Speed Test-2

1. **(a)** Acceleration of the particle $a = 2t - 1$
The particle retards when acceleration is opposite to velocity.
$\Rightarrow a\,.\,v < 0 \Rightarrow (2t-1)(t^2 - t) < 0 \Rightarrow t\,(2t-1)(t-1) < 0$
Now t is always positive
$\therefore\ (2t-1)(t-1) < 0$
or $2t - 1 < 0$ and $t - 1 > 0 \Rightarrow t < \frac{1}{2}$ and $t > 1$.
This is not possible
or $2t - 1 > 0\ \&\ t - 1 < 0 \Rightarrow 1/2 < t < 1$

2. **(b)** $x = \alpha t^3$ and $y = \beta t^3$
$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$
$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$
$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

3. **(d)** $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$
$$= \frac{x}{\frac{2x/5}{v_1} + \frac{3x/5}{v_2}} = \frac{5v_1 v_2}{3v_1 + 2v_2}$$

4. **(a)** Instantaneous speed is the distance being covered by the particle per unit time at the given instant. It is equal to the magnitude of the instantaneous velocity at the given instant.

5. **(a)** $v = \alpha\sqrt{x}$, $\quad \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha\, dt$
$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt$$
$$\left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t$$
$$\Rightarrow 2\sqrt{x} = \alpha t \ \Rightarrow x = \frac{\alpha^2}{4}t^2$$

6. **(c)** $\frac{1}{2}(1+4)\times 4 - \frac{1}{2}\times 1\times 2 - \frac{1}{2}\times 3\times 4 = 3\text{m}$

7. **(b)** The distance travel in n^{th} second is
$S_n = u + ½\,(2n-1)a$ ....(1)
so distance travel in t^{th} & $(t+1)^{th}$ second are
$S_t = u + ½\,(2t-1)a$ ....(2)
$S_{t+1} = u + ½\,(2t+1)a$ ....(3)
As per question,
$S_t + S_{t+1} = 100 = 2(u + at)$ ....(4)
Now from first equation of motion the velocity, of particle after time t, if it moves with an accleration a is
$v = u + a\,t$ ....(5)
where u is initial velocity
So from eq(4) and (5), we get v = 50 cm/sec.

8. **(d)** Relative speed of police with respect to thief
$= 10 - 9 = 1$ m/s
Instantaneous separation = 100 m
$$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{100}{1} = 100\text{sec}.$$

9. **(d)** $$x = \frac{a}{b}(1 - e^{-b\times\frac{1}{b}}) = \frac{a}{b}(1 - e^{-1}) = \frac{a}{b}(1 - \frac{1}{e})$$
$$= \frac{a}{b}\frac{(e-1)}{e} = \frac{a}{b}\frac{(2.718-1)}{2.718} = \frac{a}{b}\frac{(1.718)}{2.718} = 0.637\frac{a}{b} \simeq \frac{2}{3}a/b$$
velocity $v = \frac{dx}{dt} = ae^{-bt}$, $v_0 = a$
accleration $a = \frac{dv}{dt} = -abe^{-bt}$ & $a_0 = -ab$
At $t = 0$, $x = \frac{a}{b}(1-1) = 0$ and
At $t = \frac{1}{b}$, $x = \frac{a}{b}(1 - e^{-1}) = \frac{a}{b}(1 - \frac{1}{e}) = \frac{2}{3}a/b$
At $t = \infty$, $x = \frac{a}{b}$
It cannot go beyond this, so point $x > \frac{a}{b}$ is not reached by the particle.
At $t = 0$, $x = 0$, at $t = \infty$, $x = \frac{a}{b}$, therefore the particle does not come back to its starting point at $t = \infty$.

10. **(d)** Ist part: $u = 0$, $t = 5s$, $v = 108$ km/hr = 30 m/s
$v = u + at \Rightarrow 30 = 0 + a\times 5 \Rightarrow a = 6\ \text{m/s}^2$
$$s = ut + \frac{1}{2}at^2 = 0\times 5 + \frac{1}{2}\times 6\times 5^2 = 75\,\text{m}$$
IIIrd part: $s = 45$m, $u = 30$m/s, $v = 0$
$$a = \frac{v^2 - u^2}{2s} = \frac{-30\times 30}{2\times 45} = -10\text{m}/\text{s}^2$$
$v = u + at \Rightarrow 0 = 30 - 10\times t \Rightarrow t = 3s$
IInd part :
$s = s_1 + s_2 + s_3$
$395 = 75 + s_2 + 45 \Rightarrow s_2 = 275$ m
$$t = \frac{275}{30} = 9.16 = 9.2s.$$
Total time taken = (5 + 9.2 + 3) sec = 17.2 sec

