Sample Question Paper - 15
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

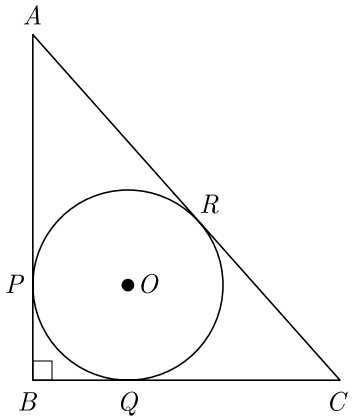
1. What is the sum of five positive integer divisible by 6.

OR

In an Arithmetic Progression, if $d = -4$, $n = 7$, $a_n = 4$, then find a.

2. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then find the length of the wire.

3. ABC is right triangle, right-angled at B, with $BC = 6$ cm and $AB = 8$ cm. A circle with centre O and radius r cm has been inscribed in ΔABC as shown in the figure. Find the value of r.



4. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

5. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere.

6. Find the mean of first 10 composite numbers.

OR

If the difference of mode and median of a data is 24, then what is the difference of median and mean?

Section B

7. Prove that the rectangle circumscribing a circle is a square.

8. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

9. The following frequency distribution shows the number of runs scored by some batsman of India in one-day cricket matches :

Run scored	2000-4000	4000-6000	6000-8000	8000-10000	10000-12000
Number of batsmen	9	8	10	2	1

Find the mode for the above data.

10. A group of students conducted a survey of their locality to collect the data regarding number of plants and recorded it in the following table :

Number of plants	0-3	3-6	6-9	9-12	12-15
Number of houses	2	4	5	1	2

Find the mode for the above data.

OR

Find the mode of the following frequency distribution:

Class-Interval	f
25-35	7
35-45	31
45-55	33
55-65	17
65-75	11
75-85	1

Section C

11. The sum of first 20 terms of an AP is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

12. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

OR

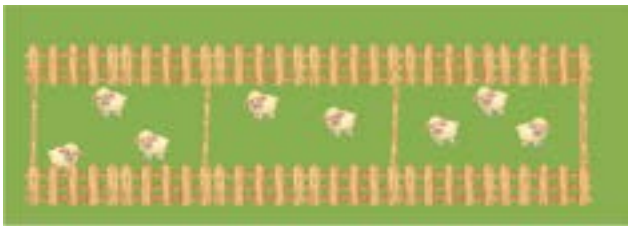
Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as center, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

13. Building Sheep Pens : Darsh and Darpan are fencing off a large rectangular area to build some temporary holding pens. To prep the males, females, and kids, they are separated into three smaller and equal-size pens partitioned

within the large rectangle.

(i) If 384 ft of fencing is available and the maximum area is desired, what will be the dimensions of the larger, outer rectangle?

(ii) What will be the dimensions of the smaller holding pens?



14. Rainfall is one of the most commonly shared experiences on Earth. Rainfall is also necessary, since it provides water to plants and ultimately fills rivers. Because rainfall is both a needed resources and a threat, it is important to better understand this natural phenomenon. The most common rainfall measurement is the total rainfall depth during a given period, expressed in millimetres (mm). For instance, we might want to know how many millimetres of rain fell over the course of 1 h, 1 day, 1 month, or 1 year.

The rain water from 22m $\times$ 20m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m.

(i) If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

(ii) If rainfall is the 1.5 cm find the hight of water collected in cylindrical vessel.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
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SECTION A

1. What is the sum of five positive integer divisible by 6.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_5 = \frac{5}{2}[2 \times 6 + (5-1)(6)]$$

$$= \frac{5}{2}[12 + 4 \times 6] = \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$

$$= 5 \times 18 = 90$$

or

In an Arithmetic Progression, if $d = -4$, $n = 7$, $a_n = 4$, then find a.

Sol :

We have, $d = -4$, $n = 7$, $a_n = 4$

Now $a_n = a + (n-1)\,d$

$4 = a + (7-1)(-4)$

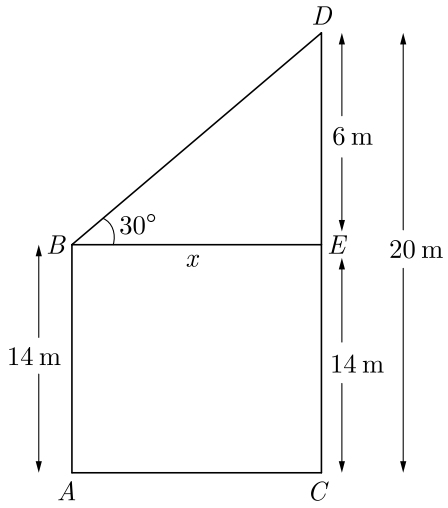
$4 = a - 24 \Rightarrow a = 28$

2. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then find the length of the wire.

