DAY FIVE

Circular Motion

Learning & Revision for the Day

- Concept of Circular Motion
- Dynamics of Uniform Circular Motion
- Forces in Circular Motion
- Applications of Circular Motion

Concept of Circular Motion

Circular motion is a two dimension motion. To bring circular motion in a body it must be given some initial velocity and a force. Circular motion can be classified into two types-Uniform circular motion and Non uniform circular motion.

When an object moves in a circular path at a constant speed then the motion is said to be a **uniform circular motion**.

When an object moves in a circular path with variable speed, then the motion is said to be **non-uniform circular motion**.

Terms Related to Circular Motion

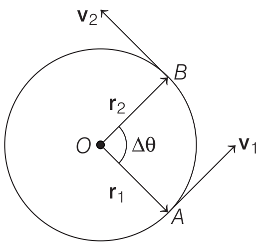
1. Angular Displacement

It is defined as the angle turned by the particle from some reference line. Angular displacement $\Delta\theta$ is usually measured in radians.

Finite angular displacement $\Delta\theta$ is a scalar but an infinitesimally small displacement is a vector.

2. Angular Velocity

It is defined as the rate of change of the angular displacement of the body.



From figure a particle moving on circular track of radius r is showing angular displacement $\Delta\theta$ in Δt time and in this time period, it covers a distance Δs along the circular track, then

Angular velocity, $\omega = \lim_{\Delta t \to 0} \left(\frac{\Delta\theta}{\Delta t}\right) = \frac{d\theta}{dt}$

It is an axial vector whose direction is given by the right hand rule. Its unit is rad/s.

3. Angular Acceleration

It is the rate of change of angular velocity.

Thus, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Its unit is rad/s^2.

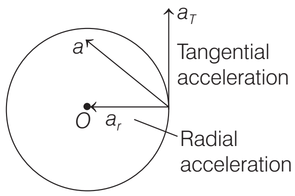
Dynamics of Uniform Circular Motion

If a particle is performing circular motion with a uniform speed, then motion of the particle is called uniform circular motion. In such a case,

$$\frac{dv}{dt} = 0 \text{ and } a_c = \omega^2 r = \frac{v^2}{r} \qquad [\because v = r\omega]$$

Thus, if a particle moves in a circle of radius r with a uniform speed v, then its acceleration is $\frac{v^2}{r}$ towards the centre. This acceleration is termed as **centripetal acceleration.**

NOTE • In non-uniform circular motion Resultant acceleration of the body is $a = \sqrt{a_r^2 + a_T^2} = \sqrt{\frac{v^4}{r^2} + r^2\alpha^2}$



Forces in Circular Motion

In circular motion of an object two kinds of forces occur which are described below

Centripetal Force

The centripetal force is the force required to move a body along a circular path with a constant speed. The direction of the centripetal force is along the radius, acting towards the centre of the circle, on which the given body is moving.

Centripetal force,

$$F = \frac{mv^2}{r} = mr\omega^2 = mr\, 4\pi^2 \nu^2 = mr\,\frac{4\pi^2}{T^2} \qquad \left[\because \nu = \frac{1}{T}\right]$$

Work done by centripetal force is always zero as it is perpendicular to velocity and hence instantaneous displacement.

Centrifugal Force

Certrifugal force can be defined as the radially directed outward force acting on a body in circular motion, as observed by a person moving with the body.

Mathematically, centrifugal force $= \frac{mv^2}{r} = mr\omega^2$

Applications of Circular Motion

Some of the most important applications of centripetal and centrifugal forces are given below

Motion of a Vehicle on a Level Circular Road

When a vehicle negotiates a circular path, it requires a centripetal force.

In such cases the lateral force of friction may provide the required centripetal force. Thus, for maintaining its circular path required centripetal force,

$$\left(\frac{mv^2}{r}\right) \le \text{frictional force } (\mu\, mg)$$

Maximum speed $v_{\max} = \sqrt{\mu r g}$

where, μ = coefficient of friction between road and vehicle tyres and r = radius of circular path.

Bending of a Cyclist

When a cyclist goes round turns in a circular track, then angle made by cyclist with vertical level is given by

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

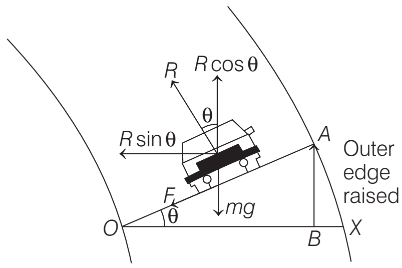
Banking of a Curved Road

For the safe journey of a vehicle on a curved (circular) road, without any risk of skidding, the road is slightly raised towards its outer end.

Let the road be banked at an angle θ from the horizontal, as shown in the figure.

