1. Algebra

Exercise 1.1

1 A. Question

Expand the following

$(5x + 2y + 3z)^2$

Answer

$(5x + 2y + 3z)^2$

The identity is

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\therefore (5x + 2y + 3z)^2$

$= (5x)^2 + (2y)^2 + (3z)^2 + 2(5x)(2y) + 2(2y)(3z) + 2(3z)(5x)$

$= 25x^2 + 4y^2 + 9z^2 + 20xy + 12yz + 30zx$

1 B. Question

Expand the following

$(2a + 3b - c)^2$

Answer

$(2a + 3b - c)^2$

Here the identity used is

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(2a + 3b - c)^2 = (2a)^2 + (3b)^2 + (-c)^2 + 2(2a)(3b) + 2(3b)(-c) + 2(-c)(2a)$

$= 4a^2 + 9b^2 + c^2 + 12ab - 6bc - 4ca$

1 C. Question

Expand the following

$(x - 2y - 4z)^2$

Answer

$(x - 2y - 4z)^2$

Here the identity used is

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\therefore (x - 2y - 4z)^2$

$= x^2 + (-2y)^2 + (-4z)^2 + 2(x)(-2y) + 2(-2y)(-4z) + 2(-4z)(x)$

$= x^2 + 4y^2 + 16z^2 - 4xy + 16yz - 8yz$

1 D. Question

Expand the following

$(P - 2q + r)^2$

Answer

$(P - 2q + r)^2$

Here we use the identity of

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\therefore (p - 2q + r)^2$

$= (p)^2 + (-2q)^2 + (r)^2 + 2(p)(-2q) + 2(-2q)(r) + 2(r)(p)$

$= p^2 + 4q^2 + r^2 - pq - 4qr + 2rp$

2 A. Question

Find the expansion of

$(x + 1)(x + 4)(x + 7)$

Answer

$(x + 1)(x + 4)(x + 7)$

we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (x + 1)(x + 4)(x + 7)$

$= x^3 + (1 + 4 + 7)x^2 + (1 \times 4 + 4 \times 7 + 7 \times 1)x + 1 \times 4 \times 7$

$= x^3 + 12x^2 + (4 + 28 + 7)x + 28$

$= x^3 + 12x^2 + 39x + 28$

2 B. Question

Find the expansion of

$(p + 2)(p - 4)(p + 6)$

Answer

$(p + 2)(p - 4)(p + 6)$

Here the identity used is

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (p + 2)(p - 4)(p + 6)$

$= p^3 + (2 - 4 + 6) p^2 + (2(- 4) + (- 4)6 + 6(2))p + (2)(- 4)(6)$

$= p^3 + 4p^2 + (- 8 - 24 + 12)p - 48$

$= p^3 + 4p^2 - 20p - 48$

2 C. Question

Find the expansion of

$(x + 5)(x - 3)(x - 1)$

Answer

$(x + 5)(x - 3)(x - 1)$

Here the identity used is

$(x + a)(x + b)(x + c) = x^3 + (a + b + c) x^2 + (ab + bc + ca)x + abc$

$\therefore (x + 5)(x - 3)(x - 1)$

$= x^3 + (5 - 3 - 1) x^2 + (5(- 3) + (- 3)(- 1)(- 1)5)x + 5(- 3)(- 1)$

$= x^3 + x^2 + (- 15 + 3 - 5) x + 15$

$= x^3 + x^2 - 17x + 15$

2 D. Question

Find the expansion of

$(x - a)(x - 2a)(x - 4a)$

Answer

$(x - a)(x - 2a)(x - 4a)$

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (x - a)(x - 2a)(x - 4a)$

$= x^3 + (- a - 2a - 3a)x^2 + [(- a)(- 2a) + (- 2a)(- 3a) + (- 3a)(- a)]x + (- a)(- 2a)(- 3a)$

$= x^3 - 6ax^2 + (2a^2 + 6a^2 + 3a^2)x - 6a^3$

$= x^3 - 6ax^2 + 11a^2x - 6a^3$

2 E. Question

Find the expansion of

$(3x + 1)(3x + 2)(3x + 5)$

Answer

$(3x + 1)(3x + 2)((3x + 5)$

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (3x + 1)(3x + 2)((3x + 5)$

$= (3x)^3 + (1 + 2 + 5)(3x)^2 + (1 \times 2 + 2 \times 5 + 5 \times 1)(3x) + 1 \times 2 \times 5$

$= 27 x^3 + (8)9x^2 + (2 + 10 + 5) (3x) + 1 \times 2 \times 5$

$= 27 x^3 + 72x^2 + 51x + 10$

2 F. Question

Find the expansion of

$(2x + 3)(2x - 5)(2x - 7)$

Answer

$(2x + 3)(2x - 5)(2x - 7)$

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (2x + 3)(2x - 5)(2x - 7)$

$= (2x)^3 + (3 - 5 - 7)(2x)^2 + [(3)(- 5) + (- 5)(- 7) + (- 7)(3)](2x) + (3)(- 5)(- 7)$

$= 8x^3 + (- 9)(4x^2) + (- 15 + 35 - 21)(2x) + 105$

$= 8x^3 - 36 x^2 - 2x + 105$

3 A. Question

Using algebraic identities find the coefficients of x^2 term, x term and constant term.

$(x + 7)(x + 3)(x + 9)$

Answer

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (x + 7)(x + 3)(x + 9)$

$= x^3 + (7 + 3 + 9)x^2 + (7 \times 3 + 3 \times 9 + 9 \times 7)x + 7 \times 3 \times 9$

$= x^3 + 19x^2 + (21 + 27 + 63)x + 189$

$= x^3 + 19x^2 + 111x + 189$

Coefficient of

$x^2 = 19$

$x = 111$

Constant term = 189

3 B. Question

Using algebraic identities find the coefficients of x^2 term, x term and constant term.

