■ Sum of counting numbers

1) As we discussed earlier the summation of counting numbers from the beginning in an order is performed by pairing the numbers from both ends.
It is illustrated below
$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ can be paired as $(1, 10), (2, 9), (3, 8), (4, 7), (5, 6)$
Pair sum is 11. The sum of numbers is $11 \times 5 = (1 + 10) \times \frac{10}{2}$
So the sum of first n counting numbers is $\frac{n(n+1)}{2}$

    a) What is the sum of first 10 counting numbers?

    b) Calculate the sum $1 + 2 + 3 + 4 + \cdots 100$

    c) Find the sum of first 25 natural numbers.

2) Look at the pattern given below
$1 = 1$
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
The sum of first n odd numbers is n^2.

    a) What is the sum of first 10 odd numbers?

    b) How many odd numbers from 1 in an orded makes the sum 400?

    c) How many odd numbers are there below 100? What is its sum?

    d) Sum of the odd numbers from 1 in an order makes the sum in between 900 and 1000. How many odd numbers make this sum? Which number is largest among them.

3) Consider the sequence of even numbers $2, 4, 6, 8, 10 \cdots$
Look at the pattern formed from this sequence
$2 + 4 = 6 = 2 \times 3 = 2(2 + 1)$
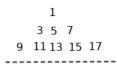$2 + 4 + 6 = 12 = 3 \times 4 = 3(3 + 1)$
$2 + 4 + 6 + 8 = 20 = 4 \times 5 = 4(4 + 1)$
$-------------------$

    a) What is the sum of first 5 even numbers?

    b) Find the sum $2 + 4 + 6 + 8 + 10 + \cdots 50$

    c) What is the sum of first 20 even numbers? Using this find the sum of first 20 counting numbers.

4) The sum of first n even numbers is $n(n + 1)$. using this formula calculate the following

    a) What is the sum of first 10 even numbers?

    b) What is the sum of all even numbers below 100?

c) How much the sum of first 50 even numbers is greater than the sum of first 50 odd numbers.

5) Look at the pattern given below

$$
\begin{array}{c}
1 \\
2 \;\; 3 \;\; 4 \\
5 \;\; 6 \;\; 7 \;\; 8 \;\; 9 \\
\hline
\end{array}
$$

a) Write the sequence of numbers at the right end of each line in the pattern.

b) Which number comes in the right end of 20 th line?

c) In which line the number 850 appears?

d) How many natural numbers are needed to make 50 lines?

e) Find the sum of all counting numbers to make 50 lines in the pattern.

6)   a) What is the sum of first 9 counting numbers?

b) Calculate the sum of first 20 counting numbers.

c) What is the sum of counting numbers from 10 to 20

7) Look at the pattern given below

$$
\begin{array}{c}
1 \\
3 \;\; 5 \;\; 7 \\
9 \;\; 11 \;\; 13 \;\; 15 \;\; 17 \\
\hline
\end{array}
$$

a) Write the number of numbers in each line as a sequence.

b) How many numbers are there in 20 th line?

c) Which number comes at the right end of 20 th line?

d) What is the sum of all numbers needed to make 20 lines?

b) $2 + 4 + 6 \cdots + 50 = 2(1 + 2 + 3 \cdots + 25) = 2 \times \frac{25(25+1)}{2} = 25 \times 26 = 650$

Another method

There are 25 odd numbers below 50 . Sum $= 25(25 + 1) = 25 \times 26 = 650$

4) **a)** $10 \times (10 + 1) = 10 \times 11 = 110$

b) There are 50 even numbers below 100. Sum of these numbers is $50(50+1) = 50 \times 51 = 2550$

c) $2550 - 2500 = 50$

5) **a)** $1, 4, 9, 16 \cdots$

b) $20^2 = 400$

c) Number at the right end of 29 th line is $= 29^2 = 841$

Number at the right end of 30 th line is $= 30^2 = 2500$

850 comes in 30 th line.

d) Number at the right end of 50th line is $50^2 = 2500$.

2500 counting numbers are needed to write 50 lines

e) Sum $\frac{2500(2500+1)}{2} = 1250 \times 2501 = 3126250$

6) **a)** Sum of first 9 counting numbers $= \frac{9 \times (9+1)}{2} = 45$

b) Sum mof first 20 counting numbers $= \frac{20 \times (2+1)}{2} = 210$

c) Sum of all counting numbers from 10 to 20 is $= 210 - 45 = 165$

7) **a)** $1, 3, 5, 7 \cdots$

b) This is an arithmetic sequence . $x_n = 2n - 1$

$x_{20} = 2 \times 20 - 1 = 799$

There are 799 numbers in 20 th line.

c) Sequence of numbers in the right end of each line

$1, 7, 17, 31 \cdots$

Algebraic form is $= 2n^2 - 1$. Number at the right end of 20 th line is $2 \times 20^2 - 1 = 799$

d) Number of odd numbers from 1 to 799 is

$2n - 1 = 799, 2n = 800, n = 400$.

Sum $= 400^2 = 160000$

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