CBSE Class 11 Mathematics

Important Questions

Chapter 5

Complex Numbers and Quadratic Equations

1 Marks Questions

1. Evaluate i^{-39}

Ans. $i^{-39}=\frac{1}{i^{39}}=\frac{1}{(i^4)^9 . i^3}$

$= \frac{1}{1\times(-i)}$ $\quad \left[\because \begin{array}{l} i^4=1 \\ i^3=-i \end{array}\right.$

$=\frac{1}{-i}\times\frac{i}{i}$

$=\frac{i}{-i^2}=\frac{i}{-(-1)}=i \quad [\because i^2=-1$

2. Solved the quadratic equation $x^2+x+\frac{1}{\sqrt{2}}=0$

Ans. $\frac{x^2}{1}+\frac{x}{1}+\frac{1}{\sqrt{2}}=0$

$\frac{\sqrt{2}x^2+\sqrt{2}x+1}{\sqrt{2}}=\frac{0}{1}$

$\sqrt{2}x^2+\sqrt{2}x+1=0$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2 - 4\sqrt{2}}}{2 \times \sqrt{2}}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2}\sqrt{1 - 2\sqrt{2}}}{2\sqrt{2}}$$

$$= \frac{-1 \pm \sqrt{2\sqrt{2} - 1}\ i}{2}$$

3. If $\left(\frac{1+i}{1-i}\right)^m$ = 1, then find the least positive integral value of m.

Ans. $\left(\frac{1+i}{1-i}\right)^m = 1$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\left(\frac{1+i^2+2i}{1-i^2}\right)^m = 1$$

$$\left(\frac{\cancel{1} - \cancel{1} + 2i}{2}\right)^m = 1 \qquad [\because i^2 = -1$$

$$i^m = 1$$

m=4

4. Evaluate (1+ i)^4

Ans. $(1+i)^4 = \left[(1+i)^2\right]^2$

$= (1+i^2+2i)^2$

$= (1-1+2i)^2$

$= (2i)^2 = 4i^2$

$= 4(-1) = -4$

5. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Ans. Let z = $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

$= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$

$= \frac{4i}{2}$

$= 2i$

$z = 0 + 2i$

$|z| = \sqrt{(0)^2 + (2)^2}$

$= 2$

6. Express in the form of a + ib. $(1+3i)^{-1}$

Ans. $(1+3i)^{-1} = \frac{1}{1+3i} \times \frac{1-3i}{1-3i}$

$$= \frac{1-3i}{(1)^2-(3i)^2}$$

$$= \frac{1-3i}{1-9i^2}$$

$$= \frac{1-3i}{1+9} \qquad [i^2 = -1$$

$$= \frac{1-3i}{10}$$

$$= \frac{1}{10} - \frac{3i}{10}$$

7. Explain the fallacy in -1 = i. i. = $\sqrt{-1}.\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$

Ans. $1 = \sqrt{1} = \sqrt{(-1)(-1)}$ is okay but

$\sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1}$ is wrong.

8. Find the conjugate of $\frac{1}{2-3i}$

Ans. Let z = $\frac{1}{2-3i}$

$$z = \frac{1}{2-3i} \times \frac{2+3i}{2+3i} i$$

$$= \frac{2+3i}{(2)^2-(3i)^2}$$

$$= \frac{2+3i}{4+9}$$

$$= \frac{2+3i}{13}$$

$$z = \frac{2}{13} + \frac{3}{13}i$$

$$\bar{z} = \frac{2}{13} - \frac{3}{13}$$

9. Find the conjugate of – 3i – 5.

Ans. Let z = 3i – 5

$$\bar{z} = 3i - 5$$

10. Let z_1 = 2 – i, z_2 = -2+i Find Re $\left(\frac{z_1 z_2}{\overline{z_1}}\right)$

Ans. z_1 z_2 = (2 – i)(-2 + i)

$$= -4 + 2i + 2i - i^2$$

$$= -4 + 4i + 1$$

$$= 4i - 3$$

$$\overline{z_1} = 2 + i$$

$$\frac{z_1 z_2}{\overline{z}_1} = \frac{4i-3}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{8i - 6 - 4i^2 + 3i}{4 - i^2}$$