11. **(a)** $\frac{dv}{dt} = -kv^3$ or $\frac{dv}{v^3} = -k\ dt$
Integrating we get, $-\frac{1}{2v^2} = -kt + c$ ...(1)
At $t = 0$, $v = v_0$ $\therefore -\frac{1}{2v_o^2} = c$

Putting in (1)

$$-\frac{1}{2v^2} = -kt - \frac{1}{2{v_0}^2} \text{ or } \frac{1}{2v_0^2} - \frac{1}{2v^2} = -kt$$

$$\text{or } \left[\frac{1}{2v_0^2} + kt\right] = \frac{1}{2v^2} \text{ or } \left[1 + 2v_0^2\, kt\right] = \frac{v_0^2}{v^2}$$

$$\text{or } v^2 = \frac{v_0^2}{1 + 2v_0^2\, kt} \text{ or } v = \frac{v_0}{\sqrt{1 + 2v_0^2\, kt}}$$

12. (c) We know that, $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$

Integrating, $\int_0^x dx = \int_0^t v\,dt$

$$\text{or } x = \int_0^t (v_0 + gt + ft^2)\,dt$$

$$= \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$$

$$\text{or, } x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

At $t = 1$, $x = v_0 + \frac{g}{2} + \frac{f}{3}$.

13. (c) Let man will catch the bus after 't' sec. So he will cover distance ut.

Similarly, distance travelled by the bus will be $\frac{1}{2}at^2$

For the given condition

$ut = 45 + \frac{1}{2}at^2 = 45 + 1.25t^2$ [As a = 2.5 m/s^2]

$\Rightarrow u = \frac{45}{t} + 1.25t$

To find the minimum value of u $\frac{du}{dt} = 0$

so we get t = 6 sec then,

$u = \frac{45}{6} + 1.25 \times 6 = 7.5 + 7.5 = 15\,\text{m/s}$

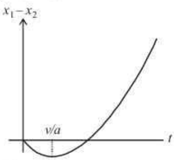
14. (b) **For the body starting from rest**

$x_1 = 0 + \frac{1}{2}at^2$

$\Rightarrow x_1 = \frac{1}{2}at^2$

For the body moving with constant speed

$x_2 = vt$



$\therefore\ x_1 - x_2 = \frac{1}{2}at^2 - vt$

at $t = 0$, $x_1 - x_2 = 0$

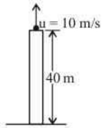
For $t < \frac{v}{a}$; the slope is negative

For $t = \frac{v}{a}$; the slope is zero

For $t > \frac{v}{a}$; the slope is positive

These characteristics are represented by graph (b).

15. (d) The stone reaches its maximum height after time t_1 given by



$t_1 = \frac{u}{g}\ (\because v = u - gt)$

$= \frac{10}{10} = 1\text{ sec}$

Again it reaches to its initial position in 1 sec and falls with same initial speed of 10 m/s.

Let t_2 be the time taken to reach the ground, then

$$v_{ground} = u + gt_2$$

$$\text{But } v_{ground}^2 = u^2 + 2gh$$

$$= (10)^2 + 2 \times 10 \times 40 = 900$$

$$\Rightarrow v_{ground} = \sqrt{900} = 30\text{ m/s}$$

$$\therefore t_2 = \frac{v_{ground} - u}{g} = \frac{30 - 10}{10} = 2\text{ sec.}$$

$\therefore$ Total required time $= (1 + 1 + 2)$ sec $= 4$ sec

16. (b)

$$L = \frac{1}{2}gt^2 - \frac{1}{2}g(t - T)^2$$

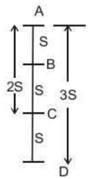
$$\Rightarrow t = \frac{T}{2} + \frac{L}{gt}.$$

17. (b) $S = AB = \frac{1}{2}g\,{t_1}^2 \Rightarrow 2S = AC = \frac{1}{2}g\,(t_1 + t_2)^2$

and $3S = AD = \frac{1}{2}g\,(t_1 + t_2 + t_3)^2$

$t_1 = \sqrt{\frac{2S}{g}}$

$t_1 + t_2 = \sqrt{\frac{4S}{g}},\ t_2 = \sqrt{\frac{4S}{g}} - \sqrt{\frac{2S}{g}}$



$t_1 + t_2 + t_3 = \sqrt{\frac{6S}{g}}$

$t_3 = \sqrt{\frac{6S}{g}} - \sqrt{\frac{4S}{g}}$

$t_1 : t_2 : t_3 :: 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

18. (c) Height of tap = 5m and (g) = 10 m/sec^2.

