RATIO AND PROPORTION

RATIO

When we say that the length of a line *AB* is 5 centimetres, we mean that a unit of length called 1 centimetre is contained in *AB* five times. If we have two lines *AB* and *CD* and their lengths be 2 and 3 centimetres respectively, we say that the length of *AB* is 2/3 of the length of *CD*.

Ratio is a relation between two quantities in the same units which shows that one quantity is how many times of another quantity. Suppose A and B are two persons who have ₹ 50 and ₹ 100 respectively. Here 50 and 100 are two quantities in the same unit, rupees. It is clear that ₹ 50 is half of ₹100. Thus we can say in term of ratio that ratio of ₹ 50 and ₹ 100 is 1: 2.

A ratio may be expressed in the form of simplest fraction (If numerator and denominator have no common factor except 1, then fraction is in the simplest or lowest form).

Sign of ratio = (:) read as "Is To" So, ratio of two quantities ₹ 50 and ₹ 100 = 50/100 = 1/2 = 1 : 2 (Pronounced as 1 is to 2).

Memorable Points

1. Here in 1 : 2, '1' is called *Antecedent* of the ratio.
2. '2' is called *Consequent* of the ratio.

PROPORTION

The equality of two ratios is called *proportion*. Suppose, we have two ratios for example, 3 : 2 and 15 : 10. Here, 3 : 2 = 15 : 10. Thus this equality of these two given ratios is called proportion.

Sign of Proportion

Sign of proportion is : :

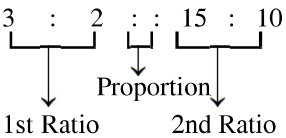
Therefore the above mentioned example is written as 3 : 2 : : 15 : 10 (it means 3/2 = 15/10)

The terms 3, 2, 15 and 10 are called proportional and named as the 1st, 2nd, 3rd and 4th proportional respectively.

In a proportion, the 1st and 4th terms are known as *extremes*, while 2nd and 3rd terms are known as *means.*

So, in given example 3 and 10 are extremes, while 2 and 15 are means.

In the concised way, all these terms are shown below :



Note : (1) It is not necessary that all four terms (proportional) are in the same unit. But in this condition, 1st and 2nd and 3rd and 4th terms must have same unit.

(2) 3 : 2 : : 15 : 10 is also written as
3/2 = 15/10

Memorable Points

We can find out the value of a unknown proportional, when values of three proportional are known by applying the following methods :

1. 1st proportional $= \frac{2nd \times 3rd}{4th}$

Example : ? : 190 : : 840 : 40

Solution : ? (1st proportional)

$$= \frac{190 \times 840}{40} = 3990.$$

2. 2nd proportional $= \frac{1st \times 4th}{3rd}$

Example : 50/ ? = 20/60

Solution : 50/ ? = 20/60

50 : ? : : 20 : 60

$$? = \frac{50 \times 60}{20} = 150$$

3. 3rd proportional $= \dfrac{\text{1st} \times \text{4th}}{\text{2nd}}$

Example : 3/4 = ? / 56

Solution : 3 : 4 :: ? : 56

$$\Rightarrow \quad ? = \frac{3 \times 56}{4} = 42$$

4. 4th proportional $= \dfrac{\text{2nd} \times \text{3rd}}{\text{1st}}$

Example : 500/1200 = 500/?

Solution : 500 : 1200 : : 500 : ?

$$\Rightarrow \quad ? = \frac{1200 \times 500}{500} = 1200$$

Some other Terms and their Formulae of Ratio and Proportion

1. **Mean proportional of '*a*' and '*b*' =** $\sqrt{ab}$

Example : Find the mean proportional between 0.32 and 0.02.

Solution : Mean proportional between 0.32 and 0.02

$$= \sqrt{0.32 \times 0.02} = \sqrt{0.0064} = 0.08$$

2. **Duplicate Ratio of $a : b = a^2 : b^2$**

Example : Find duplicate ratio of $7\sqrt{3} : 4\sqrt{2}$

Solution : Duplicate ratio of $7\sqrt{3} : 4\sqrt{2}$

$$= \left(7\sqrt{3}\right)^2 : \left(4\sqrt{2}\right)^2 = 49 \times 3 : 16 \times 2 = 147 : 32.$$

3. **Sub-duplicate Ratio of $a : b = \sqrt{a} : \sqrt{b}$**

Example : Find sub-duplicate ratio (S.D.R) of 200 : 392.