$$= \frac{11i - 2}{5}$$

$$\frac{z_1 z_2}{z_1} = \frac{11}{5}i - \frac{2}{5}$$

$$\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = -\frac{2}{5}$$

11. Express in the form of a + ib (3i-7) + (7-4i) – (6+3i) + i^{23}

Ans. Let

Z = $3i - 7 + 7 - 4i - 6 - 3i + (i^4)^5 \cdot i^3$

$= -4i - 6 - i \qquad \left[\because i^4 = 1, \; i^3 = -i\right.$

$= -5i - 6$

$= -6 + (-5i)$

12. Find the conjugate of $\sqrt{-3} + 4i^2$

Ans. $\text{Let } z = \sqrt{-3} + 4i^2$

$= \sqrt{3}\, i - 4$

$\bar{z} = -\sqrt{3}\, i - 4$

13. Solve for x and y, 3x + (2x-y) i= 6 – 3i

Ans. 3x = 6

x = 2

2x – y = - 3

2 × 2 – y = - 3

- y = - 3 – 4

y = 7

14. Find the value of $1+i^2 + i^4 + i^6 + i^8 + \text{----} + i^{20}$

Ans. $1+i^2+(i^2)^2+(i^2)^3+(i^2)^4+-----+(i^2)^{10}=1$ $\left[\because i^2 = -1\right.$

15. Multiply 3-2i by its conjugate.

Ans.Let z = 3 – 2i

$$\bar{z} = 3+2i$$

$$z\,\bar{z} = (3-2i)(3+2i)$$

$$= 9 + 6i - 6i - 4i^2$$

$$= 9 - 4(-1)$$

$$= 13$$

16. Find the multiplicative inverse 4 – 3i.

Ans. Let z = 4 – 3i

$$\bar{z} = 4+3i$$

$$|z| = \sqrt{16+9} = 5i$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$= \frac{4+3i}{25}$$

$$= \frac{4}{25} + \frac{3}{25}$$

17. Express in term of a + ib

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Ans. $= \dfrac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+i\sqrt{2}}$

$$=\frac{9+5}{2\sqrt{2}i}=\frac{14\ 7}{2\sqrt{2}i}$$

$$=\frac{7}{\sqrt{2}i}\times\frac{\sqrt{2}i}{\sqrt{2}i}=\frac{7\sqrt{2}i}{-2}$$

18. Evaluate $i^n+i^{n+1}+i^{n+2}+i^{n+3}$

Ans. $= i^n + i^n.i^1 + i^n.i^2 + i^n.i^3$

$= i^n + i^n.i - i^n + i^n.(-i)$ $\left[\begin{array}{l} i^3=-i \\ i^2=-1 \end{array}\right.$

$= 0$

19. If 1, w, w^2 are three cube root of unity, show that $(1 - w + w^2)(1 + w - w^2) = 4$

Ans. $(1 - w + w^2)(1 + w - w^2)$

$(1 + w^2 - w)(1 + w - w^2)$

$(-w-w)\left(-w^2-w^2\right)$ $\left[\begin{array}{l} \because 1+w=-w^2 \\ 1+w^2=-w \end{array}\right.$

