CHAPTER -01

RELATIONS AND FUNCTIONS

One Mark questions.

1. Define a reflexive relation [K]

2. Define a symmetric relation . [K]

3. Define a transitive relation [K]

4. Define an equivalence relation [K]

5. A relation R on A = {1, 2, 3} defined by R = {(1, 1), (1, 2), (3, 3)} is not symmetric why ? [U]

6. Give an example of a relation which is symmetric but neither reflexive nor transitive. [U]

7. Give an example of a relation which is transitive but neither reflexive and nor symmetric. [U]

8. Give an example of a relation which is reflexive and symmetric but not transitive. [U]

9. Give an example of a relation which is symmetric and transitive but not reflexive . [U]

10. Define a one-one function. [K]

11. Define an onto function [K]

12. Define a bijective function. [K]

13. Prove that $f : R \to R$ defined by $f(x) = x^2$ is many-one. [U]

14. Prove that $f : Z \to Z$ defined by $f(x) = 1 + x^2$ is not one one. [U]

15. Let A = {1, 2, 3} B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one. [U]

16. Write the number of all one – one functions from the set A = {a, b, c} to itself. [U]

17. If A contains 3 elements and B contains 2 elements, then find the number of one-one functions from A to B. [U]

18. Define a binary operation. [K]

19. Find the number of binary operations on the set {a,b}. [K]

20. On N, show that subtraction is not a binary operation. [U]

21. On Z^{+} (The set of positive integers) define * by $a*b=a-b$.Determine whether * is a binary operation or not.

[U]

22. On Z^{+}, define * by $a*b=ab$, (where Z^{+} = The set of positive integers), Determine whether * is a binary operation or not.

[U]

23. On Z^{+}, define * by $a*b=ab^{2}$,(where Z^{+} = The set of positive integers) Determine whether * is a binary operation or not.

[U]

24. On Z^{+}, define * by $a*b=a^{b}$,where Z^{+} is the set of non negative integers, Determine whether * is a binary operation or not. [U]

25. On Z^{+}, define * by $a*b=|a-b|$, (where Z^{+} = The set of positive integers)Determine whether * is a binary operation or not. [U]

26. On Z^{+}, define * by $a*b=a$, (where Z^{+} = The set of positive integers),Determine whether * is a binary operation or not. [U]

27. Let * be a binary operation on N given by a * b = H.C.F. write the value of 22 * 4. [U]

28. Is * defined on the set A={1, 2, 3, 4, 5} by a * b = L C M of a and b, a binary operation ? Justify your answer. [U]

29. Let A = {1, 2, 3, 4, 5} and * is a binary operation on A defined by a * b = H C F of a and b
Is * commutative? [U]

30. Let * be a b.o on N given by a * b = L.C.M.of a and b. find 5*7 [K]

31. Let A = {1, 2, 3, 4, 5} and * is a binary operation on A defined by a * b = H C F of a and b.
Compute (2 * 3) . [U]

32. Let * be a b.o on N given by a * b = L.C.M.of a and b. find 20*16. [U]

33. On Z^{+}, define * by $a*b=|a-b|$ where Z^{+} is the set of non negative integers, determine whether * is a binary operation or not. [U]

34. On Z^{+}, define * by $a*b=a^{b}$ where Z^{+} is the set of non negative integers, determine whether * is a binary operation or not.

[U]

35. On Z^+(the set of nonnegative integers) define * by $a*b=a-b\ \ \forall a,b\in Z^+$

Is * is a binary operation on Z^+ . [U]

36. On Z^+(the set of nonnegative integers) define * by $a*b=|a-b|\ \ \forall a,b\in Z^+$.

Is * a binary operation on Z^+ . [U]

37. Show that '0' is the identity for addition in R. [K]

38. Show that 1 is the identity for multiplication in R. [K]

39. Show that there is no identity element for subtraction (division) in R. [U]

40. On Q * is defined as , a * b = a – b .Find the identity if it exists. [U]

41. On Q * is defined as a * b = a + ab. Find the identity if it exists. [U]

42. On Q * is defined as $a*b=\frac{ab}{4}\ \ \forall\ a,b\in Q,$ find identity. [K]

43. On Q * is defined as $a*b=a+b\ \ \ \forall\ a,\ b\in N,$ find identity if it exists. [U]

44. On N, a * b = L.C.M of a and b. Find the identity of * in N. [K]

45. Show that –a is the inverse of a under addition in R. [K]

46. Show that $\frac{1}{a}$ is the inverse of a $(a\neq 0)$ under multiplication in R. [K]

47. Given a non-empty set X, consider the binary operation $*:\ P(X)\times P(X)\to P(X)$ given by $A*B=A\cap B\ \ \forall\ A,B\in P(X),$ where P(X) is the power set of X. Show that X is the identity element. [U]

48. Given a non-empty set X, let $*:\ P(X)\times P(X)\to P(X)$ defined by $A\times B=(A-B)\cup(B-A)$

Show that the empty set ϕ is the identity and all the elements of P(X). [U]

Two Mark Questions.

