$\frac{3.25}{12}$ 

# DECIMAL REPRESENTATION OF RATIONAL NUMBERS AND OPERATIONS

In the previous chapter we have seen that rational numbers can be represented in the form of  $\frac{p}{q}$  where  $q \neq o$  and p, q are integers. means  $q^{th}$  part of p or it is the number obtained on dividing p by q.

While studying rational numbers Manohar wondered as to what would happen if we divide the numerator by the denominator.

## **Division in Rational numbers**

Let us think about Manohar's question. Rational number means 5th part of 2. This

is obtained when we divide 2 by  $5 \rightarrow 5)2$ 

To divide 2 by 5 we require decimal.

$$5)\frac{0.4}{20} \\ \frac{0}{20} \\ \frac{20}{00} \\ \therefore \frac{2}{5} = 0.4$$

We can represent in the from of 0.4.

Let us see what we get if we solve or on dividing 13 by 4.

Thus 
$$\frac{13}{4} = 3.25$$
 4 /13 12

In the above the dividends 0.4 and 3.25 are decimal representation of  $\frac{2}{5}$  and  $\frac{13}{4}$  respectively. What are the decimal representations of the following rational numbers?  $\frac{10}{8}$   $\frac{20}{00}$ 

(i) 
$$\frac{3}{5}$$
 (ii)  $\frac{17}{4}$  (iii)  $\frac{15}{6}$  (iv)  $\frac{19}{2}$  (v)  
Let us think about  $\frac{20}{3}$   
 $3 \frac{6.666}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$ 

 $\frac{20}{3}$ 

 $\therefore \frac{20}{3} = 6.666...$ 

### Terminating and non-terminating decimals:-

In the earlier questions we had seen that after some finite steps division process ended. But in representation of  $\frac{20}{3}$  we will always obtain 2 as remainder and 6 in the quotient again and again. In this way we do not get 0 as remainder. Therefore, when the division gets completed in some finite steps and we reach remainder '0' then it is known as terminating. When the division does not ever get completed and we keep getting some remainder then it is known as a non-terminating number.

Find out the decimal representation of following numbers and state whether they are terminating or non- terminating.

(i) 
$$\frac{3}{8}$$
 (ii)  $\frac{15}{4}$  (iii)  $\frac{1}{6}$  (iv)  $\frac{1}{7}$  (v)  $\frac{2}{9}$  (vi)  $\frac{2}{11}$ 

On the left side some rational numbers are shown as terminating or non-terminating decimal numbers. On the right side some rational numbers are given. Follow the same method and find terminating or non-terminating numbers.

					<b>*</b>
(i)	$\frac{3}{8}$	8 ) <u>.</u>	<u>.375</u> 3	Find term from the f	inating or non-terminating ollowing fractions:-
			3 0 2 4	$\frac{5}{8}$	8)5
			60		
			<u> </u>		
			<u>40</u>		
	$\therefore \frac{3}{8} = 0.375$	Terminating	decimal		
(ii)	$\frac{15}{4}$	3	. <u>75</u> )15	$\frac{13}{4}$	4)13
			12		
			28		
			2 0		
			2 0		
			0 0		
	$\therefore  \frac{15}{4} = 0.37$	<sup>75</sup> terminating	decimal		
(ii)	$\frac{1}{6}$	$6) \frac{0.1666}{1}{0}$		$\frac{1}{12}$	$12\overline{)1}$
	0	10			
		6			
		36			
		4 0			
		36			
		4 0			
		3 6			
. 1	= 0 1666	4	na desima		
6	,		ing ucciiiia.		