$(-2w)\left(-2w^2\right)$

$4w^3$ $\left[w^3=1\right.$

4×1

$= 4$

20. Find that sum product of the complex number $-\sqrt{3}+\sqrt{-2}$ and $2\sqrt{3}-i$

Ans. $z_1+z_2=-\sqrt{3}+\sqrt{2}i+2\sqrt{3}-i$

$=\sqrt{3}+\left(\sqrt{2}-1\right)i$

$z_1 z_2=\left(-\sqrt{3}+\sqrt{2}i\right)\left(2\sqrt{3}-i\right)$

$=-6+\sqrt{3}i+2\sqrt{6}i-\sqrt{2}\ i^2$

$=-6+\sqrt{3}\ i+2\sqrt{6}\ i+\sqrt{2}$

$=\left(-6+\sqrt{2}\right)+\left(\sqrt{3}+2\sqrt{6}\right)i$

21. Write the real and imaginary part $1 - 2i^2$

Ans. Let $z = 1 - 2i^2$

$=1 - 2(-1)$

$= 1 + 2$

$= 3$

$= 3 + 0.i$

Re (z) = 3, Im (z) = 0

22. If two complex number z_1, z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$

Ans. Let $z_1 = a + ib$

$$|z_1| = \sqrt{a^2 + b^2}$$

$$z_2 = b + ia$$

$$|z_2| = \sqrt{b^2 + a^2}$$

Hence $|z_1| = |z_2|$ but $z_1 \neq z_2$

23. Find the conjugate and modulus of $\overline{9-i} + \overline{6+i^3} - \overline{9+i^2}$

Ans. Let $z = \overline{9-i} + \overline{6-i} - \overline{9-1}$

$$= 9 + i + 6 + i - 0$$
$$= 5 + 2i$$

$$\bar{z} = 5 - 2i$$

$$|z| = \sqrt{(5)^2 + (-2)^2}$$

$$= \sqrt{25 + 4}$$

$$= \sqrt{29}$$

24. Find the number of non zero integral solution of the equation $|1\text{-}i|^x = 2^x$

Ans. $|1 - i|^x = 2^x$

$$\left(\sqrt{(1)^2 + (-1)^2}\right)^x = 2^x$$

$$\left(\sqrt{2}\right)^x = 2^x$$

$$(2)^{\frac{1}{2}x} = 2^x$$

$$\frac{1}{2}x = x$$

$$\frac{1}{2} = 1$$

$$1 = 2$$

Which is false no value of x satisfies.

25. If (a + ib) (c + id) (e + if) (g + ih) = A + iB then show that

$$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$$

Ans. $(a+ib)(c+id)(e+if)(g+ih)=A+iB$

$$\Rightarrow \left|(a+ib)(c+id)(e+if)(g+ih)\right| = |A+iB|$$

$$|a+ib||c+id||e+if||g+ih| = |A+iB|$$

$$\left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right)\left(\sqrt{e^2+f^2}\right)\left(\sqrt{g^2+h^2}\right)=\sqrt{A^2+B^2}$$

sq. both side

$$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$$

CBSE Class 12 Mathematics

Important Questions

Chapter 5

Complex Numbers and Quadratic Equations

4 Marks Questions

1.If x + ι y = $\frac{a+ib}{a-ib}$ Prove that $x^2 + y^2 = 1$

Ans. $x+iy=\frac{a+ib}{a-ib}$ (i) (Given)

taking conjugate both side

$$x-iy=\frac{a-ib}{a+ib} \quad \text{(ii)}$$

(i) × (ii)

$$(x+iy)(x-iy)=\left(\frac{a+ib}{a-ib}\right)\times\left(\frac{a-ib}{a+ib}\right)$$

$$(x)^2-(iy)^2=1$$

$x^2 + y^2 = 1$

[$i^2 = -1$

2.Find real θ such that $\frac{3+2i\ \text{Sin}\,\theta}{1-2i\ \text{Sin}\,\theta}$ is purely real.

Ans. $\frac{3+2i\ Sin\theta}{1-2i\ Sin\theta}=\frac{3+2i\ Sin\theta}{1-2i\ Sin\theta}\times\frac{1+2i\ Sin\theta}{1+2i\ Sin\theta}$

$$=\frac{3+6i\ Sin\theta+2i\ Sin\theta-4Sin^2\theta}{1+4Sin^2\theta}$$

$$= \frac{3-4\ Sin^2\theta}{1+4\ Sin^2\theta} + \frac{8i\ Sin\theta}{1+4\ Sin^2\theta}$$

For purely real

Im (z) = 0

$$\frac{8Sin\theta}{1+4Sin^2\theta} = 0$$

$$Sin\theta = 0$$

$$\theta = n\pi$$

3.Find the modulus of $\frac{(1+i)(2+i)}{3+i}$

Ans. $\left|\frac{(1+i)(2+i)}{3+i}\right| = \frac{|(1+i)||2+i|}{|3+i|}$

$$= \frac{\left(\sqrt{1^2+1^2}\right)\left(\sqrt{4+1}\right)}{\sqrt{(3)^2+(1)^2}}$$

$$= \frac{\left(\sqrt{2}\right)\left(\sqrt{5}\right)}{\sqrt{10}}$$

$$= \frac{\sqrt{2}\times\sqrt{5}}{\sqrt{2}\times\sqrt{5}}$$

$$= 1$$

4.If $|a+ib| = 1$, **then Show that** $\frac{1+b+ai}{1+b-ai} = b+ai$

Ans. $|a+ib| = 1$

$$\sqrt{a^2+b^2}=1$$

$$a^2+b^2=1$$

$$\frac{1+b+ai}{1+b-ai}=\frac{(1+b)+ai}{(1+b)-ai}\times\frac{(1+b)+ai}{(1+b)+ai}$$