Sol :

Height of big pole, $CD = 20\,\text{m}$

Height of small pole, $AB = 14\,\text{m}$



$$DE = CD - CE$$

$$= CD - AB \qquad [AB = CE]$$

$$= 20 - 14 = 6\,\text{m}$$

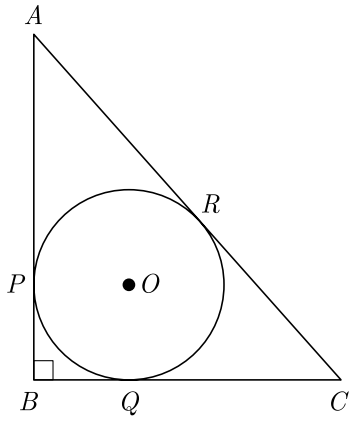
In $\triangle BDE$, $\sin 30^\circ = \frac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD}$$

$\Rightarrow$ $BD = 12\,\text{m}$

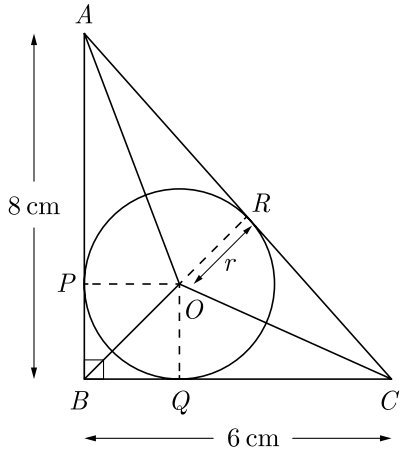
Thus length of wire is 12 m.

3. ABC is right triangle, right-angled at B, with $BC = 6\,\text{cm}$ and $AB = 8\,\text{cm}$. A circle with centre O and radius $r\,\text{cm}$ has been inscribed in ΔABC as shown in the figure. Find the value of r.



Sol :

As per question we have shown the digram below. Here, we have joined AO, BO and CO.



In ΔABC, $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{8^2 + 6^2} = 10\,\text{cm}$

Area of ΔABC $= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 6 \times 8 = 24\,\text{cm}^2$

Area of ΔABC,

A_{ABC} = Area of ΔAOB + Area of ΔBOC + Area of ΔAOC

$= \frac{1}{2} \times r \times AB + \frac{1}{2} \times r \times BC + \frac{1}{2} \times r \times AC$

$= \frac{1}{2} r(AB + BC + AC)$

$= \frac{1}{2} r(8 + 6 + 10) = 12r$

Now $12r = 24$

Thus $r = 2\,\text{cm}$

4. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Sol :

Let h_1 and h_2 be height and r_1 and r_2 be radii of two cones.

Now $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

Ratio of their volumes,

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right) = \left(\frac{3}{1}\right)^2 \left(\frac{1}{3}\right) = \frac{3}{1}$$

Hence, ratio of their volumes is 3 :1.

5. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere.

Sol :

Let R be the radius of sphere.

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (2)^2 \times 8$$

$$4R^3 = 4 \times 8$$

$$R^3 = 8 \Rightarrow R = 2\ \text{cm}$$

6. Find the mean of first 10 composite numbers.

Sol :

First 10 composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

Required mean

$$M = \frac{4 + 6 + 8 + 9 + 10 + 12 + 14 + 15 + 16 + 18}{10}$$

$$= \frac{112}{10} = 11.2$$

or

If the difference of mode and median of a data is 24, then what is the difference of median and mean?

Sol :

We have, $M_o - M_d = 24$

We know $M_o = 3M_d - 2M$

Now $M_o - M_d = 2M_d - 2M$

$24 = 2(M_d - M)$

$M_d - M = 12$

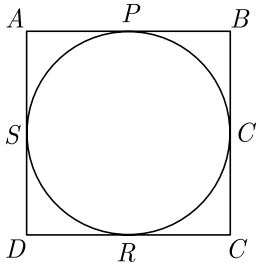
Section B

7. Prove that the rectangle circumscribing a circle is a square.

Sol :

We have a rectangle $ABCD$ circumscribe a circle which touches the circle at P, Q, R, S. We have to prove that $ABCD$ is a square.

As per given information we have drawn the figure below.