If b is width of the road and h is height of the outer edge of the road as compared to the inner edge, then

$$\tan\theta = \frac{v^2}{rg} = \frac{h}{b}$$



If friction is present between road and tyre, then Maximum speed,

$$v_{\max} = \sqrt{\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}},$$

where, μ_s = coefficient of static friction.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 For a particle in a non-uniform accelerated circular motion correct statement is

(a) velocity is radial and acceleration is transverse
(b) velocity is transverse and acceleration is radial only
(c) velocity is radial and acceleration has both radial and transverse components
(d) velocity is transverse and acceleration has both radial and transverse components

2 An athlete completes one round of a circular track of radius 10 m and time period 40 s. The distance covered by him in 2 min 20 s is

(a) 70 m (b) 140 m (c) 110 m (d) 220 m

3 A car wheel is rotated to uniform angular acceleration about its axis. Initially its angular velocity is zero. It rotates through an angle θ_1 in the first 2s, in the next 2s, it rotates through an additional angle θ_2, the ratio of $\frac{\theta_2}{\theta_1}$ is

(a) 1 (b) 2
(c) 3 (d) 5

4 Which of the following statements is false for a particle moving in a circle with a constant angular speed.

(a) The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle.
(c) The acceleration vector point to the centre of the circle
(d) The velocity and acceleration vectors are perpendicular to each other.

5 A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begain, the tangential acceleration is

(a) 160π m/s^2 (b) 40 m/s^2 (c) 40π m/s^2 (d) 640π m/s^2

6 An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rev/min, the acceleration of a point on the tip of the blade is

(a) 1600 m/s^2 (b) 47.4 m/s^2 (c) 23.7 m/s^2 (d) 50.55 m/s^2

7 An aircraft executes a horizontal loop of radius 1 km with a speed of 900 kmh^{-1}. Ratio of centripetal acceleration to acceleration due to gravity is

(a) 12.3 (b) 3.3
(c) 6.4 (d) None of these

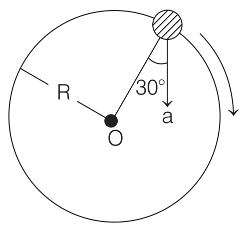
8 A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks and the stone flies off tangentially and strikes the ground after travelling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone while in circular motion?

(a) 163 ms^{-2} (b) 64 ms^{-2}
(c) 15.63 ms^{-2} (d) 125 ms^{-2}

9 A particle moves in a circle of radius 5 cm with constant speed and time period 0.2π s. The acceleration of the particle is ➜ CBSE AIPMT 2011

(a) 25 m/s^2 (b) 36 m/s^2
(c) 5 m/s^2 (d) 15 m/s^2

10 In the given figure, $a = 15$ m/s^2 represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is ➜ NEET 2016



(a) 4.5 m/s (b) 5.0 m/s (c) 5.7 m/s (d) 6.2 m/s

11 A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration a in ms^{-2} at the end of 2.0 s is a approximately ➜ NEET 2016

(a) 7.0 (b) 6.0 (c) 3.0 (d) 8.0

12 A particle of mass 100 g tied to a string is rotated along a circle of radius 0.5 m. The breaking tension of string is 10 N. The maximum speed with which particle can be rotated without breaking the string is

(a) 10 ms^{-1} (b) 9.8 ms^{-1}
(c) 7.7 ms^{-1} (d) 7.07 ms^{-1}

13 A proton of mass 1.66×10^{-27} kg goes round in a circular orbit of radius 0.10 m under a centripetal force of 4×10^{-13} N, then the frequency of revolution of the proton is about

(a) 0.08×10^8 cycle/s (b) 4×10^8 cycle/s
(c) 8×10^8 cycle/s (d) 12×10^8 cycle/s

14 If the radius of curvature of the path of two particle of same masses are in the ratio 1 : 2, then in order to have constant centripetal force, their velocity should be in the ratio of

(a) 1 : 4 (b) 4 : 1 (c) 4 : 2 (d) $1 : \sqrt{2}$

15 One end of the string of length l is connected to a particle of mass m and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed v, the net force on the particle (directed towards center) will be (T represents the tension in the string) ➔ NEET 2017

(a) T (b) $T + \frac{mv^2}{l}$ (c) $T - \frac{mv^2}{l}$ (d) Zero

16 A coin placed on a rotating turn table just slips, if it is placed at a distance of 8 cm from the centre. If angular velocity of the turn table is doubled, it will just slip at a distance of

(a) 1 cm (b) 2 cm (c) 4 cm (d) 8 cm

17 What will be the maximum speed of a car on a road turn of radius 30 m, if the coefficient of friction between, the tyres and the road is 0.4?
(Take, $g = 9.8\ \text{m/s}^2$)