$(x - 5)(x - 4)(x + 2)$

Answer

$(x - 5)(x - 4)(x + 2)$

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore (x - 5)(x - 4)(x + 2)$

$= x^3 + (-5 - 4 + 2)x^2 + \{(-5) \times (-4) + (-4) \times 2 + 2 \times (-5)\}x + (-5)(-4)2$

$= x^3 - 7x^2 + (20 - 8 - 10)x + 40$

$= x^3 - 7x^2 + 2x + 40$

Coefficient of

$x^2 = -7$

$x = 2$ and

Constant term = 40

3 C. Question

Using algebraic identities find the coefficients of x^2 term, x term and constant term.

$(2x + 3)(2x + 5)(2x + 7)$

Answer

$(2x + 3)(2x + 5)(2x + 7)$

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)(x)^2 + (ab + bc + ca)x + abc$

$\therefore (2x + 3)\ (2x + 5)(2x + 7)$

$= (2x)^3 + 4(3 + 5 + 7)x^2 + (15 + 35 + 21)2x + 105$

$= 8x^3 + 60x^2 + 142x + 105$

Coefficient of

$x^2 = 60$

$x = 142$

Constant term = 105

3 D. Question

Using algebraic identities find the coefficients of x^2 term, x term and constant term.

$(5x + 2)(1 - 5x)(5x + 3)$

Answer

$(5x + 2)(1 - 5x)(5x + 3)$

Here we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\therefore -(5x + 2)(5x - 1)(5x + 3)$

here $x = 5x$, $a = 2$, $b = -1$ and $c = 3$

$= -(5x)^3 - (2 - 1 + 3)(5x)^2 - (-2 - 3 + 6)5x + 6$

$= -125x^3 - 100x^2 - 5x + 6$

Here coefficient of

$x^2 = -100$

$x = -5$ and

constant term = 6

4. Question

If $(x + a)(x + b)(x + c) \equiv x^3 - 10x^2 + 45x - 15$ find $a + b + c$, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ and $a^2 + b^2 + c^2$.

Answer

Here, we have to use the identity

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

Comparing this with

$x^3 - 10x^2 + 45x - 15 = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

$\Rightarrow -10 = a + b + c$

Now, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{bc + ac + ab}{abc}$

$\Rightarrow -\frac{45}{15} = -3$

And $(ab + bc + ca) = 45$

Now $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (-10)^2 - 2 \times 45 = a^2 + b^2 + c^2$

$\Rightarrow 100 - 90 = a^2 + b^2 + c^2$

$\Rightarrow a^2 + b^2 + c^2 = 10$

5. Question

Expand :

(i) $(3a + 5b)^3$

(ii) $(4x - 3y)^3$

(iii) $\left(2y - \frac{3}{y}\right)^3$

Answer

(i) $(3a + 5b)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(3a + 5b)^3$

$= (3a)^3 + 3\,(3a)^2(5b) + 3(3a)(5b)^2 + (5b)^3$

$= 27a^3 + 135a^2b + 225ab^2 + 125b^3$

(ii) $(4x - 3y)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(4x - 3y)^3$

$= (4x)^3 + 3(4x)^2(-3y) + 3 \times 4x\,(-3y)^2 + (-3y)^3$

$= 64x^3 - 192\,x^2 + 108y^2 - 27y^3$

(iii) $\left(2y - \frac{3}{y}\right)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$=(2y)^3 + \left(-\frac{3}{y}\right)^3 + 3(2y)^2\left(-\frac{3}{y}\right) + 3(2y)\left(-\frac{3}{y}\right)^2$

$= 8y^3 - \frac{27}{y^3} - 36y + \frac{54}{y}$

6. Question

Evaluate :

(i) 99^3

(ii) 101^3

(iii) 98^3

(iv) $(102)^3$

(v) $(1002)^3$

Answer

(i) $(100 - 1)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(100 - 1)^3$

$= (100)^3 + 3(100)^2 (- 1) + 3(100) (- 1)^2 + (- 1)^3$

$= 1000000 - 30000 + 300 - 1$

$= 970299$

(ii) 101^3

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(101)^3 = (100 + 1)^3$

$= 100^3 + 3 \times 100^2 \times 1 + 3 \times 100 \times 1^2 + 1^3$

$= 1000000 + 30000 + 300 + 1$

$= 1030301$

(iii) 98^3

$= (100 - 2)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(100 - 2)^3$

$= 100^3 + (- 2)^3 + 3(100)^2(- 2) + 3(100)(- 2)^2$

$= 1000000 - 8 - 60000 + 1200$

$= 941192$

(iv) $(102)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(100 + 2)^3$

$= 100^3 + 3(100)^2 (2) + 3(2)^2(100) + (2)^3$

= 1000000 + 60000 + 1200 + 8

= 1061208

(vi) $(1002)^3$

$= (1000 + 2)^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(1000 + 2)^3$

$= 1000^3 + 3 \times 1000^2(2) + 3 \times 1000 \times (2)^2 + 2^3$

= 1000000000 + 6000000 + 12000 + 8

= 1006012008

7. Question

Find $8x^3 + 27y^3$ if $2x + 3y = 13$ and $xy = 6$.