Since tangent drawn from an external point to a circle are equals,

$AP = AS,\ PB = BQ,\ DR = DS,\ RC = QC$

Adding all above equation we have

$$AP + PB + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Since $ABCD$ is rectangle, $AB = CD$ and $AD = BC$,

Thus $\quad 2AB = 2BC \Rightarrow AB = BC$

Since adjacent sides are equal are equal. So, $ABCD$ is a square.

8. The following frequency distribution shows the number of runs scored by some batsman of India in one-day cricket matches :

Run scored	2000-4000	4000-6000	6000-8000	8000-10000	10000-12000
Number of batsmen	9	8	10	2	1

Find the mode for the above data.

Sol :

Class 6000-8000 has the maximum frequency 10, therefore this is model class.

Here $f_0 = 8,\ f_1 = 10,\ f_2 = 2,\ h = 2000$, and $l = 6000$

$$\text{Mode,}\quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 6000 + \left(\frac{10 - 8}{20 - 8 - 2}\right) \times 2000$$

$$= 6000 + \frac{2}{10} \times 2000$$

$$= 6000 + 400 = 6400$$

9. A group of students conducted a survey of their locality to collect the data regarding number of plants and recorded it in the following table :

Number of plants	0-3	3-6	6-9	9-12	12-15
Number of houses	2	4	5	1	2

Find the mode for the above data.

Sol :

Class 6-9 has the maximum frequency 5, therefore this is model class.

Now $l_1 = 6,\ f_1 = 5,\ f_0 = 4,\ f_2 = 1,\ h = 3$

$$\text{Mode,}\quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 6 + \frac{5 - 4}{10 - 4 - 1} \times 3$$

$$= 6 + \frac{1}{5} \times 3 = 6 + 0.6 = 6.6$$

or

Find the mode of the following frequency distribution:

Class-Interval	f
25-35	7
35-45	31
45-55	33
55-65	17
65-75	11
75-85	1

Sol :

Class 44-45 has the maximum frequency 33, therefore this is model class.

Now $\quad l_1 = 45,\ f_0 = 31,\ f_1 = 33\ \ f_2 = 17,\ h = 10$

$$\text{Mode,}\quad M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 45 + \frac{33 - 31}{66 - 31 - 17} \times 10$$

$$= 45 + \frac{2}{18} \times 10 = 46.1$$

10. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

Sol :

Let h and r be the height and radius of cylinder and cone.

Height, $h = 14$ cm

and radius, $r = 6$ cm

Volume of the remaining solid,

$$V_{\text{remain}} = V_{\text{cylinder}} - V_{\text{cone}}$$

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h = \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^3$$

Section C

11. The sum of first 20 terms of an AP is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

Sol :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We know $S_n = \frac{n}{2}\,2a + (n-1)d$

Now $S_{20} = \frac{20}{2}(2a + 19d)$

$400 = \frac{20}{2}(2a + 19d)$

$400 = 10[2a + 19d]$

$2a + 19d = 40$ ...(1)

Also, $S_{40} = \frac{40}{2}(2a + 39d)$

$1600 = 20[2a + 39d]$

$2a + 39d = 80$ ...(2)

Solving equation (1) and (2), we get $a = 1$ and $d = 2$.

Now $S_{10} = \frac{10}{2}[2 \times 1 + (10-1)(2)]$

$= 5[2 + 9 \times 2]$

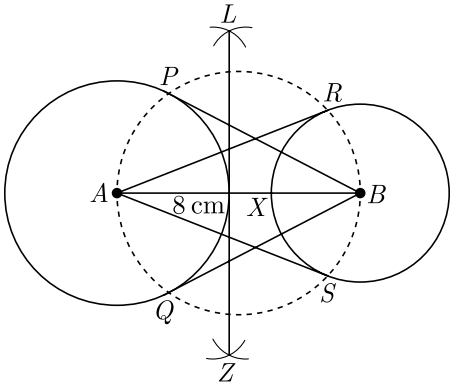
$= 5[2 + 18] = 5 \times 20 = 100$

12. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

Sol :

Steps of Construction :

1. Draw a line segment AB of length 8 cm.
2. Draw a circle with centre A and radius 4 cm.
3. Draw another circle with centre B and radius 3 cm.
4. Taking AB as diameter draw another circle, which intersects first two circles at P and Q, and R and S.
5. Join B to P, B to Q, A to R and A to S. Thus BP, BQ, AR and AS are the required tangents.