(a) 10.84 m/s (b) 9.84 m/s
(c) 8.84 m/s (d) 6.84 m/s

18 The maximum velocity with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding?

(a) 60 (b) 30
(c) 15 (d) 25

19 A cyclist goes round a circular path of length 400 m in 20 s. The angle through which he bends from vertical in order to maintain the balance is

(a) $\sin^{-1}(0.64)$ (b) $\tan^{-1}(0.64)$
(c) $\cos^{-1}(0.64)$ (d) None of these

20 A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is ➔ CBSE AIPMT 2012

(a) $20\ \text{ms}^{-1}$ (b) $30\ \text{ms}^{-1}$ (c) $5\ \text{ms}^{-1}$ (d) $10\ \text{ms}^{-1}$

21 Find the maximum velocity with which a train can be moved on a circular track of radius 200m. The banking of the track is 5.71° (Given, tan 5.71° = 0.1) ➔ AFMC 2011

(a) 1.4 m/s (b) 14 m/s (c) 140 m/s (d) 0.14 m/s

22 A cyclist riding the bicycle at a speed of $14\sqrt{3}$ m/s takes a turn a circular road of radius $20\sqrt{3}$ m without skidding. What is his inclination to the vertical?

(a) 30° (b) 90° (c) 48° (d) 60°

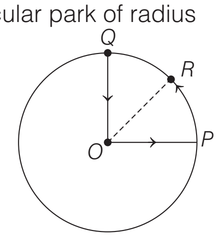
23 A road of width 20 m forms an arc of radius 15 m, its outer edge is 2 m higher than its inner edge. For what velocity the road is banked?

(a) $\sqrt{10}\ \text{ms}^{-1}$ (b) $\sqrt{14.7}\ \text{ms}^{-1}$
(c) $\sqrt{9.8}\ \text{ms}^{-1}$ (d) None of these

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A cyclist starts from centre O of a circular park of radius 1 km and moves along the path $OPRQO$ as shown in given figure. If he maintains constant speed of 10 m/s, what is his acceleration at point R in magnitude and direction?



(a) $0.1\ \text{m/s}^2$, along RO (b) $0.2\ \text{m/s}^2$, along OR
(c) $0.3\ \text{m/s}^2$, along RO (d) $0.4\ \text{m/s}^2$, along RO

2 A car rounds a curved road of radius 150 m at a speed of 20 m/s. Calculate the angle of banking, so that there is no side thrust on the tyres. Also, find the elevation of the outer wheels over the inner wheels, if the distance between them is 1 m.

(a) 15.22°, 0.428 m (b) 15.22°, 0.272 m
(c) 10°, 4.186 m (d) 10°, 3.581 m

3 A 200 kg car has to go ever a turn whose radius is 750 m and the angle of slope is 5°. The coefficient of friction between the car wheels and the road is 0.5. What should be the maximum speed of the car? So, that it may go over the turn without slipping. Take $g = 9.8\ \text{m/s}^2$.

(a) 67.2 m/s (b) 60.2 m/s (c) 72.2 m/s (d) 76.2 m/s

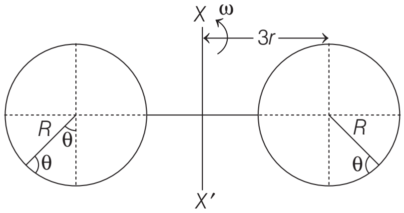
4 A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with speed 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around, if the string can withstand a maximum tension of 200 N?

(a) 4.4 N, 32.0 m/s (b) 6.6 N, 34.6 m/s
(c) 4.4 N, 42.8 m/s (d) 6.6 N, 24.8 m/s

5 Two stones of masses m and $2m$ are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is ➔ CBSE AIPMT 2015

(a) 2 (b) 3 (c) 4 (d) 1

6 The small spherical balls are free to move on the inner surface of the rotating spherical chamber of radius $R = 0.2$ m. Sphere is rotating about the axes XX' with angular velocity ω. If the balls reach a steady state at an angular position $\theta = 45°$, the angular speed ω of device is



(a) 8 rad/s (b) 2 rad/s
(c) 3.64 rad/s (d) 9.34 rad/s

7 A heavy small-sized sphere is suspended by a string of length l. The sphere rotates uniformly in a horizontal circle with the string making an angle θ with the vertical. Then, the time-period of this conical pendulum is

(a) $t = 2\pi\sqrt{\frac{l}{g}}$ (b) $t = 2\pi\sqrt{\frac{l\sin\theta}{g}}$
(c) $t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$ (d) $t = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

8 A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of acceleration of the stone?

(a) $\frac{\pi^2}{4}$ ms^{-2} and direction along the radius towards the centre
(b) π^2 ms^{-2} and direction along the radius away from centre
(c) π^2 ms^{-2} and direction along the radius towards the centre
(d) π^2ms^{-2} and direction along the tangent to the circle

9 A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude of the net acceleration of the cyclist on the circular turn?

(a) 0.5 m/s^2 (b) 0.86 m/s^2 (c) 0.72 m/s^2 (d) 0.3 m/s^2

10 A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

(a) $\sqrt{\frac{g}{R\sin\alpha}}$ (b) $\sqrt{\frac{g}{R\cos\alpha}}$ (c) $g\sqrt{\frac{1}{R\cos\alpha}}$ (d) $g\sqrt{\frac{1}{R\sin\alpha}}$

11 A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. When the floor is suddenly removed, what is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall without falling (approximately)?