Solution : S.D.R of 200 : 392 $= \sqrt{200} : \sqrt{392}$

$$= \frac{\sqrt{200}}{\sqrt{392}} = \sqrt{\frac{200}{392}}$$

$$= \sqrt{\frac{100}{196}} = \frac{10}{14} = \frac{5}{7} = 5 : 7$$

4. **Triplicate Ratio of $a : b = a^3 : b^3$**

Example : Find triplicate ratio of 4 : 5.

Solution : Triplicate ratio of 4 : 5

$= 4^3 : 5^3 = 64 : 125$

5. **Sub-Triplicate Ratio of $a : b = \sqrt[3]{a} : \sqrt[3]{b}$**

Example : Find sub-triplicate Ratio of 27 : 1.

Solution : S.T.R of 27 : 1 $= \sqrt[3]{27} : \sqrt[3]{1}$

$$= \sqrt[3]{3^3} : \sqrt[3]{1} = 3 : 1.$$

6. **Inverse or Reciprocal Ratio of $a : b = 1/a : 1/b$**

Example : Find reciprocal ratio of 4 : 5.

Solution : Reciprocal ratio of 4 : 5 = 1/4 : 1/5

7. **Third Proportional to '*a*' and '*b*' = b^2/a**

Example : Find the third proportional to 0.8 and 0.2.

Solution : Third proportional to 0.8 and 0.2

$$= \frac{(0.2)^2}{0.8} = \frac{0.04}{0.8} = \frac{4}{80} = 0.05.$$

8. **Compound Ratio of $a : b, c : d, e : f$**

$$= \frac{\text{Product of all first terms of all ratio}}{\text{Product of all second terms of all ratio}}$$

$$= \frac{a \times c \times e}{b \times d \times f}$$

Example : Find compound ratio of 8 : 2, 2 : 1 and 9 : 3.

Solution : Compound ratio $= \dfrac{8 \times 2 \times 9}{2 \times 1 \times 3}$

$$= \frac{24}{1} = 24 : 1.$$

9. **If $A : B = x : y$ and $B : C = m : n$ then**

$$(I)\ A : C = \frac{x \times m}{y \times n}$$

$$(II)\ A : B : C = mx : ym : yn$$

Example : If $A : B = 2 : 3$ and $B : C = 4 : 5$ then $C : A$ is equal to

$$A : C = \frac{2 \times 4}{3 \times 5} = \frac{8}{15} = 8 : 15$$

$$\left[\because \text{In Formula } A : C = \frac{x \times m}{y \times n} \right]$$

So, $C : A = 15 : 8$.

10. If $A : B : C = x : y : z$ and $C : D = m : n$
Then $A : B : C : D = m(x : y) : z(m : n)$

Example : If $A : B : C = 2 : 3 : 4$ and $C : D = 5 : 6$, then $A : B : C : D$ is equal to

Solution : $A : B : C : D = m(x : y) : z(m : n)$

So, $A : B : C : D = 5(2 : 3) : 4(5 : 6)$

$= 10 : 15 : 20 : 24$

EXERCISE

1. The students in three classes are in the ratio 2 : 3 : 5 . If 20 students are increased in each class, the ratio changes to 4 : 5 : 7. What is the total number in the three classes before the increase?
(*a*) 100 students (*b*) 75 students
(*c*) 150 students (*d*) 50 students
(*e*) None of these

2. The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the numbers.
(*a*) 12, 16 (*b*) 16, 12
(*c*) 12, 15 (*d*) 13, 14
(*e*) None of these

3. Divide ₹ 1540 among *A,B,C* so that *A* shall receive 2/9 as much as *B* and *C* together, and *B* 3/11 of what *A* and *C* together do. Find the share of A, B and C.
(*a*) 285, 330, 830 (*b*) 280, 330, 930
(*c*) 280, 330, 980 (*d*) 330, 380, 980
(*e*) None of these