$$=\frac{(1+b)^2+(ai)^2+2(1+b)(ai)}{(1+b)^2-(ai)^2}$$

$$=\frac{1+b^2+2b-a^2+2ai+2abc}{1+b^2+2a-a^2}$$

$$=\frac{(a^2+b^2)+b^2+2b-a^2+2ai+2abi}{(a^2+b^2)+b^2+2b-a^2}$$

$$=\frac{2b^2+2b+2ai+2abi}{2b^2+2b}$$

$$=\frac{b^2+b+ai+abi}{b^2+b}$$

$$=\frac{b(b+1)+ai(b+1)}{b(b+1)}$$

$$=b+ai$$

5.If x – iy = $\sqrt{\frac{a-ib}{c-id}}$ **Prove that** $(x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$

Ans. $x-iy=\sqrt{\frac{a-ib}{c-id}}$ (1) (Given)

Taking conjugate both side

$$x+iy=\sqrt{\frac{a+ib}{c+id}} \quad \text{(ii)}$$

(i) × (ii)

$$(x-iy)\times(x+iy)=\sqrt{\frac{a-ib}{c-id}}\times\sqrt{\frac{a+ib}{c+id}}$$

$$(x)^2-(iy)^2=\sqrt{\frac{(a)^2-(ib)^2}{(c)^2-(id)^2}}$$

$$x^2+y^2=\sqrt{\frac{a^2+b^2}{c^2+d^2}}$$

squaring both side

$$\left(x^2+y^2\right)^2=\frac{a^2+b^2}{c^2+d^2}$$

6.If $a+ib=\frac{c+i}{c-i}$ **, where a, b, c are real prove that** $a^2+b^2=1$ **and** $\frac{b}{a}=\frac{2c}{c^2-1}$

Ans. $a+ib=\frac{c+i}{c-i}$ (Given) (i)

$$a+ib=\frac{c+i}{c-i}\times\frac{c+i}{c+i}$$

$$a+ib=\frac{c^2+2ci+i^2}{c^2-i^2}$$

$$a+ib=\frac{c^2-1}{c^2+1}+\frac{2c}{c^2+1}i$$

$$a=\frac{c^2-1}{c^2+1},\ \text{b}=\frac{2c}{c^2+1}$$

$$a^2+b^2=\left(\frac{c^2-1}{c^2+1}\right)^2+\frac{4c^2}{(c^2+1)^2}$$

$$=\frac{(c^2+1)^2}{(c^2+1)^2}$$

$a^2 + b^2 = 1$

$$\frac{b}{a}=\frac{2c}{c^2-1}$$

$$\frac{b}{a}=\frac{\frac{2c}{c^2+1}}{\frac{c^2-1}{c^2+1}}$$

7.If z_1 = 2-i and Z_2 = 1+i Find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

Ans.$z_1 + z_2 + 1 = 2 - i + 1 + i + 1 = 4$

$$z_1-z_2+i=2-i-1-i+i=1-i$$

$$\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|=\left|\frac{4}{1-i}\right|$$

$$=\frac{|4|}{|1-i|}$$

$$=\frac{4}{\sqrt{1^2+(-1)^2}}$$

$$=\frac{4}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{4\sqrt{2}}{2}$$

$$=2\sqrt{2}$$

8.If $(p + iq)^2 = x + iy$ Prove that $(p^2 + q^2)^2 = x^2 + y^2$

Ans.$(p + iq)^2 = x + iy$ (i)

Taking conjugate both side

$(p - iq)^2 = x - iy$ (ii)

(i) × (ii)

$$(p+iq)^2 (p-iq)^2 = (x+iy)(x-iy)$$

$$\left[(p+iq)(p-iq)\right]^2 = (x)^2 - (iy)^2$$

$$\left[(p)^2 - (iq)^2\right]^2 = x^2 - i^2y^2$$

$$\Rightarrow (p^2+q^2)^2 = x^2 + y^2$$

9.If $a + ib = \frac{(x+i)^2}{2x^2+1}$ Prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Ans. $a + ib = \frac{(x+i)^2}{2x^2+1}$ (i) (Given)

Taking conjugate both side

$$a-ib=\frac{(x-i)^2}{2x^2+1} \quad \text{(ii)}$$

(i) × (ii)

$$(a+ib)(a-ib)=\left(\frac{(x+i)^2}{2x^2+1}\right)\times\left(\frac{(x-i)^2}{2x^2+1}\right)$$

$$(a)^2-(ib)^2=\frac{(x^2-i^2)^2}{(2x^2+1)^2}$$

$$a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2} \quad \text{proved.}$$