(a) 5 rad/s (b) 10 rad/s
(c) 15 rad/s (d) 20 rad/s

12 A national roadway bridge over a canal is in the form of an arc of a circle of radius 49 m. What is the maximum speed with which a car can move without leaving the ground at the highest point? (take, $g = 9.8$ ms^{-2})

(a) 19.6 ms^{-1} (b) 40 ms^{-1}
(c) 22 ms^{-1} (d) None of these

13 A motorcycle moving with a velocity 72 kmh^{-1} on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 m. The acceleration due to gravity is 10 ms^{-2}. In order to avoid skidding, he must not bent with respect to the vertical plane by an angle greater than

(a) $\theta = \tan^{-1}(2)$ (b) $\theta = \tan^{-1}(6)$
(c) $\theta = \tan^{-1}(4)$ (d) $\theta = \tan^{-1}(25.92)$

14 A car is moving in a circular horizontal track of radius 10.0 m with a constant speed of 10.0 ms^{-1}. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with the track is (take, $g = 10$ ms^{-2}). ➜ AFMC 2010

(a) zero (b) 30°
(c) 45° (d) 60°

15 A car is negotiating a curved road of radius R. The road is banked at angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is ➜ NEET 2016

(a) $\sqrt{gR\left(\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right)}$ (b) $\sqrt{\frac{g}{R}\left(\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right)}$
(c) $\sqrt{\frac{g}{R^2}\left(\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right)}$ (d) $\sqrt{gR^2\left(\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right)}$

ANSWERS

SESSION 1	**1** (d)	**2** (d)	**3** (c)	**4** (b)	**5** (b)	**6** (b)	**7** (c)	**8** (a)	**9** (c)	**10** (c)
	11 (d)	**12** (c)	**13** (a)	**14** (d)	**15** (a)	**16** (b)	**17** (a)	**18** (b)	**19** (b)	**20** (b)
	21 (b)	**22** (d)	**23** (b)							
SESSION 2	**1** (a)	**2** (b)	**3** (a)	**4** (b)	**5** (a)	**6** (c)	**7** (c)	**8** (d)	**9** (b)	**10** (b)
	11 (a)	**12** (c)	**13** (a)	**14** (c)	**15** (a)					

Hints and Explanations

SECTION 1

1 In non-uniform circular motion particle possess both radial as well as transverse acceleration and velocity of particle is transverse.

2 Time period = 40 sec
No of revolution
$= \dfrac{\text{Total time}}{\text{time period}} = \dfrac{140}{40} = 3.5\,\text{rev}$
So, distance
$d = 3.5 \times 2\pi R = 3.5 \times 2\pi \times 10 = 220\text{ m}$

3 $\alpha = \dfrac{\omega}{t}$ and $\omega = \dfrac{\theta}{t} \Rightarrow \alpha = \dfrac{\theta}{t^2}$

but α = constant

So, $\dfrac{\theta_1}{\theta_1 + \theta_2} = \dfrac{(2)^2}{(2+2)^2}$ or $\dfrac{\theta_1}{\theta_1+\theta_2} = \dfrac{1}{4}$

or $\dfrac{\theta_1+\theta_2}{\theta_1} = \dfrac{4}{1}$ or $1 + \dfrac{\theta_2}{\theta_1} = \dfrac{4}{1}$

$\therefore \quad \dfrac{\theta_2}{\theta_1} = 3$

4 The acceleration vector is not tangent to the circle.

5 Initial velocity, $u = 0$
Final velocity, $v = 80$ m/s
Radius of circle, $r = \left(\dfrac{20}{\pi}\right)$ m
Distance travelled
$s = 2 \times (2\pi r) = 2\left(2\pi \times \dfrac{20}{\pi}\right) = 80\text{ m}$
Now, by applying third equation of motion
$$v^2 = u^2 + 2as$$
$$(80)^2 = 0 + 2 \times a_t \times 80$$
$$a_t = 40\text{ m/s}^2$$

6 Centripetal acceleration of rotating body is given by
$$a_C = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$
$$a_C = 4r\pi^2 v^2$$
Here, $r = 30\text{ cm} = 30 \times 10^{-2}\text{ m} = 0.30\text{ m}$
$v = 120\text{ rev/min} = \dfrac{120}{60}\text{ rev/s} = 2\text{ rev/s}$
$a_C = (0.30 \times 4 \times 3.14 \times 3.14 \times 2 \times 2)$
$= 47.4\text{ m/s}^2$