Answer

Given $2x + 3y = 13$

$\Rightarrow (2x + 3y)^3 = 13^3$

Here the identity used is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(2x + 3y)^3 = (2x)^3 + 3(2x)^2 3y + 3(2x)(3y)^2 + (3y)^3$

$\Rightarrow 8x^3 + 36x^2y + 54xy^2 + 27y^3 = 2197$

$\Rightarrow 8x^3 + 27y^3 + 18xy(2x + 3y) = 2197$

$\Rightarrow 8x^3 + 27y^3 = 2197 - 108\ (2x + 3y)$ ($\because$ xy = 6 given)

$\Rightarrow 8x^3 + 27y^3 = 2197 - 108 \times 13$

$\Rightarrow 8x^3 + 27y^3 = 2197 - 1404$

$\Rightarrow 8x^3 + 27y^3 = 793$

8. Question

If x – y = – 6 and xy = 4, find the value of $x^3 - y^3$

Answer

The identity used here is

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore (x - y)^3 = x^3 + 3x^2(-y) + 3x(-y)^2 + (-y)^3$

$\Rightarrow x^3 - 3x^2y + 3xy^2 - y^3$

$\Rightarrow (x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$\Rightarrow (-6)^3 = x^3 - y^3 + 3xy(-x + y)$

$\Rightarrow -216 + 3 \times 4\,(-6) = x^3 - y^3$

$\Rightarrow -216 - 72 = x^3 - y^3$

$\Rightarrow -288 = x^3 - y^3$

9. Question

If $x + \frac{1}{x} = 4$, find the value of $x^3 + \frac{1}{x^3}$.

Answer

Here we use the identity of

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore \left(x + \frac{1}{x}\right)^3 = x^3 + \left(\frac{1}{x}\right)^3 + 3x^2\frac{1}{x} + 3x\left(\frac{1}{x}\right)^2$

$\Rightarrow (4)^3 - 3(x + \frac{1}{x}) = x^3 + \left(\frac{1}{x}\right)^3$

$\Rightarrow 64 - 12 = x^3 + \left(\frac{1}{x}\right)^3$

$\Rightarrow x^3 + \left(\frac{1}{x}\right)^3 = 52$

10. Question

If, $x - \frac{1}{x} = 4$ find the value of $x^3 - \frac{1}{x^3}$.

Answer

Here the identity used

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore \left(x - \frac{1}{x}\right)^3 = x^3 + 3x^2\left(-\frac{1}{x}\right) + 3x\left(-\frac{1}{x}\right)^2 + \left(-\frac{1}{x}\right)^3$

$\Rightarrow 4^3 = x^3 - \left(\frac{1}{x}\right)^3 - 3x + \frac{3}{x} - \left(\frac{1}{x}\right)^3$

$\Rightarrow 64 = x^3 - \left(\frac{1}{x}\right)^3 - 3\left(x - \frac{1}{x}\right) - \left(\frac{1}{x}\right)^3$

$\Rightarrow 76 = x^3 - \left(\frac{1}{x}\right)^3$

11. Question

Simplify:

(i) $(2x + y + 4z)(4x^2 + y^2 + 16z^2 - 2xy - 4yz - 8zx)$

(ii) $(x - 3y - 5z)(x^2 + 9y^2 + 25z^2 + 3xy - 15yz + 5zx)$

Answer

(i) $(2x + y + 4z)(4x^2 + y^2 + 16z^2 - 2xy - 4yz - 8zx)$

⇒ Here the identity used is

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Comparing this with given condition

$(2x + y + 4z)(4x^2 + y^2 + 16z^2 - 2xy - 4yz - 8zx) = (2x)^3 + y^3 + (4z)^3 - 3 \times 2x \times y \times 4z$

$\Rightarrow 8x^3 + y^3 + 64z^3 - 24xyz$

(ii) $(x - 3y - 5z)(x^2 + 9y^2 + 25z^2 + 3xy - 15yx + 5zx)$

⇒ Here the identity used is

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Comparing this with given condition

$(x - 3y - 5z)(x^2 + 9y^2 + 25z^2 + 3xy - 15yx + 5zx)$

$= x^3 + (-3y)^3 + (-5z)^3 - 3(x)(-3y)(-5z)$

$= x^3 - 27y^3 - 125z^3 - 45xyz$

12. Question

Evaluate using identities :

(i) $6^3 – 9^3 + 3^3$

(ii) $16^3 – 6^3 – 10^3$

Answer

(i) $6^3 – 9^3 + 3^3$

Here the identity used will be

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ac)$

$\therefore 6^3 – 9^3 + 3^3$

$\Rightarrow (6 + (– 9) + 3)(6^2 + (– 9)^2 + 3^2 – 6 \times – 9 – (– 9)3 – 6 \times 3) + 3 \times 6 \times – 9 \times 3$

$\Rightarrow 0 (36 + 81 + 9 + 54 + 27 – 18) – 486$

$\Rightarrow – 486$

(ii) $16^3 – 6^3 – 10^3$

Here the identity used will be

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ac)$

$\therefore 16^3 – 6^3 – 10^3$

$= (16 – 6 – 10) (16^2 + (– 6)^2 + (– 10)^2 – 16 \times – 6 – (– 6)(– 10) – (16)(– 10)) + 3(16) (– 6) (– 10)$

$= 0 (256 + 36 + 100 + 96 – 60 + 160) + 2880$

$= 2880$

Exercise 1.2

1 A. Question

Factorize the following expressions:

$2a^3 – 3a^2b + 2a^2c$

Answer

$= 2a^3 – 3a^2b + 2a^2c$Highest common factor is a^2

Taking a^2 common in the term $– 3a^2b + 2a^2c$

we get,$= 2a^3 + a^2(– 3b + 2c)$Taking a^2 common in both the termswe get,

$= a^2(2a + (-3b + 2c)) = a^2(2a - 3a + 2c)2a^3 - 3a^2b + 2a^2c = a^2(2a - 3a + 2c)$

1 B. Question

Factorize the following expressions:

$16x + 64x^2y$

Answer

$= 16x + 64x^2y$

Highest common factor is 16x

Taking x common in the term $16x + 64x^2y$

we get,

$= x(16 + 64xy)$

$= x(16 + 16(4)xy)$

Taking 16 common in the term $(16 + 64xy)$

we get,

$= 16x(1 + 4xy)$

$16x + 64x^2y = 16x(1 + 4xy)$

1 C. Question

Factorize the following expressions:

$10x^3 - 25x^4y$

Answer

$= 10x^3 - 25x^4y$Highest common factor is $5x^3$

Taking x^3common in the term $10x^3 - 25x^4y$

we get,

$= x^3\ (10 - 25xy)$

$= x^3\ (5(2) - 5(2)xy)$Taking 5 common in both the terms

we get,

$= 5x^3\ (2 - 5xy)10x^3 - 25x^4y = 5x^3\ (2 - 5xy)$

1 D. Question

Factorize the following expressions:

xy – xz + ay – az

Answer

= xy – xz + ay – azTaking x common in xy – xz

we get,

= x(y – z) + ay – az

Taking ‘a’ common in ay – az

we get,

= x(y – z) + a(y – z)Taking (y – z) common in both the terms

we get,= (y – z)(x + a)xy – xz + ay – az = (y – z)(x + a)

1 E. Question

Factorize the following expressions:

$p^2 + pq + pr + qr$

Answer

= $p^2 + pq + pr + qr$Taking p common in $p^2 + pq$

we get,

= p(p + q) + pr + qrTaking r common in pr + qrwe get,= p(p + q) + r(p + q)

Taking (p + q) common in both the terms

we get,= (p + q)(p + r)$p^2 + pq + pr + qr = (p + q)(p + r)$

2 A. Question

Factorize the following expressions:

$x^2 + 2x + 1$

Answer

= $x^2 + 2x + 1$

<u>Method 1</u>

It is of the form $a^2 + 2ab + b^2$Where a = x and b = 1.

Using the identity: $(a + b)^2 = a^2 + 2ab + b^2$

We get,

$\Rightarrow x^2 + 2x + 1 = (x + 1)^2$

Method 2Here 2x can be elaborated as x + x

we get,

$= x^2 + x + x + 1$

Taking x common in $x^2 + x$

we get,

$= x(x + 1) + x + 1$Taking (x + 1) common in both the terms

we get,$= (x + 1)(x + 1) = (x + 1)^2 x^2 + 2x + 1 = (x + 1)^2$

2 B. Question

Factorize the following expressions:

$9x^2 - 24xy + 16y^2$

Answer

$= 9x^2 - 24xy + 16y^2$Given equation can be simplified as,

$= (3x)^2 - 2(3x)(4y) + (4y)^2$

It is of the form $a^2 - 2ab + b^2$Where a = 3x and b = 4y∴ using the identity $(a - b)^2 = a^2 - 2ab + b^2$

We get, $\Rightarrow 9x^2 - 24xy + 16y^2 = (3x - 4y)^2$

$9x^2 - 24xy + 16y^2 = (3x - 4y)^2$

2 C. Question

Factorize the following expressions:

$b^2 - 4$

Answer

$= b^2 - 4$Given equation can be simplified as,

$= (b)^2 - (2)^2$

It is of the form $(p)^2 - (q)^2$where p = b and q = 2∴ using the identity $(p)^2 - (q)^2 = (p + q)(p - q)$

We get, $\Rightarrow b^2 - 2^2 = (b + 2)(b - 2)b^2 - 4 = (b + 2)(b - 2)$

2 D. Question

Factorize the following expressions:

$1 - 36x^2$

Answer

$= 1 - 36x^2$Given equation can be simplified as,

$= (1)^2 - (6x)^2$

It is of the form $(p)^2 - (q)^2$Where p = 1 and q = 6x∴ using the identity: $(p)^2 - (q)^2 = (p + q)(p - q)$

We get, $\Rightarrow (1)^2 - (6x)^2 = (1 + 6x)(1 - 6x)$

$1 - 36x^2 = (1 + 6x)(1 - 6x)$

3 A. Question

Factorize the following expressions:

$p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

Answer

<u>Method 1</u>$= p^2 + q^2 + r^2 + 2pq + 2qr + 2rp= (p)^2 + (q)^2 + (r)^2 + 2(p)(q) + 2(q)(r) + 2(r)(p)$It is of the from $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

where a = p, b = q, c = r

Using the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

We get,$\Rightarrow p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = (p + q + r)^2$

<u>Method 2</u>

$= p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$The above equation can be simplified as

$= p^2 + 2pq + q^2 + r^2 + 2qr + 2rp= (p + q)^2 + r^2 + 2qr + 2rp$ ($\because a^2 + 2ab + b^2 = (a + b)^2$)

Taking 2r common in the term 2qr + 2rp

we get,

$= (p + q)^2 + r^2 + 2r(q + p)$Rearranging the above equation as$= (p + q)^2 + 2r(q + p) + r^2$

This is of the from $a^2 + 2ab + b^2$

Where, a = p + q and b = rUsing the identity: $(a + b)^2 = a^2 + 2ab + b^2$

We get,

$\Rightarrow p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = ((p + q) + r)^2$

$= (p + q + r)^2 p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = (p + q + r)^2$

3 B. Question

Factorize the following expressions:

$a^2 + 4b^2 + 36 - 4ab + 24b - 12a$

Answer

$= a^2 + 4b^2 + 36 - 4ab + 24b - 12a$

Method 1

The above equation can be simplified as : $= (-a)^2 + (2b)^2 + (6)^2 + 2(-a)(2b) + 2(2b)(6) + 2(-a)(6)$This equation is of the form: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

where $p = -a$, $q = 2b$, $r = 6$Using the identity: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = (p + q + r)^2$We get,$\Rightarrow a^2 + 4b^2 + 36 - 4ab + 24b - 12a = (-a + 2b + 6)^2$

$= (-a + 2b + 6)^2$

$= (-1)^2(a - 2b - 6)^2$

$= (a - 2b - 6)^2$

Method 2

The above equation can be simplified as

$= (-a)^2 + (2b)^2 + (6)^2 + 2(-a)(2b) + 2(2b)(6) + 2(-a)(6) = \{(-a)^2 + 2(-a)(2b) + (2b)^2\} + (6)^2 + 2(2b)(6) + 2(-a)(6)$$(\because a^2 + 2ab + b^2 = (a + b)^2)$$(-a + 2b)^2 + (6)^2 + 2(2b)(6) + 2(-a)(6)$Taking 2(6) common in term $2(2b)(6) + 2(-a)(6)$$= (-a + 2b)^2 + 2(6)(-a + 2b) + (6)^2$

