Session 4

Binomial Theorem on Probability, Poisson Distribution, Expectation, Multinomial Theorem, Uncountable Uniform Spaces (Geometrical Problems)

Binomial Theorem on Probability

Suppose, a binomial experiment has probability of success p and that of failure q (i.e., $p+q=1$). If E be an event and let $X=$ number of successes i.e., number of times event E occurs in n trials. Then, the probability of occurrence of event E exactly r times in n trials is denoted by $P(X=r)$ or $P(r)$ and is given by $P(X=r)$

or $\quad P(r)={}^nC_r\,p^r q^{n-r}$

$\quad\quad = (r+1)$ th terms in the expansion of $(q+p)^n$

where, $r=0, 1, 2, 3, \ldots, n$.

Remark

1. The probability of getting atleast k success is
$$P(r \geq k) = \sum_{r=k}^{n} {}^nC_r p^r q^{n-r}.$$
2. The probability of getting atmost k success is
$$P(0 \leq r \leq k) = \sum_{r=0}^{k} {}^nC_r p^r q^{n-r}.$$
3. The probability distribution of the random variable X is as given below

X	0	1	2	...	r	...	n
$P(X)$	q^n	${}^nC_1\,pq^{n-1}$	${}^nC_2\,p^2q^{n-2}$	...	${}^nC_r\,p^rq^{n-r}$		p^n

4. The mean, the variance and the standard deviation of binomial distribution are np, npq, $\sqrt{npq}$.
5. **Mode of binomial distribution** Mode of Binomial distribution is the value of r when $P(X=r)$ is maximum.
$$(n+1)\,p-1 \leq r \leq (n+1)p$$

Example 21. If on an average, out of 10 ships, one is drowned, then what is the probability that out of 5 ships, atleast 4 reach safely?

Sol. Let p be the probability that a ship reaches safely.

$\therefore \quad p = \dfrac{9}{10}$

$\therefore \quad q =$ Probability that a ship is drowned $= 1 - p = 1 - \dfrac{9}{10}$

$\therefore \quad\quad q = \dfrac{1}{10}$

Let X be the random variable, showing the number of ships reaching safely.

Then, P (atleast 4 reaching safely) $= P(X=4 \text{ or } X=5)$

$= P(X=4) + P(X=5)$

$$= {}^5C_4\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^{5-4} + {}^5C_5\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^{5-5}$$

$$= \frac{5\times 9^4}{10^5} + \frac{9^5}{10^5} = \frac{9^4\times 14}{10^5}$$

Example 22. Numbers are selected at random one at a time, from the numbers 00, 01, 02, ..., 99 with replacement. An event E occurs, if and only if the product of the two digits of a selected number is 18. If four numbers are selected, then find the probability that E occurs atleast 3 times.

Sol. Out of the numbers 00, 01, 02, ..., 99, those numbers the product of whose digits is 18 are 29, 36, 63, 92 i.e., only 4.

$$p = P(E) = \frac{4}{100} = \frac{1}{25},\ q = P(\overline{E}) = 1 - \frac{1}{25} = \frac{24}{25}$$

Let X be the random variable, showing the number of times E occurs in 4 selections.

Then, $P(E$ occurs atleast 3 times$) = P(X=3 \text{ or } X=4)$

$$= P(X=3) + P(X=4) = {}^4C_3\,p^3q^1 + {}^4C_4\,p^4q^0$$

$$= 4p^3q + p^4 = 4\times\left(\frac{1}{25}\right)^3\times\frac{24}{25} + \left(\frac{1}{25}\right)^4$$

$$= \frac{97}{390625}$$

Example 23. A man takes a step forward with probability 0.4 and backward with probability 0.6. Then, find the probability that at the end of eleven steps he is one step away from the starting point.

Sol. Since, the man is one step away from starting point mean that either

(i) man has taken 6 steps forward and 5 steps backward.

(ii) man has taken 5 steps forward and 6 steps backward.

Taking, movement 1 step forward as success and 1 step backward as failure.