7 Centripetal acceleration is given by
$$a = \frac{v^2}{r}$$
Given, $r = 1\text{ km} = 1000\text{ m}$
$v = 900\text{ km/h} = 900 \times \dfrac{1000}{3600}\text{ ms}^{-1}$
$= 250\text{ ms}^{-1}$

$\therefore a = \dfrac{(250)^2}{1000} = 62.5\text{ms}^{-2}$ and $g = 9.8\text{ms}^{-2}$

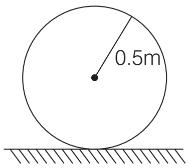
$\therefore \quad \dfrac{a}{g} = \dfrac{62.5}{9.8} = 6.4$

8 Given, $r = 1.5$ m, $h = 2$ m and $d = 10$ m
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{9.8}} = 0.64\text{ s}$$
$$v = \frac{10}{t} = 15.62\text{ ms}^{-1}$$
$$\therefore \quad a = \frac{v^2}{r} = \frac{(15.62)^2}{1.5}$$
$$= 162.65 \simeq 163\text{ ms}^{-2}$$

9 Acceleration, $a = r\omega^2$
where, ω is angular frequency given by
$$\omega = \frac{2\pi}{T}$$
As, $\quad a = \omega^2 r \Rightarrow a = \dfrac{4\pi^2}{T^2} r$
Given, $r = 5\text{ cm} = 5 \times 10^{-2}$ m,
$T = 0.2\,\pi$ s
$$\therefore \quad a = \frac{4 \times \pi^2 \times 5 \times 10^{-2}}{(0.2\,\pi)^2} = 5\text{ ms}^{-2}$$

10 Centripetal acceleration of a particle moving on a circular path is given by
$$a_c = \frac{v^2}{R}$$
In the given figure,
$a_c = a\cos 30° = 15\cos 30°\text{ m/s}^2$
$$\Rightarrow \quad \frac{v^2}{R} = 15\cos 30°$$
$$\Rightarrow \quad v^2 = R \times 15 \times \frac{\sqrt{3}}{2}$$
$$= 2.5 \times 15 \times \frac{\sqrt{3}}{2}$$
$$\therefore \quad v = 5.7\text{ m/s}$$

11 According to given question, a uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre.



This situation can be shown by the figure given below
$\therefore$ Angular acceleration, $\alpha = 2\text{ rad s}^{-2}$ (given)
Angular speed, $\omega = \alpha t = 4\text{ rad s}^{-1}$
$\because$ Centripetal acceleration, $a_c = \omega^2 r$
$= (4)^2 \times 0.5 = 16 \times 0.5$
$a_c = 8\text{ m/s}^2$

$\because$ Linear acceleration at the end of 2 s.
$a_t = \alpha r = 2 \times 0.5 \Rightarrow a_t = 1\text{ m/s}^2$
Therefore, the net acceleration at the end of 2.0 s is given by
$$a = \sqrt{a_c^2 + a_t^2}$$
$$a = \sqrt{(8)^2 + (1)^2} = \sqrt{65}$$
$$\Rightarrow \quad a \approx 8\text{ m/s}^2$$

12 Given, $m = 100$ g, $r = 0.5$ m, $F = 10$ N

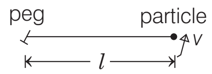
Centripetal force, $F = \dfrac{mv^2}{r}$,
$$v = \sqrt{\left(\frac{rF}{m}\right)}$$
$$= \sqrt{\frac{0.5 \times 10 \times 1000}{100}}$$
$$= \sqrt{50}\text{ ms}^{-1} = 7.07\text{ ms}^{-1}$$

13 Given $M = 1.66 \times 10^{-27}$ kg
$r = 0.10$ m
$F = 4 \times 10^{-13}$ N

Centripetal force $F = \dfrac{mv^2}{r}$
$$F = m \times 4\pi^2 n^2 r$$
$$4 \times 10^{-13} = m \times 4\pi^2 \times n^2 r$$
Frequency
$$n = \sqrt{\frac{4 \times 10^{-13}}{1.66 \times 10^{-27} \times 4 \times (3.14)^2 \times 0.10}}$$
$$= 0.08 \times 10^8\text{ cycle/s}$$

14 The centripetal force, $F = \dfrac{mv^2}{r}$
$$r = \frac{mv^2}{F}$$
$\therefore \quad r \propto v^2$ or $v \propto \sqrt{r}$
[If m and F are constant]
$$\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