10.If $(x+iy)^3=u+iv$ then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

Ans. $(x+iy)^3=4+iv$

$$x^3+(iy)^3+3x^2(iy)+3.x(iy)^2=u+iv$$

$$x^3-iy^3+3x^2yi-3xy^2=u+iv$$

$$x^3-3xy^2+(3x^2y-y^3)i=u+iv$$

$$x(x^2-3y^2)+y(3x^2-y^2)i=u+iv$$

$$x(x^2-3y^2)=u,\ \mathrm{y}(3x^2-y^2)=v$$

$$x^2-3y^2=\frac{u}{x} \quad \text{(i)} \quad \Big| \quad 3x^2-y^2=\frac{v}{y} \quad \text{(ii)}$$

(i) + (ii)

$4x^2 - 4y^2 = \frac{u}{x} + \frac{v}{y}$

$4\left(x^2 - y^2\right) = \frac{u}{x} + \frac{v}{y}$

11.Solve $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Ans. $\sqrt{3}\ x^2 - \sqrt{2}\ x + 3\ \sqrt{3} = 0$

$a = \sqrt{3}, b = -\sqrt{2}, c = 3\sqrt{3}$

$D = b^2 - 4ac$

$= \left(-\sqrt{2}\right)^2 - 4 \times \sqrt{3}\left(3\sqrt{3}\right)$

$= 2 - 36$

$= -34$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-\left(-\sqrt{2}\right) \pm \sqrt{-34}}{2 \times \sqrt{3}}$

$= \frac{\sqrt{2} \pm \sqrt{34}\ i}{2\sqrt{3}}$

12.Find the modulus $i^{25} + \left(1 + 3i\right)^3$

Ans. $i^{25} + (1+3i)^3$

$= \left(i^4\right)^6 . i + 1 + 27i^3 + 3(1)(3i)(1+3i)$

$= i + \left(1 - 27i + 9i + 27i^2\right)$

$= i + 1 - 18i - 27$

$= -26 - 17i$

$\left|i^{25} + (1+3i)^3\right| = \left|-26 - 17i\right|$

$= \sqrt{(-26)^2 + (-17)^2}$

$= \sqrt{676 + 289}$

$= \sqrt{965}$

13.If $a + ib = \dfrac{(x+i)^2}{2x-i}$ prove that $a^2 + b^2 = \dfrac{(x^2+1)^2}{4x^2+1}$

Ans. $a + ib = \dfrac{(x+i)^2}{2x-i}$ (i) (Given)

$a - ib = \dfrac{(x-i)^2}{2x+i}$ (ii) [taking conjugate both side

(i) × (ii)

$(a+ib)(a-ib) = \dfrac{(x+i)^2}{(2x-i)} \times \dfrac{(x-i)^2}{(2x+i)}$

$a^2 + b^2 = \dfrac{(x^2+1)^2}{4x^2+1}$ proved.

14.Evaluate $\left[i^{18} + \left(\dfrac{1}{i}\right)^{25}\right]^3$

Ans. $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

$\left[\left(i^4\right)^4 .i^2+\frac{1}{i^{25}}\right]^3$

$\left[i^2+\frac{1}{\left(i^4\right)^6 .i}\right]^3$

$\left[-1+\frac{1}{i}\right]^3$

$\left[-1+\frac{i^3}{i^4}\right]^3$

$[-1-i]^3=-(1+i)^3$

$=-\left[1^3+i^3+3.1.i(1+i)\right]$

$=-\left[1-i+3i+3i^2\right]$

$=-[1-i+3i-3]$

$=-[-2+2i]=2-2i$

15.Find that modulus and argument $\frac{1+i}{1-i}$

Ans. $\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$

$=\frac{(1+i)^2}{1^2-i^2}$

$$= \frac{1+i^2+2i}{1+1}$$

$$= \frac{2i}{2}$$

$$= i$$

$$z = 0 + i$$

$$r = |z| = \sqrt{(0)^2 + (1)^2} = 1$$

Let α be the acute $\angle$s

$$\tan \alpha = \left|\frac{1}{0}\right|$$

$$\alpha = \pi/2$$

$$\arg(z) = \pi/2$$

$$r = 1$$

16.For what real value of x and y are numbers equal (1+i) y^2 + (6+i) and (2+i) x