1. Define a reflexive relation and give an example of it. [K]
2. Define a symmetric relation and give an example of it. [K]
3. Define a transitive relation and give an example of it. [K]
4. Define an equivalence relation and give an example of it. [K]
5. If $f : R \to R$ is defined by f(x) = 3x – 2. Show that f is one-one. [A]
6. If $f : N \to N$ given by $f(x) = x^2$ check whether f is one-one and onto. Justify your answer.[U]
7. If $f : Z \to Z$ given by $f(x) = x^2$ check whether f is one-one and onto. Justify your answer. [U]
8. If $f : R \to R$ given by $f(x) = x^2$ check whether f is one-one and onto. Justify your answer. **[U]**
9. If $f : N \to N$ given by $f(x) = x^3$ check whether f is one-one and onto. Justify your answer. [U]
10. If $f : Z \to Z$ given by $f(x) = x^3$ check whether f is one-one and onto. Justify your answer. [U]
11. Show that the function $f : N \to N$, given by f(x) = 2x is one-one but not onto. [U]
12. Show that the function given by f(1) = f(2) = 1 and f(x) = x – 1, for every x > 2, is onto but not one-one. [U]
13. Prove that the greatest integer function $f : R \to R$ given by f(x) = [x] is neither one-one nor onto [U]
14. Show that the modulus function $f : R \to R$ given by f(x) = | x | is neither one-one nor onto.[U]
15. Show that the Signum function $f : R \to R$ defined by $f(x) = \begin{cases} \frac{|x|}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is neither one-one nor onto [U]
16. Let $f : N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$ State whether f is bijective. Justify your answer. [U]

17. Let A and B are two sets. Show that $f: A \times B \to B \times A$ such that f(a, b) = (b, a) is a bijective function. [U}

18. If $f: R \to R$ is defined by f(x) = 1 + x^2, then show that f is neither 1-1 nor onto. [U]

19. Prove that $f: R \to R$ given by f(x) = x^3 is onto. [U]

20. Let $f: \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \to \{7, 11, 15\}$ be functions defined f(2) = 3, f(3) = 4, f(4) = f(5) = 5 and g(3) = g(4) = 7 and g(5) = g(9) = 11. Find gof. [U]

21. Let $f: \{1, 3, 4\} \to \{1, 2, 5\}$ and $g: \{1, 2, 5\} \to \{1, 3\}$ given by f = {1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)} write down gof..**[U]**

22. If $f: A \to B$ and $g: B \to C$ are one-one then show that $gof: A \to C$ is also one-one.[K]

23. If $f: A \to B$ and $g: B \to C$ are onto then show that $gof: A \to C$ is also onto.[K]

24. State with reason whether $f: \{1, 2, 3, 4\} \to \{10\}$ with $f = \{(1, 10), (2, 10), (3, 10), (4,10)\}$ has inverse.[K]

25. State with reason whether $g = \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ with $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$ has inverse.[K]

26. State with reason whether $h: \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ with $h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$ has inverse.[K]

27. Consider the binary operation V on the set {1, 2, 3, 4, 5} defined by a v b = min {a, b}. Write the operation table of the operation V. [K]

28. On Z, defined by a * b = a – b Determine whether * is commutative. [U]

29. On Q, defined by a * b = ab + 1. Determine whether * is commutative [U]

30. On Q , * defined by $a * b = \frac{ab}{2}$ Determine whether * is associative. [U]

31. On Z^+, * defined by $a * b = 2^{ab}$. Determine whether * associative. [U]

32. On R – {–1}, * defined by $a * b = \frac{a}{b+1}$ Determine whether * is commutative [U]

33. Verify whether the operation * defined on Q by $a * b = \frac{ab}{2}$ is associative or not . **[U]**

Three Mark Questions.

1) A relation R on the set A = {1, 2, 3......14} is defined as R = {(x, y) : 3x – y =0}. Determine whether R is reflexive, symmetric and transitive. [U]

2) A relation R in the set N of natural number defined as R = {(x, y) : y = x + 5 and x < 4}. Determine whether R is reflexive, symmetric and transitive. [U]

3) A relation 'R' is defined on the set A = {1, 2, 3, 4, 5} as R = {(x, y) : y is divisible by x}. Determine whether R is reflexive, symmetric, transitive.

4) Relation R in the set Z of all integers is defined as R = {(x, y) : x – y is an integer}. Determine whether R is reflexive, symmetric and transitive.