15 Consider the string of length l connected to a particle as shown in the figure.



Speed of the particle is v. As the particle is in uniform circular motion, net force on the particle must be equal to centripetal force which is provided by the tension (T).
$\therefore$ Net force = Centripetal force
$$\Rightarrow \quad \frac{mv^2}{l} = T$$

16 We know that, centrifugal force

$$F = mr\omega^2$$

$$r\omega^2 = \text{constant}, \omega^2 \propto \frac{1}{r}$$

Given, $\omega_2 = 2\omega_1$ or $\left(\frac{\omega_2}{\omega_1}\right)^2 = 4 \Rightarrow \frac{r_1}{r_2} = 4$

$\therefore \quad r_2 = 2$ cm $\quad [\because r_1 = 8 \text{ cm}]$

17 If v is the velocity of the vehicle while rounding the curve, the centripetal force required $= \frac{mv^2}{r}$

As this force is provided only by the force of fricition,

$$\frac{mv^2}{r} \le \mu mg \Rightarrow v^2 \le \mu rg$$

$$v \le \sqrt{\mu rg} \Rightarrow v_{\max} = \sqrt{\mu rg}$$

$$v_{\max} = \sqrt{0.4 \times 30 \times 9.8}$$

$$= 10.84 \text{ m/s}$$

18 Given, $r = 150$ m, $\mu = 0.6$

Maximum velocity $v = \sqrt{\mu rg}$

$$= \sqrt{0.6 \times 150 \times 10} = 30 \text{ m/s}$$

19 Given, $l = 400$ m, $t = 20$ sec

Total length of circle, $l = 2\pi r$

$$400 = 2\pi r$$

$$\because \tan\theta = \frac{v^2}{rg} = \frac{\left(\frac{400}{20}\right)^2}{\left(\frac{400}{2\pi}\right)g} = \frac{6.28}{9.8} = 0.64$$

$$\therefore \quad \theta = \tan^{-1}(0.64)$$

20 Angle of banking, $\tan\theta = \frac{v^2}{rg}$

Given, $\theta = 45°$, $r = 90$ m, $g = 10 \text{ m/s}^2$

$$\therefore \quad \tan 45° = \frac{v^2}{90 \times 10}$$

$$\Rightarrow v = \sqrt{90 \times 10 \times \tan 45°}$$

$$= \sqrt{900} = 30 \text{ m/s}$$

21 Angle of banking, $\tan\theta = \frac{v^2}{rg}$

Given, $r = 200$ m, $\theta = 5.71$

$$\Rightarrow v = \sqrt{rg \tan\theta}$$

$$= \sqrt{200 \times 10 \times \tan 5.71}$$

$$= \sqrt{200 \times 10 \times 0.1}$$

$$= 14.14 \text{ m/s} = 14 \text{ m/s}$$

22 Given, $v = 14\sqrt{3}$ m/s, $r = 20\sqrt{3}$

$$\text{Angle} \quad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\left(\frac{(14\sqrt{3})^2}{20\sqrt{3} \times 9.8}\right)$$

$$= \tan^{-1}(\sqrt{3})$$

$$\theta = 60°$$

23 Condition for velocity on banked road

$$\therefore \quad \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg \tan\theta}$$

But, $\tan\theta \approx \theta \approx \sin\theta = \frac{2}{20}$

[for small angle]

Given $r = 154$ m

$$\therefore \quad v = \sqrt{15 \times 9.8 \times \frac{2}{20}}$$

$$= \sqrt{14.7} \text{ ms}^{-1}$$

SESSION 2

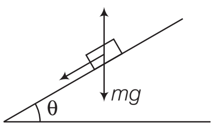
1 Acceleration of the cyclist at point R

= Centripetal acceleration (a_c)

$$a_c = \frac{v^2}{r} = \frac{(10)^2}{1000} = \frac{100}{1000}$$

$$\therefore \quad = 0.1 \text{ m/s}^2, \text{ along } RO$$

2 Here, $v = 20$ m/s, $r = 150$ m; $l = 1$ m. If θ is angle of banking, then



$$\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg} = \frac{(20)^2}{150 \times 9.8}$$

$$= 0.2721$$

$$\theta = 15.22°$$

Suppose that, h is the distance through which the outer part of the wheel has to be raised with respect to the inner rail.

$$\sin\theta = \frac{h}{l}$$

Since, θ is small, $\sin\theta \approx \tan\theta$

$$\therefore \quad \frac{h}{l} = \frac{v^2}{rg} \quad (\because \tan\theta = v^2/rg)$$

$$\Rightarrow \quad h = \frac{v^2}{rg} l = \frac{20^2 \times 1}{150 \times 9.8}$$

$$= 0.272 \text{ m}$$

3 Given, $m = 200$ kg, $r = 750$ m, $\theta = 5°$, $\mu_s = 0.5$

The maximum speed,

$$v_{\max} = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$$

$$= \left[\frac{750 \times 9.8(0.5 + \tan 5°)}{(1 - 0.5 \tan 5°)}\right]^{1/2}$$

$$= \left[\frac{750 \times 9.8(0.5 + 0.087)}{1 - 0.5 \times 0.087}\right]^{1/2}$$

$$v_{\max} = 67.2 \text{ m/s}$$

4 Mass of a stone $m = 0.25$ kg

Radius of the string $r = 1.5$ m

Frequency $\nu = 40$ rev/min

$$= \frac{40}{60} \text{ rev/s} = \frac{2}{3} \text{ rev/s}$$

Centripetal force required for circular motion is obtained from the tension in the string.