$\therefore \quad p =$ Probability of success $= 0.4$

and $q =$ Probability of failure $= 0.6$

$\therefore$ Required probability $= P(X = 6 \text{ or } X = 5)$

$$= P(X = 6) + P(X = 5) = {}^{11}C_6\, p^6 q^5 + {}^{11}C_5\, p^5 q^6$$

$$= {}^{11}C_5 (p^6 q^5 + p^5 q^6)$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \{(0 \cdot 4)^6 (0 \cdot 6)^5 + (0 \cdot 4)^5 (0 \cdot 6)^6\}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} (0 \cdot 24)^5 = 0 \cdot 37$$

Hence, the required probability is $0 \cdot 37$.

Example 24. Find the minimum number of tosses of a pair of dice, so that the probability of getting the sum of the digits on the dice equal to 7 on atleast one toss, is greater than 0.95. (Given, $\log_{10} 2 = 0 \cdot 3010$, $\log_{10} 3 = 0 \cdot 4771$)

Sol. The sample space,

$$S = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$$

$\therefore \quad n(S) = 36$ and let E be the event getting the sum of digits on the dice equal to 7, then

$$E = \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$$

$\therefore \quad n(E) = 6$

$p =$ Probability of getting the sum 7

$$p = \frac{6}{36} = \frac{1}{6} \quad \therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

$\because$ Probability of not throwing the sum 7 in first m trials $= q^m$

$\therefore P$ (atleast one 7 in m throws) $= 1 - q^m = 1 - \left(\frac{5}{6}\right)^m$

According to the question, $\quad 1 - \left(\frac{5}{6}\right)^m > 0 \cdot 95$

$$\Rightarrow \quad \left(\frac{5}{6}\right)^m < 1 - 0 \cdot 95 \Rightarrow \left(\frac{5}{6}\right)^m < 0.05$$

$$\Rightarrow \quad \left(\frac{5}{6}\right)^m < \frac{1}{20}$$

Taking logarithm,

$$\Rightarrow \quad m\{\log_{10} 5 - \log_{10} 6\} < \log_{10} 1 - \log_{10} 20$$

$$\Rightarrow m\{1 - \log_{10} 2 - \log_{10} 2 - \log_{10} 3\} < 0 - \log_{10} 2 - \log_{10} 10$$

$$\Rightarrow \quad m\{1 - 2\log_{10} 2 - \log_{10} 3\} < -\log_{10} 2 - 1$$

$$\Rightarrow \quad m\{1 - 0.6020 - 0.4771\} < -0.3010 - 1$$

$$\Rightarrow \quad -0.079\, m < -1.3010$$

$$\Rightarrow \quad m > \frac{1.3010}{0.079} = 16.44$$

$$\therefore \quad m > 16.44$$

Hence, the least number of trials is 17.

Example 25. Write probability distribution, when three coins are tossed.

Sol. Let X be a random variable denoting the number of heads occurred, then $P(X = 0) =$ Probability of occurrence of zero head

$$= P(TTT) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$P(X = 1) =$ Probability of occurrence of one head

$$= P(HTT) + P(THT) + P(TTH)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$$

$P(X = 2) =$ Probability of occurrence of two heads

$$= P(HHT) + P(HTH) + P(THH)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$$

$P(X = 3) =$ Probability of occurrence of three heads

$$= P(HHH) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

Thus, the probability distribution when three coins are tossed is as given below

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

another form,

$$\begin{pmatrix} X : & 0 & 1 & 2 & 3 \\ P(X) : & \frac{1}{3} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

Example 26. The mean and variance of a binomial variable X are 2 and 1, respectively. Find the probability that X takes values greater than 1.

Sol. Given, mean, $np = 2$ ...(i)

and variance, $npq = 1$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get $q = \frac{1}{2}$

$$\therefore \quad p = 1 - q = \frac{1}{2}$$

From Eq. (i), $n \times \frac{1}{2} = 2 \;\; \therefore \;\; n = 4$

The binomial distribution is $\left(\frac{1}{2} + \frac{1}{2}\right)^4$

Now, $P(X > 1) = P(X = 2) + P(X = 3) + P(X = 4)$

$$= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4C_4 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 + {}^4C_4 \left(\frac{1}{2}\right)^4$$

$$= \frac{6 + 4 + 1}{16} = \frac{11}{16}$$

Aliter $\quad P(X > 1) = 1 - \{P(X = 0) + P(X = 1)\}$

$$= 1 - \left\{ {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + {}^4C_3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3 \right\} = 1 - \left(\frac{1 + 4}{16}\right) = \frac{11}{16}$$