(iv) 
$$\frac{1}{7}$$
  $\frac{7}{9} \frac{)\frac{1}{10}}{10}$   $\frac{1}{14}$   
 $\frac{7}{300}$   
 $28$   
 $200$   
 $\frac{14}{40}$   
 $\frac{56}{50}$   
 $\frac{49}{10}$   
 $\frac{56}{50}$   
 $\frac{49}{10}$   
 $\frac{7}{300}$   
 $28}{2}$   
 $\frac{2}{9}$   
 $\frac{9}{9} \frac{0222}{20}$   
 $\frac{18}{200}$   
 $\frac{18}{20}$   
 $\frac{18}{20}$ 

 $\therefore \frac{2}{9} = 0.222$  non-terminating decimal

14)1

9)4

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22)1

<i>(</i> .)	2
(V1)	11

$$\begin{array}{c}
11 \\
0 \\
0 \\
20 \\
11 \\
9 \\
0 \\
8 \\
20 \\
11 \\
9 \\
0 \\
8 \\
2 \\
2
\end{array}$$

 $\therefore \frac{2}{11} = 0.1818$  non-terminating decimals

Here (i) and (ii) are terminating decimals while rational numbers (iii), (iv), (v) & (vi) are non-terminating decimals.

Think of some rational numbers and ask your friends to find whether they are terminating or non-terminating decimals.

## Non-terminating repeating decimals:

Quotient of (iii) is 0.1666.... here 6 is repeated. Quotient of (iv) is 0.14285714. When we see this carefully we find that 1,4,2,8,5,7 is repeated. Similarly, in (v) 2 and in (vi) one eight are repeated and the division is never completed. These are non-terminating digits after decimals. As there is repeated repetition of one or more digits the division would never end and therefore they are called non-terminating repeating decimals.

After the decimal sign if same digits are repeated then those digits that are repeated are denoted with a bar like.

$$\frac{1}{6} = 0.1666... = 0.1\overline{6}$$
 or  $0.1\overline{6}$ 

If after decimals one or more than one digits are repeated then a - is placed on each of the repeated number and denoted by '-----' or dots are put on the first and last digit like:-

$$\frac{1}{7} = 0.14285714.... = 0.142857 \text{ or } 0.142857$$
$$\frac{2}{9} = 0.222... = 0.227 \text{ or } 0.22$$
$$\frac{2}{11} = 0.1818... = 0.187 \text{ or } 0.18$$

Activity 1

Below some rational numbers are given find their decimal representations and say whether they are terminating or non-terminating.

S.No.	Rational number	Decimal representation	Terminating or non-terminating
1.	$\frac{1}{2}$		
2.	$\frac{1}{3}$		
3.	$\frac{1}{4}$		
4.	$\frac{1}{5}$		
5.	$\frac{1}{6}$		
6.	$\frac{1}{7}$		
7.	$\frac{1}{8}$		
8.	$\frac{1}{9}$		

Sort the terminating and the non-terminating decimals? What kind of rational numbers can be written in the form of terminating decimals? What are their characteristics? Write.

You have seen that  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  and  $\frac{1}{8}$  are the numbers whose decimal representations

are terminating.

If  $\frac{1}{2}$  is terminating then why is the decimal representation of  $\frac{1}{6}$  non-terminating. Think and find the answer.

Now you know that we get terminating or non-terminating rational numbers because of the nature of their denominators.

Thus, let us see how on the basis of the prime factors of the denominator we can say whether the number after the decimal would be terminating or non-terminating.

Activity 2

Find the decimal representations of  $\frac{5}{2}, \frac{24}{25}, \frac{3}{10}, \frac{21}{8}$  and check whether each is terminating or non- terminating?

Manohar stood up and said that they were all terminating. How did he say that? Find the Prime Factors of denominators of the rational numbers given above.

## You would see that the prime factors of the these denominators have 2,5 or both. Therefore, the decimal representations of this kind of rational numbers are

**terminating.** We have seen that the decimal representations of  $\frac{1}{6}, \frac{1}{7}, \frac{2}{9}, \frac{2}{11}$  are non-terminating. Find the prime factors of the denominators of these numbers. Do the prime factors of the denominators of these numbers have numbers other than 2 and 5? If the prime factors of denominator have numbers other than 2 and 5, then the decimal representations of such numbers would be non-terminating.