$\therefore$ Tension in the string = Centripetal force

$$T = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = mr\omega^2$$

$$= mr(2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$$

$$= mr 4\pi^2\nu^2$$

$$= 0.25 \times 1.5 \times 4 \times \left(\frac{22}{7}\right)^2 \times \left(\frac{2}{3}\right)^2 = 6.6 \text{ N}$$

Maximum tension which can be with stood by the string

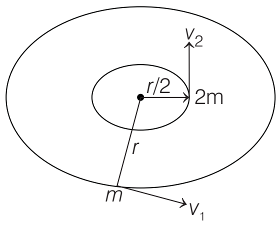
$$T_{\max} = 200 \text{ N}$$

$$\therefore \quad T_{\max} = \frac{mv^2_{\max}}{r}$$

$$\Rightarrow v^2_{\max} = \frac{T_{\max} \times r}{m} = \frac{200 \times 1.5}{0.25} = 1200$$

$$\therefore \quad v_{\max} = \sqrt{1200} = 34.6 \text{ m/s}$$

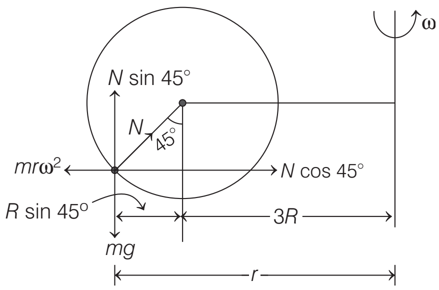
5 Given, that two stones of masses m and 2 m are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and lighter one in radius r as shown in figure.



As, lighter stone is n times that of the value of heavier stone when they experience same centripetal forces, we get $(F_c)_{\text{heavier}} = (F_c)_{\text{lighter}}$

$$\Rightarrow \frac{2m(v)^2}{(r/2)} = \frac{m(nv)^2}{r} \Rightarrow n^2 = 4 \Rightarrow n = 2$$

6 Given, $R = 0.2$ m



$$r = 3R + R\sin 45°$$

$$N\cos 45° = mr\omega^2 \text{ and } N\sin 45° = mg$$

$$\Rightarrow \quad \tan 45° = \frac{mg}{mr\omega^2} = \frac{g}{r\omega^2}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{g}{3R + \frac{R}{\sqrt{2}}}}$$

$$= 3.64 \text{ rad/s}$$

7 Radius of circular path in the horizontal plane

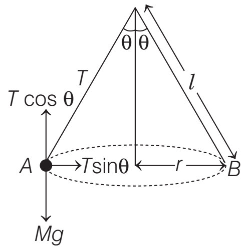
$$r = l \sin\theta$$

Resolving T along the vertical and horizontal directions, we get,

$$T \cos\theta = Mg \qquad \text{...(i)}$$

$$T \sin\theta = Mr\omega^2 = M(l \sin\theta)\,\omega^2$$

$$\text{or} \qquad T = Ml\omega^2 \qquad \text{...(ii)}$$



On dividing Eq. (ii) by Eq. (i), we get

$$\frac{1}{\cos\theta} = \frac{l\omega^2}{g} \text{ or } \omega^2 = \frac{g}{l\cos\theta}$$

$$\therefore \text{ Time period, } t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

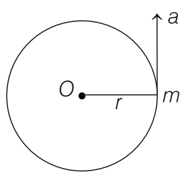
8 Since, speed is constant throughout the motion, so it is a uniform circular motion. Therefore, its radial acceleration

$$a = r\,\omega^2 = r\left(\frac{2\,\pi\,n}{t}\right)^2 = r \times \frac{4\pi^2 n^2}{t^2}$$

Given, $r = 2\,\text{m}$, $n = 22$, $t = 44$ sec

$$= \frac{1 \times 4 \times \pi^2 \times (22)^2}{(44)^2} = \pi^2 \text{ ms}^{-2}$$

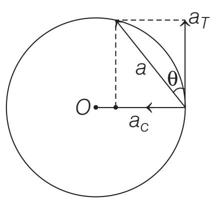
and direction along the tangent to the circle