For the first drop,

$5 = ut + \frac{1}{2}gt^2 = (0 \times t) + \frac{1}{2} \times 10t^2 = 5t^2$ or $t^2 = 1$ or $t = 1$.

It means that the third drop leaves after one second of the first drop. Or, each drop leaves after every 0.5 sec. Distance covered by the second drop in 0.5 sec

$= ut + \frac{1}{2}gt^2 = (0 \times 0.5) + \frac{1}{2} \times 10 = (0.5)^2 = 1.25m$.

Therefore, distance of the second drop above the ground $= 5 - 1.25 = 3.75$ m.

19. **(c)** $\because t = \sqrt{x} + 3$

$\Rightarrow \sqrt{x} = t - 3 \Rightarrow x = (t-3)^2$

$v = \frac{dx}{dt} = 2(t-3) = 0$

$\Rightarrow t = 3$

$\therefore x = (3-3)^2$

$\Rightarrow x = 0$.

20. **(c)** We have, $S_n = u + \frac{a}{2}(2n-1)$

or $65 = u + \frac{a}{2}(2 \times 5 - 1)$

or $65 = u + \frac{9}{2}a$ ..... (1)

Also, $105 = u + \frac{a}{2}(2 \times 9 - 1)$

or $105 = u + \frac{17}{2}a$ ..... (2)

Equation (2) – (1) gives,

$40 = \frac{17}{2}a - \frac{9}{2}a = 4a$ or $a = 10 \text{ m/s}^2$.

Substitute this value in (1) we get,

$u = 65 - \frac{9}{2} \times 10 = 65 - 45 = 20 \text{ m/s}$

$\therefore$ The distance travelled by the body in 20 s is,

$s = ut + \frac{1}{2}at^2 = 20 \times 20 + \frac{1}{2} \times 10 \times (20)^2$

$= 400 + 2000 = 2400$ m.

21. **(d)** Speed, $u = 60 \times \frac{5}{18} \text{m/s} = \frac{50}{3} \text{m/s}$

$d = 20\text{m}, u' = 120 \times \frac{5}{18} = \frac{100}{3} \text{m/s}$

Let declaration be a then $(0)^2 - u^2 = -2ad$

or $u^2 = 2ad$ ... (1)

and $(0)^2 - u'^2 = -2ad'$

or $u'^2 = 2ad'$ ...(2)

(2) divided by (1) gives,

$4 = \frac{d'}{d} \Rightarrow d' = 4 \times 20 = 80\text{m}$

22. **(b)** $8 = a\,t_1$ and $0 = 8 - a(4 - t_1)$

or $t_1 = \frac{8}{a}$ $\therefore 8 = a\left(4 - \frac{8}{a}\right)$

$8 = 4a - 8$ or $a = 4$ and $t_1 = 8/4 = 2$ sec

Now, $s_1 = 0 \times 2 + \frac{1}{2} \times 4(2)^2$ or $s_1 = 8$ m

$s_2 = 8 \times 2 - \frac{1}{2} \times 4 \times (2)^2$ or $s_2 = 8$ m

$\therefore s_1 + s_2 = 16$ m

23. **(d)**

24. **(a)** $x = \frac{1}{t+5}$

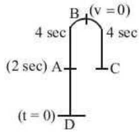
$\therefore v = \frac{dx}{dt} = \frac{-1}{(t+5)^2}$

$\therefore a = \frac{d^2x}{dt^2} = \frac{2}{(t+5)^3} = 2x^3$

Now $\frac{1}{(t+5)} \propto v^{\frac{1}{2}}$

$\therefore \frac{1}{(t+5)^3} \propto v^{\frac{3}{2}} \propto a$

25. **(d)**



As the time taken from D to A = 2 sec.
and D → A → B → C = 10 sec (given).
As ball goes from B → C (u = 0, t = 4 sec)
$v_c = 0 + 4g$.

As it moves from C to D, $s = ut + \frac{1}{2}gt^2$

$s = 4g \times 2 + \frac{1}{2}g \times 4 = 10$ g.

26. **(d)** $y = \frac{1}{2}g(n+1)^2 - \frac{1}{2}gn^2$

$= \frac{g}{2}[(n+1)^2 - n^2] = \frac{g}{2}(2n+1)$ ......(i)

Also, $h = \frac{g}{2}(2n-1)$ ......(ii)

From (i) and (ii)

$y = h + g$

27. **(b)** The stone rises up till its vertical velocity is zero and again reached the top of the tower with a speed u (downward). The speed of the stone at the base is 3u.