Ans.(1+i) y^2 + (6 + i) = (2 + i) x

$y^2 + iy^2 + 6 + i = 2x + xi$

$(y^2 + 6) + (y^2 + 1)\, i = 2x + xi$

$y^2 + 6 = 2x$

$y^2 + 1 = x$

$y^2 = x - 1$

$x - 1 + 6 = 2x$

$5 = x$

$y = \pm 2$

17.If x + iy = $\sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

Ans. $x + iy = \sqrt{\frac{1+i}{1-i}}$ (i) (Given)

taking conjugate both side

$x - iy = \sqrt{\frac{1-i}{1+i}}$ (ii)

(i) × (ii)

$(x+iy)(x-iy) = \sqrt{\frac{1+i}{1-i}} \times \sqrt{\frac{1-i}{1+i}}$

$(x)^2 - (iy)^2 = 1$

$x^2 + y^2 = 1$

Proved.

18.Convert in the polar form $\frac{1+7i}{(2-i)^2}$

Ans. $\frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{3-4i}$

$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}$

$= \frac{3+4i+21i+28i^2}{9+16}$

$$= \frac{25i - 25}{25} = i - 1$$

$$= -1 + i$$

$$r = |z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

Let α be the acute $\angle$s

$$\text{ten } \alpha = \left|\frac{1}{-1}\right|$$

$$\alpha = \pi/4$$

since Re (z) < 0, Im (z) > 0

$$\theta = \pi - \alpha$$

$$= \pi - \frac{\pi}{4} = 3\pi/4$$

$$z = r\left(\text{Cos}\theta + \text{i Sin}\theta\right)$$

$$= \sqrt{2}\left(\text{Cos } \frac{3\pi}{4} + iSin\frac{3\pi}{4}\right)$$

19.Find the real values of x and y if (x - iy) (3 + 5i) is the conjugate of – 6 – 24i

Ans.

(x – iy) (3 + 5i) = - 6 + 24i

3x + 5xi – 3yi – 5yi^2 = - 6 + 24i

$$(3x + 5y) + (5x - 3y)i = -6 + 24i$$

$$3x + 5y = -6$$

$$5x - 3y = 24$$

$x = 3$

$y = -3$

20.If $|z_1| = |z_2| = 1$, prove that $\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = |z_1 + z_2|$

Ans. If $|z_1| = |z_2| = 1$ (Given)

$\Rightarrow |z_1|^2 = |z_2|^2 = 1$

$\Rightarrow z_1 \ \overline{z_1} = 1$

$\overline{z_1} = \frac{1}{z_1}$ (1)

$z_2 \ \overline{z_2} = 1$

$\overline{z_2} = \frac{1}{z_2}$ (2)

$\left[\ \because z \ \overline{z} = |z|^2 \right.$

$\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = |\overline{z_1} + \overline{z_2}|$

$= |\overline{z_1 + z_2}|$

$= |z_1 + z_2|$

$\left[\because |\overline{z}| = |z| \right.$ proved.

CBSE Class 12 Mathematics

Important Questions

Chapter 5

Complex Numbers and Quadratic Equations

6 Marks Questions

1.If z = x + i y and w = $\frac{1-i^2}{z-i}$ Show that |w| = 1 $\Rightarrow$ z is purely real.

Ans. w = $\frac{1-iz}{z-i}$

$$=\frac{1-i(x+iy)}{x+iy-i}$$

$$=\frac{1-ix-i^2y}{x+i(y-1)}$$

$$=\frac{(1+y)-ix}{x+i(y-1)}$$

$$\therefore \quad |w|=1$$

$$\Rightarrow \left|\frac{(1+y)-ix}{x+i(y-1)}\right|=1$$

$$\frac{|(1+y)-ix|}{|x+i(y-1)|}=1$$

$$\frac{\sqrt{(1+y)^2+(-x)^2}}{\sqrt{x^2+(y-1)^2}}=1$$

$$1+y^2+2y+x^2=x^2+y^2+1-2y$$

$$4y=0$$

$y = 0$

$\therefore\ z = x + \ i$

is purely real

2.Convert into polar form $\dfrac{-16}{1+i\sqrt{3}}$

Ans. $\dfrac{-16}{1+i\sqrt{3}} = \dfrac{-16}{1+i\sqrt{3}} \times \dfrac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$= \dfrac{-16\left(1-i\sqrt{3}\right)}{(1)^2-\left(i\sqrt{3}\right)^2}$