This of the form: $(p + q)^2 + 2r(p + q) + r^2$

Where, $p = -a$, $q = 2b$ and $r = 6$

Using the identity: $(p + q)^2 = p^2 + 2pq + q^2$

We get,

$\Rightarrow a^2 + 4b^2 + 36 - 4ab + 24b - 12a = (-a + 2b + 6)^2$

$= (-a + 2b + 6)^2$

$= (-1)^2(a - 2b - 6)^2$

$= (a - 2b - 6)^2$

$a^2 + 4b^2 + 36 - 4ab + 24b - 12a = (-a + 2b + 6)^2$

3 C. Question

Factorize the following expressions:

$9x^2 + y^2 + 1 - 6xy + 6x - 2y$

Answer

Method 1

The above equation can be simplified as : $= (3x)^2 + (-y)^2 + (1)^2 + 2(3x)(-y) + 2(3x)(1) + 2(-y)(1)$This equation is of the form: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

where $p = 3x$, $q = -y$, $r = 1$Using the identity: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = (p + q + r)^2$We get,$\Rightarrow 9x^2 + y^2 + 1 - 6xy + 6x - 2y = (3x - y + 1)^2$

$= (3x - y + 1)^2$

Method 2

The above equation can be simplified as:

$= (3x)^2 + (-y)^2 + (1)^2 + 2(3x)(-y) + 2(3x)(1) + 2(-y)(1) = \{(3x)^2 + 2(3x)(-y) + (-y)^2\} + (1)^2 + 2(3x)(1) + 2(-y)(1)$($\because p^2 + 2pq + q^2 = (p + q)^2$)$= (3x - y)^2 + (1)^2 + 2(3x)(1) + 2(-y)(1)$Taking 2(1) common in term $2(3x)(1) + 2(-y)(1) = (3x - y)^2 + (1)^2 + 2(1)(3x - y) = (3x - y)^2 + 2(1)(3x - y) + (1)^2$

This of the form: $(p + q)^2 + 2r(p + q) + r^2$

Where, $p = 3x$, $q = -y$ and $r = 1$

Using the identity: $(a + b)^2 = a^2 + 2ab + b^2$

We get,

$\Rightarrow 9x^2 + y^2 + 1 - 6xy + 6x - 2y = (3x - y + 1)^2$$9x^2 + y^2 + 1 - 6xy + 6x - 2y = (3x - y + 1)^2$

3 D. Question

Factorize the following expressions:

$4a^2 + b^2 + 9c^2 - 4ab - 6bc + 12ca$

Answer

Method 1

The above equation can be simplified as : $= (2a)^2 + (-b)^2 + (3c)^2 + 2(2a)(-b) + 2(-b)(3c) + 2(3c)(2a)$This equation is of the form: $p^2 + q^2 + r^2 + 2pq + 2qr +$

2rp

where p =2a, q = – b, r = 3cUsing the identity: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = (p + q + r)^2$We get,⇒ $4a^2 + b^2 + 9c^2 - 4ab - 6bc + 12ca = (2a - b + 3c)^2$

$= (2a - b + 3c)^2$

Method 2

The above equation can be simplified as:

$= (2a)^2 + (- b)^2 + (3c)^2 + 2(2a)(- b) + 2(- b)(3c) + 2(3c)(2a)$= $\{(2a)^2 + 2(2a)(- b) + (- b)^2\} + (3c)^2 + 2(- b)(3c) + 2(3c)(2a)$($\because p^2 + 2pq + q^2 = (p + q)^2$)

$= (2a - b)^2 + (3c)^2 + 2(- b)(3c) + 2(3c)(2a)$Taking 2(3c) common in term $2(- b)(3c) + 2(3c)(2a)$= $(2a - b)^2 + (3c)^2 + 2(3c)(- b + 2a)$= $(2a - b)^2 + 2(3c)(2a - b) + (3c)^2$

This of the form: $(p + q)^2 + 2r(p + q) + r^2$

Where, p = 2a, q = – b and r = 3c

Using the identity: $(p + q)^2 = p^2 + 2pq + q^2$

We get,

⇒ $4a^2 + b^2 + 9c^2 - 4ab - 6bc + 12ca = (2a - b + 3c)^2$$4a^2 + b^2 + 9c^2 - 4ab - 6bc + 12ca = (2a - b + 3c)^2$

3 E. Question

Factorize the following expressions:

$25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30zx$

Answer

Method 1

The above equation can be simplified as : $= (- 5x)^2 + (2y)^2 + (3z)^2 + 2(- 5x)(2y) + 2(2y)(3z) + 2(3z)(- 5x)$This equation is of the form: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

where p = – 5x, q = 2y, r = 3zUsing the identity: $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp = (p + q + r)^2$We get,⇒ $25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30zx = (- 5x + 2y + 3z)^2$

$= (- 5x + 2y + 3z)^2$

$= (- 1)^2(5x - 2y - 3z)^2$

$= (5x - 2y - 3z)^2$

Method 2

The above equation can be simplified as:

$= (-5x)^2 + (2y)^2 + (3z)^2 + 2(-5x)(2y) + 2(2y)(3z) + 2(3z)(-5x) = \{(-5x)^2 + 2(-5x)(2y) + (2y)^2\} + (3z)^2 + 2(2y)(3z) + 2(3z)(-5x) (\because p^2 + 2pq + q^2 = (p + q)^2) = (-5x + 2y)^2 + (3z)^2 + 2(2y)(3z) + 2(3z)(-5x)$ Taking 2(3z) common in term $2(2y)(3z) + 2(3z)(-5x) = (-5x + 2y)^2 + (3z)^2 + 2(3z)(2y - 5x) = (-5x + 2y)^2 + 2(3z)(2y - 5x) + (3z)^2$