Poisson Distribution

It is the limiting case of binomial distribution under the following conditions :

(i) Number of trails are very large i.e. $n \to \infty$

(ii) $p \to 0$

(iii) $nq \to \lambda$, a finite quantity (λ is called parameter)

(a) Probability of r success for poisson distribution is given by $P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$, $r = 0, 1, 2, \ldots$

(b) Recurrence formula for poisson distribution is given by $P(r+1) = \frac{\lambda}{(r+1)} P(r)$

Remark

1. For poisson distribution, mean = variance = $\lambda = np$
2. If X and Y are independent poisson variates with parameters λ_1 and λ_2, then $X + Y$ has poisson distribution with parameter $\lambda_1 + \lambda_2$.

Expectation

If p be the probability of success of a person in any venture and m be the sum of money which he will receive in case of success, the sum of money denoted by pm is called his expectation.

Example 27. A random variable X has Poisson's distribution with mean 3. Then find the value of $P(X > 2.5)$

Sol. $P(X > 2.5) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$

$$\because \quad P(X = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

$$\therefore \quad P(X > 2.5) = 1 - \frac{e^{-\lambda}}{0!} - \frac{e^{-\lambda} \cdot \lambda^1}{1!} - \frac{e^{-\lambda} \cdot \lambda^2}{2!}$$

$$= 1 - e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2}\right)$$

$$= 1 - e^{-3}\left(1 + 3 + \frac{9}{2}\right) \qquad (\because \lambda = np = 3)$$

$$= 1 - \frac{17}{2e^3}$$

Example 28. ***A and B throw with one die for a stake of ₹ 11 which is to be won by the player who first throw 6. If A has the first throw, then what are their respective expectations?***

Sol. Since, A can win the game at the 1st, 3rd, 5th,..., trials.

If p be the probability of success and q be the probability of fail, then

$$p = \frac{1}{6} \quad \text{and} \quad q = \frac{5}{6}$$

$$P(A \text{ wins at the first trial}) = \frac{1}{6}$$

$$P(A \text{ wins at the 3rd trials}) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}$$

$$P(A \text{ wins at the 5th trials}) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \text{ and so on.}$$

$$\text{Therefore, } P(A \text{ wins}) = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \ldots \infty$$

$$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

$$\text{Similarly, } P(B \text{ wins}) = \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \left(\frac{5}{6}\right)^5 \frac{1}{6} + \ldots \infty$$

$$= \frac{\frac{5}{6} \cdot \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{5}{11}$$

Hence, expectations of A and B are ₹ $\frac{6}{11} \times 11$ and ₹ $\frac{5}{11} \times 11$, respectively. i.e. Expectations of A and B are ₹ 6 and ₹ 5, respectively.

Multinomial Theorem

If a dice has m faces marked 1, 2, 3,..., m and if such n dice are thrown, then the probability that the sum of the numbers of the upper faces is equal to r is given by the coefficient of x^r in $\frac{(x + x^2 + \ldots + x^m)^n}{m^n}$.

Example 29. A person throws two dice, one the common cube and the other a regular tetrahedron, the number on the lowest face being taken in the case of the tetrahedron, then find the probability that the sum of the numbers appearing on the dice is 6.

Sol. Let S be the sample space, then

$$S = \{1, 2, 3, 4\} \times \{1, 2, 3, 4, 5, 6\}$$

$$\therefore \quad n(S) = 24$$

If E be the event that the sum of the numbers on dice is 6.

Then, $n(E) =$ Coefficient of x^6 in

$$(x^1 + x^2 + x^3 + x^4) \times (x^1 + x^2 + x^3 + x^4 + x^5 + x^6)$$

$$= 1 + 1 + 1 + 1 = 4$$

$$\therefore \text{Required probability, } P(E) = \frac{n(E)}{n(S)} = \frac{4}{24} = \frac{1}{6}$$

Example 30. Five ordinary dice are rolled at random and the sum of the numbers shown on them is 16. What is the probability that the numbers shown on each is any one from 2, 3, 4 or 5?