$= \dfrac{-16\left(1-i\sqrt{3}\right)}{1+3}$

$= -4\left(1-i\sqrt{3}\right)$

$z = -4 + i4\sqrt{3}$

$r = \ |z| = \sqrt{(-4)^2 + \left(4\sqrt{3}\right)^2}$

$= \sqrt{16+48}$

$= \sqrt{64}$

$= 8$

Let α be the acute $\angle$S

$\tan\ \alpha\ = \left|\dfrac{\cancel{4}\sqrt{3}}{-\cancel{4}}\right|$

$\tan\alpha = \tan\dfrac{\pi}{3}$

$\alpha = \pi/3$

Since Re (z) < o, and Im (z) > o

$\theta = \pi - \alpha$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$z = 8\left(Cos \ \frac{2\pi}{3} + i \ \text{Sin} \ \frac{2\pi}{3} \right)$

3.Find two numbers such that their sum is 6 and the product is 14.

Ans. Let x and y be the no.

x + y = 6

xy = 14

$x^2 - 6x + 14 = 0$

$D = -20$

$x = \frac{-(-6) \pm \sqrt{-20}}{2 \times 1}$

$= \frac{6 \pm 2\sqrt{5} \ \text{i}}{2}$

$= 3 \pm \sqrt{5} \ \text{i}$

$\text{x} = 3 + \sqrt{5} \ \text{i}$

$y = 6 - \left(3 + \sqrt{5} \ \text{i} \right)$

$= 3 - \sqrt{5} \ \text{i}$

when $x = 3-\sqrt{5}\ i$

$y = 6-\left(3-\sqrt{5}\ i\right)$

$= 3+\sqrt{5}\ i$

4.Convert into polar form $z = \dfrac{i-1}{\cos\dfrac{\pi}{3}+i\ \text{Sin}\ \dfrac{\pi}{3}}$

Ans. $z = \dfrac{i-1}{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i}$

$= \dfrac{2(i-1)}{1+\sqrt{3}i}\times\dfrac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$z = \dfrac{\sqrt{3}-1}{2}+\dfrac{\sqrt{3}+1}{2}i$

$r = |z| = \left(\dfrac{\sqrt{3}-1}{2}\right)^2+\left(\dfrac{\sqrt{3}+1}{2}\right)^2$

$r = 2$

Let α be the acule $\angle$s

$\tan\ \alpha = \left|\dfrac{\dfrac{\sqrt{3}+1}{\cancel{2}}}{\dfrac{\sqrt{3}-1}{\cancel{2}}}\right|$

$= \left|\dfrac{\cancel{\sqrt{3}}\left(1+\dfrac{1}{\sqrt{3}}\right)}{\cancel{\sqrt{3}}\left(1-\dfrac{1}{\sqrt{3}}\right)}\right|$

$$= \left| \frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{6}} \right|$$

$$\tan \alpha = \left| \tan \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \right|$$

$$\alpha = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$$

$$z = 2\left(Cos \frac{5\pi}{12} + i \text{ Sin } \frac{5\pi}{12} \right)$$

5.If α and β are different complex number with |β| = 1 Then find $\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right|$

Ans. $\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right|^2 = \left(\frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right) \left(\overline{\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}} \right) \quad \left[\because |z|^2 = z\overline{z} \right]$

$$= \left(\frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right) \left(\frac{\overline{\beta} - \overline{\alpha}}{1 - \alpha\overline{\beta}} \right)$$

$$= \left(\frac{\beta\overline{\beta} - \beta\overline{\alpha} - \alpha\overline{\beta} + \alpha\overline{\alpha}}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + \alpha\overline{\alpha}\beta\overline{\beta}} \right)$$

$$= \left(\frac{|\beta|^2 - \beta\overline{\alpha} - \alpha\overline{\beta} + |\alpha|^2}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + |\alpha|^2 |\beta|^2} \right)$$

$$= \left(\frac{1 - \beta\overline{\alpha} - \alpha\overline{\beta} + |\alpha|^2}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + |\alpha|^2} \right) \qquad [\because |\beta| = 1$$

$$= 1$$

$$\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right| = \sqrt{1}$$

$$\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right| = 1$$