This of the form: $(p + q)^2 + 2r(p + q) + r^2$

Where, $p = -5x$, $q = 2y$ and $r = 3z$

Using the identity: $(p + q)^2 = p^2 + 2pq + q^2$

We get,

$\Rightarrow 25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30zx = (-5x + 2y + 3z)^2$

$= (-5x + 2y + 3z)^2$

$= (-1)^2(5x - 2y - 3z)^2$

$= (5x - 2y - 3z)^2$

$25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30zx = (5x - 2y - 3z)^2$

4 A. Question

Factorize the following expressions: $27x^3 + 64y^3$

Answer

The above equation can be simplified as

$= (3x)^3 + (4y)^3$ Both the given terms are perfect cubes, factor using the sum of cubes formula,

$(a)^3 + (b)^3 = (a + b)(a^2 - ab + b^2)$

Here $a = 3x$ and $b = 4y$

$\Rightarrow (3x)^3 + (4y)^3 = (3x + 4y)((3x)^2 - (3x)(4y) + (4y)^2)$

$= (3x + 4y)(9x^2 - 12xy + 16y^2)$ $27x^3 + 64y^3 = (3x + 4y)(9x^2 - 12xy + 16y^2)$

4 B. Question

Factorize the following expressions: $m^3 + 8$

Answer

The above equation can be simplified as

$= (m)^3 + (2)^3$Both the given terms are perfect cubes, factor using the sum of cubes formula,

$(a)^3 + (b)^3 = (a + b)(a^2 - ab + b^2)$Here a = m and b = 2

$\Rightarrow (m)^3 + (2)^3 = (m + 2)((m)^2 - (m)(2) + (2)^2)$

$= (m + 2)(m^2 - 2m + 4)m^3 + 8 = (m + 2)(m^2 - 2m + 4)$

4 C. Question

Factorize the following expressions:$a^3 + 125$

Answer

The above equation can be simplified as

$= (a)^3 + (5)^3$Both the given terms are perfect cubes, factor using the sum of cubes formula,

$(a)^3 + (b)^3 = (a + b)(a^2 - ab + b^2)$Here a = a and b = 5

$\Rightarrow (a)^3 + (5)^3 = (a + 5)((a)^2 - (a)(5) + (5)^2)$

$= (a + 5)(a^2 - 5a + 25)a^3 + 123 = (a + 5)(a^2 - 5a + 25)$

4 D. Question

Factorize the following expressions:$8x^3 - 27y^3$

Answer

The above equation can be simplified as

$= (2x)^3 - (3y)^3$Both the given terms are perfect cubes, factor using the subtraction of cubes formula,

$(a)^3 - (b)^3 = (a - b)(a^2 + ab + b^2)$Here a = 2x and b = 3y

$\Rightarrow (2x)^3 - (3y)^3 = (2x - 3y)((2x)^2 + (2x)(3y) + (3y)^2)$

$= (2x - 3y)(4x^2 + 6xy + 9y^2)8x^3 - 27y^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$

4 E. Question

Factorize the following expressions:$x^3 - 8y^3$

Answer

The above equation can be simplified as

$= (x)^3 - (2y)^3$Both the given terms are perfect cubes, factor using the subtraction of cubes formula,

$(a)^3 - (b)^3 = (a - b)(a^2 + ab + b^2)$Here a = x and b = 2y

$\Rightarrow (x)^3 - (2y)^3 = (x - 2y)((x)^2 + (x)(2y) + (2y)^2)$

$= (x - y)(x^2 + 2xy + 4y^2)x^3 - 8y^3 = (x - y)(x^2 + 2xy + 4y^2)$

Exercise 1.4

1 A. Question

Solve the following equations by substitution method.

x + 3y = 10; 2x + y = 5

Answer

[In substitution method, we have to find value of one variable in terms of the other variable from any one of the two given equations . Then, we will substitute this value in the other equation. By doing this, we will get a equation in one variable which can be solved easily]

x + 3y = 10……(1)

2x + y = 5 …..(2)

From eq(1),

X + 3y = 10

$\Rightarrow x = 10 - 3y$

Substituting the value of x in eq(2)

2x + y = 5

$\Rightarrow 2(10 - 3y) + y = 5$

$\Rightarrow 20 - 6y + y = 5$

$\Rightarrow 20 - 5y = 5$

$\Rightarrow 5y = 20 - 5$

$\Rightarrow 5y = 15$

$\Rightarrow y = \frac{15}{5}$

$\Rightarrow y = 3$

Substituting value of y in eq(1)

$x + 3y = 10$

$\Rightarrow x + 3 \times 3 = 10$

$\Rightarrow x + 9 = 10$

$\Rightarrow x = 10 - 9$

$\Rightarrow x = 1$

Hence,

$x = 1, y = 3$

1 B. Question

Solve the following equations by substitution method.

$2x + y = 1; 3x - 4y = 18$

Answer

[In substitution method, we have to find value of one variable in terms of the other variable from any one of the two given equations . Then, we will substitute this value in the other equation. By doing this, we will get a equation in one variable which can be solved easily]

$2x + y = 1$....(1)

$3x - 4y = 18$....(2)

From eq(1),

$2x + y = 1$....(1)

$\Rightarrow y = 1 - 2x$

Substituting the value of y in eq(2)

$3x - 4y = 18$....(2)

$\Rightarrow 3x - 4(1 - 2x) = 18$

$\Rightarrow 3x - 4 + 8x = 18$

$\Rightarrow 11x + 4 = 18$

$\Rightarrow 11x = 18 + 4$

$\Rightarrow 11x = 22$

$\Rightarrow x = \frac{22}{11}$

$\Rightarrow x = 2$

Substituting the value of x in eq(1),

$\Rightarrow 2x + y = 1$

$\Rightarrow 2\times2 + y = 1$

$\Rightarrow y = 1 - 4$

$\Rightarrow y = -3$

Hence,

$x = 2, y = -3$

1 C. Question

Solve the following equations by substitution method.

