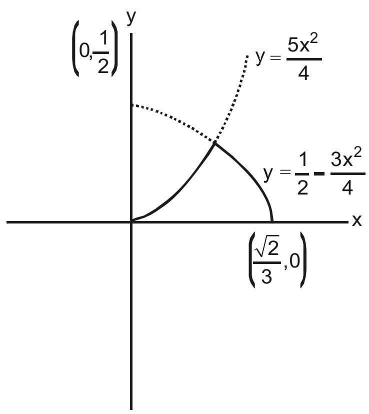
7. b Let f(x) = g(x) + h(x),

where g and h are odd functions.

$\therefore f(-x) = g(-x) + h(-x) = -g(x) - h(x) = -f(x)$.

Hence, f(x)is an odd function.

8. d



So maximum possible value will be at the point of intersection of the two graphs.

$\therefore\ \frac{1}{2}-\frac{3x^2}{4}=\frac{5x^2}{4}$

$\Rightarrow x^2=\frac{1}{4}$

Hence, required maximum value

$=\frac{5x^2}{4}=\frac{5}{4}\times\frac{1}{4}=\frac{5}{16}.$

9. b $\text{Ma}\left[\text{md}(-2),\text{mn}\left(\text{md}(-3),-2\right),\text{mn}\left(6,\ \text{md}(-8)\right)\right]$

$=\text{Ma}\left[2,\text{mn}(3,-2),\text{mn}(6,8)\right]=\text{Ma}\left[2,-2,\ 6\right]=6.$

10. a For a > b, the given equation reduces to

$\text{Ma}\left[|a|,b\right]=\text{mn}\left[a,|a|\right].$

If b < a < 0, then $|b|>|a|>0>a>b.$

$\therefore \text{Ma}\left[|a|,b\right]=|a|$ and $\text{mn}\left[a,|a|\right]=a.$

Hence, option (a)is correct.

11. b Let $g(x)=\frac{x-3}{2}$ be y. So, $fog(x)=f(y)=2y+3.$

Substituting y = $\frac{x-3}{2}$, we get

$fog(x)=(x-3)+3=x=\frac{[(2x+3)-3]}{2}=gof(x).$

12. c If $2x+3=\frac{\left[(x-3)-3\right]}{2}$, then $x=-4.$

13. b From Question 11, $fog(x)=gof(x)=x,$ you will realise that if you were to form a chain of these functions for even number of times, you would still end up getting x.

E.g. $fogofog(x)=fog(x)=x.$ Since both the brackets have the functions repeated for even number of times, each of their value will be x and their product will be x^2.

14. c From question 11, gof(x)= fog(x) = x.

fo(fog)o(gof)(x) = fo(fog)(x) = f(x) = 2x + 3.

15. c If x = 1, we have min(3, 3)= 3.

If x = 2, we have min(6, 0)= 0.

If x = 3, we have min(11, –3) = –3.

If x = 0.5, we have min(2.25, 4.5) = 2.25.

If x = 0.3, we have min(2.09, 5.1) = 2.09.

Thus, we find that as x increases above 1 and when it decreases below 1, the value of the function decreases. It is maximum at x = 1 and the corresponding value = 3.

Hint: Please note that the highest value of the given fraction will be at a point where $(2+x^2)=(6-3x)$, as even if one of the values increases beyond this, the other value will be the minimum value.

If we equate the two, we get $x^2+3x-4=0$. Solving this, we get x = 1 or x = –4.

Since x > 0, it has to be 1 and hence the result.

16. a If a = –2 and b = –3, then our expression will be me(–2 + mo(le(–2, –3)), mo(–2 + me(mo(–2), mo(–3)))

= me(–2 + mo(–3), mo(–2 + me(2, 3)))

= me(–2 + 3, mo(–2 + 3))

= me(1, mo(1)) = me(1, 1) = 1.

17. d Please note that the fastest way to solve these sums is the method of simulation, i.e., select any arbitrary values in the range given and verify whether the option holds good. E.g. a = 2, b = 3.

In this case, option (a)LHS = mo(le(2, 3)) = mo(2) = 2.

RHS = (me(mo(2), mo(3))= (me(2, 3)) = 3. Hence, LHS < RHS.

(b) LHS = mo(le(2, 3))= mo(2) = 2. RHS = (me(mo(2), mo(3)) = (me(2, 3))= 3. Hence, LHS < RHS.

(c) LHS = mo(le(2, 3))= mo(2) = 2. RHS = (le(mo(2), mo(3)) = le(2, 3)= 2. Hence, LHS = RHS.

(d) LHS = mo(le(2, 3))= mo(2) = 2. RHS = (le(mo(2), mo(3)) = le(2, 3)= 2. Hence, LHS = RHS.

18. b Let us verify by taking arbitrary values of a in the range specified.

(a) a > 3. Let a = 4.

So me(a^2 – 3a, a – 3)= me(4, 1) = 4 > 0.

(b) 0 < a < 3. Let a = 2.

So me(a^2 – 3a, a – 3)= me(–2, –1) = –1 < 0.

(c) a < 0. Let a = –1.

So me(a^2 – 3a, a – 3)= me(4, –4) = 4 > 0.

(d) a = 3, me(a^2 – 3a, a – 3) = me(0, 0) = 0.

19. d We can work this on the above lines. E.g.

(a) a > 3. Let a = 4. So le(a^2 – 3a, a – 3)= le(4, 1) = 1 > 0.

(b) 0 < a < 3. Let a = 2. So le(a^2 – 3a, a – 3) = le(–2, –1) = –2 < 0.

(c) a < 0. Let a = –1. So le(a^2 – 3a, a – 3)= le(4, –4) = –4 < 0.

20. d M(M(A(M(x, y), S(y, x)), x), A(y, x)) = M(M(A(M(2, 3), S(3, 2)), 2), A(3, 2))= M(M(A((2x3), (3 - 2)), 2), A(3, 2))

= M(M(A(6, 1), 2), A(3, 2))

= M(M((6 + 1), 2), (3 + 2))

= M(M(7, 2), 5)

= M((7 × 2), 5)

= M(14, 5)

= (14 × 5)

= 70.

21. b S[M(D(A(a,b),2),D(A(a,b),2)),M(D(S(a,b),2), D(S(a, b),2))] = S[M(D((a + b),2),D((a + b),2)), M(D((a – b),2), D((a – b),2))]

$= S\left[M\left\{\frac{(a+b)}{2}, \frac{(a+b)}{2}\right\}, M\left\{\frac{(a-b)}{2}, \frac{(a-b)}{2}\right\}\right]$

$= S\left[\left\{\frac{(a+b)}{2}\right\}^2, \left\{\frac{(a-b)}{2}\right\}^2\right]$

$= \left\{\frac{(a+b)}{2}\right\}^2 - \left\{\frac{(a-b)}{2}\right\}^2$

$= \frac{4ab}{4} = ab$

22. b This question can be done by assuming some values for x, y and z. E.g. let x = 4, y = 3 and z = 1. Thus,

la(4, 3, 1) = min(7, 4)= 4; le(x, y, z) = max(1, 2) = 2;

ma(x, y, z) = $\frac{1}{2}$(4 + 2)= 3. Hence, we can see that the only answer-choice that satisfies the relationship is ma(x, y, z)< la(x, y, x).

23. b $ma(10,\ 4,\ le(la(10,\ 5,\ 3),\ 5,\ 3))$

$= ma(10, 4,\ le(min(15,8), 5, 3))$

$= ma(10,\ 4,\ le(8,\ 5,\ 3))$

$= ma(10, 4,\ max\ (3,2))$

$= ma(10,\ 4,\ 3)$

$= \frac{1}{2}$ [le(10, 4, 3)+ la(10, 4, 3)]

$= \frac{1}{2}$ [max(6,1)+ min(14, 7)]

$= \frac{1}{2}$ (6 + 7) = 6.5

24. c $le(15, min(10, 6), le(9, 8, ma(15, 10, 9)))$

Now $ma(15, 10, 9) = \frac{1}{2}$ [le(15, 10, 9)+ la(15, 10, 9)]

$= \frac{1}{2}$ [max(5, 1)+ min(25,19)]

$= \frac{1}{2}$ (5 + 19) = 12

Hence, our original expression would now be

$le(15, min(10, 6), le(9, 8, 12))$

$= le(15, 6, max(1, -4))$

$= le(15, 6, 1)$ = max(9, 5)= 9

25. c Since both 2 and 1 are positive, (2 # 1)= 2 + 1 = 3.

$(1\nabla 2) = (1\times 2)^{1+2} = 2^3 = 8.$

Thus, the given expression is equal to $\frac{3}{8}$.

26. a Let us first simplify the numerator. Since 1 is positive, (1 # 1)is 1 + 1 = 2 which again is positive. Then

(1 # 1)# 2 = 2 # 2 = 2 + 2 = 4

Now note that $\log_{10} 0.1 = \log_{10} 10^{-1} = -1$

Then $10^{1.3} \log_{10} 0.1 = 10^{1.3} \times (-1)$ is negative.

So $10^{1.3} \nabla \log_{10} 0.1 = 1$

Hence, the numerator is equal to 4 –1 = 3

Since 1 × 2 = 2 is positive, $(1\nabla 2) = (1\times 2)^{1+2} = 2^3 = 8.$

So the denominator = 8. Hence, the answer is $\frac{3}{8}$.

27. b The best possible way to solve this is to check each of the given answer choices. In options (a), (c) and (d), either both X and Y are positive or both X and Y are negative. Since we have (–Y) in the numerator of our expression and (–X) in the denominator, X and Y will never be both positive and neither will XY be positive. Hence, both the numerator and the denominator of our expression will be 1 and the value will always be 1. Hence, the only possible answer choice is (b).

28. d $|r-6| = 11 \Rightarrow r-6 = 11,\ r = 17$

or –(r – 6) = 11, r = –5

$|2q - 12| = 8 \Rightarrow 2q - 12 = 8$, q = 10

or 2q – 12 = –8 , q =2

Hence, minimum value of $\frac{q}{r} = \frac{10}{-5} = -2.$

For questions 29 to 31:

$f(x,y) = |x+y|$ – – – This is always positive

$F(f(x,y)) = -f(x,y) = -|x+y|$ – – – This is always negative $G(f(x,y)) = -F(f(x,y)) = -(-|x+y|) = |x+y|$ – – – This is always positive

29. d $F(f(x,y))G(f(x,y)) = -|x+y|^2$

and $G(f(x,y)).G(f(x,y)) = |x+y|^2$

From the choices, we observe that:

Option (a): LHS of the expression is $-|x+y|^2$, which is always non positive. RHS of the expression is $|x+y|^2$, which is always non negative. The only situation when LHS is equal to RHS is when each is equal to zero. Hence, (a) is not necessarily true.

Option (b): The given expression can be written as

$-|x+y|^2 > |x+y|^2$ or $0 > 2|x+y|^2$.

This implies that 0 > |x + y|, which is not true. Hence, (b) is not true.

Option (c): $F(f(x,y))G(f(x,y)) = -|x+y|^2$

and $G(f(x,y)).G(f(x,y)) = |x+y|^2$

These two expressions can be equal if |x + y| = 0. Hence, (c)is not necessarily true.

Option (d): $F(f(x,y)) + G(f(x,y)) + f(x,y)$

$= -|x+y| + |x+y| + |x+y| = |x+y|$

$f(-x,-y) = |(-x)+(-y)| = |-x-y| = |-(x+y)| = |x+y|$

Therefore, the two expressions are equal.

30. c $f(G(f(1,0)), f(F(f(1,2)), G(f(1,2))))$

$= f(G(f(1,0)), f(3,-3))$

$= f(G(f(1,0)), 0)$

$= f(-1,0) = 1.$

31. c The option (c) yields x^2.

$-F(f(x,x)) \cdot G(f(x,x)) \div \log_2 16$

$= -(-2x \,.\, 2x) \div \log_2 16$

$= \frac{4x^2}{\log_2 2^4} = x^2$

32. d The graph F(x)represents the function F(x) = | x |, where x is any real number.