#### **Example 1**

Find the terminating and non-terminating numbers from the following numbers

(i) 
$$\frac{4}{125}$$
 (ii)  $\frac{5}{18}$  (iii)  $\frac{11}{8}$  (iv)  $\frac{13}{100}$ 

#### Solution

(i) In  $\frac{4}{125}$ , the prime factors of 125 are 5×5×5. The denominator has only 5 in its prime factors therefore the number has a terminating decimal representation.

(ii) In 
$$\frac{5}{18}$$
 prime factorization of 18 is 2 × 3 × 3

Here the denominator 18 has a number other than 2, 5 i.e.3 as a prime factor. Therefore, it has a non-terminating decimal.

(iii) 
$$\frac{11}{8}$$
 has a denominator 8, its prime factorization is 2×2×2

Here the prime factor is only 2 and therefore it is a terminating decimal.

(iv)  $\ln \frac{13}{100}$ , the prime factorization of the denominator 100 is =  $2 \times 2 \times 5 \times 5$ . Here the prime factors are only 2 and 5 and hence the number has a terminating decimal representation.

All the examples considered above were positive rational numbers.

How will we represent negative rational numbers in decimal form?

## Decimal representation of negative numbers:

To find the decimal representation of a negative number, first find the decimal representation of the same number without a negative sign. After that we can put the negative sign.

#### Example 2

Find the decimal representation of  $\frac{-23}{3}$ .

**Solution:** consider  $\frac{23}{3}$  in place of  $\frac{-23}{3}$ . For  $\frac{23}{3}$  the decimal representation is:

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3	$\frac{7.666}{)23}$
	21
	20
	18
	20
	18
	20
	18
	2

Or 
$$\frac{23}{3} = 7.666 = 7.\overline{6}$$

That is  $\frac{-23}{3} = -7.\overline{6}$ 



1. Without dividing find out the terminating and non-terminating decimal numbers.

$$\frac{4}{5}, \frac{8}{7}, \frac{-15}{49}, \frac{7}{50}, \frac{3}{28}$$

2. Change given rational numbers to decimal numbers.

$$\frac{3}{5}, \frac{4}{25}, \frac{7}{10}, \frac{-13}{125}, \frac{9}{40}$$

3. Write the following rational numbers as decimal numbers.

$$\frac{2}{3}, \frac{-5}{6}, \frac{8}{15}, \frac{3}{11}, \frac{19}{45}$$

## **Converting Decimal Numbers into Rational Numbers:**

You have studied how to convert rational numbers into terminating and nonterminating decimal numbers. But can decimal numbers be converted into rational numbers? Let us discuss through some examples.

$$0.25 = \frac{0.25 \times 100}{100} = \frac{25}{100} = \frac{1}{4} \quad \text{(Simplest form)}$$
$$2.6 = \frac{2.6 \times 10}{10} = \frac{26}{10} = \frac{13}{5} \quad \text{(Simplest form)}$$
$$0.317 = \frac{0.317 \times 1000}{1000} = \frac{317}{1000}$$
$$4.625 = \frac{4.625 \times 1000}{1000} = \frac{4625}{1000} = \frac{37}{8}$$

From the above examples we see that it is easy to convert decimal numbers into rational numbers. We will write 1 in the denominator and after 1 put as many zeros as the digits after the decimal point in the number, then remove the decimal point. This will give us rational numbers like:

$$7.21 = \frac{721}{100}$$
$$4.2 = \frac{42}{10} = \frac{21}{5}$$
 etc.

Geeta said that by this procedure we can only convert terminating numbers to rational number but how can we convert non-terminating and repeating decimals to rational numbers?

For e.g.  $1.666 \dots = 1.6$ 

Let us consider how we can convert non-terminating and repeating decimal numbers to rational numbers.

#### Example 3

Convert  $0.\overline{6}$  into rational number.

Let  $x = 0.\overline{6}$  x = 0.666....(i)Multiply both sides by 10, 10 x = 6.66 - (ii) Subtract (i) from (ii) Or 10  $x - x = 6.666 \dots \% 0.666 \dots$ Or 9 x = 6Or  $x = \frac{6}{9} = \frac{2}{3}$ , hence  $0.\overline{6} = \frac{2}{3}$ 

#### **Example 4**

Convert  $0.\overline{234}$  into rational number.