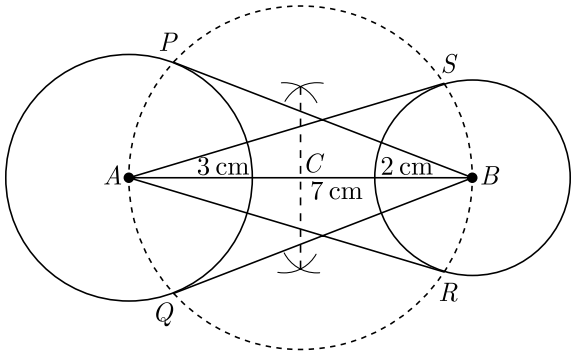
or

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as center, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Sol :

Steps of Construction :

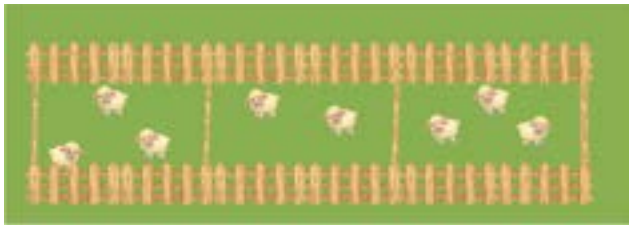
1. Draw a line segment AB of 7 cm.
2. Taking A and B as centre draw two circle of 3 cm and 2 cm radius respectively.
3. Bisect the line AB. Let mid-point of AB be C.
4. Taking C as centre draw a circle of radius AC with intersects the two circles at point P, Q, R and S.
5. Join BP, BQ, AS and AR. BP, BQ and AR, AS are the required tangents.



13. Building Sheep Pens : Darsh and Darpan are fencing off a large rectangular area to build some temporary holding pens. To prep the males, females, and kids, they are separated into three smaller and equal-size pens partitioned within the large rectangle.

(i) If 384 ft of fencing is available and the maximum area is desired, what will be the dimensions of the larger, outer rectangle?

(ii) What will be the dimensions of the smaller holding pens?



Sol :

(i) Let x represent the width, then length

$$L = \frac{384-4x}{2} = 192-2x$$

Now area,

$$\begin{aligned} A(x) &= LW = x(192-2x) \\ &= 192x-2x^2 = -2(x^2-96x) \\ &= -2(x^2-96x+48^2-48^2) \\ &= -2(x^2-96x+48^2)+2\times 48^2 \\ &= -2(x-48)^2+4608 \end{aligned}$$

From above equation it is clear that $A(x)$ is maximum at $x=48$ and this maximum value is 4608.

$$\text{Length} = (192-2x) = (192-2\times 48) = 96$$

The dimension of larger outer rectangle is 48 ft by 96 ft.

(ii) The one dimension of smaller holding pan is 48 feet. Other dimension is $\frac{96}{3} = 32$ feet.

14. Rainfall is one of the most commonly shared experiences on Earth. Rainfall is also necessary, since it provides water to plants and ultimately fills rivers. Because rainfall is both a needed resources and a threat, it is important to better understand this natural phenomenon. The most common rainfall measurement is the total rainfall depth during a given period, expressed in millimetres (mm). For instance, we might want to know how many millimetres of rain fell over the course of 1 h, 1 day, 1 month, or 1 year.

The rain water from 22m $\times$ 20m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m.

(i) If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

(ii) If rainfall is the 1.5 cm find the hight of water collected in cylindrical vessel.

Sol :

(i) Height of cylindrical vessel is 3.5 m and radius is $\frac{2}{2} = 1$ m.

Volume of water collected in cylindrical vessel,

$$V = \frac{4}{5}\pi r^2 h_{cy} = \frac{4}{5}\times\frac{22}{7}\times(1)^2\times\left(\frac{7}{2}\right) \text{ m}^3$$

$$= \frac{44}{5} \text{ m}^3$$

Let h be the rainfall.

Rain water from roof $= 22\times 20\times h$ m^3

Now $22\times 20\times h = \frac{44}{5}$

$$h = \frac{44}{5}\times\frac{1}{22\times 20} = \frac{1}{50} \text{ m}$$

$$= \frac{1}{50}\times 100 = 2 \text{ cm}$$

(ii) Since rainfall is 1.5 cm, volume of water collected from roof

$$V = 22\times 20\times h = 22\times 20\times 0.015$$

This water is collected in cylinder. Let h_1 be the height of water in cylinder. Thus volume of water in cylinder is $\pi r^2 h_1$ which is equal to the water collected from roof.

$$\pi r^2 h_1 = 22\times 20\times 0.015$$

$$\frac{22}{7}\times(1)^2\times h_1 = 22\times 20\times 0.015$$

$$h_1 = 7\times 20\times 0.015 = 2.1 \text{ m}$$