Hence $(3u)^2 = (-u)^2 + 2gh$ or $h = \frac{4u^2}{g}$

28. (b) $x = 40 + 12t - t^3$

$$v = \frac{dx}{dt} = 12 - 3t^2$$

For $v = 0$; $t = \sqrt{\frac{12}{3}} = 2$ sec

So, after 2 seconds velocity becomes zero.
Value of x in 2 secs $= 40 + 12 \times 2 - 2^3$
$= 40 + 24 - 8 = 56$ m

29. (b) The slope of v-t graph is constant and velocity decreasing for first half. It is positive and constant over next half.

30. (c) Here, $f = f_0\left(1 - \frac{t}{T}\right)$ or, $\frac{dv}{dt} = f_0\left(1 - \frac{t}{T}\right)$

or, $dv = f_0\left(1 - \frac{t}{T}\right) dt$

$$\therefore v = \int dv = \int\left[f_0\left(1 - \frac{t}{T}\right)\right]dt$$

or, $v = f_0\left(t - \frac{t^2}{2T}\right) + C$

where C is the constant of integration.
At $t = 0$, $v = 0$.

$$\therefore 0 = f_0\left(0 - \frac{0}{2T}\right) + C \Rightarrow C = 0$$

$$\therefore v = f_0\left(t - \frac{t^2}{2T}\right)$$

If $f = 0$, then

$$0 = f_0\left(1 - \frac{t}{T}\right) \Rightarrow t = T$$

Hence, particle's velocity in the time interval $t = 0$ and $t = T$ is given by

$$v_x = \int_{t=0}^{t=T} dv = \int_{t=0}^{T}\left[f_0\left(1 - \frac{t}{T}\right)\right]dt$$

$$= f_0\left[\left(t - \frac{t^2}{2T}\right)\right]_0^T$$

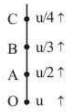
$$= f_0\left(T - \frac{T^2}{2T}\right) = f_0\left(T - \frac{T}{2}\right)$$

$$= \frac{1}{2}f_0 T.$$

31. (a) Using $v^2 = u^2 - 2gh$ i.e., $h = \frac{u^2 - v^2}{2g}$,

$$AB = \frac{\left(\frac{u}{2}\right)^2 - \left(\frac{u}{3}\right)^2}{2g}$$

and $BC = \frac{\left(\frac{u}{3}\right)^2 - \left(\frac{u}{4}\right)^2}{2g}$



$$\therefore \frac{AB}{BC} = \frac{\left(\frac{u}{2}\right)^2 - \left(\frac{u}{3}\right)^2}{\left(\frac{u}{3}\right)^2 - \left(\frac{u}{4}\right)^2} = \frac{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2 - \left(\frac{1}{4}\right)^2} = \frac{20}{7}$$

32. (a) Velocity of boat $= \frac{8+8}{2} = 8\,\text{km h}^{-1}$

Velocity of water $= 4\,\text{km h}^{-1}$

$$t = \frac{8}{8-4} + \frac{8}{8+4} = \frac{8}{3}\text{h} = 160 \text{ minutes}$$

33. (b) $v_{av} = \frac{x + 2x + 3x}{t_1 + t_2 + t_3}$

$t_1 = \frac{2x}{v_{max}}$, $t_2 = \frac{2x}{v_{max}}$, $t_3 = \frac{6x}{v_{max}}$

$$v_{av} = \frac{6x\, v_{max}}{10x}$$

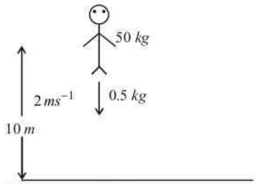
$$\frac{v_{av}}{v_{max}} = \frac{3}{5}$$

34. (b) No external force is acting, therefore,
$50\,u + 0.5 \times 2 = 0$
where u is the velocity of man.

$$u = -\frac{1}{50}\text{ms}^{-1}$$

Negative sign of u shows that man moves upward.
Time taken by the stone to reach the ground

$$= \frac{10}{2} = 5\text{S}$$



Distance moved by the man

$$= 5 \times \frac{1}{50} = 0.1\text{m}$$

$\therefore$ when the stone reaches the floor, the distance of the man above floor = 10.1 m