Sol. If the integers x_1, x_2, x_3, x_4 and x_5 are shown on the dice, then $x_1 + x_2 + x_3 + x_4 + x_5 = 16$

where, $1 \le x_i \le 6$ $\quad (i = 1, 2, 3, 4, 5)$

The number of total solutions of this equation.

$= \text{Coefficient of } x^{16} \text{ in } (x^1 + x^2 + x^3 + x^4 + x^5 + x^6)^5$

$= \text{Coefficient of } x^{16} \text{ in } x^5 (1 + x + x^2 + x^3 + x^4 + x^5)^5$

$= \text{Coefficient of } x^{11} \text{ in } (1 + x + x^2 + x^3 + x^4 + x^5)^5$

$= \text{Coefficient of } x^{11} \text{ in } \left\{\left(\frac{1 - x^6}{1 - x}\right)^5\right\}$

$= \text{Coefficient of } x^{11} \text{ in } (1 - x^6)^5 (1 - x)^{-5}$

$= \text{Coefficient of } x^{11} \text{ in}$

$(1 - 5x^6 + \dots)(1 + {}^5C_1 x + {}^6C_2 x^2 + \dots + {}^9C_{C_5} x^5 + \dots + {}^{15}C_{11} x^{11} + \dots)$

$= {}^{15}C_{11} - 5 \cdot {}^9C_5$

$= {}^{15}C_4 - 5 \cdot {}^9C_4 = \frac{15 \cdot 14 \cdot 13 \cdot 12}{1 \cdot 2 \cdot 3 \cdot 4} - 5 \cdot \frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} = 735$

If S be the sample space

$\therefore \quad n(S) = 735$

Let E be the occurrence event, then

$n(E)$ = The number of integral solutions of

$x_1 + x_2 + x_3 + x_4 + x_5 = 16$,

where $2 \le x_i \le 5$ $\quad (i = 1, 2, 3, 4, 5)$

$= \text{Coefficient of } x^{16} \text{ in } (x^2 + x^3 + x^4 + x^5)^5$

$= \text{Coefficient of } x^{16} \text{ in } x^{10} (1 + x + x^2 + x^3)^5$

$= \text{Coefficient of } x^6 \text{ in } (1 + x + x^2 + x^3)^5$

$= \text{Coefficient of } x^6 \text{ in } \left\{\left(\frac{1 - x^4}{1 - x}\right)^5\right\}$

$= \text{Coefficient of } x^6 \text{ in } (1 - x^4)^5 (1 - x)^{-5}$

$= \text{Coefficient of } x^6 \text{ in}$

$(1 - 5x^4 + \dots)(1 + {}^5C_1 x + {}^6C_2 x^2 + \dots + {}^{10}C_6 x^6 + \dots)$

$= {}^{10}C_6 - 5 \cdot {}^6C_2 = {}^{10}C_4 - 5 \cdot {}^6C_2$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} - 5 \cdot \frac{6 \cdot 5}{1 \cdot 2} = 210 - 75 = 135$

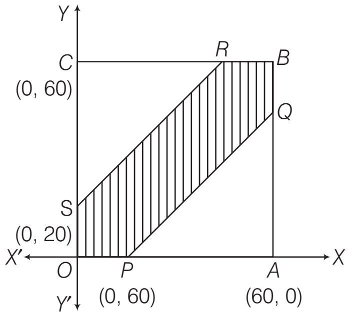
$\therefore$ The required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{135}{735} = \frac{9}{49}$

Uncountable Uniform Spaces

(Geometrical Problems)

Example 31. Two persons A and B agree to meet at a place between 11 to 12 noon. The first one to arrive waits for 20 min and then leave. If the time of their arrival be independent and at random, then what is the probability that A and B meet?

Sol. Let A and B arrive at the place of their meeting x minutes and y minutes after 11 noon.