Let  $x = 0.\overline{234}$  $x = 0.234 \ 234 \ .....(i)$ Multiply both sides by 1000

1000 x = 234.234234..... (ii)

Subtract (i) from (ii)

999 x = 234 or 
$$x = \frac{234}{999} = \frac{26}{111}$$

Hence  $0.\overline{234} = \frac{26}{111}$ 

In the above examples we adopted the following procedure:-

(i) First we considered the given decimal number as x and wrote it as equation (i).

(ii) The number repeating itself after decimal is written 2-3 times.

(iii) The number of digits repeating is counted. We multiply both sides by as many powers of 10 as the number of repeating digits. This is written as equation (2)

(iv) Subtract (ii) from (i) and find the value of x. Notice the non-terminating part gets subtracted.

Manohar asked Geeta if the repeating digits come after some digits in a decimal, e.g. 1.25666 then how do we convert it to a rational number form? Geeta did not know either?

Let us solve some such examples:-

**Example5** Write  $3.21\overline{6}$  in the form of a rational number.

**Solution:** Let  $x = 3.21\overline{6}$ 

Or x = 3.21666.... (i)

Multiply both sides by 100

We have 100 x = 321.666..... (ii)

Then multiplying both sides of equation (ii) by 10

1000x = 3216.66.... (iii)

Subtract (ii) from (iii)

Or 1000x - 100 x = 3216.66 - 321.66

900x = 2895

$$\mathbf{x} = \frac{2895}{900} = \frac{193}{60}$$

Hence

 $3.21\overline{6} = \frac{193}{60}$ 

## Example 6

Convert  $0.15\overline{23}$  into a rational form.

```
Solution: Let x = 0.15\overline{23}

Or x = 0.152323.....(i)

Multiply both sides of (1) by 100

100 \ x = 15.2323.....(ii)

Then multiply both sides of (ii) by 100

We have 10000x = 1523.2323 \dots (iii)

Subtract (ii) from (iii)

10000 \ x - 100x = 1523.2323 - 15.2323

9900 \ x = 1508

x = \frac{1508}{9900} = \frac{377}{2475}

Hence 0.15\overline{23} = \frac{377}{2475}
```

In both the examples the not repeating digits are counted and as many zeros written after 1. Multiply the given number by this. After this only the repeating digits remain. Then follow the method for numbers with only the repeating digits to find the rational number.

## Exercise 11.2

(1) Convert the following numbers into rational number forms.

(a) 0.2	(b) 0.55	(c) 6.25
(d)2.175	(e) 14.53	
(2) Write the following	in rational form.	
(a) $0.\overline{4}$	(b) 7.25	(c) $0.05\overline{6}$
(d) 0.27	(e) $0.5\overline{4}$	

## **Multiplication of Decimal Numbers:-**

You have learnt how to write rational numbers as decimal number forms. In earlier classes, we have also studied multiplication of integers. Let us learn multiplication of decimal numbers.

We want to multiply  $0.2 \times 0.3$  then

$$0.2 = \frac{2}{10}$$
 and  $0.3 = \frac{3}{10}$ 

Now 0.2 × 0.3 =  $\frac{2}{10} \times \frac{3}{10} = \frac{6}{100} = 0.06$ 

Now, we can see that there is a big difference between the products of positive integers 2 and 3 and of positive numbers 0.2 and 0.3. Clearly 6 is 100 times more than 0.06. We can also see that 6 is larger than either 2 or 3 while .06 is smaller than either of them.