The graph of F1(x)represents the function F1(x) = – x, where x is any real number.

None of the given relationships are satisfied by these two functions.

Alternate solution:

F1(–2) = 2 = F(–2)and F1(2) = –2. But F(2) = 2.

So the correct option is (d).

33. b $F(x) = \begin{cases} 0 & \text{when } x \ge 0 \\ x & \text{when } x < 0 \end{cases}$

and $F1(x) = \begin{cases} -x & \text{when } x > 0 \\ 0 & \text{when } x \le 0 \end{cases}$

Therefore, replacing x by (–x) in above functions, we get

$F(-x) = \begin{cases} 0 & \text{when } x \le 0 \\ -x & \text{when } x > 0 \end{cases}$

and $F1(-x) = \begin{cases} x & \text{when } x < 0 \\ 0 & \text{when } x \ge 0 \end{cases}$

Clearly, F1(x)= F(–x), hence, option (b) is the correct choice.

Alternate solution:

F1(–2) = 0 = F(2)and F1(2) = –2 = F(–2).

So the correct option is (b), i.e. F1(x) = F(–x).

34. b $F(x) = \begin{cases} 0 & \text{when } x \le 0 \\ -x & \text{when } x > 0 \end{cases}$

and $F1(x) = \begin{cases} x & \text{when } x < 0 \\ 0 & \text{when } x \ge 0 \end{cases}$

Therefore, replacing x by (–x) in above functions, we get

$F(-x) = \begin{cases} 0 & \text{when } x \ge 0 \\ x & \text{when } x < 0 \end{cases}$

and $F1(-x) = \begin{cases} -x & \text{when } x > 0 \\ 0 & \text{when } x \le 0 \end{cases}$

Clearly, F1(x)= F(–x), hence, option (b) is the correct choice.

Alternate solution:

F1(–2) = –2 = F(2) and F1(2) = 0 = F(–2).

So the correct option is (b), i.e. F1(x) = F(–x)

35. c $F(x) = \begin{cases} 1-x & \text{when } 0 \le x < 2 \\ 1 & \text{when } -2 < x < 0 \end{cases}$

and $F1(x) = \begin{cases} -1-x & \text{when} -2 < x < 0 \\ -1 & \text{when } 0 \le x < 2 \end{cases}$

Therefore, replacing x by (–x) in above functions, we get

$F(-x) = \begin{cases} 1+x & \text{when } -2 < x < 0 \\ 1 & \text{when } 0 \le x < 2 \end{cases}$

and $F1(-x) = \begin{cases} -1+x & \text{when } 0 \le x < 2 \\ -1 & \text{when } -2 < x < 0 \end{cases}$

$\therefore\ -F(-x) = -\begin{cases} 1+x & \text{when } -2 < x < 0 \\ 1 & \text{when } 0 \le x < 2 \end{cases}$

$= \begin{cases} -1-x & \text{when} -2 < x < 0 \\ -1 & \text{when } 0 \le x < 2 \end{cases} = F1(x)$

Hence, option (c) is the correct option.

Alternate solution:

F1(2) = –1 = F(2) and F1(–2) = 1 = F(–2).

So the correct option is (c), i.e. F1(x) = – F(–x).

36. d Using choices, (a), (b) and (c) could be both negative as well as positive, depending on the values of x and y.

37. a For (a), x, y < – 1. Then value of f(x, y) = $(x + y)^2$ and value of g(x, y) = – (x + y).

Substituting any value of x, y < – 1, we get f(x, y) always greater than g(x, y).

38. d Use choices. For the given set of questions, function j(x, y, z), n(x, y, z) means minimum of x, y, z and h(x, y, z), m(x, y, z) means maximum of x, y, z.

f(x, y, z), g(x, y, z) means the middle value.

39. a Use choices.

40. b The answer is (b) because the denominator becomes zero.

41. c From the graph, x = 2 $\Rightarrow$ f(2) = 1 and x = –2 $\Rightarrow$ f(–2) = 1

Thus, f(2)= f(–2). Hence, f(x) = f(–x)

42. d From the graph, x = 1

$\Rightarrow$ f(1) = 2 and x = –1

$\Rightarrow$ f(–1) = 1

Thus, f(1) = 2f(–1)

Hence, 3f(x) = 6f(–x)

43. b From the graph, x = 4

$\Rightarrow$ f(4) = –2 and x = –4

$\Rightarrow$ f(–4) = 2

Thus, f(4) = –f(–4)

Hence, f(x) = –f(–x)

44. c $f(2)=\frac{1}{3},\ f^2(2)=\frac{3}{4},\ f^3(2)=\frac{4}{7},\ f^4(2)=\frac{7}{11},\ f^5(2)=\frac{11}{18}$

$\therefore\ f(2)\,f^2(2)\,f^3(2)\,f^4(2)\,f^5(2)=\frac{1}{18}.$

45. b $f^1(-2) = -1$

$f^2(-2) = 0$

$f^3(-2) = \frac{1}{1}$

$\therefore f^1(-2) + f^2(-2) + f^3(-2) = -1 + 0 + 1 = 0.$

46. b $x^2 + y^2 = 0.1$

$|x - y|^2 = x^2 + y^2 - 2xy$

$(0.2)^2 = 0.1 - 2xy$

$\Rightarrow$ 2xy = 0.06 $\Rightarrow$ xy = 0.03

Now, $|x| + |y| = \sqrt{x^2 + y^2 + 2xy} = \sqrt{0.1 + 0.06}$

$\therefore$ |x| + |y| = 0.40

Hence, x = 0.3, y = 0.1 or vice versa.

47. b Solving these equations, we get 6 distinct lines.

x + y = 1, x + y = –1, x = 1, x = –1, y = 1 and y = –1. Tracing these curves, we get the area common as 3 square units.

48. b g(1) = f[f(1)] + 1 = 2 . Since f(1) has to be 1, else all the integers will not be covered. f(n) is the set of odd numbers and g(n) is the set of even numbers.

49. b f(1, 2) = f(0, f(1, 1));

Now f(1, 1)= f[0, f(1, 0)] = f[0, f(0, 1)] = f[0, 2] = 3

Hence, f(1, 2) = f(0, 3) = 4

50. c We cannot work the questions individually through I or II. But combining the two statements, we get

$(2 \oplus 0) = (0 \oplus 2) = 0$ and $0 \oplus (-5 \oplus -6) = 0$.

51. c Taking a = b = c = d = 1, we get the minimum value as (1 + 1) (1 + 1)(1 + 1) (1 + 1)

= 2 × 2 × 2 × 2 = 16.

52. c x + y = 1 and x > 0, y > 0

Taking $x = y = \frac{1}{2}$, value of $\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$

$=\left(2+\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2$

$=\frac{25}{4}+\frac{25}{4}=\frac{25}{2}$

It can be easily verified as it is the least value among options.

For questions 53 and 54:

$BA=\frac{r_1+r_2}{n_1},\ MBA_2=\frac{r_1+r_2}{n_1+n_2}$

and $MBA_1=\frac{r_1}{n_1}+\frac{n_2}{n_1}\ \max\left\{0,\ \frac{r_2}{n_2}-\frac{r_1}{n_1}\right\}$

From BA and MBA_2, we get BA $\geq MBA_2$ because $n_1+n_2 \geq n_1$.

From BA and MBA_1, we get $BA \geq MBA_1$ because

$\frac{r_1}{n_1}+\frac{r_2}{n_1}\geq\frac{r_1}{n_1}+\frac{r_2}{n_1}\times\frac{n_2}{r_2}\max\left\{0,\frac{r_2}{n_2}-\frac{r_1}{n_1}\right\}.$

Now from MBA_1 and MBA_2, we get

$\frac{r_1}{n_1}+\frac{r_2}{n_1}\times\frac{n_2}{r_2}\max\left\{0,\frac{r_2}{n_2}-\frac{r_1}{n_1}\right\}\geq\frac{r_1}{n_1+n_2}+\frac{r_2}{n_1+n_2}.$

53. d From the above information, $BA \geq MBA_1 \geq MBA_2$

None of these is the right answer.

54. b BA = 50 where there is no incomplete innings means $r_2 = n_2 = 0 \Rightarrow \frac{r_1}{n_1} = 50$

$$MBA_1 = \frac{r_1}{n_1} + \frac{n_2}{n_1}\max\left[0, \left(\frac{r_2}{n_2} - \frac{r_1}{n_1}\right)\right]$$

$$= 50 + \frac{1}{n_1}\max\left[0, \left(\frac{45}{1} - 50\right)\right]$$

$= 50 + 0$

$= 50$

$$BA = \frac{r_1 + r_2}{n_1} = \frac{50n_1 + 45}{n_1} = 50 + \frac{45}{n_1} > 50$$

$$MBA_2 = \frac{r_1 + r_2}{n_1 + n_2} = \frac{50n_1 + 45}{n_1 + 1} = 50 - \frac{5}{n+1}$$

Hence, BA will increase, MBA_2 will decrease.

55. b $f(x) + f(y) = \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$

$$= \log\left(\frac{(1+x)\ (1+y)}{(1-x)(1-y)}\right)$$

$$= \log\left(\frac{1+x+y+xy}{1+xy-x-y}\right)$$

$$= \log\left(\frac{1+xy+x+y}{1+xy-(x+y)}\right)$$

$$= \log\left(\frac{1+\left(\frac{x+y}{1+xy}\right)}{1-\left(\frac{x+y}{1+xy}\right)}\right)$$

$$= f\left(\frac{x+y}{1+xy}\right)$$

56. d $x - 1 \leq [x] \leq x$

$2x + 2y - 3 \leq L(x, y) \leq 2x + 2y \Rightarrow a - 3 \leq L \leq a$

$2x + 2y - 2 \leq R(x, y) \leq 2x + 2y \Rightarrow a - 2 \leq R \leq a$

Therefore, $L \leq R$

Note: Choice (b)is wrong, otherwise choice (a) and choice (c)are also not correct. Choose the numbers to check.

57. b Number of samosas = 200 + 20n, n is a natural number.

Price per samosa = Rs.(2 – 0.1n)

Revenue $= (200 + 20n)(2 - 0.1n)$

$= 400 + 20n - 2n^2$

$= 450 - 2(n - 5)^2$

Revenue will be maximum if $n - 5 = 0$

$\Rightarrow n = 5$

∴ Maximum revenue will be at (200 + 20 × 5) = 300 samosas.

58. b $xyz = 4$

$y - x = z - y \Rightarrow 2y = x + z$

∴ y is the AM of x, y, z.

Also, $\sqrt[3]{xyz} = 4^{\frac{2}{3}} \Rightarrow \sqrt[3]{xyz} = 2^{\frac{1}{3}}$.

∵ AM ≥ GM

∴ $y \geq 2^{\frac{2}{3}}$

Therefore, the minimum value of y is $2^{\frac{2}{3}}$.

59. b $x = -|a|\, b$

Now $a - xb = a - (-|a|\, b)b$

$= a + |a|\, b^2$

$$\therefore a - xb = \begin{cases} a + ab^2 & \text{if } a \geq 0 \\ a - ab^2 & \text{if } a < 0 \end{cases}$$

$$= \begin{cases} a\left(1 + b^2\right) & \text{if } a \geq 0 \\ a\left(1 - b^2\right) & \text{if } a < 0 \end{cases}$$

Consider first case:

As $a \geq 0$ and $|b| \geq 1$, therefore $(1 + b^2)$ is positive.

∴ $a(1 + b^2) \geq 0 \Rightarrow a - xb \geq 0$

Consider second case.

As $a < 0$ and $|b| \geq 1$, therefore $(1 - b^2) \leq 0$

∴ $a(1 - b^2) \geq 0$ (Since –ve × -ve = +ve and $1 - b^2$ can be zero also), i.e. $a - xb \geq 0$

Therefore, in both cases $a - xb \geq 0$.