$5x + 3y = 21;\ 2x - y = 4$

Answer

[In substitution method, we have to find value of one variable in terms of the other variable from any one of the two given equations . Then, we will substitute this value in the other equation. By doing this, we will get a equation in one variable which can be solved easily]

$5x + 3y = 21$……(1)

$2x - y = 4$…….(2)

From eq(2)

$2x - y = 4$

$\Rightarrow y = 2x - 4$…..(3)

Substituting value of y in eq(1)

$5x + 3y = 21$

$\Rightarrow 5x + 3(2x - 4) = 21$

$\Rightarrow 5x + 6x - 12 = 21$

$\Rightarrow 11x = 33$

$\Rightarrow x = \frac{33}{11}$

$\Rightarrow x = 3$

From eq(3)

$y = 2x - 4$

$\Rightarrow$ y = 2×3 – 4

$\Rightarrow$ y = 2

Hence,

x = 3, y = 2

1 D. Question

Solve the following equations by substitution method.

$$\frac{1}{x}+\frac{2}{y}=9;\frac{2}{x}+\frac{1}{y}=12(x\neq 0, y\neq 0)$$

Answer

In such questions, if we start question directly, then it will involve LCM of x and y, and a term of xy, which will be inconvenient for us. So, we will assume 1/x as "u "and 1/y as v"]

Let

$\frac{1}{x}$ = u

$\frac{1}{y}$ = v

Then,

$\frac{1}{x}+\frac{2}{y}$ = 9

$\Rightarrow$ u + 2v = 9……(1)

And

$\frac{2}{x}+\frac{1}{y}$ = 12

$\Rightarrow$ 2u + v = 12……(2)

From eq(1)

u + 2v = 9

$\Rightarrow$ u = 9 – 2v……(3)

Substituting value of u in eq(2)

2u + v = 12

$\Rightarrow$ 2(9 – 2v) + v = 12

$\Rightarrow$18 – 4v + v = 12

$\Rightarrow 18 - 3v = 12$

$\Rightarrow 3v = 18 - 12$

$\Rightarrow 3v = 6$

$\Rightarrow v = 2$

$\Rightarrow \frac{1}{y} = 2$

$\Rightarrow \mathbf{y = 1/2}$

Substituting value of v in eq(3)

$u = 9 - 2v$

$\Rightarrow u = 9 - 2\times2$

$\Rightarrow u = 5$

$\Rightarrow \frac{1}{x} = 5$

$\Rightarrow \mathbf{x = \frac{1}{5}}$

Hence,

$x = \frac{1}{5}, y = \frac{1}{2}$

1 E. Question

Solve the following equations by substitution method.

$$\frac{3}{x} + \frac{1}{y} = 7; \frac{5}{x} - \frac{4}{y} = 6 (x \neq 0, y \neq 0)$$

Answer

In such questions , if we start question directly, then it will involve LCM of x and y, and a term of xy, which will be inconvenient for us. So, we will assume 1/x as "u "and 1/y as v"]

$\frac{3}{x} + \frac{1}{y} = 7$

$\Rightarrow 3u + v = 7$……(1)

$\frac{5}{x} - \frac{4}{y} = 6$

$\Rightarrow 5u - 4v = 6$ ……..(2)

From eq(1)

v = 7 – 3u…..(3)

Substituting the value of v in eq(2)

5u – 4(7 – 3u) = 6

⇒ 5u – 28 + 12u = 6

⇒ 17u = 6 + 28

⇒ 17u = 34

⇒ u = 34/17

⇒ u = 2

⇒ $\frac{1}{x} = 2$

⇒ $\mathbf{x} = \frac{1}{2}$

Substituting the value of u in eq(3)

v = 7 – 3×2

⇒ v = 1

⇒ $\frac{1}{y} = 1$

⇒ y = 1

Hence, x = $\frac{1}{2}$, y = 1

2. Question

Find two numbers whose sum is 24 and difference is 8.

Answer

Let the two numbers be x and y.

Since, it is given that their sum is 24,

We can write, x + y = 24

For forming second equation, we will use their difference, which is 8.

So, we can write,

x – y = 8

(we can assume any number to be greater. Here, we assumed the greater number is x)

Here, coefficient of y is equal in magnitude and opposite in sign.

So, if we add the above two equations, it will be eliminated.

Adding equation (1)and(2)

We get

$2x = 32$

$\Rightarrow x = \frac{32}{2}$

$\Rightarrow x = 16$

Substituting the value of x in eq(1)

We get

$x + y = 24$

$\Rightarrow 16 + y = 24$

$\Rightarrow y = 24 - 16$

$\Rightarrow y = 8$

Hence, the two required numbers are 16 and 8.

3. Question

A number consists of two digits whose sum is 9. The number formed by reversing the digits exceeds twice the original number by 18. Find the original number.

Answer

Let unit digit of the number be x and its ten's digit be y.

Then the number will be $10 \times y + x = 10y + x$

Sum of its digits is 9.

Therefore, the sum of x and y is 9

$\Rightarrow x + y = 9$……(1)

On reversing the number, ten's digit will become unit's digit and unit's digit will become ten's digit.

So, after reversing the number,

Its ten's digit = x and unit's digit = y

Therefore, the new number formed = $10x + y$

New number formed exceeds the twice the original number by 18

$\Rightarrow 10x + y = 2(10y + x) + 18$

$\Rightarrow 10x + y = 20y + 2x + 18$

$\Rightarrow 8x - 19y = 18$.....(2)

From equation(1),

$x + y = 9$

$\Rightarrow x = 9 - y$.....(3)

Substituting the value of x in eq(2)

$8(9 - y) - 19y = 18$

$\Rightarrow 72 - 8y - 19y = 18$

$\Rightarrow -27y = 18 - 72$

$\Rightarrow -27y = -54$

$\Rightarrow y = 2$

Substituting the value of y in eq(3)

$x = 9 - 2$

$\Rightarrow x = 7$

Hence, the original number is $10y + x = 10\times2 + 7 = 27$

4. Question

Kavi and Kural each had a number of apples. Kavi said to Kural "If you give me 4 of your apples, my number will be thrice yours". Kural replied, "If you give me 26, my number will be twice yours". How many did each have with them?.

