266 (I)

Total No. of Questions—24

Total No. of Printed Pages—4   Regd. No.

Part III
MATHEMATICS
Paper II(A)
(English Version)

Time : 3 Hours   *Max. Marks : 75*

Note :—This question paper consists of THREE sections A, B and C.

SECTION A $10 \times 2 = 20$

I.   Very Short Answer Type Questions :

(i)   Answer ALL questions.

(ii)   Each question carries TWO marks.

1.   Write the complex number $(2 - 3i)(3 + 4i)$ in the form $A + iB$.

2.   If $z_1 = -1$ and $z_2 = i$, then find $\text{Arg}\left(\dfrac{z_1}{z_2}\right)$.

3.   If $1, w, w^2$ are the cube roots of unity, prove that :
$$(a + b)(aw + bw^2)(aw^2 + bw) = a^3 + b^3.$$

4.   Find the values of m for which the equation $x^2 - 15 - m(2x - 8) = 0$ have equal roots.

5.   If the product of the roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9, then find a.

6.   Find the number of different chains that can be prepared using 7 different coloured beads.

7.   If $^nP_r = 5040$, $^nC_r = 210$, find n and r.

8.   Find the set E of the values of x for which the binomial expansion of $(3 - 4x)^{\frac{3}{4}}$ is valid.

266   (1)   P.T.O.

9. Find the mean deviation about the median for the following data :

$$4, 6, 9, 3, 10, 13, 2.$$

10. The probability that a person chosen at random is left handed (in hand writing) is 0.1. What is the probability that in a group of 10 people, there is one who is left handed ?

SECTION B $5 \times 4 = 20$

II. Short Answer Type Questions :

   (i)   Answer ANY FIVE questions.

   (ii) Each question carries FOUR marks.

11. If the point P denotes the complex number $z = x + iy$ in the Argand plane and if $\dfrac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.

12. Prove that $\dfrac{1}{3x+1} + \dfrac{1}{x+1} - \dfrac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if x is real.

13. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order, find the rank of the word PRISON.

14. Simplify $^{34}C_5 + \displaystyle\sum_{r=0}^{4} {}^{(38-r)}C_4$.

15. Resolve $\dfrac{x^2 - 3}{(x+2)(x^2+1)}$ into partial fractions.

16. State and prove Addition theorem on probability.

17. Suppose A and B are independent events with $P(A) = 0.6$, $P(B) = 0.7$. Then compute :

   (i) $P(A \cap B)$

   (ii) $P(A \cup B)$

   (iii) $P(B/A)$

   (iv) $P(A^C \cap B^C)$.

III. Long Answer Type Questions :

(i)   Answer ANY FIVE questions.

(ii) Each question carries SEVEN marks.

18. If n is an integer, then show that :

$$(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n\left(\frac{\theta}{2}\right) \cos\left(\frac{n\theta}{2}\right).$$

19. Solve the equation :

$$2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0.$$

20. If the coefficients of 4 consecutive terms in the expansion of $(1 + x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that :

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}.$$

21. If $x = \dfrac{1.3}{3.6} + \dfrac{1.3.5}{3.6.9} + \dfrac{1.3.5.7}{3.6.9.12} + \ldots\ldots$, then prove that :

$$9x^2 + 24x = 11.$$

22. Find the mean deviation about the mean for the following data :

Marks Obtained	No. of Students
0—10	5
10—20	8
20—30	15
30—40	16
40—50	6

23. Three urns have the following composition of balls :

Urn I   :   1 white, 2 black

Urn II   :   2 white, 1 black

Urn III :   2 white, 2 black.

One of the urns is selected at random and a ball is drawn. It turns out to be white. Find the probability that it came from urn III.

24. The probability distribution of a random variable X is given below

X = x_i	P(X = x_i)
1	k
2	$2k$
3	$3k$
4	$4k$
5	$5k$

Find the value of k and the mean and variance of X.