9 Speed of the cyclist $(v) = 27$ km/h

$$= 27 \times \frac{5}{18} \text{ m/s} \qquad \left[\because 1 \text{ km/h} = \frac{5}{18} \text{m/s}\right]$$

$$= \frac{15}{2} \text{ m/s}$$



Radius of the circular turn $(r) = 80$ m

$\therefore$ Centripetal acceleration acting on the cyclist

$$a_c = \frac{v^2}{r} = \frac{(15/2)^2}{80}$$

$$= \frac{225}{4 \times 80} \text{ m/s}^2$$

$$= 0.70 \text{ m/s}^2$$

Tangential acceleration applied by brakes $\quad a_T = 0.5 \text{ m/s}^2$

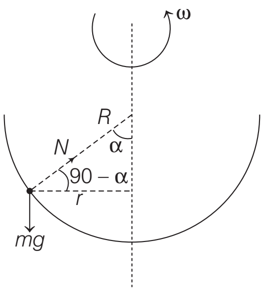
Centripetal acceleration and tangential acceleration acts perpendicular to each other.

$\therefore$ Resultant acceleration,

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(0.7)^2 + (0.5)^2}$$

$$= \sqrt{0.49 + 0.25} = \sqrt{0.74} = 0.86 \text{ m/s}^2$$

10 Balancing the forces, we get



$$N \sin\,(90 - \alpha) = mg \qquad \text{...(i)}$$

$$\text{and,} \qquad N \cos\,(90 - \alpha) = m\,\omega^2\,r$$

$$\text{Also,} \qquad r = R \sin\alpha$$

$$\text{So,} \qquad N = m\omega^2 R \qquad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\frac{m\omega^2 R}{mg} = \frac{N}{N\cos\alpha} \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$$

11 Cylinder being vertical, the normal reaction of the wall on the man acts horizontally and provides the necessary centripetal force, $N = mr\omega^2$

The frictional force f, acting upwards balances his weights $f = mg$

Man will remain stuck to the wall after the floor is removed i.e. he will continue to rotate with the cylinder without slipping, if $f \le \mu N$

$$mg \le \mu\; mr\omega^2$$

$$\omega^2 \ge \frac{g}{\mu r} \Rightarrow \omega \ge \sqrt{\frac{g}{\mu r}}$$

Minimum angular speed of rotation of the cylindrical drum is

$$\omega_{\text{min}} = \sqrt{\frac{g}{\mu r}}$$

Given, $\mu = 0.15$, $r = 3$ m, $g = 9.8$ m/s^2

$$\therefore \quad \omega_{\text{min}} = \sqrt{\frac{9.8}{0.15 \times 3}} = 4.67 \text{ rad s}^{-1}$$

$$\approx 5 \text{ rad s}^{-1}$$

12 Given, $R = 49$ m,

$$v = \sqrt{gR} = \sqrt{9.8 \times 49}$$

$$= 21.9 \text{ ms}^{-1} \approx 22 \text{ ms}^{-1}$$

13 Using the formula for motorcycle not to skid

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\text{where,} \quad r = 20 \text{ m}$$

$$v = 72 \text{ kmh}^{-1}$$

$$= 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$$

$$\therefore \qquad \theta = \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right)$$

$$\text{or} \qquad \theta = \tan^{-1}(2)$$

14 Angle of banking is

$$\tan\theta = \frac{mv^2/r}{mg} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

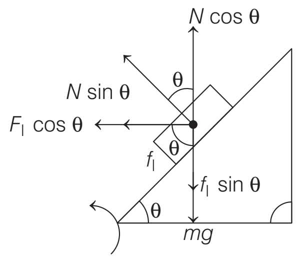
Given, $v = 10.0 \text{ ms}^{-1}$, $r = 10$ m

$$l = 1.00 \text{ m}$$

$$g = 10 \text{ ms}^{-2}$$

$$\Rightarrow \quad \tan\theta = \frac{(10)^2}{10 \times 10} = 1 \Rightarrow \theta = 45°$$

15 According to question, a car is negotiating a curved road of radius R. The road is banked at angle θ and the coefficient of friction between the tyres of car and the road is μ_s. So, this given situation can be drawn as shown in figure below.



Considering the case of vertical equilibrium

$$N\cos\theta = mg + f_l \sin\theta$$

$$\Rightarrow \qquad mg = N\cos\theta - f_l \sin\theta \qquad \text{...(i)}$$

Considering the case of horizontal equilibrium,

$$N\sin\theta + f_l \cos\theta = \frac{mv^2}{R} \qquad \text{...(ii)}$$

Divide Eqs. (i) and (ii), we get

$$\frac{v^2}{Rg} = \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} \qquad [f_1 = N\mu_s]$$

$$\Rightarrow \qquad v = \sqrt{Rg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$$

$$\Rightarrow \qquad v = \sqrt{Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}$$