35. (a) Use $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

36. (c) Downward motion

$$v^2 - 0^2 = 2 \times 9.8 \times 5$$

$\Rightarrow\ v = \sqrt{98} = 9.9$

Also for upward motion

$0^2 - u^2 = 2 \times (-9.8) \times 1.8$

$\Rightarrow\ u = \sqrt{3528} = 5.94$

Fractional loss $= \dfrac{9.9 - 5.94}{9.9} = 0.4$

37. (c) Distance travelled by the stone in the last second is

$\dfrac{9h}{25} = \dfrac{g}{2}(2t-1) \quad (\because\ u = 0)$ ...(i)

Distance travelled by the stone in t s is

$h = \dfrac{1}{2}gt^2 \quad (\text{using } s = ut + \dfrac{1}{2}at^2)$ ...(ii)

Divide (i) by (ii), we get

$\dfrac{9}{25} = \dfrac{(2t-1)}{t^2}$

$9t^2 = 50t - 25,\ 9t^2 - 50t + 25 = 0$

Solving, we get

$t = 5s \text{ or } t = \dfrac{5}{9}s$

Substituting t = 5s in (ii), we get

$h = \dfrac{1}{2} \times 9.8 \times (5)^2 = 122.5\ m$

38. (b) $y \propto t^2;\ v \propto t';\ a \propto t^\circ$

39. (b) Average velocity for the second half of the distance is

$= \dfrac{v_1 + v_2}{2} = \dfrac{4+8}{2} = 6\,ms^{-1}$

Given that first half distance is covered with a velocity of $6\,ms^{-1}$. Therefore, the average velocity for the whole time of motion is $6\,ms^{-1}$

40. (b) Bullet will take $\dfrac{100}{1000} = 0.1$ sec to reach target.

During this period vertical distance (downward) travelled by the bullet

$= \dfrac{1}{2}gt^2 = \dfrac{1}{2} \times 10 \times (0.1)^2 = 0.05\,m = 5cm$

So the gun should be aimed 5 cm above the target.

41. (c) The distance covered in n^{th} second is

$S_n = u + \dfrac{1}{2}(2n-1)a$

where u is initial velocity & a is acceleration

then $26 = u + \dfrac{19a}{2}$ ....(1)

$28 = u + \dfrac{21a}{2}$ ....(2)

$30 = u + \dfrac{23a}{2}$ ....(3)

$32 = u + \dfrac{25a}{2}$ ....(4)

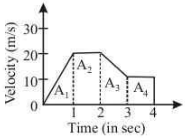
From eqs. (1) and (2) we get u = 7m/sec, a=2m/sec^2

$\therefore$ The body starts with initial velocity u = 7m/sec and moves with uniform acceleration a = 2m/sec^2

42. (a) $8 = \dfrac{x}{t_1},\ 12 = \dfrac{x}{t_2}$

$\bar{v} = \dfrac{2x}{t_1 + t_2} = \dfrac{2x}{\dfrac{x}{8} + \dfrac{x}{12}} = \dfrac{2 \times 8 \times 12}{12 + 8} = 9.6\ ms^{-1}$

43. (b) Distance = Area under v – t graph = $A_1 + A_2 + A_3 + A_4$



$= \dfrac{1}{2} \times 1 \times 20 + (20 \times 1) + \dfrac{1}{2}(20+10) \times 1 + (10 \times 1)$

$= 10 + 20 + 15 + 10 = 55\ m$

44. (a) $\because\ h = \dfrac{1}{2}gt^2$

$\therefore\ h_1 = \dfrac{1}{2}g(5)^2 = 125$

$h_1 + h_2 = \dfrac{1}{2}g(10)^2 = 500$

$\Rightarrow\ h_2 = 375$

$h_1 + h_2 + h_3 = \dfrac{1}{2}g(15)^2 = 1125$

$\Rightarrow\ h_3 = 625$

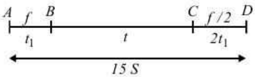
$h_2 = 3h_1,\ h_3 = 5h_1$

or $h_1 = \dfrac{h_2}{3} = \dfrac{h_3}{5}$

45. (d) Distance from A to $B = S = \dfrac{1}{2}ft_1^2$

Distance from B to $C = (ft_1)t$

Distance from C to $D = \dfrac{u^2}{2a} = \dfrac{(ft_1)^2}{2(f/2)} = ft_1^2 = 2S$



$\Rightarrow\ S + ft_1t + 2S = 15S$

$\Rightarrow\ ft_1t = 12S$ .... (i)

$\dfrac{1}{2}ft_1^2 = S$ .... (ii)

Dividing (i) by (ii), we get $t_1 = \dfrac{t}{6}$

$\Rightarrow\ S = \dfrac{1}{2}f\left(\dfrac{t}{6}\right)^2 = \dfrac{ft^2}{72}$