60. a $g^2 = g * g = h$

$g^3 = g^2 * g = h * g = f$

$g^4 = g^3 * g = f * g = e$

$\therefore n = 4$

61. d $f \oplus [f * \{f \oplus (f * f)\}] = f \oplus [f * \{f \oplus h\}]]$

$= f \oplus [f * e\}]$

$= f \oplus [f]]$

$= h$

62. a $e^8 = e^2 * e^2 * e^2$

$= e * e * e$

$= e$

If we observe a $*$ anything = a

$\therefore a^{10} = a$

$\therefore \{\underline{a^{10} * (f^{10} \oplus g^9)}\} \oplus e^8$

$= a \oplus e$

$= e$

63. d We can see that x + 2 is an increasing function and 5 – x is a decreasing function. This system of equation will have smallest value at the point of intersection of the two. i.e. 5 – x = x + 2 or x = 1.5.

Thus, smallest value of g(x) = 3.5

64. b **Case 1:** If x < 2, then y = 2 – x + 2.5 – x + 3.6 – x = 8.1 – 3x.

This will be least if x is highest i.e. just less than 2.

In this case, y will be just more than 2.1

Case 2: If $2 \le x < 2.5$, then y = x – 2 + 2.5 – x + 3.6 – x = 4.1 – x

Again, this will be least if x is the highest i.e. just less than 2.5. In this case, y will be just more than 1.6.

Case 3: If $2.5 \le x < 3.6$, then y = x – 2 + x – 2.5 + 3.6 – x = x – 0.9.

This will be least if x is least i.e. x = 2.5.

Case 4: If $x \ge 3.6$, then

y = x – 2 + x – 2.5 + x – 3.6 = 3x – 8.1

The minimum value of this will be at x = 3.6 and y = 2.7

Hence, the minimum value of y is attained at x = 2.5

Alternate method:

At x = 2, f(x) = 2.1

At x = 2.5, f(x) = 1.6

At x = 3.6, f(x) = 2.7

Hence, at x = 2.5, f(x)will be minimum.

65. d When we substitute two values of x in the above curves, at x = –2, we get

y = –8 + 4 + 5 = 1

y = 4 – 2 + 5 = 7

Hence, at x = –2 the curves do not intersect.

At x = 2, y_1 = 17 and y_2 = 11

At x = –1, y_1 = 5 and 2 and y_2 = 5

When x = 0, y_1 = 5 and y_2 = 5

And at x = 1, y_1 = 7 and y_2 = 7

Therefore, the two curves meet thrice when x = –1, 0 and 1.

66. b We have

$f(0) = 0^3 - 4(0) + p = p$

$f(1) = 1^3 - 4(1) + p = p - 3$

If p and p – 3 are of opposite signs, then p(p – 3) < 0

Hence, 0 < p < 3.

67. d When a > 0, b < 0,

ax^2 and –b |x| are non negative for all x,

i.e. $ax^2 - b|x| \ge 0$

$\therefore ax^2 - b|x|$ is minimum at x = 0 when a > 0, b < 0.

68. c $f_1f_2 = f_1(x)f_1(-x)$

$$f_1(-x) = \begin{cases} -x & 0 \le -x \le 1 \\ 1 & -x \ge 1 \\ 0 & \text{other wise} \end{cases}$$

$$= \begin{cases} -x & -1 \le x \le 0 \\ 1 & x \le -1 \\ 0 & \text{other wise} \end{cases}$$

$f_1f_1(-x) = 0 \forall x$

Similarly, $f_2f_3 = -(f_1(-x))^2 \ne 0$ for some x

$f_2f_4 = f_1(-x).\, f_3(-x) = -f_1(-x)\, f_2(-x)$

$= -f_1(-x)\, f_1(x) = 0 \;\forall x$

69. b Checking with options:

Option (b):

$f_3(-x) = -f_2(-x) = -f_1(x) \Rightarrow f_1(x) = -f_3(-x) \;\forall x$

70. d $R = \dfrac{30^{65} - (30-1)^{65}}{30^{64} + (30-1)^{64}} = \dfrac{30^{65} - 30^{65}\left(1 - \frac{1}{30}\right)^{65}}{30^{64} + 30^{64}\left(1 - \frac{1}{30}\right)^{64}}$

$$\Rightarrow R = \frac{36^{65}}{30^{64}}\left\{\frac{1 - \left(1 - \frac{1}{30}\right)^{65}}{1 + \left(1 - \frac{1}{30}\right)^{64}}\right\}$$

$$\Rightarrow R = 30\left\{\frac{1 - (0.96)^{65}}{1 + (0.96)^{64}}\right\}$$

In $\dfrac{1 - (0.96)^{65}}{1 + (0.96)^{64}}$, numerator is only slightly less than 1.

and denominator is only slightly more than 1.

Hence, R is certainly greater than 1.

71. c |x + y| + |x – y| = 4

Replacing "+x" by "– x" & "+y" by "–y" everywhere in the curve, we again get the same equation.

$\Rightarrow$ Curve is symmetric in the 4-quadrants of X–Y plane.

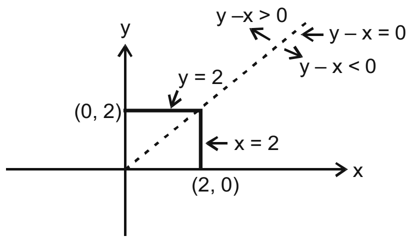
In I-quadrant (x, y > 0)

|x + y| + |x – y| = 4

$= \begin{cases}(x+y)+(y-x)=4;\ y>x\\(x+y)-(y-x)=4;\ y<x\end{cases}$

$= \begin{cases}y=2;\ y>x\\x=2;\ y<x\end{cases}$

The graph looks like below.



Area in I-quadrant = $(2)^2 = 4$

Total area of |x + y| + |x – y| = 4 is

4 × (area of I-quadrant)= 4 × 4 = 16.

72. d g(x + 1) + g(x –1) = g(x)

g(x+ 2) + g(x) = g(x + 1)

Adding these two equations, we get

g(x + 2) + g(x – 1) = 0

⇒ g(x + 3) + g(x) = 0

⇒ g(x + 4) + g(x + 1) = 0

⇒ g(x + 5) + g(x + 2) = 0

⇒ g(x + 6) + g(x + 3)= 0

⇒ g(x + 6) – g(x) = 0

73. b Going by options, we put $x = \frac{-1}{2}$

(a) $2^{-2} = \frac{1}{4}$

(b) $\frac{1}{x} \Rightarrow \frac{1}{-1/2} = -2$

(c) $\frac{1}{x^2} \Rightarrow \frac{1}{(-1/2)^2} = 4$

(d) $2^{-1/2} = \frac{1}{\sqrt{2}}$

(e) $\frac{1}{\sqrt{-x}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}.$

Clearly, $\frac{1}{x}$ bears a negative value only and hence, is the smallest.

74. d From the graph of (y – x)vs. (y + x), it is obvious that inclination is more than 45°.

Slope of line $= \frac{y-x}{y+x} = \tan(45° + \theta)$

$\Rightarrow \frac{y-x}{y+x} = \frac{1+\tan\theta}{1-\tan\theta}$

By componendo-dividendo, $\frac{y}{x} = -\tan\theta$ which is nothing but the slope of the line that shows the graph of y vs. x.

And as 0° < θ < 45°, absolute value of tan θ is less than 1.

$\frac{-1}{\tan\theta}$ is negative and also, greater than 1.

⇒ The slope of the graph y vs. x must be negative and greater than 1. Accordingly, only option (d) satisfies.

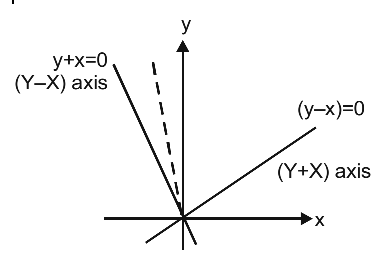
We can also try by putting the values of (y + x) = 2(say) and (y – x)= 4(anything more than 2 for that matter). We can solve for values of y and x and cross check with the given options.

Alternate method:

In the normal X-Y coordinate plane the X-axis corresponds to y = 0

And Y-axis corresponds to x = 0

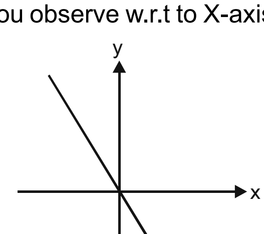
y + x = 0 and y – x = 0 are perpendicular lines on this plane.



And y-x = 0 is the axis Y+X and y+x = 0 is the axis Y-X

So, the dotted line is the graph drawn in the question.

When you observe w.r.t to X-axis it looks like



75. e f(x) = max (2x + 1, 3 - 4x)

So, the two equations are y = 2x + 1 and y = 3 - 4x

y – 2x = 1

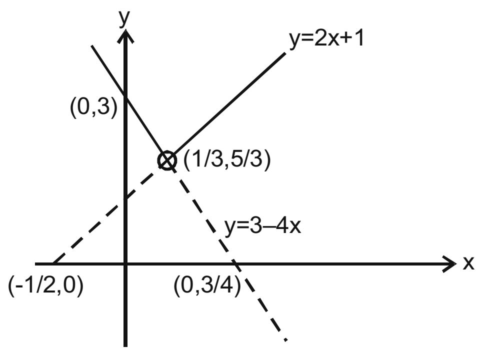
$\Rightarrow \frac{y}{1} + \frac{x}{-1/2} = 1$

Similarly, y + 4x = 3

$\Rightarrow \frac{y}{3} + \frac{x}{3/4} = 1$

Their point of intersection would be

2x + 1 = 3 - 4x $\Rightarrow$ 6x = 2 $\Rightarrow x = \frac{1}{3}$



So when $x \le \frac{1}{3}$, then $f(x)_{max} = 3 - 4x$

and when $x \ge \frac{1}{3}$, then $f(x)_{max} = 2x + 1$

Hence, the minimum of this will be at x = $\frac{1}{3}$

i.e. $y = \frac{5}{3}$

Alternative method:

As f(x)= max (2x + 1, 3 - 4x)

We know that f(x)would be minimum at the point of intersection of these curves

i.e. 2x + 1 = 3 - 4x $\Rightarrow$ 6x = 2 $\Rightarrow x = \frac{1}{3}$

Hence, min f(x) is $\frac{5}{3}$

76. a f(1) + f(2) + f(3)+ …. + f(n) = $n^2 f(n)$, f(1) = 3600.

For n = 2,

$\Rightarrow f(1) + f(2) = 2^2 f(2) \Rightarrow f(2) = \frac{f(1)}{(2^2 - 1)}$

For n = 3, $3600\left(1 + \frac{1}{(2^2 - 1)}\right) + f(3) = 3^2 f(3)$

$\Rightarrow f(3) = 3600 \times \left(\frac{2^2}{2^2 - 1}\right) \times \left(\frac{1}{3^2 - 1}\right)$

Similarly,

$$f(9) = 3600 \times \frac{2^2 \times 3^2 \times 4^2 ... \times 8^2}{(2^2 - 1)(3^2 - 1)(4^2 - 1)...(9^2 - 1)}$$

Therefore, f(9) = 80

77. b Using the given data –

$$\frac{(240 + 40b + 40^2 c) - (240 + 20b + 20^2 c)}{240 + 20b + 20^2 c} = \frac{2}{3}$$

and

$$\frac{(240 + 60b + 60^2 c) - (240 + 40b + 40^2 c)}{240 + 40b + 40^2 c} = \frac{1}{2}$$

Solving the above equations, $c = \frac{1}{10}$ and $b = 10$

So cost for producing x units = $240 + 10x + \frac{x^2}{10}$

Profit earned from x units

$= 30x - \left(240 + 10x + \frac{x^2}{10}\right) = 20x - \frac{x^2}{10} - 240$

$= 760 - \frac{1}{10}(x - 100)^2$

For maximum profit, x – 100 = 0 $\Rightarrow$ x = 100.