**Example 7** Find the value of  $0.31 \times 0.04$ 

Solution:  $0.31 = \frac{31}{100} \text{ and } 0.04 = \frac{4}{100}$ Now  $0.31 \times 0.04 = \frac{31}{100} \times \frac{4}{100} = \frac{124}{10000}$ = 0.0124 Answer

Example 8 Find the value of :  $0.015 \times 0.3 \times 0.02$ Solution:  $0.015 = \frac{15}{1000}, \ 0.3 = \frac{3}{10} \text{ and } 0.02 = \frac{2}{100}$ Now  $0.015 \times 0.3 \times 0.02 = \frac{15}{1000} \times \frac{3}{10} \times \frac{2}{100}$  $= \frac{90}{1000000} = 0.00009$ 

Activity3

Fill up the following blanks as per column 1:

S.No	o. Numbers	Multiplication process	Product as a fraction	Answer	No. of digits after decimal in answer
1.	$0.001 \times 0.02$	$\frac{1}{1000} \times \frac{2}{100}$	$\frac{2}{100000}$	0.00002	5
2.	$0.502 \times 0.45$	$\frac{502}{1000} \times {100}$	<u>22590</u> 	0.22590	5
3.	$0.22 \times 0.101$				
4.	$0.1 \times 0.003 \times 0.05$	·····			·····
5.	$0.006 \times 0.4 \times 0.08$				
6.	$0.85 \times 0.05$	· · · · · · · · · · · · · · · · · · ·	·····		·····

To place the decimal point in the product of two numbers, we count the digits after decimal point in both numbers and then put the decimal point in the product after leaving as many digits as the sum of these from right to left. If the number of digits is less than the sum then write 0's on the left of the product till we match the sum and then place the decimal point.

## Activity 4

Place the decimal point at appropriate places in the following products:

- (1) 4.283 × 3.41 = 1460503
- (2)  $326.7 \times 0.319 = 1042173$
- (3)  $9.07 \times 13.4 = 121538$
- (4)  $69.05 \times 5.044 \times 19.5 = 67916199$

## **Division in Decimal numbers**

We divide decimal numbers as we divide integers.

If the divisor is an integer then-

**Example 9** Find the value of :  $25.2025 \div 25$ 

```
Solution: 25 ) 25.2025 (1.0081

- 25 \times 2

- 0 - 20

20
```

$$\frac{-00}{202} \\
 - \frac{200}{\times \times 25} \\
 - \frac{25}{\times \times}$$

If the divisor and the dividend are both decimal numbers:-

**Example 10** Find the value of :

(1) 
$$45.27 \div 1.5 = \frac{4527}{100} \div \frac{15}{10}$$
  
 $= \frac{4527}{100} \times \frac{10}{15}$   
 $= \frac{4527}{15 \times 10}$   
 $\rightarrow 150) 4527 (30.18)$ 

(II) 
$$45.27 \div 1.5$$
  
 $\Rightarrow \frac{45.27}{1.5} = \frac{45.27}{1.5} \times \frac{10}{10}$   
 $\Rightarrow \frac{452.7}{15}$ 

= 30.18 Answer

If the divisor is a decimal number than to convert it into a whole number we multiply dividend and divisor both by 10, 100, ..... This is done so that the divisor becomes a whole number. Then the number obtained in the numerator is divided by the denominator.

**Activity** 5

S.No.	I Number × II Number	Product	Product I Number = II Number	$\frac{\text{Pr oduct}}{\text{II Number}} = \text{Distance}$
1.	$0.4 \times 0.6$	0.24	$\frac{0.24}{0.4} = 0.6$	$\frac{0.24}{0.6} = 0.4$
2.	$0.7 \times 0.02$	0.014	$\frac{0.014}{0.7} = \dots$	${0.2}$ =
3.	0.12×0.35	0.0420	${0.12} =$	$\frac{0.0420}{0.35} = \dots$
4.	7.2×0.3		$\frac{2.16}{7.2} = \dots$	$\frac{2.16}{0.3} = \dots$
5.	4.52×0.06		$\frac{0.2712}{4.52}$ =	$\frac{0.2712}{0.06} = \dots$
6.	$0.008 \times 0.0007$		$\frac{\dots}{0.008} = 0.0007$	$\frac{0.0000056}{\dots} = 0.008$

It is clear from the table that if the product of two numbers is divided by the first number, we get the second number. And if the product is divided by the second number, we get the first number. This means;

$$x \times y = p \implies x = \frac{p}{y} \text{ and } y = \frac{p}{x}$$

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Place decimal point at appropriate place in the following divisions-