The given condition $\Rightarrow$ their meeting is possible only if

$$|x - y| \le 20 \qquad \text{...(i)}$$

$OABC$ is a square, where $A \equiv (60, 0)$ and $C \equiv (0, 60)$

Considering the equality part of Eq. (i)

i.e., $\quad |x - y| = 20$

$\therefore$ The area representing the favourable cases

= Area $OPQBRSO$

= Area of square $OABC$ − Area of ΔPAQ − Area of ΔSRC

$= (60)(60) - \frac{1}{2}(40)(40) - \frac{1}{2}(40)(40)$

$= 3600 - 1600 = 2000$ sq units

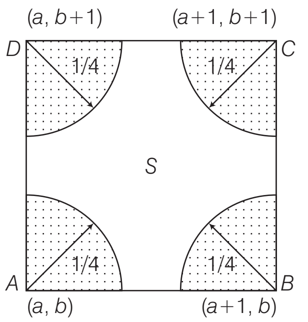
Total way = Area of square $OABC = (60)(60) = 3600$ sq units

Required probability $= \frac{2000}{3600} = \frac{5}{9}$

Example 32. Consider the cartesian plane R^2 and let X denote the subset of points for which both coordinates are integers. A coin of diameter $\frac{1}{2}$ is tossed randomly onto the plane. Find the probability p that the coin covers a point of X.

Sol. Let S denote the set of points inside a square with corners

$$(a, b), (a, b + 1), (a + 1, b), (a + 1, b + 1) \in X$$

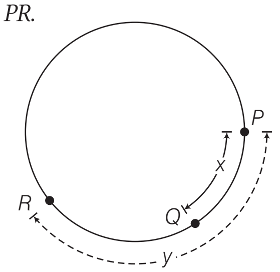


Let P denotes the set of points in S with distance less than $\frac{1}{4}$ from any corner point. (observe that the area of P is equal to the area inside a circle of radius $\frac{1}{4}$). Thus a coin, whose centre falls in S, will cover a point of X if and only if its centre falls in a point of P.

$$\text{Hence}, p = \frac{\text{area of } P}{\text{area of } S} = \frac{\pi\left(\frac{1}{4}\right)^2}{1} = \frac{\pi}{15} \approx 0.2$$

Example 33. Three points P, Q and R are selected at random from the circumference of a circle. Find the probability p that the points lie on a semi-circle.

Sol. Let the length of the circumference is $2s$. Let x denote the clockwise arc length of PQ and let y denote the clockwise arc length of PR.



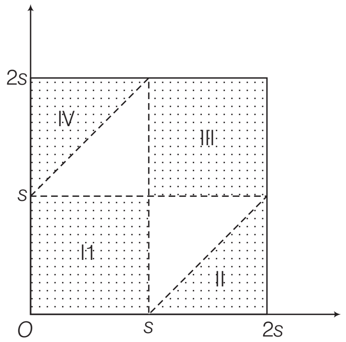
Thus, $0 < x < 2s$ and $0 < y < 2s$

Let A denotes the subset of S for which any of the following conditions holds:

(i) $x, y < s$ (ii) $x < s$ and $y - x > s$

(iii) $x, y > s$ (iv) $y < s$ and $x - y > s$

Then, A consists of those points for which P, Q and R lie on a semi-circle. Thus,



$$p = \frac{\text{area of } A}{\text{area of } S} = \frac{3s^2}{4s^2} = \frac{3}{4}$$

Example 34. A wire of length l is cut into three pieces. Find the probability that the three pieces form a triangle.

Sol. Let the lengths of three parts of the wire be x, y and $l-(x+y)$. Then, $x > 0$, $y > 0$

and $\quad l-(x+y) > 0$

i.e., $\quad x + y < l$ or $y < l - x$

Since, in a triangle, the sum of any two sides is greater than third side, so

$$x + y > l - (x+y) \Rightarrow y > \frac{l}{2} - x$$

and $\quad x + l - (x+y) > y \Rightarrow y < \frac{l}{2}$

and $\quad y + l - (x+y) > x \Rightarrow x < \frac{l}{2}$

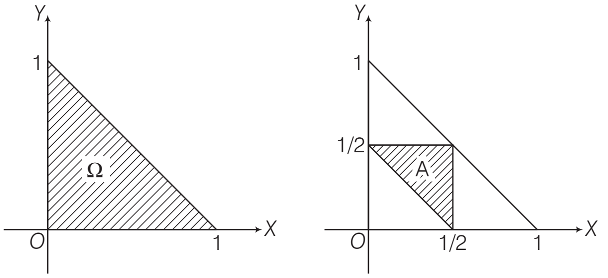
$$\Rightarrow \quad \frac{l}{2} - x < y < \frac{l}{2} \text{ and } 0 < x < \frac{l}{2}$$