5) Determine whether R, in the set A of human beings in a town at a particular time is given by R = {(x, y) : x and y work at the same place}

6) Show that the relation R in **R,** the set of reals defined as R = {(a, b) : a $\leq$ b} is reflexive and transitive but not symmetric.

7) Show that the relation R on the set of real numbers R isdefined by R = {(a, b) : a $\leq$ b^2} is neither reflexive nor symmetric nor transitive.

8) Check whether the relation R in **R** the set of real numbers defined as R = {(a, b) : a $\leq$ b^3} is reflexive, symmetric and transitive.

9) Show the relation R in the set Z of integers give by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

10) Show the relation R in the set Z of integers give by R = {(a, b) : (a – b) is divisible by 2} is an equivalence relation.

11) Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a-b| is even} is an equivalence relation.

12) Show that the relation R on the set A of point on cordinate plane given by R = {(P, Q) distance OP = OQ, where O is origin is an equivalence relation.

13) Show that the relation R on the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by R = {(a,b) $|a-b|$:is a multiple of 4 }is an equivalence relation.

14) Show that the relation R on the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by R = {(a,b) :a=b }is an equivalence relation.

15) Show that the relation R on the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by R = {(a,b) $|a-b|$:is a multiple of 4 }is an equivalence relation.

16) Let T be the set of triangles with R – a relation in T given by R = {(T_1, T_2) : T_1 is congruent to T_2}

Show that R is an equivalence relation.

17) Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is perpendicular to L_2}. Show that R is symmetric but neither reflexive nor transitive.

18) Let L be the set of all lines in the XY plane and R is the relation on L by

R = {(l_1, l_2) : l_1 is parallel to l_2}. Show that R is an equivalence relation.

Find the set of all lines related to the line y = 2x + 4.

19) Show that the relation R defined in the set A of polygons as

R = {(P_1, P_2) : P_1 and P_2 have same number of side } is an equivalence relation.

20) If R_1 and R_2 are two equivalence relations on a set, is $R_1 \cup R_2$ also an equivalence relation.? Justify your answer. [A]

21) If R_1 and R_2 are two equivalence relations on a set, then prove that $R_1 \cap R_2$ is also an equivalence relation.[A]

22) **F**ind gof and fog if $f : R \to R$ and $g : R \to R$ are given by f(x) = cos x and g(x) = $3x^2$.

Show that $gof \neq fog.$ **[U]**

23) If f & g are functions from $R \to R$ defined by $f(x) = \sin x$ and $g(x) = x^2$ Show that

$gof \neq fog.$ **[U]**

24) Find gof and fog, if $f(x) = |x|$ and $g(x) = |5x - 2|$ [U]

25) Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{1/3}$ [U]

26) If $f : R \to R$ defined by f(x) = $(3 - x^3)^{1/3}$ then find fof(x). [U]

27) Consider $f : N \to N$, $g : N \to N$ and $h : N \to R$ defined as f(x) = 2x, g(y) = 3y + 4,

h(z) = sin z $\forall$ x, y, z $\in$ N. Show that $f \circ (g \circ h) = (f \circ g) \circ h$ [U]

28) Give examples of two functions f and g such that gof is one -one but g is not one-one.[S]

29) Give examples of two functions f and g such that gof is onto but f is not onto.[S]

Five Mark Questions

1) Let $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ define $f : A \to B$ by $f(x) = \frac{3x+4}{5x-7}$ and

$g : B \to A$ by $g(x) = \frac{7x+4}{5x-3}$. Show that fog = I_B and gof = I_A. [U]

2) Consider $f : R \to R$ given by f(x) = 4x +3.Show that f is invertible.Find the inverse of f. [U]

3) Consider $f : R \to R$ given by f(x) = 10x + 7. Show tht f is invertible. Find the inverse of f. [U]

4) If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that f o f(x) = x for all $x \neq \frac{2}{3}$. What is the inverse of f

[U]

5) Consider $f : R_+ \to [4, \infty)$ given by f(x) = $x^2 + 4$. Show that f is invertible with the inverse

[U]

f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers. [U]

6) Consider $f : R_+ \to [-5, \infty)$ given f(x) = $9x^2 + 6x - 5$. Show that f is invertible with

$f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.

[U]

7) Let $f : N \to R$. be a function defined as f(x) = $4x^2 + 12x + 15$. Show that $f : N \to S$. Where S is the range of f, is invertible. Find the inverse of f

[U]

8) Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f : N \to Y$ as f(n) = n^2. Show that f is invertible.

Find the inverse of f.

[U]

9) Show that $f : [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one.

Find the inverse of the function $f : [-1, 1] \to$ Range of f.

[U]

10) Let $f : R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined by define $f\left(x\right) = \frac{4x}{3x+4}$. Find the inverse of the function $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f.

[U]