78. d Maximum profit = Rs. 760

79. b Given that f(x) = ax^2 + bx + c

Also, f(5) = –3f(2) $\Rightarrow$ f(5) + 3f(2) = 0

$\Rightarrow$ (25a + 5b + c)+ 3(4a + 2b + c) = 0

$\Rightarrow$ 37a + 11b + 4c = 0…(i)

Also, as 3 is a root of f(x)= 0, thus, f(3) = 0.

Therefore, 9a + 3b + c = 0 …(ii)

Using equation (i)and (ii), we get that a = b

Therefore, c = –12a

$\Rightarrow$ f(x) = a(x^2 + x –12)= a(x + 4) (x – 3)

Therefore, the other root of f(x) = 0 is –4.

80. e f(x) = a(x^2 + x –12)

Therefore, the value of a + b + c cannot be uniquely determined.

81. b f(x).f(y) = f(xy)

Given, f(2) = 4

We can also write,

f(2) = f(2 × 1) = f(2)× f(1)

OR f(1) × 4 = 4

$\Rightarrow$ f(1) = 1

Now we can also write,

$f(1) = f\left(2 \times \frac{1}{2}\right) = f(2) \times f\left(\frac{1}{2}\right)$

$\Rightarrow f\left(\frac{1}{2}\right) = \frac{f(1)}{f(2)} = \frac{1}{4}$

82. a f(1, 2) = f(0, f(1, 2 – 1))

(as in f(1, 2): m > 0 and n > 0)

= f(0, f(1, 1))

$= f(0, f(0, f(1, 1 – 1)))$
(as in f(1, 1): m > 0 and n > 0)
$= f(0, f(0, f(1, 0)))$
$= f(0, f(0, f(1 – 1, 1)))$
(as in f(1, 0): m > 0 and n = 0)
$= f(0, f(0, f(0, 1)))$
$= f(0, f(0, 1 + 1))$
(as in f(0, 1): m = 0)
$= f(0, f(0, 2))$
$= f(0, 2 + 1)$
(as in f(0, 2): m = 0)
$= f(0, 3)$
$= 3 + 1 = 4.$
(as in f(0, 3): m = 0)

Also, from above:
f(0, f(1, 1)) = f(0, 3) or f(1, 1) = 3.
Hence, 20[f(1, 2) + f(1, 1) + 15]
= 20 [4 + 3 + 15] = 20×22 = 440.

83. d $y_1 = \frac{(x-1)}{(x+1)}$

$y_2 = f(y_1) = -\frac{1}{x}$

$y_3 = f(y_2) = -\frac{(x+1)}{(x-1)}$

$y_4 = f(y_3) = x$

$y_5 = f(y_4) = \frac{(x-1)}{(x+1)}$

It can be concluded that the given function has the cyclicity of 4 or $y_n = y_{n+4k}$, where k is a whole number.

Hence, $y_{501} = y_1 = \frac{(x-1)}{(x+1)}$.

84. b
f(0) = 10 = c …(i)
f(1) = a + b + c and f(–1) = a – b + c
∴ b = –b = 0 …(ii)
f(2) = 4a + 2b + c = 14
∴ a = 1 …(iii)
From equations (i), (ii) and (iii), we get
f(10) = 100a + 10b + c = 110.

85. b $(x^2+y^2)^2 = 169$

$x^2 + y^2 = +13$ ...(i)

(since $x^2+y^2 \geq 0$)

$(x^2-y^2)^2 = 25$

$x^2 - y^2 = \pm 5$ ...(ii)

Case I:

$x^2 + y^2 = 13$
$x^2 – y^2 = 5$

Solving the equations, we get $x^2 = 9$ and $y^2 = 4$.

Case II:

$x^2 + y^2 = 13$
$x^2 – y^2 = –5$

Solving the equations, we get $x^2 = 4$ and $y^2 = 9$.
In both the cases, $x^2y^2 = 4 × 9 = 36$.

86. c $f(x) = \frac{1}{\log_{5-|x|}\sqrt{x^3-7x^2+14x-8}}$

$= \frac{1}{\log_{5-|x|}\sqrt{(x-1)(x-2)(x-4)}}$

Base of the logarithmic function 5 – |x| > 0 and $5-|x| \neq 1$

So, $x \in (-5,-4) \cup (-4,4) \cup (4,5)$ ...(i)

Also, (x – 1)(x – 2)(x – 4) must be greater than zero as well.

So, $x \in (1,2) \cup (4,\infty)$ ...(ii)

Combining (i) and (ii): $x \in (1,2) \cup (4,5)$

87. a Given that $F(n-1) = \frac{1}{(2-F(n))}$ and F(1)= 2.

For n = 2: $F(1) = \frac{1}{(2-F(2))}$

$\Rightarrow F(2) = \frac{3}{2},$

Similarly, one can find the values of F(3), F(4), F(5) as $\frac{4}{3}$, $\frac{5}{4}$ and $\frac{6}{5}$ respectively.

$\Rightarrow F(n) = \frac{n+1}{n}$

From this we can say that every term except [F(1)], of the series [F(1)] + [F (2)] +…………+ [F(50)] is equal to 1 as for 'n' > 0, F(n) lies between 1 and 2.

Therefore, [F(1)] + [F(2)] +……+ [F(50)] = 51.

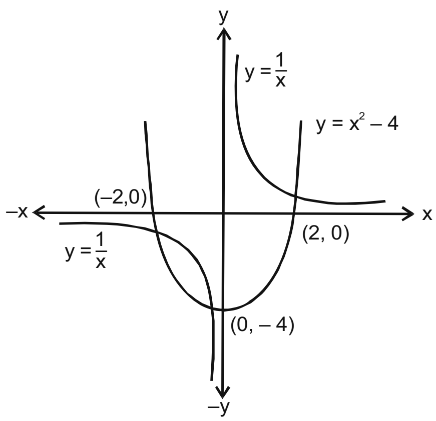
Hence, option (a) is the correct choice.

88. b AT $x = \frac{\pi}{4}$, f(x) = 0

AT $x = \frac{\pi}{2}$, f(x) = ∞

Hence, f(x) lies in the range of (0, ∞).

89. d The graphs of the two functions are shown below :



From the above figure, it is obvious that the graphs of the two functions intersect at three points.

90. c $f(2n) = 1^4 + 2^4 + 3^4 + 4^4 + 5^4 + \ldots + (2n)^4$

$\Rightarrow f(2n) = (1^4 + 3^4 + 5^4 + \ldots + (2n-1)^4) + (2^4 + 4^4 + 6^4 + \ldots + (2n)^4)$

$\therefore 1^4 + 3^4 + 5^4 + \ldots + (2n-1)^4$

$= f(2n) - (2^4 + 4^4 + 6^4 + \ldots + (2n)^4)$

$= f(2n) - 2^4 \times (1^4 + 2^4 + 3^4 + \ldots + n^4)$

$= f(2n) - 16 \times f(n)$.

91. c There would be two cases.

They are as follows:

Case I: $x \geq 0$ … (i)

The inequality becomes,

$$\frac{1}{x-2} < 0.5$$

$\Rightarrow (x-2) < 0.5\,(x-2)^2$

$\Rightarrow (x-2)^2 - 2(x-2) > 0$

$\Rightarrow (x-2)(x-4) > 0$

$\Rightarrow x > 4$ or $x < 2$

Using (i), the range becomes

$x > 4$ or $0 \leq x < 2$ ...(ii)

Case II: $x < 0$ ...(iii)

The inequality becomes,

$$\frac{1}{-x-2} < 0.5$$

$\Rightarrow \frac{1}{x+2} > -0.5$

$\Rightarrow 2(x+2) + (x+2)^2 > 0$

$\Rightarrow (x+2)(x+4) > 0$

$\Rightarrow x > -2$ or $x < -4$

Using (iii), the range becomes

$-2 < x < 0$ or $x < -4$ … (iv)

Combining (ii) and (iv),

The range is $(x < -4) \cup (-2 < x < 2) \cup (x > 4)$.

92. b Let us assume f(0) = K, where 'K' is a constant.

Then, $f(0 + y) = f(0.y) = f(0) = K$

and $f(x + 0) = f(x.0) = f(0) = K$.

This proves that the function is a constant function.

Thus, the value of

$f(-49) = f(49) = 7$

Hence, $f(-49) + f(49) = 14$.

93. b $f(x) = \frac{3}{9-x^2} + \log_{10}(x^3 - x)$

$9 - x^2 \neq 0$

$\Rightarrow x^2 \neq 9$

$\Rightarrow x \neq 3, -3$ … (i)

$x^3 - x > 0$

$\Rightarrow (x-1)x(x+1) > 0$

$\Rightarrow x > 1$

or $-1 < x < 0$ … (ii)

Combining (i) and (ii)

$x \in (-1, 0) \cup (1, 3) \cup (3, \infty)$.

94. c Replace y by – y.

The equation becomes

$\sqrt{3}x - y = \sqrt{3}$.

$\Rightarrow y = \sqrt{3}x - \sqrt{3}$.

95. c By observing we can find that x > 1 and x < 2. Else the RHS $\neq 8$.

So the combinations are [x] = 1, [2x] = 2 or 3, [3x] = 4 or 5

The combinations that give RHS = 8 are 1 + 2 + 5 or 1 + 3 + 4.

For any value of x, the case of "1 + 2 + 5" is not possible. Hence it has to be the case of "1 + 3 + 4". Which will occur

when $x \geq \frac{3}{2}$ and $x < \frac{5}{3}$.

Hence the solution is $\frac{3}{2} \leq x < \frac{5}{3}$.

96. (a) **97.** (c) **98.** (a) **99.** (d) **100.** (54) **101.** (32) **102.** (20)

96. a $(x+1) \times f(x+1) + x \times f(x) + (x-1) \times f(x-1) = 0$ …(i)

In the above equation, replacing x by x – 1, we get

$x \times f(x) + (x-1) \times f(x-1) + (x-2) \times f(x-2) = 0$ …(ii)

From equations (i) and (ii), we get

$(x+1) \times f(x+1) = (x-2) \times f(x-2)$ …(iii)

Replacing x by x + 2 in equation (iii), we get

$x \times f(x) = (x+3) \times f(x+3) = (x+6) \times f(x+6)$ and so on...

Hence, $f(1) = 4f(4) = 7f(7)\ldots$, $2f(2) = 5f(5) = 8f(8)\ldots$ and $3f(3) = 6f(6) = 9f(9)\ldots$

Also, $3f(3) = 6f(6) \Rightarrow f(3) = 2f(6) \Rightarrow f(3) = 180 \times 2 = 360$

By putting the values of f(1) and f(3) in f(1) + 2f(2) + 3f(3) = 0, we get f(2) = –560

Also, 2f(2) = 14f(14) $\Rightarrow$ f(14) = $\frac{f(2)}{7}$ = –80

97. c $f(x) = x^2$, g(x) = 2

At x = 1, f(1) = 1, g(1) = 2

So, f(f(g(1)) + g(1)))

= f(f(2) + g(1))

$f(2) = 2^2 = 4$

So f(4 + 2) = f(6)

$f(6) = 6^2 = 36$.

98. a f(ab) = f(a) f(b)

$f(1) = f(1)^2$

$f(1)^2 - f(1) = 0$

f(1) [f(1) – 1] = 0

f(1) – 1 = 0

f(1) =

99. d To satisfy this, both function should be either less than or equal to zero or greater than equal to zero

Both cannot be less than zero

$f(x) \geq 0$ and $g(x) \geq 0$

$\Rightarrow 2x - 5 \geq 0$ and $7 - 2x \geq 0$

$\Rightarrow 2x \geq 5$ and $7 \geq 2x$

$\Rightarrow x \geq \frac{5}{2}$ and $x \leq \frac{7}{2}$

Hence, $\frac{5}{2} \leq x \leq \frac{7}{2}$

100. 54 f(x + 2) = f(x) + f(x + 1)

f(15) = f(13) + f(14)

f(15) = f(13) + f(12) + f(13)

[$\because$ f(14) = f(12) + f(13)]

617 = 2f(13) + f(12)

617 = 2 [f(11) + f(12)] + f(12)

[$\because$ f(13) = f(11) + f(12)]

617 = 2 [91] + 3f(12)

435 = 3 (f(10 + f(11))

435 = 3f(10) + 273

3f(10) = 162

f(10) = 54

101. 32 $f(x) = \min\{2x^2, 52 - 5x\}$

The maximum possible value of f(x) is obtained when $52 - 5x = 2x^2$.