So, required probability $= \dfrac{\int_0^{l/2}\int_{l/2-x}^{l/2} dy\,dx}{\int_0^{l}\int_0^{l-x} dy\,dx}$

$$= \frac{\int_0^{l/2}\left\{\frac{l}{2} - \left(\frac{l}{2} - x\right)\right\}dx}{\int_0^l (l-x)dx} = \frac{\int_0^{l/2} x\,dx}{\int_0^l (l-x)\,dx} = \frac{l^2/8}{l^2/2} = \frac{1}{4}$$

Aliter

The elementary event w is characterised by two parameters x and y [since $z = l - (x+y)$]. We depict the event by a point on x, y plane. The conditions $x > 0$, $y > 0$, $x + y < l$ are imposed on the quantities x and y, the sample space is the interior of a right angled triangle with unit legs i.e. $S_\Omega = \frac{1}{2}$.



The condition A requiring that a triangle could be formed from the segments $x, y, l-(x+y)$ reduces to the following two conditions: (1) The sum of any two sides is larger than the third side, (2) The difference between any two sides is smaller than the third side. This condition is associated with the triangular domain A with area.

$$S_A = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8} \therefore P(A) = \frac{S_A}{S_\Omega} = \frac{\left(\frac{1}{8}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{4}$$

Exercise for Session 4

1 A coin is tossed three times. The probability of getting exactly 2 heads is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

2 A coin is tossed 4 times. The probability that atleast one head turns up is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{15}{16}$

3 The following is the probability distribution of a random variable X.

X	1	2	3	4	5
$P(X)$	0.1	0.2	k	0.3	$2k$

The value of k is

(a) $\frac{4}{15}$ (b) $\frac{1}{15}$ (c) $\frac{1}{5}$ (d) $\frac{2}{15}$

4 A random variable X has the distribution

X	2	3	4
$P(X = x)$	0.3	0.4	0.3

Then, variance of the distribution, is

(a) 0·6 (b) 0·7 (c) 0·77 (d) 1·55

5. In a box containing 100 bulbs, 10 bulbs are defective. Probability that out of a sample of 5 bulbs, none is defective, is

(a) 10^{-5} (b) 2^{-5} (c) $(0\cdot 9)^5$ (d) $0\cdot 9$

6. A pair of dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7, is

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) None of these

7. If X follows the binomial distribution with parameters $n = 6$ and p and $9P(X = 4) = P(X = 2)$, then p is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

8. If probability of a defective bolt is 0.1, then mean and standard deviation of distribution of bolts in a total of 400, are

(a) 30, 3 (b) 40, 5 (c) 30, 4 (d) 40, 6

9. The mean and variance of a binomial distribution are $\frac{5}{4}$ and $\frac{15}{16}$ respectively, then value of p, is

(a) $\frac{1}{2}$ (b) $\frac{15}{16}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

10. The mean and variance of a binomial distribution are 6 and 4, then n is

(a) 9 (b) 12 (c) 18 (d) 10

11. A die is thrown 100 times. Getting an even number is considered a success. Variance of number of successes, is

(a) 10 (b) 20 (c) 25 (d) 50

12. 10% of tools produced by a certain manufacturing process turn out to be defective. Assuming binomial distribution, the probability of 2 defective in sample of 10 tools chosen at random, is

(a) 0.368 (b) 0.194 (c) 0.271 (d) None of these

13. If X follows a binomial distribution with parameters $n = 100$ and $p = \frac{1}{3}$, then $P(X = r)$ is maximum, when r equals

(a) 16 (b) 32 (c) 33 (d) None of these

14. The expected value of the number of points, obtained in a single throw of die, is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

15. Two points P and Q are taken at random on a line segment OA of length a. The probability that $PQ > b$, where $0 < b < a$, is

(a) $\frac{b}{a}$ (b) $\frac{b^2}{a^2}$ (c) $\left(\frac{a-b}{a}\right)^2$ (d) $\left(\frac{a-2b}{a-b}\right)^2$

Answers

Exercise for Session 4

1. (c)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (a)
7. (a)	**8.** (d)	**9.** (c)	**10.** (c)	**11.** (c)	**12.** (b)
13. (c)	**14.** (c)	**15.** (a)			