4. In a fort there is provision for 40 days for 275 persons. If after 16 days, 125 persons leave the fort, for how many more days the provision will now last?
(*a*) 45 days (*b*) 35 days
(*c*) 44 days (*d*) 53 days
(*e*) None of these

5. A fort has provision for 35 days. If after 5 days 225 more persons joined and the food lasts 25 days, how many men are there in the fort?
(*a*) 1225 persons (*b*) 1572 persons
(*c*) 1125 persons (*d*) 1229 persons
(*e*) None of these

6. The ratio between the ages of Rahim and Karim is 3 : 5 and the sum of their ages is 56 years. What was the ratio of their ages 7 years ago?
(*a*) 1: 2 (*b*) 3 : 2
(*c*) 3 : 4 (*d*) 4 : 3
(*e*) None of these

7. The prices of a scooter and television set are in the ratio 3 : 2. If a scooter costs ₹ 6,000 more than the television set, what is the price of the television set?
(*a*) ₹ 12,000 (*b*) ₹ 8,000
(*c*) ₹ 10,000 (*d*) ₹ 5,000
(*e*) None of these

8. The prices of scooter and a moped are in the ratio of 9 : 5. If a scooter costs ₹ 4200 more than a moped, find the price of the moped.
(*a*) ₹ 5052 (*b*) ₹ 5250
(*c*) ₹ 5053 (*d*) ₹ 5060
(*e*) None of these

9. A sum of money is divided between two persons in the ratio of 3 : 5. If the share of one person is ₹ 20 less than that of the other, find the sum.
(*a*) ₹ 75 (*b*) ₹ 90
(*c*) ₹ 80 (*d*) ₹ 85
(*e*) None of these

EXPLANATORY ANSWERS

1. (*a*) : $4 - 2 = 5 - 3 = 7 - 5 = 2$.
As we know 20 students are increased in each class.

So, $(2 + 3 + 5) = \frac{20}{2} \times 10$

$= 100$ students.

2. (*a*) : Let numbers are $3x$ and $4x$

$$\frac{3x+2}{4x+2} = \frac{7}{9}$$

$\Rightarrow \quad 27x + 18 = 28x + 14$

$\Rightarrow \quad x = 4$

Hence, numbers are $3 \times 4 = 12$ and $4 \times 4 = 16$.

3. (*b*) : A's share : (B + C)'s share = 2 : 9 ... (1)
B's share : (A + C)'s share = 3 : 11 ... (2)
Now dividing ₹ 1540 in the ratio of 2 : 9 and 3 : 11
A's share 2/11 of ₹ 1540 = ₹ 280
B's share = 3/14 of ₹ 1540 = ₹ 330
C's share = ₹ 1540 – (₹ 280 + ₹ 330) = ₹ 930

4. (*c*) : Reasoning
More men less days, less men more days
So, 275 : x : : (275 – 125) : (40 – 16)

$$\text{So, } x = \frac{275 \times 24}{150} = 44 \text{ days.}$$

5. (*c*) : Let the number of persons be x.
$(35 - 5)\, x = 25\,(x + 225)$
$\Rightarrow 30\,x - 25x = 25 \times 225$

$$\Rightarrow \quad x = \frac{25 \times 225}{5}$$

$$\Rightarrow \quad x = 1125 \text{ persons.}$$

6. (*a*) : Present age of Rahim = 56/8 × 3 = 21 years
Present age of Karim = 56/8 × 5 = 35 years
So, ratio of ages 7 years ago
= (21 – 7) : (35 – 7)
= 14 : 28 = 1 : 2

7. (*a*) : Let the price of a scooter = $3x$ and the price of a television set = $2x$.
$\because \quad 3x - 2x = 6000$
$\Rightarrow \quad x = 6000$
So, price of television set = $2x = 2 \times 6000$
= ₹ 12000

8. (*b*) : We have, $9x - 5x = 4200$
$\Rightarrow \quad 4x = 4200$

$$\Rightarrow \quad x = \frac{4200}{4} = 1050$$

So, price of the moped = $5x = 5 \times 1050$
= ₹ 5250.

9. (*c*) :
$$\frac{\text{Sum}}{\text{Difference}} = \frac{\text{Sum}}{20} = \frac{3+5}{5-3}$$

$$\text{Sum} = \frac{8}{2} \times 20 = ₹\, 80$$