Solving further we get,

(x – 4) (2x + 13) = 0

x = 4 and $x = -\frac{13}{2}$

$x = -\frac{13}{2}$ will be rejected since x is a positive real number.

At x = 4, f(x) = 32.

102. 20 In order to find the point where the two graphs meet we need to equate the two equations:

$\Rightarrow 5x = 52 - 2x^2$

But we need only positive values of x, so the meeting point or x = 4

For x < 4, $52 - 2x^2$ will be maximum and for x > 4 the line 5x will be maximum.

Hence, f(x) will be minimum at x = 4 and the required value will be f(4) = 20.

Logarithm and Exponents

1. d $\log_7 \log_5 \left(\sqrt{x+5} + \sqrt{x}\right) = 0$,

We know

$\log_a b = x \Rightarrow a^x = b$

$\therefore \log_5\left(\sqrt{x+5} + \sqrt{x}\right) = 7^0$

$\Rightarrow \log_5\left(\sqrt{x+5} + \sqrt{x}\right) = 1$

$\Rightarrow \sqrt{x+5} + \sqrt{x} = 5^1$

$\Rightarrow \sqrt{x+5} = 5 - \sqrt{x}$

Squaring both sides, we get

$\Rightarrow x + 5 = 25 + x - 2 \times 5\sqrt{x}$

$\Rightarrow x + 5 = 25 + x + 10\sqrt{x}$

$\Rightarrow 10\sqrt{x} = 20$

$\Rightarrow \sqrt{x} = 2$

$\Rightarrow \left(\sqrt{x}\right)^2 = (2)^2$

$\Rightarrow x = 4$

2. c Let $\log_6 216\sqrt{6} = x$.

Then by rule, $\log_b a = x \Rightarrow b^x = a$ we have,

$6^x = 216\sqrt{6}$

$6^x = 6^3 \times 6^{\frac{1}{2}} \Rightarrow 6^x = 6^{\frac{7}{2}}$

$\therefore x = \frac{7}{2}$

3. c We know that if $\log_a x = y$, then $x = a^y$. So comparing this form with our equation, we can get $\log_7 (x^2 - x + 37) = 2^1 = 2$ and furthermore from this we can say that

$(x^2 - x + 37) = 7^2 = 49$

Thus, we have the equation

$x^2 - x - 12 = 0$

The solutions of this equation are,

$x = 4$ or $x = -3$.

The value that satisfies the given answer-choices is $x = 4$.

4. c **Statement I:** Given that x satisfies the equation,

$\log_2 x = \sqrt{x}$

$\therefore x = 2^{\sqrt{x}}$

This equation is satisfied by the values x = 4 and 16.

Hence, statement I alone is not sufficient.

Statement II: Nothing concrete can be concluded from the fact that $x \le 10$ km.

Hence, statement II alone is not sufficient.

Combining statements I and II, we get a unique value of x = 4 km.

5. b $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$

$\Rightarrow \log_{10}\left[\frac{x}{\sqrt{x}}\right] = \log_x 100$

$\Rightarrow \log_{10}\sqrt{x} = \frac{\log_{10} 100}{\log_{10} x}$

$\Rightarrow \frac{1}{2}\log_{10} x = \frac{2}{\log_{10} x}$

$\Rightarrow (\log_{10} x)^2 = 4$

$\Rightarrow \log_{10} x = \pm 2$

$\Rightarrow \log_{10} x = 2 \quad \text{or} \quad \log_{10} x = -2$

$\Rightarrow 10^2 = x \quad \text{or} \quad 10^{-2} = x$

$\therefore x = 100 \quad \text{or} \quad x = \frac{1}{100}$

6. b $\frac{1}{3}\log_3 M + 3\log_3 N = 1 + \log_{0.008} 5$

$\Rightarrow \log_3(M^{1/3}N^3) = 1 + \frac{(\log 10 - \log 2)}{\log 8 - \log 1000}$

$\Rightarrow \log_3(M^{1/3}N^3) = 1 - \frac{(1-\log 2)}{3(1-\log 2)}$

$\Rightarrow \log_3(M^{1/3}N^3) = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow M^{1/3}N^3 = 3^{2/3}$

$\Rightarrow MN^9 = 3^2$

$\Rightarrow N^9 = \frac{9}{M}$.

7. d Sum of $\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \cdots$ n terms such problem must be solved by taking the value of number of terms. Let's say 2 and check the given option. If we look at the sum of 2 terms of the given series it comes out to be

$\log m + \log\frac{m^2}{n} \Rightarrow \log\frac{m \times m^2}{n} = \log\left(\frac{m^3}{n}\right)$

Now look at the option and put number of terms as 2, only option (d)validates the above mentioned answer.

$\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{\frac{n}{2}} = \log\left[\frac{m^3}{n}\right]^1 = \log\left(\frac{m^3}{n}\right)$

8. b For the curves to intersect, $\log_{10} x = x^{-1}$

Thus, $\log_{10} x = \frac{1}{x}$ or $x^x = 10$

This is possible for only one value of x ($2 < x < 3$).

9. d Using log a – log b = $\log\left(\frac{a}{b}\right)$, $\frac{2}{y-5} = \frac{y-5}{y-3.5}$, where $y = 2^x$

Solving we get y = 4 or 8

i.e. x = 2 or 3.

It cannot be 2 as log of negative number is not defined (see the second expression).

10. b $u = (\log_2 x)^2 - 6\log_2 x + 12$

$x^u = 256$

Let $\log_2 x = y \Rightarrow x = 2^y$

$x^u = 2^8 \Rightarrow uy = 8 \Rightarrow u = \frac{8}{y}$

$\therefore \frac{8}{y} = y^2 - 6y + 12 \Rightarrow y^3 - 6y^2 + 12y - 8 = 0$

$\Rightarrow (y-2)^3 = 0 \Rightarrow y = 2$

$\therefore x = 4, \quad u = 4$

11. d $P = \log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$

$= \log_x x - \log_x y + \log_y y - \log_y x$

$= 2 - \log_x y - \log_y x$

Let $t = \log_x y$

$P = 2 - \frac{1}{t} - t = -\left[\sqrt{t} - \frac{1}{\sqrt{t}}\right]^2$

which can never be positive. Out of given options, it can't assume a value of +1.

12. e $\log_y x = a.\log_z y = b.\log_x z = a \times b$

$$a = \frac{\log_y x}{\log_z y} \text{ and } b = \frac{\log_y x}{\log_z y}$$

$$\Rightarrow a \times b = \frac{\log_y x}{\log_z y} \times \left(\frac{\log_y x}{\log_z y}\right)$$

$$= \frac{\left(\frac{\log_k x}{\log_k y}\right)}{\left(\frac{\log_k y}{\log_k z}\right)} \times \frac{\left(\frac{\log_k x}{\log_k y}\right)}{\left(\frac{\log_k y}{\log_k z}\right)} \quad [\text{For some base } k]$$

$$= \left(\frac{\log_k x}{\log_k y}\right)^3 = \left(\log_y x\right)^3 = (ab)^3$$

So, $ab - a^3b^3 = 0$

$$\Rightarrow a \times b\left(1 - a^2b^2\right) = 0$$

$$\Rightarrow ab = \pm 1$$

Only option (e)does not satisfy.

13. e Equation (ii)can be written as

$$4^{0.3x} \times 9^{0.2y} = 8 \times (81)^{1/5}$$

$$\Rightarrow (2^2)^{0.3x}(3^2)^{0.2y} = 8 \cdot (81)^{1/5}$$

$$\Rightarrow 2^{0.6x}\ 3^{0.4y} = 2^3 \cdot (3^4)^{1/5} = 2^3 \cdot 3^{4/5}$$

$$\Rightarrow 0.6x = 3$$

$$\Rightarrow x = 5 \quad \text{and } 0.4y = \frac{4}{5}$$

$$\Rightarrow y = 2$$

If we put the values of x and y in first equation these values satisfy the first equation also.

So the answer is x = 5, y = 2

Hence, option (e)is the correct option

14. c $\log_2(a+b) + \log_2(a-b) = 3$

$$\Rightarrow \quad \log_2(a+b)(a-b) = 3$$

$$\Rightarrow \quad \log_2\left(a^2 - b^2\right) = \log_2 2^3$$

$$\Rightarrow \quad a^2 - b^2 = 8$$

Solving the above equation for integer values of a and b, we get (a, b) ≡ (3, 1) or (3, –1).

Note: 'a – b' must be greater than zero.

15. a $$\log_{16} 5 = \frac{1}{4} \times \frac{\log 5}{\log 2} = m \quad \ldots\text{(i)}$$

$$\log_5 3 = \frac{\log 3}{\log 5} = n \quad \ldots\text{(ii)}$$

From equations (i) and (ii), we get

$$m \times n = \frac{\log 5}{\log 2} \times \frac{1}{4} \times \frac{\log 3}{\log 5} \text{ or } \frac{\log 2}{\log 3} = \frac{1}{4mn}$$

Let $\log_3 6$ be equal to k; therefore,

$$\log_3 6 = \frac{\log 6}{\log 3} = \frac{\log 3 + \log 2}{\log 3} = 1 + \frac{\log 2}{\log 3} = k$$

$$\therefore \quad k = \frac{1 + 4mn}{4mn}.$$

16. c $X = (\log_{10} 1 + \log_{10} 2 + \ldots + \log_{10} n)$
$- (\log_{10} 1 + \log_{10} 2 + \ldots + \log_{10} p)$
$- (\log_{10} 1 + \log_{10} 2 + \ldots + \log_{10}(n - p))$

$$\Rightarrow \quad X = \log_{10} n! - \log_{10} p! - \log_{10} (n - p)!$$

$$\Rightarrow \quad X = \log_{10} \frac{n!}{p!(n-p)!}.$$

X is maximum when $\frac{n!}{p!(n-p)!}$ is maximum.

$\Rightarrow \quad \frac{8!}{p!(8-p)!}$ is maximum, i.e. ^8C_p is maximum

$$\Rightarrow \quad p = 4$$

$$\Rightarrow \quad X = \log_{10} \frac{8!}{4!(8-4)!} = \log_{10} 70 = 1 + \log_{10} 7.$$

17. b By the question,

$$\log_3(2^x - 5) - \log_3 2 = \log_3\left(2^x - \frac{7}{2}\right) - \log_3(2^x - 5)$$

$$\Rightarrow \quad \log_3\left(\frac{2^x - 5}{2}\right) = \log_3\left(\frac{2^x - \frac{7}{2}}{2^x - 5}\right)$$

$$\Rightarrow \quad \frac{2^x - 5}{2} = \frac{2^x - \frac{7}{2}}{2^x - 5}.$$

Let $2^x = a$

$$\Rightarrow \quad \frac{a-5}{2} = \frac{a - \frac{7}{2}}{a - 5}.$$

$\Rightarrow \quad a^2 - 10a + 25 = 2a - 7$

$\Rightarrow \quad a^2 - 12a + 32 = 0$

$\Rightarrow \quad (a - 4)(a - 8) = 0$

$\Rightarrow \quad a = 4$ or 8

$\therefore \quad x = 2$ or 3.

Hence, $2^x - 5 = -1$,

when x = 2, which is not possible.

$\therefore \quad x = 3$.