Answer

Let Kavi has x apples and Kural has y apples.

According to Kavi's statement,

We can write

$x + 4 = 3(y - 4)$

$x + 4 = 3y - 12$

$\Rightarrow x = 3y - 16$.....(1)

According to Kural's statement

$y + 26 = 2(x - 26)$

$y + 26 = 2x - 52$…….(2)

Substituting the value of x from eq(1) in eq(2)

$y + 26 = 2(3y - 16) - 52$

$\Rightarrow y + 26 = 6y - 32 - 52$

$\Rightarrow y + 26 = 6y - 84$

$\Rightarrow 5y = 110$

$\Rightarrow y = 22$

Substituting value of y in eq(1)

$x = 3 \times 22 - 16$

$\Rightarrow x = 50$

Hence, Kavi has 50 apples and Kural has 22 apples.

5. Question

Solve the following inequations.

(i) $2x + 7 > 15$

(ii) $2(x - 2) < 3$

(iii) $2(x + 7) \leq 9$

(iv) $3x + 14 \geq 8$

Answer

(i) $2x + 7 > 15$

$2x > 15 - 7$

$\Rightarrow 2x > 8$

$\Rightarrow x > \frac{8}{2}$

$\Rightarrow x > 4$

(ii) $2x - 4 < 3$

$\Rightarrow 2x > 3 + 4$

$\Rightarrow 2x > 7$

$\Rightarrow x>7/2$

(iii) $2(x + 7) \le 9$

$\Rightarrow 2x + 14 \le 9$

$\Rightarrow 2x \le 9 - 14$

$\Rightarrow 2x \le -5$

$\Rightarrow x \le -\frac{5}{2}$

(iv) $3x + 14 \ge 8$

$\Rightarrow 3x \ge 8 - 14$

$\Rightarrow 3x \ge -6$

$\Rightarrow x \ge -\frac{6}{3}$

$\Rightarrow x \ge -2$

Exercise 1.5

1. Question

The expansion of $(x + 2)(x - 1)$ is

A. $x^2 - x - 2$

B. $x^2 + x + 2$

C. $x^2 + x - 2$

D. $x^2 - x + 2$

Answer

Using identity,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

We have,

$(x + 2)(x - 1)$

$= x^2 + (2 - 1)x + (2x - 1) = x^2 + x - 2$

2. Question

The expansion of $(x + 1)(x - 2)(x + 3)$ is

A. $x^3 + 2x^2 - 5x - 6$

B. $x^3 - 2x^2 + 5x - 6$

C. $x^3 + 2x^2 + 5x - 6$

D. $x^3 + 2x^2 + 5x + 6$

Answer

Using identity,

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

We have,

$(x + 1)(x - 2)(x + 3) = x^3 + (1 - 2 + 3)x^2 + (-2 - 6 + 3)x - 6$

$= x^3 + 2x^2 - 5x - 6$

3. Question

$(x - y)(x^2 + xy + y^2)$ is equal to

A. $x^3 + y^3$

B. $x^2 + y^2$

C. $x^2 - y^2$

D. $x^3 - y^3$

Answer

$(x - y)(x^2 + xy + y^2)$ is simply the expansion of $x^3 - y^3$.

we can also obtain this result by multiplying them

i.e. $(x - y)(x^2 + xy + y^2) = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3$

4. Question

Factorization of $x^2 + 2x - 8$ is

A. $(x + 4)(x - 2)$

B. $(x - 4)(x + 2)$

C. $(x + 4)(x + 2)$

D. (x – 4)(x – 2)

Answer

Suppose (x + p)(x + q) are two factors of $x^2 + 2x - 8$.

Then, $x^2 + 2x - 8 = (x + p)(x + q)$

$= x^2 + (p + q)x + pq$

So, to factorize we have to find p and q, such that pq = – 8 and p + q = 2.

Factors of – 8	Sum of factors
– 2, 4	2
2, – 4	– 2
The required factors are – 2 and 4	

∴ $x^2 + 2x - 8 = (x + 4)(x - 2)$

5. Question

If one of the factors of $x^2 - 6x - 16$ is (x + 2) then other factor is

A. x + 5

B. x – 5

C. x + 8

D. x – 8

Answer

Let the other factor be (x + p)

Then, $x^2 - 6x - 16 = (x + p)(x + 2)$

$= x^2 + (p + 2)x + 2p$

On comparing the coefficients on both sides, we get,

p + 2 = – 6

⇒ p = – 8

∴ The other factor is (x – 8).

6. Question

If (2x + 1) and (x – 3) are the factors of $ax^2 - 5x + c$, then the values of a and c are respectively

A. 2, 3

B. – 2, 3

C. 2, – 3

D. 1, – 3

Answer

$ax^2 - 5x + c = (2x + 1)(x - 3)$

$= 2x^2 - 6x + x - 3$

$= 2x^2 - 5x - 3$

On comparing the coefficients on both the sides, we get,

a = 2 and c = – 3

7. Question

If x + y = 10 and x – y = 2, then value of x is

A. 4

B. – 6

C. – 4

D. 6

Answer

x + y = 10

⇒ y = 10 – x substituting this value in x – y = 2

We get,

x – (10 – x) = 2

⇒ 2x – 10 = 2

⇒ 2x = 12

⇒ x = 6

8. Question

The solution of 2 – x <5 is

A. x > – 3

B. x < – 3

C. x > 3

D. $x < 3$

Answer

$2 - x < 5$

Adding x on both the sides,

$2 < 5 + x$

$\Rightarrow -3 < x$

$\Rightarrow x > -3$