1.  $68.64 \div 4.4 = 156$ 2. 400.14 ÷ 85.5 = 468 3.  $0.735 \div 0.7 = 105$ 4.  $51.1875 \div 1.05 = 4875$ 5. 3.773 ÷0.98 = 385 **Example 11**: Solve 0.512 × 4.375  $\frac{512}{1000} \times \frac{4375}{1000}$ Solution  $=\frac{2240000}{1000000}$  $=\frac{224}{100}$ = 2.24 Answer **Example 12**. Solve 3.15 ÷0.02  $3.15 \div 0.02$ Solution:  $=\frac{3.15}{0.02}=\frac{3.15}{0.02}\times\frac{100}{100}$  $=\frac{315}{2}$ 2) 315 (157.5 <u>- 2</u> <u>11</u> - <u>10</u> ×15 -<u>14</u> ×10 -<u>10</u> ×× Answer = 157.5

**Example 13:** Find the value of  $0.3942 \div 1.8$ 

**Solution:**  $0.3942 \div 1.8 = \frac{0.3942}{1.8} \times \frac{10}{10}$ 

$$=\frac{3.942}{18}$$
18) 3.942 (0.219
$$\frac{-0}{39}$$

$$\frac{-36}{\times 34}$$

$$\frac{-18}{162}$$

$$\frac{-162}{\times \times \times}$$

Answer = 0.219

Enomalo 14	$0.005 \times 0.84 \times 2.25$
Example 14	$0.021 \times 0.05 \times 1.10$
Solution:	$\frac{0.005 \times 0.84 \times 2.25}{0.021 \times 0.05 \times 1.10} \times \frac{10^7}{10^7}$
	$=\frac{5\times84\times225}{21\times5\times110}$
	$=\frac{900}{110}$
	$=\frac{90}{11}$ = $8.\overline{19}$ Answer

Exercise 11.3

1. Add

- (i) 1.0087 + 0.321
  (ii) 0.2+0.02+0.0202+0.20204
  (iii) 3.81+0.009+10.0023
  (iv) 2.45+6.908+0.125 +1.0074
  2. Find the value
  (i) 7.89-2.324
  (ii) 5.01-0.00729
  - (iii) 1.01-0.1-0.001+10.001 (iv) 7.802-1.4+2.8-0.00107
- 3. Solve

```
(i) 243 \times 0.15 (ii) 0.85 \times 0.022 (iii) 0.1 \times 0.1 \times 0.1 \times 0.1
```

- 4. Solve
  - (i) $2.25 \div 15$ (ii) $10.206 \div 0.06$ (iii) $0.324 \div 1.8$ (iv) $46.225 \div 2.15$
- 5. Suneeta purchased oil for Rs. 23 and 50 paisa, soap for Rs. 8 and 15 paisa and powder for Rs. 12 and 39 paisa. How much was the bill for?
- 6. Simran usually pays Rs. 472.5 as the electricity bill of the house. Per unit Rs. 3.50 is charged for electricity. How many units does she use in her house?
- 7. Rahim pays Rs. 2075 as house rent per month. How much will he pay in two and a half years?
- 8. Find in cubic meters the air contained in a room of length, breadth and height 5.5m, 4.6m and 3.2 m respectively.

## We Have Learnt

- 1. Each rational number can be expressed as a decimal number form.
- 2. We can convert decimal numbers into rational form.
- 3. In converting rational numbers into decimal number forms if the division ends after a finite number of steps, then it is terminating other wise nonterminating
- 4. In a terminating decimal, the denominator has only 2 and 5 as the prime factors.
- 5. If in converting a rational number form into a decimal number, we see that one or more digits start being repeated after the decimal point. Then these digits are called repeating numbers and the number is a non- terminating decimal. Mark a line or a bar on the repeating digit or mark a point on the first and the last digit of the repeating digits in the numbers to avoid repeatedly writing them.
- 6. In multiplication of decimal numbers, we count the digits after decimal in each of the given decimal numbers. We place the decimal in the product after adding the two and counting the required digits from right to left.