18. 0 Clearly, x > 0

The equation reduces to $2x = x^2 + 2x + 1$

$\Rightarrow 0 = x^2 + 1$

Which does not have real roots.

Hence, there are no solutions.

19. $\log_c a = \frac{1}{2}$ or $c = a^2$ ...(i)

$\log_d b = \frac{1}{3}$ or $d = b^3$ ...(ii)

Clearly, 'a' and 'b' cannot be equal to 1.

Therefore, $a_{min.} = 2$ and $b_{min.} = 3$

The minimum possible value of a + b + c + d

= 2 + 3 + 4 + 27 = 36

20. $\log_3 5 = \log_5 (2 + x)$

Only 3 < x < 23 is true.

21. $\frac{\log(2^3 \times 3^3 \times 5) + \log(2^6 \times 3 \times 5^7) + \log(2 \times 3^2 \times 5^4)}{3}$

$= \log(2^a \times 3^b \times 5^c)$

$\Rightarrow \log(2^8 \times 3^4 \times 5^8) + \log(2 \times 3^2 \times 5^4)$

$= 3\log(2^a \times 3^b \times 5^c)$

$\log(2^9 \times 3^6 \times 5^{12}) = 3\log(2^a \times 3^b \times 5^c)$.

$\Rightarrow 2^9 \times 3^6 \times 5^{12} = (2^a \times 3^b \times 5^c)^3$

$\Rightarrow 3a = 9, a = 3.$

22. $\log_2 (5 + \log_3 a) = 3$

$2^3 = 5 + \log_3 a$

$\log_3 a = 3$

$3^3 = a$

$\Rightarrow a = 27$

$\log_5 (4a + 12 + \log_2 b) = 3$

$5^3 = 4a + 12 + \log_2 b.$

$\Rightarrow \log_2 b = 125 - 120$

$\log_2 b = 5$

$b = 2^5 = 32$

Hence, a + b = 27 + 32 = 59

23. $\log_{12} 81 = p$

$$3\left[\frac{4-p}{4+p}\right] = 3\left[\frac{4 - \frac{\log 81}{\log 12}}{4 + \frac{\log 81}{\log 12}}\right] = 3\left[\frac{4\log 12 - \log 81}{4\log 12 + \log 81}\right]$$

$$= 3\left[\frac{\log\frac{12^4}{81}}{\log(12)^4 \times 81}\right] = 3\frac{\log 4^4}{\log(12)^4 \times (3)^4}$$

$$= \frac{12\log 4}{4\log 36} = \frac{24\log 2}{8\log 6}$$

$$= \frac{3\log 2}{\log 6} = \frac{\log 8}{\log 6} = \log_6 8$$

24. $2^x = 3^{\log_5 2}$

Taking log both sides

$\log 2^x = \log 3^{\log_5 2}$

$x\log 2 = \log_5 2 \times \log 3$

$x\log 2 = \frac{\log 2}{\log 5} \times \log 3$

$x = \frac{\log 3}{\log 5} + 1 - 1$

$$x = \frac{\log 3 - \log 5}{\log 5} + 1 = \frac{\log\frac{3}{5}}{\log 5} + 1 = 1 + \log_5 \frac{3}{5}$$

25. $0.25 \le 2x \le 200$

So, the possible values of x

= –2, –1, 0, 1, 2, 3, 4, 5, 6, 7.

But $2^x + 2$ is perfectly divisible by either 3 or 4, so, on putting x = 0, 1, 2, 4, 6 in $2^x + 2$, the numbers obtained will be perfectly divisible by either 3 or 4. Hence, 5 values of x are possible.

26. Given that $x^{2018}\, y^{2017} = \frac{1}{2}$ and $x^{2016}\, y^{2019} = 8$

$x = \frac{1}{2}$ and y = 2 will satisfy the above equations.

As $\left(\frac{1}{2}\right)^{2018} \times (2)^{2017} = \frac{1}{2}$

$x^{2016}\, y^{2019} = \left(\frac{1}{2}\right)^2 \times (2)^{2019} = 8$

$\therefore x^2 + y^3 = \left(\frac{1}{2}\right)^2 + (2)^3$

$= \frac{1}{4} + 8 = \frac{33}{4}$

27. Let $p^3 = q^4 = r^5 = s^6 = k$

$\Rightarrow p = k^{1/3}, q = k^{1/4}, r = k^{1/5}$ and $s = k^{1/6}$

$\Rightarrow \log_s(pqr)$

$$= \frac{\log k^{\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)}}{\log k^{\frac{1}{6}}} = \frac{\frac{47}{60}\log k}{\frac{1}{6}\log k} = \frac{47}{60} \times 6 = \frac{47}{10}.$$

28. The given expression can be written as:

$$\frac{1}{\log 100} \times (\log 2 - \log 4 + \log 5 - \log 10 + \log 20 - \log 25 + \log 50)$$

$$= \frac{1}{\log 100} \times (\log 2 \times 5 \times 20 \times 50 - \log 4 \times 10 \times 25)$$

$$= \frac{1}{2} \times \frac{1}{\log 10} \times \log \frac{10000}{1000} = \frac{1}{2}.$$

Progressions

1. c $\frac{1}{(1 \times 2)} + \frac{1}{(2 \times 3)} + \frac{1}{(3 \times 4)} + - - - + \frac{1}{(100 \times 101)}$

$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$+ --- + \left(\frac{1}{99} - \frac{1}{100}\right) + \left(\frac{1}{100} - \frac{1}{101}\right) = 1 - \frac{1}{101} = \frac{100}{101}.$$

2. d First elements of each set = 1, 2, 4, 7, 11, 16, …

This series is neither an AP nor a GP, but the difference between the terms viz.1, 2, 3, 4, 5,….. is in AP with both first term and common difference as 1.

Hence, to find the 50^{th} term of the original series we have to add the sum of 49 terms of the second series to the first term of the original series.

The sum of first 49 terms = $\frac{(49 \times 50)}{2}$ = 1225.

Therefore, the 50^{th} term of the original series = (1225 + 1) = 1226.

This will be the first element of the set S_{50}, which will have 50 elements.

The last element of S_{50} will be = 1226 + 49 = 1275 .

So the sum of the elements in this set

$$= \frac{50 \times (1226 + 1275)}{2} = 62525.$$

3. a $U_0 = 2^0 - 1 = 0$

$U_1 = 2^1 - 1 = 1$

$U_2 = 2^2 - 1 = 3$

$U_3 = 2^3 - 1 = 7$ and so on.

Hence, $U_{10} = 2^{10} - 1 = 1023$.

4. c The harmonic mean of two numbers *x* and *y* is $\frac{2xy}{(x + y)}$ and the geometric mean is $\sqrt{xy}$.

$$\therefore \frac{\frac{2xy}{(x+y)}}{\sqrt{xy}} = \frac{12}{13} \Rightarrow \frac{4xy}{(x+y)^2} = \frac{144}{169}.$$

Although this can be simplified to get the answer, the best way to proceed from here would be to look out for the answer choices and figure out which pair of x & y satisfies the above equation. You will find the answer is (c).

HINT : Students please note that this sum is a classic example of how you could have gone for intelligent guess work. Since we know that the denominator of the ratio is the geometric mean, which is $\sqrt{xy}$, the two numbers should be in such a ratio that their product should be a perfect square. The only pair from the answer choices that supports this is 4 & 9, as $\sqrt{4 \times 9} = \sqrt{36} = 6$.

5. c Middle term of an A.P. is average of all the terms in A.P.

Number of terms = 7

Middle term = Fourth term = 8

Therefore, sum of all the terms = 56.

6. d Let the number of stones be ‘n’.

As the person covers 4.8 km, he covers 2.4 km on one side and 2.4 km on other side.

So total distance covered by him = 20 + 40 + 60 +…….

$$\therefore 2400 = \frac{n}{2}\left[2 \times 20 + (n-1)20\right] = 10n(n+1)$$

(Here n is the number of stones)

After solving, we get n = 15

$\therefore$ Total number of stones = 15 + 15 + 1 = 31

7. b Let the number of toffees with the first, second and third boy be x, (x + 4) and (x + 8) respectively.

Hence, total number of toffees = (3x + 12).

The statement I merely suggests that (3x + 12) is a multiple of 2, which means that x is a multiple of 2. Nothing concrete can be concluded on the basis of this statement.

The statement II suggests that (x – 4 + 2), (x + 4 – 6 + 2) and (x + 8 – 4)are in GP or (x – 2), x and (x + 4) is in GP.

$\therefore x^2 = (x+4)(x-2)$

$\Rightarrow x = 4$

$\Rightarrow$ (3x + 12) = 24

Question can be answered using statement II alone.

8. a Since 116 is less than 11^2, it can be figured out that both the first two terms of the AP should be less than 10.

There is only one pair of positive integers whose squares add up to 116 and they are 10 and 4.

Thus, these two should be the first two terms of the AP. Hence, the first term is 4, and can be obtained only from statement I.

Statement II merely suggests that the fifth term is of the form 7k. Nothing correct can be concluded from this.

For questions 9 to 13:

First series: (S_1)= x, y, $\frac{x}{2}$, z, x + 20

Second series: (S_2)= a_1, a_2, a_3, a_4

Now $a_1 = y - x$, $a_2 = \frac{x}{2} - y$, $a_3 = z - \frac{x}{2}$

and $a_4 = x + 20 - z$

$a_2 - a_1 = 30$ gives $3x - 4y = 60$ ... (i)

$a_4 - a_3 = 30$ gives $3x - 4z = 20$ ... (ii)

and $a_4 - a_2 = 60$ gives $x - 2z + 2y = 80$ .. (iii)

Solving these equations we get the values of x = 100, y = 60, z = 70

$\therefore S_1$ = 100, 60, 50, 70, 120

S_2 = –40, –10, 20, 50

14. a Let there be x bacteria in the first generation i.e. $n_1 = x$.

$\therefore n_2 = 8x$, but only 50% survives

$\Rightarrow n_{2,\text{survived}} = \frac{8x}{2} = 4x$

$n_3 = 8(4x)$, but only 50% survives

$n_{3,\text{survived}} = 4^2x$

Similarly,

$n_{7\text{survived}} = 4^{7-1}x$ = 4096 million

$\therefore$ x = 1 million.

15. a Statement II is not required at all as no way can we express X in terms of 'a'.

Statement I implies that X + Y = 2a and $XY = a^2$.

Solving these two, we can say that X = a.

Hence, statement I alone is sufficient to answer the question.

16. d The ideal approach is to pick up the options one by one.

Option (a)– Let S1 and S2 be two sequences of positive numbers. After change of sign, S1 will consist of negative numbers while S2 remains unchanged. Clearly, the members of S1 would be less than that of S2. Hence, option (a) is not correct.

Option (b)- Let S1 and S2 be two sequences of positive numbers. After change of sign, S1 will consist of negative numbers while S2 remains unchanged. Clearly, G would remain in S2 itself. Hence, option (b)is not correct.

Option (c)– If S1 and S2 had same sign, say positive initially, then the largest number of S1 and S2 would be in S2. Then after the change of sign, every member of S1 will be negative and therefore, less than every member of S2. This implies that the largest number would remain in S2. Hence, option (c) is not correct.

17. a The elements of S1 are in the order : $a_1 < a_2 < a_3 < a_4 < . . . < a_{24}$

The elements of S2 are in the order: $a_{25} > a_{26} > . . . > a_{49} > a_{50}$

Even if a_{24} and a_{25} are interchanged, the elements of S1 continues to be in ascending order. However, nothing can be concluded about the elements of S2.

18. d Since every element of S1 is less than or equal to each member of S2, L will be in S1 and G in S2.

For some i $(1 \le i \le 24)$, a_i = L and for some j $(25 \le j \le 50)$, a_j = G.

Every other element of S1 is greater than a_i and every other member of S2 is less than a_j.

Therefore, to make every element of S1 greater than or equal to that of S2, we need to add a minimum of $(a_j - a_i)$= G – L.

19. d $a_1 = 1$, $a_2 = 7$, $a_3 = 19$, $a_4 = 43$.

The difference between successive terms is in series 6, 12, 24, 48, ..., i.e. they are in GP. Hence,

$$a_{100} = a_1 + a\left(\frac{r^n - 1}{r - 1}\right) = 1 + 6\frac{\left(2^{99} - 1\right)}{(2-1)} = 6 \times 2^{99} - 5$$

20. c $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ... + \frac{1}{19.21}$

$$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + ... + \frac{1}{2}\left(\frac{1}{19} - \frac{1}{21}\right)$$

$$= \frac{1}{2} - \frac{1}{42} = \frac{(21-1)}{42} = \frac{20}{42} = \frac{10}{21}$$

21. a Each term has to be either 1 or –1.

Hence, if the sum of n such terms is 0, then n is even.

22. a Amount of money given to X

= 12 × 300 + 12 × 330 + ... + 12 × 570

= 12[300 + 330 + ... + 540 + 570]

$= 12 \times \frac{10}{2}[600 + 9 \times 30]$ = 52200

Amount of money given to Y is

6 × 200 + 6 × 215 + 6 × 230 + 6 × 245 + ... to 20 terms

= 6[200 + 215 + 280 + ... 485]

$= 6 \times \frac{20}{2}[400 + 19 \times 15]$

= 6 × 10[400 + 285]

= 60 × 685 = 41100

$\therefore$ Total amount paid = 52200 + 41100 = Rs. 93,300.

23. c Let the total number of pages in the book be n.

Let page number x be repeated.

Then $\sum_{i=1}^{n} i + x = 1000$

$\frac{n(n+1)}{2} + x = 1000$

Thus, $\frac{n(n+1)}{2} \le 1000$ gives n = 44

Since $\frac{n(n+1)}{2} = 990$ (for n = 44), hence, x = 10.

24. c Let the 6th and the 7th terms be x and y.

Then 8th term = x + y

Also $y^2 - x^2 = 517$

$\Rightarrow$ (y + x)(y – x)= 517 = 47 × 11

So y + x = 47

y – x = 11

Taking y = 29 and x = 18, we have 8th term = 47,

9th term = 47 + 29 = 76 and 10th term = 76 + 47 = 123.

25. d $x_0 = x$

$x_1 = -x$

$x_2 = -x$

$x_3 = x$

$x_4 = x$

$x_5 = -x$

$x_6 = -x$

………..

$\Rightarrow$ Choices (a), (b), (c)are incorrect.

26. a Coefficient of $x^n = \frac{1}{2}(n+1)(n+4)$

$S = 2 + 5x + 9x^2 + 14x^3 + ...$

$xS = 2x + 5x^2 + ...$

$S(1-x) = 2 + 3x + 4x^2 + 5x^3 + ...$

Let $S_1 = S(1-x) \Rightarrow S_1 = 2 + 3x + 4x^2 + ...$

$xS_1 = 2x + 3x^2 + ...$

$S_1(1-x) = 2 + x + x^2 + ...$

$S_1(1-x) = 2 + \frac{x}{1-x}$

$S(1-x^2) = 2 + \frac{x}{1-x} \Rightarrow S = \frac{2-x}{(1-x)^3}$

27. d $575 = \frac{n^2+n}{2} - x$

$1150 = n^2 + n - 2x$

$n(n+1) \ge 1150$

$n^2 + n \ge 1150$

The smallest value for it is n = 34.

For n = 34,

40 = 2x $\Rightarrow$ x = 20

28. c $\frac{P + \frac{P}{\sqrt{2}} + \cdots\infty}{A + \frac{A}{2} + \cdots\infty} = \frac{\frac{P}{1-\frac{1}{\sqrt{2}}}}{2A} = \frac{P\sqrt{2}}{(\sqrt{2}-1)} \times \frac{1}{2A}$

$= \frac{\sqrt{2}P(\sqrt{2}+1)}{2A} = \frac{\sqrt{2} \times 4a(\sqrt{2}+1)}{2 \times a^2}$

$= \frac{\sqrt{2} \times 2(\sqrt{2}+1)}{a} = \frac{2(2+\sqrt{2})}{a}$

29. c Let $S = 1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4}$ … (i)

$\therefore \frac{1}{7}S = \frac{1}{7} + \frac{4}{7^2} + \frac{9}{7^3} + \frac{16}{7^4}$ … (ii)

(i) – (ii) gives,

$S\left(1-\frac{1}{7}\right) = 1 + \frac{3}{7} + \frac{5}{7^2} + \frac{7}{7^3} + \frac{9}{7^4}$ … (iii)

$\frac{1}{7} \times S\left(1-\frac{1}{7}\right) = \frac{1}{7} + \frac{3}{7^2} + \frac{5}{7^3} + \frac{7}{7^4}$ … (iv)

(iii) – (iv) gives,

$S\left(1-\frac{1}{7}\right) - \frac{1}{7}S\left(1-\frac{1}{7}\right) = 1 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \frac{2}{7^4} \cdots$

$\Rightarrow S\left(1-\frac{1}{7}\right)\left(1-\frac{1}{7}\right) = 1 + \frac{2}{7}\left[1 + \frac{1}{7} + \frac{1}{7^2} + \cdots\infty\right]$

$\Rightarrow S\left(1-\frac{1}{7}\right)^2 = 1 + \frac{2}{7} \times \frac{1}{1-\frac{1}{7}}$

$\Rightarrow S\left(\frac{6}{7}\right)^2 = 1 + \frac{2}{7} \times \frac{7}{6} \Rightarrow S \times \frac{36}{49} = 1 + \frac{1}{3}$

$\Rightarrow S = \frac{49}{36} \times \frac{4}{3} \Rightarrow S = \frac{49}{27}$

30. c Let the 1st term be 'a' and common difference be 'd' then we have 3^{rd} term = a + 2d

15^{th} term = a + 14d

6^{th} term = a + 5d

11^{th} term = a + 10d

13^{th} term = a + 12d

Since sum of 3^{rd} and 15^{th} term = sum of 6^{th}, 11^{th} and 13^{th} term, therefore, we have

2a + 16d = 3a + 27d

$\Rightarrow$ a + 11d = 0

which is the 12^{th} term.

31. d The number of terms of the series forms the sum of first n natural numbers i.e. $\frac{n(n+1)}{2}$.

Thus the first 23 letters will account for the first $\frac{23 \times 24}{2}$ = 276 terms of the series.

The 288th term will be the 24th letter which is x.

32. c Assume the number of horizontal layers in the pile be n.

So $\Sigma \frac{n(n+1)}{2} = 8436$

$\Rightarrow \frac{1}{2}\ [\Sigma n^2 + \Sigma n] = 8436$

$\Rightarrow \frac{n(n+1)\ (2n+1)}{12} + \frac{n(n+1)}{4} = 8436$

$\Rightarrow n(n+1)\ \left[\frac{2n+4}{12}\right] = 8436$

$\Rightarrow \frac{n(n+1)(n+2)}{6} = 8436$

$\Rightarrow n(n+1)(n+2) = 36 \times 37 \times 38$

So n = 36

33. c The best way to do this is to take some value and verify.

E.g. 2, $\frac{1}{2}$ and 1. Thus, n = 3 and the sum of the three numbers = 3.5.

Thus, options (a), (b)and (d) get eliminated.

Alternative method:

Let the n positive numbers be $a_1, a_2, a_3, \ldots a_n$

$a_1, a_2, a_3, \ldots a_n = 1$

We know that AM $\geq$ GM

Hence, $\frac{1}{n}(a_1 + a_2 + a_3 + \ldots + a_n) \geq (a_1 a_2 \ldots a_n)^{1/n}$

or $(a_1 + a_2 + a_3 + \ldots + a_n) \geq n$

34. d $T_n = a + (n - 1)d$

$\Rightarrow 467 = 3 + (n - 1)8$

$\Rightarrow n = 59$

Half of n = 29 terms

29th term is 227 and 30th term is 235 and when these two terms are added, the sum is less than 470.

Hence, the maximum possible values the set S can have is 30.

35. a Both the series are infinitely diminishing series.

For the first series: First term = $\frac{1}{a^2}$ and r = $\frac{1}{a^2}$

For the second series: First term = $\frac{1}{a}$ and r = $\frac{1}{a^2}$

The sum of the first series = $\frac{\frac{1}{a^2}}{1-\frac{1}{a^2}} = \frac{1}{a^2-1}$

The sum of the second series = $\frac{\frac{1}{a}}{1-\frac{1}{a^2}} = \frac{a}{a^2-1}$

Now, from the first statement, the relation can be anything (depending on whether a is positive or negative).

But the second statement tells us, $4a^2 - 4a + 1 = 0$ or a = $\frac{1}{2}$. For this value of a, the sum of second series will always be greater than that of the first.

36. a Given

$t_1 + t_2 + \ldots + t_{11} = t_1 + t_2 + \ldots. + t_{19}$ (for an A.P.)

$\Rightarrow \frac{11}{2}[2a + (11-1)d] = \frac{19}{2}[2a + (19-1)d]$

$\Rightarrow$ 22a + 110d = 38a + 342d

$\Rightarrow$ 16a + 232d = 0

$\Rightarrow$ 2a + 29d = 0

$\Rightarrow \frac{30}{2}[2a + (30-1)d] = 0$

$\Rightarrow S_{30 terms} = 0$

37. a There will be an increase of 6 times.

Number of members s_1 will be in A.P.

On July 2nd , 2004, s_1 will have n + 6 b members = n + 6 × 10.5 n = 64n

Number of members in s_2 will be in G.P

On July 2nd, 2004, number of members in $s_2 = nr^6$

They are equal.

Hence, 64 n = nr^6

$\Rightarrow 64 = r^6 \Rightarrow r = 2$

38. c Given $a_1 = 81.33$; $a_2 = -19$

Also:

$a_j = a_{j-1} - a_{j-2}$, for $j \geq 3$

$\Rightarrow a_3 = a_2 - a_1 = -100.33$

$a_4 = a_3 - a_2 = -81.33$

$a_5 = a_4 - a_3 = 19$

$a_6 = a_5 - a_4 = +100.33$

$a_7 = a_6 - a_5 = +81.33$

$a_8 = a_7 - a_6 = -19$

Clearly, a_7 onwards there is a cycle of 6 and the sum of terms in every such cycle = 0. Therefore, when we add $a_1, a_2, a_3, \ldots$upto a_{6002}, we will eventually be left with $a_1 + a_2$ only i.e. 81.33 – 19 = 62.33.

39. c $a_1 = 1$, $a_{n+1} - 3a_n + 2 = 4n$

$a_{n+1} = 3a_n + 4n - 2$

when n = 2 then $a_2 = 3 + 4 - 2 = 5$

when n = 3 then $a_3 = 3 \times 5 + 4 \times 2 - 2 = 21$

from the options, we get an idea that a_n can be expressed in a combination of some power of 3 & some multiple of 100.

(a) $3^{99} - 200$; tells us that a_n could be: $3^{n-1} - 2 \times n$; but it does not fit a_1 or a_2 or a_3

(b) $3^{99} + 200$; tells us that a_n could be: $3^{n-1} + 2 \times n$; again, not valid for a_1, a_2 etc.

(c) $3^{100} - 200$; tells $3^n - 2n$: valid for all a_1, a_2, a_3.

(d) $3^{100} + 200$; tells $3^n + 2n$: again not valid.

Hence, (c)is the correct answer.

40. a $t_3 \times t_4 \times t_5 \times .... \times t_{53}$

$= \frac{3}{5}\times\frac{4}{6}\times\frac{5}{7}\times.......\times\frac{51}{53}\times\frac{52}{54}\times\frac{53}{55} = \frac{3\times4}{54\times55} = \frac{2}{495}$

Hence, option (a)is the correct answer.

41. d Let the no. of students in front row be x.

So, the no. of students in next rows be x – 3, x - 6, x – 9,…. so on

If n i.e. no. of rows be 3, then

x + (x – 3) + (x – 6) = 630

$\Rightarrow$ 3x = 639 $\Rightarrow$ x = 213

So possible.

Similarly, for n = 4,

x + (x – 3) + (x – 6)+ (x - 9) = 630

$\Rightarrow$ 4x – 18 = 630

$\Rightarrow x = \frac{648}{4} = 162$

$\therefore$ x = 4 to possible.

If n = 5,

(4x – 18) + (x - 12) = 630

$\Rightarrow$ 5x – 30 = 630

$\Rightarrow$ x = 120

Again n = 5 is possible.

If n = 6,

(5x - 30) + (x - 15) = 630

$\Rightarrow$ 6x - 45 = 630

$\Rightarrow$ 6x = 675

$\Rightarrow x \neq$ Integer

Hence, $n \neq 6$.

42. d Let number of elements in progression be n, then

$1000 = 1 + (n-1)d$

$\Rightarrow (n-1)d = 999 = 3^3 \times 37$

Possible values of d = 3, 37, 9, 111, 27, 333, 999

Hence, 7 progressions are possible.

43. b Sum of the odd integers in the set S

$= \frac{n}{2}(2\times3 + (n-1)\times2)$

$= \frac{n}{2}(2n+4) = n\times(n+2)$

Therefore, the average of the odd integers in set S = n + 2

Sum of the even integers in the set S

$= \frac{n}{2}(2\times2 + (n-1)\times2)$

$= \frac{n}{2}(2n+2) = n\times(n+1)$

Therefore, the average of the even integers in the set S = n + 1

Hence, X – Y = (n + 2)– (n + 1) = 1

44. c Price of Darjeeling tea (in rupees per kilo gram)is 100 + 0.10n

Price of Ooty tea (in rupees per kilo gram)is 89 + 0.15n

Price of the Darjeeling tea on the 100th day = 100 + 0.1 × 100 = 110 $\Rightarrow 89 + 0.15n = 110 \Rightarrow n = 140$

Number of days in the months of January, February, March and April in the year 2007 = 31 + 28 + 31 + 30 = 120.

Therefore, the price of both the tea will be equal on 20th May.

45. a $a_n + b_n\,(n \text{ is even}) = p^{\frac{n}{2}}q^{\frac{n}{2}} + p^{\frac{n}{2}-1}q^{\frac{n}{2}+1}$

$= q(pq)^{\frac{n}{2}-1}(p+q)$

46. d $a_n + b_n\,(n \text{ is odd}) = p^{\frac{n+1}{2}}q^{\frac{n-1}{2}} + p^{\frac{n-1}{2}}q^{\frac{n+1}{2}}$

$= (p+q)(pq)^{\frac{n-1}{2}}$

Substituting $p = \frac{1}{3}$ and $q = \frac{2}{3}$

$a_n + b_n = \left(\frac{2}{9}\right)^{\frac{n-1}{2}}$

Substituting n = 7, $a_n + b_n > 0.01$

Substituting n = 9, $a_n + b_n < 0.01$

Hence, smallest value of n is 9.

47. c Total sum of the numbers written on the blackboard = $\frac{40\times41}{2} = 820$

When two numbers 'a' and 'b' are erased and replaced by a new number a + b – 1, the total sum of the numbers written on the blackboard is reduced by 1.

Since this operation is repeated 39 times, therefore, the total sum of the numbers will be reduced by 1 × 39 = 39.

Therefore, after 39 operations there will be only 1 number that will be left on the blackboard and that will be 820 – 39 = 781.

48. c Total number of terms in the sequence 17, 21, 25, … ,417 is equal to $\frac{417-17}{4}+1=101$.

Total number of terms in the sequence 16, 21, 26, …, 466 is equal to $\frac{466-16}{5}+1=91$.

n^{th} term of the first sequence = 4n + 13.

m^{th} term of the second sequence = 5m + 11.

As per the information given in the question 4n + 13 = 5m + 11

$\Rightarrow$ 5m – 4n = 2.

Possible integral values of n that satisfy 5m = 2 + 4n are (2, 7, 12, …, 97)

Therefore, the total number of terms common in both the sequences is 20.

49. a $S=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$

$$T_n=\sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}$$

$$=\sqrt{\frac{n^4+2n^3+3n^2+2n+1}{n^2(n+1)^2}}$$

$$=\frac{n^2+n+1}{n^2+n}=1+\frac{1}{n^2+n}$$

$$S=\sum_{n=1}^{2007}T_n=2007+\sum_{n=1}^{2007}\left\{\frac{1}{n}-\frac{1}{n+1}\right\}$$

$$=2008-\frac{1}{2008}$$

50. b Let the n^{th} term of the A.P. be a_n and common difference be d.

$a_{n+1} = a_1 + nd$

$a_{n+2} = a + (n + 1)d = (a + d) + nd = a_2 + nd$

$. . . a_{2n} = a + (2n – 1)d = (a + (n – 1)d) + nd = a_n + nd$

Sum of first n terms = $a_1 + a_2 \ldots\ldots + a_n$=100

Sum of next n terms = $a_{n+1} + a_{n+2} \ldots\ldots + a_{2n}$

$= a_1 + a_2 \ldots\ldots + a_n + n(nd)$

$= 100 + n^2d = 300$

Hence, $n^2d = 200$.

Also, sum of the first n terms

$$=\left(\frac{n}{2}\right)(2a + (n-1)d)$$

$$= an + n^2\frac{d}{2} - n\frac{d}{2}$$

$$= an + \frac{200}{2} - n\frac{d}{2} = 100$$

Hence, $an = n\frac{d}{2}$

$\Rightarrow$ a : d = 1 : 2

51. c Sum of $a_0 + a_1 + \ldots\ldots + a_{50}$ = 1 + 3 + ... + 101 = 2601

52. b E = 3 + 8 + 15 + 24 + … + 195 = 1 × 3 + 2 × 4 + 3 × 5 + 4 × 6 + …+ 13 × 15

$\therefore$ $T_n = n(n + 2)$ and n = 13

$$\therefore\quad E=\sum_{n=1}^{13}T_n=\sum_{n=1}^{13}n(n+2)$$

$$=\frac{n(n+1)(2n+1)}{6}+2\times\frac{n(n+1)}{2}$$

$$=\frac{13\times14\times27}{6}+2\times\frac{13\times14}{2}=1001$$

= 7 × 11 × 13

Hence, the sum of the prime factors of E

= 7 + 11 + 13 = 31.

53. d Say you have an equation,

(x – 1) (x – 2) (x – 3) = 0

$\Rightarrow$ $(x^2 – 3x + 2) (x – 3) = 0$

$\Rightarrow$ $x^3 – 6x^2 + 11x – 6 = 0$

By the question, x = 3, $a_n = –6$, $a_1 = –6$.

Substitute the above values in the option.

Option (a): $(-6)^3 \geq 3^3 \times -6$

$\Rightarrow$ $-216 \geq -168$

This is incorrect, thus option (a) is incorrect.

Option (b): $3^3 \geq (-6)^3 \times -6$

$27 \geq 6^4$

This is incorrect, thus option (b) is incorrect.

Option (c): $(-6)^3 \geq 3^3 \times -6$

$\Rightarrow$ $-216 \geq -168$

This is incorrect, thus option (c) is incorrect.

Thus, none of the options is necessarily true.

54. d When common difference is 1, the first term can be anything from 1 to 46 i.e. 46 values.

When common difference is 2, the first term can be anything from 1 to 42 i.e. 42 values.

Similarly, when common difference is 12, the first term can be anything from 1 to 2 i.e. 2 values.

$\therefore$ Total possible AP's = 46 + 42 + … + 2

$$= 12\left(\frac{2+46}{2}\right) = 288.$$

55. a nth term of the series can be written as

$$\frac{1}{n! + (n+1)!} = \frac{1}{n!(n+2)} = \frac{(n+1)}{(n+2)!}$$

$$= \frac{1}{(n+1)!} - \frac{1}{(n+2)!}.$$

Put n = 1, 2, … 10 to get

$$\left(\frac{1}{2!} - \frac{1}{3!}\right) + \left(\frac{1}{3!} - \frac{1}{4!}\right) + \ldots + \left(\frac{1}{10!} - \frac{1}{11!}\right)$$

$$= \frac{1}{2!} - \frac{1}{11!}.$$

56. d If we take the 1st option, and delete all perfect squares and perfect cubes, a total of 22 perfect square will be deleted (1^2, 2^2 ....22^{22}) and a total of 7 perfect cubes will be deleted (1^3, 2^3, … 7^3).

Two numbers are common in between them viz. 1^6 and 2^6 which are perfect squares as well as perfect cubes.

Thus, 500 is the (500 – 22 – 7 + 2) = 473rd term.

Hence, the 476th term will be 500 + 3 = 503.

57. c Let the number of balls with

$P_i = a_i$ (i = 1 to 11)

$a_1 + a_3 + a_5 \ldots + a_{11} = 6(a_6) = 72.$

As a_6 would be the arithmetic mean of these 11 numbers and

$$2(a_6) = (a_1 + a_{11}) = (a_2 + a_{10})$$
$$= (a_3 + a_9) = (a_4 + a_8)$$
$$= (a_5 + a_7)$$

$\therefore \quad a_1 + a_6 + a_{11} = 3(a_6) = 36.$

58. c $a_1 = 3(a_2 + a_3 + \ldots\infty)$

let $a_2 + a_3 + \ldots\infty = x$

$a_1 = 3x$

So, $a_1 + a_2 + a_3 + \ldots = 32$

$\Rightarrow 4x = 32$

$\Rightarrow x = 8$

$\Rightarrow a_1 = 24$ and $a_2 + a_3 + \ldots = 8$

Now $a_2 = 3(a_3 + a_4 + \ldots)$

Let $a_3 + a_4 + \ldots = y$

$\Rightarrow a_2 = 3y$

$\Rightarrow 24 + 3y + y = 32$

$\Rightarrow 4y = 8$

$\Rightarrow y = 2$

$\Rightarrow a_2 = 6$

As $a_1 = 24$ and $a_2 = 6$

$$\Rightarrow a_5 = 24 \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = 24\left(\frac{1}{4}\right)^4 = \frac{3}{32}.$$

59. a $a_1 + a_2 + a_3 + \ldots a_{100}$

$$= \frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \ldots \frac{1}{299\times302}$$

$$= \frac{1}{3}\left[\frac{3}{2\times5} + \frac{3}{5\times8} + \frac{3}{8\times11} + \ldots \frac{3}{299\times302}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \ldots \frac{1}{299} - \frac{1}{302}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{302}\right] = \frac{1}{3}\left[\frac{151-1}{302}\right] = \frac{150}{3\times302} = \frac{25}{151}.$$

60. a x, y and z are in G.P.

$\therefore \quad y^2 = xz$

$$x = \frac{y^2}{z}.$$

5x, 16y and 12z are in A.P.

32y = 12z + 5x

$$32y = 12z + \frac{5y^2}{z} \quad \left[\because x = \frac{y^2}{z}\right]$$

$32yz = 12z^2 + 5y^2.$

$12z^2 - 32yz + 5y^2 = 0$

Divide by y^2

$$12\left(\frac{z}{y}\right)^2 32\left(\frac{z}{y}\right) + 5 = 0$$

On solving this quadratic equation, we get

$$\frac{z}{y} = \frac{1}{6} \text{ or } \frac{5}{2}$$

But as z > y, therefore $\frac{z}{y} = \frac{5}{2}$ is the only possible value, which is the common ratio of G.P.

61. b Let the series starting with a_1 be A and that with a_2 be B

For a_1 to be maximum we need a_2 to be maximum as well which will be 50

Therefore, arithmetic mean of B = $\frac{50+100}{2} = 75$

and thus the arithmetic mean of A will be 75 – 1 = 74

Hence, maximum value of a_1 = 74 – 51 × 1 = 23.

■■